

# Contents

<b>1 Basic</b>	<b>1</b>	<b>8 Geometry</b>	<b>18</b>
1.1 Shell script	1	8.1 Default Code	18
1.2 Default code	1	8.2 Convex hull*	19
1.3 vimrc	1	8.3 External bisector	19
1.4 readchar	1	8.4 Heart	19
1.5 Black Magic	2	8.5 Minimum Enclosing Circle*	19
<b>2 Graph</b>	<b>2</b>	8.6 Polar Angle Sort*	19
2.1 BCC Vertex*	2	8.7 Intersection of two circles*	19
2.2 Bridge*	2	8.8 Intersection of polygon and circle	19
2.3 2SAT (SCC)*	2	8.9 Intersection of line and circle	20
2.4 MinimumMeanCycle*	3	8.10 point in circle	20
2.5 Virtual Tree*	3	8.11 Half plane intersection	20
2.6 Maximum Clique Dyn*	3	8.12 CircleCover*	20
2.7 Minimum Steiner Tree*	3	8.13 3Dpoint*	21
2.8 Dominator Tree*	4	8.14 Convexhull3D*	21
2.9 Minimum Arborescence*	4	8.15 Delaunay Triangulation*	22
2.10 Vizing's theorem	4	8.16 Triangulation Voronoi*	23
2.11 Minimum Clique Cover*	5	8.17 Tangent line of two circles	23
2.12 Number of Maximal Clique*	5	8.18 minMaxEnclosingRectangle	23
<b>3 Data Structure</b>	<b>5</b>	8.19 minDistOfTwoConvex	23
3.1 Leftist Tree	5	8.20 Minkowski Sum*	23
3.2 Heavy light Decomposition	6	8.21 RotatingSweepLine	24
3.3 Centroid Decomposition*	6	<b>9 Else</b>	<b>24</b>
3.4 Link cut tree*	6	9.1 Mo's Alogrithm(With modification)	24
3.5 KDTree	7	9.2 Mo's Alogrithm On Tree	24
<b>4 Flow/Matching</b>	<b>7</b>	9.3 DynamicConvexTrick*	24
4.1 Kuhn Munkres	7	9.4 DLX*	25
4.2 MincostMaxflow	8	9.5 Matroid Intersection	25
4.3 Maximum Simple Graph Matching*	8	9.6 AdaptiveSimpson	25
4.4 Minimum Weight Matching (Clique version)*	9	<b>1 Basic</b>	
4.5 SW-mincut	9	<b>1.1 Shell script</b>	
4.6 BoundedFlow(Dinic*)	9	g++ -O2 -std=c++17 -Dbbq -Wall -Wextra -Wshadow -o \$1	
4.7 Gomory Hu tree	10	\$1.cpp	
4.8 Flow Models	10	chmod +x compile.sh	
<b>5 String</b>	<b>10</b>	<b>1.2 Default code</b>	
5.1 KMP	10	#include <bits/stdc++.h>	
5.2 Z-value	10	using namespace std;	
5.3 Manacher*	11	typedef long long ll;	
5.4 Suffix Array	11	typedef pair<int, int> pii;	
5.5 SAIS*	11	typedef pair<ll, ll> pll;	
5.6 Aho-Corasick Automatan	11	#define X first	
5.7 Smallest Rotation	12	#define Y second	
5.8 De Bruijn sequence*	12	#define SZ(a) ((int)a.size())	
5.9 SAM	12	#define ALL(v) v.begin(), v.end()	
5.10 PalTree	12	#define pb push_back	
5.11 cyclic LCS	13	<b>1.3 vimrc</b>	
<b>6 Math</b>	<b>13</b>	"This file should be placed at ~/.vimrc"	
6.1 ax+by=gcd*	13	se nu ai hls et ru ic is sc cul	
6.2 floor and ceil	13	se re=1 ts=4 sts=4 sw=4 ls=2 mouse=a	
6.3 Miller Rabin*	13	syntax on	
6.4 Fraction	13	hi cursorline cterm=none ctermbg=89	
6.5 Simultaneous Equations	14	set bg=dark	
6.6 Pollard Rho	14	inoremap {<ENTER> {}<LEFT><ENTER><ENTER><UP><TAB>	
6.7 Simplex Algorithm	14	<b>1.4 readchar</b>	
6.7.1 Construction	14	inline char readchar() {	
6.8 Schreier-Sims Algorithm*	14	static const size_t bufsize = 65536;	
6.9 chineseRemainder	15	static char buf[bufsize];	
6.10 Quadratic Residue	15	static char *p = buf, *end = buf;	
6.11 PiCount	15	if (p == end) end = buf + fread_unlocked(buf, 1,	
6.12 Primes	16	bufsize, stdin), p = buf;	
6.13 Theorem	16	return *p++;	
6.13.1 Kirchhoff's Theorem	16	}	
6.13.2 Tutte's Matrix	16		
6.13.3 Cayley's Formula	16		
6.13.4 Erdős-Gallai theorem	16		
6.13.5 Gale-Ryser theorem	16		
6.13.6 Fulkerson-Chen-Anstee theorem	16		
6.14 Euclidean Algorithms	16		
6.15 General Purpose Numbers	16		
6.15.1 Bernoulli numbers	16		
6.15.2 Stirling numbers of the second kind	16		
<b>7 Polynomial</b>	<b>16</b>		
7.1 Fast Fourier Transform	16		
7.2 Number Theory Transform	17		
7.3 Fast Walsh Transform*	17		
7.4 Polynomial Operation	17		
7.5 Newton's Method	18		

