ECE 5553 Final Project

Brian

Kevin

Ovee

Contents

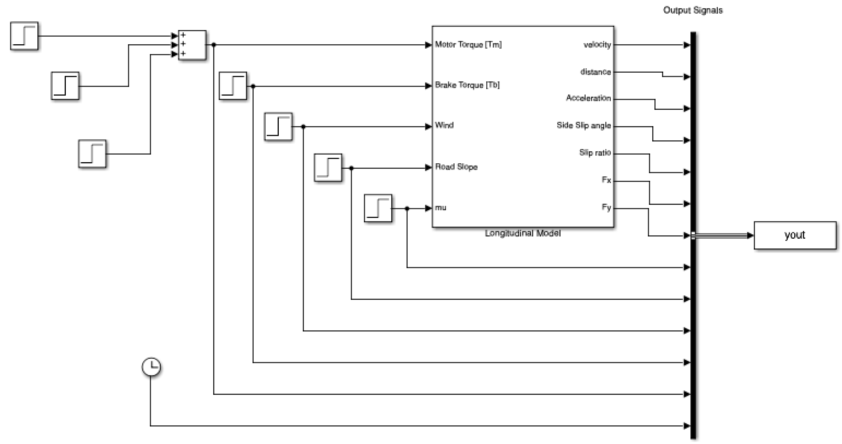
Longitudinal dynamics parameters

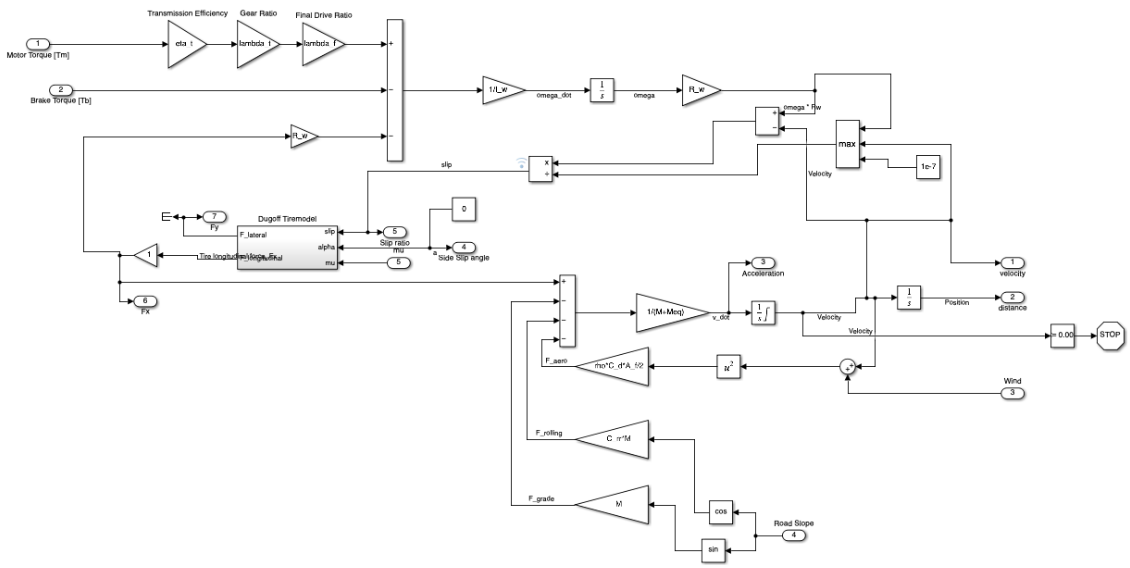
Deriving more accurate parameters by modifying parameters of a sedan, used in a previous project, below:

```
clc; clear; close all;

% longitudinal dynamics parameters for a sedan
    % M=2000; % Mass of the vehicle[Kg]
    % eta_t=0.9; % Transmission Efficiency
    % lambda_t=1.0; % Gear Ratio
    % lambda_f=4.1; % Final Drive Ratio
    % I_w=1; % Inertia of the Wheel[kgm2]
    % R_w=0.3; % Wheel Radius[m]
    % Meq=0.1*M; % Equivalent Mass Factor
    % C_d=0.29; % Drag coefficient
    % rho=1.225; % Air density[kg/m3]
```

```
    % A_f=2.8;% Frontal area    [m2]
    % g=9.81;% [N/m2]
    % C_rr=0.015;%Rolling resistance coefficient
    % C_x=3e5; %Longitudinal Stiffness[N]
    % C_alpha = 1.5e5; % Cornering Stiffness for 1 tire [N/rad]
    % L=2.85;% Wheelbase[m]
    % l_f=1.3;% Distance from the center of gravity of the vehicle (CG) to
the front axle [m]
    % l_r=1.55;% Distance from the center of gravity of the vehicle (CG) to
the rear axle [m]
    % C_s=1.5e5;% Cornering Stiffness of Front and Rear Tires [N/rad]
    % I_z=3700;% Inertia moment around z axis J or Iz [kg/m2]
    % mu=0.7; % Road friction coefficient

% for the shuttle
    M=3500; % Mass of the vehicle[Kg] based on Tesla Model X size and
weight, the shuttle will be 2k kg heavier
    eta_t=0.8; % Transmission Efficiency, less efficient than a sedan
    lambda_t=1.0; % Gear Ratio, the same as the sedan
    lambda_f=4; % Final Drive Ratio, same as the sedan
    I_w=1; % Inertia of the Wheel[kgm2], same as the sedan
    R_w=0.3; % Wheel Radius[m], same as the sedan
    Meq=0.1*M; % Equivalent Mass Factor, same as the sedan
    C_d=0.4; % Drag coefficient, Worse drag than the sedan
    rho=1.225; % Air density[kg/m3], same as the sedan (obviously)
    A_f=6;% Frontal area [m2], more frontal area than the sedan
    g=9.81;% [N/m2], same as the sedan
    C_rr=0.015;%Rolling resistance coefficient, same as the sedan

    C_x=3e5; %Longitudinal Stiffness[N], same as the sedan
    C_alpha = 1.5e5; % Cornering Stiffness for 1 tire [N/rad], same as the
sedan

    L=4.75;% Wheelbase[m], longer by 2 meters than the sedan
    l_f=2.25;% Distance from the center of gravity of the vehicle (CG) to
the front axle [m]
    l_r=2.5;% Distance from the center of gravity of the vehicle (CG) to
the rear axle [m]
    C_s=1.5e5;% Cornering Stiffness of Front and Rear Tires [N/rad]
    I_z=4000;% Inertia moment around z axis J or Iz [kg/m2] Higher
    mu=0.7; % Road friction coefficient same

[M, eta_t, lambda_t, lambda_f, I_w, R_w, Meq, C_d, rho, A_f, g, C_rr, C_x,
C_alpha,L, l_f, l_r, C_s, I_z, mu] = getLogitudinalParameters();
```

