Contents

1 Basic

1.1 Shell script

```
g++ -O2 -std=c++17 -Dbbq -Wall -Wextra -Wshadow -o $1
    $1.cpp
chmod +x compile.sh
```

1.2 Default code

```
#include<bits/stdc++.h>
using namespace std;
typedef long long ll;
typedef pair<int, int> pii;
typedef pair<ll, ll> pll;
#define X first
#define Y second
#define SZ(a) ((int)a.size())
#define ALL(v) v.begin(), v.end()
#define pb push_back
```

1.3 vimrc

```
"This file should be placed at ~/.vimrc"
se nu ai hls et ru ic is sc cul
se re=1 ts=4 sts=4 sw=4 ls=2 mouse=a
syntax on
hi cursorline cterm=none ctermbg=89
set bg=dark
inoremap {<CR> {<CR>}<Esc>ko<tab>
```

1.4 readchar

```
inline char readchar() {
  static const size_t bufsize = 65536;
  static char buf[bufsize];
  static char *p = buf, *end = buf;
  if (p == end) end = buf + fread_unlocked(buf, 1,
      bufsize, stdin), p = buf;
  return *p++;
}
```

1.5 Black Magic

```
#include <ext/pb_ds/priority_queue.hpp>
#include <ext/pb_ds/assoc_container.hpp> // rb_tree
#include <ext/rope> // rope
using namespace __gnu_pbds;
using namespace __gnu_cxx; // rope
typedef __gnu_pbds::priority_queue<int> heap;
int main() {
  heap h1, h2; // max heap
  h1.push(1), h1.push(3), h2.push(2), h2.push(4);
  h1.join(h2); // h1 = {1, 2, 3, 4}, h2 = {};
  tree<ll, null_type, less<ll>, rb_tree_tag,
      tree_order_statistics_node_update> st;
  tree<ll, ll, less<ll>, rb_tree_tag,
      tree_order_statistics_node_update> mp;
  for (int x : {0, 2, 3, 4}) st.insert(x);
```

```
  cout << *st.find_by_order(2) << st.order_of_key(1) <<
       endl; //31
  rope<char> *root[10]; // nsqrt(n)
  root[0] = new rope<char>();
  root[1] = new rope<char>(*root[0]);
  // root[1]->insert(pos, 'a');
  // root[1]->at(pos); 0-base
  // root[1]->erase(pos, size);
}
// __int128_t,__float128_t
// for (int i = bs._Find_first(); i < bs.size(); i = bs
    ._Find_next(i));
```

2 Graph

2.1 BCC Vertex*

```
vector<int> G[N]; // 1-base
vector<int> nG[N], bcc[N];
int low[N], dfn[N], Time;
int bcc_id[N], bcc_cnt; // 1-base
bool is_cut[N]; // whether is av
bool cir[N];
int st[N], top;

void dfs(int u, int pa = -1) {
  int child = 0;
  low[u] = dfn[u] = ++Time;
  st[top++] = u;
  for (int v : G[u])
    if (!dfn[v]) {
      dfs(v, u), ++child;
      low[u] = min(low[u], low[v]);
      if (dfn[u] <= low[v]) {
        is_cut[u] = 1;
        bcc[++bcc_cnt].clear();
        int t;
        do {
          bcc_id[t = st[--top]] = bcc_cnt;
          bcc[bcc_cnt].push_back(t);
        } while (t != v);
        bcc_id[u] = bcc_cnt;
        bcc[bcc_cnt].pb(u);
      }
    } else if (dfn[v] < dfn[u] && v != pa)
      low[u] = min(low[u], dfn[v]);
  if (pa == -1 && child < 2) is_cut[u] = 0;
}

void bcc_init(int n) {
  Time = bcc_cnt = top = 0;
  for (int i = 1; i <= n; ++i)
    G[i].clear(), dfn[i] = bcc_id[i] = is_cut[i] = 0;
}

void bcc_solve(int n) {
  for (int i = 1; i <= n; ++i)
    if (!dfn[i]) dfs(i);
  // block-cut tree
  for (int i = 1; i <= n; ++i)
    if (is_cut[i])
      bcc_id[i] = ++bcc_cnt, cir[bcc_cnt] = 1;
  for (int i = 1; i <= bcc_cnt && !cir[i]; ++i)
    for (int j : bcc[i])
      if (is_cut[j])
        nG[i].pb(bcc_id[j]), nG[bcc_id[j]].pb(i);
}
```

2.2 Bridge*

```
int low[N], dfn[N], Time; // 1-base
vector<pii> G[N], edge;
vector<bool> is_bridge;

void init(int n) {
  Time = 0;
  for (int i = 1; i <= n; ++i)
    G[i].clear(), low[i] = dfn[i] = 0;
}

void add_edge(int a, int b) {
  G[a].pb(pii(b, SZ(edge))), G[b].pb(pii(a, SZ(edge)));
  edge.pb(pii(a, b));
}

void dfs(int u, int f) {
  dfn[u] = low[u] = ++Time;
  for (auto i : G[u])
    if (!dfn[i.X])
      dfs(i.X, i.Y), low[u] = min(low[u], low[i.X]);
    else if (i.Y != f) low[u] = min(low[u], dfn[i.X]);
  if (low[u] == dfn[u] && f != -1) is_bridge[f] = 1;
}

void solve(int n) {
  is_bridge.resize(SZ(edge));
  for (int i = 1; i <= n; ++i)
    if (!dfn[i]) dfs(i, -1);
}
```

2.3 2SAT (SCC)*

```
struct SAT { // 0-base
  int low[N], dfn[N], bln[N], n, Time, nScc;
  bool instack[N], istrue[N];
  stack<int> st;
  vector<int> G[N], SCC[N];
  void init(int _n) {
    n = _n; // assert(n * 2 <= N);
    for (int i = 0; i < n + n; ++i) G[i].clear();
  }
  void add_edge(int a, int b) { G[a].pb(b); }
  int rv(int a) {
    if (a >= n) return a - n;
    return a + n;
  }
  void add_clause(int a, int b) {
    add_edge(rv(a), b), add_edge(rv(b), a);
  }
  void dfs(int u) {
    dfn[u] = low[u] = ++Time;
    instack[u] = 1, st.push(u);
    for (int i : G[u])
      if (!dfn[i])
        dfs(i), low[u] = min(low[i], low[u]);
      else if (instack[i] && dfn[i] < dfn[u])
        low[u] = min(low[u], dfn[i]);
    if (low[u] == dfn[u]) {
      int tmp;
      do {
        tmp = st.top(), st.pop();
        instack[tmp] = 0, bln[tmp] = nScc;
      } while (tmp != u);
      ++nScc;
    }
  }
  bool solve() {
    Time = nScc = 0;
    for (int i = 0; i < n + n; ++i)
      SCC[i].clear(), low[i] = dfn[i] = bln[i] = 0;
    for (int i = 0; i < n + n; ++i)
      if (!dfn[i]) dfs(i);
    for (int i = 0; i < n + n; ++i) SCC[bln[i]].pb(i);
    for (int i = 0; i < n; ++i) {
      if (bln[i] == bln[i + n]) return false;
      istrue[i] = bln[i] < bln[i + n];
      istrue[i + n] = !istrue[i];
    }
    return true;
  }
};
```

2.4 MinimumMeanCycle*

```
ll road[N][N]; // input here
struct MinimumMeanCycle {
  ll dp[N + 5][N], n;
  pll solve() {
    ll a = -1, b = -1, L = n + 1;
    for (int i = 2; i <= L; ++i)
```

```
      for (int k = 0; k < n; ++k)
        for (int j = 0; j < n; ++j)
          dp[i][j] =
            min(dp[i - 1][k] + road[k][j], dp[i][j]);
    for (int i = 0; i < n; ++i) {
      if (dp[L][i] >= INF) continue;
      ll ta = 0, tb = 1;
      for (int j = 1; j < n; ++j)
        if (dp[j][i] < INF &&
          ta * (L - j) < (dp[L][i] - dp[j][i]) * tb)
          ta = dp[L][i] - dp[j][i], tb = L - j;
      if (ta == 0) continue;
      if (a == -1 || a * tb > ta * b) a = ta, b = tb;
    }
    if (a != -1) {
      ll g = __gcd(a, b);
      return pll(a / g, b / g);
    }
    return pll(-1LL, -1LL);
  }
  void init(int _n) {
    n = _n;
    for (int i = 0; i < n; ++i)
      for (int j = 0; j < n; ++j) dp[i + 2][j] = INF;
  }
};
```

2.5 Virtual Tree*

```
vector<int> vG[N];
int top, st[N];

void insert(int u) {
  if (top == -1) return st[++top] = u, void();
  int p = LCA(st[top], u);
  if (p == st[top]) return st[++top] = u, void();
  while (top >= 1 && dep[st[top - 1]] >= dep[p])
    vG[st[top - 1]].pb(st[top]), --top;
  if (st[top] != p)
    vG[p].pb(st[top]), --top, st[++top] = p;
  st[++top] = u;
}

void reset(int u) {
  for (int i : vG[u]) reset(i);
  vG[u].clear();
}

void solve(vector<int> &v) {
  top = -1;
  sort(ALL(v),
    [&](int a, int b) { return dfn[a] < dfn[b]; });
  for (int i : v) insert(i);
  while (top > 0) vG[st[top - 1]].pb(st[top]), --top;
  // do something
  reset(v[0]);
}
```

2.6 Maximum Clique Dyn*

```
const int N = 150;
struct MaxClique { // Maximum Clique
  bitset<N> a[N], cs[N];
  int ans, sol[N], q, cur[N], d[N], n;
  void init(int _n) {
    n = _n;
    for (int i = 0; i < n; i++) a[i].reset();
  }
  void addEdge(int u, int v) { a[u][v] = a[v][u] = 1; }
  void csort(vector<int> &r, vector<int> &c) {
    int mx = 1, km = max(ans - q + 1, 1), t = 0,
        m = r.size();
    cs[1].reset(), cs[2].reset();
    for (int i = 0; i < m; i++) {
      int p = r[i], k = 1;
      while ((cs[k] & a[p]).count()) k++;
      if (k > mx) mx++, cs[mx + 1].reset();
      cs[k][p] = 1;
      if (k < km) r[t++] = p;
    }
    c.resize(m);
    if (t) c[t - 1] = 0;
    for (int k = km; k <= mx; k++)
      for (int p = cs[k]._Find_first(); p < N;
           p = cs[k]._Find_next(p))
        r[t] = p, c[t] = k, t++;
  }
  void dfs(vector<int> &r, vector<int> &c, int l,
    bitset<N> mask) {
    while (!r.empty()) {
      int p = r.back();
      r.pop_back(), mask[p] = 0;
      if (q + c.back() <= ans) return;
      cur[q++] = p;
      vector<int> nr, nc;
      bitset<N> nmask = mask & a[p];
      for (int i : r)
        if (a[p][i]) nr.push_back(i);
      if (!nr.empty()) {
        if (l < 4) {
          for (int i : nr)
            d[i] = (a[i] & nmask).count();
          sort(nr.begin(), nr.end(),
            [&](int x, int y) { return d[x] > d[y]; });
        }
        csort(nr, nc), dfs(nr, nc, l + 1, nmask);
      } else if (q > ans) ans = q, copy_n(cur, q, sol);
      c.pop_back(), q--;
    }
  }
  int solve(bitset<N> mask = bitset<N>(
              string(N, '1'))) { // vertex mask
    vector<int> r, c;
    ans = q = 0;
    for (int i = 0; i < n; i++)
      if (mask[i]) r.push_back(i);
    for (int i = 0; i < n; i++)
      d[i] = (a[i] & mask).count();
    sort(r.begin(), r.end(),
      [&](int i, int j) { return d[i] > d[j]; });
    csort(r, c), dfs(r, c, 1, mask);
    return ans; // sol[0 ~ ans-1]
  }
} graph;
```

2.7 Minimum Steiner Tree*

```
// Minimum Steiner Tree
// O(V 3^T + V^2 2^T)
struct SteinerTree { // 0-base
  static const int T = 10, N = 105, INF = 1e9;
  int n, dst[N][N], dp[1 << T][N], tdst[N];
  int vcost[N]; // the cost of vertexs
  void init(int _n) {
    n = _n;
    for (int i = 0; i < n; ++i) {
      for (int j = 0; j < n; ++j) dst[i][j] = INF;
      dst[i][i] = vcost[i] = 0;
    }
  }
  void add_edge(int ui, int vi, int wi) {
    dst[ui][vi] = min(dst[ui][vi], wi);
  }
  void shortest_path() {
    for (int k = 0; k < n; ++k)
      for (int i = 0; i < n; ++i)
        for (int j = 0; j < n; ++j)
          dst[i][j] =
            min(dst[i][j], dst[i][k] + dst[k][j]);
  }
  int solve(const vector<int> &ter) {
    shortest_path();
    int t = SZ(ter);
    for (int i = 0; i < (1 << t); ++i)
      for (int j = 0; j < n; ++j) dp[i][j] = INF;
    for (int i = 0; i < n; ++i) dp[0][i] = vcost[i];
    for (int msk = 1; msk < (1 << t); ++msk) {
      if (!(msk & (msk - 1))) {
        int who = __lg(msk);
        for (int i = 0; i < n; ++i)
          dp[msk][i] =
            vcost[ter[who]] + dst[ter[who]][i];
```

```
    }
    for (int i = 0; i < n; ++i)
      for (int submsk = (msk - 1) & msk; submsk;
           submsk = (submsk - 1) & msk)
        dp[msk][i] = min(dp[msk][i],
          dp[submsk][i] + dp[msk ^ submsk][i] -
            vcost[i]);
    for (int i = 0; i < n; ++i) {
      tdst[i] = INF;
      for (int j = 0; j < n; ++j)
        tdst[i] =
          min(tdst[i], dp[msk][j] + dst[j][i]);
    }
    for (int i = 0; i < n; ++i) dp[msk][i] = tdst[i];
  }
  int ans = INF;
  for (int i = 0; i < n; ++i)
    ans = min(ans, dp[(1 << t) - 1][i]);
  return ans;
  }
};
```

2.8 Dominator Tree*

```
struct dominator_tree { // 1-base
  vector<int> G[N], rG[N];
  int n, pa[N], dfn[N], id[N], Time;
  int semi[N], idom[N], best[N];
  vector<int> tree[N]; // dominator_tree
  void init(int _n) {
    n = _n;
    for (int i = 1; i <= n; ++i)
      G[i].clear(), rG[i].clear();
  }
  void add_edge(int u, int v) {
    G[u].pb(v), rG[v].pb(u);
  }
  void dfs(int u) {
    id[dfn[u] = ++Time] = u;
    for (auto v : G[u])
      if (!dfn[v]) dfs(v), pa[dfn[v]] = dfn[u];
  }
  int find(int y, int x) {
    if (y <= x) return y;
    int tmp = find(pa[y], x);
    if (semi[best[y]] > semi[best[pa[y]]])
      best[y] = best[pa[y]];
    return pa[y] = tmp;
  }
  void tarjan(int root) {
    Time = 0;
    for (int i = 1; i <= n; ++i) {
      dfn[i] = idom[i] = 0;
      tree[i].clear();
      best[i] = semi[i] = i;
    }
    dfs(root);
    for (int i = Time; i > 1; --i) {
      int u = id[i];
      for (auto v : rG[u])
        if (v = dfn[v]) {
          find(v, i);
          semi[i] = min(semi[i], semi[best[v]]);
        }
      tree[semi[i]].pb(i);
      for (auto v : tree[pa[i]]) {
        find(v, pa[i]);
        idom[v] =
          semi[best[v]] == pa[i] ? pa[i] : best[v];
      }
      tree[pa[i]].clear();
    }
    for (int i = 2; i <= Time; ++i) {
      if (idom[i] != semi[i]) idom[i] = idom[idom[i]];
      tree[id[idom[i]]].pb(id[i]);
    }
  }
};
```

2.9 Minimum Arborescence*

```
struct zhu_liu { // O(VE)
  struct edge {
    int u, v;
    ll w;
  };
  vector<edge> E; // 0-base
  int pe[N], id[N], vis[N];
  ll in[N];
  void init() { E.clear(); }
  void add_edge(int u, int v, ll w) {
    if (u != v) E.pb(edge{u, v, w});
  }
  ll build(int root, int n) {
    ll ans = 0;
    for (;;) {
      fill_n(in, n, INF);
      for (int i = 0; i < SZ(E); ++i)
        if (E[i].u != E[i].v && E[i].w < in[E[i].v])
          pe[E[i].v] = i, in[E[i].v] = E[i].w;
      for (int u = 0; u < n; ++u) // no solution
        if (u != root && in[u] == INF) return -INF;
      int cntnode = 0;
      fill_n(id, n, -1), fill_n(vis, n, -1);
      for (int u = 0; u < n; ++u) {
        if (u != root) ans += in[u];
        int v = u;
        while (vis[v] != u && !~id[v] && v != root)
          vis[v] = u, v = E[pe[v]].u;
        if (v != root && !~id[v]) {
          for (int x = E[pe[v]].u; x != v;
               x = E[pe[x]].u)
            id[x] = cntnode;
          id[v] = cntnode++;
        }
      }
      if (!cntnode) break; // no cycle
      for (int u = 0; u < n; ++u)
        if (!~id[u]) id[u] = cntnode++;
      for (int i = 0; i < SZ(E); ++i) {
        int v = E[i].v;
        E[i].u = id[E[i].u], E[i].v = id[E[i].v];
        if (E[i].u != E[i].v) E[i].w -= in[v];
      }
      n = cntnode, root = id[root];
    }
    return ans;
  }
};
```

