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## 1.1 Shell script

## 1.2 Default code

### 1.3 vimrc

## 1.4 readchar

```
inline char readchar() {
    static const size_t bufsize = 65536;
    static char buf[bufsize];
    static char *p = buf, *end = buf;
    if (p == end) end = buf + fread_unlocked(buf, 1,
        bufsize, stdin), p = buf;
    return *p++;
}
```

## 1.5 Black Magic

```
#include <ext/pb_ds/priority_queue.hpp>
#include <ext/pb_ds/assoc_container.hpp> //rb_tree
using namespace __gnu_pbds;
typedef __gnu_pbds::priority_queue<int> heap;
int main() {
    heap h1, h2;
    h1.push(1), h1.push(3);
    h2.push(2), h2.push(4);
    h1.join(h2);
    cout << h1.size() << h2.size() << h1.top() << endl;
    //404
    tree<ll, null_type, less<ll>, rb_tree_tag,
    tree_order_statistics_node_update> st;
    tree<ll, ll, less<ll>, rb_tree_tag,
    tree_order_statistics_node_update> mp;
    for (int x : {0, 2, 3, 4}) st.insert(x);
    cout << *st.find_by_order(2) << st.order_of_key(1) <<
    endl; //31
}
//__int128_t, __float128_t
```

## 2 Graph

### 2.1 BCC Vertex\*

```
vector<int> G[N]; // 1-base
vector<int> nG[N], bcc[N];
int low[N], dfn[N], Time;
int bcc_id[N], bcc_cnt; // 1-base
bool is_cut[N]; // whether is av
bool cir[N];
int st[N], top;

void dfs(int u, int pa = -1) {
    int child = 0;
    low[u] = dfn[u] = ++Time;
    st[top++] = u;
    for (int v : G[u])
        if (!dfn[v]) {
            dfs(v, u), ++child;
            low[u] = min(low[u], low[v]);
            if (dfn[u] <= low[v]) {
                is_cut[u] = 1;
                bcc[++bcc_cnt].clear();
                int t;
                do {
                    bcc_id[t = st[--top]] = bcc_cnt;
                    bcc[bcc_cnt].push_back(t);
                } while (t != v);
                bcc_id[u] = bcc_cnt;
                bcc[bcc_cnt].pb(u);
            }
        } else if (dfn[v] < dfn[u] && v != pa)
            low[u] = min(low[u], dfn[v]);
    if (pa == -1 && child < 2) is_cut[u] = 0;
}

void bcc_init(int n) {
    Time = bcc_cnt = top = 0;
    for (int i = 1; i <= n; ++i)
        G[i].clear(), dfn[i] = bcc_id[i] = is_cut[i] = 0;
}

void bcc_solve(int n) {
    for (int i = 1; i <= n; ++i)
        if (!dfn[i]) dfs(i);
    // block-cut tree
    for (int i = 1; i <= n; ++i)
        if (is_cut[i])
            bcc_id[i] = ++bcc_cnt, cir[bcc_cnt] = 1;
    for (int i = 1; i <= bcc_cnt && !cir[i]; ++i)
        for (int j : bcc[i])
            if (is_cut[j])
                nG[i].pb(bcc_id[j]), nG[bcc_id[j]].pb(i);
}
```

### 2.2 Bridge\*

```
int low[N], dfn[N], Time; // 1-base
vector<pii> G[N], edge;
vector<bool> is_bridge;

void init(int n) {
    Time = 0;
    for (int i = 1; i <= n; ++i)
        G[i].clear(), low[i] = dfn[i] = 0;
}

void add_edge(int a, int b) {
    G[a].pb(pii(b, SZ(edge))), G[b].pb(pii(a, SZ(edge)));
    edge.pb(pii(a, b));
}

void dfs(int u, int f) {
    dfn[u] = low[u] = ++Time;
    for (auto i : G[u])
        if (!dfn[i.X])
            dfs(i.X, i.Y), low[u] = min(low[u], low[i.X]);
        else if (i.Y != f) low[u] = min(low[u], dfn[i.X]);
    if (low[u] == dfn[u] && f != -1) is_bridge[f] = 1;
}

void solve(int n) {
    is_bridge.resize(SZ(edge));
    for (int i = 1; i <= n; ++i)
        if (!dfn[i]) dfs(i, -1);
}
```

### 2.3 2SAT (SCC)\*

```
struct SAT { // 0-base
    int low[N], dfn[N], bln[N], n, Time, nScc;
    bool instack[N], istrue[N];
    stack<int> st;
    vector<int> G[N], SCC[N];
    void init(int _n) {
        n = _n; // assert(n * 2 <= N);
        for (int i = 0; i < n + n; ++i) G[i].clear();
    }
    void add_edge(int a, int b) { G[a].pb(b); }
    int rv(int a) {
        if (a >= n) return a - n;
        return a + n;
    }
    void add_clause(int a, int b) {
        add_edge(rv(a), b), add_edge(rv(b), a);
    }
    void dfs(int u) {
        dfn[u] = low[u] = ++Time;
        instack[u] = 1, st.push(u);
        for (int i : G[u])
            if (!dfn[i])
                dfs(i), low[u] = min(low[u], low[i]);
            else if (instack[i] && dfn[i] < dfn[u])
                low[u] = min(low[u], dfn[i]);
        if (low[u] == dfn[u]) {
            int tmp;
            do {
                tmp = st.top(), st.pop();
                instack[tmp] = 0, bln[tmp] = nScc;
            } while (tmp != u);
            ++nScc;
        }
    }
    bool solve() {
        Time = nScc = 0;
        for (int i = 0; i < n + n; ++i)
            SCC[i].clear(), low[i] = dfn[i] = bln[i] = 0;
        for (int i = 0; i < n + n; ++i)
            if (!dfn[i]) dfs(i);
        for (int i = 0; i < n + n; ++i) SCC[bln[i]].pb(i);
        for (int i = 0; i < n; ++i) {
            if (bln[i] == bln[i + n]) return false;
            istrue[i] = bln[i] < bln[i + n];
            istrue[i + n] = !istrue[i];
        }
        return true;
    }
}
```

```
};
```

## 2.4 MinimumMeanCycle\*

```
ll road[N][N]; // input here
struct MinimumMeanCycle {
    ll dp[N + 5][N], n;
    pll solve() {
        ll a = -1, b = -1, L = n + 1;
        for (int i = 2; i <= L; ++i)
            for (int k = 0; k < n; ++k)
                for (int j = 0; j < n; ++j)
                    dp[i][j] =
                        min(dp[i - 1][k] + road[k][j], dp[i][j]);
        for (int i = 0; i < n; ++i) {
            if (dp[L][i] >= INF) continue;
            ll ta = 0, tb = 1;
            for (int j = 1; j < n; ++j)
                if (dp[j][i] < INF &&
                    ta * (L - j) < (dp[L][i] - dp[j][i]) * tb)
                    ta = dp[L][i] - dp[j][i], tb = L - j;
            if (ta == 0) continue;
            if (a == -1 || a * tb > ta * b) a = ta, b = tb;
        }
        if (a != -1) {
            ll g = __gcd(a, b);
            return pll(a / g, b / g);
        }
        return pll(-1LL, -1LL);
    }
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i)
            for (int j = 0; j < n; ++j) dp[i + 2][j] = INF;
    }
};
```

## 2.5 Virtual Tree\*

```
vector<int> vG[N];
int top, st[N];

void insert(int u) {
    if (top == -1) return st[++top] = u, void();
    int p = LCA(st[top], u);
    if (p == st[top]) return st[++top] = u, void();
    while (top >= 1 && dep[st[top - 1]] >= dep[p])
        vG[st[top - 1]].pb(st[top]), --top;
    if (st[top] != p)
        vG[p].pb(st[top]), --top, st[++top] = p;
    st[++top] = u;
}

void reset(int u) {
    for (int i : vG[u]) reset(i);
    vG[u].clear();
}

void solve(vector<int> &v) {
    top = -1;
    sort(ALL(v),
        [&](int a, int b) { return dfn[a] < dfn[b]; });
    for (int i : v) insert(i);
    while (top > 0) vG[st[top - 1]].pb(st[top]), --top;
    // do something
    reset(v[0]);
}
```

## 2.6 Maximum Clique Dyn\*

```
const int N = 150;
struct MaxClique { // Maximum Clique
    bitset<N> a[N], cs[N];
    int ans, sol[N], q, cur[N], d[N], n;
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i) a[i].reset();
    }
    void addEdge(int u, int v) { a[u][v] = a[v][u] = 1; }
    void csort(vector<int> &r, vector<int> &c) {
```

```
int mx = 1, km = max(ans - q + 1, 1), t = 0,
    m = r.size();
cs[1].reset(), cs[2].reset();
for (int i = 0; i < m; i++) {
    int p = r[i], k = 1;
    while ((cs[k] & a[p]).count()) k++;
    if (k > mx) mx++, cs[mx + 1].reset();
    cs[k][p] = 1;
    if (k < km) r[t++] = p;
}
c.resize(m);
if (t) c[t - 1] = 0;
for (int k = km; k <= mx; k++)
    for (int p = cs[k]._Find_first(); p < N;
        p = cs[k]._Find_next(p))
        r[t] = p, c[t] = k, t++;
}

void dfs(vector<int> &r, vector<int> &c, int l,
    bitset<N> mask) {
    while (!r.empty()) {
        int p = r.back();
        r.pop_back(), mask[p] = 0;
        if (q + c.back() <= ans) return;
        cur[q++] = p;
        vector<int> nr, nc;
        bitset<N> nmask = mask & a[p];
        for (int i : r)
            if (a[p][i]) nr.push_back(i);
        if (!nr.empty()) {
            if (l < 4) {
                for (int i : nr)
                    d[i] = (a[i] & nmask).count();
                sort(nr.begin(), nr.end(),
                    [&](int x, int y) { return d[x] > d[y]; });
            }
            csort(nr, nc), dfs(nr, nc, l + 1, nmask);
        } else if (q > ans) ans = q, copy_n(cur, q, sol);
        c.pop_back(), q--;
    }
}

int solve(bitset<N> mask = bitset<N>()
    string(N, '1')) { // vertex mask
    vector<int> r, c;
    ans = q = 0;
    for (int i = 0; i < n; i++)
        if (mask[i]) r.push_back(i);
    for (int i = 0; i < n; i++)
        d[i] = (a[i] & mask).count();
    sort(r.begin(), r.end(),
        [&](int i, int j) { return d[i] > d[j]; });
    csort(r, c), dfs(r, c, 1, mask);
    return ans; // sol[0 ~ ans-1]
}

} graph;
```

## 2.7 Minimum Steiner Tree\*

```
// Minimum Steiner Tree
//  $O(V \cdot 3^T + V^2 \cdot 2^T)$ 
struct SteinerTree { // 0-base
    static const int T = 10, N = 105, INF = 1e9;
    int n, dst[N][N], dp[1 << T][N], tdst[N];
    int vcost[N]; // the cost of vertexs
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i) {
            for (int j = 0; j < n; ++j) dst[i][j] = INF;
            dst[i][i] = vcost[i] = 0;
        }
    }
    void add_edge(int ui, int vi, int wi) {
        dst[ui][vi] = min(dst[ui][vi], wi);
    }
    void shortest_path() {
        for (int k = 0; k < n; ++k)
            for (int i = 0; i < n; ++i)
                for (int j = 0; j < n; ++j)
                    dst[i][j] =
                        min(dst[i][j], dst[i][k] + dst[k][j]);
    }
    int solve(const vector<int> &ter) {
```

```

shortest_path();
int t = SZ(ter);
for (int i = 0; i < (1 << t); ++i)
    for (int j = 0; j < n; ++j) dp[i][j] = INF;
for (int i = 0; i < n; ++i) dp[0][i] = vcost[i];
for (int msk = 1; msk < (1 << t); ++msk) {
    if (!(msk & (msk - 1))) {
        int who = __lg(msk);
        for (int i = 0; i < n; ++i)
            dp[msk][i] =
                vcost[ter[who]] + dst[ter[who]][i];
    }
    for (int i = 0; i < n; ++i)
        for (int submsk = (msk - 1) & msk; submsk; submsk = (submsk - 1) & msk)
            dp[msk][i] = min(dp[msk][i],
                dp[submsk][i] + dp[msk ^ submsk][i] -
                vcost[i]);
    for (int i = 0; i < n; ++i) {
        tdst[i] = INF;
        for (int j = 0; j < n; ++j)
            tdst[i] =
                min(tdst[i], dp[msk][j] + dst[j][i]);
    }
    for (int i = 0; i < n; ++i) dp[msk][i] = tdst[i];
}
int ans = INF;
for (int i = 0; i < n; ++i)
    ans = min(ans, dp[(1 << t) - 1][i]);
return ans;
}
};

```

## 2.8 Dominator Tree\*

```

struct dominator_tree { // 1-base
    vector<int> G[N], rG[N];
    int n, pa[N], dfn[N], id[N], Time;
    int semi[N], idom[N], best[N];
    vector<int> tree[N]; // dominator_tree
    void init(int _n) {
        n = _n;
        for (int i = 1; i <= n; ++i)
            G[i].clear(), rG[i].clear();
    }
    void add_edge(int u, int v) {
        G[u].pb(v), rG[v].pb(u);
    }
    void dfs(int u) {
        id[dfn[u] = ++Time] = u;
        for (auto v : G[u])
            if (!dfn[v]) dfs(v), pa[dfn[v]] = dfn[u];
    }
    int find(int y, int x) {
        if (y <= x) return y;
        int tmp = find(pa[y], x);
        if (semi[best[y]] > semi[best[pa[y]]])
            best[y] = best[pa[y]];
        return pa[y] = tmp;
    }
    void tarjan(int root) {
        Time = 0;
        for (int i = 1; i <= n; ++i) {
            dfn[i] = idom[i] = 0;
            tree[i].clear();
            best[i] = semi[i] = i;
        }
        dfs(root);
        for (int i = Time; i > 1; --i) {
            int u = id[i];
            for (auto v : rG[u])
                if (v = dfn[v]) {
                    find(v, i);
                    semi[i] = min(semi[i], semi[best[v]]);
                }
            tree[semi[i]].pb(i);
            for (auto v : tree[pa[i]]) {
                find(v, pa[i]);
                idom[v] =
                    semi[best[v]] == pa[i] ? pa[i] : best[v];
            }
        }
    }
};

```

```

        tree[pa[i]].clear();
    }
    for (int i = 2; i <= Time; ++i) {
        if (idom[i] != semi[i]) idom[i] = idom[idom[i]];
        tree[id[idom[i]]].pb(i);
    }
}
};

```

## 2.9 Minimum Arborescence\*

```

struct zhu_liu { // 0(VE)
    struct edge {
        int u, v;
        ll w;
    };
    vector<edge> E; // 0-base
    int pe[N], id[N], vis[N];
    ll in[N];
    void init() { E.clear(); }
    void add_edge(int u, int v, ll w) {
        if (u != v) E.pb(edge{u, v, w});
    }
    ll build(int root, int n) {
        ll ans = 0;
        for (;;) {
            fill_n(in, n, INF);
            for (int i = 0; i < SZ(E); ++i)
                if (E[i].u != E[i].v && E[i].w < in[E[i].v])
                    pe[E[i].v] = i, in[E[i].v] = E[i].w;
            for (int u = 0; u < n; ++u) // no solution
                if (u != root && in[u] == INF) return -INF;
            int cntnode = 0;
            fill_n(id, n, -1), fill_n(vis, n, -1);
            for (int u = 0; u < n; ++u) {
                if (u != root) ans += in[u];
                int v = u;
                while (vis[v] != u && !~id[v] && v != root)
                    vis[v] = u, v = E[pe[v]].u;
                if (v != root && !~id[v]) {
                    for (int x = E[pe[v]].u; x != v;
                        x = E[pe[x]].u)
                        id[x] = cntnode;
                    id[v] = cntnode++;
                }
            }
            if (!cntnode) break; // no cycle
            for (int u = 0; u < n; ++u)
                if (!~id[u]) id[u] = cntnode++;
            for (int i = 0; i < SZ(E); ++i) {
                int v = E[i].v;
                E[i].u = id[E[i].u], E[i].v = id[E[i].v];
                if (E[i].u != E[i].v) E[i].w -= in[v];
            }
            n = cntnode, root = id[root];
        }
        return ans;
    }
};

```

## 2.10 Vizing's theorem

```

namespace vizing { // returns edge coloring in adjacent
// matrix G. 1 - based
int C[kN][kN], G[kN][kN];
void clear(int N) {
    for (int i = 0; i <= N; ++i) {
        for (int j = 0; j <= N; ++j) C[i][j] = G[i][j] = 0;
    }
}
void solve(vector<pair<int, int>> &E, int N, int M) {
    int X[kN] = {}, a;
    auto update = [&](int u) {
        for (X[u] = 1; C[u][X[u]]; X[u]++)
            ;
    };
    auto color = [&](int u, int v, int c) {
        int p = G[u][v];
        G[u][v] = G[v][u] = c;
        C[u][c] = v, C[v][c] = u;
        C[u][p] = C[v][p] = 0;
    };
};

```

```

    if (p) X[u] = X[v] = p;
    else update(u), update(v);
    return p;
};
auto flip = [&](int u, int c1, int c2) {
    int p = C[u][c1];
    swap(C[u][c1], C[u][c2]);
    if (p) G[u][p] = G[p][u] = c2;
    if (!C[u][c1]) X[u] = c1;
    if (!C[u][c2]) X[u] = c2;
    return p;
};
for (int i = 1; i <= N; i++) X[i] = 1;
for (int t = 0; t < E.size(); t++) {
    int u = E[t].first, v0 = E[t].second, v = v0,
        c0 = X[u], c = c0, d;
    vector<pair<int, int>> L;
    int vst[kN] = {};
    while (!G[u][v0]) {
        L.emplace_back(v, d = X[v]);
        if (!C[v][c])
            for (a = (int)L.size() - 1; a >= 0; a--)
                c = color(u, L[a].first, c);
        else if (!C[u][d])
            for (a = (int)L.size() - 1; a >= 0; a--)
                color(u, L[a].first, L[a].second);
        else if (vst[d]) break;
        else vst[d] = 1, v = C[u][d];
    }
    if (!G[u][v0]) {
        for (; v; v = flip(v, c, d), swap(c, d))
            ;
        if (C[u][c0]) {
            for (a = (int)L.size() - 2;
                a >= 0 && L[a].second != c; a--)
                ;
            for (; a >= 0; a--)
                color(u, L[a].first, L[a].second);
        } else t--;
    }
}
} // namespace vizing