## 1.5 Black Magic

```
#include <ext/pb_ds/priority_queue.hpp>
#include <ext/pb_ds/assoc_container.hpp> //rb_tree
using namespace __gnu_pbds;
typedef __gnu_pbds::priority_queue<int> heap;
int main() {
    heap h1, h2;
    h1.push(1), h1.push(3);
    h2.push(2), h2.push(4);
    h1.join(h2);
    cout << h1.size() << h2.size() << h1.top() << endl;
    //404
    tree<ll, null_type, less<ll>, rb_tree_tag,
        tree_order_statistics_node_update> st;
    tree<ll, ll, less<ll>, rb_tree_tag,
        tree_order_statistics_node_update> mp;
    for (int x : {0, 2, 3, 4}) st.insert(x);
    cout << *st.find_by_order(2) << st.order_of_key(1) <<
        endl; //31
}
//__int128_t, __float128_t
```

## 2 Graph

### 2.1 BCC Vertex\*

```
vector<int> G[N]; // 1-base
vector<int> nG[N], bcc[N];
int low[N], dfn[N], Time;
int bcc_id[N], bcc_cnt; // 1-base
bool is_cut[N]; // whether is av
bool cir[N];
int st[N], top;

void dfs(int u, int pa = -1) {
    int child = 0;
    low[u] = dfn[u] = ++Time;
    st[top++] = u;
    for (int v : G[u])
        if (!dfn[v]) {
            dfs(v, u), ++child;
            low[u] = min(low[u], low[v]);
            if (dfn[u] <= low[v]) {
                is_cut[u] = 1;
                bcc[++bcc_cnt].clear();
                int t;
                do {
                    bcc_id[t = st[--top]] = bcc_cnt;
                    bcc[bcc_cnt].push_back(t);
                } while (t != v);
                bcc_id[u] = bcc_cnt;
                bcc[bcc_cnt].pb(u);
            }
        } else if (dfn[v] < dfn[u] && v != pa)
            low[u] = min(low[u], dfn[v]);
    if (pa == -1 && child < 2) is_cut[u] = 0;
}

void bcc_init(int n) {
    Time = bcc_cnt = top = 0;
    for (int i = 1; i <= n; ++i)
        G[i].clear(), dfn[i] = bcc_id[i] = is_cut[i] = 0;
}

void bcc_solve(int n) {
    for (int i = 1; i <= n; ++i)
        if (!dfn[i]) dfs(i);
    // circle-square tree
    for (int i = 1; i <= n; ++i)
        if (is_cut[i])
            bcc_id[i] = ++bcc_cnt, cir[bcc_cnt] = 1;
    for (int i = 1; i <= bcc_cnt && !cir[i]; ++i)
        for (int j : bcc[i])
            if (is_cut[j])
                nG[i].pb(bcc_id[j]), nG[bcc_id[j]].pb(i);
}
```