2.10 Vizing's theorem

```
namespace vizing { // returns edge coloring in adjacent
                   // matrix G. 1 - based
int C[kN][kN], G[kN][kN];
void clear(int N) {
  for (int i = 0; i <= N; i++) {
    for (int j = 0; j <= N; j++) C[i][j] = G[i][j] = 0;
  }
}
void solve(vector<pair<int, int>> &E, int N, int M) {
  int X[kN] = {}, a;
  auto update = [&](int u) {
    for (X[u] = 1; C[u][X[u]]; X[u]++)
      ;
  };
  auto color = [&](int u, int v, int c) {
    int p = G[u][v];
    G[u][v] = G[v][u] = c;
    C[u][c] = v, C[v][c] = u;
    C[u][p] = C[v][p] = 0;
    if (p) X[u] = X[v] = p;
    else update(u), update(v);
    return p;
  };
  auto flip = [&](int u, int c1, int c2) {
    int p = C[u][c1];
    swap(C[u][c1], C[u][c2]);
    if (p) G[u][p] = G[p][u] = c2;
    if (!C[u][c1]) X[u] = c1;
    if (!C[u][c2]) X[u] = c2;
    return p;
```

```
  };
  for (int i = 1; i <= N; i++) X[i] = 1;
  for (int t = 0; t < E.size(); t++) {
    int u = E[t].first, v0 = E[t].second, v = v0,
        c0 = X[u], c = c0, d;
    vector<pair<int, int>> L;
    int vst[kN] = {};
    while (!G[u][v0]) {
      L.emplace_back(v, d = X[v]);
      if (!C[v][c])
        for (a = (int)L.size() - 1; a >= 0; a--)
          c = color(u, L[a].first, c);
      else if (!C[u][d])
        for (a = (int)L.size() - 1; a >= 0; a--)
          color(u, L[a].first, L[a].second);
      else if (vst[d]) break;
      else vst[d] = 1, v = C[u][d];
    }
    if (!G[u][v0]) {
      for (; v; v = flip(v, c, d), swap(c, d))
        ;
      if (C[u][c0]) {
        for (a = (int)L.size() - 2;
             a >= 0 && L[a].second != c; a--)
          ;
        for (; a >= 0; a--)
          color(u, L[a].first, L[a].second);
      } else t--;
    }
  }
}
} // namespace vizing
```

2.11 Minimum Clique Cover*

```
struct Clique_Cover { // 0-base, O(n2^n)
  int co[1 << N], n, E[N];
  int dp[1 << N];
  void init(int _n) {
    n = _n, fill_n(dp, 1 << n, 0);
    fill_n(E, n, 0), fill_n(co, 1 << n, 0);
  }
  void add_edge(int u, int v) {
    E[u] |= 1 << v, E[v] |= 1 << u;
  }
  int solve() {
    for (int i = 0; i < n; ++i)
      co[1 << i] = E[i] | (1 << i);
    co[0] = (1 << n) - 1;
    dp[0] = (n & 1) * 2 - 1;
    for (int i = 1; i < (1 << n); ++i) {
      int t = i & -i;
      dp[i] = -dp[i ^ t];
      co[i] = co[i ^ t] & co[t];
    }
    for (int i = 0; i < (1 << n); ++i)
      co[i] = (co[i] & i) == i;
    fwt(co, 1 << n, 1);
    for (int ans = 1; ans < n; ++ans) {
      int sum = 0; // probabilistic
      for (int i = 0; i < (1 << n); ++i)
        sum += (dp[i] *= co[i]);
      if (sum) return ans;
    }
    return n;
  }
};
```

2.12 NumberofMaximalClique*

```
struct BronKerbosch { // 1-base
  int n, a[N], g[N][N];
  int S, all[N][N], some[N][N], none[N][N];
  void init(int _n) {
    n = _n;
    for (int i = 1; i <= n; ++i)
      for (int j = 1; j <= n; ++j) g[i][j] = 0;
  }
  void add_edge(int u, int v) {
    g[u][v] = g[v][u] = 1;
  }
  void dfs(int d, int an, int sn, int nn) {
    if (S > 1000) return; // pruning
    if (sn == 0 && nn == 0) ++S;
    int u = some[d][0];
    for (int i = 0; i < sn; ++i) {
      int v = some[d][i];
      if (g[u][v]) continue;
      int tsn = 0, tnn = 0;
      copy_n(all[d], an, all[d + 1]);
      all[d + 1][an] = v;
      for (int j = 0; j < sn; ++j)
        if (g[v][some[d][j]])
          some[d + 1][tsn++] = some[d][j];
      for (int j = 0; j < nn; ++j)
        if (g[v][none[d][j]])
          none[d + 1][tnn++] = none[d][j];
      dfs(d + 1, an + 1, tsn, tnn);
      some[d][i] = 0, none[d][nn++] = v;
    }
  }
  int solve() {
    iota(some[0], some[0] + n, 1);
    S = 0, dfs(0, 0, n, 0);
    return S;
  }
};
```

3 Data Structure

3.1 Leftist Tree

```
struct node {
  ll v, data, sz, sum;
  node *l, *r;
  node(ll k)
    : v(0), data(k), sz(1), l(0), r(0), sum(k) {}
};
ll sz(node *p) { return p ? p->sz : 0; }
ll V(node *p) { return p ? p->v : -1; }
ll sum(node *p) { return p ? p->sum : 0; }
node *merge(node *a, node *b) {
  if (!a || !b) return a ? a : b;
  if (a->data < b->data) swap(a, b);
  a->r = merge(a->r, b);
  if (V(a->r) > V(a->l)) swap(a->r, a->l);
  a->v = V(a->r) + 1, a->sz = sz(a->l) + sz(a->r) + 1;
  a->sum = sum(a->l) + sum(a->r) + a->data;
  return a;
}
void pop(node *&o) {
  node *tmp = o;
  o = merge(o->l, o->r);
  delete tmp;
}
```

3.2 Heavy light Decomposition

```
struct Heavy_light_Decomposition { // 1-base
  int n, ulink[10005], deep[10005], mxson[10005],
    w[10005], pa[10005];
  int t, pl[10005], data[10005], dt[10005], bln[10005],
    edge[10005], et;
  vector<pii> G[10005];
  void init(int _n) {
    n = _n, t = 0, et = 1;
    for (int i = 1; i <= n; ++i)
      G[i].clear(), mxson[i] = 0;
  }
  void add_edge(int a, int b, int w) {
    G[a].pb(pii(b, et)), G[b].pb(pii(a, et)),
      edge[et++] = w;
  }
  void dfs(int u, int f, int d) {
    w[u] = 1, pa[u] = f, deep[u] = d++;
    for (auto &i : G[u])
      if (i.X != f) {
        dfs(i.X, u, d), w[u] += w[i.X];
        if (w[mxson[u]] < w[i.X]) mxson[u] = i.X;
      } else bln[i.Y] = u, dt[u] = edge[i.Y];
```

```
  }
  void cut(int u, int link) {
    data[pl[u] = t++] = dt[u], ulink[u] = link;
    if (!mxson[u]) return;
    cut(mxson[u], link);
    for (auto i : G[u])
      if (i.X != pa[u] && i.X != mxson[u])
        cut(i.X, i.X);
  }
  void build() { dfs(1, 1, 1), cut(1, 1), /*build*/; }
  int query(int a, int b) {
    int ta = ulink[a], tb = ulink[b], re = 0;
    while (ta != tb)
      if (deep[ta] < deep[tb])
        /*query*/, tb = ulink[b = pa[tb]];
      else /*query*/, ta = ulink[a = pa[ta]];
    if (a == b) return re;
    if (pl[a] > pl[b]) swap(a, b);
    /*query*/
    return re;
  }
};
```

3.3 Centroid Decomposition*

```
struct Cent_Dec { // 1-base
  vector<pll> G[N];
  pll info[N]; // store info. of itself
  pll upinfo[N]; // store info. of climbing up
  int n, pa[N], layer[N], sz[N], done[N];
  ll dis[__lg(N) + 1][N];
  void init(int _n) {
    n = _n, layer[0] = -1;
    fill_n(pa + 1, n, 0), fill_n(done + 1, n, 0);
    for (int i = 1; i <= n; ++i) G[i].clear();
  }
  void add_edge(int a, int b, int w) {
    G[a].pb(pll(b, w)), G[b].pb(pll(a, w));
  }
  void get_cent(
    int u, int f, int &mx, int &c, int num) {
    int mxsz = 0;
    sz[u] = 1;
    for (pll e : G[u])
      if (!done[e.X] && e.X != f) {
        get_cent(e.X, u, mx, c, num);
        sz[u] += sz[e.X], mxsz = max(mxsz, sz[e.X]);
      }
    if (mx > max(mxsz, num - sz[u]))
      mx = max(mxsz, num - sz[u]), c = u;
  }
  void dfs(int u, int f, ll d, int org) {
    // if required, add self info or climbing info
    dis[layer[org]][u] = d;
    for (pll e : G[u])
      if (!done[e.X] && e.X != f)
        dfs(e.X, u, d + e.Y, org);
  }
  int cut(int u, int f, int num) {
    int mx = 1e9, c = 0, lc;
    get_cent(u, f, mx, c, num);
    done[c] = 1, pa[c] = f, layer[c] = layer[f] + 1;
    for (pll e : G[c])
      if (!done[e.X]) {
        if (sz[e.X] > sz[c])
          lc = cut(e.X, c, num - sz[c]);
        else lc = cut(e.X, c, sz[e.X]);
        upinfo[lc] = pll(), dfs(e.X, c, e.Y, c);
      }
    return done[c] = 0, c;
  }
  void build() { cut(1, 0, n); }
  void modify(int u) {
    for (int a = u, ly = layer[a]; a;
         a = pa[a], --ly) {
      info[a].X += dis[ly][u], ++info[a].Y;
      if (pa[a])
        upinfo[a].X += dis[ly - 1][u], ++upinfo[a].Y;
    }
  }
  ll query(int u) {
    ll rt = 0;
    for (int a = u, ly = layer[a]; a;
         a = pa[a], --ly) {
      rt += info[a].X + info[a].Y * dis[ly][u];
      if (pa[a])
        rt -=
          upinfo[a].X + upinfo[a].Y * dis[ly - 1][u];
    }
    return rt;
  }
};
```

3.4 Link cut tree*

```
struct Splay { // xor-sum
  static Splay nil;
  Splay *ch[2], *f;
  int val, sum, rev, size;
  Splay(int _val = 0)
    : val(_val), sum(_val), rev(0), size(1) {
    f = ch[0] = ch[1] = &nil;
  }
  bool isr() {
    return f->ch[0] != this && f->ch[1] != this;
  }
  int dir() { return f->ch[0] == this ? 0 : 1; }
  void setCh(Splay *c, int d) {
    ch[d] = c;
    if (c != &nil) c->f = this;
    pull();
  }
  void push() {
    if (!rev) return;
    swap(ch[0], ch[1]);
    if (ch[0] != &nil) ch[0]->rev ^= 1;
    if (ch[1] != &nil) ch[1]->rev ^= 1;
    rev = 0;
  }
  void pull() {
    // take care of the nil!
    size = ch[0]->size + ch[1]->size + 1;
    sum = ch[0]->sum ^ ch[1]->sum ^ val;
    if (ch[0] != &nil) ch[0]->f = this;
    if (ch[1] != &nil) ch[1]->f = this;
  }
} Splay::nil;
Splay *nil = &Splay::nil;
void rotate(Splay *x) {
  Splay *p = x->f;
  int d = x->dir();
  if (!p->isr()) p->f->setCh(x, p->dir());
  else x->f = p->f;
  p->setCh(x->ch[!d], d);
  x->setCh(p, !d);
  p->pull(), x->pull();
}
void splay(Splay *x) {
  vector<Splay *> splayVec;
  for (Splay *q = x;; q = q->f) {
    splayVec.pb(q);
    if (q->isr()) break;
  }
  reverse(ALL(splayVec));
  for (auto it : splayVec) it->push();
  while (!x->isr()) {
    if (x->f->isr()) rotate(x);
    else if (x->dir() == x->f->dir())
      rotate(x->f), rotate(x);
    else rotate(x), rotate(x);
  }
}
Splay *access(Splay *x) {
  Splay *q = nil;
  for (; x != nil; x = x->f)
    splay(x), x->setCh(q, 1), q = x;
  return q;
}
void root_path(Splay *x) { access(x), splay(x); }
void chroot(Splay *x) {
  root_path(x), x->rev ^= 1;
  x->push(), x->pull();
```

```
}
void split(Splay *x, Splay *y) {
  chroot(x), root_path(y);
}
void link(Splay *x, Splay *y) {
  root_path(x), chroot(y);
  x->setCh(y, 1);
}
void cut(Splay *x, Splay *y) {
  split(x, y);
  if (y->size != 5) return;
  y->push();
  y->ch[0] = y->ch[0]->f = nil;
}
Splay *get_root(Splay *x) {
  for (root_path(x); x->ch[0] != nil; x = x->ch[0])
    x->push();
  splay(x);
  return x;
}
bool conn(Splay *x, Splay *y) {
  return get_root(x) == get_root(y);
}
Splay *lca(Splay *x, Splay *y) {
  access(x), root_path(y);
  if (y->f == nil) return y;
  return y->f;
}
void change(Splay *x, int val) {
  splay(x), x->val = val, x->pull();
}
int query(Splay *x, Splay *y) {
  split(x, y);
  return y->sum;
}
```

3.5 KDTree

```
namespace kdt {
int root, lc[maxn], rc[maxn], xl[maxn], xr[maxn],
  yl[maxn], yr[maxn];
point p[maxn];
int build(int l, int r, int dep = 0) {
  if (l == r) return -1;
  function<bool(const point &, const point &)> f =
    [dep](const point &a, const point &b) {
      if (dep & 1) return a.x < b.x;
      else return a.y < b.y;
    };
  int m = (l + r) >> 1;
  nth_element(p + l, p + m, p + r, f);
  xl[m] = xr[m] = p[m].x;
  yl[m] = yr[m] = p[m].y;
  lc[m] = build(l, m, dep + 1);
  if (~lc[m]) {
    xl[m] = min(xl[m], xl[lc[m]]);
    xr[m] = max(xr[m], xr[lc[m]]);
    yl[m] = min(yl[m], yl[lc[m]]);
    yr[m] = max(yr[m], yr[lc[m]]);
  }
  rc[m] = build(m + 1, r, dep + 1);
  if (~rc[m]) {
    xl[m] = min(xl[m], xl[rc[m]]);
    xr[m] = max(xr[m], xr[rc[m]]);
    yl[m] = min(yl[m], yl[rc[m]]);
    yr[m] = max(yr[m], yr[rc[m]]);
  }
  return m;
}
bool bound(const point &q, int o, long long d) {
  double ds = sqrt(d + 1.0);
  if (q.x < xl[o] - ds || q.x > xr[o] + ds ||
    q.y < yl[o] - ds || q.y > yr[o] + ds)
    return false;
  return true;
}
long long dist(const point &a, const point &b) {
  return (a.x - b.x) * 1ll * (a.x - b.x) +
    (a.y - b.y) * 1ll * (a.y - b.y);
}
void dfs(
  const point &q, long long &d, int o, int dep = 0) {
  if (!bound(q, o, d)) return;
  long long cd = dist(p[o], q);
  if (cd != 0) d = min(d, cd);
  if ((dep & 1) && q.x < p[o].x ||
    !(dep & 1) && q.y < p[o].y) {
    if (~lc[o]) dfs(q, d, lc[o], dep + 1);
    if (~rc[o]) dfs(q, d, rc[o], dep + 1);
  } else {
    if (~rc[o]) dfs(q, d, rc[o], dep + 1);
    if (~lc[o]) dfs(q, d, lc[o], dep + 1);
  }
}
void init(const vector<point> &v) {
  for (int i = 0; i < v.size(); ++i) p[i] = v[i];
  root = build(0, v.size());
}
long long nearest(const point &q) {
  long long res = 1e18;
  dfs(q, res, root);
  return res;
}
} // namespace kdt
```

4 Flow/Matching

4.1 Kuhn Munkres

```
struct KM { // 0-base
  int w[MAXN][MAXN], hl[MAXN], hr[MAXN], slk[MAXN], n;
  int fl[MAXN], fr[MAXN], pre[MAXN], qu[MAXN], ql, qr;
  bool vl[MAXN], vr[MAXN];
  void init(int _n) {
    n = _n;
    for (int i = 0; i < n; ++i)
      for (int j = 0; j < n; ++j) w[i][j] = -INF;
  }
  void add_edge(int a, int b, int wei) {
    w[a][b] = wei;
  }
  bool Check(int x) {
    if (vl[x] = 1, ~fl[x])
      return vr[qu[qr++] = fl[x]] = 1;
    while (~x) swap(x, fr[fl[x] = pre[x]]);
    return 0;
  }
  void Bfs(int s) {
    fill(slk, slk + n, INF);
    fill(vl, vl + n, 0), fill(vr, vr + n, 0);
    ql = qr = 0, qu[qr++] = s, vr[s] = 1;
    while (1) {
      int d;
      while (ql < qr)
        for (int x = 0, y = qu[ql++]; x < n; ++x)
          if (!vl[x] &&
            slk[x] >= (d = hl[x] + hr[y] - w[x][y]))
            if (pre[x] = y, d) slk[x] = d;
            else if (!Check(x)) return;
      d = INF;
      for (int x = 0; x < n; ++x)
        if (!vl[x] && d > slk[x]) d = slk[x];
      for (int x = 0; x < n; ++x) {
        if (vl[x]) hl[x] += d;
        else slk[x] -= d;
        if (vr[x]) hr[x] -= d;
      }
      for (int x = 0; x < n; ++x)
        if (!vl[x] && !slk[x] && !Check(x)) return;
    }
  }
  int Solve() {
    fill(fl, fl + n, -1), fill(fr, fr + n, -1),
      fill(hr, hr + n, 0);
    for (int i = 0; i < n; ++i)
      hl[i] = *max_element(w[i], w[i] + n);
    for (int i = 0; i < n; ++i) Bfs(i);
    int res = 0;
    for (int i = 0; i < n; ++i) res += w[i][fl[i]];
    return res;
```

```
  }
};
```