```

## 2.11 Minimum Clique Cover\*

```

struct Clique_Cover { // 0-base, O(n2^n)
    int co[1 << N], n, E[N];
    int dp[1 << N];
    void init(int _n) {
        n = _n, fill_n(dp, 1 << n, 0);
        fill_n(E, n, 0), fill_n(co, 1 << n, 0);
    }
    void add_edge(int u, int v) {
        E[u] |= 1 << v, E[v] |= 1 << u;
    }
    int solve() {
        for (int i = 0; i < n; ++i)
            co[1 << i] = E[i] | (1 << i);
        co[0] = (1 << n) - 1;
        dp[0] = (n & 1) * 2 - 1;
        for (int i = 1; i < (1 << n); ++i) {
            int t = i & -i;
            dp[i] = -dp[i ^ t];
            co[i] = co[i ^ t] & co[t];
        }
        for (int i = 0; i < (1 << n); ++i)
            co[i] = (co[i] & i) == i;
        fwt(co, 1 << n);
        for (int ans = 1; ans < n; ++ans) {
            int sum = 0;
            for (int i = 0; i < (1 << n); ++i)
                sum += (dp[i] * co[i]);
            if (sum) return ans;
        }
        return n;
    }
};

```

## 2.12 NumberofMaximalClique\*

```

struct BronKerbosch { // 1-base
    int n, a[N], g[N][N];
    int S, all[N][N], some[N][N], none[N][N];
    void init(int _n) {
        n = _n;
        for (int i = 1; i <= n; ++i)
            for (int j = 1; j <= n; ++j) g[i][j] = 0;
    }
    void add_edge(int u, int v) {
        g[u][v] = g[v][u] = 1;
    }
    void dfs(int d, int an, int sn, int nn) {
        if (S > 1000) return; // pruning
        if (sn == 0 && nn == 0) ++S;
        int u = some[d][0];
        for (int i = 0; i < sn; ++i) {
            int v = some[d][i];
            if (g[u][v]) continue;
            int tsu = 0, tnn = 0;
            copy_n(all[d], an, all[d + 1]);
            all[d + 1][an] = v;
            for (int j = 0; j < sn; ++j)
                if (g[v][some[d][j]])
                    some[d + 1][tsu++] = some[d][j];
            for (int j = 0; j < nn; ++j)
                if (g[v][none[d][j]])
                    none[d + 1][tnn++] = none[d][j];
            dfs(d + 1, an + 1, tsu, tnn);
            some[d][i] = 0, none[d][nn++] = v;
        }
    }
    int solve() {
        iota(some[0], some[0] + n, 1);
        S = 0, dfs(0, 0, n, 0);
        return S;
    }
};

```

## 3 Data Structure

### 3.1 Leftist Tree

```

struct node {
    ll v, data, sz, sum;
    node *l, *r;
    node(ll k) {
        : v(0), data(k), sz(1), l(0), r(0), sum(k) {}
    };
    ll sz(node *p) { return p ? p->sz : 0; }
    ll V(node *p) { return p ? p->v : -1; }
    ll sum(node *p) { return p ? p->sum : 0; }
    node *merge(node *a, node *b) {
        if (!a || !b) return a ? a : b;
        if (a->data < b->data) swap(a, b);
        a->r = merge(a->r, b);
        if (V(a->r) > V(a->l)) swap(a->r, a->l);
        a->v = V(a->r) + 1, a->sz = sz(a->l) + sz(a->r) + 1;
        a->sum = sum(a->l) + sum(a->r) + a->data;
        return a;
    }
    void pop(node *&o) {
        node *tmp = o;
        o = merge(o->l, o->r);
        delete tmp;
    }
};

```

### 3.2 Heavy light Decomposition

```

struct Heavy_light_Decomposition { // 1-base
    int n, ulink[10005], deep[10005], mxson[10005],
        w[10005], pa[10005];
    int t, pl[10005], data[10005], dt[10005], bln[10005],
        edge[10005], et;
    vector<pii> G[10005];
    void init(int _n) {
        n = _n, t = 0, et = 1;
        for (int i = 1; i <= n; ++i)
            G[i].clear(), mxson[i] = 0;
    }
};

```

```

void add_edge(int a, int b, int w) {
    G[a].pb(pii(b, et)), G[b].pb(pii(a, et)),
    edge[et++] = w;
}
void dfs(int u, int f, int d) {
    w[u] = 1, pa[u] = f, deep[u] = d++;
    for (auto &i : G[u])
        if (i.X != f) {
            dfs(i.X, u, d), w[u] += w[i.X];
            if (w[mxson[u]] < w[i.X]) mxson[u] = i.X;
        } else bln[i.Y] = u, dt[u] = edge[i.Y];
}
void cut(int u, int link) {
    data[pl[u] = t++] = dt[u], ulink[u] = link;
    if (!mxson[u]) return;
    cut(mxson[u], link);
    for (auto i : G[u])
        if (i.X != pa[u] && i.X != mxson[u])
            cut(i.X, i.X);
}
void build() { dfs(1, 1, 1), cut(1, 1), /*build*/; }
int query(int a, int b) {
    int ta = ulink[a], tb = ulink[b], re = 0;
    while (ta != tb)
        if (deep[ta] < deep[tb])
            /*query*/, tb = ulink[b = pa[tb]];
        else /*query*/, ta = ulink[a = pa[ta]];
    if (a == b) return re;
    if (pl[a] > pl[b]) swap(a, b);
    /*query*/
    return re;
}
};

```

### 3.3 Centroid Decomposition\*

```

struct Cent_Dec { // 1-base
    vector<pll> G[N];
    pll info[N]; // store info. of itself
    pll upinfo[N]; // store info. of climbing up
    int n, pa[N], layer[N], sz[N], done[N];
    ll dis[___lg(N) + 1][N];
    void init(int _n) {
        n = _n, layer[0] = -1;
        fill_n(pa + 1, n, 0), fill_n(done + 1, n, 0);
        for (int i = 1; i <= n; ++i) G[i].clear();
    }
    void add_edge(int a, int b, int w) {
        G[a].pb(pll(b, w)), G[b].pb(pll(a, w));
    }
    void get_cent(
        int u, int f, int &mx, int &c, int num) {
        int mxsz = 0;
        sz[u] = 1;
        for (pll e : G[u])
            if (!done[e.X] && e.X != f) {
                get_cent(e.X, u, mx, c, num);
                sz[u] += sz[e.X], mxsz = max(mxsz, sz[e.X]);
            }
        if (mx > max(mxsz, num - sz[u]))
            mx = max(mxsz, num - sz[u]), c = u;
    }
    void dfs(int u, int f, ll d, int org) {
        // if required, add self info or climbing info
        dis[layer[org]][u] = d;
        for (pll e : G[u])
            if (!done[e.X] && e.X != f)
                dfs(e.X, u, d + e.Y, org);
    }
    int cut(int u, int f, int num) {
        int mx = 1e9, c = 0, lc;
        get_cent(u, f, mx, c, num);
        done[c] = 1, pa[c] = f, layer[c] = layer[f] + 1;
        for (pll e : G[c])
            if (!done[e.X]) {
                if (sz[e.X] > sz[c])
                    lc = cut(e.X, c, num - sz[c]);
                else lc = cut(e.X, c, sz[e.X]);
                upinfo[lc] = pll(), dfs(e.X, c, e.Y, c);
            }
        return done[c] = 0, c;
    }
};

```

```

}
void build() { cut(1, 0, n); }
void modify(int u) {
    for (int a = u, ly = layer[a]; a;
        a = pa[a], --ly) {
        info[a].X += dis[ly][u], ++info[a].Y;
        if (pa[a])
            upinfo[a].X += dis[ly - 1][u], ++upinfo[a].Y;
    }
}
ll query(int u) {
    ll rt = 0;
    for (int a = u, ly = layer[a]; a;
        a = pa[a], --ly) {
        rt += info[a].X + info[a].Y * dis[ly][u];
        if (pa[a])
            rt -=
                upinfo[a].X + upinfo[a].Y * dis[ly - 1][u];
    }
    return rt;
}
};

```

### 3.4 Link cut tree\*

```

struct Splay { // xor-sum
    static Splay nil;
    Splay *ch[2], *f;
    int val, sum, rev, size;
    Splay(int _val = 0)
        : val(_val), sum(_val), rev(0), size(1) {
        f = ch[0] = ch[1] = &nil;
    }
    bool isr() {
        return f->ch[0] != this && f->ch[1] != this;
    }
    int dir() { return f->ch[0] == this ? 0 : 1; }
    void setCh(Splay *c, int d) {
        ch[d] = c;
        if (c != &nil) c->f = this;
        pull();
    }
    void push() {
        if (!rev) return;
        swap(ch[0], ch[1]);
        if (ch[0] != &nil) ch[0]->rev ^= 1;
        if (ch[1] != &nil) ch[1]->rev ^= 1;
        rev = 0;
    }
    void pull() {
        // take care of the nil!
        size = ch[0]->size + ch[1]->size + 1;
        sum = ch[0]->sum ^ ch[1]->sum ^ val;
        if (ch[0] != &nil) ch[0]->f = this;
        if (ch[1] != &nil) ch[1]->f = this;
    }
} Splay::nil;
Splay *nil = &Splay::nil;
void rotate(Splay *x) {
    Splay *p = x->f;
    int d = x->dir();
    if (!p->isr()) p->f->setCh(x, p->dir());
    else x->f = p->f;
    p->setCh(x->ch[!d], d);
    x->setCh(p, !d);
    p->pull(), x->pull();
}
void splay(Splay *x) {
    vector<Splay*> splayVec;
    for (Splay *q = x;; q = q->f) {
        splayVec.pb(q);
        if (q->isr()) break;
    }
    reverse(ALL(splayVec));
    for (auto it : splayVec) it->push();
    while (!x->isr()) {
        if (x->f->isr()) rotate(x);
        else if (x->dir() == x->f->dir())
            rotate(x->f), rotate(x);
        else rotate(x), rotate(x);
    }
}

```



```

}
Splay *access(Splay *x) {
    Splay *q = nil;
    for (; x != nil; x = x->f)
        splay(x), x->setCh(q, 1), q = x;
    return q;
}
void root_path(Splay *x) { access(x), splay(x); }
void chroot(Splay *x) {
    root_path(x), x->rev ^= 1;
    x->push(), x->pull();
}
void split(Splay *x, Splay *y) {
    chroot(x), root_path(y);
}
void link(Splay *x, Splay *y) {
    root_path(x), chroot(y);
    x->setCh(y, 1);
}
void cut(Splay *x, Splay *y) {
    split(x, y);
    if (y->size != 5) return;
    y->push();
    y->ch[0] = y->ch[0]->f = nil;
}
Splay *get_root(Splay *x) {
    for (root_path(x); x->ch[0] != nil; x = x->ch[0])
        x->push();
    splay(x);
    return x;
}
bool conn(Splay *x, Splay *y) {
    return get_root(x) == get_root(y);
}
Splay *lca(Splay *x, Splay *y) {
    access(x), root_path(y);
    if (y->f == nil) return y;
    return y->f;
}
void change(Splay *x, int val) {
    splay(x), x->val = val, x->pull();
}
int query(Splay *x, Splay *y) {
    split(x, y);
    return y->sum;
}
}

```

### 3.5 KDTree

```

namespace kdt {
    int root, lc[maxn], rc[maxn], xl[maxn], xr[maxn],
        yl[maxn], yr[maxn];
    point p[maxn];
    int build(int l, int r, int dep = 0) {
        if (l == r) return -1;
        function<bool(const point &, const point &> f =
            [dep](const point &a, const point &b) {
                if (dep & 1) return a.x < b.x;
                else return a.y < b.y;
            });
        int m = (l + r) >> 1;
        nth_element(p + l, p + m, p + r, f);
        xl[m] = xr[m] = p[m].x;
        yl[m] = yr[m] = p[m].y;
        lc[m] = build(l, m, dep + 1);
        if (~lc[m]) {
            xl[m] = min(xl[m], xl[lc[m]]);
            xr[m] = max(xr[m], xr[lc[m]]);
            yl[m] = min(yl[m], yl[lc[m]]);
            yr[m] = max(yr[m], yr[lc[m]]);
        }
        rc[m] = build(m + 1, r, dep + 1);
        if (~rc[m]) {
            xl[m] = min(xl[m], xl[rc[m]]);
            xr[m] = max(xr[m], xr[rc[m]]);
            yl[m] = min(yl[m], yl[rc[m]]);
            yr[m] = max(yr[m], yr[rc[m]]);
        }
        return m;
    }
    bool bound(const point &q, int o, long long d) {

```

```

        double ds = sqrt(d + 1.0);
        if (q.x < xl[o] - ds || q.x > xr[o] + ds ||
            q.y < yl[o] - ds || q.y > yr[o] + ds)
            return false;
        return true;
    }
    long long dist(const point &a, const point &b) {
        return (a.x - b.x) * 1ll * (a.x - b.x) +
            (a.y - b.y) * 1ll * (a.y - b.y);
    }
    void dfs(
        const point &q, long long &d, int o, int dep = 0) {
        if (!bound(q, o, d)) return;
        long long cd = dist(p[o], q);
        if (cd != 0) d = min(d, cd);
        if ((dep & 1) && q.x < p[o].x ||
            !(dep & 1) && q.y < p[o].y) {
            if (~lc[o]) dfs(q, d, lc[o], dep + 1);
            if (~rc[o]) dfs(q, d, rc[o], dep + 1);
        } else {
            if (~rc[o]) dfs(q, d, rc[o], dep + 1);
            if (~lc[o]) dfs(q, d, lc[o], dep + 1);
        }
    }
    void init(const vector<point> &v) {
        for (int i = 0; i < v.size(); ++i) p[i] = v[i];
        root = build(0, v.size());
    }
    long long nearest(const point &q) {
        long long res = 1e18;
        dfs(q, res, root);
        return res;
    }
} // namespace kdt

```

## 4 Flow/Matching

### 4.1 Kuhn Munkres

```

struct KM { // 0-base
    int w[MAXN][MAXN], h1[MAXN], hr[MAXN], slk[MAXN], n;
    int fl[MAXN], fr[MAXN], pre[MAXN], qu[MAXN], ql, qr;
    bool vl[MAXN], vr[MAXN];
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i)
            for (int j = 0; j < n; ++j) w[i][j] = -INF;
    }
    void add_edge(int a, int b, int wei) {
        w[a][b] = wei;
    }
    bool Check(int x) {
        if (vl[x] = 1, ~fl[x])
            return vr[qu[qr++]] = fl[x] = 1;
        while (~x) swap(x, fr[fl[x] = pre[x]]);
        return 0;
    }
    void Bfs(int s) {
        fill(slk, slk + n, INF);
        fill(vl, vl + n, 0), fill(vr, vr + n, 0);
        ql = qr = 0, qu[qr++] = s, vr[s] = 1;
        while (1) {
            int d;
            while (ql < qr)
                for (int x = 0, y = qu[ql++]; x < n; ++x)
                    if (!vl[x] &&
                        slk[x] >= (d = h1[x] + hr[y] - w[x][y]))
                        if (pre[x] = y, d) slk[x] = d;
                        else if (!Check(x)) return;
            d = INF;
            for (int x = 0; x < n; ++x)
                if (!vl[x] && d > slk[x]) d = slk[x];
            for (int x = 0; x < n; ++x) {
                if (vl[x]) h1[x] += d;
                else slk[x] -= d;
                if (vr[x]) hr[x] -= d;
            }
            for (int x = 0; x < n; ++x)
                if (!vl[x] && !slk[x] && !Check(x)) return;

```

```

    }
}
int Solve() {
    fill(fl, fl + n, -1), fill(fr, fr + n, -1),
    fill(hr, hr + n, 0);
    for (int i = 0; i < n; ++i)
        hl[i] = *max_element(w[i], w[i] + n);
    for (int i = 0; i < n; ++i) Bfs(i);
    int res = 0;
    for (int i = 0; i < n; ++i) res += w[i][fl[i]];
    return res;
}
};

```

## 4.2 MincostMaxflow

```

struct MCMF { // 0-base
    struct edge {
        ll from, to, cap, flow, cost, rev;
    } * past[MAXN];
    vector<edge> G[MAXN];
    bitset<MAXN> inq;
    ll dis[MAXN], up[MAXN], s, t, mx, n;
    bool BellmanFord(ll &flow, ll &cost) {
        fill(dis, dis + n, INF);
        queue<ll> q;
        q.push(s), inq.reset(), inq[s] = 1;
        up[s] = mx - flow, past[s] = 0, dis[s] = 0;
        while (!q.empty()) {
            ll u = q.front();
            q.pop(), inq[u] = 0;
            if (!up[u]) continue;
            for (auto &e : G[u])
                if (e.flow != e.cap &&
                    dis[e.to] > dis[u] + e.cost) {
                    dis[e.to] = dis[u] + e.cost, past[e.to] = &e;
                    up[e.to] = min(up[u], e.cap - e.flow);
                    if (!inq[e.to]) inq[e.to] = 1, q.push(e.to);
                }
        }
        if (dis[t] == INF) return 0;
        flow += up[t], cost += up[t] * dis[t];
        for (ll i = t; past[i]; i = past[i]->from) {
            auto &e = *past[i];
            e.flow += up[t], G[e.to][e.rev].flow -= up[t];
        }
        return 1;
    }
    ll MinCostMaxFlow(ll _s, ll _t, ll &cost) {
        s = _s, t = _t, cost = 0;
        ll flow = 0;
        while (BellmanFord(flow, cost))
            ;
        return flow;
    }
    void init(ll _n, ll _mx) {
        n = _n, mx = _mx;
        for (int i = 0; i < n; ++i) G[i].clear();
    }
    void add_edge(ll a, ll b, ll cap, ll cost) {
        G[a].pb(edge{a, b, cap, 0, cost, G[b].size()});
        G[b].pb(edge{b, a, 0, 0, -cost, G[a].size() - 1});
    }
};

