

1 Basic	1	8.3 External Disector	17
1.1 Shell script	1	8.4 Heart	17
1.2 Default code	1	8.5 Minimum Enclosing Circle*	18
1.3 vimrc	1	8.6 Polar Angle Sort*	18
1.4 readchar	2	8.7 Intersection of two circles*	18
1.5 Black Magic	2	8.8 Intersection of polygon and circle	18
2 Graph	2	8.9 Intersection of line and circle	18
2.1 BCC Vertex*	2	8.10 point in circle	18
2.2 Bridge*	2	8.11 Half plane intersection	19
2.3 2SAT (SCC)*	2	8.12 Circle Cover*	19
2.4 Minimum Mean Cycle*	3	8.13 3D point*	19
2.5 Maximum Clique Dyn*	3	8.14 Convex hull 3D*	20
2.6 Maximum Clique*	3	8.15 Tangent line of two circles	20
2.7 Minimum Steiner Tree*	4	8.16 minMax Enclosing Rectangle	21
2.8 Minimum Arborescence*	4	8.17 minDistOfTwoConvex	21
2.9 Vizing's theorem	4	8.18 Minkowski Sum*	21
2.10 Minimum Clique Cover*	5	8.19 Rotating Sweep Line	21
2.11 Number of Maximal Clique*	5	9 Else	21
2.12 Theory	5	9.1 Mo's Algorithm (With modification)	21
3 Data Structure	5	9.2 Mo's Algorithm On Tree	22
3.1 Leftist Tree	5	9.3 Dynamic Convex Trick*	22
3.2 Heavy light Decomposition	5	9.4 Matroid Intersection	22
3.3 Centroid Decomposition*	6	9.5 Adaptive Simpson	22
3.4 Link cut tree*	6	10 2A1B	22
3.5 KDTree	7	10.1 run.sh	22
4 Flow/Matching	7	10.2 vimrc	23
4.1 Kuhn Munkres	7	10.3 Binary Indexed Tree.cpp	23
4.2 MinCostMaxflow	8	10.4 Bipartite Matching.cpp	23
4.3 Minimum Weight Matching (Clique version)*	8	10.5 Closest Pair.cpp	23
4.4 SW-mincut	8	10.6 Dijkstra.cpp	23
4.5 BoundedFlow(Dinic*)	8	10.7 Dinic.cpp	24
4.6 Flow Models	9	10.8 Kosaraju.cpp	24
5 String	9	10.9 Segment Tree with tag.cpp	24
5.1 KMP	9	10.10 suffix Array.cpp	24
5.2 Z-value	10	10.11 Treap.cpp	25
5.3 Manacher*	10	10.12 Simple Graph Matching.cpp	25
5.4 SAIS*	10	1 Basic	
5.5 Aho-Corasick Automatan	10	1.1 Shell script	
5.6 Smallest Rotation	11	g++ -O2 -std=c++17 -Dbbq -Wall -Wextra -Wshadow -o \$1	
5.7 De Bruijn sequence*	11	\$1.cpp	
5.8 SAM	11	chmod +x compile.sh	
5.9 PalTree	11	1.2 Default code	
5.10 cyclic LCS	11	#include <bits/stdc++.h>	
6 Math	12	using namespace std;	
6.1 ax+by=gcd*	12	typedef long long ll;	
6.2 floor and ceil	12	typedef pair<int, int> pii;	
6.3 Miller Rabin*	12	typedef pair<ll, ll> pll;	
6.4 Big number	12	#define X first	
6.5 Fraction	13	#define Y second	
6.6 Simultaneous Equations	13	#define SZ(a) ((int)a.size())	
6.7 Pollard Rho	13	#define ALL(v) v.begin(), v.end()	
6.8 Simplex Algorithm	14	#define pb push_back	
6.8.1 Construction	14	1.3 vimrc	
6.9 Schreier-Sims Algorithm*	14	"This file should be placed at ~/.vimrc"	
6.10 chinese Remainder	15	se nu ai hls et ru ic is sc cul	
6.11 Quadratic Residue	15	se re=1 ts=4 sts=4 sw=4 ls=2 mouse=a	
6.12 Discrete Log	15	syntax on	
6.13 PiCount	15	hi cursorline cterm=none ctermbg=89	
6.14 Primes	16	set bg=dark	
6.15 Theorem	16	inoremap {<ENTER> {<LEFT><ENTER><ENTER><UP><TAB>	
6.15.1 Kirchhoff's Theorem	16		
6.15.2 Tutte's Matrix	16		
6.15.3 Cayley's Formula	16		
6.15.4 Erdős-Gallai theorem	16		
6.15.5 Gale-Ryser theorem	16		
6.15.6 Fulkerson-Chen-Anstee theorem	16		
6.15.7 Pick's theorem	16		
6.16 Euclidean Algorithms	16		
7 Polynomial	16		
7.1 Fast Fourier Transform	16		
7.2 Number Theory Transform	16		
7.3 Fast Walsh Transform*	17		
7.4 Newton's Method	17		

1.4 readchar

```
inline char readchar() {
    static const size_t bufsize = 65536;
    static char buf[bufsize];
    static char *p = buf, *end = buf;
    if (p == end) end = buf + fread_unlocked(buf, 1,
        bufsize, stdin), p = buf;
    return *p++;
}
```

1.5 Black Magic

```
#include <ext/pb_ds/priority_queue.hpp>
#include <ext/pb_ds/assoc_container.hpp> //rb_tree
using namespace __gnu_pbds;
typedef __gnu_pbds::priority_queue<int> heap;
int main() {
    heap h1, h2;
    h1.push(1), h1.push(3);
    h2.push(2), h2.push(4);
    h1.join(h2);
    cout << h1.size() << h2.size() << h1.top() << endl;
    //404
    tree<ll, null_type, less<ll>, rb_tree_tag,
        tree_order_statistics_node_update> st;
    tree<ll, ll, less<ll>, rb_tree_tag,
        tree_order_statistics_node_update> mp;
    for (int x : {0, 2, 3, 4}) st.insert(x);
    cout << *st.find_by_order(2) << st.order_of_key(1) <<
        endl; //31
}
//__int128_t, __float128_t
```

2 Graph

2.1 BCC Vertex*

```
vector<int> G[N]; // 1-base
vector<int> nG[N], bcc[N];
int low[N], dfn[N], Time;
int bcc_id[N], bcc_cnt; // 1-base
bool is_cut[N]; // whether is av
bool cir[N];
int st[N], top;

void dfs(int u, int pa = -1) {
    int child = 0;
    low[u] = dfn[u] = ++Time;
    st[top++] = u;
    for (int v : G[u])
        if (!dfn[v]) {
            dfs(v, u), ++child;
            low[u] = min(low[u], low[v]);
            if (dfn[u] <= low[v]) {
                is_cut[u] = 1;
                bcc[++bcc_cnt].clear();
                int t;
                do {
                    bcc_id[t = st[--top]] = bcc_cnt;
                    bcc[bcc_cnt].push_back(t);
                } while (t != v);
                bcc_id[u] = bcc_cnt;
                bcc[bcc_cnt].pb(u);
            }
        } else if (dfn[v] < dfn[u] && v != pa)
            low[u] = min(low[u], dfn[v]);
    if (pa == -1 && child < 2) is_cut[u] = 0;
}

void bcc_init(int n) {
    Time = bcc_cnt = top = 0;
    for (int i = 1; i <= n; ++i)
        G[i].clear(), dfn[i] = bcc_id[i] = is_cut[i] = 0;
}
```

```
void bcc_solve(int n) {
    for (int i = 1; i <= n; ++i)
        if (!dfn[i]) dfs(i);
    // circle-square tree
    for (int i = 1; i <= n; ++i)
        if (is_cut[i])
            bcc_id[i] = ++bcc_cnt, cir[bcc_cnt] = 1;
    for (int i = 1; i <= bcc_cnt && !cir[i]; ++i)
        for (int j : bcc[i])
            if (is_cut[j])
                nG[i].pb(bcc_id[j]), nG[bcc_id[j]].pb(i);
}
```

2.2 Bridge*

```
int low[N], dfn[N], Time; // 1-base
vector<pii> G[N], edge;
vector<bool> is_bridge;

void init(int n) {
    Time = 0;
    for (int i = 1; i <= n; ++i)
        G[i].clear(), low[i] = dfn[i] = 0;
}

void add_edge(int a, int b) {
    G[a].pb(pii(b, SZ(edge))), G[b].pb(pii(a, SZ(edge)));
    edge.pb(pii(a, b));
}

void dfs(int u, int f) {
    dfn[u] = low[u] = ++Time;
    for (auto i : G[u])
        if (!dfn[i.X])
            dfs(i.X, i.Y), low[u] = min(low[u], low[i.X]);
        else if (i.Y != f) low[u] = min(low[u], dfn[i.X]);
    if (low[u] == dfn[u] && f != -1) is_bridge[f] = 1;
}

void solve(int n) {
    is_bridge.resize(SZ(edge));
    for (int i = 1; i <= n; ++i)
        if (!dfn[i]) dfs(i, -1);
}
```

2.3 2SAT (SCC)*

```
struct SAT { // 0-base
    int low[N], dfn[N], bln[N], n, Time, nScc;
    bool instack[N], istrue[N];
    stack<int> st;
    vector<int> G[N], SCC[N];
    void init(int _n) {
        n = _n; // assert(n * 2 <= N);
        for (int i = 0; i < n + n; ++i) G[i].clear();
    }
    void add_edge(int a, int b) { G[a].pb(b); }
    int rv(int a) {
        if (a > n) return a - n;
        return a + n;
    }
    void add_clause(int a, int b) {
        add_edge(rv(a), b), add_edge(rv(b), a);
    }
    void dfs(int u) {
        dfn[u] = low[u] = ++Time;
        instack[u] = 1, st.push(u);
        for (int i : G[u])
            if (!dfn[i])
                dfs(i), low[u] = min(low[i], low[u]);
            else if (instack[i] && dfn[i] < dfn[u])
                low[u] = min(low[u], dfn[i]);
        if (low[u] == dfn[u]) {
            int tmp;
            do {
                tmp = st.top(), st.pop();
                instack[tmp] = 0, bln[tmp] = nScc;
            } while (tmp != u);
        }
    }
}
```

```

    ++nScc;
}
}
bool solve() {
    Time = nScc = 0;
    for (int i = 0; i < n + n; ++i)
        SCC[i].clear(), low[i] = dfn[i] = bln[i] = 0;
    for (int i = 0; i < n + n; ++i)
        if (!dfn[i]) dfs(i);
    for (int i = 0; i < n + n; ++i) SCC[bln[i]].pb(i);
    for (int i = 0; i < n; ++i) {
        if (bln[i] == bln[i + n]) return false;
        istrue[i] = bln[i] < bln[i + n];
        istrue[i + n] = !istrue[i];
    }
    return true;
}
};

```

2.4 MinimumMeanCycle*

```

ll road[N][N]; // input here
struct MinimumMeanCycle {
    ll dp[N + 5][N], n;
    pll solve() {
        ll a = -1, b = -1, L = n + 1;
        for (int i = 2; i <= L; ++i)
            for (int k = 0; k < n; ++k)
                for (int j = 0; j < n; ++j)
                    dp[i][j] =
                        min(dp[i - 1][k] + road[k][j], dp[i][j]);
        for (int i = 0; i < n; ++i) {
            if (dp[L][i] >= INF) continue;
            ll ta = 0, tb = 1;
            for (int j = 1; j < n; ++j)
                if (dp[j][i] < INF &&
                    ta * (L - j) < (dp[L][i] - dp[j][i]) * tb)
                    ta = dp[L][i] - dp[j][i], tb = L - j;
            if (ta == 0) continue;
            if (a == -1 || a * tb > ta * b) a = ta, b = tb;
        }
        if (a != -1) {
            ll g = __gcd(a, b);
            return pll(a / g, b / g);
        }
        return pll(-1LL, -1LL);
    }
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i)
            for (int j = 0; j < n; ++j) dp[i + 2][j] = INF;
    }
};

```

2.5 Maximum Clique Dyn*

```

const int N = 150;
struct MaxClique { // Maximum Clique
    bitset<N> a[N], cs[N];
    int ans, sol[N], q, cur[N], d[N], n;
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; i++) a[i].reset();
    }
    void addEdge(int u, int v) { a[u][v] = a[v][u] = 1; }
    void csort(vector<int> &r, vector<int> &c) {
        int mx = 1, km = max(ans - q + 1, 1), t = 0,
            m = r.size();
        cs[1].reset(), cs[2].reset();
        for (int i = 0; i < m; i++) {
            int p = r[i], k = 1;
            while ((cs[k] & a[p]).count()) k++;
            if (k > mx) mx++, cs[mx + 1].reset();
            cs[k][p] = 1;
            if (k < km) r[t++] = p;
        }
        c.resize(m);
        if (t) c[t - 1] = 0;
    }
};

```

```

    for (int k = km; k <= mx; k++)
        for (int p = cs[k]._Find_first(); p < N;
            p = cs[k]._Find_next(p))
            r[t] = p, c[t] = k, t++;
}
void dfs(vector<int> &r, vector<int> &c, int l,
    bitset<N> mask) {
    while (!r.empty()) {
        int p = r.back();
        r.pop_back(), mask[p] = 0;
        if (q + c.back() <= ans) return;
        cur[q++] = p;
        vector<int> nr, nc;
        bitset<N> nmask = mask & a[p];
        for (int i : r)
            if (a[p][i]) nr.push_back(i);
        if (!nr.empty()) {
            if (l < 4) {
                for (int i : nr)
                    d[i] = (a[i] & nmask).count();
                sort(nr.begin(), nr.end(),
                    [&](int x, int y) { return d[x] > d[y]; });
            }
            csort(nr, nc), dfs(nr, nc, l + 1, nmask);
        } else if (q > ans) ans = q, copy_n(cur, q, sol);
        c.pop_back(), q--;
    }
}
int solve(bitset<N> mask = bitset<N>(
    string(N, '1')) { // vertex mask
    vector<int> r, c;
    ans = q = 0;
    for (int i = 0; i < n; i++)
        if (mask[i]) r.push_back(i);
    for (int i = 0; i < n; i++)
        d[i] = (a[i] & mask).count();
    sort(r.begin(), r.end(),
        [&](int i, int j) { return d[i] > d[j]; });
    csort(r, c), dfs(r, c, 1, mask);
    return ans; // sol[0 ~ ans-1]
}
} graph;

```

2.6 Maximum Clique*

```

struct Maximum_Clique {
    typedef bitset<MAXN> bst;
    bst N[MAXN], empty;
    int p[MAXN], n, ans;
    void BronKerbosch2(bst R, bst P, bst X) {
        if (P == empty && X == empty)
            return ans = max(ans, (int)R.count()), void();
        bst tmp = P | X;
        int u;
        if ((R | P | X).count() <= ans) return;
        for (int uu = 0; uu < n; ++uu) {
            u = p[uu];
            if (tmp[u] == 1) break;
        }
        // if (double(clock())/CLOCKS_PER_SEC > .999)
        // return;
        bst now2 = P & ~N[u];
        for (int vv = 0; vv < n; ++vv) {
            int v = p[vv];
            if (now2[v] == 1) {
                R[v] = 1;
                BronKerbosch2(R, P & N[v], X & N[v]);
                R[v] = 0, P[v] = 0, X[v] = 1;
            }
        }
    }
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i) N[i].reset();
    }
    void add_edge(int u, int v) {
        N[u][v] = N[v][u] = 1;
    }
    int solve() { // remember srand
        bst R, P, X;
    }
};

```

```

    ans = 0, P.flip();
    for (int i = 0; i < n; ++i) p[i] = i;
    random_shuffle(p, p + n), BronKerbosch2(R, P, X);
    return ans;
}
};

```

2.7 Minimum Steiner Tree*

```

// Minimum Steiner Tree
// O(V 3^T + V^2 2^T)
struct SteinerTree { // 0-base
    static const int T = 10, N = 105, INF = 1e9;
    int n, dst[N][N], dp[1 << T][N], tdst[N];
    int vcost[N]; // the cost of vertexs
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i) {
            for (int j = 0; j < n; ++j) dst[i][j] = INF;
            dst[i][i] = vcost[i] = 0;
        }
    }
    void add_edge(int ui, int vi, int wi) {
        dst[ui][vi] = min(dst[ui][vi], wi);
    }
    void shortest_path() {
        for (int k = 0; k < n; ++k)
            for (int i = 0; i < n; ++i)
                for (int j = 0; j < n; ++j)
                    dst[i][j] =
                        min(dst[i][j], dst[i][k] + dst[k][j]);
    }
    int solve(const vector<int> &ter) {
        shortest_path();
        int t = SZ(ter);
        for (int i = 0; i < (1 << t); ++i)
            for (int j = 0; j < n; ++j) dp[i][j] = INF;
        for (int i = 0; i < n; ++i) dp[0][i] = vcost[i];
        for (int msk = 1; msk < (1 << t); ++msk) {
            if (!(msk & (msk - 1))) {
                int who = __lg(msk);
                for (int i = 0; i < n; ++i)
                    dp[msk][i] =
                        vcost[ter[who]] + dst[ter[who]][i];
            }
            for (int i = 0; i < n; ++i)
                for (int submsk = (msk - 1) & msk; submsk;
                    submsk = (submsk - 1) & msk)
                    dp[msk][i] = min(dp[msk][i],
                        dp[submsk][i] + dp[msk ^ submsk][i] -
                        vcost[i]);
            for (int i = 0; i < n; ++i) {
                tdst[i] = INF;
                for (int j = 0; j < n; ++j)
                    tdst[i] =
                        min(tdst[i], dp[msk][j] + dst[j][i]);
            }
            for (int i = 0; i < n; ++i) dp[msk][i] = tdst[i];
        }
        int ans = INF;
        for (int i = 0; i < n; ++i)
            ans = min(ans, dp[(1 << t) - 1][i]);
        return ans;
    }
};