4.2 MincostMaxflow

```
struct MCMF { // 0-base
  struct edge {
    ll from, to, cap, flow, cost, rev;
  } * past[MAXN];
  vector<edge> G[MAXN];
  bitset<MAXN> inq;
  ll dis[MAXN], up[MAXN], s, t, mx, n;
  bool BellmanFord(ll &flow, ll &cost) {
    fill(dis, dis + n, INF);
    queue<ll> q;
    q.push(s), inq.reset(), inq[s] = 1;
    up[s] = mx - flow, past[s] = 0, dis[s] = 0;
    while (!q.empty()) {
      ll u = q.front();
      q.pop(), inq[u] = 0;
      if (!up[u]) continue;
      for (auto &e : G[u])
        if (e.flow != e.cap &&
          dis[e.to] > dis[u] + e.cost) {
          dis[e.to] = dis[u] + e.cost, past[e.to] = &e;
          up[e.to] = min(up[u], e.cap - e.flow);
          if (!inq[e.to]) inq[e.to] = 1, q.push(e.to);
        }
    }
    if (dis[t] == INF) return 0;
    flow += up[t], cost += up[t] * dis[t];
    for (ll i = t; past[i]; i = past[i]->from) {
      auto &e = *past[i];
      e.flow += up[t], G[e.to][e.rev].flow -= up[t];
    }
    return 1;
  }
  ll MinCostMaxFlow(ll _s, ll _t, ll &cost) {
    s = _s, t = _t, cost = 0;
    ll flow = 0;
    while (BellmanFord(flow, cost))
      ;
    return flow;
  }
  void init(ll _n, ll _mx) {
    n = _n, mx = _mx;
    for (int i = 0; i < n; ++i) G[i].clear();
  }
  void add_edge(ll a, ll b, ll cap, ll cost) {
    G[a].pb(edge{a, b, cap, 0, cost, G[b].size()});
    G[b].pb(edge{b, a, 0, 0, -cost, G[a].size() - 1});
  }
};
```

4.3 Maximum Simple Graph Matching*

```
struct GenMatch { // 1-base
  int V, pr[N];
  bool el[N][N], inq[N], inp[N], inb[N];
  int st, ed, nb, bk[N], djs[N], ans;
  void init(int _V) {
    V = _V;
    for (int i = 0; i <= V; ++i) {
      for (int j = 0; j <= V; ++j) el[i][j] = 0;
      pr[i] = bk[i] = djs[i] = 0;
      inq[i] = inp[i] = inb[i] = 0;
    }
  }
  void add_edge(int u, int v) {
    el[u][v] = el[v][u] = 1;
  }
  int lca(int u, int v) {
    fill_n(inp, V + 1, 0);
    while (1)
      if (u = djs[u], inp[u] = true, u == st) break;
      else u = bk[pr[u]];
    while (1)
      if (v = djs[v], inp[v]) return v;
      else v = bk[pr[v]];
    return v;
  }
  void upd(int u) {
    for (int v; djs[u] != nb;) {
      v = pr[u], inb[djs[u]] = inb[djs[v]] = true;
      u = bk[v];
      if (djs[u] != nb) bk[u] = v;
    }
  }
  void blo(int u, int v, queue<int> &qe) {
    nb = lca(u, v), fill_n(inb, V + 1, 0);
    upd(u), upd(v);
    if (djs[u] != nb) bk[u] = v;
    if (djs[v] != nb) bk[v] = u;
    for (int tu = 1; tu <= V; ++tu)
      if (inb[djs[tu]])
        if (djs[tu] = nb, !inq[tu])
          qe.push(tu), inq[tu] = 1;
  }
  void flow() {
    fill_n(inq + 1, V, 0), fill_n(bk + 1, V, 0);
    iota(djs + 1, djs + V + 1, 1);
    queue<int> qe;
    qe.push(st), inq[st] = 1, ed = 0;
    while (!qe.empty()) {
      int u = qe.front();
      qe.pop();
      for (int v = 1; v <= V; ++v)
        if (el[u][v] && djs[u] != djs[v] &&
          pr[u] != v) {
          if ((v == st) ||
            (pr[v] > 0 && bk[pr[v]] > 0)) {
            blo(u, v, qe);
          } else if (!bk[v]) {
            if (bk[v] = u, pr[v] > 0) {
              if (!inq[pr[v]]) qe.push(pr[v]);
            } else {
              return ed = v, void();
            }
          }
        }
    }
  }
  void aug() {
    for (int u = ed, v, w; u > 0;)
      v = bk[u], w = pr[v], pr[v] = u, pr[u] = v,
      u = w;
  }
  int solve() {
    fill_n(pr, V + 1, 0), ans = 0;
    for (int u = 1; u <= V; ++u)
      if (!pr[u])
        if (st = u, flow(), ed > 0) aug(), ++ans;
    return ans;
  }
};
```

4.4 Minimum Weight Matching (Clique version)*

```
struct Graph { // 0-base (Perfect Match), n is even
  int n, match[N], onstk[N], stk[N], tp;
  ll edge[N][N], dis[N];
  void init(int _n) {
    n = _n, tp = 0;
    for (int i = 0; i < n; ++i) fill_n(edge[i], n, 0);
  }
  void add_edge(int u, int v, ll w) {
    edge[u][v] = edge[v][u] = w;
  }
  bool SPFA(int u) {
    stk[tp++] = u, onstk[u] = 1;
    for (int v = 0; v < n; ++v)
      if (!onstk[v] && match[u] != v) {
        int m = match[v];
        if (dis[m] >
          dis[u] - edge[v][m] + edge[u][v]) {
          dis[m] = dis[u] - edge[v][m] + edge[u][v];
          onstk[v] = 1, stk[tp++] = v;
          if (onstk[m] || SPFA(m)) return 1;
          --tp, onstk[v] = 0;
        }
      }
```

```
    onstk[u] = 0, --tp;
    return 0;
  }
  ll solve() { // find a match
    for (int i = 0; i < n; ++i) match[i] = i ^ 1;
    while (1) {
      int found = 0;
      fill_n(dis, n, 0);
      fill_n(onstk, n, 0);
      for (int i = 0; i < n; ++i)
        if (tp = 0, !onstk[i] && SPFA(i))
          for (found = 1; tp >= 2;) {
            int u = stk[--tp];
            int v = stk[--tp];
            match[u] = v, match[v] = u;
          }
      if (!found) break;
    }
    ll ret = 0;
    for (int i = 0; i < n; ++i)
      ret += edge[i][match[i]];
    return ret >> 1;
  }
};
```

4.5 SW-mincut

```
// global min cut
struct SW { // O(V^3)
  static const int MXN = 514;
  int n, vst[MXN], del[MXN];
  int edge[MXN][MXN], wei[MXN];
  void init(int _n) {
    n = _n, MEM(edge, 0), MEM(del, 0);
  }
  void addEdge(int u, int v, int w) {
    edge[u][v] += w, edge[v][u] += w;
  }
  void search(int &s, int &t) {
    MEM(vst, 0), MEM(wei, 0), s = t = -1;
    while (1) {
      int mx = -1, cur = 0;
      for (int i = 0; i < n; ++i)
        if (!del[i] && !vst[i] && mx < wei[i])
          cur = i, mx = wei[i];
      if (mx == -1) break;
      vst[cur] = 1, s = t, t = cur;
      for (int i = 0; i < n; ++i)
        if (!vst[i] && !del[i]) wei[i] += edge[cur][i];
    }
  }
  int solve() {
    int res = INF;
    for (int i = 0, x, y; i < n - 1; ++i) {
      search(x, y), res = min(res, wei[y]), del[y] = 1;
      for (int j = 0; j < n; ++j)
        edge[x][j] = (edge[j][x] += edge[y][j]);
    }
    return res;
  }
};
```

4.6 BoundedFlow(Dinic*)

```
struct BoundedFlow { // 0-base
  struct edge {
    int to, cap, flow, rev;
  };
  vector<edge> G[N];
  int n, s, t, dis[N], cur[N], cnt[N];
  void init(int _n) {
    n = _n;
    for (int i = 0; i < n + 2; ++i)
      G[i].clear(), cnt[i] = 0;
  }
  void add_edge(int u, int v, int lcap, int rcap) {
    cnt[u] -= lcap, cnt[v] += lcap;
    G[u].pb(edge{v, rcap, lcap, SZ(G[v])});
    G[v].pb(edge{u, 0, 0, SZ(G[u]) - 1});
  }
  void add_edge(int u, int v, int cap) {
    G[u].pb(edge{v, cap, 0, SZ(G[v])});
    G[v].pb(edge{u, 0, 0, SZ(G[u]) - 1});
  }
  int dfs(int u, int cap) {
    if (u == t || !cap) return cap;
    for (int &i = cur[u]; i < SZ(G[u]); ++i) {
      edge &e = G[u][i];
      if (dis[e.to] == dis[u] + 1 && e.cap != e.flow) {
        int df = dfs(e.to, min(e.cap - e.flow, cap));
        if (df) {
          e.flow += df, G[e.to][e.rev].flow -= df;
          return df;
        }
      }
    }
    dis[u] = -1;
    return 0;
  }
  bool bfs() {
    fill_n(dis, n + 3, -1);
    queue<int> q;
    q.push(s), dis[s] = 0;
    while (!q.empty()) {
      int u = q.front();
      q.pop();
      for (edge &e : G[u])
        if (!~dis[e.to] && e.flow != e.cap)
          q.push(e.to), dis[e.to] = dis[u] + 1;
    }
    return dis[t] != -1;
  }
  int maxflow(int _s, int _t) {
    s = _s, t = _t;
    int flow = 0, df;
    while (bfs()) {
      fill_n(cur, n + 3, 0);
      while ((df = dfs(s, INF))) flow += df;
    }
    return flow;
  }
  bool solve() {
    int sum = 0;
    for (int i = 0; i < n; ++i)
      if (cnt[i] > 0)
        add_edge(n + 1, i, cnt[i]), sum += cnt[i];
      else if (cnt[i] < 0) add_edge(i, n + 2, -cnt[i]);
    if (sum != maxflow(n + 1, n + 2)) sum = -1;
    for (int i = 0; i < n; ++i)
      if (cnt[i] > 0)
        G[n + 1].pop_back(), G[i].pop_back();
      else if (cnt[i] < 0)
        G[i].pop_back(), G[n + 2].pop_back();
    return sum != -1;
  }
  int solve(int _s, int _t) {
    add_edge(_t, _s, INF);
    if (!solve()) return -1; // invalid flow
    int x = G[_t].back().flow;
    return G[_t].pop_back(), G[_s].pop_back(), x;
  }
};
```

4.7 Gomory Hu tree*

```
struct Gomory_Hu_tree { // 0-base
  MaxFlow Dinic;
  int n;
  vector<pii> G[MAXN];
  void init(int _n) {
    n = _n;
    for (int i = 0; i < n; ++i) G[i].clear();
  }
  void solve(vector<int> &v) {
    if (v.size() <= 1) return;
    int s = rand() % SZ(v);
    swap(v.back(), v[s]), s = v.back();
    int t = v[rand() % (SZ(v) - 1)];
    vector<int> L, R;
    int x = (Dinic.reset(), Dinic.maxflow(s, t));
    G[s].pb(pii(t, x)), G[t].pb(pii(s, x));
    for (int i : v)
```

```
      if (~Dinic.dis[i]) L.pb(i);
      else R.pb(i);
    solve(L), solve(R);
  }
  void build() {
    vector<int> v(n);
    for (int i = 0; i < n; ++i) v[i] = i;
    solve(v);
  }
} ght;
MaxFlow &Dinic = ght.Dinic;
```

4.8 Minimum Cost Circulation

```
struct Edge { int to, cap, rev, cost; };
vector<Edge> g[kN];
int dist[kN], pv[kN], ed[kN];
bool mark[kN];
int NegativeCycle(int n) {
  memset(mark, false, sizeof(mark));
  memset(dist, 0, sizeof(dist));
  int upd = -1;
  for (int i = 0; i <= n; ++i) {
    for (int j = 0; j < n; ++j) {
      int idx = 0;
      for (auto &e : g[j]) {
        if (e.cap > 0 && dist[e.to] > dist[j] + e.cost)
            {
          dist[e.to] = dist[j] + e.cost;
          pv[e.to] = j, ed[e.to] = idx;
          if (i == n) {
            upd = j;
            while (!mark[upd]) mark[upd] = true, upd =
                pv[upd];
            return upd;
          }
        }
        idx++;
      }
    }
  }
  return -1;
}
int Solve(int n) {
  int rt = -1, ans = 0;
  while ((rt = NegativeCycle(n)) >= 0) {
    memset(mark, false, sizeof(mark));
    vector<pair<int, int>> cyc;
    while (!mark[rt]) {
      cyc.emplace_back(pv[rt], ed[rt]);
      mark[rt] = true;
      rt = pv[rt];
    }
    reverse(cyc.begin(), cyc.end());
    int cap = kInf;
    for (auto &i : cyc) {
      auto &e = g[i.first][i.second];
      cap = min(cap, e.cap);
    }
    for (auto &i : cyc) {
      auto &e = g[i.first][i.second];
      e.cap -= cap;
      g[e.to][e.rev].cap += cap;
      ans += e.cost * cap;
    }
  }
  return ans;
}
```

4.9 Flow Models

- Maximum/Minimum flow with lower bound / Circulation problem
  1. Construct super source S and sink T.
  2. For each edge (x, y, l, u), connect $x \to y$ with capacity $u - l$.
  3. For each vertex v, denote by $in(v)$ the difference between the sum of incoming lower bounds and the sum of outgoing lower bounds.
  4. If $in(v) > 0$, connect $S \to v$ with capacity $in(v)$, otherwise, connect $v \to T$ with capacity $-in(v)$.
     - To maximize, connect $t \to s$ with capacity ∞ (skip this in circulation problem), and let f be the maximum flow from S to T. If $f \neq \sum_{v\in V, in(v)>0} in(v)$, there's no solution. Otherwise, the maximum flow from s to t is the answer.
     - To minimize, let f be the maximum flow from S to T. Connect $t \to s$ with capacity ∞ and let the flow from S to T be f'. If $f + f' \neq \sum_{v\in V, in(v)>0} in(v)$, there's no solution. Otherwise, f' is the answer.
  5. The solution of each edge e is $l_e + f_e$, where f_e corresponds to the flow of edge e on the graph.
- Construct minimum vertex cover from maximum matching M on bipartite graph (X, Y)
  1. Redirect every edge: $y \to x$ if $(x, y) \in M$, $x \to y$ otherwise.
  2. DFS from unmatched vertices in X.
  3. $x \in X$ is chosen iff x is unvisited.
  4. $y \in Y$ is chosen iff y is visited.
- Minimum cost cyclic flow
  1. Consruct super source S and sink T
  2. For each edge (x, y, c), connect $x \to y$ with $(cost, cap) = (c, 1)$ if $c > 0$, otherwise connect $y \to x$ with $(cost, cap) = (-c, 1)$
  3. For each edge with $c < 0$, sum these cost as K, then increase $d(y)$ by 1, decrease $d(x)$ by 1
  4. For each vertex v with $d(v) > 0$, connect $S \to v$ with $(cost, cap) = (0, d(v))$
  5. For each vertex v with $d(v) < 0$, connect $v \to T$ with $(cost, cap) = (0, -d(v))$
  6. Flow from S to T, the answer is the cost of the flow $C + K$
- Maximum density induced subgraph
  1. Binary search on answer, suppose we're checking answer T
  2. Construct a max flow model, let K be the sum of all weights
  3. Connect source $s \to v$, $v \in G$ with capacity K
  4. For each edge (u, v, w) in G, connect $u \to v$ and $v \to u$ with capacity w
  5. For $v \in G$, connect it with sink $v \to t$ with capacity $K + 2T - (\sum_{e\in E(v)} w(e)) - 2w(v)$
  6. T is a valid answer if the maximum flow $f < K|V|$
- Minimum weight edge cover
  1. For each $v \in V$ create a copy v', and connect $u' \to v'$ with weight $w(u, v)$.
  2. Connect $v \to v'$ with weight $2\mu(v)$, where $\mu(v)$ is the cost of the cheapest edge incident to v.
  3. Find the minimum weight perfect matching on G'.
- Project selection problem
  1. If $p_v > 0$, create edge (s, v) with capacity p_v; otherwise, create edge (v, t) with capacity $-p_v$.
  2. Create edge (u, v) with capacity w with w being the cost of choosing u without choosing v.
  3. The mincut is equivalent to the maximum profit of a subset of projects.
- 0/1 quadratic programming

$$\sum_x c_x x + \sum_y c_y \bar{y} + \sum_{xy} c_{xy} x\bar{y} + \sum_{xyx'y'} c_{xyx'y'}(x\bar{y} + x'\bar{y'})$$

  can be minimized by the mincut of the following graph:

  1. Create edge (x, t) with capacity c_x and create edge (s, y) with capacity c_y.
  2. Create edge (x, y) with capacity c_{xy}.
  3. Create edge (x, y) and edge (x', y') with capacity $c_{xyx'y'}$.