### 2.2 Bridge\*

```
int low[N], dfn[N], Time; // 1-base
vector<pii> G[N], edge;
vector<bool> is_bridge;

void init(int n) {
    Time = 0;
    for (int i = 1; i <= n; ++i)
        G[i].clear(), low[i] = dfn[i] = 0;
}

void add_edge(int a, int b) {
    G[a].pb(pii(b, SZ(edge))), G[b].pb(pii(a, SZ(edge)));
    edge.pb(pii(a, b));
}

void dfs(int u, int f) {
    dfn[u] = low[u] = ++Time;
    for (auto i : G[u])
        if (!dfn[i.X])
            dfs(i.X, i.Y), low[u] = min(low[u], low[i.X]);
        else if (i.Y != f) low[u] = min(low[u], dfn[i.X]);
    if (low[u] == dfn[u] && f != -1) is_bridge[f] = 1;
}

void solve(int n) {
    is_bridge.resize(SZ(edge));
    for (int i = 1; i <= n; ++i)
        if (!dfn[i]) dfs(i, -1);
}
```

### 2.3 2SAT (SCC)\*

```
struct SAT { // 0-base
    int low[N], dfn[N], bln[N], n, Time, nScc;
    bool instack[N], istrue[N];
    stack<int> st;
    vector<int> G[N], SCC[N];
    void init(int _n) {
        n = _n; // assert(n * 2 <= N);
        for (int i = 0; i < n + n; ++i) G[i].clear();
    }
    void add_edge(int a, int b) { G[a].pb(b); }
    int rv(int a) {
        if (a > n) return a - n;
        return a + n;
    }
    void add_clause(int a, int b) {
        add_edge(rv(a), b), add_edge(rv(b), a);
    }
    void dfs(int u) {
        dfn[u] = low[u] = ++Time;
        instack[u] = 1, st.push(u);
        for (int i : G[u])
            if (!dfn[i])
                dfs(i), low[u] = min(low[i], low[u]);
            else if (instack[i] && dfn[i] < dfn[u])
                low[u] = min(low[u], dfn[i]);
        if (low[u] == dfn[u]) {
            int tmp;
            do {
                tmp = st.top(), st.pop();
                instack[tmp] = 0, bln[tmp] = nScc;
            } while (tmp != u);
            ++nScc;
        }
    }
    bool solve() {
        Time = nScc = 0;
        for (int i = 0; i < n + n; ++i)
            SCC[i].clear(), low[i] = dfn[i] = bln[i] = 0;
        for (int i = 0; i < n + n; ++i)
            if (!dfn[i]) dfs(i);
        for (int i = 0; i < n + n; ++i) SCC[bln[i]].pb(i);
        for (int i = 0; i < n; ++i) {
            if (bln[i] == bln[i + n]) return false;
            istrue[i] = bln[i] < bln[i + n];
            istrue[i + n] = !istrue[i];
        }
    }
};
```

```

    }
    return true;
}
};

```

## 2.4 MinimumMeanCycle\*

```

ll road[N][N]; // input here
struct MinimumMeanCycle {
    ll dp[N + 5][N], n;
    pll solve() {
        ll a = -1, b = -1, L = n + 1;
        for (int i = 2; i < n; ++i)
            for (int k = 0; k < n; ++k)
                for (int j = 0; j < n; ++j)
                    dp[i][j] =
                        min(dp[i - 1][k] + road[k][j], dp[i][j]);
        for (int i = 0; i < n; ++i) {
            if (dp[L][i] >= INF) continue;
            ll ta = 0, tb = 1;
            for (int j = 1; j < n; ++j)
                if (dp[j][i] < INF &&
                    ta * (L - j) < (dp[L][i] - dp[j][i]) * tb)
                    ta = dp[L][i] - dp[j][i], tb = L - j;
            if (ta == 0) continue;
            if (a == -1 || a * tb > ta * b) a = ta, b = tb;
        }
        if (a != -1) {
            ll g = __gcd(a, b);
            return pll(a / g, b / g);
        }
        return pll(-1LL, -1LL);
    }
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i)
            for (int j = 0; j < n; ++j) dp[i + 2][j] = INF;
    }
};