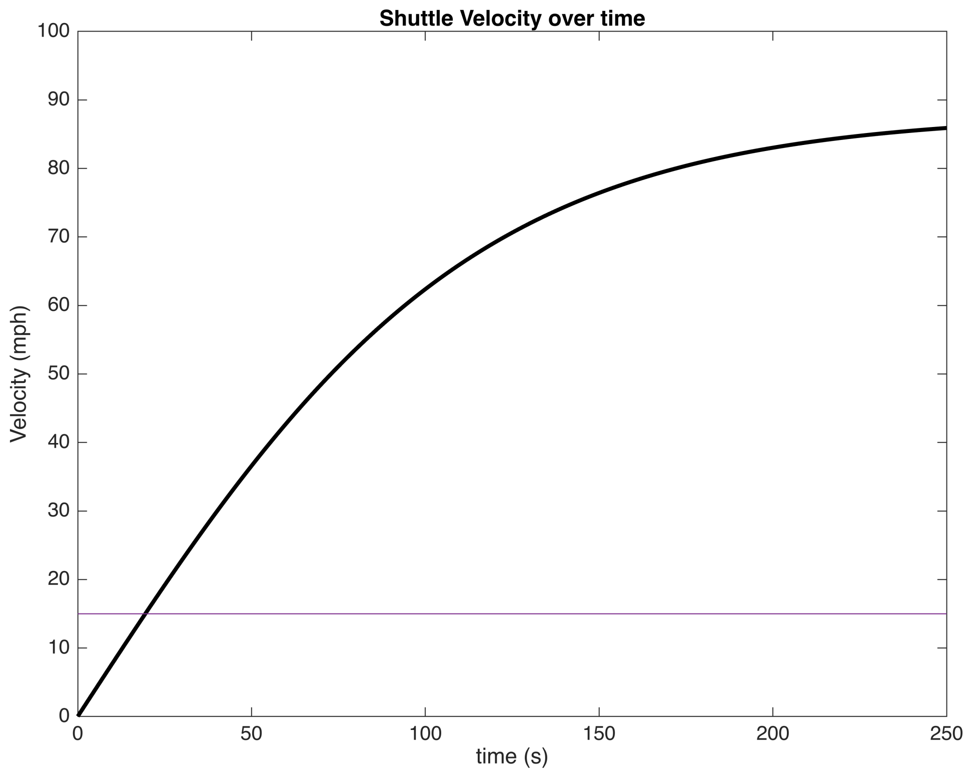
Longitudinal Dynamics via Duggoff model






```
% unchanging Sim parameters for pure acceleration
[motorStart,motorStartTime,motorIncrease,motorIncreaseTime,motorDecrease,mot
orDecreaseTime,brakeTime,breakStrength,headWindTime,headWind,roadSlopeTime,r
oadSlope] = setLongitudinalInputsToZero();
[M, eta_t, lambda_t, lambda_f, I_w, R_w, Meq, C_d, rho, A_f, g, C_rr, C_x,
C_alpha,L, l_f, l_r, C_s, I_z, mu] = getLogitudinalParameters();

motorStart = 65;
addpath("Simulink_Models/")
sim("longitudinal_modelV2.slx")
```

```
ynames = ["V","X","A","Side Slip Angle","Slip Ratio","Tire force
Fx","Tire Force Fy","Motor Torque","Brake Torque","Wind Force","Road
Slope","Mu","time"];
sim1 = yout;
time1 = sim1(:,length(ynames));
plot(time1,2.23694*sim1(:,1),'k','Linewidth',2), ylabel('Velocity (mph)'),
xlabel('time (s)'), title('Shuttle Velocity over time'), hold on
axis([0 250 0 100])
% horizontal line
line_x = linspace(0,250,4); line_y = 15 * ones(1,length(line_x));
plot(line_x,line_y)
```



The longitudinal dynamics model used to make this curve originates from the duggoff model which uses the parameters chosen in the previous section. With the desired cruising velocity of 15 mph, the shuttle can reach this speed after ~ 20 seconds, which is reasonable for the low speed application of this shuttle. Overally, this is likely a reasonable velocity time curve for standard use. It is however built off the nonlinear duggoff tire model and is higher fidelity than needed in ongoing steps of the project. Therefor, in the next sectioon this curve's transfer function will be described and approximated using a simple linear system.

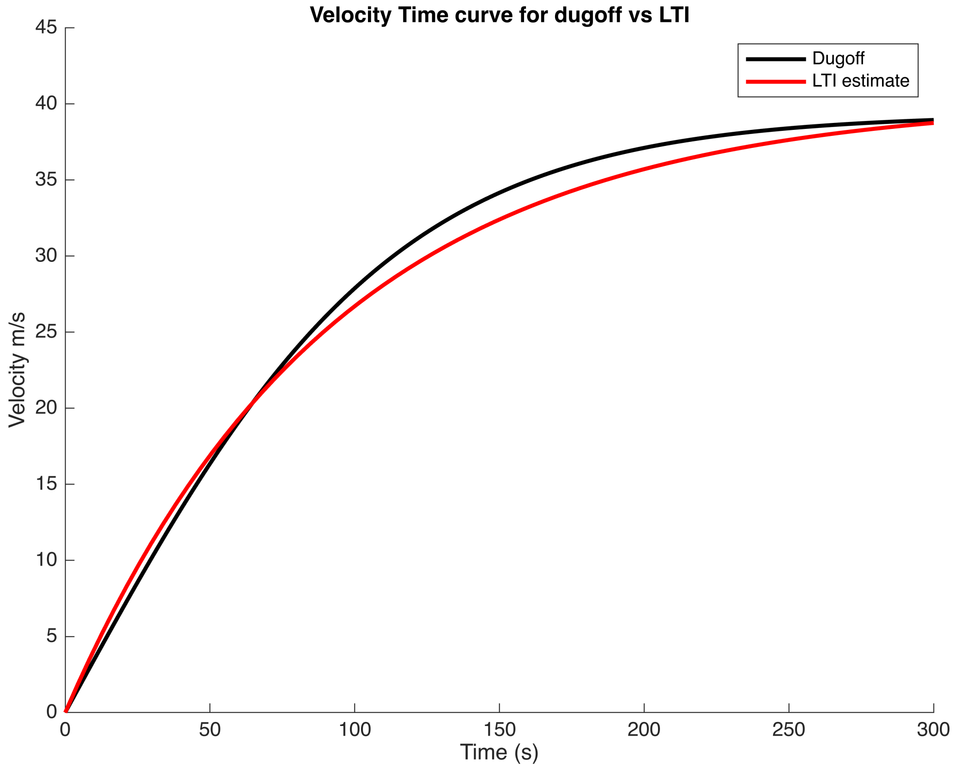
LTI System Approximation

The system that creates the transfer function above can be described as a linear single input single output transfer function with Velocity as the output and Motor torque as the input. Therefor, the simulated transfer function can be displayed by plotting the simulation Velocity values divided by the motor torque, which we chose as constant in the last section.

```
motor_torque = motorStart;
velocity = sim1(:,1);

figure()
plot(time1,velocity/motor_torque,'k','Linewidth',2), ylabel('System'),
xlabel(''), title('Shuttle Velocity / Motor Torque'), hold on

K = 0.62;
T = 92;

s = tf('s');
V_tf = K/ (T*s + 1);

[h,g] = step(V_tf);
plot(g, squeeze(h), 'b','LineWidth',2)

legend('Experimental V/tm Transfer function','model estimate transfer
function')
title('System Transfer Functions')
ylabel('V/Tm')
axis([0 300 0 .75])
```

The LTI system approximates the duggoff model around our motor torque of interest of 65 Nm, lets reproduce the velocity time curve from this LTI system and compare it to the simulated duggoff model

```
figure(), hold on
plot(time1,sim1(:,1),'k','Linewidth',2)
plot(g,h*motor_torque,'Linewidth',2,'Color',[1 0 0])
xlabel('Time (s)'), ylabel('Velocity m/s'), title('Velocity Time curve for
dugoff vs LTI'), legend('Dugoff','LTI estimate'), axis([0 300 0 45])
```



This curve is just a scaled version of the previous one.

Lateral Dynamics Parameters

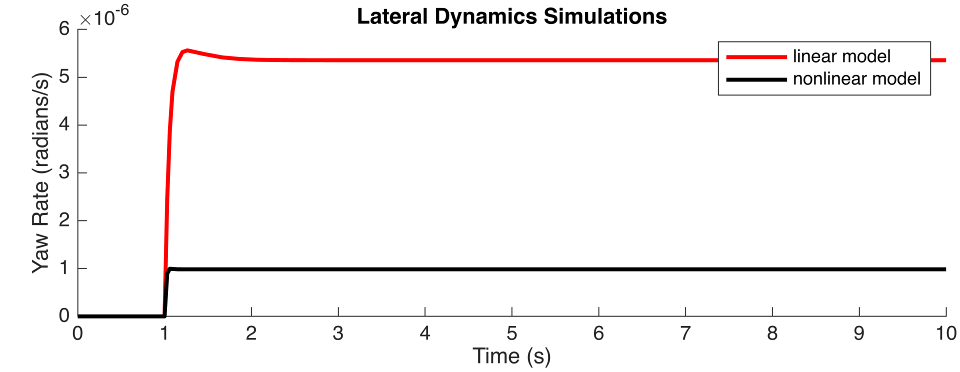
```
% These Lateral parameters have overlap with the longitudinal values
% chosen, and have been set to match

    % L=4.75; % Distance between the axles [m], matched with first set
    % g=9.81; %
    % Lr = 2.5;% Distance from the center of gravity of the vehicle (CG) to
the rear axle, matched with first set
    % Lf = 2.25; % Distance from the center of gravity of the vehicle (CG)
to the front axle, matched with first set
    % Cf = 1.5e5; % Cornering Stiffness of Front Tires x2, matched with
first set
    % Cr = 1.5e5; % Cornering Stiffness of Rear Tires x2, matched with
first set
    % Cs = 1.5e5; % Cornering Stiffness, matched with first set
    % m = 3500; %Mass of the vehicle [kg], matched with first set
    % J = 4000; %Yaw moment of Inertia, matched with first set
    % mu = 0.7; %Dry coefficient of Friction, matched with first set
    % R = 0.3; % Wheel radius, matched with first set
    % Vref = 20; % Constant vehicle velocity.
    % alphaf = 0.1; % Steering wheel angle rad/sec
    %
```