5 String

5.1 KMP

```
int F[MAXN];
vector<int> match(string A, string B) {
  vector<int> ans;
  F[0] = -1, F[1] = 0;
  for (int i = 1, j = 0; i < SZ(B); F[++i] = ++j) {
    if (B[i] == B[j]) F[i] = F[j]; // optimize
    while (j != -1 && B[i] != B[j]) j = F[j];
  }
  for (int i = 0, j = 0; i < SZ(A); ++i) {
    while (j != -1 && A[i] != B[j]) j = F[j];
    if (++j == SZ(B)) ans.pb(i + 1 - j), j = F[j];
  }
  return ans;
}
```

5.2 Z-value*

```
int z[MAXn];
void make_z(const string &s) {
  int l = 0, r = 0;
  for (int i = 1; i < SZ(s); ++i) {
    for (z[i] = max(0, min(r - i + 1, z[i - l]));
         i + z[i] < SZ(s) && s[i + z[i]] == s[z[i]];
         ++z[i])
      ;
    if (i + z[i] - 1 > r) l = i, r = i + z[i] - 1;
  }
}
```

5.3 Manacher*

```
int z[MAXN];
int Manacher(string tmp) {
  string s = "&";
  int l = 0, r = 0, x, ans;
  for (char c : tmp) s.pb(c), s.pb('%');
  ans = 0, x = 0;
  for (int i = 1; i < SZ(s); ++i) {
    z[i] = r > i ? min(z[2 * l - i], r - i) : 1;
    while (s[i + z[i]] == s[i - z[i]]) ++z[i];
    if (z[i] + i > r) r = z[i] + i, l = i;
  }
  for (int i = 1; i < SZ(s); ++i)
    if (s[i] == '%') x = max(x, z[i]);
  ans = x / 2 * 2, x = 0;
  for (int i = 1; i < SZ(s); ++i)
    if (s[i] != '%') x = max(x, z[i]);
  return max(ans, (x - 1) / 2 * 2 + 1);
}
```

5.4 Suffix Array

```
struct suffix_array {
  int box[MAXN], tp[MAXN], m;
  bool not_equ(int a, int b, int k, int n) {
    return ra[a] != ra[b] || a + k >= n ||
      b + k >= n || ra[a + k] != ra[b + k];
  }
  void radix(int *key, int *it, int *ot, int n) {
    fill_n(box, m, 0);
    for (int i = 0; i < n; ++i) ++box[key[i]];
    partial_sum(box, box + m, box);
    for (int i = n - 1; i >= 0; --i)
      ot[--box[key[it[i]]]] = it[i];
  }
  void make_sa(const string &s, int n) {
    int k = 1;
    for (int i = 0; i < n; ++i) ra[i] = s[i];
    do {
      iota(tp, tp + k, n - k), iota(sa + k, sa + n, 0);
      radix(ra + k, sa + k, tp + k, n - k);
      radix(ra, tp, sa, n);
      tp[sa[0]] = 0, m = 1;
      for (int i = 1; i < n; ++i) {
        m += not_equ(sa[i], sa[i - 1], k, n);
        tp[sa[i]] = m - 1;
      }
      copy_n(tp, n, ra);
      k *= 2;
    } while (k < n && m != n);
  }
  void make_he(const string &s, int n) {
    for (int j = 0, k = 0; j < n; ++j) {
      if (ra[j])
        for (; s[j + k] == s[sa[ra[j] - 1] + k]; ++k)
          ;
      he[ra[j]] = k, k = max(0, k - 1);
    }
  }
  int sa[MAXN], ra[MAXN], he[MAXN];
  void build(const string &s) {
    int n = SZ(s);
    fill_n(sa, n, 0), fill_n(ra, n, 0), fill_n(he, n,
        0);
    fill_n(box, n, 0), fill_n(tp, n, 0), m = 256;
    make_sa(s, n), make_he(s, n);
  }
};
```

5.5 SAIS

```
class SAIS {
public:
  int *SA, *H;
  // zero based, string content MUST > 0
  // result height H[i] is LCP(SA[i - 1], SA[i])
  // string, length, |sigma|
  void build(int *s, int n, int m = 128) {
    copy_n(s, n, _s);
    _h[0] = _s[n++] = 0;
    sais(_s, _sa, _p, _q, _t, _c, n, m);
    mkhei(n);
    SA = _sa + 1;
    H = _h + 1;
  }

private:
  bool _t[N * 2];
  int _s[N * 2], _c[N * 2], x[N], _p[N], _q[N * 2],
    r[N], _sa[N * 2], _h[N];
  void mkhei(int n) {
    for (int i = 0; i < n; i++) r[_sa[i]] = i;
    for (int i = 0; i < n; i++)
      if (r[i]) {
        int ans = i > 0 ? max(_h[r[i - 1]] - 1, 0) : 0;
        while (_s[i + ans] == _s[_sa[r[i] - 1] + ans])
          ans++;
        _h[r[i]] = ans;
      }
  }
  void sais(int *s, int *sa, int *p, int *q, bool *t,
    int *c, int n, int z) {
    bool uniq = t[n - 1] = 1, neq;
    int nn = 0, nmxz = -1, *nsa = sa + n, *ns = s + n,
        lst = -1;

#define MAGIC(XD)                                          \
  fill_n(sa, n, 0);                                        \
  copy_n(c, z, x);                                         \
  XD;                                                      \
  copy_n(c, z - 1, x + 1);                                 \
  for (int i = 0; i < n; i++)                              \
    if (sa[i] && !t[sa[i] - 1])                            \
      sa[x[s[sa[i] - 1]]++] = sa[i] - 1;                   \
  copy_n(c, z, x);                                         \
  for (int i = n - 1; i >= 0; i--)                         \
    if (sa[i] && t[sa[i] - 1])                             \
      sa[--x[s[sa[i] - 1]]] = sa[i] - 1;

    fill_n(c, z, 0);
    for (int i = 0; i < n; i++) uniq &= ++c[s[i]] < 2;
    partial_sum(c, c + z, c);
    if (uniq) {
      for (int i = 0; i < n; i++) sa[--c[s[i]]] = i;
      return;
    }
    for (int i = n - 2; i >= 0; i--)
      t[i] = (s[i] == s[i + 1] ? t[i + 1]
                                 : s[i] < s[i + 1]);
    MAGIC(for (int i = 1; i <= n - 1;
               i++) if (t[i] && !t[i - 1])
            sa[--x[s[i]]] = p[q[i] = nn++] = i);
    for (int i = 0; i < n; i++)
      if (sa[i] && t[sa[i]] && !t[sa[i] - 1]) {
        neq = (lst < 0) ||
          !equal(s + lst,
            s + lst + p[q[sa[i]] + 1] - sa[i],
            s + sa[i]);
        ns[q[lst = sa[i]]] = nmxz += neq;
      }
    sais(ns, nsa, p + nn, q + n, t + n, c + z, nn,
      nmxz + 1);
    MAGIC(for (int i = nn - 1; i >= 0; i--)
            sa[--x[s[p[nsa[i]]]]] = p[nsa[i]]);
  }
} sa;
```

5.6 Aho-Corasick Automatan

```
const int len = 400000, sigma = 26;
struct AC_Automatan {
  int nx[len][sigma], fl[len], cnt[len], pri[len], top;
  int newnode() {
    fill(nx[top], nx[top] + sigma, -1);
    return top++;
  }
  void init() { top = 1, newnode(); }
  int input(
    string &s) { // return the end_node of string
    int X = 1;
    for (char c : s) {
      if (!~nx[X][c - 'a']) nx[X][c - 'a'] = newnode();
      X = nx[X][c - 'a'];
    }
    return X;
  }
  void make_fl() {
    queue<int> q;
    q.push(1), fl[1] = 0;
    for (int t = 0; !q.empty();) {
      int R = q.front();
      q.pop(), pri[t++] = R;
      for (int i = 0; i < sigma; ++i)
        if (~nx[R][i]) {
          int X = nx[R][i], Z = fl[R];
          for (; Z && !~nx[Z][i];) Z = fl[Z];
          fl[X] = Z ? nx[Z][i] : 1, q.push(X);
        }
    }
  }
  void get_v(string &s) {
    int X = 1;
    fill(cnt, cnt + top, 0);
    for (char c : s) {
      while (X && !~nx[X][c - 'a']) X = fl[X];
      X = X ? nx[X][c - 'a'] : 1, ++cnt[X];
    }
    for (int i = top - 2; i > 0; --i)
      cnt[fl[pri[i]]] += cnt[pri[i]];
  }
};
```

5.7 Smallest Rotation

```
string mcp(string s) {
  int n = SZ(s), i = 0, j = 1;
  s += s;
  while (i < n && j < n) {
    int k = 0;
    while (k < n && s[i + k] == s[j + k]) ++k;
    if (s[i + k] <= s[j + k]) j += k + 1;
    else i += k + 1;
    if (i == j) ++j;
  }
  int ans = i < n ? i : j;
  return s.substr(ans, n);
}
```

5.8 De Bruijn sequence*

```
constexpr int MAXC = 10, MAXN = 1e5 + 10;
struct DBSeq {
  int C, N, K, L, buf[MAXC * MAXN]; // K <= C^N
  void dfs(int *out, int t, int p, int &ptr) {
    if (ptr >= L) return;
    if (t > N) {
      if (N % p) return;
      for (int i = 1; i <= p && ptr < L; ++i)
        out[ptr++] = buf[i];
    } else {
      buf[t] = buf[t - p], dfs(out, t + 1, p, ptr);
      for (int j = buf[t - p] + 1; j < C; ++j)
        buf[t] = j, dfs(out, t + 1, t, ptr);
    }
  }
  void solve(int _c, int _n, int _k, int *out) {
    int p = 0;
    C = _c, N = _n, K = _k, L = N + K - 1;
    dfs(out, 1, 1, p);
    if (p < L) fill(out + p, out + L, 0);
  }
} dbs;
```

5.9 SAM

```
const int MAXM = 1000010;
struct SAM {
  int tot, root, lst, mom[MAXM], mx[MAXM];
  int acc[MAXM], nxt[MAXM][33];
  int newNode() {
    int res = ++tot;
    fill(nxt[res], nxt[res] + 33, 0);
    mom[res] = mx[res] = acc[res] = 0;
    return res;
  }
  void init() {
    tot = 0;
    root = newNode();
    mom[root] = 0, mx[root] = 0;
    lst = root;
  }
  void push(int c) {
    int p = lst;
    int np = newNode();
    mx[np] = mx[p] + 1;
    for (; p && nxt[p][c] == 0; p = mom[p])
      nxt[p][c] = np;
    if (p == 0) mom[np] = root;
    else {
      int q = nxt[p][c];
      if (mx[p] + 1 == mx[q]) mom[np] = q;
      else {
        int nq = newNode();
        mx[nq] = mx[p] + 1;
        for (int i = 0; i < 33; i++)
          nxt[nq][i] = nxt[q][i];
        mom[nq] = mom[q];
        mom[q] = nq;
        mom[np] = nq;
        for (; p && nxt[p][c] == q; p = mom[p])
          nxt[p][c] = nq;
      }
    }
    lst = np;
  }
  void push(char *str) {
    for (int i = 0; str[i]; i++)
      push(str[i] - 'a' + 1);
  }
} sam;
```

5.10 PalTree*

```
struct palindromic_tree {
  struct node {
    int next[26], fail, len;
    int cnt, num; // cnt: appear times, num: number of
                  // pal. suf.
    node(int l = 0) : fail(0), len(l), cnt(0), num(0) {
      for (int i = 0; i < 26; ++i) next[i] = 0;
    }
  };
  vector<node> St;
  vector<char> s;
  int last, n;
  palindromic_tree() : St(2), last(1), n(0) {
    St[0].fail = 1, St[1].len = -1, s.pb(-1);
  }
  inline void clear() {
    St.clear(), s.clear(), last = 1, n = 0;
    St.pb(0), St.pb(-1);
    St[0].fail = 1, s.pb(-1);
  }
  inline int get_fail(int x) {
    while (s[n - St[x].len - 1] != s[n])
      x = St[x].fail;
    return x;
  }
  inline void add(int c) {
```

```
    s.push_back(c -= 'a'), ++n;
    int cur = get_fail(last);
    if (!St[cur].next[c]) {
      int now = SZ(St);
      St.pb(St[cur].len + 2);
      St[now].fail =
        St[get_fail(St[cur].fail)].next[c];
      St[cur].next[c] = now;
      St[now].num = St[St[now].fail].num + 1;
    }
    last = St[cur].next[c], ++St[last].cnt;
  }
  inline void count() { // counting cnt
    auto i = St.rbegin();
    for (; i != St.rend(); ++i) {
      St[i->fail].cnt += i->cnt;
    }
  }
  inline int size() { // The number of diff. pal.
    return SZ(St) - 2;
  }
};
```

6 Math

6.1 ax+by=gcd*

```
pll exgcd(ll a, ll b) {
  if(b == 0) return pll(1, 0);
  else {
    ll p = a / b;
    pll q = exgcd(b, a % b);
    return pll(q.Y, q.X - q.Y * p);
  }
}
```

6.2 floor and ceil

```
int floor(int a,int b){
  return a/b-(a%b&&a<0^b<0);
}
int ceil(int a,int b){
  return a/b+(a%b&&a<0^b>0);
}
```

6.3 Gaussian integer gcd

```
cpx gaussian_gcd(cpx a, cpx b) {
#define rnd(a, b) ((a >= 0 ? a * 2 + b : a * 2 - b) / (
    b * 2))
    ll c = a.real() * b.real() + a.imag() * b.imag();
    ll d = a.imag() * b.real() - a.real() * b.imag();
    ll r = b.real() * b.real() + b.imag() * b.imag();
    if (c % r == 0 && d % r == 0) return b;
    return gaussian_gcd(b, a - cpx(rnd(c, r), rnd(d, r)
        ) * b);
}
```

6.4 Miller Rabin*

```
// n < 4,759,123,141        3 : 2, 7, 61
// n < 1,122,004,669,633   4 : 2, 13, 23, 1662803
// n < 3,474,749,660,383   6 : pirmes <= 13
// n < 2^64                7 :
// 2, 325, 9375, 28178, 450775, 9780504, 1795265022
bool Miller_Rabin(ll a, ll n) {
  if((a = a % n) == 0) return 1;
  if((n & 1) ^ 1) return n == 2;
  ll tmp = (n - 1) / ((n - 1) & (1 - n));
  ll t = __lg(((n - 1) & (1 - n))), x = 1;
  for(; tmp; tmp >>= 1, a = mul(a, a, n))
    if(tmp & 1) x = mul(x, a, n);
  if(x == 1 || x == n - 1) return 1;
  while(--t)
    if((x = mul(x, x, n)) == n - 1) return 1;
  return 0;
}
```

6.5 Fraction

```
struct fraction{
  ll n,d;
  fraction(const ll &_n=0,const ll &_d=1):n(_n),d(_d){
    ll t=__gcd(n,d);
    n/=t,d/=t;
    if(d<0) n=-n,d=-d;
  }
  fraction operator-()const{
    return fraction(-n,d);
  }
  fraction operator+(const fraction &b)const{
    return fraction(n*b.d+b.n*d,d*b.d);
  }
  fraction operator-(const fraction &b)const{
    return fraction(n*b.d-b.n*d,d*b.d);
  }
  fraction operator*(const fraction &b)const{
    return fraction(n*b.n,d*b.d);
  }
  fraction operator/(const fraction &b)const{
    return fraction(n*b.d,d*b.n);
  }
  void print(){
    cout << n;
    if(d!=1) cout << "/" << d;
  }
};
```

6.6 Simultaneous Equations

```
struct matrix { //m variables, n equations
  int n, m;
  fraction M[MAXN][MAXN + 1], sol[MAXN];
  int solve() { //-1: inconsistent, >= 0: rank
    for (int i = 0; i < n; ++i) {
      int piv = 0;
      while (piv < m && !M[i][piv].n) ++piv;
      if (piv == m) continue;
      for (int j = 0; j < n; ++j) {
        if (i == j) continue;
        fraction tmp = -M[j][piv] / M[i][piv];
        for (int k = 0; k <= m; ++k) M[j][k] = tmp * M[
            i][k] + M[j][k];
      }
    }
    int rank = 0;
    for (int i = 0; i < n; ++i) {
      int piv = 0;
      while (piv < m && !M[i][piv].n) ++piv;
      if (piv == m && M[i][m].n) return -1;
      else if (piv < m) ++rank, sol[piv] = M[i][m] / M[
          i][piv];
    }
    return rank;
  }
};
```

6.7 Pollard Rho*

```
map<ll, int> cnt;
void PollardRho(ll n) {
  if (n == 1) return;
  if (prime(n)) return ++cnt[n], void();
  if (n % 2 == 0) return PollardRho(n / 2), ++cnt[2],
      void();
  ll x = 2, y = 2, d = 1, p = 1;
  #define f(x, n, p) ((mul(x, x, n) + p) % n)
  while (true) {
    if (d != n && d != 1) {
      PollardRho(n / d);
      PollardRho(d);
      return;
    }
    if (d == n) ++p;
    x = f(x, n, p); y = f(f(y, n, p), n, p);
    d = __gcd(abs(x - y), n);
  }
}
```

6.8 Simplex Algorithm

```
const int MAXN = 111;
const int MAXM = 111;
const double eps = 1E-10;
double a[MAXN][MAXM], b[MAXN], c[MAXM], d[MAXN][MAXM];
double x[MAXM];
int ix[MAXN + MAXM]; // !!! array all indexed from 0
// max{cx} subject to {Ax<=b,x>=0}
// n: constraints, m: vars !!!
// x[] is the optimal solution vector
// usage :
// value = simplex(a, b, c, N, M);
double simplex(double a[MAXN][MAXM], double b[MAXN],
    double c[MAXM], int n, int m){
  ++m;
  int r = n, s = m - 1;
  memset(d, 0, sizeof(d));
  for (int i = 0; i < n + m; ++i) ix[i] = i;
  for (int i = 0; i < n; ++i) {
    for (int j = 0; j < m - 1; ++j) d[i][j] = -a[i][j];
    d[i][m - 1] = 1;
    d[i][m] = b[i];
    if (d[r][m] > d[i][m]) r = i;
  }
  for (int j = 0; j < m - 1; ++j) d[n][j] = c[j];
  d[n + 1][m - 1] = -1;
  for (double dd;; ) {
    if (r < n) {
      int t = ix[s]; ix[s] = ix[r + m]; ix[r + m] = t;
      d[r][s] = 1.0 / d[r][s];
      for (int j = 0; j <= m; ++j)
        if (j != s) d[r][j] *= -d[r][s];
      for (int i = 0; i <= n + 1; ++i) if (i != r) {
        for (int j = 0; j <= m; ++j) if (j != s)
          d[i][j] += d[r][j] * d[i][s];
        d[i][s] *= d[r][s];
      }
    }
    r = -1; s = -1;
    for (int j = 0; j < m; ++j)
      if (s < 0 || ix[s] > ix[j]) {
        if (d[n + 1][j] > eps ||
            (d[n + 1][j] > -eps && d[n][j] > eps))
          s = j;
      }
    if (s < 0) break;
    for (int i = 0; i < n; ++i) if (d[i][s] < -eps) {
      if (r < 0 ||
          (dd = d[r][m] / d[r][s] - d[i][m] / d[i][s])
              < -eps ||
          (dd < eps && ix[r + m] > ix[i + m]))
        r = i;
    }
    if (r < 0) return -1; // not bounded
  }
  if (d[n + 1][m] < -eps) return -1; // not executable
  double ans = 0;
  for(int i=0; i<m; i++) x[i] = 0;
  for (int i = m; i < n + m; ++i) { // the missing
      enumerated x[i] = 0
    if (ix[i] < m - 1){
      ans += d[i - m][m] * c[ix[i]];
      x[ix[i]] = d[i-m][m];
    }
  }
  return ans;
}
```

6.8.1 Construction

Standard form: maximize $\mathbf{c}^T\mathbf{x}$ subject to $A\mathbf{x} \le \mathbf{b}$ and $\mathbf{x} \ge 0$.
Dual LP: minimize $\mathbf{b}^T\mathbf{y}$ subject to $A^T\mathbf{y} \ge \mathbf{c}$ and $\mathbf{y} \ge 0$.
$\bar{\mathbf{x}}$ and $\bar{\mathbf{y}}$ are optimal if and only if for all $i \in [1, n]$, either $\bar{x}_i = 0$ or $\sum_{j=1}^{m} A_{ji}\bar{y}_j = c_i$ holds and for all $i \in [1, m]$ either $\bar{y}_i = 0$ or $\sum_{j=1}^{n} A_{ij}\bar{x}_j = b_j$ holds.