```

## 4.3 Maximum Simple Graph Matching\*

```

struct GenMatch { // 1-base
    int V, pr[N];
    bool el[N][N], inq[N], inp[N], inb[N];
    int st, ed, nb, bk[N], djs[N], ans;
    void init(int _V) {
        V = _V;
        for (int i = 0; i <= V; ++i) {
            for (int j = 0; j <= V; ++j) el[i][j] = 0;
            pr[i] = bk[i] = djs[i] = 0;
            inq[i] = inp[i] = inb[i] = 0;
        }
    }
    void add_edge(int u, int v) {
        el[u][v] = el[v][u] = 1;
    }
};

```

```

}
int lca(int u, int v) {
    fill_n(inp, V + 1, 0);
    while (1)
        if (u = djs[u], inp[u] = true, u == st) break;
        else u = bk[pr[u]];
    while (1)
        if (v = djs[v], inp[v]) return v;
        else v = bk[pr[v]];
    return v;
}
void upd(int u) {
    for (int v; djs[u] != nb; ) {
        v = pr[u], inb[djs[u]] = inb[djs[v]] = true;
        u = bk[v];
        if (djs[u] != nb) bk[u] = v;
    }
}
void blo(int u, int v, queue<int> &qe) {
    nb = lca(u, v), fill_n(inb, V + 1, 0);
    upd(u), upd(v);
    if (djs[u] != nb) bk[u] = v;
    if (djs[v] != nb) bk[v] = u;
    for (int tu = 1; tu <= V; ++tu)
        if (inb[djs[tu]])
            if (djs[tu] == nb, !inq[tu])
                qe.push(tu), inq[tu] = 1;
}
void flow() {
    fill_n(inq + 1, V, 0), fill_n(bk + 1, V, 0);
    iota(djs + 1, djs + V + 1, 1);
    queue<int> qe;
    qe.push(st), inq[st] = 1, ed = 0;
    while (!qe.empty()) {
        int u = qe.front();
        qe.pop();
        for (int v = 1; v <= V; ++v)
            if (el[u][v] && djs[u] != djs[v] &&
                pr[u] != v) {
                if ((v == st) ||
                    (pr[v] > 0 && bk[pr[v]] > 0))
                    blo(u, v, qe);
                else if (!bk[v]) {
                    if (bk[v] = u, pr[v] > 0) {
                        if (!inq[pr[v]]) qe.push(pr[v]);
                    } else return ed = v, void();
                }
            }
    }
}
void aug() {
    for (int u = ed, v, w; u > 0; )
        v = bk[u], w = pr[v], pr[v] = u, pr[u] = v,
        u = w;
}
int solve() {
    fill_n(pr, V + 1, 0), ans = 0;
    for (int u = 1; u <= V; ++u)
        if (!pr[u])
            if (st = u, flow(), ed > 0) aug(), ++ans;
    return ans;
}
};

```

## 4.4 Minimum Weight Matching (Clique version)\*

```

struct Graph { // 0-base (Perfect Match), n is even
    int n, match[N], onstk[N], stk[N], tp;
    ll edge[N][N], dis[N];
    void init(int _n) {
        n = _n, tp = 0;
        for (int i = 0; i < n; ++i) fill_n(edge[i], n, 0);
    }
    void add_edge(int u, int v, ll w) {
        edge[u][v] = edge[v][u] = w;
    }
    bool SPFA(int u) {
        stk[tp++] = u, onstk[u] = 1;
        for (int v = 0; v < n; ++v)
            if (!onstk[v] && match[u] != v) {

```



```

    int m = match[v];
    if (dis[m] >
        dis[u] - edge[v][m] + edge[u][v]) {
        dis[m] = dis[u] - edge[v][m] + edge[u][v];
        onstk[v] = 1, stk[tp++] = v;
        if (onstk[m] || SPFA(m)) return 1;
        --tp, onstk[v] = 0;
    }
}
onstk[u] = 0, --tp;
return 0;
}
ll solve() { // find a match
    for (int i = 0; i < n; ++i) match[i] = i ^ 1;
    while (1) {
        int found = 0;
        fill_n(dis, n, 0);
        fill_n(onstk, n, 0);
        for (int i = 0; i < n; ++i)
            if (tp = 0, !onstk[i] && SPFA(i))
                for (found = 1; tp >= 2;) {
                    int u = stk[--tp];
                    int v = stk[--tp];
                    match[u] = v, match[v] = u;
                }
        if (!found) break;
    }
    ll ret = 0;
    for (int i = 0; i < n; ++i)
        ret += edge[i][match[i]];
    return ret >> 1;
}
};

```

#### 4.5 SW-mincut

```

// global min cut
struct SW { //  $O(V^3)$ 
    static const int MXN = 514;
    int n, vst[MXN], del[MXN];
    int edge[MXN][MXN], wei[MXN];
    void init(int _n) {
        n = _n, MEM(edge, 0), MEM(del, 0);
    }
    void addEdge(int u, int v, int w) {
        edge[u][v] += w, edge[v][u] += w;
    }
    void search(int &s, int &t) {
        MEM(vst, 0), MEM(wei, 0), s = t = -1;
        while (1) {
            int mx = -1, cur = 0;
            for (int i = 0; i < n; ++i)
                if (!del[i] && !vst[i] && mx < wei[i])
                    cur = i, mx = wei[i];
            if (mx == -1) break;
            vst[cur] = 1, s = t, t = cur;
            for (int i = 0; i < n; ++i)
                if (!vst[i] && !del[i]) wei[i] += edge[cur][i];
        }
    }
    int solve() {
        int res = INF;
        for (int i = 0, x, y; i < n - 1; ++i) {
            search(x, y), res = min(res, wei[y]), del[y] = 1;
            for (int j = 0; j < n; ++j)
                edge[x][j] = (edge[j][x] += edge[y][j]);
        }
        return res;
    }
};

```

#### 4.6 BoundedFlow(Dinic\*)

```

struct BoundedFlow { // 0-base
    struct edge {
        int to, cap, flow, rev;
    };
    vector<edge> G[N];
    int n, s, t, dis[N], cur[N], cnt[N];
    void init(int _n) {
        n = _n;
    }
};

```

```

    for (int i = 0; i < n + 2; ++i)
        G[i].clear(), cnt[i] = 0;
}
void add_edge(int u, int v, int lcap, int rcap) {
    cnt[u] -= lcap, cnt[v] += lcap;
    G[u].pb(edge{v, rcap, lcap, SZ(G[v])});
    G[v].pb(edge{u, 0, 0, SZ(G[u]) - 1});
}
void add_edge(int u, int v, int cap) {
    G[u].pb(edge{v, cap, 0, SZ(G[v])});
    G[v].pb(edge{u, 0, 0, SZ(G[u]) - 1});
}
int dfs(int u, int cap) {
    if (u == t || !cap) return cap;
    for (int &i = cur[u]; i < SZ(G[u]); ++i) {
        edge &e = G[u][i];
        if (dis[e.to] == dis[u] + 1 && e.cap != e.flow) {
            int df = dfs(e.to, min(e.cap - e.flow, cap));
            if (df) {
                e.flow += df, G[e.to][e.rev].flow -= df;
                return df;
            }
        }
    }
    dis[u] = -1;
    return 0;
}
bool bfs() {
    fill_n(dis, n + 3, -1);
    queue<int> q;
    q.push(s), dis[s] = 0;
    while (!q.empty()) {
        int u = q.front();
        q.pop();
        for (edge &e : G[u])
            if (!dis[e.to] && e.flow != e.cap)
                q.push(e.to), dis[e.to] = dis[u] + 1;
    }
    return dis[t] != -1;
}
int maxflow(int _s, int _t) {
    s = _s, t = _t;
    int flow = 0, df;
    while (bfs()) {
        fill_n(cur, n + 3, 0);
        while ((df = dfs(s, INF))) flow += df;
    }
    return flow;
}
bool solve() {
    int sum = 0;
    for (int i = 0; i < n; ++i)
        if (cnt[i] > 0)
            add_edge(n + 1, i, cnt[i]), sum += cnt[i];
        else if (cnt[i] < 0) add_edge(i, n + 2, -cnt[i]);
    if (sum != maxflow(n + 1, n + 2)) sum = -1;
    for (int i = 0; i < n; ++i)
        if (cnt[i] > 0)
            G[n + 1].pop_back(), G[i].pop_back();
        else if (cnt[i] < 0)
            G[i].pop_back(), G[n + 2].pop_back();
    return sum != -1;
}
int solve(int _s, int _t) {
    add_edge(_t, _s, INF);
    if (!solve()) return -1; // invalid flow
    int x = G[_t].back().flow;
    return G[_t].pop_back(), G[_s].pop_back(), x;
}
};

```

#### 4.7 Gomory Hu tree

```

struct Gomory_Hu_tree { // 0-base
    MaxFlow Dinic;
    int n;
    vector<pii> G[MAXN];
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i) G[i].clear();
    }
};

```

```

void solve(vector<int> &v) {
    if (v.size() <= 1) return;
    int s = rand() % SZ(v);
    swap(v.back(), v[s]), s = v.back();
    int t = v[rand() % (SZ(v) - 1)];
    vector<int> L, R;
    int x = (Dinic.reset(), Dinic.maxflow(s, t));
    G[s].pb(pii(t, x)), G[t].pb(pii(s, x));
    for (int i : v)
        if (~Dinic.dis[i]) L.pb(i);
        else R.pb(i);
    solve(L), solve(R);
}
void build() {
    vector<int> v(n);
    for (int i = 0; i < n; ++i) v[i] = i;
    solve(v);
}
} ght; // test by BZOJ 4519
MaxFlow &Dinic = ght.Dinic;

```

## 4.8 Flow Models

- Maximum/Minimum flow with lower bound / Circulation problem
  - Construct super source  $S$  and sink  $T$ .
  - For each edge  $(x, y, l, u)$ , connect  $x \rightarrow y$  with capacity  $u - l$ .
  - For each vertex  $v$ , denote by  $in(v)$  the difference between the sum of incoming lower bounds and the sum of outgoing lower bounds.
  - If  $in(v) > 0$ , connect  $S \rightarrow v$  with capacity  $in(v)$ , otherwise, connect  $v \rightarrow T$  with capacity  $-in(v)$ .
    - To maximize, connect  $t \rightarrow s$  with capacity  $\infty$  (skip this in circulation problem), and let  $f$  be the maximum flow from  $S$  to  $T$ . If  $f \neq \sum_{v \in V, in(v) > 0} in(v)$ , there's no solution. Otherwise, the maximum flow from  $s$  to  $t$  is the answer.
    - To minimize, let  $f$  be the maximum flow from  $S$  to  $T$ . Connect  $t \rightarrow s$  with capacity  $\infty$  and let the flow from  $S$  to  $T$  be  $f'$ . If  $f + f' \neq \sum_{v \in V, in(v) > 0} in(v)$ , there's no solution. Otherwise,  $f'$  is the answer.
  - The solution of each edge  $e$  is  $l_e + f_e$ , where  $f_e$  corresponds to the flow of edge  $e$  on the graph.
- Construct minimum vertex cover from maximum matching  $M$  on bipartite graph  $(X, Y)$ 
  - Redirect every edge:  $y \rightarrow x$  if  $(x, y) \in M$ ,  $x \rightarrow y$  otherwise.
  - Dfs from unmatched vertices in  $X$ .
  - $x \in X$  is chosen iff  $x$  is unvisited.
  - $y \in Y$  is chosen iff  $y$  is visited.
- Minimum cost cyclic flow
  - Construct super source  $S$  and sink  $T$
  - For each edge  $(x, y, c)$ , connect  $x \rightarrow y$  with  $(cost, cap) = (c, 1)$  if  $c > 0$ , otherwise connect  $y \rightarrow x$  with  $(cost, cap) = (-c, 1)$
  - For each edge with  $c < 0$ , sum these cost as  $K$ , then increase  $d(y)$  by 1, decrease  $d(x)$  by 1
  - For each vertex  $v$  with  $d(v) > 0$ , connect  $S \rightarrow v$  with  $(cost, cap) = (0, d(v))$
  - For each vertex  $v$  with  $d(v) < 0$ , connect  $v \rightarrow T$  with  $(cost, cap) = (0, -d(v))$
  - Flow from  $S$  to  $T$ , the answer is the cost of the flow  $C + K$
- Maximum density induced subgraph
  - Binary search on answer, suppose we're checking answer  $T$
  - Construct a max flow model, let  $K$  be the sum of all weights
  - Connect source  $s \rightarrow v$ ,  $v \in G$  with capacity  $K$
  - For each edge  $(u, v, w)$  in  $G$ , connect  $u \rightarrow v$  and  $v \rightarrow u$  with capacity  $w$
  - For  $v \in G$ , connect it with sink  $v \rightarrow t$  with capacity  $K + 2T - (\sum_{e \in E(v)} w(e)) - 2w(v)$
  - $T$  is a valid answer if the maximum flow  $f < K|V|$
- Minimum weight edge cover
  - For each  $v \in V$  create a copy  $v'$ , and connect  $u' \rightarrow v'$  with weight  $w(u, v)$ .
  - Connect  $v \rightarrow v'$  with weight  $2\mu(v)$ , where  $\mu(v)$  is the cost of the cheapest edge incident to  $v$ .
  - Find the minimum weight perfect matching on  $G'$ .
- Project selection problem
  - If  $p_v > 0$ , create edge  $(s, v)$  with capacity  $p_v$ ; otherwise, create edge  $(v, t)$  with capacity  $-p_v$ .
  - Create edge  $(u, v)$  with capacity  $w$  with  $w$  being the cost of choosing  $u$  without choosing  $v$ .
  - The mincut is equivalent to the maximum profit of a subset of projects.
- 0/1 quadratic programming
 
$$\sum_x c_x x + \sum_y c_y \bar{y} + \sum_{xy} c_{xy} x \bar{y} + \sum_{xyx'y'} c_{xyx'y'} (x \bar{y} + x' \bar{y}')$$

can be minimized by the mincut of the following graph:

  - Create edge  $(x, t)$  with capacity  $c_x$  and create edge  $(s, y)$  with capacity  $c_y$ .
  - Create edge  $(x, y)$  with capacity  $c_{xy}$ .
  - Create edge  $(x, y)$  and edge  $(x', y')$  with capacity  $c_{xyx'y'}$ .

## 5 String

### 5.1 KMP

```

int F[MAXN];
vector<int> match(string A, string B) {
    vector<int> ans;
    F[0] = -1, F[1] = 0;
    for (int i = 1, j = 0; i < SZ(B); F[++i] = ++j) {
        if (B[i] == B[j]) F[i] = F[j]; // optimize
        while (j != -1 && B[i] != B[j]) j = F[j];
    }
    for (int i = 0, j = 0; i < SZ(A); ++i) {
        while (j != -1 && A[i] != B[j]) j = F[j];
        if (++j == SZ(B)) ans.pb(i + 1 - j), j = F[j];
    }
    return ans;
}

```

### 5.2 Z-value\*

```

int z[MAXN];
void make_z(const string &s) {
    int l = 0, r = 0;
    for (int i = 1; i < SZ(s); ++i) {
        for (z[i] = max(0, min(r - i + 1, z[i - 1]));
             i + z[i] < SZ(s) && s[i + z[i]] == s[z[i]];
             ++z[i])
            ;
        if (i + z[i] - 1 > r) l = i, r = i + z[i] - 1;
    }
}

```

### 5.3 Manacher\*

```

int z[MAXN];
int Manacher(string tmp) {
    string s = "&";
    int l = 0, r = 0, x, ans;
    for (char c : tmp) s.pb(c), s.pb('%');
    ans = 0, x = 0;
    for (int i = 1; i < SZ(s); ++i) {
        z[i] = r > i ? min(z[2 * l - i], r - i) : 1;
        while (s[i + z[i]] == s[i - z[i]]) ++z[i];
        if (z[i] + i > r) r = z[i] + i, l = i;
    }
    for (int i = 1; i < SZ(s); ++i)
        if (s[i] == '%') x = max(x, z[i]);
    ans = x / 2 * 2, x = 0;
    for (int i = 1; i < SZ(s); ++i)
        if (s[i] != '%') x = max(x, z[i]);
    return max(ans, (x - 1) / 2 * 2 + 1);
}

```

### 5.4 Suffix Array

```

struct suffix_array {
    int box[MAXN], tp[MAXN], m;
    bool not_equ(int a, int b, int k, int n) {
        return ra[a] != ra[b] || a + k >= n ||
            b + k >= n || ra[a + k] != ra[b + k];
    }
    void radix(int *key, int *it, int *ot, int n) {
        fill_n(box, m, 0);
        for (int i = 0; i < n; ++i) ++box[key[i]];
        partial_sum(box, box + m, box);
        for (int i = n - 1; i >= 0; --i)
            ot[--box[key[it[i]]]] = it[i];
    }
    void make_sa(const string &s, int n) {
        int k = 1;
        for (int i = 0; i < n; ++i) ra[i] = s[i];
        do {
            iota(tp, tp + k, n - k), iota(sa + k, sa + n, 0);
            radix(ra + k, sa + k, tp + k, n - k);
            radix(ra, tp, sa, n);
            tp[sa[0]] = 0, m = 1;
            for (int i = 1; i < n; ++i) {
                m += not_equ(sa[i], sa[i - 1], k, n);
                tp[sa[i]] = m - 1;
            }
        } while (m < n);
    }
}

```

```

    }
    copy_n(tp, n, ra);
    k *= 2;
  } while (k < n && m != n);
}
void make_he(const string &s, int n) {
  for (int j = 0, k = 0; j < n; ++j) {
    if (ra[j])
      for (; s[j + k] == s[sa[ra[j]] - 1] + k; ++k)
        ;
    he[ra[j]] = k, k = max(0, k - 1);
  }
}
int sa[MAXN], ra[MAXN], he[MAXN];
void build(const string &s) {
  int n = SZ(s);
  fill_n(sa, n, 0), fill_n(ra, n, 0),
  fill_n(he, n, 0);
  fill_n(box, n, 0), fill_n(tp, n, 0), m = 256;
  make_sa(s, n), make_he(s, n);
}
};