```

2.8 Minimum Arborescence*

```

struct zhu_liu { // O(VE)
    struct edge {
        int u, v;
        ll w;
    };
    vector<edge> E; // 0-base
    int pe[N], id[N], vis[N];
    ll in[N];
    void init() { E.clear(); }
    void add_edge(int u, int v, ll w) {

```

```

        if (u != v) E.pb(edge{u, v, w});
    }
    ll build(int root, int n) {
        ll ans = 0;
        for (;;) {
            fill_n(in, n, INF);
            for (int i = 0; i < SZ(E); ++i)
                if (E[i].u != E[i].v && E[i].w < in[E[i].v])
                    pe[E[i].v] = i, in[E[i].v] = E[i].w;
            for (int u = 0; u < n; ++u) // no solution
                if (u != root && in[u] == INF) return -INF;
            int cntnode = 0;
            fill_n(id, n, -1), fill_n(vis, n, -1);
            for (int u = 0; u < n; ++u) {
                if (u != root) ans += in[u];
                int v = u;
                while (vis[v] != u && !~id[v] && v != root)
                    vis[v] = u, v = E[pe[v]].u;
                if (v != root && !~id[v]) {
                    for (int x = E[pe[v]].u; x != v;
                        x = E[pe[x]].u)
                        id[x] = cntnode;
                    id[v] = cntnode++;
                }
            }
            if (!cntnode) break; // no cycle
            for (int u = 0; u < n; ++u)
                if (!~id[u]) id[u] = cntnode++;
            for (int i = 0; i < SZ(E); ++i) {
                int v = E[i].v;
                E[i].u = id[E[i].u], E[i].v = id[E[i].v];
                if (E[i].u != E[i].v) E[i].w -= in[v];
            }
            n = cntnode, root = id[root];
        }
        return ans;
    }
};

```

2.9 Vizing's theorem

```

namespace vizing { // returns edge coloring in adjacent
// matrix G. 1 - based
int C[kN][kN], G[kN][kN];
void clear(int N) {
    for (int i = 0; i <= N; i++) {
        for (int j = 0; j <= N; j++) C[i][j] = G[i][j] = 0;
    }
}
void solve(vector<pair<int, int>> &E, int N, int M) {
    int X[kN] = {}, a;
    auto update = [&](int u) {
        for (X[u] = 1; C[u][X[u]]; X[u]++)
            ;
    };
    auto color = [&](int u, int v, int c) {
        int p = G[u][v];
        G[u][v] = G[v][u] = c;
        C[u][c] = v, C[v][c] = u;
        C[u][p] = C[v][p] = 0;
        if (p) X[u] = X[v] = p;
        else update(u), update(v);
        return p;
    };
    auto flip = [&](int u, int c1, int c2) {
        int p = C[u][c1];
        swap(C[u][c1], C[u][c2]);
        if (p) G[u][p] = G[p][u] = c2;
        if (!C[u][c1]) X[u] = c1;
        if (!C[u][c2]) X[u] = c2;
        return p;
    };
    for (int i = 1; i <= N; i++) X[i] = 1;
    for (int t = 0; t < E.size(); t++) {
        int u = E[t].first, v0 = E[t].second, v = v0,
            c0 = X[u], c = c0, d;
        vector<pair<int, int>> L;
        int vst[kN] = {};
        while (!G[u][v0]) {
            L.emplace_back(v, d = X[v]);

```

```

    if (!C[v][c])
        for (a = (int)L.size() - 1; a >= 0; a--)
            c = color(u, L[a].first, c);
    else if (!C[u][d])
        for (a = (int)L.size() - 1; a >= 0; a--)
            color(u, L[a].first, L[a].second);
    else if (vst[d]) break;
    else vst[d] = 1, v = C[u][d];
}
if (!G[u][v0]) {
    for (; v; v = flip(v, c, d), swap(c, d))
        ;
    if (C[u][c0]) {
        for (a = (int)L.size() - 2;
            a >= 0 && L[a].second != c; a--)
            ;
        for (; a >= 0; a--)
            color(u, L[a].first, L[a].second);
        else t--;
    }
}
}
} // namespace vizing

```

2.10 Minimum Clique Cover*

```

struct Clique_Cover { // 0-base, O(n2^n)
    int co[1 << N], n, E[N];
    int dp[1 << N];
    void init(int _n) {
        n = _n, fill_n(dp, 1 << n, 0);
        fill_n(E, n, 0), fill_n(co, 1 << n, 0);
    }
    void add_edge(int u, int v) {
        E[u] |= 1 << v, E[v] |= 1 << u;
    }
    int solve() {
        for (int i = 0; i < n; ++i)
            co[1 << i] = E[i] | (1 << i);
        co[0] = (1 << n) - 1;
        dp[0] = (n & 1) * 2 - 1;
        for (int i = 1; i < (1 << n); ++i) {
            int t = i & -i;
            dp[i] = -dp[i ^ t];
            co[i] = co[i ^ t] & co[t];
        }
        for (int i = 0; i < (1 << n); ++i)
            co[i] = (co[i] & i) == i;
        fwt(co, 1 << n);
        for (int ans = 1; ans < n; ++ans) {
            int sum = 0;
            for (int i = 0; i < (1 << n); ++i)
                sum += (dp[i] * co[i]);
            if (sum) return ans;
        }
        return n;
    }
};

```

2.11 NumberofMaximalClique*

```

struct BronKerbosch { // 1-base
    int n, a[N], g[N][N];
    int S, all[N][N], some[N][N], none[N][N];
    void init(int _n) {
        n = _n;
        for (int i = 1; i <= n; ++i)
            for (int j = 1; j <= n; ++j) g[i][j] = 0;
    }
    void add_edge(int u, int v) {
        g[u][v] = g[v][u] = 1;
    }
    void dfs(int d, int an, int sn, int nn) {
        if (S > 1000) return; // pruning
        if (sn == 0 && nn == 0) ++S;
        int u = some[d][0];
        for (int i = 0; i < sn; ++i) {
            int v = some[d][i];

```

```

            if (g[u][v]) continue;
            int tsu = 0, tnn = 0;
            copy_n(all[d], an, all[d + 1]);
            all[d + 1][an] = v;
            for (int j = 0; j < sn; ++j)
                if (g[v][some[d][j]])
                    some[d + 1][tsu++] = some[d][j];
            for (int j = 0; j < nn; ++j)
                if (g[v][none[d][j]])
                    none[d + 1][tnn++] = none[d][j];
            dfs(d + 1, an + 1, tsu, tnn);
            some[d][i] = 0, none[d][nn++] = v;
        }
    }
    int solve() {
        iota(some[0], some[0] + n, 1);
        S = 0, dfs(0, 0, n, 0);
        return S;
    }
};

```

2.12 Theory

$$|\text{Maximum independent edge set}| = |V| - |\text{Minimum edge cover}|$$

$$|\text{Maximum independent set}| = |V| - |\text{Minimum vertex cover}|$$

3 Data Structure

3.1 Leftist Tree

```

struct node {
    ll v, data, sz, sum;
    node *l, *r;
    node(ll k)
        : v(0), data(k), sz(1), l(0), r(0), sum(k) {}
};
ll sz(node *p) { return p ? p->sz : 0; }
ll V(node *p) { return p ? p->v : -1; }
ll sum(node *p) { return p ? p->sum : 0; }
node *merge(node *a, node *b) {
    if (!a || !b) return a ? a : b;
    if (a->data < b->data) swap(a, b);
    a->r = merge(a->r, b);
    if (V(a->r) > V(a->l)) swap(a->r, a->l);
    a->v = V(a->r) + 1, a->sz = sz(a->l) + sz(a->r) + 1;
    a->sum = sum(a->l) + sum(a->r) + a->data;
    return a;
}
void pop(node *&o) {
    node *tmp = o;
    o = merge(o->l, o->r);
    delete tmp;
}

```

3.2 Heavy light Decomposition

```

struct Heavy_light_Decomposition { // 1-base
    int n, ulink[10005], deep[10005], mxson[10005],
        w[10005], pa[10005];
    int t, pl[10005], data[10005], dt[10005], bln[10005],
        edge[10005], et;
    vector<pii> G[10005];
    void init(int _n) {
        n = _n, t = 0, et = 1;
        for (int i = 1; i <= n; ++i)
            G[i].clear(), mxson[i] = 0;
    }
    void add_edge(int a, int b, int w) {
        G[a].pb(pii(b, et)), G[b].pb(pii(a, et)),
            edge[et++] = w;
    }
    void dfs(int u, int f, int d) {
        w[u] = 1, pa[u] = f, deep[u] = d++;
        for (auto &i : G[u])
            if (i.X != f) {

```

```

        dfs(i.X, u, d), w[u] += w[i.X];
        if (w[mxson[u]] < w[i.X]) mxson[u] = i.X;
    } else bln[i.Y] = u, dt[u] = edge[i.Y];
}
void cut(int u, int link) {
    data[pl[u] = t++] = dt[u], ulink[u] = link;
    if (!mxson[u]) return;
    cut(mxson[u], link);
    for (auto i : G[u])
        if (i.X != pa[u] && i.X != mxson[u])
            cut(i.X, i.X);
}
void build() { dfs(1, 1, 1), cut(1, 1), /*build*/; }
int query(int a, int b) {
    int ta = ulink[a], tb = ulink[b], re = 0;
    while (ta != tb)
        if (deep[ta] < deep[tb])
            /*query*/, tb = ulink[b = pa[tb]];
        else /*query*/, ta = ulink[a = pa[ta]];
    if (a == b) return re;
    if (pl[a] > pl[b]) swap(a, b);
    /*query*/
    return re;
}
};

```

3.3 Centroid Decomposition*

```

struct Cent_Dec { // 1-base
    vector<pll> G[N];
    pll info[N]; // store info. of itself
    pll upinfo[N]; // store info. of climbing up
    int n, pa[N], layer[N], sz[N], done[N];
    ll dis[lg(N) + 1][N];
    void init(int _n) {
        n = _n, layer[0] = -1;
        fill_n(pa + 1, n, 0), fill_n(done + 1, n, 0);
        for (int i = 1; i <= n; ++i) G[i].clear();
    }
    void add_edge(int a, int b, int w) {
        G[a].pb(pll(b, w)), G[b].pb(pll(a, w));
    }
    void get_cent(
        int u, int f, int &mx, int &c, int num) {
        int mxsz = 0;
        sz[u] = 1;
        for (pll e : G[u])
            if (!done[e.X] && e.X != f) {
                get_cent(e.X, u, mx, c, num);
                sz[u] += sz[e.X], mxsz = max(mxsz, sz[e.X]);
            }
        if (mx > max(mxsz, num - sz[u]))
            mx = max(mxsz, num - sz[u]), c = u;
    }
    void dfs(int u, int f, ll d, int org) {
        // if required, add self info or climbing info
        dis[layer[org]][u] = d;
        for (pll e : G[u])
            if (!done[e.X] && e.X != f)
                dfs(e.X, u, d + e.Y, org);
    }
    int cut(int u, int f, int num) {
        int mx = 1e9, c = 0, lc;
        get_cent(u, f, mx, c, num);
        done[c] = 1, pa[c] = f, layer[c] = layer[f] + 1;
        for (pll e : G[c])
            if (!done[e.X]) {
                if (sz[e.X] > sz[c])
                    lc = cut(e.X, c, num - sz[c]);
                else lc = cut(e.X, c, sz[e.X]);
                upinfo[lc] = pll(), dfs(e.X, c, e.Y, c);
            }
        return done[c] = 0, c;
    }
    void build() { cut(1, 0, n); }
    void modify(int u) {
        for (int a = u, ly = layer[a]; a;
            a = pa[a], --ly) {
            info[a].X += dis[ly][u], ++info[a].Y;
            if (pa[a])

```

```

                upinfo[a].X += dis[ly - 1][u], ++upinfo[a].Y;
            }
        }
        ll query(int u) {
            ll rt = 0;
            for (int a = u, ly = layer[a]; a;
                a = pa[a], --ly) {
                rt += info[a].X + info[a].Y * dis[ly][u];
                if (pa[a])
                    rt -=
                        upinfo[a].X + upinfo[a].Y * dis[ly - 1][u];
            }
            return rt;
        }
    };
};

```

3.4 Link cut tree*

```

struct Splay { // xor-sum
    static Splay nil;
    Splay *ch[2], *f;
    int val, sum, rev, size;
    Splay(int _val = 0)
        : val(_val), sum(_val), rev(0), size(1) {
        f = ch[0] = ch[1] = &nil;
    }
    bool isr() {
        return f->ch[0] != this && f->ch[1] != this;
    }
    int dir() { return f->ch[0] == this ? 0 : 1; }
    void setCh(Splay *c, int d) {
        ch[d] = c;
        if (c != &nil) c->f = this;
        pull();
    }
    void push() {
        if (!rev) return;
        swap(ch[0], ch[1]);
        if (ch[0] != &nil) ch[0]->rev ^= 1;
        if (ch[1] != &nil) ch[1]->rev ^= 1;
        rev = 0;
    }
    void pull() {
        // take care of the nil!
        size = ch[0]->size + ch[1]->size + 1;
        sum = ch[0]->sum ^ ch[1]->sum ^ val;
        if (ch[0] != &nil) ch[0]->f = this;
        if (ch[1] != &nil) ch[1]->f = this;
    }
} Splay::nil;
Splay *nil = &Splay::nil;
void rotate(Splay *x) {
    Splay *p = x->f;
    int d = x->dir();
    if (!p->isr()) p->f->setCh(x, p->dir());
    else x->f = p->f;
    p->setCh(x->ch[!d], d);
    x->setCh(p, !d);
    p->pull(), x->pull();
}
void splay(Splay *x) {
    vector<Splay*> splayVec;
    for (Splay *q = x; q; q = q->f) {
        splayVec.pb(q);
        if (q->isr()) break;
    }
    reverse(ALL(splayVec));
    for (auto it : splayVec) it->push();
    while (!x->isr()) {
        if (x->f->isr()) rotate(x);
        else if (x->dir() == x->f->dir())
            rotate(x->f), rotate(x);
        else rotate(x), rotate(x);
    }
}
Splay *access(Splay *x) {
    Splay *q = nil;
    for (; x != nil; x = x->f)
        splay(x), x->setCh(q, 1), q = x;
    return q;
}

```



```

}
void root_path(Splay *x) { access(x), splay(x); }
void chroot(Splay *x) {
    root_path(x), x->rev ^= 1;
    x->push(), x->pull();
}
void split(Splay *x, Splay *y) {
    chroot(x), root_path(y);
}
void link(Splay *x, Splay *y) {
    root_path(x), chroot(y);
    x->setCh(y, 1);
}
void cut(Splay *x, Splay *y) {
    split(x, y);
    if (y->size != 5) return;
    y->push();
    y->ch[0] = y->ch[0]->f = nil;
}
Splay *get_root(Splay *x) {
    for (root_path(x); x->ch[0] != nil; x = x->ch[0])
        x->push();
    splay(x);
    return x;
}
bool conn(Splay *x, Splay *y) {
    return get_root(x) == get_root(y);
}
Splay *lca(Splay *x, Splay *y) {
    access(x), root_path(y);
    if (y->f == nil) return y;
    return y->f;
}
void change(Splay *x, int val) {
    splay(x), x->val = val, x->pull();
}
int query(Splay *x, Splay *y) {
    split(x, y);
    return y->sum;
}
}

```

3.5 KDTree

```

namespace kdt {
int root, lc[maxn], rc[maxn], xl[maxn], xr[maxn],
    yl[maxn], yr[maxn];
point p[maxn];
int build(int l, int r, int dep = 0) {
    if (l == r) return -1;
    function<bool(const point &, const point &)> f =
        [dep](const point &a, const point &b) {
            if (dep & 1) return a.x < b.x;
            else return a.y < b.y;
        };
    int m = (l + r) >> 1;
    nth_element(p + l, p + m, p + r, f);
    xl[m] = xr[m] = p[m].x;
    yl[m] = yr[m] = p[m].y;
    lc[m] = build(l, m, dep + 1);
    if (~lc[m]) {
        xl[m] = min(xl[m], xl[lc[m]]);
        xr[m] = max(xr[m], xr[lc[m]]);
        yl[m] = min(yl[m], yl[lc[m]]);
        yr[m] = max(yr[m], yr[lc[m]]);
    }
    rc[m] = build(m + 1, r, dep + 1);
    if (~rc[m]) {
        xl[m] = min(xl[m], xl[rc[m]]);
        xr[m] = max(xr[m], xr[rc[m]]);
        yl[m] = min(yl[m], yl[rc[m]]);
        yr[m] = max(yr[m], yr[rc[m]]);
    }
    return m;
}
bool bound(const point &q, int o, long long d) {
    double ds = sqrt(d + 1.0);
    if (q.x < xl[o] - ds || q.x > xr[o] + ds ||
        q.y < yl[o] - ds || q.y > yr[o] + ds)
        return false;
    return true;
}
}

```

```

}
long long dist(const point &a, const point &b) {
    return (a.x - b.x) * 1ll * (a.x - b.x) +
        (a.y - b.y) * 1ll * (a.y - b.y);
}
void dfs(
    const point &q, long long &d, int o, int dep = 0) {
    if (!bound(q, o, d)) return;
    long long cd = dist(p[o], q);
    if (cd != 0) d = min(d, cd);
    if ((dep & 1) && q.x < p[o].x ||
        !(dep & 1) && q.y < p[o].y) {
        if (~lc[o]) dfs(q, d, lc[o], dep + 1);
        if (~rc[o]) dfs(q, d, rc[o], dep + 1);
    } else {
        if (~rc[o]) dfs(q, d, rc[o], dep + 1);
        if (~lc[o]) dfs(q, d, lc[o], dep + 1);
    }
}
void init(const vector<point> &v) {
    for (int i = 0; i < v.size(); ++i) p[i] = v[i];
    root = build(0, v.size());
}
long long nearest(const point &q) {
    long long res = 1e18;
    dfs(q, res, root);
    return res;
}
} // namespace kdt