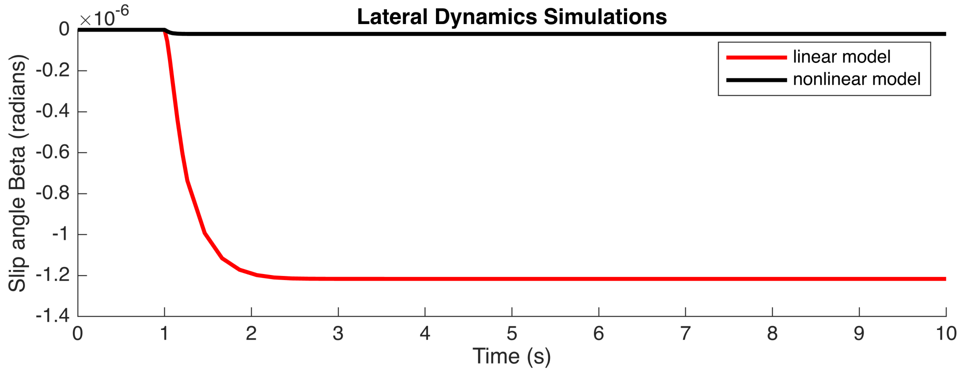
```
    % %Linear Parameters Calculation
    % a11 = -(Cr+Cf)/(mVref);
    % a12 = -1-((CfLf-CrLr)/(mVref^2));
    % a21 = (LrCr-LfCf)/J;
    % a22 = -((CfLf^2)+(CrLr^2))/(VrefJ);
    % b11 = Cf/(mVref);
    % b12 = Cr/(mVref); %delta_r parameter
    % b21 = CfLf/J;
    % b22 = Cr*Lr/J; %delta_r parameter
    % e2 = 1/J; % For yaw moment term

[L,g,Lr,Lf,Cf,Cr,Cs,m,J,mu,R,Vref,alphaf,a11,a12,a21,a22,b11,b12,b21,b22,e2]
 = getLateralParameters();
Vref = 6.7;
```

Lateral Dynamics Simulation

Lets use the lateral model comparison simulink from an earlier project

```
sim('lateral_models')
figure
subplot(2,1,1), hold on
plot(time,yawRateLin,'r','linewidth',2)
plot(time,yawRateNonLin,'k','linewidth',2)
legend('linear model','nonlinear model'), title('Lateral Dynamics
Simulations'),xlabel('Time (s)'), ylabel('Yaw Rate (radians/s)')
subplot(2,1,2), hold on
plot(time,betaLin,'r','linewidth',2)
plot(time,betaNonLin,'k','linewidth',2)
legend('linear model','nonlinear model'), title('Lateral Dynamics
Simulations'),xlabel('Time (s)'), ylabel('Slip angle Beta (radians)')
```





These figures showcase the difference between the bicycle model with linear tires vs nonlinear dugoff tire models. The linear tires overestimate both the yaw rate and side slip angle, but resemble the same shape.

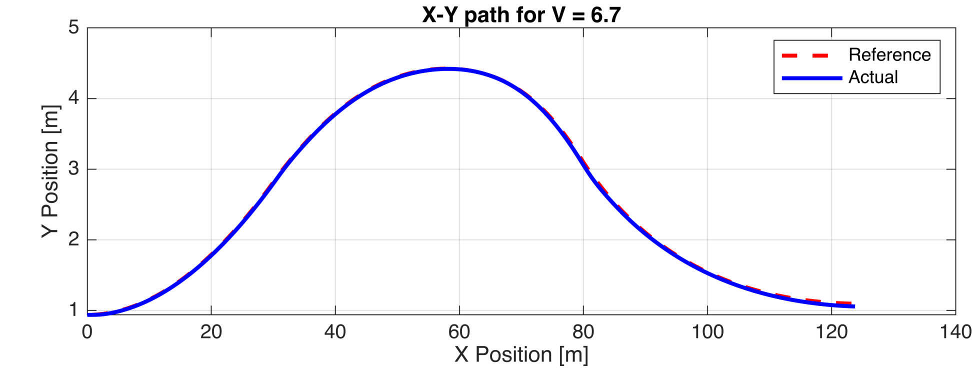
Path Following model

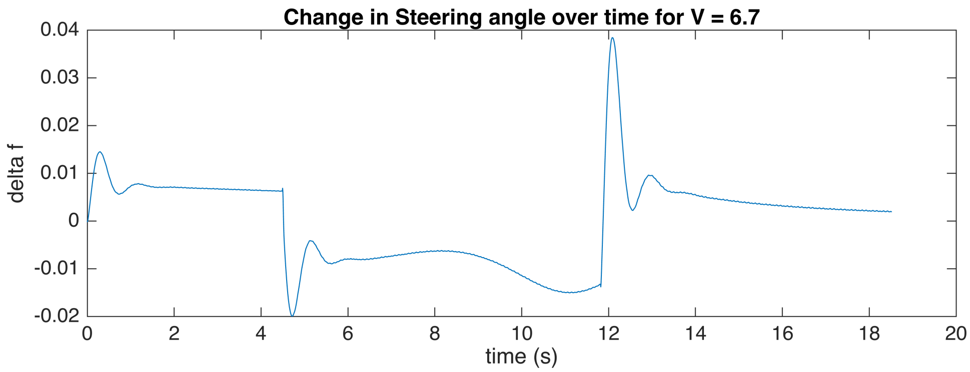
Using the bicycle model with linear tires, the same as the red curves in the previous section

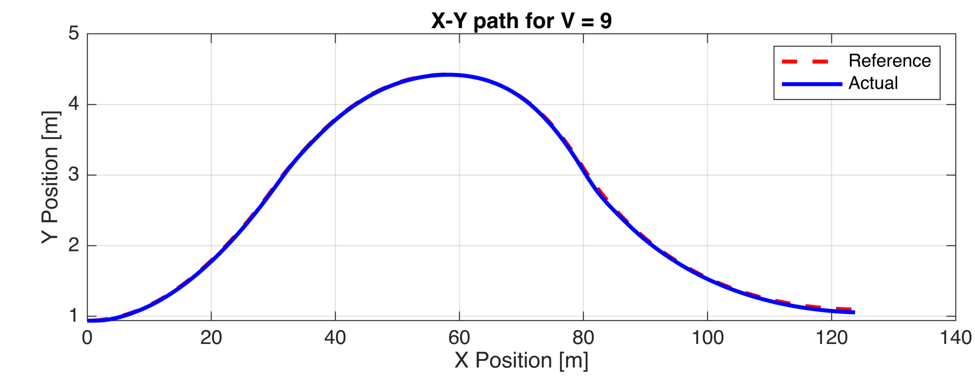
```
[L,g,Lr,Lf,Cf,Cr,Cs,m,J,mu,R,V,alphaf,a11,a12,a21,a22,b11,b12,b21,b22,e2] =
getLateralParameters();
ls = 2; % m
V = 6.7; % m/s
%%%%State Space Representation needs to be added to the existing parameters
    A_matrix = [a11 a12 0 0; a21 a22 0 0; 0 1 0 0; V ls V 0];
    B_matrix = [b11 0; b21 0; 0 -V; 0 -ls*V];
    C_matrix = [1 0 0 0; 0 1 0 0; 0 0 1 0; 0 0 0 1]; %Output is beta, r and
ey
    D_matrix = [0 0; 0 0; 0 0; 0 0];

% Controller parameters
P = -1;
I = -4.5;
D = 0.0113;
N = 200;

% Path Generation
```