```

## 5.5 SAIS

```

class SAIS {
public:
  int *SA, *H;
  // zero based, string content MUST > 0
  // result height H[i] is LCP(SA[i - 1], SA[i])
  // string, length, |sigma|
  void build(int *s, int n, int m = 128) {
    copy_n(s, n, _s);
    _h[0] = _s[n++] = 0;
    sais(_s, _sa, _p, _q, _t, _c, n, m);
    mkhei(n);
    SA = _sa + 1;
    H = _h + 1;
  }
private:
  bool _t[N * 2];
  int _s[N * 2], _c[N * 2], x[N], _p[N], _q[N * 2],
  r[N], _sa[N * 2], _h[N];
  void mkhei(int n) {
    for (int i = 0; i < n; i++) r[_sa[i]] = i;
    for (int i = 0; i < n; i++)
      if (r[i]) {
        int ans = i > 0 ? max(_h[r[i] - 1] - 1, 0) : 0;
        while (_s[i + ans] == _s[_sa[r[i]] - 1] + ans)
          ans++;
        _h[r[i]] = ans;
      }
  }
  void sais(int *s, int *sa, int *p, int *q, bool *t,
  int *c, int n, int z) {
    bool uniq = t[n - 1] = 1, neq;
    int nn = 0, nmzx = -1, *nsa = sa + n, *ns = s + n,
    lst = -1;

#define MAGIC(XD) \
    fill_n(sa, n, 0); \
    copy_n(c, z, x); \
    XD; \
    copy_n(c, z - 1, x + 1); \
    for (int i = 0; i < n; i++) \
      if (sa[i] && !t[sa[i] - 1]) \
        sa[x[s[sa[i]] - 1]++] = sa[i] - 1; \
    copy_n(c, z, x); \
    for (int i = n - 1; i >= 0; i--) \
      if (sa[i] && t[sa[i] - 1]) \
        sa[--x[s[sa[i]] - 1]] = sa[i] - 1; \
    fill_n(c, z, 0); \
    for (int i = 0; i < n; i++) uniq &= ++c[s[i]] < 2; \
    partial_sum(c, c + z, c); \
    if (uniq) { \
      for (int i = 0; i < n; i++) sa[--c[s[i]]] = i; \
      return; \
    } \
    for (int i = n - 2; i >= 0; i--)

```

```

    t[i] = (s[i] == s[i + 1] ? t[i + 1]
    : s[i] < s[i + 1]);
    MAGIC(for (int i = 1; i <= n - 1;
    i++) if (t[i] && !t[i - 1])
    sa[--x[s[i]]] = p[q[i] = nn++] = i);
    for (int i = 0; i < n; i++)
      if (sa[i] && t[sa[i]] && !t[sa[i] - 1]) {
        neq = (lst < 0) ||
        !equal(s + lst,
        s + lst + p[q[sa[i]] + 1] - sa[i],
        s + sa[i]);
        ns[q[lst = sa[i]]] = nmzx += neq;
      }
    sais(ns, nsa, p + nn, q + n, t + n, c + z, nn,
    nmzx + 1);
    MAGIC(for (int i = nn - 1; i >= 0; i--)
    sa[--x[s[p[nsa[i]]]]] = p[nsa[i]]);
  }
} sa;

```

## 5.6 Aho-Corasick Automatan

```

const int len = 400000, sigma = 26;
struct AC_Automatan {
  int nx[len][sigma], fl[len], cnt[len], pri[len], top;
  int newnode() {
    fill(nx[top], nx[top] + sigma, -1);
    return top++;
  }
  void init() { top = 1, newnode(); }
  int input(
  string &s) { // return the end_node of string
    int X = 1;
    for (char c : s) {
      if (!nx[X][c - 'a']) nx[X][c - 'a'] = newnode();
      X = nx[X][c - 'a'];
    }
    return X;
  }
  void make_fl() {
    queue<int> q;
    q.push(1), fl[1] = 0;
    for (int t = 0; !q.empty(); t++) {
      int R = q.front();
      q.pop(), pri[t++] = R;
      for (int i = 0; i < sigma; ++i)
        if (~nx[R][i]) {
          int X = nx[R][i], Z = fl[R];
          for (; Z && !nx[Z][i];) Z = fl[Z];
          fl[X] = Z ? nx[Z][i] : 1, q.push(X);
        }
    }
  }
  void get_v(string &s) {
    int X = 1;
    fill(cnt, cnt + top, 0);
    for (char c : s) {
      while (X && !nx[X][c - 'a']) X = fl[X];
      X = X ? nx[X][c - 'a'] : 1, ++cnt[X];
    }
    for (int i = top - 2; i > 0; --i)
      cnt[fl[pri[i]]] += cnt[pri[i]];
  }
};

```

## 5.7 Smallest Rotation

```

string mcp(string s) {
  int n = SZ(s), i = 0, j = 1;
  s += s;
  while (i < n && j < n) {
    int k = 0;
    while (k < n && s[i + k] == s[j + k]) ++k;
    if (s[i + k] <= s[j + k]) j += k + 1;
    else i += k + 1;
    if (i == j) ++j;
  }
  int ans = i < n ? i : j;
  return s.substr(ans, n);
}

```

## 5.8 De Bruijn sequence\*

```
constexpr int MAXC = 10, MAXN = 1e5 + 10;
struct DBSeq {
    int C, N, K, L, buf[MAXC * MAXN]; // K <= C^N
    void dfs(int *out, int t, int p, int &ptr) {
        if (ptr >= L) return;
        if (t > N) {
            if (N % p) return;
            for (int i = 1; i <= p && ptr < L; ++i)
                out[ptr++] = buf[i];
        } else {
            buf[t] = buf[t - p], dfs(out, t + 1, p, ptr);
            for (int j = buf[t - p] + 1; j < C; ++j)
                buf[t] = j, dfs(out, t + 1, t, ptr);
        }
    }
    void solve(int _c, int _n, int _k, int *out) {
        int p = 0;
        C = _c, N = _n, K = _k, L = N + K - 1;
        dfs(out, 1, 1, p);
        if (p < L) fill(out + p, out + L, 0);
    }
} dbs;
```

## 5.9 SAM

```
const int MAXM = 1000010;
struct SAM {
    int tot, root, lst, mom[MAXM], mx[MAXM];
    int acc[MAXM], nxt[MAXM][33];
    int newNode() {
        int res = ++tot;
        fill(nxt[res], nxt[res] + 33, 0);
        mom[res] = mx[res] = acc[res] = 0;
        return res;
    }
    void init() {
        tot = 0;
        root = newNode();
        mom[root] = 0, mx[root] = 0;
        lst = root;
    }
    void push(int c) {
        int p = lst;
        int np = newNode();
        mx[np] = mx[p] + 1;
        for (; p && nxt[p][c] == 0; p = mom[p])
            nxt[p][c] = np;
        if (p == 0) mom[np] = root;
        else {
            int q = nxt[p][c];
            if (mx[p] + 1 == mx[q]) mom[np] = q;
            else {
                int nq = newNode();
                mx[nq] = mx[p] + 1;
                for (int i = 0; i < 33; i++)
                    nxt[nq][i] = nxt[q][i];
                mom[nq] = mom[q];
                mom[q] = nq;
                mom[np] = nq;
                for (; p && nxt[p][c] == q; p = mom[p])
                    nxt[p][c] = nq;
            }
        }
        lst = np;
    }
    void push(char *str) {
        for (int i = 0; str[i]; i++)
            push(str[i] - 'a' + 1);
    }
} sam;
```

## 5.10 PalTree

```
struct palindromic_tree { // Check by APIO 2014
    // palindrome
    struct node {
        int next[26], fail, len;
        int cnt, num; // cnt: appear times, num: number of
        // pal. suf.
    };
    vector<node> St;
```

```
node(int l = 0) : fail(0), len(l), cnt(0), num(0) {
    for (int i = 0; i < 26; ++i) next[i] = 0;
};
vector<node> St;
vector<char> s;
int last, n;
palindromic_tree() : St(2), last(1), n(0) {
    St[0].fail = 1, St[1].len = -1, s.pb(-1);
}
inline void clear() {
    St.clear(), s.clear(), last = 1, n = 0;
    St.pb(0), St.pb(-1);
    St[0].fail = 1, s.pb(-1);
}
inline int get_fail(int x) {
    while (s[n - St[x].len - 1] != s[n])
        x = St[x].fail;
    return x;
}
inline void add(int c) {
    s.push_back(c - 'a'), ++n;
    int cur = get_fail(last);
    if (!St[cur].next[c]) {
        int now = SZ(St);
        St.pb(St[cur].len + 2);
        St[now].fail =
            St[get_fail(St[cur].fail)].next[c];
        St[cur].next[c] = now;
        St[now].num = St[St[now].fail].num + 1;
    }
    last = St[cur].next[c], ++St[last].cnt;
}
inline void count() { // counting cnt
    auto i = St.rbegin();
    for (; i != St.rend(); ++i) {
        St[i->fail].cnt += i->cnt;
    }
}
inline int size() { // The number of diff. pal.
    return SZ(St) - 2;
}
};
```

## 5.11 cyclicLCS

```
#define L 0
#define LU 1
#define U 2
const int mov[3][2] = {0, -1, -1, -1, -1, 0};
int al, bl;
char a[MAXL * 2], b[MAXL * 2]; // 0-indexed
int dp[MAXL * 2][MAXL];
char pred[MAXL * 2][MAXL];
inline int lcs_length(int r) {
    int i = r + al, j = bl, l = 0;
    while (i > r) {
        char dir = pred[i][j];
        if (dir == LU) l++;
        i += mov[dir][0];
        j += mov[dir][1];
    }
    return l;
}
inline void reroot(int r) { // r = new base row
    int i = r, j = 1;
    while (j <= bl && pred[i][j] != LU) j++;
    if (j > bl) return;
    pred[i][j] = L;
    while (i < 2 * al && j <= bl) {
        if (pred[i + 1][j] == U) {
            i++;
            pred[i][j] = L;
        } else if (j < bl && pred[i + 1][j + 1] == LU) {
            i++;
            j++;
            pred[i][j] = L;
        } else {
            j++;
        }
    }
}
```

```

}
int cyclic_lcs() {
    // a, b, al, bl should be properly filled
    // note: a WILL be altered in process
    //      -- concatenated after itself
    char tmp[MAXL];
    if (al > bl) {
        swap(al, bl);
        strcpy(tmp, a);
        strcpy(a, b);
        strcpy(b, tmp);
    }
    strcpy(tmp, a);
    strcat(a, tmp);
    // basic lcs
    for (int i = 0; i <= 2 * al; i++) {
        dp[i][0] = 0;
        pred[i][0] = U;
    }
    for (int j = 0; j <= bl; j++) {
        dp[0][j] = 0;
        pred[0][j] = L;
    }
    for (int i = 1; i <= 2 * al; i++) {
        for (int j = 1; j <= bl; j++) {
            if (a[i - 1] == b[j - 1])
                dp[i][j] = dp[i - 1][j - 1] + 1;
            else dp[i][j] = max(dp[i - 1][j], dp[i][j - 1]);
            if (dp[i][j - 1] == dp[i - 1][j]) pred[i][j] = L;
            else if (a[i - 1] == b[j - 1]) pred[i][j] = LU;
            else pred[i][j] = U;
        }
    }
    // do cyclic lcs
    int clcs = 0;
    for (int i = 0; i < al; i++) {
        clcs = max(clcs, lcs_length(i));
        reroot(i + 1);
    }
    // recover a
    a[al] = '\0';
    return clcs;
}

```

## 6 Math

### 6.1 $ax+by=\gcd^*$

```

pll exgcd(ll a, ll b) {
    if (b == 0) return pll(1, 0);
    else {
        ll p = a / b;
        pll q = exgcd(b, a % b);
        return pll(q.Y, q.X - q.Y * p);
    }
}

```

### 6.2 floor and ceil

```

int floor(int a, int b) {
    return a / b - (a % b && a < 0 ^ b < 0);
}
int ceil(int a, int b) {
    return a / b + (a % b && a < 0 ^ b > 0);
}

```

### 6.3 Miller Rabin\*

```

// n < 4,759,123,141      3 : 2, 7, 61
// n < 1,122,004,669,633  4 : 2, 13, 23, 1662803
// n < 3,474,749,660,383  6 : pimes <= 13
// n < 2^64              7 :
// 2, 325, 9375, 28178, 450775, 9780504, 1795265022
bool Miller_Rabin(ll a, ll n) {
    if ((a = a % n) == 0) return 1;
    if ((n & 1) ^ 1) return n == 2;
    ll tmp = (n - 1) / ((n - 1) & (1 - n));
    ll t = __lg(((n - 1) & (1 - n))), x = 1;

```

```

    for (; tmp; tmp >>= 1, a = mul(a, a, n))
        if (tmp & 1) x = mul(x, a, n);
    if (x == 1 || x == n - 1) return 1;
    while (--t)
        if ((x = mul(x, x, n)) == n - 1) return 1;
    return 0;
}

```

### 6.4 Fraction

```

struct fraction {
    ll n, d;
    fraction(const ll &n = 0, const ll &d = 1)
        : n(_n), d(_d) {
        ll t = __gcd(n, d);
        n /= t, d /= t;
        if (d < 0) n = -n, d = -d;
    }
    fraction operator-(const fraction &b) const {
        return fraction(-n, d);
    }
    fraction operator+(const fraction &b) const {
        return fraction(n * b.d + b.n * d, d * b.d);
    }
    fraction operator-(const fraction &b) const {
        return fraction(n * b.d - b.n * d, d * b.d);
    }
    fraction operator*(const fraction &b) const {
        return fraction(n * b.n, d * b.d);
    }
    fraction operator/(const fraction &b) const {
        return fraction(n * b.d, d * b.n);
    }
    void print() {
        cout << n;
        if (d != 1) cout << "/" << d;
    }
};

```

### 6.5 Simultaneous Equations

```

struct matrix { // m variables, n equations
    int n, m;
    fraction M[MAXN][MAXN + 1], sol[MAXN];
    int solve() { // -1: inconsistent, >= 0: rank
        for (int i = 0; i < n; ++i) {
            int piv = 0;
            while (piv < m && !M[i][piv].n) ++piv;
            if (piv == m) continue;
            for (int j = 0; j < n; ++j) {
                if (i == j) continue;
                fraction tmp = -M[j][piv] / M[i][piv];
                for (int k = 0; k <= m; ++k)
                    M[j][k] = tmp * M[i][k] + M[j][k];
            }
        }
        int rank = 0;
        for (int i = 0; i < n; ++i) {
            int piv = 0;
            while (piv < m && !M[i][piv].n) ++piv;
            if (piv == m && M[i][m].n) return -1;
            else if (piv < m)
                ++rank, sol[piv] = M[i][m] / M[i][piv];
        }
        return rank;
    }
};

```

### 6.6 Pollard Rho\*

```

map<ll, int> cnt;
void PollardRho(ll n) {
    if (n == 1) return;
    if (prime(n)) return ++cnt[n], void();
    if (n % 2 == 0)
        return PollardRho(n / 2), ++cnt[2], void();
    ll x = 2, y = 2, d = 1, p = 1;
    #define f(x, n, p) ((mul(x, x, n) + p) % n)
    while (true) {
        if (d != n && d != 1) {

```

```

    PollardRho(n / d);
    PollardRho(d);
    return;
}
if (d == n) ++p;
x = f(x, n, p);
y = f(f(y, n, p), n, p);
d = __gcd(abs(x - y), n);
}
}

```

## 6.7 Simplex Algorithm

```

const int MAXN = 111;
const int MAXM = 111;
const double eps = 1E-10;
double a[MAXN][MAXM], b[MAXN], c[MAXM], d[MAXN][MAXM];
double x[MAXM];
int ix[MAXN + MAXM]; // !!! array all indexed from 0
// max{cx} subject to {Ax<=b,x>=0}
// n: constraints, m: vars !!!
// x[] is the optimal solution vector
// usage :
// value = simplex(a, b, c, N, M);
double simplex(double a[MAXN][MAXM], double b[MAXN],
    double c[MAXM], int n, int m) {
    ++m;
    int r = n, s = m - 1;
    memset(d, 0, sizeof(d));
    for (int i = 0; i < n + m; ++i) ix[i] = i;
    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < m - 1; ++j) d[i][j] = -a[i][j];
        d[i][m - 1] = 1;
        d[i][m] = b[i];
        if (d[r][m] > d[i][m]) r = i;
    }
    for (int j = 0; j < m - 1; ++j) d[n][j] = c[j];
    d[n + 1][m - 1] = -1;
    for (double dd;;) {
        if (r < n) {
            int t = ix[s];
            ix[s] = ix[r + m];
            ix[r + m] = t;
            d[r][s] = 1.0 / d[r][s];
            for (int j = 0; j <= m; ++j)
                if (j != s) d[r][j] *= -d[r][s];
            for (int i = 0; i <= n + 1; ++i)
                if (i != r) {
                    for (int j = 0; j <= m; ++j)
                        if (j != s) d[i][j] += d[r][j] * d[i][s];
                    d[i][s] *= d[r][s];
                }
        }
        r = -1;
        s = -1;
        for (int j = 0; j < m; ++j)
            if (s < 0 || ix[s] > ix[j]) {
                if (d[n + 1][j] > eps ||
                    (d[n + 1][j] > -eps && d[n][j] > eps))
                    s = j;
            }
        if (s < 0) break;
        for (int i = 0; i < n; ++i)
            if (d[i][s] < -eps) {
                if (r < 0 ||
                    (dd = d[r][m] / d[r][s] -
                     d[i][m] / d[i][s]) < -eps ||
                    (dd < eps && ix[r + m] > ix[i + m]))
                    r = i;
            }
        if (r < 0) return -1; // not bounded
    }
    if (d[n + 1][m] < -eps) return -1; // not executable
    double ans = 0;
    for (int i = 0; i < m; i++) x[i] = 0;
    for (int i = m; i < n + m;
        ++i) { // the missing enumerated x[i] = 0
        if (ix[i] < m - 1) {
            ans += d[i - m][m] * c[ix[i]];
            x[ix[i]] = d[i - m][m];
        }
    }
}

```

```

}
return ans;
}

```

### 6.7.1 Construction

Standard form: maximize  $\mathbf{c}^T \mathbf{x}$  subject to  $A\mathbf{x} \leq \mathbf{b}$  and  $\mathbf{x} \geq 0$ .  
 Dual LP: minimize  $\mathbf{b}^T \mathbf{y}$  subject to  $A^T \mathbf{y} \geq \mathbf{c}$  and  $\mathbf{y} \geq 0$ .  
 $\bar{\mathbf{x}}$  and  $\bar{\mathbf{y}}$  are optimal if and only if for all  $i \in [1, n]$ , either  $\bar{x}_i = 0$  or  $\sum_{j=1}^m A_{ji} \bar{y}_j = c_i$  holds and for all  $i \in [1, m]$  either  $\bar{y}_i = 0$  or  $\sum_{j=1}^n A_{ij} \bar{x}_j = b_j$  holds.