```

4 Flow/Matching

4.1 Kuhn Munkres

```

struct KM { // 0-base
int w[MAXN][MAXN], hl[MAXN], hr[MAXN], slk[MAXN], n;
int fl[MAXN], fr[MAXN], pre[MAXN], qu[MAXN], ql, qr;
bool vl[MAXN], vr[MAXN];
void init(int _n) {
    n = _n;
    for (int i = 0; i < n; ++i)
        for (int j = 0; j < n; ++j) w[i][j] = -INF;
}
void add_edge(int a, int b, int wei) {
    w[a][b] = wei;
}
bool Check(int x) {
    if (vl[x] = 1, ~fl[x])
        return vr[qu[qr++]] = fl[x] = 1;
    while (~x) swap(x, fr[fl[x] = pre[x]]);
    return 0;
}
void Bfs(int s) {
    fill(slk, slk + n, INF);
    fill(vl, vl + n, 0), fill(vr, vr + n, 0);
    ql = qr = 0, qu[qr++] = s, vr[s] = 1;
    while (1) {
        int d;
        while (ql < qr)
            for (int x = 0; x < n; ++x)
                if (!vl[x] &&
                    slk[x] >= (d = hl[x] + hr[qu[ql++]] - w[x][qu[ql]]))
                    if (pre[x] = qu[ql], slk[x] = d;
                        else if (!Check(x)) return;
                d = INF;
            for (int x = 0; x < n; ++x)
                if (!vl[x] && d > slk[x]) d = slk[x];
            for (int x = 0; x < n; ++x) {
                if (vl[x]) hl[x] += d;
                else slk[x] -= d;
                if (vr[x]) hr[x] -= d;
            }
            for (int x = 0; x < n; ++x)
                if (!vl[x] && !slk[x] && !Check(x)) return;
    }
}
int Solve() {
    fill(fl, fl + n, -1), fill(fr, fr + n, -1),
}

```

```

    fill(hr, hr + n, 0);
    for (int i = 0; i < n; ++i)
        hl[i] = *max_element(w[i], w[i] + n);
    for (int i = 0; i < n; ++i) Bfs(i);
    int res = 0;
    for (int i = 0; i < n; ++i) res += w[i][fl[i]];
    return res;
}
};

```

4.2 MincostMaxflow

```

struct MCMF { // 0-base
    struct edge {
        ll from, to, cap, flow, cost, rev;
    } * past[MAXN];
    vector<edge> G[MAXN];
    bitset<MAXN> inq;
    ll dis[MAXN], up[MAXN], s, t, mx, n;
    bool BellmanFord(ll &flow, ll &cost) {
        fill(dis, dis + n, INF);
        queue<ll> q;
        q.push(s), inq.reset(), inq[s] = 1;
        up[s] = mx - flow, past[s] = 0, dis[s] = 0;
        while (!q.empty()) {
            ll u = q.front();
            q.pop(), inq[u] = 0;
            if (!up[u]) continue;
            for (auto &e : G[u])
                if (e.flow != e.cap &&
                    dis[e.to] > dis[u] + e.cost) {
                    dis[e.to] = dis[u] + e.cost, past[e.to] = &e;
                    up[e.to] = min(up[u], e.cap - e.flow);
                    if (!inq[e.to]) inq[e.to] = 1, q.push(e.to);
                }
        }
        if (dis[t] == INF) return 0;
        flow += up[t], cost += up[t] * dis[t];
        for (ll i = t; past[i]; i = past[i]->from) {
            auto &e = *past[i];
            e.flow += up[t], G[e.to][e.rev].flow -= up[t];
        }
        return 1;
    }
    ll MinCostMaxFlow(ll _s, ll _t, ll &cost) {
        s = _s, t = _t, cost = 0;
        ll flow = 0;
        while (BellmanFord(flow, cost))
            ;
        return flow;
    }
    void init(ll _n, ll _mx) {
        n = _n, mx = _mx;
        for (int i = 0; i < n; ++i) G[i].clear();
    }
    void add_edge(ll a, ll b, ll cap, ll cost) {
        G[a].pb(edge{a, b, cap, 0, cost, G[b].size()});
        G[b].pb(edge{b, a, 0, 0, -cost, G[a].size() - 1});
    }
};

```

4.3 Minimum Weight Matching (Clique version)*

```

struct Graph { // 0-base (Perfect Match), n is even
    int n, match[N], onstk[N], stk[N], tp;
    ll edge[N][N], dis[N];
    void init(int _n) {
        n = _n, tp = 0;
        for (int i = 0; i < n; ++i) fill_n(edge[i], n, 0);
    }
    void add_edge(int u, int v, ll w) {
        edge[u][v] = edge[v][u] = w;
    }
    bool SPFA(int u) {
        stk[tp++] = u, onstk[u] = 1;
        for (int v = 0; v < n; ++v)

```

```

        if (!onstk[v] && match[u] != v) {
            int m = match[v];
            if (dis[m] >
                dis[u] - edge[v][m] + edge[u][v]) {
                dis[m] = dis[u] - edge[v][m] + edge[u][v];
                onstk[v] = 1, stk[tp++] = v;
                if (onstk[m] || SPFA(m)) return 1;
                --tp, onstk[v] = 0;
            }
        }
        onstk[u] = 0, --tp;
        return 0;
    }
    ll solve() { // find a match
        for (int i = 0; i < n; ++i) match[i] = i ^ 1;
        while (1) {
            int found = 0;
            fill_n(dis, n, 0);
            fill_n(onstk, n, 0);
            for (int i = 0; i < n; ++i)
                if (tp = 0, !onstk[i] && SPFA(i))
                    for (found = 1; tp >= 2; ) {
                        int u = stk[--tp];
                        int v = stk[--tp];
                        match[u] = v, match[v] = u;
                    }
            if (!found) break;
        }
        ll ret = 0;
        for (int i = 0; i < n; ++i)
            ret += edge[i][match[i]];
        return ret >> 1;
    }
};

```

4.4 SW-mincut

```

// global min cut
struct SW { // O(V^3)
    static const int MXN = 514;
    int n, vst[MXN], del[MXN];
    int edge[MXN][MXN], wei[MXN];
    void init(int _n) {
        n = _n, MEM(edge, 0), MEM(del, 0);
    }
    void addEdge(int u, int v, int w) {
        edge[u][v] += w, edge[v][u] += w;
    }
    void search(int &s, int &t) {
        MEM(vst, 0), MEM(wei, 0), s = t = -1;
        while (1) {
            int mx = -1, cur = 0;
            for (int i = 0; i < n; ++i)
                if (!del[i] && !vst[i] && mx < wei[i])
                    cur = i, mx = wei[i];
            if (mx == -1) break;
            vst[cur] = 1, s = t, t = cur;
            for (int i = 0; i < n; ++i)
                if (!vst[i] && !del[i]) wei[i] += edge[cur][i];
        }
    }
    int solve() {
        int res = INF;
        for (int i = 0, x, y; i < n - 1; ++i) {
            search(x, y), res = min(res, wei[y]), del[y] = 1;
            for (int j = 0; j < n; ++j)
                edge[x][j] = (edge[j][x] += edge[y][j]);
        }
        return res;
    }
};

```

4.5 BoundedFlow(Dinic*)

```

struct BoundedFlow { // 0-base
    struct edge {
        int to, cap, flow, rev;
    };

```



```

vector<edge> G[N];
int n, s, t, dis[N], cur[N], cnt[N];
void init(int _n) {
    n = _n;
    for (int i = 0; i < n + 2; ++i)
        G[i].clear(), cnt[i] = 0;
}
void add_edge(int u, int v, int lcap, int rcap) {
    cnt[u] -= lcap, cnt[v] += lcap;
    G[u].pb(edge{v, rcap, lcap, SZ(G[v])});
    G[v].pb(edge{u, 0, 0, SZ(G[u]) - 1});
}
void add_edge(int u, int v, int cap) {
    G[u].pb(edge{v, cap, 0, SZ(G[v])});
    G[v].pb(edge{u, 0, 0, SZ(G[u]) - 1});
}
int dfs(int u, int cap) {
    if (u == t || !cap) return cap;
    for (int &i = cur[u]; i < SZ(G[u]); ++i) {
        edge &e = G[u][i];
        if (dis[e.to] == dis[u] + 1 && e.cap != e.flow) {
            int df = dfs(e.to, min(e.cap - e.flow, cap));
            if (df) {
                e.flow += df, G[e.to][e.rev].flow -= df;
                return df;
            }
        }
    }
    dis[u] = -1;
    return 0;
}
bool bfs() {
    fill_n(dis, n + 3, -1);
    queue<int> q;
    q.push(s), dis[s] = 0;
    while (!q.empty()) {
        int u = q.front();
        q.pop();
        for (edge &e : G[u])
            if (!dis[e.to] && e.flow != e.cap)
                q.push(e.to), dis[e.to] = dis[u] + 1;
    }
    return dis[t] != -1;
}
int maxflow(int _s, int _t) {
    s = _s, t = _t;
    int flow = 0, df;
    while (bfs()) {
        fill_n(cur, n + 3, 0);
        while ((df = dfs(s, INF))) flow += df;
    }
    return flow;
}
bool solve() {
    int sum = 0;
    for (int i = 0; i < n; ++i)
        if (cnt[i] > 0)
            add_edge(n + 1, i, cnt[i]), sum += cnt[i];
        else if (cnt[i] < 0) add_edge(i, n + 2, -cnt[i]);
    if (sum != maxflow(n + 1, n + 2)) sum = -1;
    for (int i = 0; i < n; ++i)
        if (cnt[i] > 0)
            G[n + 1].pop_back(), G[i].pop_back();
        else if (cnt[i] < 0)
            G[i].pop_back(), G[n + 2].pop_back();
    return sum != -1;
}
int solve(int _s, int _t) {
    add_edge(_t, _s, INF);
    if (!solve()) return -1; // invalid flow
    int x = G[_t].back().flow;
    return G[_t].pop_back(), G[_s].pop_back(), x;
}
};

```

4.6 Flow Models

- Maximum/Minimum flow with lower bound / Circulation problem
 - Construct super source S and sink T .
 - For each edge (x, y, l, u) , connect $x \rightarrow y$ with capacity $u - l$.

- For each vertex v , denote by $in(v)$ the difference between the sum of incoming lower bounds and the sum of outgoing lower bounds.
- If $in(v) > 0$, connect $S \rightarrow v$ with capacity $in(v)$, otherwise, connect $v \rightarrow T$ with capacity $-in(v)$.
 - To maximize, connect $t \rightarrow s$ with capacity ∞ (skip this in circulation problem), and let f be the maximum flow from S to T . If $f \neq \sum_{v \in V, in(v) > 0} in(v)$, there's no solution. Otherwise, the maximum flow from s to t is the answer.
 - To minimize, let f be the maximum flow from S to T . Connect $t \rightarrow s$ with capacity ∞ and let the flow from S to T be f' . If $f + f' \neq \sum_{v \in V, in(v) > 0} in(v)$, there's no solution. Otherwise, f' is the answer.
- The solution of each edge e is $l_e + f_e$, where f_e corresponds to the flow of edge e on the graph.

- Construct minimum vertex cover from maximum matching M on bipartite graph (X, Y)
 - Redirect every edge: $y \rightarrow x$ if $(x, y) \in M$, $x \rightarrow y$ otherwise.
 - DFS from unmatched vertices in X .
 - $x \in X$ is chosen iff x is unvisited.
 - $y \in Y$ is chosen iff y is visited.
- Minimum cost cyclic flow
 - Construct super source S and sink T
 - For each edge (x, y, c) , connect $x \rightarrow y$ with $(cost, cap) = (c, 1)$ if $c > 0$, otherwise connect $y \rightarrow x$ with $(cost, cap) = (-c, 1)$
 - For each edge with $c < 0$, sum these cost as K , then increase $d(y)$ by 1, decrease $d(x)$ by 1
 - For each vertex v with $d(v) > 0$, connect $S \rightarrow v$ with $(cost, cap) = (0, d(v))$
 - For each vertex v with $d(v) < 0$, connect $v \rightarrow T$ with $(cost, cap) = (0, -d(v))$
 - Flow from S to T , the answer is the cost of the flow $C + K$
- Maximum density induced subgraph
 - Binary search on answer, suppose we're checking answer T
 - Construct a max flow model, let K be the sum of all weights
 - Connect source $s \rightarrow v$, $v \in G$ with capacity K
 - For each edge (u, v, w) in G , connect $u \rightarrow v$ and $v \rightarrow u$ with capacity w
 - For $v \in G$, connect it with sink $v \rightarrow t$ with capacity $K + 2T - (\sum_{e \in E(v)} w(e)) - 2w(v)$
 - T is a valid answer if the maximum flow $f < K|V|$
- Minimum weight edge cover
 - For each $v \in V$ create a copy v' , and connect $u' \rightarrow v'$ with weight $w(u, v)$.
 - Connect $v \rightarrow v'$ with weight $2\mu(v)$, where $\mu(v)$ is the cost of the cheapest edge incident to v .
 - Find the minimum weight perfect matching on G' .
- Project selection problem
 - If $p_v > 0$, create edge (s, v) with capacity p_v ; otherwise, create edge (v, t) with capacity $-p_v$.
 - Create edge (u, v) with capacity w with w being the cost of choosing u without choosing v .
 - The mincut is equivalent to the maximum profit of a subset of projects.

$$\sum_x c_x x + \sum_y c_y \bar{y} + \sum_{xy} c_{xy} x \bar{y} + \sum_{xyx'y'} c_{xyx'y'} (x \bar{y} + x' \bar{y}')$$

can be minimized by the mincut of the following graph:

- Create edge (x, t) with capacity c_x and create edge (s, y) with capacity c_y .
- Create edge (x, y) with capacity c_{xy} .
- Create edge (x, y) and edge (x', y') with capacity $c_{xyx'y'}$.

5 String

5.1 KMP

```

int F[MAXN];
vector<int> match(string A, string B) {
    vector<int> ans;
    F[0] = -1, F[1] = 0;
    for (int i = 1, j = 0; i < SZ(B); F[++i] = ++j) {
        if (B[i] == B[j]) F[i] = F[j]; // optimize
        while (j != -1 && B[i] != B[j]) j = F[j];
    }
    for (int i = 0, j = 0; i < SZ(A); ++i) {
        while (j != -1 && A[i] != B[j]) j = F[j];
        if (++j == SZ(B)) ans.pb(i + 1 - j), j = F[j];
    }
    return ans;
}

```

5.2 Z-value

```
const int MAXn = 1e5 + 5;
int z[MAXn];
void make_z(string s) {
    int l = 0, r = 0;
    for (int i = 1; i < s.size(); i++) {
        for (z[i] = max(0, min(r - i + 1, z[i - 1]));
             i + z[i] < s.size() && s[i + z[i]] == s[z[i]]);
            z[i]++;
        ;
        if (i + z[i] - 1 > r) l = i, r = i + z[i] - 1;
    }
}
```

5.3 Manacher*

```
int z[MAXN];
int Manacher(string tmp) {
    string s = "&";
    int l = 0, r = 0, x, ans;
    for (char c : tmp) s.pb(c), s.pb('%');
    ans = 0, x = 0;
    for (int i = 1; i < SZ(s); ++i) {
        z[i] = r > i ? min(z[2 * l - i], r - i) : 1;
        while (s[i + z[i]] == s[i - z[i]]) ++z[i];
        if (z[i] + i > r) r = z[i] + i, l = i;
    }
    for (int i = 1; i < SZ(s); ++i)
        if (s[i] == '%') x = max(x, z[i]);
    ans = x / 2 * 2, x = 0;
    for (int i = 1; i < SZ(s); ++i)
        if (s[i] != '%') x = max(x, z[i]);
    return max(ans, (x - 1) / 2 * 2 + 1);
}
```