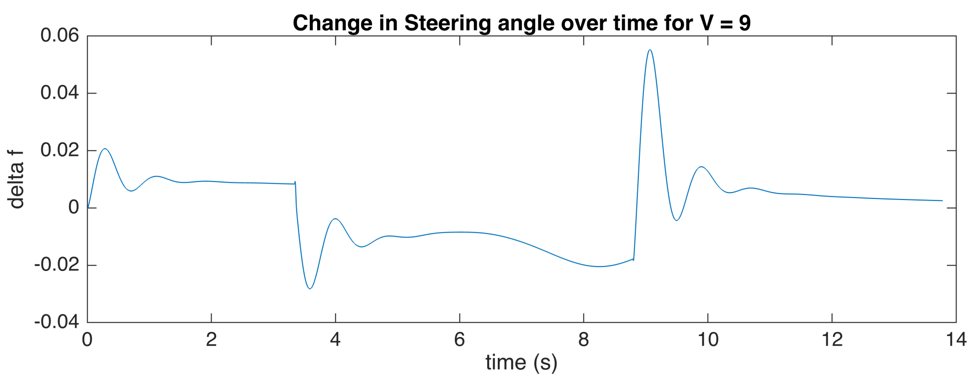
```
[Xp,Yp] = getC0Path();
plotPathQuality(Xp,Yp)
% Variables Needed for simulink model
path_curvatures = getPathCurvature(Xp,Yp);
cumulative_distances = getCumulativeDistances(Xp,Yp); % the simulink uses
this variable in a lookup table
dis_vec = cumulative_distances(1:end-1); % the model uses only n-2 entries
in its lookup table because curvature loses two entries
```

```
%%Operating condition
ls = 2; %Preview Distance [m]

loop = 1;
Velocities = [6.7,9];
n = length(Velocities);
for V = Velocities
    figure(), hold off

    [A_matrix, B_matrix, C_matrix, D_matrix] = changeModel(V,ls);
    sim('PathFollowingModel');

    % actual vs refrence path
    subplot(2,1,1), plot(X_ref,Y_ref,'r--','Linewidth',2); hold on; grid
on; plot(X_actual,Y_actual,'b','Linewidth',2); grid on; hold off
    legend('Reference','Actual'),xlabel('X Position [m]'),ylabel('Y
Position [m]'),title(['X-Y path for V = ', num2str(V)]);

    % plot Steering angle change
    subplot(2,1,2), plot(time,delta_f_out)
    xlabel('time (s)'), ylabel('delta f'), title(['Change in Steering angle
over time for V = ', num2str(V)]);

    loop = loop + 1;
end
```





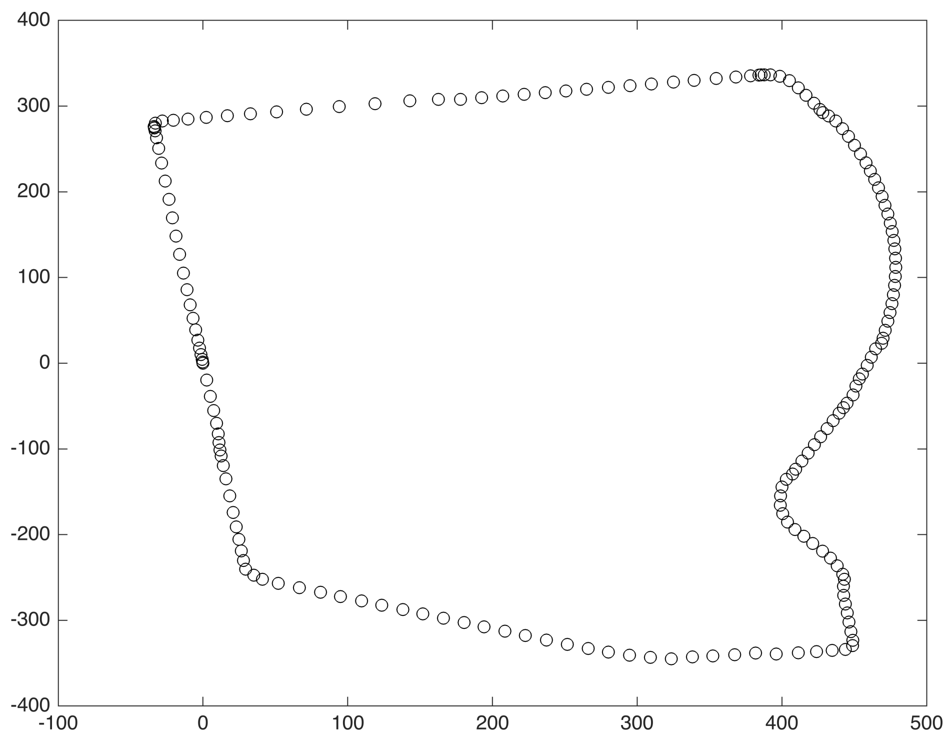




Long Path following, Attempt 1

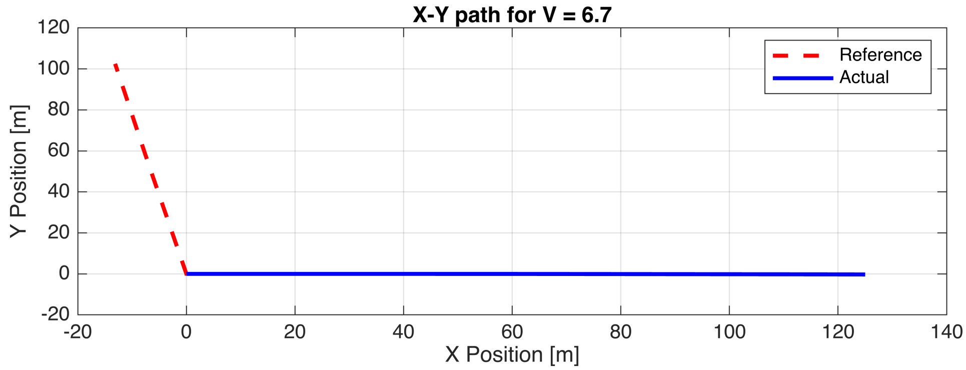
This first attempt uses the sparse waypoints as the input path, where curvature and cumulative distance are calculated numerically and by straight line path respectively. This attempt may be problematic because the path is sparse and so numerical calculation of the curvature values is suboptimal.

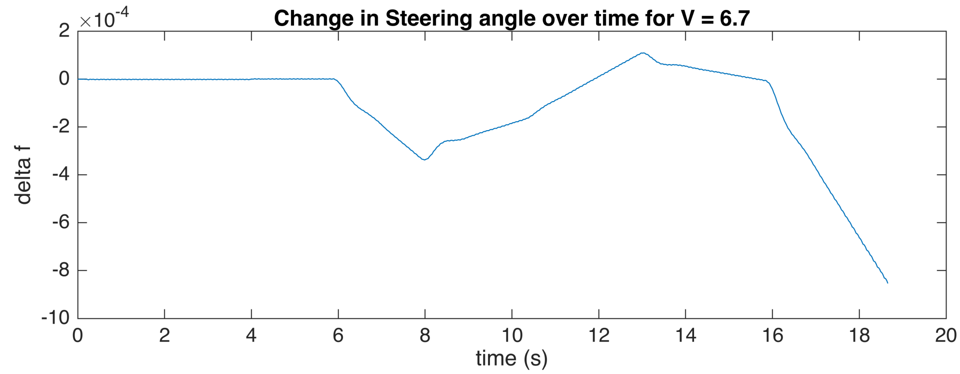
```
points = load('center_points.mat');
% Assuming the variable name in the .mat file is center_points
points = points.center_points;

% Plot the points
figure
plot(points(:,1), points(:,2),'ko');
```




```
figure
Xp = points(:,1); Yp = points(:,2);

% Variables Needed for simulink model
path_curvatures = getPathCurvature(Xp,Yp);
cumulative_distances = abs(getCumulativeDistances(Xp,Yp)); % the simulink
uses this variable in a lookup table

% cumulative distances was not monotomically increasing due to some
% truncation error, this fixes it
for i = 1:1:length(cumulative_distances)
    cumulative_distances(i) = cumulative_distances(i) + i*.0001;
end
```

```
dis_vec = cumulative_distances(1:end-1); % the model uses only n-2 entries
in its lookup table because curvature loses two entries

[L,g,Lr,Lf,Cf,Cr,Cs,m,J,mu,R,V,alphaf,a11,a12,a21,a22,b11,b12,b21,b22,e2] =
getLateralParameters();
ls = 2; % m
V = 6.7; % m/s
%%%%State Space Representation needs to be added to the existing parameters
    A_matrix = [a11 a12 0 0; a21 a22 0 0; 0 1 0 0; V ls V 0];
    B_matrix = [b11 0; b21 0; 0 -V; 0 -ls*V];
    C_matrix = [1 0 0 0; 0 1 0 0; 0 0 1 0; 0 0 0 1]; %Output is beta, r and
ey
    D_matrix = [0 0; 0 0; 0 0; 0 0];

% Controller parameters
P = -1;
I = -4.5;
D = 0.0113;
N = 200;

%%Operating condition
ls = 2; %Preview Distance [m]

figure()
loop = 1;
Velocities = [6.7];
n = length(Velocities);
for V = Velocities
    figure(), hold off

    [A_matrix, B_matrix, C_matrix, D_matrix] = changeModel(V,ls);
    sim('PathFollowingModel');

    % actual vs refrence path
    subplot(2,1,1), plot(X_ref,Y_ref,'r--','Linewidth',2); hold on; grid
on; plot(X_actual,Y_actual,'b','Linewidth',2); grid on; hold off
    legend('Reference','Actual'),xlabel('X Position [m]'),ylabel('Y
Position [m]'),title(['X-Y path for V = ', num2str(V)]);

    % plot Steering angle change
    subplot(2,1,2), plot(time,delta_f_out)
    xlabel('time (s)'), ylabel('delta f'), title(['Change in Steering angle
over time for V = ', num2str(V)]);

    loop = loop + 1;
end
```