```

## 2.5 Virtual Tree\*

```

vector<int> vG[N];
int top, st[N];

void insert(int u) {
    if (top == -1) return st[++top] = u, void();
    int p = LCA(st[top], u);
    if (p == st[top]) return st[++top] = u, void();
    while (top >= 1 && dep[st[top - 1]] >= dep[p])
        vG[st[top - 1]].pb(st[top]), --top;
    if (st[top] != p)
        vG[p].pb(st[top]), --top, st[++top] = p;
    st[++top] = u;
}

void reset(int u) {
    for (int i : vG[u]) reset(i);
    vG[u].clear();
}

void solve(vector<int> &v) {
    top = -1;
    sort(ALL(v),
        [&](int a, int b) { return dfn[a] < dfn[b]; });
    for (int i : v) insert(i);
    while (top > 0) vG[st[top - 1]].pb(st[top]), --top;
    // do something
    reset(v[0]);
}

```

## 2.6 Maximum Clique Dyn\*

```

const int N = 150;
struct MaxClique { // Maximum Clique
    bitset<N> a[N], cs[N];

```

```

int ans, sol[N], q, cur[N], d[N], n;
void init(int _n) {
    n = _n;
    for (int i = 0; i < n; i++) a[i].reset();
}
void addEdge(int u, int v) { a[u][v] = a[v][u] = 1; }
void csort(vector<int> &r, vector<int> &c) {
    int mx = 1, km = max(ans - q + 1, 1), t = 0,
        m = r.size();
    cs[1].reset(), cs[2].reset();
    for (int i = 0; i < m; i++) {
        int p = r[i], k = 1;
        while ((cs[k] & a[p]).count()) k++;
        if (k > mx) mx++, cs[mx + 1].reset();
        cs[k][p] = 1;
        if (k < km) r[t++] = p;
    }
    c.resize(m);
    if (t) c[t - 1] = 0;
    for (int k = km; k <= mx; k++)
        for (int p = cs[k]._Find_first(); p < N;
            p = cs[k]._Find_next(p))
            r[t] = p, c[t] = k, t++;
}
void dfs(vector<int> &r, vector<int> &c, int l,
    bitset<N> mask) {
    while (!r.empty()) {
        int p = r.back();
        r.pop_back(), mask[p] = 0;
        if (q + c.back() <= ans) return;
        cur[q++] = p;
        vector<int> nr, nc;
        bitset<N> nmask = mask & a[p];
        for (int i : r)
            if (a[p][i]) nr.push_back(i);
        if (!nr.empty()) {
            if (l < 4) {
                for (int i : nr)
                    d[i] = (a[i] & nmask).count();
                sort(nr.begin(), nr.end(),
                    [&](int x, int y) { return d[x] > d[y]; });
            }
            csort(nr, nc), dfs(nr, nc, l + 1, nmask);
        } else if (q > ans) ans = q, copy_n(cur, q, sol);
        c.pop_back(), q--;
    }
}
int solve(bitset<N> mask = bitset<N>(),
    string(N, '1')) { // vertex mask
    vector<int> r, c;
    ans = q = 0;
    for (int i = 0; i < n; i++)
        if (mask[i]) r.push_back(i);
    for (int i = 0; i < n; i++)
        d[i] = (a[i] & mask).count();
    sort(r.begin(), r.end(),
        [&](int i, int j) { return d[i] > d[j]; });
    csort(r, c), dfs(r, c, 1, mask);
    return ans; // sol[0 ~ ans-1]
}
} graph;

```

## 2.7 Minimum Steiner Tree\*

```

// Minimum Steiner Tree
// O(V 3^T + V^2 2^T)
struct SteinerTree { // 0-base
    static const int T = 10, N = 105, INF = 1e9;
    int n, dst[N][N], dp[1 << T][N], tdst[N];
    int vcost[N]; // the cost of vertices
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i) {
            for (int j = 0; j < n; ++j) dst[i][j] = INF;
            dst[i][i] = vcost[i] = 0;
        }
    }
    void add_edge(int ui, int vi, int wi) {
        dst[ui][vi] = min(dst[ui][vi], wi);
    }
}