1. In case of minimization, let  $c'_i = -c_i$
2.  $\sum_{1 \leq i \leq n} A_{ji} x_i \geq b_j \rightarrow \sum_{1 \leq i \leq n} -A_{ji} x_i \leq -b_j$
3.  $\sum_{1 \leq i \leq n} A_{ji} x_i = b_j$ 
  - $\sum_{1 \leq i \leq n} A_{ji} x_i \leq b_j$
  - $\sum_{1 \leq i \leq n} A_{ji} x_i \geq b_j$
4. If  $x_i$  has no lower bound, replace  $x_i$  with  $x_i - x'_i$

## 6.8 Schreier-Sims Algorithm\*

```

namespace schreier {
int n;
vector<vector<vector<int>>> bkets, binv;
vector<vector<int>> lk;
vector<int> operator*(
    const vector<int> &a, const vector<int> &b) {
    vector<int> res(SZ(a));
    for (int i = 0; i < SZ(a); ++i) res[i] = b[a[i]];
    return res;
}
vector<int> inv(const vector<int> &a) {
    vector<int> res(SZ(a));
    for (int i = 0; i < SZ(a); ++i) res[a[i]] = i;
    return res;
}
int filter(const vector<int> &g, bool add = true) {
    n = SZ(bkets);
    vector<int> p = g;
    for (int i = 0; i < n; ++i) {
        assert(p[i] >= 0 && p[i] < SZ(lk[i]));
        if (lk[i][p[i]] == -1) {
            if (add) {
                bkets[i].pb(p);
                binv[i].pb(inv(p));
                lk[i][p[i]] = SZ(bkets[i]) - 1;
            }
            return i;
        }
        p = p * binv[i][lk[i][p[i]]];
    }
    return -1;
}
bool inside(const vector<int> &g) {
    return filter(g, false) == -1;
}
void solve(const vector<vector<int>> &gen, int _n) {
    n = _n;
    bkets.clear(), bkets.resize(n);
    binv.clear(), binv.resize(n);
    lk.clear(), lk.resize(n);
    vector<int> iden(n);
    iota(iden.begin(), iden.end(), 0);
    for (int i = 0; i < n; ++i) {
        lk[i].resize(n, -1);
        bkets[i].pb(iden);
        binv[i].pb(iden);
        lk[i][i] = 0;
    }
    for (int i = 0; i < SZ(gen); ++i) filter(gen[i]);
    queue<pair<pii, pii>> upd;
    for (int i = 0; i < n; ++i)
        for (int j = i; j < n; ++j)
            for (int k = 0; k < SZ(bkets[i]); ++k)
                for (int l = 0; l < SZ(bkets[j]); ++l)
                    upd.emplace(pii(i, k), pii(j, l));
    while (!upd.empty()) {
        auto a = upd.front().X;
        auto b = upd.front().Y;
        upd.pop();
    }
}
}

```



```

    int res = filter(bkts[a.X][a.Y] * bkts[b.X][b.Y]);
    if (res == -1) continue;
    pii pr = pii(res, SZ(bkts[res]) - 1);
    for (int i = 0; i < n; ++i)
        for (int j = 0; j < SZ(bkts[i]); ++j) {
            if (i <= res) upd.emplace(pii(i, j), pr);
            if (res <= i) upd.emplace(pr, pii(i, j));
        }
}
}
long long size() {
    long long res = 1;
    for (int i = 0; i < n; ++i) res = res * SZ(bkts[i]);
    return res;
}
// namespace schreier

```

## 6.9 chineseRemainder

```

LL solve(LL x1, LL m1, LL x2, LL m2) {
    LL g = __gcd(m1, m2);
    if ((x2 - x1) % g) return -1; // no sol
    m1 /= g;
    m2 /= g;
    pair<LL, LL> p = gcd(m1, m2);
    LL lcm = m1 * m2 * g;
    LL res = p.first * (x2 - x1) * m1 + x1;
    return (res % lcm + lcm) % lcm;
}

```

## 6.10 QuadraticResidue\*

```

int Jacobi(int a, int m) {
    int s = 1;
    for (; m > 1;) {
        a %= m;
        if (a == 0) return 0;
        const int r = __builtin_ctz(a);
        if ((r & 1) && ((m + 2) & 4)) s = -s;
        a >>= r;
        if (a & m & 2) s = -s;
        swap(a, m);
    }
    return s;
}

int QuadraticResidue(int a, int p) {
    if (p == 2) return a & 1;
    const int jc = Jacobi(a, p);
    if (jc == 0) return 0;
    if (jc == -1) return -1;
    int b, d;
    for (;;) {
        b = rand() % p;
        d = (1LL * b * b + p - a) % p;
        if (Jacobi(d, p) == -1) break;
    }
    int f0 = b, f1 = 1, g0 = 1, g1 = 0, tmp;
    for (int e = (1LL + p) >> 1; e; e >>= 1) {
        if (e & 1) {
            tmp = (1LL * g0 * f0 +
                    1LL * d * (1LL * g1 * f1 % p)) %
                p;
            g1 = (1LL * g0 * f1 + 1LL * g1 * f0) % p;
            g0 = tmp;
        }
        tmp =
            (1LL * f0 * f0 + 1LL * d * (1LL * f1 * f1 % p)) %
                p;
        f1 = (2LL * f0 * f1) % p;
        f0 = tmp;
    }
    return g0;
}

```

## 6.11 PiCount

```

int64_t PrimeCount(int64_t n) {
    if (n <= 1) return 0;
    const int v = sqrt(n);

```

```

    vector<int> smalls(v + 1);
    for (int i = 2; i <= v; ++i) smalls[i] = (i + 1) / 2;
    int s = (v + 1) / 2;
    vector<int> roughs(s);
    for (int i = 0; i < s; ++i) roughs[i] = 2 * i + 1;
    vector<int64_t> larges(s);
    for (int i = 0; i < s; ++i)
        larges[i] = (n / (2 * i + 1) + 1) / 2;
    vector<bool> skip(v + 1);
    int pc = 0;
    for (int p = 3; p <= v; ++p) {
        if (smalls[p] > smalls[p - 1]) {
            int q = p * p;
            pc++;
            if (1LL * q * q > n) break;
            skip[p] = true;
            for (int i = q; i <= v; i += 2 * p)
                skip[i] = true;
            int ns = 0;
            for (int k = 0; k < s; ++k) {
                int i = roughs[k];
                if (skip[i]) continue;
                int64_t d = 1LL * i * p;
                larges[ns] = larges[k] -
                    (d <= v ? larges[smalls[d] - pc]
                     : smalls[n / d]) +
                    pc;
                roughs[ns++] = i;
            }
            s = ns;
            for (int j = v / p; j >= p; --j) {
                int c = smalls[j] - pc;
                for (int i = j * p, e = min(i + p, v + 1);
                     i < e; ++i)
                    smalls[i] -= c;
            }
        }
        for (int k = 1; k < s; ++k) {
            const int64_t m = n / roughs[k];
            int64_t s = larges[k] - (pc + k - 1);
            for (int l = 1; l < k; ++l) {
                int p = roughs[l];
                if (1LL * p * p > m) break;
                s -= smalls[m / p] - (pc + l - 1);
            }
            larges[0] -= s;
        }
        return larges[0];
    }
}

```

## 6.12 Primes

```

/* 12721 13331 14341 75577 123457 222557 556679 999983
 * 1097774749 1076767633 100102021 999997771 1001010013
 * 1000512343 987654361 999991231 999888733 98789101
 * 987777733 999991921 1010101333 1010102101
 * 100000000039 100000000000037 2305843009213693951
 * 4611686018427387847 9223372036854775783
 * 18446744073709551557 */

```

## 6.13 Theorem

### 6.13.1 Kirchhoff's Theorem

Denote  $L$  be a  $n \times n$  matrix as the Laplacian matrix of graph  $G$ , where  $L_{ii} = d(i)$ ,  $L_{ij} = -c$  where  $c$  is the number of edge  $(i, j)$  in  $G$ .

- The number of undirected spanning in  $G$  is  $|\det(\tilde{L}_{11})|$ .
- The number of directed spanning tree rooted at  $r$  in  $G$  is  $|\det(\tilde{L}_{rr})|$ .

### 6.13.2 Tutte's Matrix

Let  $D$  be a  $n \times n$  matrix, where  $d_{ij} = x_{ij}$  ( $x_{ij}$  is chosen uniformly at random) if  $i < j$  and  $(i, j) \in E$ , otherwise  $d_{ij} = -d_{ji}$ .  $\frac{\text{rank}(D)}{2}$  is the maximum matching on  $G$ .

### 6.13.3 Cayley's Formula

- Given a degree sequence  $d_1, d_2, \dots, d_n$  for each labeled vertices, there are  $\frac{(n-2)!}{(d_1-1)!(d_2-1)! \dots (d_n-1)!}$  spanning trees.

- Let  $T_{n,k}$  be the number of *labeled* forests on  $n$  vertices with  $k$  components, such that vertex  $1, 2, \dots, k$  belong to different components. Then  $T_{n,k} = kn^{n-k-1}$ .

### 6.13.4 Erdős–Gallai theorem

A sequence of nonnegative integers  $d_1 \geq \dots \geq d_n$  can be represented as the degree sequence of a finite simple graph on  $n$  vertices if and only if  $d_1 + \dots + d_n$  is even and  $\sum_{i=1}^k d_i \leq k(k-1) + \sum_{i=k+1}^n \min(d_i, k)$  holds for every  $1 \leq k \leq n$ .

### 6.13.5 Gale–Ryser theorem

A pair of sequences of nonnegative integers  $a_1 \geq \dots \geq a_n$  and  $b_1, \dots, b_n$  is bigraphic if and only if  $\sum_{i=1}^n a_i = \sum_{i=1}^n b_i$  and  $\sum_{i=1}^k a_i \leq \sum_{i=1}^n \min(b_i, k)$  holds for every  $1 \leq k \leq n$ .

### 6.13.6 Fulkerson–Chen–Anstee theorem

A sequence  $(a_1, b_1), \dots, (a_n, b_n)$  of nonnegative integer pairs with  $a_1 \geq \dots \geq a_n$  is digraphic if and only if  $\sum_{i=1}^n a_i = \sum_{i=1}^n b_i$  and  $\sum_{i=1}^k a_i \leq \sum_{i=1}^n \min(b_i, k) + \sum_{i=k+1}^n \min(b_i, k)$  holds for every  $1 \leq k \leq n$ .

## 6.14 Euclidean Algorithms

- $m = \lfloor \frac{a+b}{c} \rfloor$
- Time complexity:  $O(\log n)$

$$f(a, b, c, n) = \sum_{i=0}^n \lfloor \frac{ai+b}{c} \rfloor$$

$$= \begin{cases} \lfloor \frac{a}{c} \rfloor \cdot \frac{n(n+1)}{2} + \lfloor \frac{b}{c} \rfloor \cdot (n+1) \\ + f(a \bmod c, b \bmod c, c, n), & a \geq c \vee b \geq c \\ 0, & n < 0 \vee a = 0 \\ nm - f(c, c-b-1, a, m-1), & \text{otherwise} \end{cases}$$

$$g(a, b, c, n) = \sum_{i=0}^n i \lfloor \frac{ai+b}{c} \rfloor$$

$$= \begin{cases} \lfloor \frac{a}{c} \rfloor \cdot \frac{n(n+1)(2n+1)}{6} + \lfloor \frac{b}{c} \rfloor \cdot \frac{n(n+1)}{2} \\ + g(a \bmod c, b \bmod c, c, n), & a \geq c \vee b \geq c \\ 0, & n < 0 \vee a = 0 \\ \frac{1}{2} \cdot (n(n+1)m - f(c, c-b-1, a, m-1) \\ - h(c, c-b-1, a, m-1)), & \text{otherwise} \end{cases}$$

$$h(a, b, c, n) = \sum_{i=0}^n i \lfloor \frac{ai+b}{c} \rfloor^2$$

$$= \begin{cases} \lfloor \frac{a}{c} \rfloor^2 \cdot \frac{n(n+1)(2n+1)}{6} + \lfloor \frac{b}{c} \rfloor^2 \cdot (n+1) \\ + \lfloor \frac{a}{c} \rfloor \cdot \lfloor \frac{b}{c} \rfloor \cdot n(n+1) \\ + h(a \bmod c, b \bmod c, c, n) \\ + 2 \lfloor \frac{a}{c} \rfloor \cdot g(a \bmod c, b \bmod c, c, n) \\ + 2 \lfloor \frac{b}{c} \rfloor \cdot f(a \bmod c, b \bmod c, c, n), & a \geq c \vee b \geq c \\ 0, & n < 0 \vee a = 0 \\ nm(m+1) - 2g(c, c-b-1, a, m-1) \\ - 2f(c, c-b-1, a, m-1) - f(a, b, c, n), & \text{otherwise} \end{cases}$$

## 6.15 General Purpose Numbers

### 6.15.1 Bernoulli numbers

$$B_0 = 1, B_1^{\pm} = \pm \frac{1}{2}, B_2 = \frac{1}{6}, B_3 = 0$$

$$\sum_{j=0}^m \binom{m+1}{j} B_j = 0, \text{ EGF is } B(x) = \frac{x}{e^x - 1} = \sum_{n=0}^{\infty} B_n \frac{x^n}{n!}.$$

$$S_m(n) = \sum_{k=1}^n k^m = \frac{1}{m+1} \sum_{k=0}^m \binom{m+1}{k} B_k^+ n^{m+1-k}$$

### 6.15.2 Stirling numbers of the second kind

Partitions of  $n$  distinct elements into exactly  $k$  groups.

$$S(n, k) = S(n-1, k-1) + kS(n-1, k), S(n, 1) = S(n, n) = 1$$

$$S(n, k) = \frac{1}{k!} \sum_{i=0}^k (-1)^{k-i} \binom{k}{i} i^n$$

## 7 Polynomial

### 7.1 Fast Fourier Transform

```
template <int MAXN> struct FFT {
    using val_t = complex<double>;
    const double PI = acos(-1);
    val_t w[MAXN];
    FFT() {
        for (int i = 0; i < MAXN; ++i) {
            double arg = 2 * PI * i / MAXN;
            w[i] = val_t(cos(arg), sin(arg));
        }
    }
    void bitrev(val_t *a, int n); // see NTT
    void trans(
        val_t *a, int n, bool inv = false); // see NTT;
        // remember to replace LL with val_t
    };
```

### 7.2 Number Theory Transform\*

```
//(2^16)+1, 65537, 3
// 7*17*(2^23)+1, 998244353, 3
// 1255*(2^20)+1, 1315962881, 3
// 51*(2^25)+1, 1711276033, 29
template <int MAXN, ll P, ll RT> // MAXN must be 2^k
struct NTT {
    ll w[MAXN];
    ll mpow(ll a, ll n);
    ll minv(ll a) { return mpow(a, P - 2); }
    NTT() {
        ll dw = mpow(RT, (P - 1) / MAXN);
        w[0] = 1;
        for (int i = 1; i < MAXN; ++i)
            w[i] = w[i - 1] * dw % P;
    }
    void bitrev(ll *a, int n) {
        int i = 0;
        for (int j = 1; j < n - 1; ++j) {
            for (int k = n >> 1; (i ^= k) < k; k >>= 1)
                if (j < i) swap(a[i], a[j]);
        }
    }
    void operator()(
        ll *a, int n, bool inv = false) { // 0 <= a[i] < P
        bitrev(a, n);
        for (int L = 2; L <= n; L <= 1) {
            int dx = MAXN / L, dl = L >> 1;
            for (int i = 0; i < n; i += L) {
                for (int j = i, x = 0; j < i + dl;
                    ++j, x += dx) {
                    ll tmp = a[j + dl] * w[x] % P;
                    if ((a[j + dl] = a[j] - tmp) < 0)
                        a[j + dl] += P;
                    if ((a[j] += tmp) >= P) a[j] -= P;
                }
            }
        }
        if (inv) {
            reverse(a + 1, a + n);
            ll invn = minv(n);
            for (int i = 0; i < n; ++i)
                a[i] = a[i] * invn % P;
        }
    }
};
```

### 7.3 Fast Walsh Transform\*

```
/* x: a[j], y: a[j + (L >> 1)]
or: (y += x * op), and: (x += y * op)
xor: (x, y = (x + y) * op, (x - y) * op)
invop: or, and, xor = -1, -1, 1/2 */
void fwt(int *a, int n, int op) { // or
    for (int L = 2; L <= n; L <= 1)
        for (int i = 0; i < n; i += L)
            for (int j = i; j < i + (L >> 1); ++j)
                a[j + (L >> 1)] += a[j] * op;
```

```

}
const int N = 21;
int f[N][1 << N], g[N][1 << N], h[N][1 << N],
    ct[1 << N];
void subset_convolution(
    int *a, int *b, int *c, int L) {
    // c_k = \sum_{i+j=k, i&j=0} a_i * b_j
    int n = 1 << L;
    for (int i = 1; i < n; ++i)
        ct[i] = ct[i & (i - 1)] + 1;
    for (int i = 0; i < n; ++i)
        f[ct[i]][i] = a[i], g[ct[i]][i] = b[i];
    for (int i = 0; i <= L; ++i)
        fwt(f[i], n, 1), fwt(g[i], n, 1);
    for (int i = 0; i <= L; ++i)
        for (int j = 0; j <= i; ++j)
            for (int x = 0; x < n; ++x)
                h[i][x] += f[j][x] * g[i - j][x];
    for (int i = 0; i <= L; ++i) fwt(h[i], n, -1);
    for (int i = 0; i < n; ++i) c[i] = h[ct[i]][i];
}