5.4 SAIS*

```
class SAIS {
public:
    int *SA, *H;
    // zero based, string content MUST > 0
    // result height H[i] is LCP(SA[i - 1], SA[i])
    // string, length, |sigma|
    void build(int *s, int n, int m = 128) {
        copy_n(s, n, _s);
        _h[0] = _s[n++] = 0;
        sais(_s, _sa, _p, _q, _t, _c, n, m);
        mkhei(n);
        SA = _sa + 1;
        H = _h + 1;
    }
private:
    bool _t[N * 2];
    int _s[N * 2], _c[N * 2], x[N], _p[N], _q[N * 2],
        r[N], _sa[N * 2], _h[N];
    void mkhei(int n) {
        for (int i = 0; i < n; i++) r[_sa[i]] = i;
        for (int i = 0; i < n; i++)
            if (r[i]) {
                int ans = i > 0 ? max(_h[r[i] - 1] - 1, 0) : 0;
                while (_s[i + ans] == _s[_sa[r[i] - 1] + ans])
                    ans++;
                _h[r[i]] = ans;
            }
    }
    void sais(int *s, int *sa, int *p, int *q, bool *t,
              int *c, int n, int z) {
        bool uniq = t[n - 1] = 1, neq;
        int nn = 0, nmzx = -1, *nsa = sa + n, *ns = s + n,
            lst = -1;
#define MAGIC(XD)
        fill_n(sa, n, 0);
        copy_n(c, z, x);
        XD;
        copy_n(c, z - 1, x + 1);
        for (int i = 0; i < n; i++)
            if (sa[i] && !t[sa[i] - 1])
                sa[x[s[sa[i] - 1]]++] = sa[i] - 1;
        copy_n(c, z, x);
        for (int i = n - 1; i >= 0; i--)
            if (sa[i] && t[sa[i] - 1])
                sa[--x[s[sa[i] - 1]]] = sa[i] - 1;

        fill_n(c, z, 0);
        for (int i = 0; i < n; i++) uniq &= ++c[s[i]] < 2;
        partial_sum(c, c + z, c);
        if (uniq) {
            for (int i = 0; i < n; i++) sa[--c[s[i]]] = i;
            return;
        }
        for (int i = n - 2; i >= 0; i--)
            t[i] = (s[i] == s[i + 1] ? t[i + 1]
                    : s[i] < s[i + 1]);
        MAGIC(for (int i = 1; i <= n - 1;
                  i++) if (t[i] && !t[i - 1])
                        sa[--x[s[i]]] = p[q[i] = nn++] = i);
        for (int i = 0; i < n; i++)
            if (sa[i] && t[sa[i]] && !t[sa[i] - 1]) {
                neq = (lst < 0) ||
                    !equal(s + lst,
                          s + lst + p[q[sa[i]] + 1] - sa[i],
                          s + sa[i]);
                ns[q[lst = sa[i]]] = nmzx += neq;
            }
        sais(ns, nsa, p + nn, q + n, t + n, c + z, nn,
            nmzx + 1);
        MAGIC(for (int i = nn - 1; i >= 0; i--)
              sa[--x[s[p[nsa[i]]]]] = p[nsa[i]]);
    }
} sa;
```

```
copy_n(c, z - 1, x + 1);
for (int i = 0; i < n; i++)
    if (sa[i] && !t[sa[i] - 1])
        sa[x[s[sa[i] - 1]]++] = sa[i] - 1;
copy_n(c, z, x);
for (int i = n - 1; i >= 0; i--)
    if (sa[i] && t[sa[i] - 1])
        sa[--x[s[sa[i] - 1]]] = sa[i] - 1;

fill_n(c, z, 0);
for (int i = 0; i < n; i++) uniq &= ++c[s[i]] < 2;
partial_sum(c, c + z, c);
if (uniq) {
    for (int i = 0; i < n; i++) sa[--c[s[i]]] = i;
    return;
}
for (int i = n - 2; i >= 0; i--)
    t[i] = (s[i] == s[i + 1] ? t[i + 1]
            : s[i] < s[i + 1]);
MAGIC(for (int i = 1; i <= n - 1;
          i++) if (t[i] && !t[i - 1])
                sa[--x[s[i]]] = p[q[i] = nn++] = i);
for (int i = 0; i < n; i++)
    if (sa[i] && t[sa[i]] && !t[sa[i] - 1]) {
        neq = (lst < 0) ||
            !equal(s + lst,
                  s + lst + p[q[sa[i]] + 1] - sa[i],
                  s + sa[i]);
        ns[q[lst = sa[i]]] = nmzx += neq;
    }
sais(ns, nsa, p + nn, q + n, t + n, c + z, nn,
    nmzx + 1);
MAGIC(for (int i = nn - 1; i >= 0; i--)
      sa[--x[s[p[nsa[i]]]]] = p[nsa[i]]);
}
} sa;
```

5.5 Aho-Corasick Automatan

```
const int len = 400000, sigma = 26;
struct AC_Automatan {
    int nx[len][sigma], fl[len], cnt[len], pri[len], top;
    int newnode() {
        fill(nx[top], nx[top] + sigma, -1);
        return top++;
    }
    void init() { top = 1, newnode(); }
    int input(
        string &s) { // return the end_node of string
        int X = 1;
        for (char c : s) {
            if (!nx[X][c - 'a']) nx[X][c - 'a'] = newnode();
            X = nx[X][c - 'a'];
        }
        return X;
    }
    void make_fl() {
        queue<int> q;
        q.push(1), fl[1] = 0;
        for (int t = 0; !q.empty(); ) {
            int R = q.front();
            q.pop(), pri[t++] = R;
            for (int i = 0; i < sigma; ++i)
                if (~nx[R][i]) {
                    int X = nx[R][i], Z = fl[R];
                    for (; Z && !nx[Z][i];) Z = fl[Z];
                    fl[X] = Z ? nx[Z][i] : 1, q.push(X);
                }
        }
    }
    void get_v(string &s) {
        int X = 1;
        fill(cnt, cnt + top, 0);
        for (char c : s) {
            while (X && !nx[X][c - 'a']) X = fl[X];
            X = X ? nx[X][c - 'a'] : 1, ++cnt[X];
        }
        for (int i = top - 2; i > 0; --i)
            cnt[fl[pri[i]]] += cnt[pri[i]];
    }
}
```

```
};
```

5.6 Smallest Rotation

```
string mcp(string s) {
    int n = SZ(s), i = 0, j = 1;
    s += s;
    while (i < n && j < n) {
        int k = 0;
        while (k < n && s[i + k] == s[j + k]) ++k;
        if (s[i + k] <= s[j + k]) j += k + 1;
        else i += k + 1;
        if (i == j) ++j;
    }
    int ans = i < n ? i : j;
    return s.substr(ans, n);
}
```

5.7 De Bruijn sequence*

```
constexpr int MAXC = 10, MAXN = 1e5 + 10;
struct DBSeq {
    int C, N, K, L, buf[MAXC * MAXN]; // K <= C^N
    void dfs(int *out, int t, int p, int &ptr) {
        if (ptr >= L) return;
        if (t > N) {
            if (N % p) return;
            for (int i = 1; i <= p && ptr < L; ++i)
                out[ptr++] = buf[i];
        } else {
            buf[t] = buf[t - p], dfs(out, t + 1, p, ptr);
            for (int j = buf[t - p] + 1; j < C; ++j)
                buf[t] = j, dfs(out, t + 1, t, ptr);
        }
    }
    void solve(int _c, int _n, int _k, int *out) {
        int p = 0;
        C = _c, N = _n, K = _k, L = N + K - 1;
        dfs(out, 1, 1, p);
        if (p < L) fill(out + p, out + L, 0);
    }
} dbs;
```

5.8 SAM

```
const int MAXM = 1000010;
struct SAM {
    int tot, root, lst, mom[MAXM], mx[MAXM];
    int acc[MAXM], nxt[MAXM][33];
    int newNode() {
        int res = ++tot;
        fill(nxt[res], nxt[res] + 33, 0);
        mom[res] = mx[res] = acc[res] = 0;
        return res;
    }
    void init() {
        tot = 0;
        root = newNode();
        mom[root] = 0, mx[root] = 0;
        lst = root;
    }
    void push(int c) {
        int p = lst;
        int np = newNode();
        mx[np] = mx[p] + 1;
        for (; p && nxt[p][c] == 0; p = mom[p])
            nxt[p][c] = np;
        if (p == 0) mom[np] = root;
        else {
            int q = nxt[p][c];
            if (mx[p] + 1 == mx[q]) mom[np] = q;
            else {
                int nq = newNode();
                mx[nq] = mx[p] + 1;
                for (int i = 0; i < 33; i++)
                    nxt[nq][i] = nxt[q][i];
```

```
                mom[nq] = mom[q];
                mom[q] = nq;
                mom[np] = nq;
                for (; p && nxt[p][c] == q; p = mom[p])
                    nxt[p][c] = nq;
            }
        }
        lst = np;
    }
    void push(char *str) {
        for (int i = 0; str[i]; i++)
            push(str[i] - 'a' + 1);
    }
} sam;
```

5.9 PalTree

```
struct palindromic_tree { // Check by APIO 2014
    // palindrome
    struct node {
        int next[26], fail, len;
        int cnt, num; // cnt: appear times, num: number of
        // pal. suf.
        node(int l = 0) : fail(0), len(l), cnt(0), num(0) {
            for (int i = 0; i < 26; ++i) next[i] = 0;
        }
    };
    vector<node> St;
    vector<char> s;
    int last, n;
    palindromic_tree() : St(2), last(1), n(0) {
        St[0].fail = 1, St[1].len = -1, s.pb(-1);
    }
    inline void clear() {
        St.clear(), s.clear(), last = 1, n = 0;
        St.pb(0), St.pb(-1);
        St[0].fail = 1, s.pb(-1);
    }
    inline int get_fail(int x) {
        while (s[n - St[x].len - 1] != s[n])
            x = St[x].fail;
        return x;
    }
    inline void add(int c) {
        s.push_back(c - 'a'), ++n;
        int cur = get_fail(last);
        if (!St[cur].next[c]) {
            int now = SZ(St);
            St.pb(St[cur].len + 2);
            St[now].fail =
                St[get_fail(St[cur].fail)].next[c];
            St[cur].next[c] = now;
            St[now].num = St[St[now].fail].num + 1;
        }
        last = St[cur].next[c], ++St[last].cnt;
    }
    inline void count() { // counting cnt
        auto i = St.rbegin();
        for (; i != St.rend(); ++i) {
            St[i->fail].cnt += i->cnt;
        }
    }
    inline int size() { // The number of diff. pal.
        return SZ(St) - 2;
    }
};
```

5.10 cyclicLCS

```
#define L 0
#define LU 1
#define U 2
const int mov[3][2] = {0, -1, -1, -1, -1, 0};
int a1, b1;
char a[MAXL * 2], b[MAXL * 2]; // 0-indexed
int dp[MAXL * 2][MAXL];
char pred[MAXL * 2][MAXL];
inline int lcs_length(int r) {
```

```

int i = r + al, j = bl, l = 0;
while (i > r) {
    char dir = pred[i][j];
    if (dir == LU) l++;
    i += mov[dir][0];
    j += mov[dir][1];
}
return l;
}

inline void reroot(int r) { // r = new base row
    int i = r, j = 1;
    while (j <= bl && pred[i][j] != LU) j++;
    if (j > bl) return;
    pred[i][j] = L;
    while (i < 2 * al && j <= bl) {
        if (pred[i + 1][j] == U) {
            i++;
            pred[i][j] = L;
        } else if (j < bl && pred[i + 1][j + 1] == LU) {
            i++;
            j++;
            pred[i][j] = L;
        } else {
            j++;
        }
    }
}

int cyclic_lcs() {
    // a, b, al, bl should be properly filled
    // note: a WILL be altered in process
    // -- concatenated after itself
    char tmp[MAXL];
    if (al > bl) {
        swap(al, bl);
        strcpy(tmp, a);
        strcpy(a, b);
        strcpy(b, tmp);
    }
    strcpy(tmp, a);
    strcat(a, tmp);
    // basic lcs
    for (int i = 0; i <= 2 * al; i++) {
        dp[i][0] = 0;
        pred[i][0] = U;
    }
    for (int j = 0; j <= bl; j++) {
        dp[0][j] = 0;
        pred[0][j] = L;
    }
    for (int i = 1; i <= 2 * al; i++) {
        for (int j = 1; j <= bl; j++) {
            if (a[i - 1] == b[j - 1])
                dp[i][j] = dp[i - 1][j - 1] + 1;
            else dp[i][j] = max(dp[i - 1][j], dp[i][j - 1]);
            if (dp[i][j - 1] == dp[i][j]) pred[i][j] = L;
            else if (a[i - 1] == b[j - 1]) pred[i][j] = LU;
            else pred[i][j] = U;
        }
    }
    // do cyclic lcs
    int clcs = 0;
    for (int i = 0; i < al; i++) {
        clcs = max(clcs, lcs_length(i));
        reroot(i + 1);
    }
    // recover a
    a[al] = '\0';
    return clcs;
}

```

6 Math

6.1 $ax+by=\gcd^*$

```

pll exgcd(ll a, ll b) {
    if (b == 0) return pll(1, 0);
    else {
        ll p = a / b;

```

```

        pll q = exgcd(b, a % b);
        return pll(q.Y, q.X - q.Y * p);
    }
}

```

6.2 floor and ceil

```

int floor(int a, int b) {
    return a/b - (a%b && a < 0 ^ b < 0);
}

int ceil(int a, int b) {
    return a/b + (a%b && a < 0 ^ b > 0);
}

```

6.3 Miller Rabin*

```

// n < 4,759,123,141      3 : 2, 7, 61
// n < 1,122,004,669,633  4 : 2, 13, 23, 1662803
// n < 3,474,749,660,383  6 : pimes <= 13
// n < 2^64              7 :
// 2, 325, 9375, 28178, 450775, 9780504, 1795265022
bool Miller_Rabin(ll a, ll n) {
    if ((a = a % n) == 0) return 1;
    if ((n & 1) ^ 1) return n == 2;
    ll tmp = (n - 1) / ((n - 1) & (1 - n));
    ll t = __lg(((n - 1) & (1 - n))), x = 1;
    for (; tmp; tmp >>= 1, a = mul(a, a, n))
        if (tmp & 1) x = mul(x, a, n);
    if (x == 1 || x == n - 1) return 1;
    while (--t)
        if ((x = mul(x, x, n)) == n - 1) return 1;
    return 0;
}

```

6.4 Big number

```

template<typename T>
inline string to_string(const T& x) {
    stringstream ss;
    return ss<<x, ss.str();
}

struct bigN: vector<ll> {
    const static int base=1000000000, width=log10(base);
    bool negative;
    bigN(const_iterator a, const_iterator b): vector<ll>(a, b) {}
    bigN(string s) {
        if (s.empty()) return;
        if (s[0] == '-') negative=1, s=s.substr(1);
        else negative=0;
        for (int i=int(s.size())-1; i>=0; i-=width) {
            ll t=0;
            for (int j=max(0, i-width+1); j<=i; ++j)
                t=t*10+s[j]-'0';
            push_back(t);
        }
        trim();
    }
    template<typename T>
    bigN(const T &x): bigN(to_string(x)) {}
    bigN(): negative(0) {}
    void trim() {
        while (size() && !back()) pop_back();
        if (empty()) negative=0;
    }
    void carry(int _base=base) {
        for (size_t i=0; i<size(); ++i) {
            if (at(i) >= 0 && at(i) < _base) continue;
            if (i+1u==size()) push_back(0);
            int r=at(i)%_base;
            if (r<0) r+=_base;
            at(i+1)+=(at(i)-r)/_base, at(i)=r;
        }
    }
    int abscmp(const bigN &b) const {
        if (size() > b.size()) return 1;

```

```

    if(size()<b.size())return -1;
    for(int i=int(size())-1;i>=0;--i){
        if(at(i)>b[i])return 1;
        if(at(i)<b[i])return -1;
    }
    return 0;
}
int cmp(const bigN &b)const{
    if(negative!=b.negative)return negative?-1:1;
    return negative?-abscmp(b):abscmp(b);
}
bool operator<(const bigN&b)const{return cmp(b)<0;}
bool operator>(const bigN&b)const{return cmp(b)>0;}
bool operator<=(const bigN&b)const{return cmp(b)<=0;}
bool operator>=(const bigN&b)const{return cmp(b)>=0;}
bool operator==(const bigN&b)const{return !cmp(b);}
bool operator!=(const bigN&b)const{return cmp(b)!=0;}
bigN abs()const{
    bigN res=*this;
    return res.negative=0, res;
}
bigN operator-()const{
    bigN res=*this;
    return res.negative=!negative,res.trim(),res;
}
bigN operator+(const bigN &b)const{
    if(negative)return -(-(*this)+(-b));
    if(b.negative)return *this-(-b);
    bigN res=*this;
    if(b.size()>size())res.resize(b.size());
    for(size_t i=0;i<b.size();++i)res[i]+=b[i];
    return res.carry(),res.trim(),res;
}
bigN operator-(const bigN &b)const{
    if(negative)return -(-(*this)-(-b));
    if(b.negative)return *this+(-b);
    if(abscmp(b)<0)return -(b-(*this));
    bigN res=*this;
    if(b.size()>size())res.resize(b.size());
    for(size_t i=0;i<b.size();++i)res[i]-=b[i];
    return res.carry(),res.trim(),res;
}
bigN operator*(const bigN &b)const{
    bigN res;
    res.negative=negative!=b.negative;
    res.resize(size()+b.size());
    for(size_t i=0;i<size();++i)
        for(size_t j=0;j<b.size();++j)
            if((res[i+j]+=at(i)*b[j])>=base){
                res[i+j+1]+=res[i+j]/base;
                res[i+j]%=base;
            }//¼k¥carry|.|□
    return res.trim(),res;
}
bigN operator/(const bigN &b)const{
    int norm=base/(b.back()+1);
    bigN x=abs()*norm;
    bigN y=b.abs()*norm;
    bigN q,r;
    q.resize(x.size());
    for(int i=int(x.size())-1;i>=0;--i){
        r=r*base+x[i];
        int s1=r.size()<=y.size()?0:r[y.size()];
        int s2=r.size()<y.size()?0:r[y.size()-1];
        int d=(ll(base)*s1+s2)/y.back();
        r=r-y*d;
        while(r.negative)r=r+y,--d;
        q[i]=d;
    }
    q.negative=negative!=b.negative;
    return q.trim(),q;
}
bigN operator%(const bigN &b)const{
    return *this-(*this/b)*b;
}
friend istream& operator>>(istream &ss, bigN &b){
    string s;
    return ss>>s, b=s, ss;
}
friend ostream& operator<<(ostream &ss, const bigN &b)
{
    if(b.negative)ss<<'-';

```

```

    ss<<(b.empty()?0:b.back());
    for(int i=int(b.size())-2;i>=0;--i)
        ss<<setw(width)<<setfill('0')<<b[i];
    return ss;
}
template<typename T>
operator T(){
    stringstream ss;
    ss<<*this;
    T res;
    return ss>>res,res;
}
};

```

6.5 Fraction

```

struct fraction{
    ll n,d;
    fraction(const ll &n=0,const ll &d=1):n(_n),d(_d){
        ll t=__gcd(n,d);
        n/=t,d/=t;
        if(d<0) n=-n,d=-d;
    }
    fraction operator-()const{
        return fraction(-n,d);
    }
    fraction operator+(const fraction &b)const{
        return fraction(n*b.d+b.n*d,d*b.d);
    }
    fraction operator-(const fraction &b)const{
        return fraction(n*b.d-b.n*d,d*b.d);
    }
    fraction operator*(const fraction &b)const{
        return fraction(n*b.n,d*b.d);
    }
    fraction operator/(const fraction &b)const{
        return fraction(n*b.d,d*b.n);
    }
    void print(){
        cout << n;
        if(d!=1) cout << "/" << d;
    }
};