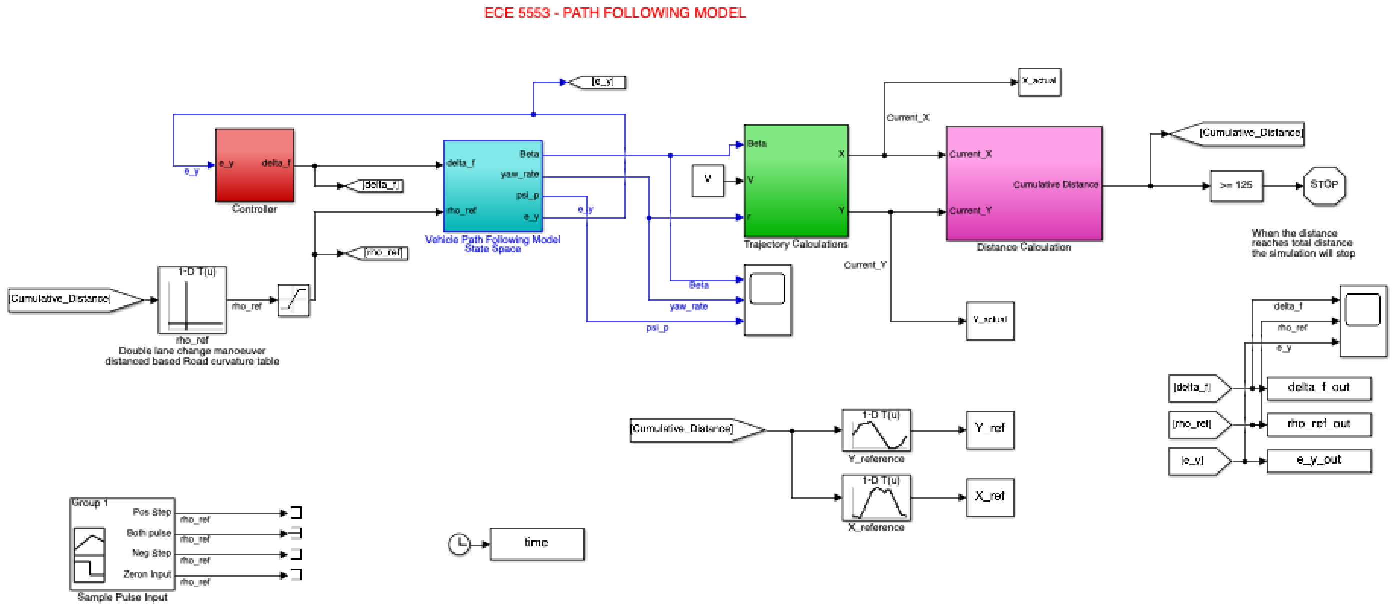




Long Path Following Next steps

Should this project continue the following steps would be taken

Following from the linearized wheels bicycle model, the commanded longitudinal velocity is kept constant here. This can be modified to utilize the LTI system approximation from earlier in this report. The bicycle model used here is the same as the linear lateral simulations from earlier in this report, the issue is that the velocity is now able to change, and so the vehicle state space model must change and have velocity as an input, this is simple in theory, but the state space model used utilizes 4 matricies, which must be constructed in simulink and then input into the state space model block - this was the roadblock encountered and unsolved by the project deadline. Perhaps there are alternate solutions, such as scheduling the bicycle model used based on current velocity, so that the model is switched rather than recalculated.



Localization and Perception

Localization and perception will not be explored in depth in this report, instead, a brief section is included to go over potential leads for future solutions. This section is inspired by an undergraduate honors thesis by Brian Lesko, which demonstrates a Reccurrent Nueral network called Yolo. In this specific refrence, perception is done through a spherical camera to reduce system complexity while retaining 360 degree perception.

Normal Camera

Ohio State Campus, photo taken by Brian Lesko, processed by Yolo V5, taken on iphone 12

Spherical Footage, unfolded

Photo by Brian Lesko, processed by Adobe after effects and Yolo - V5 , shot on Gopro Max

The localization of each detection is not demonstrated and represents a field of its own.

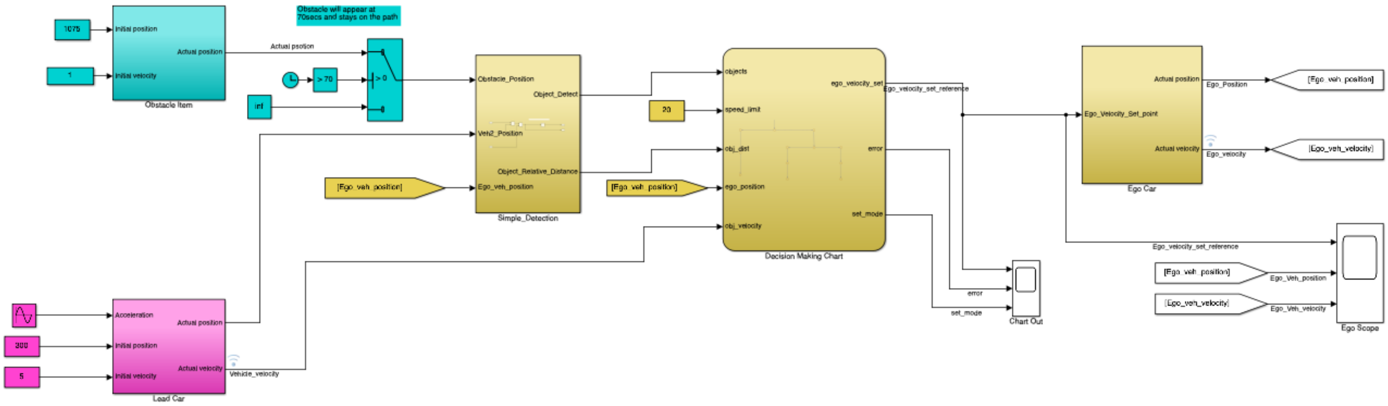
Collision Avoidance, State Flow (Kevin)

State flow diagram to detect collision risk and decide when to stop and wait.

blob:https://buckeyemailosu-my.sharepoint.com/54a5e52e-1bc0-470e-9378-5dbea50f65a2

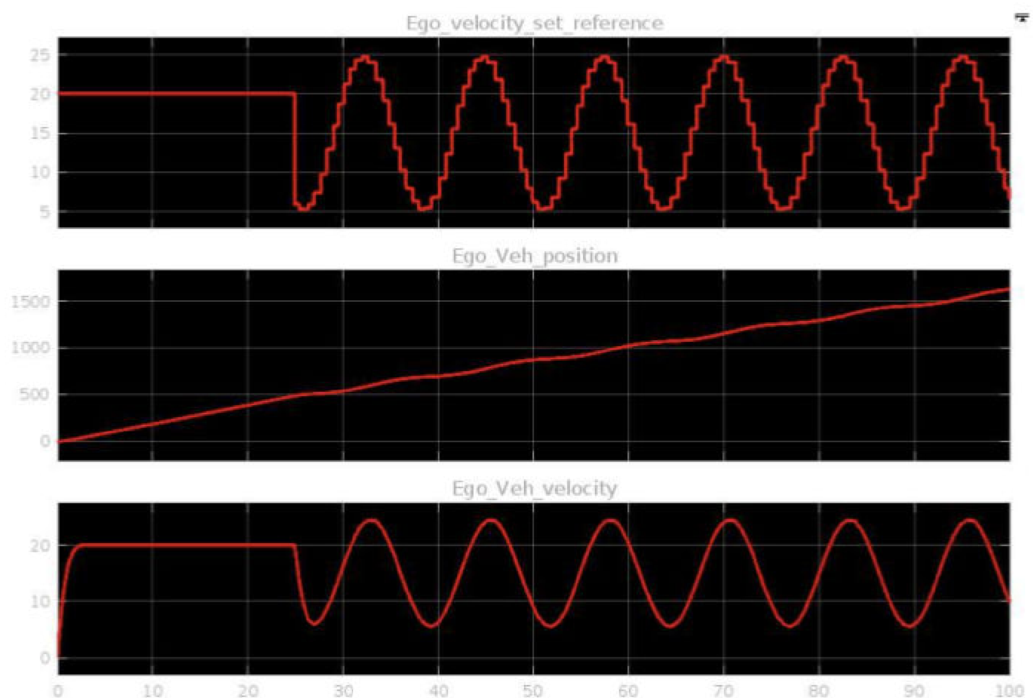
Potential Obstacle distances are input on the left half of the simulink model. The output is the state of the vehicle, which