1. In case of minimization, let $c'_i = -c_i$
2. $\sum_{1\le i\le n} A_{ji}x_i \ge b_j \to \sum_{1\le i\le n} -A_{ji}x_i \le -b_j$
3. $\sum_{1\le i\le n} A_{ji}x_i = b_j$
   - $\sum_{1\le i\le n} A_{ji}x_i \le b_j$
   - $\sum_{1\le i\le n} A_{ji}x_i \ge b_j$
4. If x_i has no lower bound, replace x_i with $x_i - x'_i$

6.9 Schreier-Sims Algorithm*

```
namespace schreier {
int n;
vector<vector<vector<int>>> bkts, binv;
vector<vector<int>> lk;
vector<int> operator*(const vector<int> &a, const
    vector<int> &b) {
    vector<int> res(SZ(a));
    for (int i = 0; i < SZ(a); ++i) res[i] = b[a[i]];
    return res;
}
vector<int> inv(const vector<int> &a) {
    vector<int> res(SZ(a));
    for (int i = 0; i < SZ(a); ++i) res[a[i]] = i;
    return res;
}
int filter(const vector<int> &g, bool add = true) {
    n = SZ(bkts);
    vector<int> p = g;
    for (int i = 0; i < n; ++i) {
        assert(p[i] >= 0 && p[i] < SZ(lk[i]));
        if (lk[i][p[i]] == -1) {
            if (add) {
                bkts[i].pb(p);
                binv[i].pb(inv(p));
                lk[i][p[i]] = SZ(bkts[i]) - 1;
            }
            return i;
        }
        p = p * binv[i][lk[i][p[i]]];
    }
    return -1;
}
bool inside(const vector<int> &g) { return filter(g,
    false) == -1; }
void solve(const vector<vector<int>> &gen, int _n) {
    n = _n;
    bkts.clear(), bkts.resize(n);
    binv.clear(), binv.resize(n);
    lk.clear(), lk.resize(n);
    vector<int> iden(n);
    iota(iden.begin(), iden.end(), 0);
    for (int i = 0; i < n; ++i) {
        lk[i].resize(n, -1);
        bkts[i].pb(iden);
        binv[i].pb(iden);
        lk[i][i] = 0;
    }
    for (int i = 0; i < SZ(gen); ++i) filter(gen[i]);
    queue<pair<pii, pii>> upd;
    for (int i = 0; i < n; ++i)
        for (int j = i; j < n; ++j)
            for (int k = 0; k < SZ(bkts[i]); ++k)
                for (int l = 0; l < SZ(bkts[j]); ++l)
                    upd.emplace(pii(i, k), pii(j, l));
    while (!upd.empty()) {
        auto a = upd.front().X;
        auto b = upd.front().Y;
        upd.pop();
        int res = filter(bkts[a.X][a.Y] * bkts[b.X][b.Y
            ]);
        if (res == -1) continue;
        pii pr = pii(res, SZ(bkts[res]) - 1);
        for (int i = 0; i < n; ++i)
            for (int j = 0; j < SZ(bkts[i]); ++j) {
                if (i <= res) upd.emplace(pii(i, j), pr
                    );
                if (res <= i) upd.emplace(pr, pii(i, j)
                    );
            }
    }
}
long long size() {
    long long res = 1;
    for (int i = 0; i < n; ++i) res = res * SZ(bkts[i])
        ;
    return res;
}
}}
```

6.10 chineseRemainder

```
LL solve(LL x1, LL m1, LL x2, LL m2) {
  LL g = __gcd(m1, m2);
  if((x2 - x1) % g) return -1;// no sol
  m1 /= g; m2 /= g;
  pair<LL,LL> p = gcd(m1, m2);
  LL lcm = m1 * m2 * g;
  LL res = p.first * (x2 - x1) * m1 + x1;
  return (res % lcm + lcm) % lcm;
}
```

6.11 QuadraticResidue*

```
int Jacobi(int a, int m) {
  int s = 1;
  for (; m > 1; ) {
    a %= m;
    if (a == 0) return 0;
    const int r = __builtin_ctz(a);
    if ((r & 1) && ((m + 2) & 4)) s = -s;
    a >>= r;
    if (a & m & 2) s = -s;
    swap(a, m);
  }
  return s;
}

int QuadraticResidue(int a, int p) {
  if (p == 2) return a & 1;
  const int jc = Jacobi(a, p);
  if (jc == 0) return 0;
  if (jc == -1) return -1;
  int b, d;
  for (; ; ) {
    b = rand() % p;
    d = (1LL * b * b + p - a) % p;
    if (Jacobi(d, p) == -1) break;
  }
  int f0 = b, f1 = 1, g0 = 1, g1 = 0, tmp;
  for (int e = (1LL + p) >> 1; e; e >>= 1) {
    if (e & 1) {
      tmp = (1LL * g0 * f0 + 1LL * d * (1LL * g1 * f1 %
           p)) % p;
      g1 = (1LL * g0 * f1 + 1LL * g1 * f0) % p;
      g0 = tmp;
    }
    tmp = (1LL * f0 * f0 + 1LL * d * (1LL * f1 * f1 % p
        )) % p;
    f1 = (2LL * f0 * f1) % p;
    f0 = tmp;
  }
  return g0;
}
```

6.12 PiCount

```
int64_t PrimeCount(int64_t n) {
  if (n <= 1) return 0;
  const int v = sqrt(n);
  vector<int> smalls(v + 1);
  for (int i = 2; i <= v; ++i) smalls[i] = (i + 1) / 2;
  int s = (v + 1) / 2;
  vector<int> roughs(s);
  for (int i = 0; i < s; ++i) roughs[i] = 2 * i + 1;
  vector<int64_t> larges(s);
  for (int i = 0; i < s; ++i) larges[i] = (n / (2 * i +
      1) + 1) / 2;
  vector<bool> skip(v + 1);
  int pc = 0;
  for (int p = 3; p <= v; ++p) {
    if (smalls[p] > smalls[p - 1]) {
      int q = p * p;
      pc++;
      if (1LL * q * q > n) break;
      skip[p] = true;
      for (int i = q; i <= v; i += 2 * p) skip[i] =
          true;
      int ns = 0;
      for (int k = 0; k < s; ++k) {
        int i = roughs[k];
        if (skip[i]) continue;
        int64_t d = 1LL * i * p;
        larges[ns] = larges[k] - (d <= v ? larges[
            smalls[d] - pc] : smalls[n / d]) + pc;
        roughs[ns++] = i;
      }
      s = ns;
      for (int j = v / p; j >= p; --j) {
        int c = smalls[j] - pc;
        for (int i = j * p, e = min(i + p, v + 1); i <
            e; ++i) smalls[i] -= c;
      }
    }
  }
  for (int k = 1; k < s; ++k) {
    const int64_t m = n / roughs[k];
    int64_t s = larges[k] - (pc + k - 1);
    for (int l = 1; l < k; ++l) {
      int p = roughs[l];
      if (1LL * p * p > m) break;
      s -= smalls[m / p] - (pc + l - 1);
    }
    larges[0] -= s;
  }
  return larges[0];
}
```

6.13 Discrete Log*

```
int DiscreteLog(int s, int x, int y, int m) {
  constexpr int kStep = 32000;
  unordered_map<int, int> p;
  int b = 1;
  for (int i = 0; i < kStep; ++i) {
    p[y] = i;
    y = 1LL * y * x % m;
    b = 1LL * b * x % m;
  }
  for (int i = 0; i < m + 10; i += kStep) {
    s = 1LL * s * b % m;
    if (p.find(s) != p.end()) return i + kStep - p[s];
  }
  return -1;
}
int DiscreteLog(int x, int y, int m) {
  if (m == 1) return 0;
  int s = 1;
  for (int i = 0; i < 100; ++i) {
    if (s == y) return i;
    s = 1LL * s * x % m;
  }
  if (s == y) return 100;
  int p = 100 + DiscreteLog(s, x, y, m);
  if (fpow(x, p, m) != y) return -1;
  return p;
}
```

6.14 Primes

```
/* 12721 13331 14341 75577 123457 222557 556679 999983
   1097774749 1076767633 100102021 999997771
   1001010013 1000512343 987654361 999991231 999888733
    98789101 987777733 999991921 1010101333 1010102101
    1000000000039 1000000000000037 2305843009213693951
    4611686018427387847 9223372036854775783
   18446744073709551557 */
```

6.15 Theorem

- Kirchhoff's Theorem

  Denote L be a $n \times n$ matrix as the Laplacian matrix of graph G, where $L_{ii} = d(i)$, $L_{ij} = -c$ where c is the number of edge (i, j) in G.

  - The number of undirected spanning in G is $|\det(\tilde{L}_{11})|$.
  - The number of directed spanning tree rooted at r in G is $|\det(\tilde{L}_{rr})|$.

- Tutte's Matrix

  Let D be a $n \times n$ matrix, where $d_{ij} = x_{ij}$ (x_{ij} is chosen uniformly at random) if $i < j$ and $(i, j) \in E$, otherwise $d_{ij} = -d_{ji}$. $\frac{rank(D)}{2}$ is the maximum matching on G.

- Cayley's Formula
  - Given a degree sequence $d_1, d_2, \ldots, d_n$ for each *labeled* vertices, there are $\frac{(n-2)!}{(d_1-1)!(d_2-1)!\cdots(d_n-1)!}$ spanning trees.
  - Let $T_{n,k}$ be the number of *labeled* forests on n vertices with k components, such that vertex $1, 2, \ldots, k$ belong to different components. Then $T_{n,k} = kn^{n-k-1}$.
- Erdős–Gallai theorem

  A sequence of nonnegative integers $d_1 \geq \cdots \geq d_n$ can be represented as the degree sequence of a finite simple graph on n vertices if and only if $d_1 + \cdots + d_n$ is even and $\sum_{i-1}^{k} d_i \leq k(k-1) + \sum_{i=k+1}^{n} \min(d_i, k)$ holds for every $1 \leq k \leq n$.
- Gale–Ryser theorem

  A pair of sequences of nonnegative integers $a_1 \geq \cdots \geq a_n$ and $b_1, \ldots, b_n$ is bigraphic if and only if $\sum_{i=1}^{n} a_i = \sum_{i=1}^{n} b_i$ and $\sum_{i=1}^{k} a_i \leq \sum_{i=1}^{n} \min(b_i, k)$ holds for every $1 \leq k \leq n$.
- Fulkerson–Chen–Anstee theorem

  A sequence $(a_1, b_1), \ldots, (a_n, b_n)$ of nonnegative integer pairs with $a_1 \geq \cdots \geq a_n$ is digraphic if and only if $\sum_{i=1}^{n} a_i = \sum_{i=1}^{n} b_i$ and $\sum_{i=1}^{k} a_i \leq \sum_{i=1}^{k} \min(b_i, k-1) + \sum_{i=k+1}^{n} \min(b_i, k)$ holds for every $1 \leq k \leq n$.
- Möbius inversion formula
  - $f(n) = \sum_{d|n} g(d) \Leftrightarrow g(n) = \sum_{d|n} \mu(d) f(\frac{n}{d})$
  - $f(n) = \sum_{n|d} g(d) \Leftrightarrow g(n) = \sum_{n|d} \mu(\frac{d}{n}) f(d)$

6.16 Euclidean Algorithms

- $m = \lfloor \frac{an+b}{c} \rfloor$
- Time complexity: $O(\log n)$

$$f(a,b,c,n) = \sum_{i=0}^{n} \lfloor \frac{ai+b}{c} \rfloor$$
$$= \begin{cases} \lfloor \frac{a}{c} \rfloor \cdot \frac{n(n+1)}{2} + \lfloor \frac{b}{c} \rfloor \cdot (n+1) & \\ +f(a \bmod c, b \bmod c, c, n), & a \geq c \lor b \geq c \\ 0, & n < 0 \lor a = 0 \\ nm - f(c, c-b-1, a, m-1), & \text{otherwise} \end{cases}$$

$$g(a,b,c,n) = \sum_{i=0}^{n} i \lfloor \frac{ai+b}{c} \rfloor$$
$$= \begin{cases} \lfloor \frac{a}{c} \rfloor \cdot \frac{n(n+1)(2n+1)}{6} + \lfloor \frac{b}{c} \rfloor \cdot \frac{n(n+1)}{2} & \\ +g(a \bmod c, b \bmod c, c, n), & a \geq c \lor b \geq c \\ 0, & n < 0 \lor a = 0 \\ \frac{1}{2} \cdot (n(n+1)m - f(c, c-b-1, a, m-1) & \\ -h(c, c-b-1, a, m-1)), & \text{otherwise} \end{cases}$$

$$h(a,b,c,n) = \sum_{i=0}^{n} \lfloor \frac{ai+b}{c} \rfloor^2$$
$$= \begin{cases} \lfloor \frac{a}{c} \rfloor^2 \cdot \frac{n(n+1)(2n+1)}{6} + \lfloor \frac{b}{c} \rfloor^2 \cdot (n+1) & \\ +\lfloor \frac{a}{c} \rfloor \cdot \lfloor \frac{b}{c} \rfloor \cdot n(n+1) & \\ +h(a \bmod c, b \bmod c, c, n) & \\ +2\lfloor \frac{a}{c} \rfloor \cdot g(a \bmod c, b \bmod c, c, n) & \\ +2\lfloor \frac{b}{c} \rfloor \cdot f(a \bmod c, b \bmod c, c, n), & a \geq c \lor b \geq c \\ 0, & n < 0 \lor a = 0 \\ nm(m+1) - 2g(c, c-b-1, a, m-1) & \\ -2f(c, c-b-1, a, m-1) - f(a,b,c,n), & \text{otherwise} \end{cases}$$

6.17 General Purpose Numbers

- Bernoulli numbers

  $B_0 - 1, B_1^{\pm} = \pm\frac{1}{2}, B_2 = \frac{1}{6}, B_3 = 0$

  $\sum_{j=0}^{m} \binom{m+1}{j} B_j = 0$, EGF is $B(x) = \frac{x}{e^x - 1} = \sum_{n=0}^{\infty} B_n \frac{x^n}{n!}$.

  $$S_m(n) = \sum_{k=1}^{n} k^m = \frac{1}{m+1} \sum_{k=0}^{m} \binom{m+1}{k} B_k^+ n^{m+1-k}$$
- Stirling numbers of the second kind Partitions of n distinct elements into exactly k groups.

  $$S(n,k) = S(n-1,k-1) + kS(n-1,k), S(n,1) = S(n,n) = 1$$
  $$S(n,k) = \frac{1}{k!} \sum_{i=0}^{k} (-1)^{k-i} \binom{k}{i} i^n$$
- Pentagonal number theorem

  $$\prod_{n=1}^{\infty} (1 - x^n) = 1 + \sum_{k=1}^{\infty} (-1)^k \left(x^{k(3k+1)/2} + x^{k(3k-1)/2} \right)$$
- Catalan numbers

  $$C_n^{(k)} = \frac{1}{(k-1)n+1} \binom{kn}{n}$$
  $$C^{(k)}(x) = 1 + x[C^{(k)}(x)]^k$$

7 Polynomial

7.1 Fast Fourier Transform

```cpp
template<int MAXN>
struct FFT {
  using val_t = complex<double>;
  const double PI = acos(-1);
  val_t w[MAXN];
  FFT() {
    for (int i = 0; i < MAXN; ++i) {
      double arg = 2 * PI * i / MAXN;
      w[i] = val_t(cos(arg), sin(arg));
    }
  }
  void bitrev(val_t *a, int n); // see NTT
  void trans(val_t *a, int n, bool inv = false); // see NTT;
  // remember to replace LL with val_t
};
```

7.2 Number Theory Transform*

```cpp
//(2^16)+1, 65537, 3
//7*17*(2^23)+1, 998244353, 3
//1255*(2^20)+1, 1315962881, 3
//51*(2^25)+1, 1711276033, 29
template<int MAXN, ll P, ll RT> //MAXN must be 2^k
struct NTT {
  ll w[MAXN];
  ll mpow(ll a, ll n);
  ll minv(ll a) { return mpow(a, P - 2); }
  NTT() {
    ll dw = mpow(RT, (P - 1) / MAXN);
    w[0] = 1;
    for (int i = 1; i < MAXN; ++i) w[i] = w[i - 1] * dw % P;
  }
  void bitrev(ll *a, int n) {
    int i = 0;
    for (int j = 1; j < n - 1; ++j) {
      for (int k = n >> 1; (i ^= k) < k; k >>= 1);
      if (j < i) swap(a[i], a[j]);
    }
  }
  void operator()(ll *a, int n, bool inv = false) { //0 <= a[i] < P
    bitrev(a, n);
    for (int L = 2; L <= n; L <<= 1) {
      int dx = MAXN / L, dl = L >> 1;
      for (int i = 0; i < n; i += L) {
        for (int j = i, x = 0; j < i + dl; ++j, x += dx) {
          ll tmp = a[j + dl] * w[x] % P;
          if ((a[j + dl] = a[j] - tmp) < 0) a[j + dl] += P;
          if ((a[j] += tmp) >= P) a[j] -= P;
        }
      }
    }
    if (inv) {
      reverse(a + 1, a + n);
      ll invn = minv(n);
      for (int i = 0; i < n; ++i) a[i] = a[i] * invn % P;
    }
  }
};
```