```

6.6 Simultaneous Equations

```

struct matrix { //m variables, n equations
    int n, m;
    fraction M[MAXN][MAXN + 1], sol[MAXN];
    int solve() { //-1: inconsistent, >= 0: rank
        for (int i = 0; i < n; ++i) {
            int piv = 0;
            while (piv < m && !M[i][piv].n) ++piv;
            if (piv == m) continue;
            for (int j = 0; j < n; ++j) {
                if (i == j) continue;
                fraction tmp = -M[j][piv] / M[i][piv];
                for (int k = 0; k <= m; ++k) M[j][k] = tmp * M[i][k] + M[j][k];
            }
        }
        int rank = 0;
        for (int i = 0; i < n; ++i) {
            int piv = 0;
            while (piv < m && !M[i][piv].n) ++piv;
            if (piv == m && M[i][m].n) return -1;
            else if (piv < m) ++rank, sol[piv] = M[i][m] / M[i][piv];
        }
        return rank;
    }
};

```

6.7 Pollard Rho

```
// does not work when n is prime
ll f(ll x, ll mod){ return add(mul(x,x,mod),1,mod); }
ll pollard_rho(ll n){
    if(!(n&1)) return 2;
    while(1){
        ll y=2,x=rand()%(n-1)+1,res=1;
        for(int sz=2;res==1;y=x,sz*=2)
            for(int i=0;i<sz&&res<=1;++i)
                x=f(x,n),res=__gcd(abs(x-y),n);
        if(res!=0&&res!=n) return res;
    }
}
```

6.8 Simplex Algorithm

```
const int MAXN = 111;
const int MAXM = 111;
const double eps = 1E-10;
double a[MAXN][MAXM], b[MAXN], c[MAXN], d[MAXN][MAXM];
double x[MAXN];
int ix[MAXN + MAXM]; // !!! array all indexed from 0
// max{cx} subject to {Ax<=b,x>=0}
// n: constraints, m: vars !!!
// x[] is the optimal solution vector
// usage :
// value = simplex(a, b, c, N, M);
double simplex(double a[MAXN][MAXM], double b[MAXN],
    double c[MAXN], int n, int m){
    ++m;
    int r = n, s = m - 1;
    memset(d, 0, sizeof(d));
    for (int i = 0; i < n + m; ++i) ix[i] = i;
    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < m - 1; ++j) d[i][j] = -a[i][j];
        d[i][m - 1] = 1;
        d[i][m] = b[i];
        if (d[r][m] > d[i][m]) r = i;
    }
    for (int j = 0; j < m - 1; ++j) d[n][j] = c[j];
    d[n + 1][m - 1] = -1;
    for (double dd;; ) {
        if (r < n) {
            int t = ix[s]; ix[s] = ix[r + m]; ix[r + m] = t;
            d[r][s] = 1.0 / d[r][s];
            for (int j = 0; j <= m; ++j)
                if (j != s) d[r][j] *= -d[r][s];
            for (int i = 0; i <= n + 1; ++i) if (i != r) {
                for (int j = 0; j <= m; ++j) if (j != s)
                    d[i][j] += d[r][j] * d[i][s];
                d[i][s] *= d[r][s];
            }
        }
        r = -1; s = -1;
        for (int j = 0; j < m; ++j)
            if (s < 0 || ix[s] > ix[j]) {
                if (d[n + 1][j] > eps ||
                    (d[n + 1][j] > -eps && d[n][j] > eps))
                    s = j;
            }
        if (s < 0) break;
        for (int i = 0; i < n; ++i) if (d[i][s] < -eps) {
            if (r < 0 ||
                (dd = d[r][m] / d[r][s] - d[i][m] / d[i][s]) <
                -eps ||
                (dd < eps && ix[r + m] > ix[i + m]))
                r = i;
        }
        if (r < 0) return -1; // not bounded
    }
    if (d[n + 1][m] < -eps) return -1; // not executable
    double ans = 0;
    for(int i=0; i<m; i++) x[i] = 0;
    for (int i = m; i < n + m; ++i) { // the missing
        enumerated x[i] = 0
        if (ix[i] < m - 1){
            ans += d[i - m][m] * c[ix[i]];
            x[ix[i]] = d[i - m][m];
        }
    }
    return ans;
}
```

```
}
```

6.8.1 Construction

Standard form: maximize $c^T x$ subject to $Ax \leq b$ and $x \geq 0$.

Dual LP: minimize $b^T y$ subject to $A^T y \geq c$ and $y \geq 0$.

\bar{x} and \bar{y} are optimal if and only if for all $i \in [1, n]$, either $\bar{x}_i = 0$ or $\sum_{j=1}^m A_{ji} \bar{y}_j = c_i$ holds and for all $i \in [1, m]$ either $\bar{y}_i = 0$ or $\sum_{j=1}^n A_{ij} \bar{x}_j = b_j$ holds.

1. In case of minimization, let $c'_i = -c_i$
2. $\sum_{1 \leq i \leq n} A_{ji} x_i \geq b_j \rightarrow \sum_{1 \leq i \leq n} -A_{ji} x_i \leq -b_j$
3. $\sum_{1 \leq i \leq n} A_{ji} x_i = b_j$

$$\begin{aligned} & \bullet \sum_{1 \leq i \leq n} A_{ji} x_i \leq b_j \\ & \bullet \sum_{1 \leq i \leq n} A_{ji} x_i \geq b_j \end{aligned}$$

4. If x_i has no lower bound, replace x_i with $x_i - x'_i$

6.9 Schreier-Sims Algorithm*

```
namespace schreier {
    int n;
    vector<vector<vector<int>>> bkets, binv;
    vector<vector<int>> lk;
    vector<int> operator*(const vector<int> &a, const
        vector<int> &b) {
        vector<int> res(SZ(a));
        for (int i = 0; i < SZ(a); ++i) res[i] = b[a[i]];
        return res;
    }
    vector<int> inv(const vector<int> &a) {
        vector<int> res(SZ(a));
        for (int i = 0; i < SZ(a); ++i) res[a[i]] = i;
        return res;
    }
    int filter(const vector<int> &g, bool add = true) {
        n = SZ(bkets);
        vector<int> p = g;
        for (int i = 0; i < n; ++i) {
            assert(p[i] >= 0 && p[i] < SZ(lk[i]));
            if (lk[i][p[i]] == -1) {
                if (add) {
                    bkets[i].pb(p);
                    binv[i].pb(inv(p));
                    lk[i][p[i]] = SZ(bkets[i]) - 1;
                }
                return i;
            }
            p = p * binv[i][lk[i][p[i]]];
        }
        return -1;
    }
    bool inside(const vector<int> &g) { return filter(g,
        false) == -1; }
    void solve(const vector<vector<int>> &gen, int _n) {
        n = _n;
        bkets.clear(), bkets.resize(n);
        binv.clear(), binv.resize(n);
        lk.clear(), lk.resize(n);
        vector<int> iden(n);
        iota(iden.begin(), iden.end(), 0);
        for (int i = 0; i < n; ++i) {
            lk[i].resize(n, -1);
            bkets[i].pb(iden);
            binv[i].pb(iden);
            lk[i][i] = 0;
        }
        for (int i = 0; i < SZ(gen); ++i) filter(gen[i]);
        queue<pair<pii, pii>> upd;
        for (int i = 0; i < n; ++i)
            for (int j = i; j < n; ++j)
                for (int k = 0; k < SZ(bkets[i]); ++k)
                    for (int l = 0; l < SZ(bkets[j]); ++l)
                        upd.emplace(pii(i, k), pii(j, l));
        while (!upd.empty()) {
            auto a = upd.front().X;
            auto b = upd.front().Y;
            upd.pop();
            int res = filter(bkets[a.X][a.Y] * bkets[b.X][b.Y]);
        }
    }
}
```



```

        if (res == -1) continue;
        pii pr = pii(res, SZ(bkts[res]) - 1);
        for (int i = 0; i < n; ++i)
            for (int j = 0; j < SZ(bkts[i]); ++j) {
                if (i <= res) upd.emplace(pii(i, j), pr);
                if (res <= i) upd.emplace(pr, pii(i, j));
            }
    }
}
long long size() {
    long long res = 1;
    for (int i = 0; i < n; ++i) res = res * SZ(bkts[i]);
    return res;
}
}

```

6.10 chineseRemainder

```

LL solve(LL x1, LL m1, LL x2, LL m2) {
    LL g = __gcd(m1, m2);
    if ((x2 - x1) % g) return -1; // no sol
    m1 /= g; m2 /= g;
    pair<LL, LL> p = gcd(m1, m2);
    LL lcm = m1 * m2 * g;
    LL res = p.first * (x2 - x1) * m1 + x1;
    return (res % lcm + lcm) % lcm;
}

```

6.11 QuadraticResidue

```

int Jacobi(int a, int m) {
    int s = 1;
    for (; m > 1; ) {
        a %= m;
        if (a == 0) return 0;
        const int r = __builtin_ctz(a);
        if ((r & 1) && ((m + 2) & 4)) s = -s;
        a >>= r;
        if (a & m & 2) s = -s;
        swap(a, m);
    }
    return s;
}

int QuadraticResidue(int a, int p) {
    if (p == 2) return a & 1;
    const int jc = Jacobi(a, p);
    if (jc == 0) return 0;
    if (jc == -1) return -1;
    int b, d;
    for (; ) {
        b = rand() % p;
        d = (1LL * b * b + p - a) % p;
        if (Jacobi(d, p) == -1) break;
    }
    int f0 = b, f1 = 1, g0 = 1, g1 = 0, tmp;
    for (int e = (1LL + p) >> 1; e; e >>= 1) {
        if (e & 1) {
            tmp = (1LL * g0 * f0 + 1LL * d * (1LL * g1 * f1 % p)) % p;
            g1 = (1LL * g0 * f1 + 1LL * g1 * f0) % p;
            g0 = tmp;
        }
        tmp = (1LL * f0 * f0 + 1LL * d * (1LL * f1 * f1 % p)) % p;
        f1 = (2LL * f0 * f1) % p;
        f0 = tmp;
    }
    return g0;
}

```

6.12 Discrete Log

```

int DiscreteLog(int s, int x, int y, int m) {
    constexpr int kStep = 32000;
    unordered_map<int, int> p;
    int b = 1;
    for (int i = 0; i < kStep; ++i) {
        p[y] = i;
        y = 1LL * y * x % m;
        b = 1LL * b * x % m;
    }
    for (int i = 0; i < m + 10; i += kStep) {
        s = 1LL * s * b % m;
        if (p.find(s) != p.end()) return i + kStep - p[s];
    }
    return -1;
}

int DiscreteLog(int x, int y, int m) {
    if (m == 1) return 0;
    int s = 1;
    for (int i = 0; i < 100; ++i) {
        if (s == y) return i;
        s = 1LL * s * x % m;
    }
    if (s == y) return 100;
    int p = 100 + DiscreteLog(s, x, y, m);
    if (fpow(x, p, m) != y) return -1;
    return p;
}

```

6.13 PiCount

```

int64_t PrimeCount(int64_t n) {
    if (n <= 1) return 0;
    const int v = sqrt(n);
    vector<int> smalls(v + 1);
    for (int i = 2; i <= v; ++i) smalls[i] = (i + 1) / 2;
    int s = (v + 1) / 2;
    vector<int> roughs(s);
    for (int i = 0; i < s; ++i) roughs[i] = 2 * i + 1;
    vector<int64_t> larges(s);
    for (int i = 0; i < s; ++i) larges[i] = (n / (2 * i + 1) + 1) / 2;
    vector<bool> skip(v + 1);
    int pc = 0;
    for (int p = 3; p <= v; ++p) {
        if (smalls[p] > smalls[p - 1]) {
            int q = p * p;
            pc++;
            if (1LL * q * q > n) break;
            skip[p] = true;
            for (int i = q; i <= v; i += 2 * p) skip[i] = true;
            int ns = 0;
            for (int k = 0; k < s; ++k) {
                int i = roughs[k];
                if (skip[i]) continue;
                int64_t d = 1LL * i * p;
                larges[ns] = larges[k] - (d <= v ? larges[smalls[d] - pc] : smalls[n / d]) + pc;
                roughs[ns++] = i;
            }
            s = ns;
            for (int j = v / p; j >= p; --j) {
                int c = smalls[j] - pc;
                for (int i = j * p, e = min(i + p, v + 1); i < e; ++i) smalls[i] -= c;
            }
        }
        for (int k = 1; k < s; ++k) {
            const int64_t m = n / roughs[k];
            int64_t s = larges[k] - (pc + k - 1);
            for (int l = 1; l < k; ++l) {
                int p = roughs[l];
                if (1LL * p * p > m) break;
                s -= smalls[m / p] - (pc + l - 1);
            }
            larges[0] -= s;
        }
    }
    return larges[0];
}

```

6.14 Primes

```
/*
12721 13331 14341 75577 123457 222557 556679 999983
1097774749 1076767633 100102021 999997771
1001010013 1000512343 987654361 999991231
999888733 98789101 98777733 999991921
1010101333 1010102101 100000000039
100000000000037 2305843009213693951
4611686018427387847 9223372036854775783
18446744073709551557
*/
```

6.15 Theorem

6.15.1 Kirchhoff's Theorem

Denote L be a $n \times n$ matrix as the Laplacian matrix of graph G , where $L_{ii} = d(i)$, $L_{ij} = -c$ where c is the number of edge (i, j) in G .

- The number of undirected spanning in G is $|\det(\tilde{L}_{11})|$.
- The number of directed spanning tree rooted at r in G is $|\det(\tilde{L}_{rr})|$.

6.15.2 Tutte's Matrix

Let D be a $n \times n$ matrix, where $d_{ij} = x_{ij}$ (x_{ij} is chosen uniformly at random) if $i < j$ and $(i, j) \in E$, otherwise $d_{ij} = -d_{ji}$. $\frac{\text{rank}(D)}{2}$ is the maximum matching on G .

6.15.3 Cayley's Formula

- Given a degree sequence d_1, d_2, \dots, d_n for each labeled vertices, there are $\frac{(n-2)!}{(d_1-1)!(d_2-1)!\dots(d_n-1)!}$ spanning trees.
- Let $T_{n,k}$ be the number of labeled forests on n vertices with k components, such that vertex $1, 2, \dots, k$ belong to different components. Then $T_{n,k} = kn^{n-k-1}$.

6.15.4 Erdős–Gallai theorem

A sequence of nonnegative integers $d_1 \geq \dots \geq d_n$ can be represented as the degree sequence of a finite simple graph on n vertices if and only if

if $d_1 + \dots + d_n$ is even and $\sum_{i=1}^k d_i \leq k(k-1) + \sum_{i=k+1}^n \min(d_i, k)$ holds for every $1 \leq k \leq n$.

6.15.5 Gale–Ryser theorem

A pair of sequences of nonnegative integers $a_1 \geq \dots \geq a_n$ and b_1, \dots, b_n is bigraphic if and only if $\sum_{i=1}^n a_i = \sum_{i=1}^n b_i$ and $\sum_{i=1}^k a_i \leq \sum_{i=1}^n \min(b_i, k)$ holds for every $1 \leq k \leq n$.

6.15.6 Fulkerson–Chen–Anstee theorem

A sequence $(a_1, b_1), \dots, (a_n, b_n)$ of nonnegative integer pairs with $a_1 \geq \dots \geq a_n$ is digraphic if and only if $\sum_{i=1}^n a_i = \sum_{i=1}^n b_i$ and $\sum_{i=1}^k a_i \leq \sum_{i=1}^n \min(b_i, k-1) + \sum_{i=k+1}^n \min(b_i, k)$ holds for every $1 \leq k \leq n$.