ECE 5553 DECISION MAKING AND COLLISION AVOIDANCE

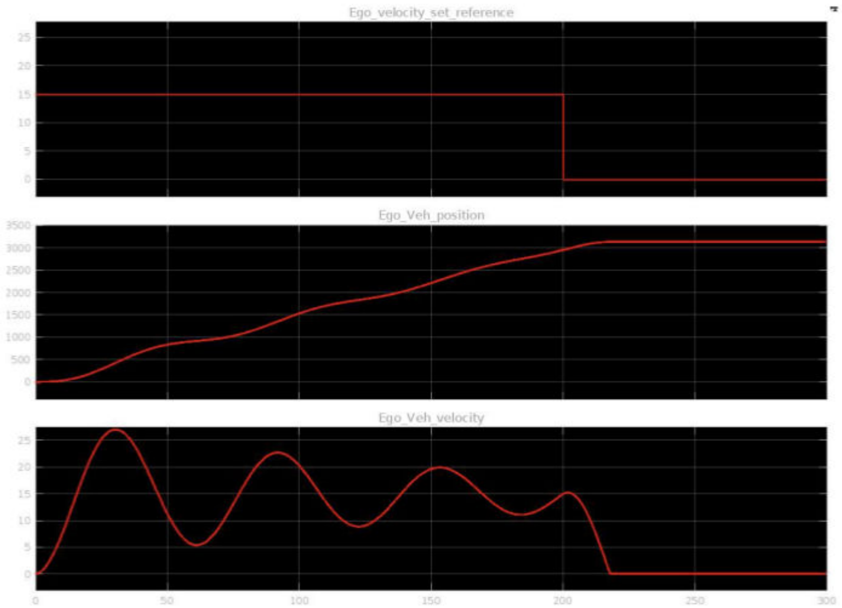


State Flow and Collision Avoidance Results

The Advanced Emergency Breaking System (AEBS) was created based off of the Homework 7 AEBS Simulink model. This model has been adjusted to be in line with our car's parameters including reducing the speed limit to 15 mph as well as changing the Ego Car's parameters. The state flow block in the AEBS model was created to follow the speed limit, identify objects, adjust to a lead car's velocity, and output an emergency break in the event an object appears too close to the ego car. From the resulting output, the vehicle can be seen adjusting its velocity in response to the lead vehicle's velocity as it reduces and increases in speed. A sudden object is presented at 200 seconds into the route where the ego vehicle needed to apply its AEBS. This can be seen in the output as the car suddenly drops to 0 mph at the time of the object being present. To follow these conditions, the decision-making block uses the inputs of the presence of objects, distance of the object, speed limit, ego car position, and the lead car's velocity. With these input conditions, the state flow chart created then outputs the ego car's velocity, error, and set mode. The set mode determines at any point in time if the ego car needs to go the speed limit, follow the velocity of a lead car, or apply its AEBS.



State Flow Lead Vehicle Following



State Flow Lead Vehicle Following with Collision Avoidance at 200 Seconds

Shuttle Vehicle, Sensors, and Implementation problems, (Ovee)

EZ10 specifications at a glance:

- Passenger capacity per vehicle: 12 (900kg max.)
- Disabled access: built-in automated electric ramp
- Service operation modes: scheduled (fixed route, network) or on-demand
- Fleet management and supervision system: EZFleet
- Weather conditions: heavy rain, snow, fog, temperature from -15°C to 45 °C
- Net vehicle weight (estimate): 2,130 kg (4 battery packs and enhanced A/C)
- Gross Vehicle Weight (GVW): 3,130 kg (4 batteries and enhanced A/C)
- Dimensions (LxWxH): 4,050 x 1,892 x 2,871 mm

This vehicle was obtained from the Easymile website. The vehicle seats 12 people and has the dimensions listed above.

Autonomous Architecture:

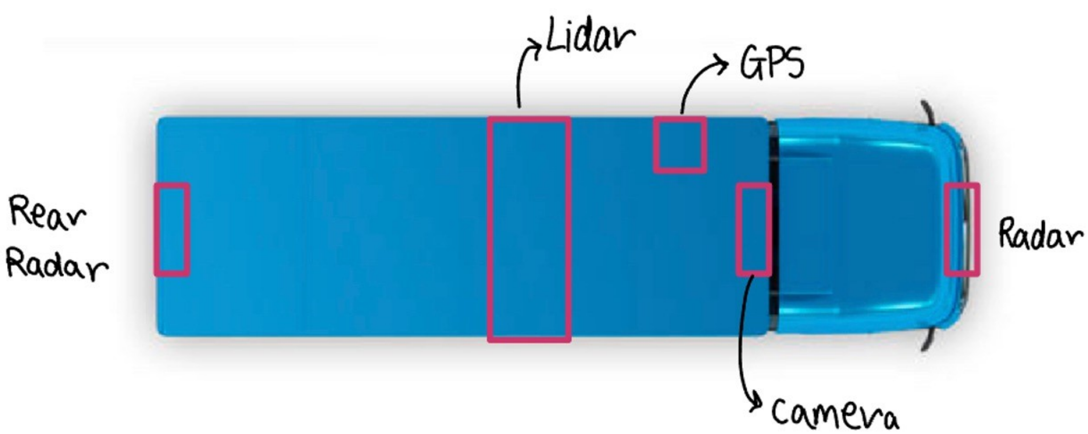
Choice of sensors:

Sensor

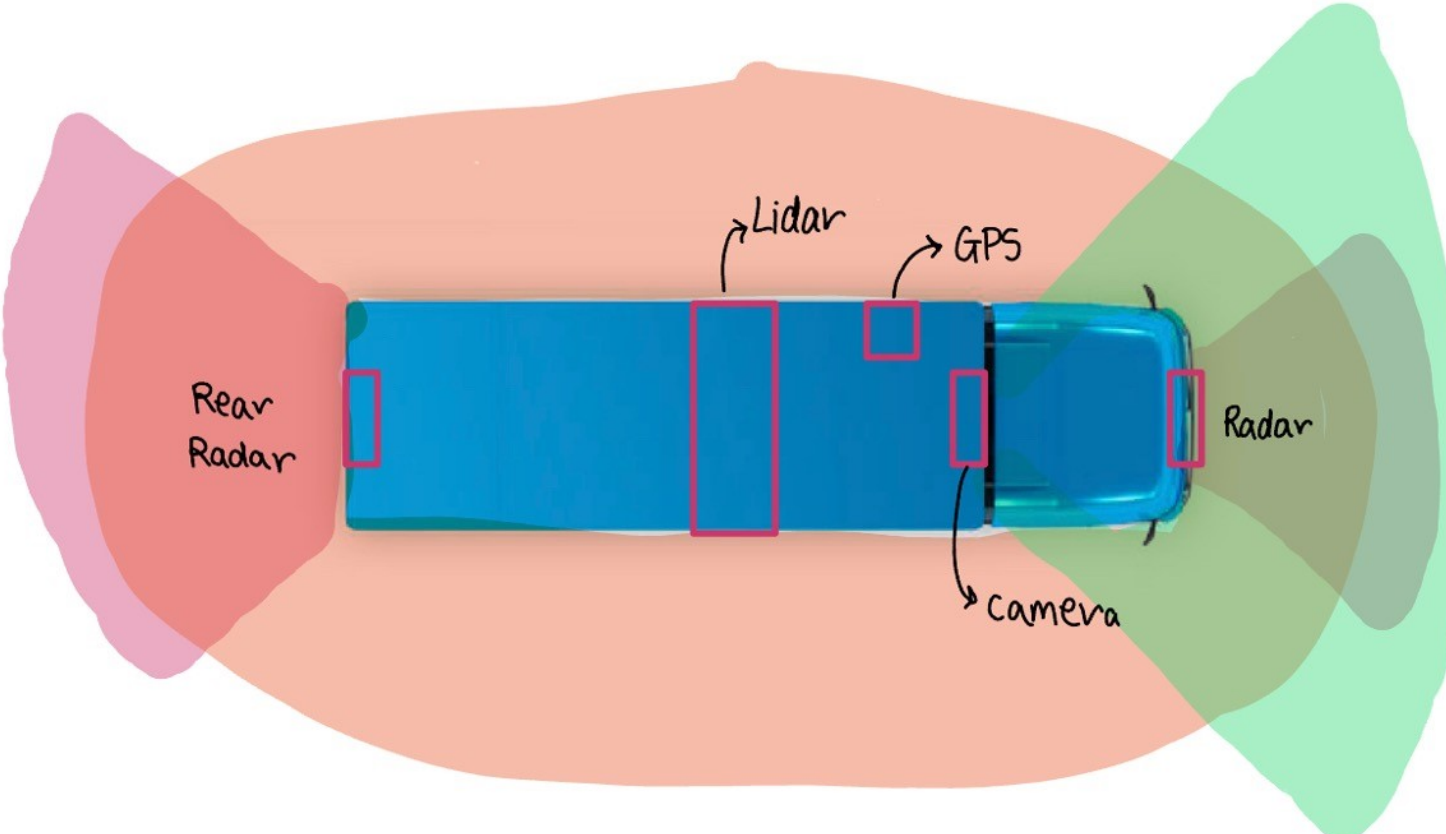
Characteristics

Sensor	Characteristics
Radar (Continental ARS 300 Radar)	Angular Resolution: 60° / 17° x 3.2° FOV Max. Distance (m): 60/200
Lidar (SICK LMS30206)	Angular Resolution: 1° / 0,5° / 0,25° Max. Distance (m): 80 Response Time (ms): 13 / 26 / 53
GPS	Sub-meter accuracy with Omnistar VBS corrections Tightly coupled inertial
Camera	

Location of the sensors:



Area covered by the sensors:



List of real world implementations challenges:

Implementation Problem	Severity (1-5)
Icy Conditions	3
Too many pedestrians	4
Unexpected cars out of the traffic pattern	5
Unexpected debris on the road	5
Potholes	1
Rerouting during road blockages	2
Passing a car in 2 lane or highway	4

Another implementation challenge would be creating the state change diagram and implementing all the models used in this report.

Summary

overall, this report demonstrates some development of vehicle system models and touches upon the main aspects necessary for an low level autonomous vehicle. Future work includes implementing and combining models used here.

Functions

Fetch longitudinal parameters

```
function [M, eta_t, lambda_t, lambda_f, I_w, R_w, Meq, C_d, rho, A_f, g,
C_rr, C_x, C_alpha,L, l_f, l_r, C_s, I_z, mu] = getLogitudinalParameters()
    M=2000; % Mass of the vehicle[Kg]
    eta_t=0.9; % Transmission Efficiency
    lambda_t=1.0; % Gear Ratio
    lambda_f=4.1; % Final Drive Ratio
    I_w=1; % Inertia of the Wheel[kgm2]
    R_w=0.3; % Wheel Radius[m]
    Meq=0.1*M; % Equivalent Mass Factor
    C_d=0.29; % Drag coefficient
    rho=1.225; % Air density[kg/m3]
    A_f=2.8;% Frontal area    [m2]
    g=9.81;% [N/m2]
    C_rr=0.015;%Rolling resistance coefficient
```

```
    C_x=3e5; %Longitudinal Stiffness[N]
    C_alpha = 1.5e5; % Cornering Stiffness for 1 tire [N/rad]
    L=2.85;% Wheelbase[m]
    l_f=1.3;% Distance from the center of gravity of the vehicle (CG) to
the front axle [m]
    l_r=1.55;% Distance from the center of gravity of the vehicle (CG) to
the rear axle [m]
    C_s=1.5e5;% Cornering Stiffness of Front and Rear Tires [N/rad]
    I_z=3700;% Inertia moment around z axis J or Iz [kg/m2]
    mu=0.7; % Road friction coefficient
end
function
[motorStart,motorStartTime,motorIncrease,motorIncreaseTime,motorDecrease,mot
orDecreaseTime,brakeTime,breakStrength,headWindTime,headWind,roadSlopeTime,r
oadSlope] = setLongitudinalInputsToZero()
    motorStart = 0; motorStartTime = 0;
    motorIncrease = 0;motorIncreaseTime = 0;
    motorDecrease = 0;motorDecreaseTime = 0;
    brakeTime = 0;breakStrength = 0;
    headWindTime = 0;headWind = 0;
    roadSlopeTime = 0;roadSlope = 0;
end
```

Sample a vector evenly to make it length b

```
function y = sampVec(x, b)
    % x: input vector
    % b: length of output vector
    a = length(x);
    y = linspace(1, a, b)';
    y = round(y);
    y = x(y);
end
```

Fetch stored Lateral Parameters

```
function
[L,g,Lr,Lf,Cf,Cr,Cs,m,J,mu,R,Vref,alphaf,a11,a12,a21,a22,b11,b12,b21,b22,e2]
 = getLateralParameters()
    L=4.75; % Distance between the axles [m], matched with first set
    g=9.81; %
    Lr = 2.5;% Distance from the center of gravity of the vehicle (CG) to
the rear axle, matched with first set
    Lf = 2.25; % Distance from the center of gravity of the vehicle (CG) to
the front axle, matched with first set
    Cf = 1.5e5; % Cornering Stiffness of Front Tires x2, matched with first
set
    Cr = 1.5e5; % Cornering Stiffness of Rear Tires x2, matched with first
set
    Cs = 1.5e5; % Cornering Stiffness, matched with first set
    m = 3500; %Mass of the vehicle [kg], matched with first set
```

```
    J = 4000; %Yaw moment of Inertia, matched with first set
    mu = 0.7; %Dry coefficient of Friction, matched with first set
    R = 0.3; % Wheel radius, matched with first set
    Vref = 20; % Constant vehicle velocity.
    alphaf = 0.1; % Steering wheel angle rad/sec

    %Linear Parameters Calculation
    a11 = -(Cr+Cf)/(m*Vref);
    a12 = -1-((Cf*Lf-Cr*Lr)/(m*Vref^2));
    a21 = (Lr*Cr-Lf*Cf)/J;
    a22 = -((Cf*Lf^2)+(Cr*Lr^2))/(Vref*J);
    b11 = Cf/(m*Vref);
    b12 = Cr/(m*Vref); %delta_r parameter
    b21 = Cf*Lf/J;
    b22 = Cr*Lr/J; %delta_r parameter

    e2 = 1/J; % For yaw moment term
end
function [A_matrix, B_matrix, C_matrix, D_matrix] = changeModel(V,ls)
    L=4.75; % Distance between the axles [m], matched with first set
    g=9.81; %
    Lr = 2.5;% Distance from the center of gravity of the vehicle (CG) to
the rear axle, matched with first set
    Lf = 2.25; % Distance from the center of gravity of the vehicle (CG) to
the front axle, matched with first set
    Cf = 1.5e5; % Cornering Stiffness of Front Tires x2, matched with first
set
    Cr = 1.5e5; % Cornering Stiffness of Rear Tires x2, matched with first
set
    Cs = 1.5e5; % Cornering Stiffness, matched with first set
    m = 3500; %Mass of the vehicle [kg], matched with first set
    J = 4000; %Yaw moment of Inertia, matched with first set
    mu = 0.7; %Dry coefficient of Friction, matched with first set
    R = 0.3; % Wheel radius, matched with first set
    Vref = 20; % Constant vehicle velocity.
    alphaf = 0.1; % Steering wheel angle rad/sec

    %Linear Parameters Calculation
    a11 = -(Cr+Cf)/(m*Vref);
    a12 = -1-((Cf*Lf-Cr*Lr)/(m*Vref^2));
    a21 = (Lr*Cr-Lf*Cf)/J;
    a22 = -((Cf*Lf^2)+(Cr*Lr^2))/(Vref*J);
    b11 = Cf/(m*Vref);
    b12 = Cr/(m*Vref); %delta_r parameter
    b21 = Cf*Lf/J;
    b22 = Cr*Lr/J; %delta_r parameter

    e2 = 1/J; % For yaw moment term

    %%%%State Space Representation
```

```
    A_matrix = [a11 a12 0 0; a21 a22 0 0; 0 1 0 0; V ls V 0];
    B_matrix = [b11 0; b21 0; 0 -V; 0 -ls*V];
    C_matrix = [1 0 0 0; 0 1 0 0; 0 0 1 0; 0 0 0 1]; %Output is beta, r and
ey
    D_matrix = [0 0; 0 0; 0 0; 0 0];
end
```