```

## 7.4 Polynomial Operation

```

#define fi(s, n)
    for (int i = (int)(s); i < (int)(n); ++i)
template <int MAXN, LL P, LL RT> // MAXN = 2^k
struct Poly : vector<LL> { // coefficients in [0, P)
    using vector<LL>::vector;
    static NTT<MAXN, P, RT> ntt;
    int n() const { return (int)size(); } // n() >= 1
    Poly(const Poly &p, int _n) : vector<LL>(_n) {
        copy_n(p.data(), min(p.n(), _n), data());
    }
    Poly &irev() {
        return reverse(data(), data() + n()), *this;
    }
    Poly &isz(int _n) { return resize(_n), *this; }
    Poly &iadd(const Poly &rhs) { // n() == rhs.n()
        fi(0, n()) if (((*this)[i] += rhs[i]) >= P)
            (*this)[i] -= P;
        return *this;
    }
    Poly &imul(LL k) {
        fi(0, n()) (*this)[i] = (*this)[i] * k % P;
        return *this;
    }
    Poly Mul(const Poly &rhs) const {
        int _n = 1;
        while (_n < n() + rhs.n() - 1) _n <<= 1;
        Poly X(*this, _n), Y(rhs, _n);
        ntt(X.data(), _n), ntt(Y.data(), _n);
        fi(0, _n) X[i] = X[i] * Y[i] % P;
        ntt(X.data(), _n, true);
        return X.isz(n() + rhs.n() - 1);
    }
    Poly Inv() const { // (*this)[0] != 0
        if (n() == 1) return {ntt.minv((*this)[0])};
        int _n = 1;
        while (_n < n() * 2) _n <<= 1;
        Poly Xi = Poly(*this, (n() + 1) / 2).Inv().isz(_n);
        Poly Y(*this, _n);
        ntt(Xi.data(), _n), ntt(Y.data(), _n);
        fi(0, _n) {
            Xi[i] *= (2 - Xi[i] * Y[i]) % P;
            if ((Xi[i] % P) < 0) Xi[i] += P;
        }
        ntt(Xi.data(), _n, true);
        return Xi.isz(n());
    }
    Poly Sqrt() const { // Jacobi((*this)[0], P) = 1
        if (n() == 1)
            return {QuadraticResidue((*this)[0], P)};
        Poly X =
            Poly(*this, (n() + 1) / 2).Sqrt().isz(n());
        return X.iadd(Mul(X.Inv()).isz(n()))
            .imul(P / 2 + 1);
    }
    pair<Poly, Poly> DivMod(
        const Poly &rhs) const { // (rhs.)back() != 0
        if (n() < rhs.n()) return {{0}, *this};

```

```

        const int _n = n() - rhs.n() + 1;
        Poly X(rhs);
        X.irev().isz(_n);
        Poly Y(*this);
        Y.irev().isz(_n);
        Poly Q = Y.Mul(X.Inv()).isz(_n).irev();
        X = rhs.Mul(Q), Y = *this;
        fi(0, n()) if ((Y[i] - X[i]) < 0) Y[i] += P;
        return {Q, Y.isz(max(1, rhs.n() - 1))};
    }
    Poly Dx() const {
        Poly ret(n() - 1);
        fi(0, ret.n()) ret[i] =
            (i + 1) * (*this)[i + 1] % P;
        return ret.isz(max(1, ret.n()));
    }
    Poly Sx() const {
        Poly ret(n() + 1);
        fi(0, n()) ret[i + 1] =
            ntt.minv(i + 1) * (*this)[i] % P;
        return ret;
    }
    Poly _tmul(int nn, const Poly &rhs) const {
        Poly Y = Mul(rhs).isz(n() + nn - 1);
        return Poly(Y.data() + n() - 1, Y.data() + Y.n());
    }
    vector<LL> _eval(const vector<LL> &x,
        const vector<Poly> &up) const {
        const int _n = (int)x.size();
        if (!_n) return {};
        vector<Poly> down(_n * 2);
        down[1] = DivMod(up[1]).second;
        fi(2, _n * 2) down[i] =
            down[i / 2].DivMod(up[i]).second;
        /* down[1] =
            Poly(up[1]).irev().isz(n()).Inv().irev()._tmul(_n,
            *this); fi(2, _n * 2) down[i] = up[i] ^
            1]._tmul(up[i].n() - 1, down[i / 2]); */
        vector<LL> y(_n);
        fi(0, _n) y[i] = down[_n + i][0];
        return y;
    }
    static vector<Poly> _tree1(const vector<LL> &x) {
        const int _n = (int)x.size();
        vector<Poly> up(_n * 2);
        fi(0, _n) up[_n + i] = {(x[i] ? P - x[i] : 0), 1};
        for (int i = _n - 1; i > 0; --i)
            up[i] = up[i * 2].Mul(up[i * 2 + 1]);
        return up;
    }
    vector<LL> Eval(const vector<LL> &x) const {
        auto up = _tree1(x);
        return _eval(x, up);
    }
    static Poly Interpolate(
        const vector<LL> &x, const vector<LL> &y) {
        const int _n = (int)x.size();
        vector<Poly> up = _tree1(x), down(_n * 2);
        vector<LL> z = up[1].Dx()._eval(x, up);
        fi(0, _n) z[i] = y[i] * ntt.minv(z[i]) % P;
        fi(0, _n) down[_n + i] = {z[i]};
        for (int i = _n - 1; i > 0; --i)
            down[i] =
                down[i * 2]
                .Mul(up[i * 2 + 1])
                .iadd(down[i * 2 + 1].Mul(up[i * 2]));
        return down[1];
    }
    Poly Ln() const { // (*this)[0] == 1
        return Dx().Mul(Inv()).Sx().isz(n());
    }
    Poly Exp() const { // (*this)[0] == 0
        if (n() == 1) return {1};
        Poly X = Poly(*this, (n() + 1) / 2).Exp().isz(n());
        Poly Y = X.Ln();
        Y[0] = P - 1;
        fi(0, n()) if ((Y[i] = (*this)[i] - Y[i]) < 0)
            Y[i] += P;
        return X.Mul(Y).isz(n());
    }
    Poly Pow(const string &K) const {
        int nz = 0;

```

```

while (nz < n()) && !(*this)[nz]) ++nz;
LL nk = 0, nk2 = 0;
for (char c : K) {
    nk = (nk * 10 + c - '0') % P;
    nk2 = nk2 * 10 + c - '0';
    if (nk2 * nz >= n()) return Poly(n());
    nk2 %= P - 1;
}
if (!nk && !nk2) return Poly(Poly{1}, n());
Poly X(data() + nz, data() + nz + n() - nz * nk2);
LL x0 = X[0];
return X.imul(ntt.minv(x0))
    .Ln()
    .imul(nk)
    .Exp()
    .imul(ntt.mpow(x0, nk2))
    .irev()
    .isz(n())
    .irev();
}
static LL LinearRecursion(const vector<LL> &a,
    const vector<LL> &coef,
    LL n) { // a_n = \sum c_j a_{n-j}
    const int k = (int)a.size();
    assert((int)coef.size() == k + 1);
    Poly C(k + 1), W(Poly{1}, k), M = {0, 1};
    fi(1, k + 1) C[k - i] = coef[i] ? P - coef[i] : 0;
    C[k] = 1;
    while (n) {
        if (n % 2) W = W.Mul(M).DivMod(C).second;
        n /= 2, M = M.Mul(M).DivMod(C).second;
    }
    LL ret = 0;
    fi(0, k) ret = (ret + W[i] * a[i]) % P;
    return ret;
}
};
#undef fi
using Poly_t = Poly<131072 * 2, 998244353, 3>;
template <> decltype(Poly_t::ntt) Poly_t::ntt = {};

```

## 7.5 Newton's Method

Given  $F(x)$  where

$$F(x) = \sum_{i=0}^{\infty} \alpha_i (x - \beta)^i$$

for  $\beta$  being some constant. Polynomial  $P$  such that  $F(P) = 0$  can be found iteratively. Denote by  $Q_k$  the polynomial such that  $F(Q_k) = 0 \pmod{x^{2^k}}$ , then

$$Q_{k+1} = Q_k - \frac{F(Q_k)}{F'(Q_k)} \pmod{x^{2^{k+1}}}$$

## 8 Geometry

### 8.1 Default Code

```

typedef pair<double, double> pdd;
typedef pair<pdd, pdd> Line;
struct Cir {
    pdd o;
    double R;
};
const double eps = 1e-8;
pdd operator+(const pdd &a, const pdd &b) {
    return pdd(a.X + b.X, a.Y + b.Y);
}
pdd operator-(const pdd &a, const pdd &b) {
    return pdd(a.X - b.X, a.Y - b.Y);
}
pdd operator*(const pdd &a, const double &b) {
    return pdd(a.X * b, a.Y * b);
}
pdd operator/(const pdd &a, const double &b) {
    return pdd(a.X / b, a.Y / b);
}
double dot(const pdd &a, const pdd &b) {
    return a.X * b.X + a.Y * b.Y;
}

```

```

}
double cross(const pdd &a, const pdd &b) {
    return a.X * b.Y - a.Y * b.X;
}
double abs2(const pdd &a) { return dot(a, a); }
double abs(const pdd &a) { return sqrt(dot(a, a)); }
int sign(const double &a) {
    return fabs(a) < eps ? 0 : a > 0 ? 1 : -1;
}
int ori(const pdd &a, const pdd &b, const pdd &c) {
    return sign(cross(b - a, c - a));
}
bool collinearity(
    const pdd &p1, const pdd &p2, const pdd &p3) {
    return fabs(cross(p1 - p3, p2 - p3)) < eps;
}
bool btw(const pdd &p1, const pdd &p2, const pdd &p3) {
    if (!collinearity(p1, p2, p3)) return 0;
    return dot(p1 - p3, p2 - p3) < eps;
}
bool seg_intersect(const pdd &p1, const pdd &p2,
    const pdd &p3, const pdd &p4) {
    int a123 = ori(p1, p2, p3);
    int a124 = ori(p1, p2, p4);
    int a341 = ori(p3, p4, p1);
    int a342 = ori(p3, p4, p2);
    if (a123 == 0 && a124 == 0)
        return btw(p1, p2, p3) || btw(p1, p2, p4) ||
            btw(p3, p4, p1) || btw(p3, p4, p2);
    return a123 * a124 <= 0 && a341 * a342 <= 0;
}
pdd intersect(const pdd &p1, const pdd &p2,
    const pdd &p3, const pdd &p4) {
    double a123 = cross(p2 - p1, p3 - p1);
    double a124 = cross(p2 - p1, p4 - p1);
    return (p4 * a123 - p3 * a124) / (a123 - a124);
}
pdd perp(const pdd &p1) { return pdd(-p1.Y, p1.X); }
pdd foot(const pdd &p1, const pdd &p2, const pdd &p3) {
    return intersect(p1, p2, p3, p3 + perp(p2 - p1));
}
}

```

### 8.2 Convex hull\*

```

void hull(vector<pdd> &dots) {
    sort(dots.begin(), dots.end());
    vector<pdd> ans(1, dots[0]);
    for (int ct = 0; ct < 2; ++ct, reverse(ALL(dots)))
        for (int i = 1, t = SZ(ans); i < SZ(dots);
            ans.pb(dots[i++]))
            while (SZ(ans) > t &&
                ori(ans[SZ(ans) - 2], ans.back(), dots[i]) <=
                    0)
                ans.pop_back();
    ans.pop_back(), ans.swap(dots);
}

```

### 8.3 External bisector

```

pdd external_bisector(pdd p1, pdd p2, pdd p3) { // 213
    pdd L1 = p2 - p1, L2 = p3 - p1;
    L2 = L2 * abs(L1) / abs(L2);
    return L1 + L2;
}

```

### 8.4 Heart

```

pdd excenter(pdd p0, pdd p1, pdd p2, double &radius) {
    p1 = p1 - p0, p2 = p2 - p0;
    double x1 = p1.X, y1 = p1.Y, x2 = p2.X, y2 = p2.Y;
    double m = 2. * (x1 * y2 - y1 * x2);
    center.X = (x1 * x1 * y2 - x2 * x2 * y1 +
        y1 * y2 * (y1 - y2)) /
        m;
    center.Y = (x1 * x2 * (x2 - x1) - y1 * y1 * x2 +
        x1 * y2 * y2) /
        m;
    return radius = abs(center), center + p0;
}

```

```

pdd incenter(pdd p1, pdd p2, pdd p3, double &radius) {
    double a = abs(p2 - p1), b = abs(p3 - p1),
           c = abs(p3 - p2);
    double s = (a + b + c) / 2,
           area = sqrt(s * (s - a) * (s - b) * (s - c));
    pdd L1 = external_bisector(p1, p2, p3),
        L2 = external_bisector(p2, p1, p3);
    return radius = area / s,
           intersect(p1, p1 + L1, p2, p2 + L2),
}

pdd escenter(pdd p1, pdd p2, pdd p3) { // 213
    pdd L1 = external_bisector(p1, p2, p3),
        L2 = external_bisector(p2, p2 + p2 - p1, p3);
    return intersect(p1, p1 + L1, p2, p2 + L2);
}

pdd barycenter(pdd p1, pdd p2, pdd p3) {
    return (p1 + p2 + p3) / 3;
}

pdd orthocenter(pdd p1, pdd p2, pdd p3) {
    pdd L1 = p3 - p2, L2 = p3 - p1;
    swap(L1.X, L1.Y), L1.X *= -1;
    swap(L2.X, L2.Y), L2.X *= -1;
    return intersect(p1, p1 + L1, p2, p2 + L2);
}

```

## 8.5 Minimum Enclosing Circle\*

```

pdd Minimum_Enclosing_Circle(
    vector<pdd> dots, double &r) {
    pdd cent;
    random_shuffle(ALL(dots));
    cent = dots[0], r = 0;
    for (int i = 1; i < SZ(dots); ++i)
        if (abs(dots[i] - cent) > r) {
            cent = dots[i], r = 0;
            for (int j = 0; j < i; ++j)
                if (abs(dots[j] - cent) > r) {
                    cent = (dots[i] + dots[j]) / 2;
                    r = abs(dots[i] - cent);
                    for (int k = 0; k < j; ++k)
                        if (abs(dots[k] - cent) > r)
                            cent =
                                excenter(dots[i], dots[j], dots[k], r);
                }
        }
    return cent;
}

```

## 8.6 Polar Angle Sort\*

```

pdd center; // sort base
int Quadrant(pdd a) {
    if (a.X > 0 && a.Y >= 0) return 1;
    if (a.X <= 0 && a.Y > 0) return 2;
    if (a.X < 0 && a.Y <= 0) return 3;
    if (a.X >= 0 && a.Y < 0) return 4;
}

bool cmp(p11 a, p11 b) {
    a = a - center, b = b - center;
    if (Quadrant(a) != Quadrant(b))
        return Quadrant(a) < Quadrant(b);
    if (cross(b, a) == 0) return abs2(a) < abs2(b);
    return cross(a, b) > 0;
}

bool cmp(pdd a, pdd b) {
    a = a - center, b = b - center;
    if (fabs(atan2(a.Y, a.X) - atan2(b.Y, b.X)) > eps)
        return atan2(a.Y, a.X) < atan2(b.Y, b.X);
    return abs(a) < abs(b);
}

```

## 8.7 Intersection of two circles\*

```

bool CCinter(Cir &a, Cir &b, pdd &p1, pdd &p2) {
    pdd o1 = a.O, o2 = b.O;
    double r1 = a.R, r2 = b.R, d2 = abs2(o1 - o2),
           d = sqrt(d2);

```

```

    if (d < max(r1, r2) - min(r1, r2) || d > r1 + r2)
        return 0;
    pdd u = (o1 + o2) * 0.5 +
           (o1 - o2) * ((r2 * r2 - r1 * r1) / (2 * d2));
    double A = sqrt((r1 + r2 + d) * (r1 - r2 + d) *
                    (r1 + r2 - d) * (-r1 + r2 + d));
    pdd v =
        pdd(o1.Y - o2.Y, -o1.X + o2.X) * A / (2 * d2);
    p1 = u + v, p2 = u - v;
    return 1;
}

```

## 8.8 Intersection of polygon and circle

```

// Divides into multiple triangle, and sum up
// test by HDU2892
const double PI = acos(-1);
double _area(pdd pa, pdd pb, double r) {
    if (abs(pa) < abs(pb)) swap(pa, pb);
    if (abs(pb) < eps) return 0;
    double S, h, theta;
    double a = abs(pb), b = abs(pa), c = abs(pb - pa);
    double cosB = dot(pb, pb - pa) / a / c,
           B = acos(cosB);
    double cosC = dot(pa, pb) / a / b, C = acos(cosC);
    if (a > r) {
        S = (C / 2) * r * r;
        h = a * b * sin(C) / c;
        if (h < r && B < PI / 2)
            S -= (acos(h / r) * r * r -
                  h * sqrt(r * r - h * h));
    } else if (b > r) {
        theta = PI - B - asin(sin(B) / r * a);
        S = .5 * a * r * sin(theta) +
            (C - theta) / 2 * r * r;
    } else S = .5 * sin(C) * a * b;
    return S;
}

double area_poly_circle(const vector<pdd> poly,
    const pdd &o, const double r) {
    double S = 0;
    for (int i = 0; i < SZ(poly); ++i)
        S += _area(poly[i] - o,
                    poly[(i + 1) % SZ(poly)] - o, r) *
            ori(0, poly[i], poly[(i + 1) % SZ(poly)]);
    return fabs(S);
}

```

## 8.9 Intersection of line and circle

```

vector<pdd> line_interCircle(const pdd &p1,
    const pdd &p2, const pdd &c, const double r) {
    pdd ft = foot(p1, p2, c), vec = p2 - p1;
    double dis = abs(c - ft);
    if (fabs(dis - r) < eps) return vector<pdd>{ft};
    if (dis > r) return {};
    vec = vec * sqrt(r * r - dis * dis) / abs(vec);
    return vector<pdd>{ft + vec, ft - vec};
}

```

## 8.10 point in circle

```

// return p4 is strictly in circumcircle of
// tri(p1,p2,p3)
long long sqr(long long x) { return x * x; }
bool in_cc(const p11 &p1, const p11 &p2, const p11 &p3,
    const p11 &p4) {
    long long u11 = p1.X - p4.X;
    long long u12 = p1.Y - p4.Y;
    long long u21 = p2.X - p4.X;
    long long u22 = p2.Y - p4.Y;
    long long u31 = p3.X - p4.X;
    long long u32 = p3.Y - p4.Y;
    long long u13 =
        sqr(p1.X) - sqr(p4.X) + sqr(p1.Y) - sqr(p4.Y);
    long long u23 =
        sqr(p2.X) - sqr(p4.X) + sqr(p2.Y) - sqr(p4.Y);
    long long u33 =
        sqr(p3.X) - sqr(p4.X) + sqr(p3.Y) - sqr(p4.Y);
    __int128 det = (__int128)-u13 * u22 * u31 +