6.15.7 Pick's theorem

給定頂點座標均是整點（或正方形格子點）的簡單多邊形，皮克定理說明了其面積 A 和內部格點數目 i 、邊上格點數目 b 的關係： $A = i + \frac{b}{2} - 1$

6.16 Euclidean Algorithms

- $m = \lfloor \frac{a+b}{c} \rfloor$
- Time complexity: $O(\log n)$

$$f(a, b, c, n) = \sum_{i=0}^n \lfloor \frac{ai+b}{c} \rfloor$$

$$= \begin{cases} \lfloor \frac{a}{c} \rfloor \cdot \frac{n(n+1)}{2} + \lfloor \frac{b}{c} \rfloor \cdot (n+1) \\ + f(a \bmod c, b \bmod c, c, n), & a \geq c \vee b \geq c \\ 0, & n < 0 \vee a = 0 \\ nm - f(c, c-b-1, a, m-1), & \text{otherwise} \end{cases}$$

$$g(a, b, c, n) = \sum_{i=0}^n i \lfloor \frac{ai+b}{c} \rfloor$$

$$= \begin{cases} \lfloor \frac{a}{c} \rfloor \cdot \frac{n(n+1)(2n+1)}{6} + \lfloor \frac{b}{c} \rfloor \cdot \frac{n(n+1)}{2} \\ + g(a \bmod c, b \bmod c, c, n), & a \geq c \vee b \geq c \\ 0, & n < 0 \vee a = 0 \\ \frac{1}{2} \cdot (n(n+1)m - f(c, c-b-1, a, m-1) \\ - h(c, c-b-1, a, m-1)), & \text{otherwise} \end{cases}$$

$$h(a, b, c, n) = \sum_{i=0}^n \lfloor \frac{ai+b}{c} \rfloor^2$$

$$= \begin{cases} \lfloor \frac{a}{c} \rfloor^2 \cdot \frac{n(n+1)(2n+1)}{6} + \lfloor \frac{b}{c} \rfloor^2 \cdot (n+1) \\ + \lfloor \frac{a}{c} \rfloor \cdot \lfloor \frac{b}{c} \rfloor \cdot n(n+1) \\ + h(a \bmod c, b \bmod c, c, n) \\ + 2 \lfloor \frac{a}{c} \rfloor \cdot g(a \bmod c, b \bmod c, c, n) \\ + 2 \lfloor \frac{b}{c} \rfloor \cdot f(a \bmod c, b \bmod c, c, n), & a \geq c \vee b \geq c \\ 0, & n < 0 \vee a = 0 \\ nm(m+1) - 2g(c, c-b-1, a, m-1) \\ - 2f(c, c-b-1, a, m-1) - f(a, b, c, n), & \text{otherwise} \end{cases}$$

7 Polynomial

7.1 Fast Fourier Transform

```
template<int MAXN>
struct FFT {
    using val_t = complex<double>;
    const double PI = acos(-1);
    val_t w[MAXN];
    FFT() {
        for (int i = 0; i < MAXN; ++i) {
            double arg = 2 * PI * i / MAXN;
            w[i] = val_t(cos(arg), sin(arg));
        }
    }
    void bitrev(val_t *a, int n); // see NTT
    void trans(val_t *a, int n, bool inv = false); // see
    NTT;
    // remember to replace LL with val_t
};
```

7.2 Number Theory Transform

```
//(2^16)+1, 65537, 3
//7*17*(2^23)+1, 998244353, 3
//1255*(2^20)+1, 1315962881, 3
//51*(2^25)+1, 1711276033, 29
template<int MAXN, LL P, LL RT> //MAXN must be 2^k
struct NTT {
    LL w[MAXN];
    LL mpow(LL a, LL n);
    LL minv(LL a) { return mpow(a, P - 2); }
    NTT() {
        LL dw = mpow(RT, (P - 1) / MAXN);
        w[0] = 1;
        for (int i = 1; i < MAXN; ++i) w[i] = w[i - 1] * dw
            % P;
    }
    void bitrev(LL *a, int n) {
        int i = 0;
        for (int j = 1; j < n - 1; ++j) {
            for (int k = n >> 1; (i ^ k) < k; k >>= 1);
            if (j < i) swap(a[i], a[j]);
        }
    }
    void operator()(LL *a, int n, bool inv = false) { //0
        <= a[i] < P
        bitrev(a, n);
        for (int L = 2; L <= n; L <= 1) {
            int dx = MAXN / L, dl = L >> 1;
            for (int i = 0; i < n; i += L) {
                for (int j = i, x = 0; j < i + dl; ++j, x += dx) {
                    LL tmp = a[j + dl] * w[x] % P;
                    if ((a[j + dl] = a[j] - tmp) < 0) a[j + dl]
                        += P;
                    if ((a[j] += tmp) >= P) a[j] -= P;
                }
            }
        }
    }
};
```

```

    }
}
if (inv) {
    reverse(a + 1, a + n);
    LL invn = minv(n);
    for (int i = 0; i < n; ++i) a[i] = a[i] * invn % P;
}
}
};

```

7.3 Fast Walsh Transform*

```

/* x: a[j], y: a[j + (L >> 1)]
or: (y += x * op), and: (x += y * op)
xor: (x, y = (x + y) * op, (x - y) * op)
invop: or, and, xor = -1, -1, 1/2 */
void fwt(int *a, int n, int op) { //or
    for (int L = 2; L <= n; L <= 1)
        for (int i = 0; i < n; i += L)
            for (int j = i; j < i + (L >> 1); ++j)
                a[j + (L >> 1)] += a[j] * op;
}
const int N = 21;
int f[N][1 << N], g[N][1 << N], h[N][1 << N], ct[1 << N];
void subset_convolution(int *a, int *b, int *c, int L)
{
    // c_k = \sum_{i+j=k, i&j=0} a_i * b_j
    int n = 1 << L;
    for (int i = 1; i < n; ++i)
        ct[i] = ct[i & (i - 1)] + 1;
    for (int i = 0; i < n; ++i)
        f[ct[i]][i] = a[i], g[ct[i]][i] = b[i];
    for (int i = 0; i <= L; ++i)
        fwt(f[i], n, 1), fwt(g[i], n, 1);
    for (int i = 0; i <= L; ++i)
        for (int j = 0; j <= i; ++j)
            for (int x = 0; x < n; ++x)
                h[i][x] += f[j][x] * g[i - j][x];
    for (int i = 0; i <= L; ++i)
        fwt(h[i], n, -1);
    for (int i = 0; i < n; ++i)
        c[i] = h[ct[i]][i];
}

```

7.4 Newton' s Method

Given $F(x)$ where

$$F(x) = \sum_{i=0}^{\infty} \alpha_i (x - \beta)^i$$

for β being some constant. Polynomial P such that $F(P) = 0$ can be found iteratively. Denote by Q_k the polynomial such that $F(Q_k) = 0 \pmod{x^{2^k}}$, then

$$Q_{k+1} = Q_k - \frac{F(Q_k)}{F'(Q_k)} \pmod{x^{2^{k+1}}}$$

8 Geometry

8.1 Default Code

```

typedef pair<double,double> pdd;
typedef pair<pdd,pdd> Line;
struct Cir{pdd O; double R;};
const double eps=1e-8;
pdd operator+(const pdd &a, const pdd &b)
{ return pdd(a.X + b.X, a.Y + b.Y);}
pdd operator-(const pdd &a, const pdd &b)
{ return pdd(a.X - b.X, a.Y - b.Y);}
pdd operator*(const pdd &a, const double &b)
{ return pdd(a.X * b, a.Y * b);}
pdd operator/(const pdd &a, const double &b)

```

```

{ return pdd(a.X / b, a.Y / b);}
double dot(const pdd &a,const pdd &b)
{ return a.X * b.X + a.Y * b.Y;}
double cross(const pdd &a,const pdd &b)
{ return a.X * b.Y - a.Y * b.X;}
double abs2(const pdd &a)
{ return dot(a, a);}
double abs(const pdd &a)
{ return sqrt(dot(a, a));}
int sign(const double &a)
{ return fabs(a) < eps ? 0 : a > 0 ? 1 : -1;}
int ori(const pdd &a,const pdd &b,const pdd &c)
{ return sign(cross(b - a, c - a));}
bool collinearity(const pdd &p1, const pdd &p2, const pdd &p3)
{ return fabs(cross(p1 - p3, p2 - p3)) < eps;}
bool btw(const pdd &p1,const pdd &p2,const pdd &p3) {
    if(!collinearity(p1, p2, p3)) return 0;
    return dot(p1 - p3, p2 - p3) < eps;
}
bool seg_intersect(const pdd &p1,const pdd &p2,const pdd &p3,const pdd &p4) {
    int a123 = ori(p1, p2, p3);
    int a124 = ori(p1, p2, p4);
    int a341 = ori(p3, p4, p1);
    int a342 = ori(p3, p4, p2);
    if(a123 == 0 && a124 == 0)
        return btw(p1, p2, p3) || btw(p1, p2, p4) ||
            btw(p3, p4, p1) || btw(p3, p4, p2);
    return a123 * a124 <= 0 && a341 * a342 <= 0;
}
pdd intersect(const pdd &p1, const pdd &p2, const pdd &p3, const pdd &p4) {
    double a123 = cross(p2 - p1, p3 - p1);
    double a124 = cross(p2 - p1, p4 - p1);
    return (p4 * a123 - p3 * a124) / (a123 - a124);
}
pdd perp(const pdd &p1)
{ return pdd(-p1.Y, p1.X);}
pdd foot(const pdd &p1, const pdd &p2, const pdd &p3)
{ return intersect(p1, p2, p3, p3 + perp(p2 - p1));}

```

8.2 Convex hull*

```

void hull(vector<p11> &dots) {
    sort(dots.begin(), dots.end());
    vector<p11> ans(1, dots[0]);
    for (int ct = 0; ct < 2; ++ct, reverse(ALL(dots)))
        for (int i = 1, t = SZ(ans); i < SZ(dots); ans.pb(dots[i++]))
            while (SZ(ans) > t && ori(ans[SZ(ans) - 2], ans.back(), dots[i]) <= 0)
                ans.pop_back();
    ans.pop_back(), ans.swap(dots);
}

```

8.3 External bisector

```

pdd external_bisector(pdd p1,pdd p2,pdd p3){//213
    pdd L1=p2-p1,L2=p3-p1;
    L2=L2*abs(L1)/abs(L2);
    return L1+L2;
}

```

8.4 Heart

```

pdd excenter(pdd p0,pdd p1,pdd p2,double &radius){
    p1=p1-p0,p2=p2-p0;
    double x1=p1.X,y1=p1.Y,x2=p2.X,y2=p2.Y;
    double m=2.*(x1*y2-y1*x2);
    center.X=(x1*x1*y2-x2*x2*y1+y1*y2*(y1-y2))/m;
    center.Y=(x1*x2*(x2-x1)-y1*y1*x2+x1*y2*y2)/m;
    return radius=abs(center),center+p0;
}
pdd incenter(pdd p1,pdd p2,pdd p3,double &radius){

```

```

double a=abs(p2-p1),b=abs(p3-p1),c=abs(p3-p2);
double s=(a+b+c)/2,area=sqrt(s*(s-a)*(s-b)*(s-c));
pdd l1=external_bisector(p1,p2,p3),l2=
    external_bisector(p2,p1,p3);
return radius=area/s,intersect(p1,p1+l1,p2,p2+l2),
}

pdd escenter(pdd p1,pdd p2,pdd p3){//213
    pdd l1=external_bisector(p1,p2,p3),l2=
        external_bisector(p2,p2+p2-p1,p3);
    return intersect(p1,p1+l1,p2,p2+l2);
}

pdd barycenter(pdd p1,pdd p2,pdd p3){
    return (p1+p2+p3)/3;
}

pdd orthocenter(pdd p1,pdd p2,pdd p3){
    pdd l1=p3-p2,l2=p3-p1;
    swap(l1.X,l1.Y),l1.X*=-1;
    swap(l2.X,l2.Y),l2.X*=-1;
    return intersect(p1,p1+l1,p2,p2+l2);
}

```

8.5 Minimum Enclosing Circle*

```

pdd Minimum_Enclosing_Circle(vector<pdd> dots, double &
    r) {
    pdd cent;
    random_shuffle(ALL(dots));
    cent = dots[0], r = 0;
    for (int i = 1; i < SZ(dots); ++i)
        if (abs(dots[i] - cent) > r) {
            cent = dots[i], r = 0;
            for (int j = 0; j < i; ++j)
                if (abs(dots[j] - cent) > r) {
                    cent = (dots[i] + dots[j]) / 2;
                    r = abs(dots[i] - cent);
                    for (int k = 0; k < j; ++k)
                        if (abs(dots[k] - cent) > r)
                            cent = excenter(dots[i], dots[j], dots[k], r);
                }
        }
    return cent;
}

```

8.6 Polar Angle Sort*

```

pdd center;//sort base
int Quadrant(pdd a) {
    if(a.X > 0 && a.Y >= 0) return 1;
    if(a.X <= 0 && a.Y > 0) return 2;
    if(a.X < 0 && a.Y <= 0) return 3;
    if(a.X >= 0 && a.Y < 0) return 4;
}
bool cmp(p11 a, p11 b) {
    a = a - center, b = b - center;
    if (Quadrant(a) != Quadrant(b))
        return Quadrant(a) < Quadrant(b);
    if (cross(b, a) == 0) return abs2(a) < abs2(b);
    return cross(a, b) > 0;
}
bool cmp(pdd a, pdd b) {
    a = a - center, b = b - center;
    if (fabs(atan2(a.Y, a.X) - atan2(b.Y, b.X)) > eps)
        return atan2(a.Y, a.X) < atan2(b.Y, b.X);
    return abs(a) < abs(b);
}

```

8.7 Intersection of two circles*

```

bool CCinter(Cir &a, Cir &b, pdd &p1, pdd &p2) {
    pdd o1 = a.O, o2 = b.O;
    double r1 = a.R, r2 = b.R, d2 = abs2(o1 - o2), d =
        sqrt(d2);

```

```

if(d < max(r1, r2) - min(r1, r2) || d > r1 + r2)
    return 0;
pdd u = (o1 + o2) * 0.5 + (o1 - o2) * ((r2 * r2 - r1
    * r1) / (2 * d2));
double A = sqrt((r1 + r2 + d) * (r1 - r2 + d) * (r1 +
    r2 - d) * (-r1 + r2 + d));
pdd v = pdd(o1.Y - o2.Y, -o1.X + o2.X) * A / (2 * d2)
    ;
p1 = u + v, p2 = u - v;
return 1;
}

```

8.8 Intersection of polygon and circle

```

// Divides into multiple triangle, and sum up
// test by HDU2892
const double PI=acos(-1);
double _area(pdd pa, pdd pb, double r){
    if(abs(pa)<abs(pb)) swap(pa, pb);
    if(abs(pb)<eps) return 0;
    double S, h, theta;
    double a=abs(pb),b=abs(pa),c=abs(pb-pa);
    double cosB = dot(pb,pb-pa) / a / c, B = acos(cosB);
    double cosC = dot(pa,pb) / a / b, C = acos(cosC);
    if(a > r){
        S = (C/2)*r*r;
        h = a*b*sin(C)/c;
        if (h < r && B < PI/2) S -= (acos(h/r)*r*r - h*sqrt
            (r*r-h*h));
    }
    else if(b > r){
        theta = PI - B - asin(sin(B)/r*a);
        S = .5*a*r*sin(theta) + (C-theta)/2*r*r;
    }
    else S = .5*sin(C)*a*b;
    return S;
}
double area_poly_circle(const vector<pdd> poly,const
    pdd &o,const double r){
    double S=0;
    for(int i=0;i<SZ(poly);++i)
        S+=_area(poly[i]-o,poly[(i+1)%SZ(poly)]-o,r)*ori(0,
            poly[i],poly[(i+1)%SZ(poly)]);
    return fabs(S);
}

```

8.9 Intersection of line and circle

```

vector<pdd> line_interCircle(const pdd &p1,const pdd &
    p2,const pdd &c,const double r){
    pdd ft=foot(p1,p2,c),vec=p2-p1;
    double dis=abs(c-ft);
    if(fabs(dis-r)<eps) return vector<pdd>{ft};
    if(dis>r) return {};
    vec=vec*sqrt(r*r-dis*dis)/abs(vec);
    return vector<pdd>{ft+vec,ft-vec};
}

```

8.10 point in circle

```

// return p4 is strictly in circumcircle of tri(p1,p2,
    p3)
long long sqr(long long x) { return x * x; }
bool in_cc(const p11& p1, const p11& p2, const p11& p3,
    const p11& p4) {
    long long u11 = p1.X - p4.X; long long u12 = p1.Y -
        p4.Y;
    long long u21 = p2.X - p4.X; long long u22 = p2.Y -
        p4.Y;
    long long u31 = p3.X - p4.X; long long u32 = p3.Y -
        p4.Y;
    long long u13 = sqr(p1.X) - sqr(p4.X) + sqr(p1.Y) -
        sqr(p4.Y);
    long long u23 = sqr(p2.X) - sqr(p4.X) + sqr(p2.Y) -
        sqr(p4.Y);

```

```

    long long u33 = sqr(p3.X) - sqr(p4.X) + sqr(p3.Y) -
        sqr(p4.Y);
    __int128 det = (__int128)-u13 * u22 * u31 + (
        __int128)u12 * u23 * u31 + (__int128)u13 * u21
        * u32 - (__int128)u11 * u23 * u32 - (__int128)
        u12 * u21 * u33 + (__int128)u11 * u22 * u33;
    return det > eps;
}

```

8.11 Half plane intersection

```

bool isin( Line l0, Line l1, Line l2 ){
    // Check inter(l1, l2) in l0
    pdd p = intersect(l1.X, l1.Y, l2.X, l2.Y);
    return cross(l0.Y - l0.X, p - l0.X) > eps;
}
/* If no solution, check: 1. ret.size() < 3
 * Or more precisely, 2. interPnt(ret[0], ret[1])
 * in all the lines. (use (l1.Y - l1.X) ^ (p - l1.X) > 0
 */
/* ---^-- Line.X ---^-- Line.Y ---^-- */
vector<Line> halfPlaneInter(vector<Line> lines){
    int sz = lines.size();
    vector<double> ata(sz), ord(sz);
    for(int i=0; i<sz; ++i) {
        ord[i] = i;
        pdd d = lines[i].Y - lines[i].X;
        ata[i] = atan2(d.Y, d.X);
    }
    sort(ord.begin(), ord.end(), [&](int i, int j){
        if( fabs(ata[i] - ata[j]) < eps )
            return (cross(lines[i].Y-lines[i].X,
                lines[j].Y-lines[i].X)<0;
        return ata[i] < ata[j];
    });
    vector<Line> fin;
    for (int i=0; i<sz; ++i)
        if (!i || fabs(ata[ord[i]] - ata[ord[i-1]]) > eps)
            fin.pb(lines[ord[i]]);
    deque<Line> dq;
    for (int i=0; i<SZ(fin); i++){
        while(SZ(dq)>=2&&!isin(fin[i], dq[SZ(dq)-2], dq.back()))
            dq.pop_back();
        while(SZ(dq)>=2&&!isin(fin[i], dq[0], dq[1]))
            dq.pop_front();
        dq.push_back(fin[i]);
    }
    while(SZ(dq)>=3&&!isin(dq[0], dq[SZ(dq)-2], dq.back()))
        dq.pop_back();
    while(SZ(dq)>=3&&!isin(dq.back(), dq[0], dq[1]))
        dq.pop_front();
    vector<Line> res(ALL(dq));
    return res;
}

```

8.12 CircleCover*

```

const int N = 1021;
struct CircleCover {
    int C;
    Cir c[N];
    bool g[N][N], overlap[N][N];
    // Area[i] : area covered by at least i circles
    double Area[ N ];
    void init(int _C){ C = _C; }
    struct Teve {
        pdd p; double ang; int add;
        Teve() {}
        Teve(pdd _a, double _b, int _c):p(_a), ang(_b), add
            (_c){}
        bool operator<(const Teve &a)const
        {return ang < a.ang;}
    }eve[N * 2];
    // strict: x = 0, otherwise x = -1
    bool disjunct(Cir &a, Cir &b, int x)
    {return sign(abs(a.O - b.O) - a.R - b.R) > x;}
    bool contain(Cir &a, Cir &b, int x)