7.3 Fast Walsh Transform*

```
/* x: a[j], y: a[j + (L >> 1)]
or: (y += x * op), and: (x += y * op)
xor: (x, y = (x + y) * op, (x - y) * op)
invop: or, and, xor = -1, -1, 1/2 */
void fwt(int *a, int n, int op) { //or
    for (int L = 2; L <= n; L <<= 1)
        for (int i = 0; i < n; i += L)
            for (int j = i; j < i + (L >> 1); ++j)
                a[j + (L >> 1)] += a[j] * op;
}
const int N = 21;
int f[N][1 << N], g[N][1 << N], h[N][1 << N], ct[1 << N
    ];
void subset_convolution(int *a, int *b, int *c, int L)
    {
    // c_k = \sum_{i | j = k, i & j = 0} a_i * b_j
    int n = 1 << L;
    for (int i = 1; i < n; ++i)
        ct[i] = ct[i & (i - 1)] + 1;
    for (int i = 0; i < n; ++i)
        f[ct[i]][i] = a[i], g[ct[i]][i] = b[i];
    for (int i = 0; i <= L; ++i)
        fwt(f[i], n, 1), fwt(g[i], n, 1);
    for (int i = 0; i <= L; ++i)
        for (int j = 0; j <= i; ++j)
            for (int x = 0; x < n; ++x)
                h[i][x] += f[j][x] * g[i - j][x];
    for (int i = 0; i <= L; ++i)
        fwt(h[i], n, -1);
    for (int i = 0; i < n; ++i)
        c[i] = h[ct[i]][i];
}
```

7.4 Polynomial Operation

```
#define fi(s, n) for (int i = (int)(s); i < (int)(n);
    ++i)
template<int MAXN, ll P, ll RT> // MAXN = 2^k
struct Poly : vector<ll> { // coefficients in [0, P)
  using vector<ll>::vector;
  static NTT<MAXN, P, RT> ntt;
  int n() const { return (int)size(); } // n() >= 1
  Poly(const Poly &p, int _n) : vector<ll>(_n) {
    copy_n(p.data(), min(p.n(), _n), data());
  }
  Poly& irev() { return reverse(data(), data() + n()),
      *this; }
  Poly& isz(int _n) { return resize(_n), *this; }
  Poly& iadd(const Poly &rhs) { // n() == rhs.n()
    fi(0, n()) if (((*this)[i] += rhs[i]) >= P) (*this)
        [i] -= P;
    return *this;
  }
  Poly& imul(ll k) {
    fi(0, n()) (*this)[i] = (*this)[i] * k % P;
    return *this;
  }
  Poly Mul(const Poly &rhs) const {
    int _n = 1;
    while (_n < n() + rhs.n() - 1) _n <<= 1;
    Poly X(*this, _n), Y(rhs, _n);
    ntt(X.data(), _n), ntt(Y.data(), _n);
    fi(0, _n) X[i] = X[i] * Y[i] % P;
    ntt(X.data(), _n, true);
    return X.isz(n() + rhs.n() - 1);
  }
  Poly Inv() const { // (*this)[0] != 0, 1e5/95ms
    if (n() == 1) return {ntt.minv((*this)[0])};
    int _n = 1;
    while (_n < n() * 2) _n <<= 1;
    Poly Xi = Poly(*this, (n() + 1) / 2).Inv().isz(_n);
    Poly Y(*this, _n);
    ntt(Xi.data(), _n), ntt(Y.data(), _n);
    fi(0, _n) {
      Xi[i] *= (2 - Xi[i] * Y[i]) % P;
      if ((Xi[i] %= P) < 0) Xi[i] += P;
    }
    ntt(Xi.data(), _n, true);
    return Xi.isz(n());
  }
  Poly Sqrt() const { // Jacobi((*this)[0], P) = 1, 1e5
      /235ms
    if (n() == 1) return {QuadraticResidue((*this)[0],
        P)};
    Poly X = Poly(*this, (n() + 1) / 2).Sqrt().isz(n())
        ;
    return X.iadd(Mul(X.Inv()).isz(n())).imul(P / 2 +
        1);
  }
  pair<Poly, Poly> DivMod(const Poly &rhs) const { // (
      rhs.)back() != 0
    if (n() < rhs.n()) return {{0}, *this};
    const int _n = n() - rhs.n() + 1;
    Poly X(rhs); X.irev().isz(_n);
    Poly Y(*this); Y.irev().isz(_n);
    Poly Q = Y.Mul(X.Inv()).isz(_n).irev();
    X = rhs.Mul(Q), Y = *this;
    fi(0, n()) if ((Y[i] -= X[i]) < 0) Y[i] += P;
    return {Q, Y.isz(max(1, rhs.n() - 1))};
  }
  Poly Dx() const {
    Poly ret(n() - 1);
    fi(0, ret.n()) ret[i] = (i + 1) * (*this)[i + 1] %
        P;
    return ret.isz(max(1, ret.n()));
  }
  Poly Sx() const {
    Poly ret(n() + 1);
    fi(0, n()) ret[i + 1] = ntt.minv(i + 1) * (*this)[i
        ] % P;
    return ret;
  }
  Poly _tmul(int nn, const Poly &rhs) const {
    Poly Y = Mul(rhs).isz(n() + nn - 1);
    return Poly(Y.data() + n() - 1, Y.data() + Y.n());
  }
  vector<ll> _eval(const vector<ll> &x, const vector<
      Poly> &up) const {
    const int _n = (int)x.size();
    if (!_n) return {};
    vector<Poly> down(_n * 2);
    // down[1] = DivMod(up[1]).second;
    // fi(2, _n * 2) down[i] = down[i / 2].DivMod(up[i
        ]).second;
    down[1] = Poly(up[1]).irev().isz(n()).Inv().irev().
        _tmul(_n, *this);
    fi(2, _n * 2) down[i] = up[i ^ 1]._tmul(up[i].n() -
         1, down[i / 2]);
    vector<ll> y(_n);
    fi(0, _n) y[i] = down[_n + i][0];
    return y;
  }
  static vector<Poly> _tree1(const vector<ll> &x) {
    const int _n = (int)x.size();
    vector<Poly> up(_n * 2);
    fi(0, _n) up[_n + i] = {(x[i] ? P - x[i] : 0), 1};
    for (int i = _n - 1; i > 0; --i) up[i] = up[i * 2].
        Mul(up[i * 2 + 1]);
    return up;
  }
  vector<ll> Eval(const vector<ll> &x) const { // 1e5,
      1s
    auto up = _tree1(x); return _eval(x, up);
  }
  static Poly Interpolate(const vector<ll> &x, const
      vector<ll> &y) { // 1e5, 1.4s
    const int _n = (int)x.size();
    vector<Poly> up = _tree1(x), down(_n * 2);
    vector<ll> z = up[1].Dx()._eval(x, up);
    fi(0, _n) z[i] = y[i] * ntt.minv(z[i]) % P;
    fi(0, _n) down[_n + i] = {z[i]};
    for (int i = _n - 1; i > 0; --i) down[i] = down[i *
         2].Mul(up[i * 2 + 1]).iadd(down[i * 2 + 1].Mul
        (up[i * 2]));
    return down[1];
  }
  Poly Ln() const { // (*this)[0] == 1, 1e5/170ms
    return Dx().Mul(Inv()).Sx().isz(n());
  }
  Poly Exp() const { // (*this)[0] == 0, 1e5/360ms
    if (n() == 1) return {1};
    Poly X = Poly(*this, (n() + 1) / 2).Exp().isz(n());
```

```
    Poly Y = X.Ln(); Y[0] = P - 1;
    fi(0, n()) if ((Y[i] = (*this)[i] - Y[i]) < 0) Y[i]
       += P;
    return X.Mul(Y).isz(n());
  }
  // M := P(P - 1). If k >= M, k := k % M + M.
  Poly Pow(ll k) const {
    int nz = 0;
    while (nz < n() && !(*this)[nz]) ++nz;
    if (nz * min(k, (ll)n()) >= n()) return Poly(n());
    if (!k) return Poly(Poly {1}, n());
    Poly X(data() + nz, data() + nz + n() - nz * k);
    const ll c = ntt.mpow(X[0], k % (P - 1));
    return X.Ln().imul(k % P).Exp().imul(c).irev().isz(
        n()).irev();
  }
  static ll LinearRecursion(const vector<ll> &a, const
      vector<ll> &coef, ll n) { // a_n = \sum c_j a_(n-
      j)
    const int k = (int)a.size();
    assert((int)coef.size() == k + 1);
    Poly C(k + 1), W(Poly {1}, k), M = {0, 1};
    fi(1, k + 1) C[k - i] = coef[i] ? P - coef[i] : 0;
    C[k] = 1;
    while (n) {
      if (n % 2) W = W.Mul(M).DivMod(C).second;
      n /= 2, M = M.Mul(M).DivMod(C).second;
    }
    ll ret = 0;
    fi(0, k) ret = (ret + W[i] * a[i]) % P;
    return ret;
  }
};
#undef fi
using Poly_t = Poly<131072 * 2, 998244353, 3>;
template<> decltype(Poly_t::ntt) Poly_t::ntt = {};
```

7.5 Newton's Method

Given $F(x)$ where

$$F(x) = \sum_{i=0}^{\infty} \alpha_i (x - \beta)^i$$

for β being some constant. Polynomial P such that $F(P) = 0$ can be found iteratively. Denote by Q_k the polynomial such that $F(Q_k) = 0 \pmod{x^{2^k}}$, then

$$Q_{k+1} = Q_k - \frac{F(Q_k)}{F'(Q_k)} \pmod{x^{2^{k+1}}}$$

8 Geometry

8.1 Default Code

```
typedef pair<double,double> pdd;
typedef pair<pdd,pdd> Line;
struct Cir{pdd O; double R;};
const double eps=1e-8;
pdd operator+(const pdd &a, const pdd &b)
{ return pdd(a.X + b.X, a.Y + b.Y);}
pdd operator-(const pdd &a, const pdd &b)
{ return pdd(a.X - b.X, a.Y - b.Y);}
pdd operator*(const pdd &a, const double &b)
{ return pdd(a.X * b, a.Y * b);}
pdd operator/(const pdd &a, const double &b)
{ return pdd(a.X / b, a.Y / b);}
double dot(const pdd &a,const pdd &b)
{ return a.X * b.X + a.Y * b.Y;}
double cross(const pdd &a,const pdd &b)
{ return a.X * b.Y - a.Y * b.X;}
double abs2(const pdd &a)
{ return dot(a, a);}
double abs(const pdd &a)
{ return sqrt(dot(a, a));}
int sign(const double &a)
{ return fabs(a) < eps ? 0 : a > 0 ? 1 : -1;}
int ori(const pdd &a,const pdd &b,const pdd &c)
{ return sign(cross(b - a, c - a));}
bool collinearity(const pdd &p1, const pdd &p2, const
    pdd &p3)
{ return fabs(cross(p1 - p3, p2 - p3)) < eps;}
bool btw(const pdd &p1,const pdd &p2,const pdd &p3) {
  if(!collinearity(p1, p2, p3)) return 0;
  return dot(p1 - p3, p2 - p3) < eps;
}
bool seg_intersect(const pdd &p1,const pdd &p2,const
    pdd &p3,const pdd &p4) {
  int a123 = ori(p1, p2, p3);
  int a124 = ori(p1, p2, p4);
  int a341 = ori(p3, p4, p1);
  int a342 = ori(p3, p4, p2);
  if(a123 == 0 && a124 == 0)
    return btw(p1, p2, p3) || btw(p1, p2, p4) ||
      btw(p3, p4, p1) || btw(p3, p4, p2);
  return a123 * a124 <= 0 && a341 * a342 <= 0;
}
pdd intersect(const pdd &p1, const pdd &p2, const pdd &
    p3, const pdd &p4) {
  double a123 = cross(p2 - p1, p3 - p1);
  double a124 = cross(p2 - p1, p4 - p1);
  return (p4 * a123 - p3 * a124) / (a123 - a124);
}
pdd perp(const pdd &p1)
{ return pdd(-p1.Y, p1.X);}
pdd foot(const pdd &p1, const pdd &p2, const pdd &p3)
{ return intersect(p1, p2, p3, p3 + perp(p2 - p1));}
```

8.2 Convex hull*

```
void hull(vector<pll> &dots) { // n=1 => ans = {}
  sort(dots.begin(), dots.end());
  vector<pll> ans(1, dots[0]);
  for (int ct = 0; ct < 2; ++ct, reverse(ALL(dots)))
    for (int i = 1, t = SZ(ans); i < SZ(dots); ans.pb(
        dots[i++]))
      while (SZ(ans) > t && ori(ans[SZ(ans) - 2], ans.
          back(), dots[i]) <= 0)
        ans.pop_back();
  ans.pop_back(), ans.swap(dots);
}
```

8.3 External bisector

```
pdd external_bisector(pdd p1,pdd p2,pdd p3){//213
  pdd L1=p2-p1,L2=p3-p1;
  L2=L2*abs(L1)/abs(L2);
  return L1+L2;
}
```

8.4 Heart

```
pdd circenter(pdd p0, pdd p1, pdd p2) { // radius = abs
    (center)
  p1 = p1 - p0, p2 = p2 - p0;
  double x1 = p1.X, y1 = p1.Y, x2 = p2.X, y2 = p2.Y;
  double m = 2. * (x1 * y2 - y1 * x2);
  center.X = (x1 * x1 * y2 - x2 * x2 * y1 + y1 * y2 * (
      y1 - y2)) / m;
  center.Y = (x1 * x2 * (x2 - x1) - y1 * y1 * x2 + x1 *
       y2 * y2) / m;
  return center + p0;
}
pdd incenter(pdd p1, pdd p2, pdd p3) { // radius = area
     / s * 2
  double a = abs(p2 - p3), b = abs(p1 - p3), c = abs(p1
      - p2);
  double s = a + b + c;
  return (a * p1 + b * p2 + c * p3) / s;
}
pdd masscenter(pdd p1, pdd p2, pdd p3)
{ return (p1 + p2 + p3) / 3; }
pdd orthcenter(pdd p1, pdd p2, pdd p3)
{ return masscenter(p1, p2, p3) * 3 - circenter(p1, p2,
     p3) * 2; }
```

8.5 Minimum Enclosing Circle*

```
pdd Minimum_Enclosing_Circle(vector<pdd> dots, double &
    r) {
  pdd cent;
  random_shuffle(ALL(dots));
  cent = dots[0], r = 0;
  for (int i = 1; i < SZ(dots); ++i)
    if (abs(dots[i] - cent) > r) {
      cent = dots[i], r = 0;
      for (int j = 0; j < i; ++j)
        if (abs(dots[j] - cent) > r) {
          cent = (dots[i] + dots[j]) / 2;
          r = abs(dots[i] - cent);
          for(int k = 0; k < j; ++k)
            if(abs(dots[k] - cent) > r)
              cent = excenter(dots[i], dots[j], dots[k
                  ], r);
        }
    }
  return cent;
}
```

8.6 Polar Angle Sort*

```
bool cmp(pdd a, pdd b) {
#define is_neg(k) (sign(k.Y) < 0 || (sign(k.Y) == 0 &&
    sign(k.X) < 0))
  int A = is_neg(a), B = is_neg(b);
  if (A != B)
    return A < B;
  if (sign(cross(a, b)) == 0)
    return abs2(a) < abs2(b);
  return sign(cross(a, b)) > 0;
}
bool cmp(pdd a, pdd b) {
  if(sign(atan2(a.Y, a.X) - atan2(b.Y, b.X)) != 0)
    return atan2(a.Y, a.X) < atan2(b.Y, b.X);
  return abs2(a) < abs2(b);
}
```

8.7 Intersection of two circles*

```
bool CCinter(Cir &a, Cir &b, pdd &p1, pdd &p2) {
  pdd o1 = a.O, o2 = b.O;
  double r1 = a.R, r2 = b.R, d2 = abs2(o1 - o2), d =
      sqrt(d2);
  if(d < max(r1, r2) - min(r1, r2) || d > r1 + r2)
      return 0;
  pdd u = (o1 + o2) * 0.5 + (o1 - o2) * ((r2 * r2 - r1
      * r1) / (2 * d2));
  double A = sqrt((r1 + r2 + d) * (r1 - r2 + d) * (r1 +
       r2 - d) * (-r1 + r2 + d));
  pdd v = pdd(o1.Y - o2.Y, -o1.X + o2.X) * A / (2 * d2)
      ;
  p1 = u + v, p2 = u - v;
  return 1;
}
```

8.8 Intersection of polygon and circle*

```
// Divides into multiple triangle, and sum up
const double PI=acos(-1);
double _area(pdd pa, pdd pb, double r){
  if(abs(pa)<abs(pb)) swap(pa, pb);
  if(abs(pb)<eps) return 0;
  double S, h, theta;
  double a=abs(pb),b=abs(pa),c=abs(pb-pa);
  double cosB = dot(pb,pb-pa) / a / c, B = acos(cosB);
  double cosC = dot(pa,pb) / a / b, C = acos(cosC);
  if(a > r){
    S = (C/2)*r*r;
    h = a*b*sin(C)/c;
    if (h < r && B < PI/2) S -= (acos(h/r)*r*r - h*sqrt
        (r*r-h*h));
  }
  else if(b > r){
    theta = PI - B - asin(sin(B)/r*a);
    S = .5*a*r*sin(theta) + (C-theta)/2*r*r;
  }
  else S = .5*sin(C)*a*b;
  return S;
}
double area_poly_circle(const vector<pdd> poly,const
    pdd &O,const double r){
  double S=0;
  for(int i=0;i<SZ(poly);++i)
    S+=_area(poly[i]-O,poly[(i+1)%SZ(poly)]-O,r)*ori(O,
        poly[i],poly[(i+1)%SZ(poly)]);
  return fabs(S);
}
```

8.9 Intersection of line and circle

```
vector<pdd> line_interCircle(const pdd &p1,const pdd &
    p2,const pdd &c,const double r){
  pdd ft=foot(p1,p2,c),vec=p2-p1;
  double dis=abs(c-ft);
  if(fabs(dis-r)<eps) return vector<pdd>{ft};
  if(dis>r) return {};
  vec=vec*sqrt(r*r-dis*dis)/abs(vec);
  return vector<pdd>{ft+vec,ft-vec};
}
```