Path Following Functions

```
function cumulative_distances = getCumulativeDistances(Xp,Yp)
    % This function calculates the cumulative distances traveled based on
Xp and Yp arrays of coordinates

    % Initialize the total distance traveled to 0
    Tot_d = 0;

    % Loop through all the points in the input arrays (from the first to
the second to last)
    cumulative_distances = zeros(length(Xp)-1,1);
    for j = 1:length(Xp)-1
        % Calculate the Euclidean distance between the consecutive points
(j and j+1)
        d(j) = sqrt((Xp(j+1)-Xp(j))^2+(Yp(j+1)-Yp(j))^2);

        % Update the total distance with the calculated distance
        Tot_d = Tot_d + d(j);

        % Store the cumulative distance traveled so far in the output array
        cumulative_distances(j) = Tot_d;
    end
end
function path_curvatures = getPathCurvature(Xp, Yp)
    % Road Curvature Calculations

    % Input: Xp and Yp are vectors containing the X and Y coordinates of
the path points
    % Output: path_curvatures is a vector containing the curvature values
for each point in the path

    % Check if Xp and Yp have the same length
    if length(Xp) ~= length(Yp)
        error('Input vectors Xp and Yp must have the same length.');
    end

    % Compute the first-order differences (derivatives)
    d_Xp = diff(Xp);
    d_Yp = diff(Yp);

    % Compute the second-order differences (second derivatives)
    dd_Xp = diff(Xp, 2);
```

```
    dd_Yp = diff(Yp, 2);

    % Curvature formula given in Lecture 21, Slide 27
    % (d_Xp * dd_Yp - d_Yp * dd_Xp) / ((d_Xp^2 + d_Yp^2)^(3/2))
    path_curvatures = (d_Xp(2:end) .* dd_Yp - d_Yp(2:end) .* dd_Xp) ./
(d_Xp(2:end).^2 + d_Yp(2:end).^2).^1.5;
end
function [Xp,Yp] = getC0Path()
    %Initialize the variables
    Xp = [];
    Yp = [];

    %%%%%%%
    %DOUBLE LANE CHANGE MANOEUVER - ISO 3888-1
    %%%%%%%

    Veh_width = 1.57; %For a Mid-size Sedan Vehicle, width in m
    L1 = Veh_width*1.1+0.15;
    L2 = Veh_width*1.2+0.15;
    L3 = Veh_width*1.3+0.15;

    p = [];

    %Change the points here to modify the shape of the curve.
    %If you are adding new points, make sure you modify the p & p_ub_pts
array
    %points accordingly
    p_lb_pts = [0,0; 15,0; 31,2; 45,3.5; 53,3.5; 63,3.5; 70,3.5; 80,2;
95,0; 125,0;]; %Points on the curve considering 0,0 starting point and no
offset L1,L2,L3
    %p_lb_pts = [0,0; 15,0; 31.25,1.95; 45,3.5; 53,3.5; 63,3.5;
70,3.5;81.75,1.95; 95,0; 125,0;]; % good path!
    %p_lb_pts = [0,0; 15,0; 31,2; 45,3.5; 53,3.5; 63,3.5; 70,3.5; 80,2;
95,0;125,0;]; %best path so far

    p_ub_pts = p_lb_pts;
    p = p_lb_pts;

    %Upper Bound
    p_ub_pts(1:3,2) = p_lb_pts(1:3,2)+(L1); %Offset of the path. upper bound
    p_ub_pts(4:7,2) = p_lb_pts(4:7,2)+(L2); %Offset of the path. upper bound
    p_ub_pts(8:10,2) = p_lb_pts(8:10,2)+(L3); %Offset of the path. upper
bound

    %Mid-path
    p(1:3,2) = p_lb_pts(1:3,2)+(L1/2); %Offset of the path to the centre of
the vehicle
    p(4:7,2) = p_lb_pts(4:7,2)+(L2/2); %Offset of the path to the centre of
the vehicle
```

```
    p(8:10,2) = p_lb_pts(8:10,2)+(L3/2); %Offset of the path to the centre
of the vehicle

    %Split the points into three bezier curves%

    %Beizer Curves passes through first and last points and approximates
    %through the middle points. This is the characteristic of the Beizer
curve
    p1 = p(1:3,:);
    p2 = p(3:8,:);
    p3 = p(8:10,:);


    [Xp_1,Yp_1] = bezier_curve(p1); %For first curve
    [Xp_2,Yp_2] = bezier_curve(p2); %For second curve
    [Xp_3,Yp_3] = bezier_curve(p3); %For third curve

    Xp = [Xp_1(1:end-1); Xp_2(1:end-1); Xp_3(1:end-1);];
    Yp = [Yp_1(1:end-1); Yp_2(1:end-1); Yp_3(1:end-1);];
end
function [Xp,Yp] = bezier_curve(p1)
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    %%ECE 5553 - Autonomy in Vehicles
    %%HW 4 - Path Following Linear Model
    %%Spring 2019
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    %%Bezier curve for higher order polynomials%

% https://ocw.mit.edu/courses/electrical-engineering-and-computer-science/
6-837-computer-graphics-fall-2012/lecture-notes/MIT6_837F12_Lec01.pdf
    % Refer to Slide 62

    %   Detailed explanation goes here
    p = p1;

    n = length(p); %number of points
    n1=n-1;

    for i=0:1:n1
    sigma(i+1)=factorial(n1)/(factorial(i)*factorial(n1-i));  % for
calculating (x!/(y!(x-y)!)) values
    end
    l=[];
    UB=[];
    for u=0:0.002:1
        for d=1:n
        UB(d)=sigma(d)*((1-u)^(n-d))*(u^(d-1));
        end
    l=cat(1,l,UB);
    end
```

```
    P=l*p;

    Xp = [P(:,1)]; %X_reference Points
    Yp = [P(:,2)]; %Y_reference Points
end
function plotPathQuality(Xp,Yp)
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    % Written by Brian Lesko
    % 4/11/23
    % This function plots the path quality of a planned vehicle path in
terms of the path's various characteristics,
    % including its first and second derivatives, curvature, and curvature
derivative.
    % It accepts input parameters Xp and Yp, which are arrays representing
the X and Y coordinates of the planned vehicle path.
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

    figure()
    subplot(5,1,1),plot(Xp,Yp,'k','Linewidth',2), title('Planned Vehicle
Path'), xlabel('X (m)'), ylabel('Y (m)'),set(gca,'FontSize',14)

    % Path derivative dy/dx
    subplot(5,1,2), plot(Xp(1:end-1),diff(Yp),'k','Linewidth',2),
title('Path derivative'), xlabel('X path'),ylabel('dy/
dx'),set(gca,'FontSize',14)

    % Path second derivative
    subplot(5,1,3), plot(Xp(1:end-2),diff(diff(Yp)),'k','Linewidth',2),
title('Path 2nd derivative'), xlabel('X path'),ylabel('dy^2/
dx^2'),set(gca,'FontSize',14)

    % Road Curvature c
    path_curvature = getPathCurvature(Xp,Yp); subplot(5,1,4),
plot(Xp(1:end-2),path_curvature,'k','Linewidth',2), title('Path
Curvature'),xlabel('Xp'),ylabel('curvature'),set(gca,'FontSize',14)

    cumulative_distances = getCumulativeDistances(Xp,Yp);

    % Path curvative derivative dc/dx
    subplot(5,1,5), plot(diff(path_curvature),'k','Linewidth',2);
xlabel('points'), ylabel('dc/dx'), title('Derivative of Path
Curvature'),set(gca,'FontSize',14)

    % increase the figure size
    set(gcf, 'Position', [100, 100, 800, 800]);
end
```