```

```

{return sign(a.R - b.R - abs(a.O - b.O)) > x;}
bool contain(int i, int j) {
    /* c[j] is non-strictly in c[i]. */
    return (sign(c[i].R - c[j].R) > 0 || (sign(c[i].R -
        c[j].R) == 0 && i < j)) && contain(c[i], c[j],
        -1);
}
void solve(){
    fill_n(Area, C + 2, 0);
    for(int i = 0; i < C; ++i)
        for(int j = 0; j < C; ++j)
            overlap[i][j] = contain(i, j);
    for(int i = 0; i < C; ++i)
        for(int j = 0; j < C; ++j)
            g[i][j] = !(overlap[i][j] || overlap[j][i] ||
                disjunct(c[i], c[j], -1));
    for(int i = 0; i < C; ++i){
        int E = 0, cnt = 1;
        for(int j = 0; j < C; ++j)
            if(j != i && overlap[j][i])
                ++cnt;
        for(int j = 0; j < C; ++j)
            if(i != j && g[i][j]) {
                pdd aa, bb;
                CCinter(c[i], c[j], aa, bb);
                double A = atan2(aa.Y - c[i].O.Y, aa.X - c[i]
                    ].O.X);
                double B = atan2(bb.Y - c[i].O.Y, bb.X - c[i]
                    ].O.X);
                eve[E++] = Teve(bb, B, 1), eve[E++] = Teve(aa
                    , A, -1);
                if(B > A) ++cnt;
            }
        if(E == 0) Area[cnt] += pi * c[i].R * c[i].R;
        else{
            sort(eve, eve + E);
            eve[E] = eve[0];
            for(int j = 0; j < E; ++j){
                cnt += eve[j].add;
                Area[cnt] += cross(eve[j].p, eve[j + 1].p) *
                    .5;
                double theta = eve[j + 1].ang - eve[j].ang;
                if (theta < 0) theta += 2. * pi;
                Area[cnt] += (theta - sin(theta)) * c[i].R *
                    c[i].R * .5;
            }
        }
    }
}

```

8.13 3Dpoint*

```

struct Point {
    double x, y, z;
    Point(double _x = 0, double _y = 0, double _z = 0): x
        (_x), y(_y), z(_z){}
    Point(pdd p) { x = p.X, y = p.Y, z = abs2(p); }
};
Point operator-(const Point &p1, const Point &p2)
{ return Point(p1.x - p2.x, p1.y - p2.y, p1.z - p2.z);}
Point cross(const Point &p1, const Point &p2)
{ return Point(p1.y * p2.z - p1.z * p2.y, p1.z * p2.x -
    p1.x * p2.z, p1.x * p2.y - p1.y * p2.x);}
double dot(const Point &p1, const Point &p2)
{ return p1.x * p2.x + p1.y * p2.y + p1.z * p2.z;}
double abs(const Point &a)
{ return sqrt(dot(a, a));}
Point cross3(const Point &a, const Point &b, const
    Point &c)
{ return cross(b - a, c - a);}
double area(Point a, Point b, Point c)
{ return abs(cross3(a, b, c));}
double volume(Point a, Point b, Point c, Point d)
{return dot(cross3(a, b, c), d - a);}
pdd proj(Point a, Point b, Point c, Point u) {
    // proj. u to the plane of a, b, and c
    Point e1 = b - a;
    Point e2 = c - a;
    e1 = e1 / abs(e1);

```

```

    e2 = e2 - e1 * dot(e2, e1);
    e2 = e2 / abs(e2);
    Point p = u - a;
    return pdd(dot(p, e1), dot(p, e2));
}

```

8.14 Convexhull3D*

```

struct CH3D {
    struct face{int a, b, c; bool ok;} F[8 * N];
    double dblcmp(Point &p, face &f)
    {return dot(cross3(P[f.a], P[f.b], P[f.c]), p - P[f.a]);}
    int g[N][N], num, n;
    Point P[N];
    void deal(int p, int a, int b) {
        int f = g[a][b];
        face add;
        if (F[f].ok) {
            if (dblcmp(P[p], F[f]) > eps) dfs(p, f);
            else
                add.a = b, add.b = a, add.c = p, add.ok = 1, g[p][b] = g[a][p] = g[b][a] = num, F[num++] = add;
        }
    }
    void dfs(int p, int now) {
        F[now].ok = 0;
        deal(p, F[now].b, F[now].a), deal(p, F[now].c, F[now].b), deal(p, F[now].a, F[now].c);
    }
    bool same(int s, int t){
        Point &a = P[F[s].a];
        Point &b = P[F[s].b];
        Point &c = P[F[s].c];
        return fabs(volume(a, b, c, P[F[t].a])) < eps &&
            fabs(volume(a, b, c, P[F[t].b])) < eps && fabs(
                volume(a, b, c, P[F[t].c])) < eps;
    }
    void init(int _n){n = _n, num = 0;}
    void solve() {
        face add;
        num = 0;
        if(n < 4) return;
        if([&](){
            for (int i = 1; i < n; ++i)
                if (abs(P[0] - P[i]) > eps)
                    return swap(P[1], P[i]), 0;
            return 1;
        }() || [&](){
            for (int i = 2; i < n; ++i)
                if (abs(cross3(P[i], P[0], P[1])) > eps)
                    return swap(P[2], P[i]), 0;
            return 1;
        }() || [&](){
            for (int i = 3; i < n; ++i)
                if (fabs(dot(cross(P[0] - P[1], P[1] - P[2]), P[0] - P[i])) > eps)
                    return swap(P[3], P[i]), 0;
            return 1;
        }())return;
        for (int i = 0; i < 4; ++i) {
            add.a = (i + 1) % 4, add.b = (i + 2) % 4, add.c = (i + 3) % 4, add.ok = true;
            if (dblcmp(P[i], add) > 0) swap(add.b, add.c);
            g[add.a][add.b] = g[add.b][add.c] = g[add.c][add.a] = num;
            F[num++] = add;
        }
        for (int i = 4; i < n; ++i)
            for (int j = 0; j < num; ++j)
                if (F[j].ok && dblcmp(P[i], F[j]) > eps) {
                    dfs(i, j);
                    break;
                }
        for (int tmp = num, i = (num = 0); i < tmp; ++i)
            if (F[i].ok) F[num++] = F[i];
    }
    double get_area() {
        double res = 0.0;

```

```

        if (n == 3)
            return abs(cross3(P[0], P[1], P[2])) / 2.0;
        for (int i = 0; i < num; ++i)
            res += area(P[F[i].a], P[F[i].b], P[F[i].c]);
        return res / 2.0;
    }
    double get_volume() {
        double res = 0.0;
        for (int i = 0; i < num; ++i)
            res += volume(Point(0, 0, 0), P[F[i].a], P[F[i].b], P[F[i].c]);
        return fabs(res / 6.0);
    }
    int triangle() {return num;}
    int polygon() {
        int res = 0;
        for (int i = 0, flag = 1; i < num; ++i, res += flag, flag = 1)
            for (int j = 0; j < i && flag; ++j)
                flag &= !same(i, j);
        return res;
    }
    Point getcent(){
        Point ans(0, 0, 0), temp = P[F[0].a];
        double v = 0.0, t2;
        for (int i = 0; i < num; ++i)
            if (F[i].ok == true) {
                Point p1 = P[F[i].a], p2 = P[F[i].b], p3 = P[F[i].c];
                t2 = volume(temp, p1, p2, p3) / 6.0;
                if (t2 > 0)
                    ans.x += (p1.x + p2.x + p3.x + temp.x) * t2,
                    ans.y += (p1.y + p2.y + p3.y + temp.y) * t2,
                    ans.z += (p1.z + p2.z + p3.z + temp.z) * t2, v += t2;
            }
        ans.x /= (4 * v), ans.y /= (4 * v), ans.z /= (4 * v);
        return ans;
    }
    double pointmindis(Point p) {
        double rt = 99999999;
        for(int i = 0; i < num; ++i)
            if(F[i].ok == true) {
                Point p1 = P[F[i].a], p2 = P[F[i].b], p3 = P[F[i].c];
                double a = (p2.y - p1.y) * (p3.z - p1.z) - (p2.z - p1.z) * (p3.y - p1.y);
                double b = (p2.z - p1.z) * (p3.x - p1.x) - (p2.x - p1.x) * (p3.z - p1.z);
                double c = (p2.x - p1.x) * (p3.y - p1.y) - (p2.y - p1.y) * (p3.x - p1.x);
                double d = 0 - (a * p1.x + b * p1.y + c * p1.z);
                double temp = fabs(a * p.x + b * p.y + c * p.z + d) / sqrt(a * a + b * b + c * c);
                rt = min(rt, temp);
            }
        return rt;
    }
};

```

8.15 Tangent line of two circles

```

vector<Line> go( const Cir& c1 , const Cir& c2 , int sign1 ){
    // sign1 = 1 for outer tang, -1 for inter tang
    vector<Line> ret;
    double d_sq = norm2( c1.O - c2.O );
    if( d_sq < eps ) return ret;
    double d = sqrt( d_sq );
    Pt v = ( c2.O - c1.O ) / d;
    double c = ( c1.R - sign1 * c2.R ) / d;
    if( c * c > 1 ) return ret;
    double h = sqrt( max( 0.0 , 1.0 - c * c ) );
    for( int sign2 = 1 ; sign2 >= -1 ; sign2 -= 2 ){
        Pt n = { v.X * c - sign2 * h * v.Y ,
            v.Y * c + sign2 * h * v.X };
        Pt p1 = c1.O + n * c1.R;
        Pt p2 = c2.O + n * ( c2.R * sign1 );

```



```

    if( fabs( p1.X - p2.X ) < eps and
        fabs( p1.Y - p2.Y ) < eps )
        p2 = p1 + perp( c2.0 - c1.0 );
    ret.push_back( { p1 , p2 } );
}
return ret;
}

```

8.16 minMaxEnclosingRectangle

```

pdd solve(vector<p11> &dots){
    vector<p11> hull;
    const double INF=1e18,qi=acos(-1)/2*3;
    cv.dots=dots;
    hull=cv.hull();
    double Max=0,Min=INF,deg;
    ll n=hull.size();
    hull.pb(hull[0]);
    for(int i=0,u=1,r=1,l;i<n;++i){
        p11 nw=hull[i+1]-hull[i];
        while(cross(nw,hull[u+1]-hull[i])>cross(nw,hull[u]-hull[i]))
            u=(u+1)%n;
        while(dot(nw,hull[r+1]-hull[i])>dot(nw,hull[r]-hull[i]))
            r=(r+1)%n;
        if(!i) l=(r+1)%n;
        while(dot(nw,hull[l+1]-hull[i])<dot(nw,hull[l]-hull[i]))
            l=(l+1)%n;
        Min=min(Min,(double)(dot(nw,hull[r]-hull[i])-dot(nw,hull[l]-hull[i]))*cross(nw,hull[u]-hull[i])/abs2(nw));
        deg=acos((double)dot(hull[r]-hull[l],hull[u]-hull[i])/abs(hull[r]-hull[l])/abs(hull[u]-hull[i]));
        deg=(qi-deg)/2;
        Max=max(Max,(double)abs(hull[r]-hull[l])*abs(hull[u]-hull[i])*sin(deg)*sin(deg));
    }
    return pdd(Min,Max);
}

```

8.17 minDistOfTwoConvex

```

// p, q is convex
double TwoConvexHullMinDist(Point P[], Point Q[], int n, int m) {
    int YMinP = 0, YMaxQ = 0;
    double tmp, ans = 999999999;
    for (i = 0; i < n; ++i) if (P[i].y < P[YMinP].y) YMinP = i;
    for (i = 0; i < m; ++i) if (Q[i].y > Q[YMaxQ].y) YMaxQ = i;
    P[n] = P[0], Q[m] = Q[0];
    for (int i = 0; i < n; ++i) {
        while (tmp = Cross(Q[YMaxQ + 1] - P[YMinP + 1], P[YMinP] - P[YMinP + 1]) > Cross(Q[YMaxQ] - P[YMinP + 1], P[YMinP] - P[YMinP + 1])) YMaxQ = (YMaxQ + 1) % m;
        if (tmp < 0) ans = min(ans, PointToSegDist(P[YMinP], P[YMinP + 1], Q[YMaxQ]));
        else ans = min(ans, TwoSegMinDist(P[YMinP], P[YMinP + 1], Q[YMaxQ], Q[YMaxQ + 1]));
        YMinP = (YMinP + 1) % n;
    }
    return ans;
}

```

8.18 Minkowski Sum*

```

vector<p11> Minkowski(vector<p11> A, vector<p11> B) {
    hull(A), hull(B);
    vector<p11> C(1, A[0] + B[0]), s1, s2;
    for(int i = 0; i < SZ(A); ++i)
        s1.pb(A[(i + 1) % SZ(A)] - A[i]);
    for(int i = 0; i < SZ(B); i++)

```

```

        s2.pb(B[(i + 1) % SZ(B)] - B[i]);
    for(int p1 = 0, p2 = 0; p1 < SZ(A) || p2 < SZ(B);)
        if (p2 >= SZ(B) || (p1 < SZ(A) && cross(s1[p1], s2[p2]) >= 0))
            C.pb(C.back() + s1[p1++]);
        else
            C.pb(C.back() + s2[p2++]);
    return hull(C), C;
}

```

8.19 RotatingSweepLine

```

void rotatingSweepLine(vector<pii> &ps) {
    int n = SZ(ps);
    vector<int> id(n), pos(n);
    vector<pii> line(n * (n - 1) / 2);
    int m = 0;
    for (int i = 0; i < n; ++i)
        for (int j = i + 1; j < n; ++j)
            line[m++] = pii(i,j);
    sort(ALL(line), [&](const pii &a, const pii &b)->
        bool {
            if (ps[a.X].X == ps[a.Y].X)
                return 0;
            if (ps[b.X].X == ps[b.Y].X)
                return 1;
            return (double)(ps[a.X].Y - ps[a.Y].Y) / (ps[a.X].X - ps[a.Y].X) < (double)(ps[b.X].Y - ps[b.Y].Y) / (ps[b.X].X - ps[b.Y].X);
        });
    iota(id, id + n, 0);
    sort(ALL(id), [&](const int &a, const int &b){ return ps[a] < ps[b]; });
    for (int i = 0; i < n; ++i) pos[id[i]] = i;
    for (int i = 0; i < m; ++i) {
        auto l = line[i];
        // meow
        tie(pos[l.X], pos[l.Y], id[pos[l.X]], id[pos[l.Y]]) = make_tuple(pos[l.Y], pos[l.X], l.Y, l.X);
    }
}

```

9 Else

9.1 Mo' s Alogrithm(With modification)

```

struct QUERY{//BLOCK=N^{2/3}
    int L,R,id,LBId,RBId,T;
    QUERY(int l,int r,int id,int lb,int rb,int t):
        L(l),R(r),id(id),LBId(lb),RBId(rb),T(t){}
    bool operator<(const QUERY &b)const{
        if(LBId!=b.LBId) return LBId<b.LBId;
        if(RBId!=b.RBId) return RBId<b.RBId;
        return T<b.T;
    }
};
vector<QUERY> query;
int cur_ans,arr[MAXN],ans[MAXN];
void addTime(int L,int R,int T){}
void subTime(int L,int R,int T){}
void add(int x){}
void sub(int x){}
void solve(){
    sort(ALL(query));
    int L=0,R=0,T=-1;
    for(auto q:query){
        while(T<q.T) addTime(L,R,++T);
        while(T>q.T) subTime(L,R,T--);
        while(R<q.R) add(arr[++R]);
        while(L>q.L) add(arr[--L]);
        while(R>q.R) sub(arr[R--]);
        while(L<q.L) sub(arr[L--]);
        ans[q.id]=cur_ans;
    }
}

```

9.2 Mo' s Alogrithm On Tree

```
const int MAXN=40005;
vector<int> G[MAXN]; //1-base
int n,B,arr[MAXN],ans[100005],cur_ans;
int in[MAXN],out[MAXN],dfn[MAXN*2],dft;
int deep[MAXN],sp[___lg(MAXN*2)+1][MAXN*2],bln[MAXN],spt;
bitset<MAXN> inset;
struct QUERY{
    int L,R,Lid,id,lca;
    QUERY(int l,int r,int _id):L(l),R(r),lca(0),id(_id){}
    bool operator<(const QUERY &b){
        if(Lid!=b.Lid) return Lid<b.Lid;
        return R<b.R;
    }
};
vector<QUERY> query;
void dfs(int u,int f,int d){
    deep[u]=d,sp[0][spt]=u,bln[u]=spt++;
    dfn[dft]=u,in[u]=dft++;
    for(int v:G[u])
        if(v!=f)
            dfs(v,u,d+1),sp[0][spt]=u,bln[u]=spt++;
    dfn[dft]=u,out[u]=dft++;
}
int lca(int u,int v){
    if(bln[u]>bln[v]) swap(u,v);
    int t=___lg(bln[v]-bln[u]+1);
    int a=sp[t][bln[u]],b=sp[t][bln[v]-(1<<t)+1];
    if(deep[a]<deep[b]) return a;
    return b;
}
void sub(int x){}
void add(int x){}
void flip(int x){
    if(inset[x]) sub(arr[x]);
    else add(arr[x]);
    inset[x]=~inset[x];
}
void solve(){
    B=sqrt(2*n),dft=spt=cur_ans=0,dfs(1,1,0);
    for(int i=1,x=2;x<2*n;++i,x<=1)
        for(int j=0;j+x<=2*n;++j)
            if(deep[sp[i-1][j]]<deep[sp[i-1][j+x/2]])
                sp[i][j]=sp[i-1][j];
            else sp[i][j]=sp[i-1][j+x/2];
    for(auto &q:query){
        int c=lca(q.L,q.R);
        if(c==q.L||c==q.R)
            q.L=out[c==q.L?q.R:q.L],q.R=out[c];
        else if(out[q.L]<in[q.R])
            q.lca=c,q.L=out[q.L],q.R=in[q.R];
        else q.lca=c,c=in[q.L],q.L=out[q.R],q.R=c;
        q.Lid=q.L/B;
    }
    sort(ALL(query));
    int L=0,R=-1;
    for(auto q:query){
        while(R<q.R) flip(dfn[++R]);
        while(L>q.L) flip(dfn[--L]);
        while(R>q.R) flip(dfn[R--]);
        while(L<q.L) flip(dfn[L++]);
        if(q.lca) add(arr[q.lca]);
        ans[q.id]=cur_ans;
        if(q.lca) sub(arr[q.lca]);
    }
}
```

9.3 DynamicConvexTrick*

```
// only works for integer coordinates!!
struct Line {
    mutable ll a, b, p;
    bool operator<(const Line &rhs) const { return a <
        rhs.a; }
    bool operator<(ll x) const { return p < x; }
};
struct DynamicHull : multiset<Line, less<>> {
```

```
static const ll kInf = 1e18;
ll Div(ll a, ll b) { return a / b - ((a ^ b) < 0 &&
    a % b); }
bool isect(iterator x, iterator y) {
    if (y == end()) { x -> p = kInf; return 0; }
    if (x -> a == y -> a) x -> p = x -> b > y -> b
        ? kInf : -kInf;
    else x -> p = Div(y -> b - x -> b, x -> a - y
        -> a);
    return x -> p >= y -> p;
}
void addline(ll a, ll b) {
    auto z = insert({a, b, 0}), y = z++, x = y;
    while (isect(y, z)) z = erase(z);
    if (x != begin() && isect(--x, y)) isect(x, y =
        erase(y));
    while ((y = x) != begin() && (--x) -> p >= y ->
        p) isect(x, erase(y));
}
ll query(ll x) {
    auto l = *lower_bound(x);
    return l.a * x + l.b;
}
};
```

9.4 Matroid Intersection

Start from $S = \emptyset$. In each iteration, let

- $Y_1 = \{x \notin S \mid S \cup \{x\} \in I_1\}$
- $Y_2 = \{x \notin S \mid S \cup \{x\} \in I_2\}$

If there exists $x \in Y_1 \cap Y_2$, insert x into S . Otherwise for each $x \in S, y \notin S$, create edges

- $x \rightarrow y$ if $S - \{x\} \cup \{y\} \in I_1$.
- $y \rightarrow x$ if $S - \{x\} \cup \{y\} \in I_2$.

