KS System Research Notes

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1 Introduction

This is a short summary of the paper "A Kochen-Specker system has at least 22 vertices" by Sander Uijlen and Bas Westerbaan.

2 Definitions and results

Definition 2.1 *A **010-coloring** is a $\{0,1\}$-coloring of the points such that*

- *No pair of orthogonal points are both colored* 1
- *Of three pairwise orthogonal points, exactly* 1 *is colored* 1

Definition 2.2

*A **kochen-Specker (KS) system** is a finite set of points on the sphere with no antipodal pair and no 010-coloring.*

Definition 2.3 *The **orthogonality graph** $G(S)$ of a finite subset S of the projective plane are constructed such that*

- *Let vertices of $G(S)$ be the points of S*
- *Two vertices are connected if their corresponding points are orthogonal.*

Definition 2.4 *A graph G is **embeddable**, if it is a subgraph of an orthogonality graph.*

Definition 2.5 *A graph is **010-colorable** if there exists a $\{0,1\}$-coloring of the vertices such that*

- *for each C_3 subgraph, there is exactly one vertex that is colored* 1
- *adjacent vertices cannot be both* 1

Definition 2.6 *A **KS graph** is an embeddable graph that is not 010-colorable.*

Proposition 2.1 *An embeddable graph does not contain subgraph of C_4.*

Proposition 2.2 *A minimal KS graph is connected*

Proposition 2.3 *A minimal KS graph has minimal vertex-order three.*

Proposition 2.4 *In a minimal KS graph, every vertex is part of a triangle.*

Proposition 2.5 *A minimal KS graph is edge-biconnected, removing a single edge leave the graph connected.*

Proposition 2.6 *A minmal KS graph is edge-triconnected, removing two edges keep the graph connected.*

Their approach is to yield all non-010-colorable graphs of certain size. If it is shown that one of them is embeddable, then a new KS system is found.

3 Conclusion and future research

Enumerating all candidate graphs of less than 31 vertices is computationally infeasible using the current approach. The result could be improved by combining satisfiability checking and symbolic computation and incorporate various filtering parameters.

4 Recommended further reading

The master thesis (click here) of Arends was recommended to further investigate some of the properties of KS graphs.