8.10 point in circle

```
// return p4 is strictly in circumcircle of tri(p1,p2,
    p3)
long long sqr(long long x) { return x * x; }
bool in_cc(const pll& p1, const pll& p2, const pll& p3,
     const pll& p4) {
    long long u11 = p1.X - p4.X; long long u12 = p1.Y -
         p4.Y;
    long long u21 = p2.X - p4.X; long long u22 = p2.Y -
         p4.Y;
    long long u31 = p3.X - p4.X; long long u32 = p3.Y -
         p4.Y;
    long long u13 = sqr(p1.X) - sqr(p4.X) + sqr(p1.Y) -
         sqr(p4.Y);
    long long u23 = sqr(p2.X) - sqr(p4.X) + sqr(p2.Y) -
         sqr(p4.Y);
    long long u33 = sqr(p3.X) - sqr(p4.X) + sqr(p3.Y) -
         sqr(p4.Y);
    __int128 det = (__int128)-u13 * u22 * u31 + (
        __int128)u12 * u23 * u31 + (__int128)u13 * u21
        * u32 - (__int128)u11 * u23 * u32 - (__int128)
        u12 * u21 * u33 + (__int128)u11 * u22 * u33;
    return det > eps;
}
```

8.11 Half plane intersection

```
bool isin( Line l0, Line l1, Line l2 ){
  // Check inter(l1, l2) in l0
  pdd p = intersect(l1.X,l1.Y,l2.X,l2.Y);
  return cross(l0.Y - l0.X,p - l0.X) > eps;
}
/* If no solution, check intersect(ret[0], ret[1])
 * in all the lines. (use (l.Y - l.X) ^ (p - l.X) > 0
 */
/* --^-- Line.X --^-- Line.Y --^-- */
vector<Line> halfPlaneInter(vector<Line> lines){
  int sz = lines.size();
  vector<double> ata(sz),ord(sz);
  for(int i=0; i<sz; ++i) {
    ord[i] = i;
    pdd d = lines[i].Y - lines[i].X;
    ata[i] = atan2(d.Y, d.X);
  }
  sort(ord.begin(), ord.end(), [&](int i,int j){
      if( fabs(ata[i] - ata[j]) < eps )
      return (cross(lines[i].Y-lines[i].X,
            lines[j].Y-lines[i].X))<0;
      return ata[i] < ata[j];
      });
  vector<Line> fin;
  for (int i=0; i<sz; ++i)
    if (!i || fabs(ata[ord[i]] - ata[ord[i-1]]) > eps)
      fin.pb(lines[ord[i]]);
  deque<Line> dq;
  for (int i=0; i<SZ(fin); i++){
```

```
    while(SZ(dq)>=2&&!isin(fin[i],dq[SZ(dq)-2],dq.back
        ()))
      dq.pop_back();
    while(SZ(dq)>=2&&!isin(fin[i],dq[0],dq[1]))
      dq.pop_front();
    dq.push_back(fin[i]);
  }
  while(SZ(dq)>=3&&!isin(dq[0],dq[SZ(dq)-2],dq.back()))
    dq.pop_back();
  while(SZ(dq)>=3&&!isin(dq.back(), dq[0], dq[1]))
    dq.pop_front();
  vector<Line> res(ALL(dq));
  return res;
}
```

8.12 CircleCover*

```
const int N = 1021;
struct CircleCover {
  int C;
  Cir c[N];
  bool g[N][N], overlap[N][N];
  // Area[i] : area covered by at least i circles
  double Area[ N ];
  void init(int _C){ C = _C;}
  struct Teve {
    pdd p; double ang; int add;
    Teve() {}
    Teve(pdd _a, double _b, int _c):p(_a), ang(_b), add
        (_c){}
    bool operator<(const Teve &a)const
    {return ang < a.ang;}
  }eve[N * 2];
  // strict: x = 0, otherwise x = -1
  bool disjuct(Cir &a, Cir &b, int x)
  {return sign(abs(a.O - b.O) - a.R - b.R) > x;}
  bool contain(Cir &a, Cir &b, int x)
  {return sign(a.R - b.R - abs(a.O - b.O)) > x;}
  bool contain(int i, int j) {
    /* c[j] is non-strictly in c[i]. */
    return (sign(c[i].R - c[j].R) > 0 || (sign(c[i].R -
         c[j].R) == 0 && i < j)) && contain(c[i], c[j],
         -1);
  }
  void solve(){
    fill_n(Area, C + 2, 0);
    for(int i = 0; i < C; ++i)
      for(int j = 0; j < C; ++j)
        overlap[i][j] = contain(i, j);
    for(int i = 0; i < C; ++i)
      for(int j = 0; j < C; ++j)
        g[i][j] = !(overlap[i][j] || overlap[j][i] ||
            disjuct(c[i], c[j], -1));
    for(int i = 0; i < C; ++i){
      int E = 0, cnt = 1;
      for(int j = 0; j < C; ++j)
        if(j != i && overlap[j][i])
          ++cnt;
      for(int j = 0; j < C; ++j)
        if(i != j && g[i][j]) {
          pdd aa, bb;
          CCinter(c[i], c[j], aa, bb);
          double A = atan2(aa.Y - c[i].O.Y, aa.X - c[i
              ].O.X);
          double B = atan2(bb.Y - c[i].O.Y, bb.X - c[i
              ].O.X);
          eve[E++] = Teve(bb, B, 1), eve[E++] = Teve(aa
              , A, -1);
          if(B > A) ++cnt;
        }
      if(E == 0) Area[cnt] += pi * c[i].R * c[i].R;
      else{
        sort(eve, eve + E);
        eve[E] = eve[0];
        for(int j = 0; j < E; ++j){
          cnt += eve[j].add;
          Area[cnt] += cross(eve[j].p, eve[j + 1].p) *
              .5;
          double theta = eve[j + 1].ang - eve[j].ang;
          if (theta < 0) theta += 2. * pi;
          Area[cnt] += (theta - sin(theta)) * c[i].R *
              c[i].R * .5;
        }
      }
    }
  }
};
```

8.13 3Dpoint*

```
struct Point {
  double x, y, z;
  Point(double _x = 0, double _y = 0, double _z = 0): x
      (_x), y(_y), z(_z){}
  Point(pdd p) { x = p.X, y = p.Y, z = abs2(p); }
};
Point operator-(const Point &p1, const Point &p2)
{ return Point(p1.x - p2.x, p1.y - p2.y, p1.z - p2.z);}
Point cross(const Point &p1, const Point &p2)
{ return Point(p1.y * p2.z - p1.z * p2.y, p1.z * p2.x -
     p1.x * p2.z, p1.x * p2.y - p1.y * p2.x);}
double dot(const Point &p1, const Point &p2)
{ return p1.x * p2.x + p1.y * p2.y + p1.z * p2.z;}
double abs(const Point &a)
{ return sqrt(dot(a, a));}
Point cross3(const Point &a, const Point &b, const
    Point &c)
{ return cross(b - a, c - a);}
double area(Point a, Point b, Point c)
{ return abs(cross3(a, b, c));}
double volume(Point a, Point b, Point c, Point d)
{return dot(cross3(a, b, c), d - a);}
pdd proj(Point a, Point b, Point c, Point u) {
// proj. u to the plane of a, b, and c
    Point e1 = b - a;
    Point e2 = c - a;
    e1 = e1 / abs(e1);
    e2 = e2 - e1 * dot(e2, e1);
    e2 = e2 / abs(e2);
    Point p = u - a;
    return pdd(dot(p, e1), dot(p, e2));
}
```

8.14 Convexhull3D*

```
struct CH3D {
  struct face{int a, b, c; bool ok;} F[8 * N];
  double dblcmp(Point &p,face &f)
  {return dot(cross3(P[f.a], P[f.b], P[f.c]), p - P[f.a
      ]);}
  int g[N][N], num, n;
  Point P[N];
  void deal(int p,int a,int b) {
    int f = g[a][b];
    face add;
    if (F[f].ok) {
      if (dblcmp(P[p],F[f]) > eps) dfs(p,f);
      else
        add.a = b, add.b = a, add.c = p, add.ok = 1, g[
            p][b] = g[a][p] = g[b][a] = num, F[num++]=
            add;
    }
  }
  void dfs(int p, int now) {
    F[now].ok = 0;
    deal(p, F[now].b, F[now].a), deal(p, F[now].c, F[
        now].b), deal(p, F[now].a, F[now].c);
  }
  bool same(int s,int t){
    Point &a = P[F[s].a];
    Point &b = P[F[s].b];
    Point &c = P[F[s].c];
    return fabs(volume(a, b, c, P[F[t].a])) < eps &&
        fabs(volume(a, b, c, P[F[t].b])) < eps && fabs(
        volume(a, b, c, P[F[t].c])) < eps;
  }
  void init(int _n){n = _n, num = 0;}
  void solve() {
    face add;
    num = 0;
    if(n < 4) return;
    if([&](){
        for (int i = 1; i < n; ++i)
```

```
      if (abs(P[0] - P[i]) > eps)
      return swap(P[1], P[i]), 0;
      return 1;
      }() || [&](){
      for (int i = 2; i < n; ++i)
      if (abs(cross3(P[i], P[0], P[1])) > eps)
      return swap(P[2], P[i]), 0;
      return 1;
      }() || [&](){
      for (int i = 3; i < n; ++i)
      if (fabs(dot(cross(P[0] - P[1], P[1] - P[2]), P
          [0] - P[i])) > eps)
      return swap(P[3], P[i]), 0;
      return 1;
      }())return;
  for (int i = 0; i < 4; ++i) {
    add.a = (i + 1) % 4, add.b = (i + 2) % 4, add.c =
        (i + 3) % 4, add.ok = true;
    if (dblcmp(P[i],add) > 0) swap(add.b, add.c);
    g[add.a][add.b] = g[add.b][add.c] = g[add.c][add.
        a] = num;
    F[num++] = add;
  }
  for (int i = 4; i < n; ++i)
    for (int j = 0; j < num; ++j)
      if (F[j].ok && dblcmp(P[i],F[j]) > eps) {
        dfs(i, j);
        break;
      }
  for (int tmp = num, i = (num = 0); i < tmp; ++i)
    if (F[i].ok) F[num++] = F[i];
}
double get_area() {
  double res = 0.0;
  if (n == 3)
    return abs(cross3(P[0], P[1], P[2])) / 2.0;
  for (int i = 0; i < num; ++i)
    res += area(P[F[i].a], P[F[i].b], P[F[i].c]);
  return res / 2.0;
}
double get_volume() {
  double res = 0.0;
  for (int i = 0; i < num; ++i)
    res += volume(Point(0, 0, 0), P[F[i].a], P[F[i].b
        ], P[F[i].c]);
  return fabs(res / 6.0);
}
int triangle() {return num;}
int polygon() {
  int res = 0;
  for (int i = 0, flag = 1; i < num; ++i, res += flag
      , flag = 1)
    for (int j = 0; j < i && flag; ++j)
      flag &= !same(i,j);
  return res;
}
Point getcent(){
  Point ans(0, 0, 0), temp = P[F[0].a];
  double v = 0.0, t2;
  for (int i = 0; i < num; ++i)
    if (F[i].ok == true) {
      Point p1 = P[F[i].a], p2 = P[F[i].b], p3 = P[F[
          i].c];
      t2 = volume(temp, p1, p2, p3) / 6.0;
      if (t2>0)
        ans.x += (p1.x + p2.x + p3.x + temp.x) * t2,
            ans.y += (p1.y + p2.y + p3.y + temp.y) *
            t2, ans.z += (p1.z + p2.z + p3.z + temp.z
            ) * t2, v += t2;
    }
  ans.x /= (4 * v), ans.y /= (4 * v), ans.z /= (4 * v
      );
  return ans;
}
double pointmindis(Point p) {
  double rt = 99999999;
  for(int i = 0; i < num; ++i)
    if(F[i].ok == true) {
      Point p1 = P[F[i].a], p2 = P[F[i].b], p3 = P[F[
          i].c];
      double a = (p2.y - p1.y) * (p3.z - p1.z) - (p2.
          z - p1.z) * (p3.y - p1.y);
      double b = (p2.z - p1.z) * (p3.x - p1.x) - (p2.
          x - p1.x) * (p3.z - p1.z);
      double c = (p2.x - p1.x) * (p3.y - p1.y) - (p2.
          y - p1.y) * (p3.x - p1.x);
      double d = 0 - (a * p1.x + b * p1.y + c * p1.z)
          ;
      double temp = fabs(a * p.x + b * p.y + c * p.z
          + d) / sqrt(a * a + b * b + c * c);
      rt = min(rt, temp);
    }
    return rt;
  }
};
```

8.15 DelaunayTriangulation*

```
/* Delaunay Triangulation:
Given a sets of points on 2D plane, find a
triangulation such that no points will strictly
inside circumcircle of any triangle.
find : return a triangle contain given point
add_point : add a point into triangulation
A Triangle is in triangulation iff. its has_chd is 0.
Region of triangle u: iterate each u.edge[i].tri,
each points are u.p[(i+1)%3], u.p[(i+2)%3]
Voronoi diagram: for each triangle in triangulation,
the bisector of all its edges will split the region.
nearest point will belong to the triangle containing it
 */
const ll inf = MAXC * MAXC * 100; // lower_bound
    unknown
struct Tri;
struct Edge {
    Tri* tri; int side;
    Edge(): tri(0), side(0){}
    Edge(Tri* _tri, int _side): tri(_tri), side(_side)
        {}
};
struct Tri {
    pll p[3];
    Edge edge[3];
    Tri* chd[3];
    Tri() {}
    Tri(const pll& p0, const pll& p1, const pll& p2) {
        p[0] = p0; p[1] = p1; p[2] = p2;
        chd[0] = chd[1] = chd[2] = 0;
    }
    bool has_chd() const { return chd[0] != 0; }
    int num_chd() const {
        return !!chd[0] + !!chd[1] + !!chd[2];
    }
    bool contains(pll const& q) const {
        for (int i = 0; i < 3; ++i)
            if (ori(p[i], p[(i + 1) % 3], q) < 0)
                return 0;
        return 1;
    }
} pool[N * 10], *tris;
void edge(Edge a, Edge b) {
    if(a.tri) a.tri -> edge[a.side] = b;
    if(b.tri) b.tri -> edge[b.side] = a;
}
struct Trig { // Triangulation
    Trig() {
        the_root = // Tri should at least contain all
            points
            new(tris++) Tri(pll(-inf, -inf), pll(inf +
                inf, -inf), pll(-inf, inf + inf));
    }
    Tri* find(pll p) { return find(the_root, p); }
    void add_point(const pll &p) { add_point(find(
        the_root, p), p); }
    Tri* the_root;
    static Tri* find(Tri* root, const pll &p) {
        while (1) {
            if (!root -> has_chd())
                return root;
            for (int i = 0; i < 3 && root -> chd[i]; ++
                i)
                if (root -> chd[i] -> contains(p)) {
                    root = root -> chd[i];
```

```
                    break;
                }
            }
        assert(0); // "point not found"
    }
    void add_point(Tri* root, pll const& p) {
        Tri* t[3];
        /* split it into three triangles */
        for (int i = 0; i < 3; ++i)
            t[i] = new(tris++) Tri(root -> p[i], root
                -> p[(i + 1) % 3], p);
        for (int i = 0; i < 3; ++i)
            edge(Edge(t[i], 0), Edge(t[(i + 1) % 3], 1)
                );
        for (int i = 0; i < 3; ++i)
            edge(Edge(t[i], 2), root -> edge[(i + 2) %
                3]);
        for (int i = 0; i < 3; ++i)
            root -> chd[i] = t[i];
        for (int i = 0; i < 3; ++i)
            flip(t[i], 2);
    }
    void flip(Tri* tri, int pi) {
        Tri* trj = tri -> edge[pi].tri;
        int pj = tri -> edge[pi].side;
        if (!trj) return;
        if (!in_cc(tri -> p[0], tri -> p[1], tri -> p
            [2], trj -> p[pj])) return;
        /* flip edge between tri,trj */
        Tri* trk = new(tris++) Tri(tri -> p[(pi + 1) %
            3], trj -> p[pj], tri -> p[pi]);
        Tri* trl = new(tris++) Tri(trj -> p[(pj + 1) %
            3], tri -> p[pi], trj -> p[pj]);
        edge(Edge(trk, 0), Edge(trl, 0));
        edge(Edge(trk, 1), tri->edge[(pi + 2) % 3]);
        edge(Edge(trk, 2), trj->edge[(pj + 1) % 3]);
        edge(Edge(trl, 1), trj->edge[(pj + 2) % 3]);
        edge(Edge(trl, 2), tri->edge[(pi + 1) % 3]);
        tri -> chd[0] = trk; tri -> chd[1] = trl; tri
            -> chd[2] = 0;
        trj -> chd[0] = trk; trj -> chd[1] = trl; trj
            -> chd[2] = 0;
        flip(trk, 1); flip(trk, 2);
        flip(trl, 1); flip(trl, 2);
    }
};
vector<Tri*> triang; // vector of all triangle
set<Tri*> vst;
void go(Tri* now) { // store all tri into triang
    if (vst.find(now) != vst.end())
        return;
    vst.insert(now);
    if (!now -> has_chd())
        return triang.push_back(now);
    for (int i = 0; i < now->num_chd(); ++i)
        go(now -> chd[i]);
}
void build(int n, pll* ps) { // build triangulation
    tris = pool; triang.clear(); vst.clear();
    random_shuffle(ps, ps + n);
    Trig tri; // the triangulation structure
    for (int i = 0; i < n; ++i)
        tri.add_point(ps[i]);
    go(tri.the_root);
}
```

8.16 Triangulation Vonoroi*

```
vector<Line> ls[N];
pll arr[N];
Line make_line(pdd p, Line l) {
    pdd d = l.Y - l.X; d = perp(d);
    pdd m = (l.X + l.Y) / 2;
    l = Line(m, m + d);
    if (ori(l.X, l.Y, p) < 0)
        l = Line(m + d, m);
    return l;
}
double calc_area(int id) {
    // use to calculate the area of point "strictly in
        the convex hull"
    vector<Line> hpi = halfPlaneInter(ls[id]);
    vector<pdd> ps;
    for (int i = 0; i < SZ(hpi); ++i)
        ps.pb(intersect(hpi[i].X, hpi[i].Y, hpi[(i + 1)
            % SZ(hpi)].X, hpi[(i + 1) % SZ(hpi)].Y));
    double rt = 0;
    for (int i = 0; i < SZ(ps); ++i)
        rt += cross(ps[i], ps[(i + 1) % SZ(ps)]);
    return fabs(rt) / 2;
}
void solve(int n, pii *oarr) {
    map<pll, int> mp;
    for (int i = 0; i < n; ++i)
        arr[i] = pll(oarr[i].X, oarr[i].Y), mp[arr[i]]
            = i;
    build(n, arr); // Triangulation
    for (auto *t : triang) {
        vector<int> p;
        for (int i = 0; i < 3; ++i)
            if (mp.find(t -> p[i]) != mp.end())
                p.pb(mp[t -> p[i]]);
        for (int i = 0; i < SZ(p); ++i)
            for (int j = i + 1; j < SZ(p); ++j) {
                Line l(oarr[p[i]], oarr[p[j]]);
                ls[p[i]].pb(make_line(oarr[p[i]], l));
                ls[p[j]].pb(make_line(oarr[p[j]], l));
            }
    }
}
```