Find a shortest path (with BFS) starting from a vertex in Y_1 and ending at a vertex in Y_2 which doesn't pass through any other vertices in Y_2 , and alternate the path. The size of S will be incremented by 1 in each iteration. For the weighted case, assign weight $w(x)$ to vertex x if $x \in S$ and $-w(x)$ if $x \notin S$. Find the path with the minimum number of edges among all minimum length paths and alternate it.

9.5 AdaptiveSimpson

```
using F_t = function<double(double)>;
pdd simpson(const F_t &f, double l, double r,
    double fl, double fr, double fm = nan("")) {
    if (isnan(fm)) fm = f((l + r) / 2);
    return {fm, (r - l) / 6 * (fl + 4 * fm + fr)};
}
double simpson_ada(const F_t &f, double l, double r,
    double fl, double fm, double fr, double eps) {
    double m = (l + r) / 2,
        s = simpson(f, l, r, fl, fr, fm).second;
    auto [flm, sl] = simpson(f, l, m, fl, fm);
    auto [fmr, sr] = simpson(f, m, r, fm, fr);
    double delta = sl + sr - s;
    if (abs(delta) <= 15 * eps)
        return sl + sr + delta / 15;
    return simpson_ada(f, l, m, fl, flm, fm, eps / 2) +
        simpson_ada(f, m, r, fm, fmr, fr, eps / 2);
}
double simpson_ada(const F_t &f, double l, double r) {
    return simpson_ada(
        f, l, r, f(l), f((l + r) / 2), f(r), 1e-9 / 7122);
}
double simpson_ada2(const F_t &f, double l, double r) {
    double h = (r - l) / 7122, s = 0;
    for (int i = 0; i < 7122; ++i, l += h)
        s += simpson_ada(f, l, l + h);
    return s;
}
```

10 2A1B

10.1 run.sh

```
rm -f a.out
g++ -std=c++11 -O2 -Wextra -Wall $1

if [ -f "a.out" ]; then
    echo "[successfully compiled]"
    ./a.out
else
    echo "[compile error]"
fi
```

10.2 vimrc

```
" for 204 (ubuntu)
inoremap {<ENTER> {<LEFT><ENTER><ENTER><UP><TAB>
se nu ai hls et ru is is sc cul
se re=1 ts=4 sts=4 sw=4 ls=2 mouse=a
syntax on
hi CursorLine cterm=NONE ctermbg=237
set bg=dark

set whichwrap+=<,>,h,l,[,]
set selection=exclusive
vnoremap < <gv
vnoremap > >gv
```

10.3 Binary Indexed Tree.cpp

```
int sum(int i) {
    int s = 0;
    while (i > 0) {
        s += bit[i];
        i -= i & -i;
    }
    return s;
}

void add(int i, int x) {
    while (i <= n) {
        bit[i] += x;
        i += i & -i;
    }
}
```

10.4 Bipartite Matching.cpp

```
// x nodes are numbered 1 to n, y nodes are numbered n
// +1 to n+m
// g[X].push_back[Y] / g[u].push_back(nx + v)
vector<int> g[200007];
int nx, ny, ma[200007], d[200007];
bool bfs() {
    int i, u, v, len;
    queue<int> Q;
    for(i=1; i<=nx; i++) {
        if(ma[i]==0) {
            d[i] = 0;
            Q.push(i);
        }
        else d[i] = INF;
    }
    d[0] = INF;
    while(!Q.empty()) {
        u = Q.front(); Q.pop();
        if(u!=0) {
            len = g[u].size();
            for(i=0; i<len; i++) {
                v = g[u][i];
                if(d[ma[v]]==INF) {
                    d[ma[v]] = d[u] + 1;
                    Q.push(ma[v]);
                }
            }
        }
    }
    return (d[0]!=INF);
}

bool dfs(int u){
```

```
int i, v, len;
if(u!=0) {
    len = g[u].size();
    for(i=0; i<len; i++) {
        v = g[u][i];
        if(d[ma[v]]==d[u]+1) {
            if(dfs(ma[v])) {
                ma[v] = u;
                ma[u] = v;
                return true;
            }
        }
    }
    d[u] = INF;
    return false;
}
return true;
}

int hopcroft_karp() {
    int res = 0, i;
    while(bfs())
        for(i=1; i<=nx; i++)
            if(ma[i]==0 && dfs(i))
                res++;
    return res;
}
```

10.5 Closest Pair.cpp

```
pair<double, double> p[50007], t[50007];
double solve(int l, int r) {
    if (l == r) return INF;
    int mid = (l + r) >> 1;
    double x = p[mid].first;
    double d = min(solve(l, mid), solve(mid + 1, r));
    int i = l, j = mid + 1, id = 1;
    while (i <= mid || j <= r) {
        if (i <= mid && (j > r || p[i].second < p[j].second))
            t[id++] = p[i++];
        else t[id++] = p[j++];
    }
    for (int i = l; i <= r; i++) p[i] = t[i];
    vector<pair<double, double> > v;
    for (int i = l; i <= r; i++) if (abs(p[i].first - x) < d) v.push_back(p[i]);
    for (int i = 0; i < v.size(); i++) {
        for (int j = i + 1; j < v.size(); j++) {
            if (v[j].second - v[i].second >= d) break;
            d = min(d, sqrt((v[i].first - v[j].first) * (v[i].first - v[j].first) + (v[i].second - v[j].second) * (v[i].second - v[j].second)));
        }
    }
    return d;
}

main(){
    sort(p + 1, p + n + 1);
    solve(1, n);
}
```

10.6 Dijkstra.cpp

```
// luogu4779
vector<pii> edge[100020];
int dis[100020];
int vis[100020];
void dijkstra(int s)
{
    CLR(dis, 0x3f);
    dis[s] = 0;
    priority_queue<pii, vector<pii>, greater<pii>> pq;
    pq.emplace(0, s);
    while(pq.size())
    {
        int now = pq.top().Y;
        pq.pop();
        if(vis[now]) continue;
        vis[now] = 1;
```

```

    for(pii e:edge[now])
    {
        if(!vis[e.X]&&dis[e.X]>dis[now]+e.Y)
        {
            dis[e.X]=dis[now]+e.Y;
            pq.emplace(dis[e.X],e.X);
        }
    }
}
}

```

10.7 Dinic.cpp

```

struct MaxFlow { // 0-base
    struct edge {
        int to, cap, flow, rev;
    };
    vector<edge> G[MAXN];
    int s, t, dis[MAXN], cur[MAXN], n;
    int dfs(int u, int cap) {
        if (u == t || !cap) return cap;
        for (int &i = cur[u]; i < (int)G[u].size(); ++i) {
            edge &e = G[u][i];
            if (dis[e.to] == dis[u] + 1 && e.flow != e.cap) {
                int df = dfs(e.to, min(e.cap - e.flow, cap));
                if (df) {
                    e.flow += df;
                    G[e.to][e.rev].flow -= df;
                    return df;
                }
            }
        }
        dis[u] = -1;
        return 0;
    }
    bool bfs() {
        FILL(dis, -1);
        queue<int> q;
        q.push(s), dis[s] = 0;
        while (!q.empty()) {
            int tmp = q.front();
            q.pop();
            for (auto &u : G[tmp])
                if (!dis[u.to] && u.flow != u.cap) {
                    q.push(u.to);
                    dis[u.to] = dis[tmp] + 1;
                }
        }
        return dis[t] != -1;
    }
    int maxflow(int _s, int _t) {
        s = _s, t = _t;
        int flow = 0, df;
        while (bfs()) {
            FILL(cur, 0);
            while (df = dfs(s, INF)) flow += df;
        }
        return flow;
    }
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i) G[i].clear();
    }
    void reset() {
        for (int i = 0; i < n; ++i)
            for (auto &j : G[i]) j.flow = 0;
    }
    void add_edge(int u, int v, int cap) {
        G[u].pb(edge{v, cap, 0, (int)G[v].size()});
        G[v].pb(edge{u, 0, 0, (int)G[u].size() - 1});
    }
};

```

10.8 Kosaraju.cpp

```

vector<pii>edge[100020],redge[100020];
int vis[100020],scc[100020];
void dfs1(int x,vector<int>&stk){

```

```

    vis[x]=1;
    for(pii i:edge[x])
        if(!vis[i.X])dfs1(i.X,stk);
    stk.emplace_back(x);
}
void dfs2(int x,int id){
    scc[x]=id;
    for(pii i:redge[x])
        if(!scc[i.X])dfs2(i.X,id);
}
void kosaraju(){
    int nsc=0;
    vector<int>stk;
    for(int i=1;i<=n;i++)
        if(!vis[i])dfs1(i,stk);
    while(stk.size()){
        if(!scc[stk.back()])
            dfs2(stk.back(),++nsc);
        stk.pop_back();
    }
}

```

10.9 Segment Tree with tag.cpp

```

void push(int id, int l, int r) {
    int mid = (l + r) >> 1;
    seg[id * 2] += tag[id] * (mid - l + 1);
    seg[id * 2 + 1] += tag[id] * (r - mid);
    tag[id * 2] += tag[id];
    tag[id * 2 + 1] += tag[id];
    tag[id] = 0;
}
void modify(int id, int l, int r, int ql, int qr, int val) {
    if (ql > r || qr < l) return;
    if (ql <= l && r <= qr) {
        seg[id] += val * (r - l + 1);
        tag[id] += val;
        return;
    }
    if (l == r) return;
    push(id, l, r);
    int mid = (l + r) >> 1;
    modify(id * 2, l, mid, ql, qr, val);
    modify(id * 2 + 1, mid + 1, r, ql, qr, val);
    seg[id] = seg[id * 2] + seg[id * 2 + 1];
}
int query(int id, int l, int r, int ql, int qr) {
    if (ql > r || qr < l) return 0;
    if (ql <= l && r <= qr) return seg[id];
    push(id, l, r);
    int mid = (l + r) >> 1;
    return query(id * 2, l, mid, ql, qr) + query(id * 2 + 1, mid + 1, r, ql, qr);
}

```

10.10 Suffix Array.cpp

```

// array c is eventually equal to the position of the
// suffixes in the suffix array
// don't add another '$' to the string
int sa[400007], c[400007], sa_new[400007], c_new[400007], cnt[400007], pos[400007], lcp[400007];
pair<char, int> P[400007];
void calc_suffix_array(string s) {
    s += '$';
    int n = s.size();
    for (int i = 0; i < n; i++) P[i] = {s[i], i};
    sort(P, P + n);
    for (int i = 0; i < n; i++) sa[i] = P[i].second;
    c[sa[0]] = 0;
    for (int i = 1; i < n; i++) c[sa[i]] = c[sa[i - 1]] + (P[i].first > P[i - 1].first ? 1 : 0);
    int k = 1;
    while (k < n) {
        for (int i = 0; i < n; i++) sa[i] = (sa[i] - k + n) % n;
        for (int i = 0; i < n; i++) cnt[i] = 0;

```

```

    for (int i = 0; i < n; i++) cnt[c[i]]++;
    pos[0] = cnt[0] - 1;
    for (int i = 1; i < n; i++) pos[i] = pos[i - 1] +
        cnt[i];
    for (int i = n - 1; i >= 0; i--) sa_new[pos[c[sa[i]
        ]]]-- = sa[i];
    for (int i = 0; i < n; i++) sa[i] = sa_new[i];
    c_new[sa[0]] = 0;
    for (int i = 1; i < n; i++) {
        c_new[sa[i]] = c_new[sa[i - 1]];
        pair<int, int> prev = {c[sa[i - 1]], c[(sa[i - 1]
            + k) % n]};
        pair<int, int> now = {c[sa[i]], c[(sa[i] + k) % n
            ]};
        if (now > prev) c_new[sa[i]]++;
    }
    for (int i = 0; i < n; i++) c[i] = c_new[i];
    k *= 2;
}
}
void calc_lcp_array(string s) {
    int n = s.size(), k = 0;
    for (int i = 0; i < n; i++) {
        int j = sa[c[i] - 1];
        while (i + k < n && j + k < n && s[i + k] == s[j +
            k]) k++;
        lcp[c[i] - 1] = k;
        k = max(k - 1, 011);
    }
}
}

```

10.11 Treap.cpp

```

// Zerojudge a063: subsequence reversal
struct Treap {
    Treap *lc, *rc;
    int pri, sz, val;
    bool tag;
    Treap (int x) {
        lc = rc = NULL;
        pri = rand(), sz = 1, val = x, tag = 0;
    }
};
inline int size(Treap* t) {
    return t ? t->sz : 0;
}
inline void pull(Treap* t) {
    t->sz = size(t->lc) + 1 + size(t->rc);
}
void push(Treap* t) {
    if (t->tag) {
        swap(t->lc, t->rc);
        if (t->lc) t->lc->tag = !t->lc->tag;
        if (t->rc) t->rc->tag = !t->rc->tag;
        t->tag = 0;
    }
}
Treap* merge(Treap* a, Treap* b) {
    if (!a || !b) return a ? a : b;
    if (a->pri > b->pri) {
        push(a);
        a->rc = merge(a->rc, b);
        pull(a);
        return a;
    }
    else {
        push(b);
        b->lc = merge(a, b->lc);
        pull(b);
        return b;
    }
}
void split(Treap* t, int k, Treap *&a, Treap *&b) {
    if (!t) a = b = NULL;
    else {
        push(t);
        if (size(t->lc) + 1 <= k) {
            a = t;
            split(t->rc, k - size(t->lc) - 1, a->rc, b);
            pull(a);
        }
    }
}

```

```

    }
    else {
        b = t;
        split(t->lc, k, a, b->lc);
        pull(b);
    }
}
}

```

10.12 Simple Graph Matching.cpp

```

#include <bits/stdc++.h>
using namespace std;
#define FOR(i,a,b) for(int i=a;i<=b;i++)
#define REP(u) for(int i=h[u],v=v[e[i].t,i;i=e[i].n)
const int N=520,M=2e5+1;
queue<int>q;
int n,m,tot,qwq,ans;
int h[N],lk[N],tag[N],fa[N],pre[N],dfn[N];
struct edge{int t,n;e[M];
void link(int x,int y){lk[x]=y,lk[y]=x;}
void add_edge(int x,int y){
    if(!lk[x]&&!lk[y])link(x,y),ans++;
    e[++tot]=(edge){y,h[x]},h[x]=tot;
    e[++tot]=(edge){x,h[y]},h[y]=tot;
}
void rev(int x){if(x)rev(x[pre][lk]),link(x,pre[x]);}
int find(int x){return fa[x]==x?x:fa[x]=find(fa[x]);}
int lca(int x,int y){
    for(qwq++;x=x[lk][pre],swap(x,y))
        if(dfn[x=find(x)]=qwq)return x;
    else if(x)dfn[x]=qwq;
}
int shrink(int x,int y,int p){
    for(;find(x)!=p;x=pre[y]){
        pre[x]=y,y=lk[x],fa[x]=fa[y]=p;
        if(tag[y]==2)tag[y]=1,q.push(y);
    }
}
int blossom(int u){
    FOR(i,1,n)tag[i]=pre[i]=0,fa[i]=i;
    tag[u]=1,q=queue<int>(),q.push(u);
    for(int p;!q.empty();q.pop())REP(u=q.front())
        if(tag[v]==1)
            p=lca(u,v),shrink(u,v,p),shrink(v,u,p);
        else if(!tag[v]){
            pre[v]=u,tag[v]=2;
            if(!lk[v])return rev(v),1;
            else tag[lk[v]]=1,q.push(lk[v]);
        }
    return 0;
}
int main(){
    scanf("%d%d",&n,&m);
    for(int x,y;m--;add_edge(x,y))scanf("%d%d",&x,&y);
    FOR(i,1,n)ans+=!lk[i]&&blossom(i);
    cout<<ans<<'\n';
    for(int i=1;i<=n;i++)cout<<i<<' '<<lk[i]<<'\n'
    return 0;
}

```