```

```

    (__int128)u12 * u23 * u31 +
    (__int128)u13 * u21 * u32 -
    (__int128)u11 * u23 * u32 -
    (__int128)u12 * u21 * u33 +
    (__int128)u11 * u22 * u33;
    return det > eps;
}

```

### 8.11 Half plane intersection

```

bool isin(Line l0, Line l1, Line l2) {
    // Check inter(l1, l2) in l0
    pdd p = intersect(l1.X, l1.Y, l2.X, l2.Y);
    return cross(l0.Y - l0.X, p - l0.X) > eps;
}

/* If no solution, check: 1. ret.size() < 3
 * Or more precisely, 2. interPnt(ret[0], ret[1])
 * in all the lines. (use (l.Y - l.X) ^ (p - l.X) > 0
 */
/* --^-- Line.X --^-- Line.Y --^-- */
vector<Line> halfPlaneInter(vector<Line> lines) {
    int sz = lines.size();
    vector<double> ata(sz), ord(sz);
    for (int i = 0; i < sz; ++i) {
        ord[i] = i;
        pdd d = lines[i].Y - lines[i].X;
        ata[i] = atan2(d.Y, d.X);
    }
    sort(ord.begin(), ord.end(), [&](int i, int j) {
        if (fabs(ata[i] - ata[j]) < eps)
            return (cross(lines[i].Y - lines[i].X,
                          lines[j].Y - lines[i].X)) < 0;
        return ata[i] < ata[j];
    });
    vector<Line> fin;
    for (int i = 0; i < sz; ++i)
        if (!i ||
            fabs(ata[ord[i]] - ata[ord[i - 1]]) > eps)
            fin.pb(lines[ord[i]]);
    deque<Line> dq;
    for (int i = 0; i < SZ(fin); i++) {
        while (SZ(dq) >= 2 &&
            !isin(fin[i], dq[SZ(dq) - 2], dq.back()))
            dq.pop_back();
        while (SZ(dq) >= 2 && !isin(fin[i], dq[0], dq[1]))
            dq.pop_front();
        dq.push_back(fin[i]);
    }
    while (SZ(dq) >= 3 &&
        !isin(dq[0], dq[SZ(dq) - 2], dq.back()))
        dq.pop_back();
    while (SZ(dq) >= 3 && !isin(dq.back(), dq[0], dq[1]))
        dq.pop_front();
    vector<Line> res(ALL(dq));
    return res;
}

```

### 8.12 CircleCover\*

```
const int N = 1021;
struct CircleCover {
    int C;
    Cir c[N];
    bool g[N][N], overlap[N][N];
    // Area[i] : area covered by at least i circles
    double Area[N];
    void init(int _C) { C = _C; }
    struct Teve {
        pdd p;
        double ang;
        int add;
        Teve() {}
        Teve(pdd _a, double _b, int _c)
            : p(_a), ang(_b), add(_c) {}
        bool operator<(const Teve &a) const {
            return ang < a.ang;
        }
    } eve[N * 2];
    // strict: x = 0, otherwise x = -1
    bool disjunct(Cir &a, Cir &b, int x) {
        return sign(abs(a.O - b.O) - a.R - b.R) > x;
    }
};
```

```

}
bool contain(Cir &a, Cir &b, int x) {
    return sign(a.R - b.R - abs(a.O - b.O)) > x;
}
bool contain(int i, int j) {
    /* c[j] is non-strictly in c[i]. */
    return (sign(c[i].R - c[j].R) > 0 ||
            (sign(c[i].R - c[j].R) == 0 && i < j)) &&
        contain(c[i], c[j], -1);
}
void solve() {
    fill_n(Area, C + 2, 0);
    for (int i = 0; i < C; ++i)
        for (int j = 0; j < C; ++j)
            overlap[i][j] = contain(i, j);
    for (int i = 0; i < C; ++i)
        for (int j = 0; j < C; ++j)
            g[i][j] = !(overlap[i][j] || overlap[j][i] ||
                        disjunct(c[i], c[j], -1));
    for (int i = 0; i < C; ++i) {
        int E = 0, cnt = 1;
        for (int j = 0; j < C; ++j)
            if (j != i && overlap[j][i]) ++cnt;
        for (int j = 0; j < C; ++j)
            if (i != j && g[i][j]) {
                pdd aa, bb;
                CCinter(c[i], c[j], aa, bb);
                double A =
                    atan2(aa.Y - c[i].O.Y, aa.X - c[i].O.X);
                double B =
                    atan2(bb.Y - c[i].O.Y, bb.X - c[i].O.X);
                eve[E++] = Teve(bb, B, 1),
                eve[E++] = Teve(aa, A, -1);
                if (B > A) ++cnt;
            }
        if (E == 0) Area[cnt] += pi * c[i].R * c[i].R;
        else {
            sort(eve, eve + E);
            eve[E] = eve[0];
            for (int j = 0; j < E; ++j) {
                cnt += eve[j].add;
                Area[cnt] +=
                    cross(eve[j].p, eve[j + 1].p) * .5;
                double theta = eve[j + 1].ang - eve[j].ang;
                if (theta < 0) theta += 2. * pi;
                Area[cnt] += (theta - sin(theta)) * c[i].R *
                    c[i].R * .5;
            }
        }
    }
}
};

```

### 8.13 3Dpoint\*

```

struct Point {
    double x, y, z;
    Point(double _x = 0, double _y = 0, double _z = 0)
        : x(_x), y(_y), z(_z) {}
    Point(pdd p) { x = p.X, y = p.Y, z = abs2(p); }
};

Point operator-(const Point &p1, const Point &p2) {
    return Point(p1.x - p2.x, p1.y - p2.y, p1.z - p2.z);
}

Point cross(const Point &p1, const Point &p2) {
    return Point(p1.y * p2.z - p1.z * p2.y,
                p1.z * p2.x - p1.x * p2.z,
                p1.x * p2.y - p1.y * p2.x);
}

double dot(const Point &p1, const Point &p2) {
    return p1.x * p2.x + p1.y * p2.y + p1.z * p2.z;
}

double abs(const Point &a) { return sqrt(dot(a, a)); }

Point cross3(
    const Point &a, const Point &b, const Point &c) {
    return cross(b - a, c - a);
}

double area(Point a, Point b, Point c) {
    return abs(cross3(a, b, c));
}

double volume(Point a, Point b, Point c, Point d) {

```



```

    return dot(cross3(a, b, c), d - a);
}
pdd proj(Point a, Point b, Point c, Point u) {
    // proj. u to the plane of a, b, and c
    Point e1 = b - a;
    Point e2 = c - a;
    e1 = e1 / abs(e1);
    e2 = e2 - e1 * dot(e2, e1);
    e2 = e2 / abs(e2);
    Point p = u - a;
    return pdd(dot(p, e1), dot(p, e2));
}

```

## 8.14 Convexhull3D\*

```

struct CH3D {
    struct face {
        int a, b, c;
        bool ok;
    } F[8 * N];
    double dblcmp(Point &p, face &f) {
        return dot(
            cross3(P[f.a], P[f.b], P[f.c]), p - P[f.a]);
    }
    int g[N][N], num, n;
    Point P[N];
    void deal(int p, int a, int b) {
        int f = g[a][b];
        face add;
        if (F[f].ok) {
            if (dblcmp(P[p], F[f]) > eps) dfs(p, f);
        } else
            add.a = b, add.b = a, add.c = p, add.ok = 1,
            g[p][b] = g[a][p] = g[b][a] = num,
            F[num++] = add;
    }
    void dfs(int p, int now) {
        F[now].ok = 0;
        deal(p, F[now].b, F[now].a),
        deal(p, F[now].c, F[now].b),
        deal(p, F[now].a, F[now].c);
    }
    bool same(int s, int t) {
        Point &a = P[F[s].a];
        Point &b = P[F[s].b];
        Point &c = P[F[s].c];
        return fabs(volume(a, b, c, P[F[t].a])) < eps &&
            fabs(volume(a, b, c, P[F[t].b])) < eps &&
            fabs(volume(a, b, c, P[F[t].c])) < eps;
    }
    void init(int _n) { n = _n, num = 0; }
    void solve() {
        face add;
        num = 0;
        if (n < 4) return;
        if ([&]() {
            for (int i = 1; i < n; ++i)
                if (abs(P[0] - P[i]) > eps)
                    return swap(P[1], P[i]), 0;
            return 1;
        }) ||
        [&]() {
            for (int i = 2; i < n; ++i)
                if (abs(cross3(P[i], P[0], P[1])) > eps)
                    return swap(P[2], P[i]), 0;
            return 1;
        }) ||
        [&]() {
            for (int i = 3; i < n; ++i)
                if (fabs(dot(cross(P[0] - P[1], P[1] - P[2]),
                    P[0] - P[i])) > eps)
                    return swap(P[3], P[i]), 0;
            return 1;
        })
        return;
        for (int i = 0; i < 4; ++i) {
            add.a = (i + 1) % 4, add.b = (i + 2) % 4,
            add.c = (i + 3) % 4, add.ok = true;
            if (dblcmp(P[i], add) > 0) swap(add.b, add.c);
            g[add.a][add.b] = g[add.b][add.c] =

```

```

            g[add.c][add.a] = num;
            F[num++] = add;
        }
        for (int i = 4; i < n; ++i)
            for (int j = 0; j < num; ++j)
                if (F[j].ok && dblcmp(P[i], F[j]) > eps) {
                    dfs(i, j);
                    break;
                }
        for (int tmp = num, i = (num = 0); i < tmp; ++i)
            if (F[i].ok) F[num++] = F[i];
    }
    double get_area() {
        double res = 0.0;
        if (n == 3)
            return abs(cross3(P[0], P[1], P[2])) / 2.0;
        for (int i = 0; i < num; ++i)
            res += area(P[F[i].a], P[F[i].b], P[F[i].c]);
        return res / 2.0;
    }
    double get_volume() {
        double res = 0.0;
        for (int i = 0; i < num; ++i)
            res += volume(Point(0, 0, 0), P[F[i].a],
                P[F[i].b], P[F[i].c]);
        return fabs(res / 6.0);
    }
    int triangle() { return num; }
    int polygon() {
        int res = 0;
        for (int i = 0, flag = 1; i < num; ++i, res += flag, flag = 1)
            for (int j = 0; j < i && flag; ++j)
                flag &= !same(i, j);
        return res;
    }
    Point getcent() {
        Point ans(0, 0, 0), temp = P[F[0].a];
        double v = 0.0, t2;
        for (int i = 0; i < num; ++i)
            if (F[i].ok == true) {
                Point p1 = P[F[i].a], p2 = P[F[i].b],
                    p3 = P[F[i].c];
                t2 = volume(temp, p1, p2, p3) / 6.0;
                if (t2 > 0)
                    ans.x += (p1.x + p2.x + p3.x + temp.x) * t2,
                    ans.y +=
                        (p1.y + p2.y + p3.y + temp.y) * t2,
                    ans.z +=
                        (p1.z + p2.z + p3.z + temp.z) * t2,
                    v += t2;
            }
        ans.x /= (4 * v), ans.y /= (4 * v),
        ans.z /= (4 * v);
        return ans;
    }
    double pointmindis(Point p) {
        double rt = 99999999;
        for (int i = 0; i < num; ++i)
            if (F[i].ok == true) {
                Point p1 = P[F[i].a], p2 = P[F[i].b],
                    p3 = P[F[i].c];
                double a = (p2.y - p1.y) * (p3.z - p1.z) -
                    (p2.z - p1.z) * (p3.y - p1.y);
                double b = (p2.z - p1.z) * (p3.x - p1.x) -
                    (p2.x - p1.x) * (p3.z - p1.z);
                double c = (p2.x - p1.x) * (p3.y - p1.y) -
                    (p2.y - p1.y) * (p3.x - p1.x);
                double d =
                    0 - (a * p1.x + b * p1.y + c * p1.z);
                double temp =
                    fabs(a * p.x + b * p.y + c * p.z + d) /
                    sqrt(a * a + b * b + c * c);
                rt = min(rt, temp);
            }
        return rt;
    }
};

```

## 8.15 DelaunayTriangulation\*

```
/* Delaunay Triangulation:
```

Given a sets of points on 2D plane, find a triangulation such that no points will strictly inside circumcircle of any triangle.  
 find : return a triangle contain given point  
 add\_point : add a point into triangulation  
 A Triangle is in triangulation iff. its has\_chd is 0.  
 Region of triangle u: iterate each u.edge[i].tri, each points are u.p[(i+1)%3], u.p[(i+2)%3]  
 Voronoi diagram: for each triangle in triangulation, the bisector of all its edges will split the region. nearest point will belong to the triangle containing it  
 \*/

```
const ll inf =
    MAXC * MAXC * 100; // Lower_bound unknown
struct Tri;
struct Edge {
    Tri *tri;
    int side;
    Edge() : tri(0), side(0) {}
    Edge(Tri *_tri, int _side)
        : tri(_tri), side(_side) {}
};
struct Tri {
    pll p[3];
    Edge edge[3];
    Tri *chd[3];
    Tri() {}
    Tri(const pll &p0, const pll &p1, const pll &p2) {
        p[0] = p0;
        p[1] = p1;
        p[2] = p2;
        chd[0] = chd[1] = chd[2] = 0;
    }
    bool has_chd() const { return chd[0] != 0; }
    int num_chd() const {
        return !!chd[0] + !!chd[1] + !!chd[2];
    }
    bool contains(pll const &q) const {
        for (int i = 0; i < 3; ++i)
            if (ori(p[i], p[(i + 1) % 3], q) < 0) return 0;
        return 1;
    }
} pool[N * 10], *tris;
void edge(Edge a, Edge b) {
    if (a.tri) a.tri->edge[a.side] = b;
    if (b.tri) b.tri->edge[b.side] = a;
}
struct Trig { // Triangulation
    Trig() {
        the_root = // Tri should at least contain all
                    // points
                    new (tris++) Tri(pll(-inf, -inf),
                    pll(inf + inf, -inf), pll(-inf, inf + inf));
    }
    Tri *find(pll p) { return find(the_root, p); }
    void add_point(const pll &p) {
        add_point(find(the_root, p), p);
    }
    Tri *the_root;
    static Tri *find(Tri *root, const pll &p) {
        while (1) {
            if (!root->has_chd()) return root;
            for (int i = 0; i < 3 && root->chd[i]; ++i)
                if (root->chd[i]->contains(p)) {
                    root = root->chd[i];
                    break;
                }
        }
        assert(0); // "point not found"
    }
    void add_point(Tri *root, pll const &p) {
        Tri *t[3];
        /* split it into three triangles */
        for (int i = 0; i < 3; ++i)
            t[i] = new (tris++)
                Tri(root->p[i], root->p[(i + 1) % 3], p);
        for (int i = 0; i < 3; ++i)
            edge(Edge(t[i], 0), Edge(t[(i + 1) % 3], 1));
        for (int i = 0; i < 3; ++i)
            edge(Edge(t[i], 2), root->edge[(i + 2) % 3]);
        for (int i = 0; i < 3; ++i) root->chd[i] = t[i];
        for (int i = 0; i < 3; ++i) flip(t[i], 2);
    }
};
```

```

    }
    void flip(Tri *tri, int pi) {
        Tri *trj = tri->edge[pi].tri;
        int pj = tri->edge[pi].side;
        if (!trj) return;
        if (!in_cc(
            tri->p[0], tri->p[1], tri->p[2], trj->p[pj]))
            return;
        /* flip edge between tri, trj */
        Tri *trk = new (tris++) Tri(
            tri->p[(pi + 1) % 3], trj->p[pj], tri->p[pi]);
        Tri *trl = new (tris++) Tri(
            trj->p[(pj + 1) % 3], tri->p[pi], trj->p[pj]);
        edge(Edge(trk, 0), Edge(trl, 0));
        edge(Edge(trk, 1), tri->edge[(pi + 2) % 3]);
        edge(Edge(trk, 2), trj->edge[(pj + 1) % 3]);
        edge(Edge(trl, 1), trj->edge[(pj + 2) % 3]);
        edge(Edge(trl, 2), tri->edge[(pi + 1) % 3]);
        tri->chd[0] = trk;
        tri->chd[1] = trl;
        tri->chd[2] = 0;
        trj->chd[0] = trk;
        trj->chd[1] = trl;
        trj->chd[2] = 0;
        flip(trk, 1);
        flip(trk, 2);
        flip(trl, 1);
        flip(trl, 2);
    }
};
vector<Tri *> triang; // vector of all triangle
set<Tri *> vst;
void go(Tri *now) { // store all tri into triang
    if (vst.find(now) != vst.end()) return;
    vst.insert(now);
    if (!now->has_chd()) return triang.push_back(now);
    for (int i = 0; i < now->num_chd(); ++i)
        go(now->chd[i]);
}
void build(int n, pll *ps) { // build triangulation
    tris = pool;
    triang.clear();
    vst.clear();
    random_shuffle(ps, ps + n);
    Trig tri; // the triangulation structure
    for (int i = 0; i < n; ++i) tri.add_point(ps[i]);
    go(tri.the_root);
}
}
```

## 8.16 Triangulation Voronoi\*

```
vector<Line> ls[N];
pll arr[N];
Line make_line(pdd p, Line l) {
    pdd d = 1.Y - 1.X;
    d = perp(d);
    pdd m = (1.X + 1.Y) / 2;
    l = Line(m, m + d);
    if (ori(1.X, 1.Y, p) < 0) l = Line(m + d, m);
    return l;
}
double calc_area(int id) {
    // use to calculate the area of point "strictly in
    // the convex hull"
    vector<Line> hpi = halfPlaneInter(ls[id]);
    vector<pdd> ps;
    for (int i = 0; i < SZ(hpi); ++i)
        ps.pb(intersect(hpi[i].X, hpi[i].Y,
            hpi[(i + 1) % SZ(hpi)].X,
            hpi[(i + 1) % SZ(hpi)].Y));
    double rt = 0;
    for (int i = 0; i < SZ(ps); ++i)
        rt += cross(ps[i], ps[(i + 1) % SZ(ps)]);
    return fabs(rt) / 2;
}
void solve(int n, pii *oarr) {
    map<pll, int> mp;
    for (int i = 0; i < n; ++i)
        arr[i] = pll(oarr[i].X, oarr[i].Y), mp[arr[i]] = i;
    build(n, arr); // Triangulation
    for (auto *t : triang) {
```

```

vector<int> p;
for (int i = 0; i < 3; ++i)
    if (mp.find(t->p[i]) != mp.end())
        p.pb(mp[t->p[i]]);
for (int i = 0; i < SZ(p); ++i)
    for (int j = i + 1; j < SZ(p); ++j) {
        Line l(oarr[p[i]], oarr[p[j]]);
        ls[p[i]].pb(make_line(oarr[p[i]], l));
        ls[p[j]].pb(make_line(oarr[p[j]], l));
    }
}