8.17 Tangent line of two circles

```
vector<Line> go( const Cir& c1 , const Cir& c2 , int
    sign1 ){
  // sign1 = 1 for outer tang, -1 for inter tang
  vector<Line> ret;
  double d_sq = norm2( c1.O - c2.O );
  if( d_sq < eps ) return ret;
  double d = sqrt( d_sq );
  Pt v = ( c2.O - c1.O ) / d;
  double c = ( c1.R - sign1 * c2.R ) / d;
  if( c * c > 1 ) return ret;
  double h = sqrt( max( 0.0 , 1.0 - c * c ) );
  for( int sign2 = 1 ; sign2 >= -1 ; sign2 -= 2 ){
    Pt n = { v.X * c - sign2 * h * v.Y ,
      v.Y * c + sign2 * h * v.X };
    Pt p1 = c1.O + n * c1.R;
    Pt p2 = c2.O + n * ( c2.R * sign1 );
    if( fabs( p1.X - p2.X ) < eps and
        fabs( p1.Y - p2.Y ) < eps )
      p2 = p1 + perp( c2.O - c1.O );
    ret.push_back( { p1 , p2 } );
  }
  return ret;
}
```

8.18 minMaxEnclosingRectangle

```
pdd solve(vector<pll> &dots){
  vector<pll> hull;
  const double INF=1e18,qi=acos(-1)/2*3;
  cv.dots=dots;
  hull=cv.hull();
  double Max=0,Min=INF,deg;
  ll n=hull.size();
  hull.pb(hull[0]);
  for(int i=0,u=1,r=1,l;i<n;++i){
    pll nw=hull[i+1]-hull[i];
    while(cross(nw,hull[u+1]-hull[i])>cross(nw,hull[u]-
        hull[i]))
      u=(u+1)%n;
    while(dot(nw,hull[r+1]-hull[i])>dot(nw,hull[r]-hull
        [i]))
      r=(r+1)%n;
    if(!i) l=(r+1)%n;
    while(dot(nw,hull[l+1]-hull[i])<dot(nw,hull[l]-hull
        [i]))
      l=(l+1)%n;
    Min=min(Min,(double)(dot(nw,hull[r]-hull[i])-dot(nw
        ,hull[l]-hull[i]))*cross(nw,hull[u]-hull[i])/
        abs2(nw));
```

```
    deg=acos((double)dot(hull[r]-hull[l],hull[u]-hull[i
        ])/abs(hull[r]-hull[l])/abs(hull[u]-hull[i]));
    deg=(qi-deg)/2;
    Max=max(Max,(double)abs(hull[r]-hull[l])*abs(hull[u
        ]-hull[i])*sin(deg)*sin(deg));
  }
  return pdd(Min,Max);
}
```

8.19 PointSegDist

```
double PointSegDist(pdd q0, pdd q1, pdd p) {
    if (sign(abs(q0 - q1)) == 0) return abs(q0 - p);
    if (sign(dot(q1 - q0, p - q0)) >= 0 && sign(dot(q0
        - q1, p - q1)) >= 0)
        return fabs(cross(q1 - q0, p - q0) / abs(q0 -
            q1));
    return min(abs(p - q0), abs(p - q1));
}
```

8.20 PointInConvex

```
bool PointInConvex(const vector<pll> &C, pdd p) {
    if (SZ(C) == 0) return false;
    if (SZ(C) == 1) return abs(C[0] - p) < eps;
    if (SZ(C) == 2) return btw(C[0], C[1], p);
    for (int i = 0; i < SZ(C); ++i) {
        const int j = i + 1 == SZ(C) ? 0 : i + 1;
        if (cross(C[j] - C[i], p - C[i]) < -eps)
            return false;
    }
    return true;
}
```

8.21 Minkowski Sum*

```
vector<pll> Minkowski(vector<pll> A, vector<pll> B) {
  hull(A), hull(B);
  vector<pll> C(1, A[0] + B[0]), s1, s2;
  for(int i = 0; i < SZ(A); ++i)
    s1.pb(A[(i + 1) % SZ(A)] - A[i]);
  for(int i = 0; i < SZ(B); i++)
    s2.pb(B[(i + 1) % SZ(B)] - B[i]);
  for(int p1 = 0, p2 = 0; p1 < SZ(A) || p2 < SZ(B);)
    if (p2 >= SZ(B) || (p1 < SZ(A) && cross(s1[p1], s2[
        p2]) >= 0))
      C.pb(C.back() + s1[p1++]);
    else
      C.pb(C.back() + s2[p2++]);
  return hull(C), C;
}
```

8.22 RotatingSweepLine

```
void rotatingSweepLine(vector<pii> &ps) {
  int n = SZ(ps), m = 0;
  vector<int> id(n), pos(n);
  vector<pii> line(n * (n - 1));
  for (int i = 0; i < n; ++i)
    for (int j = 0; j < n; ++j)
      if (i != j) line[m++] = pii(i, j);
  sort(ALL(line), [&](pii a, pii b) {
    return cmp(ps[a.Y] - ps[a.X], ps[b.Y] - ps[b.X]);
  }); // cmp(): polar angle compare
  iota(ALL(id), 0);
  sort(ALL(id), [&](int a, int b) {
    if (ps[a].Y != ps[b].Y) return ps[a].Y < ps[b].Y;
    return ps[a] < ps[b];
  }); // initial order, since (1, 0) is the smallest
  for (int i = 0; i < n; ++i) pos[id[i]] = i;
  for (int i = 0; i < m; ++i) {
    auto l = line[i];
    // do something
    tie(pos[l.X], pos[l.Y], id[pos[l.X]], id[pos[l.Y]])
        = make_tuple(pos[l.Y], pos[l.X], l.Y, l.X);
  }
}
```

9 Else

9.1 Mo's Alogrithm(With modification)

```
struct QUERY{//BLOCK=N^{2/3}
  int L,R,id,LBid,RBid,T;
  QUERY(int l,int r,int id,int lb,int rb,int t):
    L(l),R(r),id(id),LBid(lb),RBid(rb),T(t){}
  bool operator<(const QUERY &b)const{
    if(LBid!=b.LBid) return LBid<b.LBid;
    if(RBid!=b.RBid) return RBid<b.RBid;
    return T<b.T;
  }
};
vector<QUERY> query;
int cur_ans,arr[MAXN],ans[MAXN];
void solve(){
  sort(ALL(query));
  int L=0,R=0,T=-1;
  for(auto q:query){
    while(T<q.T) addTime(L,R,++T); // TODO
    while(T>q.T) subTime(L,R,T--); // TODO
    while(R<q.R) add(arr[++R]); // TODO
    while(L>q.L) add(arr[--L]); // TODO
    while(R>q.R) sub(arr[R--]); // TODO
    while(L<q.L) sub(arr[L++]); // TODO
    ans[q.id]=cur_ans;
  }
}
```

9.2 Mo's Alogrithm On Tree

```
const int MAXN=40005;
vector<int> G[MAXN];//1-base
int n,B,arr[MAXN],ans[100005],cur_ans;
int in[MAXN],out[MAXN],dfn[MAXN*2],dft;
int deep[MAXN],sp[__lg(MAXN*2)+1][MAXN*2],bln[MAXN],spt
    ;
bitset<MAXN> inset;
struct QUERY{
  int L,R,Lid,id,lca;
  QUERY(int l,int r,int _id):L(l),R(r),lca(0),id(_id){}
  bool operator<(const QUERY &b){
    if(Lid!=b.Lid) return Lid<b.Lid;
    return R<b.R;
  }
};
vector<QUERY> query;
void dfs(int u,int f,int d){
  deep[u]=d,sp[0][spt]=u,bln[u]=spt++;
  dfn[dft]=u,in[u]=dft++;
  for(int v:G[u])
    if(v!=f)
      dfs(v,u,d+1),sp[0][spt]=u,bln[u]=spt++;
  dfn[dft]=u,out[u]=dft++;
}
int lca(int u,int v){
  if(bln[u]>bln[v]) swap(u,v);
  int t=__lg(bln[v]-bln[u]+1);
  int a=sp[t][bln[u]],b=sp[t][bln[v]-(1<<t)+1];
  if(deep[a]<deep[b]) return a;
  return b;
}
void flip(int x){
  if(inset[x]) sub(arr[x]); // TODO
  else add(arr[x]); // TODO
  inset[x]=~inset[x];
}
void solve(){
  B=sqrt(2*n),dft=spt=cur_ans=0,dfs(1,1,0);
  for(int i=1,x=2;x<2*n;++i,x<<=1)
    for(int j=0;j+x<=2*n;++j)
      if(deep[sp[i-1][j]]<deep[sp[i-1][j+x/2]])
        sp[i][j]=sp[i-1][j];
      else sp[i][j]=sp[i-1][j+x/2];
  for(auto &q:query){
    int c=lca(q.L,q.R);
    if(c==q.L||c==q.R)
      q.L=out[c==q.L?q.R:q.L],q.R=out[c];
    else if(out[q.L]<in[q.R])
      q.lca=c,q.L=out[q.L],q.R=in[q.R];
```

```
    else q.lca=c,c=in[q.L],q.L=out[q.R],q.R=c;
    q.Lid=q.L/B;
  }
  sort(ALL(query));
  int L=0,R=-1;
  for(auto q:query){
    while(R<q.R) flip(dfn[++R]);
    while(L>q.L) flip(dfn[--L]);
    while(R>q.R) flip(dfn[R--]);
    while(L<q.L) flip(dfn[L++]);
    if(q.lca) add(arr[q.lca]);
    ans[q.id]=cur_ans;
    if(q.lca) sub(arr[q.lca]);
  }
}
```

9.3 Hilbert Curve

```
ll hilbert(int n, int x, int y) {
  ll res = 0;
  for (int s = n / 2; s; s >>= 1) {
    int rx = (x & s) > 0;
    int ry = (y & s) > 0;
    res += s * 1ll * s * ((3 * rx) ^ ry);
    if (ry == 0) {
      if (rx == 1) x = s - 1 - x, y = s - 1 - y;
      swap(x, y);
    }
  }
  return res;
}
```

9.4 DynamicConvexTrick*

```
// only works for integer coordinates!!
struct Line {
    mutable ll a, b, p;
    bool operator<(const Line &rhs) const { return a <
        rhs.a; }
    bool operator<(ll x) const { return p < x; }
};
struct DynamicHull : multiset<Line, less<>> {
    static const ll kInf = 1e18;
    ll Div(ll a, ll b) { return a / b - ((a ^ b) < 0 &&
         a % b); }
    bool isect(iterator x, iterator y) {
        if (y == end()) { x -> p = kInf; return 0; }
        if (x -> a == y -> a) x -> p = x -> b > y -> b
            ? kInf : -kInf;
        else x -> p = Div(y -> b - x -> b, x -> a - y
            -> a);
        return x -> p >= y -> p;
    }
    void addline(ll a, ll b) {
        auto z = insert({a, b, 0}), y = z++, x = y;
        while (isect(y, z)) z = erase(z);
        if (x != begin() && isect(--x, y)) isect(x, y =
             erase(y));
        while ((y = x) != begin() && (--x) -> p >= y ->
             p) isect(x, erase(y));
    }
    ll query(ll x) {
        auto l = *lower_bound(x);
        return l.a * x + l.b;
    }
};
```

9.5 All LCS*

```
void all_lcs(string s, string t) { // 0-base
    vector<int> h(SZ(t));
    iota(ALL(h), 0);
    for (int a = 0; a < SZ(s); ++a) {
        int v = -1;
        for (int c = 0; c < SZ(t); ++c)
            if (s[a] == t[c] || h[c] < v)
                swap(h[c], v);
        // LCS(s[0, a], t[b, c]) =
        // c - b + 1 - sum([h[i] >= b] | i <= c)
        // h[i] might become -1 !!
    }
}
```

9.6 DLX*

```
#define TRAV(i, link, start) for (int i = link[start];
    i != start; i = link[i])
template<bool A, bool B = !A> // A: Exact
struct DLX {
  int lt[NN], rg[NN], up[NN], dn[NN], cl[NN], rw[NN],
      bt[NN], s[NN], head, sz, ans;
  int columns;
  bool vis[NN];
  void remove(int c) {
    if (A) lt[rg[c]] = lt[c], rg[lt[c]] = rg[c];
    TRAV(i, dn, c) {
      if (A) {
        TRAV(j, rg, i)
          up[dn[j]] = up[j], dn[up[j]] = dn[j], --s[cl[
              j]];
      } else {
        lt[rg[i]] = lt[i], rg[lt[i]] = rg[i];
      }
    }
  }
  void restore(int c) {
    TRAV(i, up, c) {
      if (A) {
        TRAV(j, lt, i)
          ++s[cl[j]], up[dn[j]] = j, dn[up[j]] = j;
      } else {
        lt[rg[i]] = rg[lt[i]] = i;
      }
    }
    if (A) lt[rg[c]] = c, rg[lt[c]] = c;
  }
  void init(int c) {
    columns = c;
    for (int i = 0; i < c; ++i) {
      up[i] = dn[i] = bt[i] = i;
      lt[i] = i == 0 ? c : i - 1;
      rg[i] = i == c - 1 ? c : i + 1;
      s[i] = 0;
    }
    rg[c] = 0, lt[c] = c - 1;
    up[c] = dn[c] = -1;
    head = c, sz = c + 1;
  }
  void insert(int r, const vector<int> &col) {
    if (col.empty()) return;
    int f = sz;
    for (int i = 0; i < (int)col.size(); ++i) {
      int c = col[i], v = sz++;
      dn[bt[c]] = v;
      up[v] = bt[c], bt[c] = v;
      rg[v] = (i + 1 == (int)col.size() ? f : v + 1);
      rw[v] = r, cl[v] = c;
      ++s[c];
      if (i > 0) lt[v] = v - 1;
    }
    lt[f] = sz - 1;
  }
  int h() {
    int ret = 0;
    memset(vis, 0, sizeof(bool) * sz);
    TRAV(x, rg, head) {
      if (vis[x]) continue;
      vis[x] = true, ++ret;
      TRAV(i, dn, x) TRAV(j, rg, i) vis[cl[j]] = true;
    }
    return ret;
  }
  void dfs(int dep) {
    if (dep + (A ? 0 : h()) >= ans) return;
    if (rg[head] == head) return ans = dep, void();
    if (dn[rg[head]] == rg[head]) return;
    int w = rg[head];
    TRAV(x, rg, head) if (s[x] < s[w]) w = x;
    if (A) remove(w);
    TRAV(i, dn, w) {
      if (B) remove(i);
      TRAV(j, rg, i) remove(A ? cl[j] : j);
      dfs(dep + 1);
      TRAV(j, lt, i) restore(A ? cl[j] : j);
      if (B) restore(i);
```

```
    }
    if (A) restore(w);
  }
  int solve() {
    for (int i = 0; i < columns; ++i)
      dn[bt[i]] = i, up[i] = bt[i];
    ans = 1e9, dfs(0);
    return ans;
  }
};
```

9.7 Matroid Intersection

Start from $S = \emptyset$. In each iteration, let

- $Y_1 = \{x \notin S \mid S \cup \{x\} \in I_1\}$
- $Y_2 = \{x \notin S \mid S \cup \{x\} \in I_2\}$

If there exists $x \in Y_1 \cap Y_2$, insert x into S. Otherwise for each $x \in S, y \notin S$, create edges

- $x \to y$ if $S - \{x\} \cup \{y\} \in I_1$.
- $y \to x$ if $S - \{x\} \cup \{y\} \in I_2$.

Find a *shortest* path (with BFS) starting from a vertex in Y_1 and ending at a vertex in Y_2 which doesn't pass through any other vertices in Y_2, and alternate the path. The size of S will be incremented by 1 in each iteration. For the weighted case, assign weight $w(x)$ to vertex x if $x \in S$ and $-w(x)$ if $x \notin S$. Find the path with the minimum number of edges among all minimum length paths and alternate it.

9.8 AdaptiveSimpson

```
using F_t = function<double(double)>;
pdd simpson(const F_t &f, double l, double r,
  double fl, double fr, double fm = nan("")) {
  if (isnan(fm)) fm = f((l + r) / 2);
  return {fm, (r - l) / 6 * (fl + 4 * fm + fr)};
}
double simpson_ada(const F_t &f, double l, double r,
  double fl, double fm, double fr, double eps) {
  double m = (l + r) / 2,
         s = simpson(f, l, r, fl, fr, fm).second;
  auto [flm, sl] = simpson(f, l, m, fl, fm);
  auto [fmr, sr] = simpson(f, m, r, fm, fr);
  double delta = sl + sr - s;
  if (abs(delta) <= 15 * eps)
    return sl + sr + delta / 15;
  return simpson_ada(f, l, m, fl, flm, fm, eps / 2) +
    simpson_ada(f, m, r, fm, fmr, fr, eps / 2);
}
double simpson_ada(const F_t &f, double l, double r) {
  return simpson_ada(
    f, l, r, f(l), f((l + r) / 2), f(r), 1e-9 / 7122);
}
double simpson_ada2(const F_t &f, double l, double r) {
    double h = (r - l) / 7122, s = 0;
    for (int i = 0; i < 7122; ++i, l += h)
        s += simpson_ada(f, l, l + h);
    return s;
}
```

10 Python

10.1 Misc

```
from decimal import *
setcontext(Context(prec=MAX_PREC, Emax=MAX_EMAX,
    rounding=ROUND_FLOOR))
print(Decimal(input()) * Decimal(input()))
from fractions import Fraction
Fraction('3.1415926535897932').limit_denominator(1000)
```