```

## 8.17 Tangent line of two circles

```

vector<Line> go(
    const Cir &c1, const Cir &c2, int sign1) {
    // sign1 = 1 for outer tang, -1 for inner tang
    vector<Line> ret;
    double d_sq = norm2(c1.O - c2.O);
    if (d_sq < eps) return ret;
    double d = sqrt(d_sq);
    Pt v = (c2.O - c1.O) / d;
    double c = (c1.R - sign1 * c2.R) / d;
    if (c * c > 1) return ret;
    double h = sqrt(max(0.0, 1.0 - c * c));
    for (int sign2 = 1; sign2 >= -1; sign2 -= 2) {
        Pt n = {v.X * c - sign2 * h * v.Y,
                v.Y * c + sign2 * h * v.X};
        Pt p1 = c1.O + n * c1.R;
        Pt p2 = c2.O + n * (c2.R * sign1);
        if (fabs(p1.X - p2.X) < eps and
            fabs(p1.Y - p2.Y) < eps)
            p2 = p1 + perp(c2.O - c1.O);
        ret.push_back({p1, p2});
    }
    return ret;
}

```

## 8.18 minMaxEnclosingRectangle

```

pdd solve(vector<pll> &dots) {
    vector<pll> hull;
    const double INF = 1e18, qi = acos(-1) / 2 * 3;
    cv.dots = dots;
    hull = cv.hull();
    double Max = 0, Min = INF, deg;
    ll n = hull.size();
    hull.pb(hull[0]);
    for (int i = 0, u = 1, r = 1, l; i < n; ++i) {
        pll nw = hull[i + 1] - hull[i];
        while (cross(nw, hull[u + 1] - hull[i]) >
                cross(nw, hull[u] - hull[i]))
            u = (u + 1) % n;
        while (dot(nw, hull[r + 1] - hull[i]) >
                dot(nw, hull[r] - hull[i]))
            r = (r + 1) % n;
        if (!i) l = (r + 1) % n;
        while (dot(nw, hull[l + 1] - hull[i]) <
                dot(nw, hull[l] - hull[i]))
            l = (l + 1) % n;
        Min = min(Min,
            (double)(dot(nw, hull[r] - hull[i]) -
                dot(nw, hull[l] - hull[i])) *
                cross(nw, hull[u] - hull[i]) / abs2(nw));
        deg = acos((double)dot(hull[r] - hull[l],
            hull[u] - hull[i]) /
            abs(hull[r] - hull[l]) / abs(hull[u] - hull[i]));
        deg = (qi - deg) / 2;
        Max = max(Max,
            (double)abs(hull[r] - hull[l]) *
            abs(hull[u] - hull[i]) * sin(deg) * sin(deg));
    }
    return pdd(Min, Max);
}

```

## 8.19 minDistOfTwoConvex

```

// p, q is convex
double TwoConvexHullMinDist(

```

```

Point P[], Point Q[], int n, int m) {
    int YMinP = 0, YMaxQ = 0;
    double tmp, ans = 999999999;
    for (i = 0; i < n; ++i)
        if (P[i].y < P[YMinP].y) YMinP = i;
    for (i = 0; i < m; ++i)
        if (Q[i].y > Q[YMaxQ].y) YMaxQ = i;
    P[n] = P[0], Q[m] = Q[0];
    for (int i = 0; i < n; ++i) {
        while (tmp = Cross(Q[YMaxQ + 1] - P[YMinP + 1],
            P[YMinP] - P[YMinP + 1]) >
            Cross(Q[YMaxQ] - P[YMinP + 1],
            P[YMinP] - P[YMinP + 1]))
            YMaxQ = (YMaxQ + 1) % m;
        if (tmp < 0)
            ans = min(ans,
                PointToSegDist(
                    P[YMinP], P[YMinP + 1], Q[YMaxQ]));
        else
            ans = min(ans,
                TwoSegMinDist(P[YMinP], P[YMinP + 1], Q[YMaxQ],
                    Q[YMaxQ + 1]));
        YMinP = (YMinP + 1) % n;
    }
    return ans;
}

```

## 8.20 Minkowski Sum\*

```

vector<pll> Minkowski(vector<pll> A, vector<pll> B) {
    hull(A), hull(B);
    vector<pll> C(1, A[0] + B[0]), s1, s2;
    for (int i = 0; i < SZ(A); ++i)
        s1.pb(A[(i + 1) % SZ(A)] - A[i]);
    for (int i = 0; i < SZ(B); ++i)
        s2.pb(B[(i + 1) % SZ(B)] - B[i]);
    for (int p1 = 0, p2 = 0; p1 < SZ(A) || p2 < SZ(B);)
        if (p2 >= SZ(B) ||
            (p1 < SZ(A) && cross(s1[p1], s2[p2]) >= 0))
            C.pb(C.back() + s1[p1++]);
        else C.pb(C.back() + s2[p2++]);
    return hull(C), C;
}

```

## 8.21 RotatingSweepLine

```

void rotatingSweepLine(vector<pii> &ps) {
    int n = SZ(ps);
    vector<int> id(n), pos(n);
    vector<pii> line(n * (n - 1) / 2);
    int m = 0;
    for (int i = 0; i < n; ++i)
        for (int j = i + 1; j < n; ++j)
            line[m++] = pii(i, j);
    sort(ALL(line),
        [&](const pii &a, const pii &b) -> bool {
            if (ps[a.X].X == ps[a.Y].X) return 0;
            if (ps[b.X].X == ps[b.Y].X) return 1;
            return (double)(ps[a.X].Y - ps[a.Y].Y) /
                (ps[a.X].X - ps[a.Y].X) <
                (double)(ps[b.X].Y - ps[b.Y].Y) /
                (ps[b.X].X - ps[b.Y].X);
        });
    iota(id, id + n, 0);
    sort(ALL(id), [&](const int &a, const int &b) {
        return ps[a] < ps[b];
    });
    for (int i = 0; i < n; ++i) pos[id[i]] = i;
    for (int i = 0; i < m; ++i) {
        auto l = line[i];
        // meow
        tie(
            pos[l.X], pos[l.Y], id[pos[l.X]], id[pos[l.Y]]) =
            make_tuple(pos[l.Y], pos[l.X], l.Y, l.X);
    }
}

```

## 9 Else

### 9.1 Mo's Algorithm(With modification)

```
struct QUERY { // BLOCK=N^{2/3}
    int L, R, id, Lbid, Rbid, T;
    QUERY(int l, int r, int id, int lb, int rb, int t)
        : L(l), R(r), id(id), Lbid(lb), Rbid(rb), T(t) {}
    bool operator<(const QUERY &b) const {
        if (Lbid != b.Lbid) return Lbid < b.Lbid;
        if (Rbid != b.Rbid) return Rbid < b.Rbid;
        return T < b.T;
    }
};
vector<QUERY> query;
int cur_ans, arr[MAXN], ans[MAXN];
void addTime(int L, int R, int T) {}
void subTime(int L, int R, int T) {}
void add(int x) {}
void sub(int x) {}
void solve() {
    sort(ALL(query));
    int L = 0, R = 0, T = -1;
    for (auto q : query) {
        while (T < q.T) addTime(L, R, ++T);
        while (T > q.T) subTime(L, R, T--);
        while (R < q.R) add(arr[++R]);
        while (L > q.L) add(arr[--L]);
        while (R > q.R) sub(arr[R--]);
        while (L < q.L) sub(arr[L++]);
        ans[q.id] = cur_ans;
    }
}
```

### 9.2 Mo's Algorithm On Tree

```
const int MAXN = 40005;
vector<int> G[MAXN]; // 1-base
int n, B, arr[MAXN], ans[100005], cur_ans;
int in[MAXN], out[MAXN], dfn[MAXN * 2], dft;
int deep[MAXN], sp[___lg(MAXN * 2) + 1][MAXN * 2],
    bln[MAXN], spt;
bitset<MAXN> inset;
struct QUERY {
    int L, R, Lid, id, lca;
    QUERY(int l, int r, int _id)
        : L(l), R(r), lca(0), id(_id) {}
    bool operator<(const QUERY &b) {
        if (Lid != b.Lid) return Lid < b.Lid;
        return R < b.R;
    }
};
vector<QUERY> query;
void dfs(int u, int f, int d) {
    deep[u] = d, sp[0][spt] = u, bln[u] = spt++;
    dfn[dft] = u, in[u] = dft++;
    for (int v : G[u])
        if (v != f)
            dfs(v, u, d + 1), sp[0][spt] = u, bln[u] = spt++;
    dft = dfn[u];
}
int lca(int u, int v) {
    if (bln[u] > bln[v]) swap(u, v);
    int t = ___lg(bln[v] - bln[u] + 1);
    int a = sp[t][bln[u]],
        b = sp[t][bln[v] - (1 << t) + 1];
    if (deep[a] < deep[b]) return a;
    return b;
}
void sub(int x) {}
void add(int x) {}
void flip(int x) {
    if (inset[x]) sub(arr[x]);
    else add(arr[x]);
    inset[x] = ~inset[x];
}
void solve() {
    B = sqrt(2 * n), dft = spt = cur_ans = 0,
    dfs(1, 1, 0);
    for (int i = 1, x = 2; x < 2 * n; ++i, x <= 1)
        for (int j = 0; j + x <= 2 * n; ++j)
```

```
        if (deep[sp[i - 1][j]] <
            deep[sp[i - 1][j + x / 2]])
            sp[i][j] = sp[i - 1][j];
        else sp[i][j] = sp[i - 1][j + x / 2];
    for (auto &q : query) {
        int c = lca(q.L, q.R);
        if (c == q.L || c == q.R)
            q.L = out[c == q.L ? q.R : q.L], q.R = out[c];
        else if (out[q.L] < in[q.R])
            q.lca = c, q.L = out[q.L], q.R = in[q.R];
        else
            q.lca = c, c = in[q.L], q.L = out[q.R], q.R = c;
        q.Lid = q.L / B;
    }
    sort(ALL(query));
    int L = 0, R = -1;
    for (auto q : query) {
        while (R < q.R) flip(dfn[++R]);
        while (L > q.L) flip(dfn[--L]);
        while (R > q.R) flip(dfn[R--]);
        while (L < q.L) flip(dfn[L++]);
        if (q.lca) add(arr[q.lca]);
        ans[q.id] = cur_ans;
        if (q.lca) sub(arr[q.lca]);
    }
}
```

### 9.3 Hilbert Curve

```
ll hilbert(int n, int x, int y) {
    ll res = 0;
    for (int s = n / 2; s; s >>= 1) {
        int rx = (x & s) > 0;
        int ry = (y & s) > 0;
        res += s * 111 * s * ((3 * rx) ^ ry);
        if (ry == 0) {
            if (rx == 1) x = s - 1 - x, y = s - 1 - y;
            swap(x, y);
        }
    }
    return res;
}
```

### 9.4 DynamicConvexTrick\*

```
// only works for integer coordinates!!
struct Line {
    mutable ll a, b, p;
    bool operator<(const Line &rhs) const {
        return a < rhs.a;
    }
    bool operator<(ll x) const { return p < x; }
};
struct DynamicHull : multiset<Line, less<>> {
    static const ll kInf = 1e18;
    ll Div(ll a, ll b) {
        return a / b - ((a ^ b) < 0 && a % b);
    }
    bool isect(iterator x, iterator y) {
        if (y == end()) {
            x->p = kInf;
            return 0;
        }
        if (x->a == y->a)
            x->p = x->b > y->b ? kInf : -kInf;
        else x->p = Div(y->b - x->b, x->a - y->a);
        return x->p >= y->p;
    }
    void addline(ll a, ll b) {
        auto z = insert({a, b, 0}), y = z++, x = y;
        while (isect(y, z)) z = erase(z);
        if (x != begin() && isect(--x, y))
            isect(x, y = erase(y));
        while ((y = x) != begin() && (--x)->p >= y->p)
            isect(x, erase(y));
    }
    ll query(ll x) {
        auto l = *lower_bound(x);
        return l.a * x + l.b;
    }
};
```

## 9.5 DLX\*

```
#define TRAV(i, link, start) \
    for (int i = link[start]; i != start; i = link[i])
template<bool A, bool B = !A> // A: Exact
struct DLX {
    int lt[NN], rg[NN], up[NN], dn[NN], cl[NN], rw[NN],
        bt[NN], s[NN], head, sz, ans;
    int columns;
    bool vis[NN];
    void remove(int c) {
        if (A) lt[rg[c]] = lt[c], rg[lt[c]] = rg[c];
        TRAV(i, dn, c) {
            if (A) {
                TRAV(j, rg, i)
                up[dn[j]] = up[j], dn[up[j]] = dn[j],
                --s[cl[j]];
            } else {
                lt[rg[i]] = lt[i], rg[lt[i]] = rg[i];
            }
        }
    }
    void restore(int c) {
        TRAV(i, up, c) {
            if (A) {
                TRAV(j, lt, i)
                ++s[cl[j]], up[dn[j]] = j, dn[up[j]] = j;
            } else {
                lt[rg[i]] = rg[lt[i]] = i;
            }
        }
        if (A) lt[rg[c]] = c, rg[lt[c]] = c;
    }
    void init(int c) {
        columns = c;
        for (int i = 0; i < c; ++i) {
            up[i] = dn[i] = bt[i] = i;
            lt[i] = i == 0 ? c : i - 1;
            rg[i] = i == c - 1 ? c : i + 1;
            s[i] = 0;
        }
        rg[c] = 0, lt[c] = c - 1;
        up[c] = dn[c] = -1;
        head = c, sz = c + 1;
    }
    void insert(int r, const vector<int> &col) {
        if (col.empty()) return;
        int f = sz;
        for (int i = 0; i < (int)col.size(); ++i) {
            int c = col[i], v = sz++;
            dn[bt[c]] = v;
            up[v] = bt[c], bt[c] = v;
            rg[v] = (i + 1 == (int)col.size()) ? f : v + 1;
            rw[v] = r, cl[v] = c;
            ++s[c];
            if (i > 0) lt[v] = v - 1;
        }
        lt[f] = sz - 1;
    }
    int h() {
        int ret = 0;
        memset(vis, 0, sizeof(bool) * sz);
        TRAV(x, rg, head) {
            if (vis[x]) continue;
            vis[x] = true, ++ret;
            TRAV(i, dn, x) TRAV(j, rg, i) vis[cl[j]] = true;
        }
        return ret;
    }
    void dfs(int dep) {
        if (dep + (A ? 0 : h()) >= ans) return;
        if (rg[head] == head) return ans = dep, void();
        if (dn[rg[head]] == rg[head]) return;
        int w = rg[head];
        TRAV(x, rg, head) if (s[x] < s[w]) w = x;
        if (A) remove(w);
        TRAV(i, dn, w) {
            if (B) remove(i);
            TRAV(j, rg, i) remove(A ? cl[j] : j);
            dfs(dep + 1);
            TRAV(j, lt, i) restore(A ? cl[j] : j);
            if (B) restore(i);
        }
    }
};
```

```
    }
    if (A) restore(w);
}
int solve() {
    for (int i = 0; i < columns; ++i)
        dn[bt[i]] = i, up[i] = bt[i];
    ans = 1e9, dfs(0);
    return ans;
}
};
```

## 9.6 Matroid Intersection

Start from  $S = \emptyset$ . In each iteration, let

- $Y_1 = \{x \notin S \mid S \cup \{x\} \in I_1\}$
- $Y_2 = \{x \notin S \mid S \cup \{x\} \in I_2\}$

If there exists  $x \in Y_1 \cap Y_2$ , insert  $x$  into  $S$ . Otherwise for each  $x \in S, y \notin S$ , create edges

- $x \rightarrow y$  if  $S - \{x\} \cup \{y\} \in I_1$ .
- $y \rightarrow x$  if  $S - \{x\} \cup \{y\} \in I_2$ .

Find a *shortest* path (with BFS) starting from a vertex in  $Y_1$  and ending at a vertex in  $Y_2$  which doesn't pass through any other vertices in  $Y_2$ , and alternate the path. The size of  $S$  will be incremented by 1 in each iteration. For the weighted case, assign weight  $w(x)$  to vertex  $x$  if  $x \in S$  and  $-w(x)$  if  $x \notin S$ . Find the path with the minimum number of edges among all minimum length paths and alternate it.

## 9.7 AdaptiveSimpson

```
using F_t = function<double(double)>;
pdd simpson(const F_t &f, double l, double r,
    double fl, double fr, double fm = nan("")) {
    if (isnan(fm)) fm = f((l + r) / 2);
    return {fm, (r - l) / 6 * (fl + 4 * fm + fr)};
}
double simpson_ada(const F_t &f, double l, double r,
    double fl, double fm, double fr, double eps) {
    double m = (l + r) / 2,
        s = simpson(f, l, r, fl, fr, fm).second;
    auto [flm, sl] = simpson(f, l, m, fl, fm);
    auto [fmr, sr] = simpson(f, m, r, fm, fr);
    double delta = sl + sr - s;
    if (abs(delta) <= 15 * eps)
        return sl + sr + delta / 15;
    return simpson_ada(f, l, m, fl, flm, fm, eps / 2) +
        simpson_ada(f, m, r, fm, fmr, fr, eps / 2);
}
double simpson_ada(const F_t &f, double l, double r) {
    return simpson_ada(
        f, l, r, f(l), f((l + r) / 2), f(r), 1e-9 / 7122);
}
double simpson_ada2(const F_t &f, double l, double r) {
    double h = (r - l) / 7122, s = 0;
    for (int i = 0; i < 7122; ++i, l += h)
        s += simpson_ada(f, l, l + h);
    return s;
}
```

## 10 Python

### 10.1 Misc

```
from decimal import *
setcontext(Context(prec=MAX_PREC, Emax=MAX_EMAX,
    rounding=ROUND_FLOOR))
print(Decimal(input()) * Decimal(input()))
from fractions import Fraction
Fraction('3.1415926535897932').limit_denominator(1000)
```