```

```

void shortest_path() {
    for (int k = 0; k < n; ++k)
        for (int i = 0; i < n; ++i)
            for (int j = 0; j < n; ++j)
                dst[i][j] =
                    min(dst[i][j], dst[i][k] + dst[k][j]);
}
int solve(const vector<int> &ter) {
    shortest_path();
    int t = SZ(ter);
    for (int i = 0; i < (1 << t); ++i)
        for (int j = 0; j < n; ++j) dp[i][j] = INF;
    for (int i = 0; i < n; ++i) dp[0][i] = vcost[i];
    for (int msk = 1; msk < (1 << t); ++msk) {
        if (!(msk & (msk - 1))) {
            int who = __lg(msk);
            for (int i = 0; i < n; ++i)
                dp[msk][i] =
                    vcost[ter[who]] + dst[ter[who]][i];
        }
        for (int i = 0; i < n; ++i)
            for (int submsk = (msk - 1) & msk; submsk;
                submsk = (submsk - 1) & msk)
                dp[msk][i] = min(dp[msk][i],
                    dp[submsk][i] + dp[msk ^ submsk][i] -
                    vcost[i]);
        for (int i = 0; i < n; ++i) {
            tdst[i] = INF;
            for (int j = 0; j < n; ++j)
                tdst[i] =
                    min(tdst[i], dp[msk][j] + dst[j][i]);
        }
        for (int i = 0; i < n; ++i) dp[msk][i] = tdst[i];
    }
    int ans = INF;
    for (int i = 0; i < n; ++i)
        ans = min(ans, dp[(1 << t) - 1][i]);
    return ans;
}
};

```

## 2.8 Dominator Tree\*

```

struct dominator_tree { // 1-base
    vector<int> G[N], rG[N];
    int n, pa[N], dfn[N], id[N], Time;
    int semi[N], idom[N], best[N];
    vector<int> tree[N]; // dominator_tree
    void init(int _n) {
        n = _n;
        for (int i = 1; i <= n; ++i)
            G[i].clear(), rG[i].clear();
    }
    void add_edge(int u, int v) {
        G[u].pb(v), rG[v].pb(u);
    }
    void dfs(int u) {
        id[dfn[u] = ++Time] = u;
        for (auto v : G[u])
            if (!dfn[v]) dfs(v), pa[dfn[v]] = dfn[u];
    }
    int find(int y, int x) {
        if (y <= x) return y;
        int tmp = find(pa[y], x);
        if (semi[best[y]] > semi[best[pa[y]]])
            best[y] = best[pa[y]];
        return pa[y] = tmp;
    }
    void tarjan(int root) {
        Time = 0;
        for (int i = 1; i <= n; ++i) {
            dfn[i] = idom[i] = 0;
            tree[i].clear();
            best[i] = semi[i] = i;
        }
        dfs(root);
        for (int i = Time; i > 1; --i) {
            int u = id[i];
            for (auto v : rG[u])
                if (v == dfn[v]) {

```

```

                    find(v, i);
                    semi[i] = min(semi[i], semi[best[v]]);
                }
            tree[semi[i]].pb(i);
            for (auto v : tree[pa[i]]) {
                find(v, pa[i]);
                idom[v] =
                    semi[best[v]] == pa[i] ? pa[i] : best[v];
            }
            tree[pa[i]].clear();
        }
        for (int i = 2; i <= Time; ++i) {
            if (idom[i] != semi[i]) idom[i] = idom[idom[i]];
            tree[id[idom[i]]].pb(id[i]);
        }
    }
};

```

## 2.9 Minimum Arborescence\*

```

struct zhu_liu { // 0(VE)
    struct edge {
        int u, v;
        ll w;
    };
    vector<edge> E; // 0-base
    int pe[N], id[N], vis[N];
    ll in[N];
    void init() { E.clear(); }
    void add_edge(int u, int v, ll w) {
        if (u != v) E.pb(edge{u, v, w});
    }
    ll build(int root, int n) {
        ll ans = 0;
        for (;;) {
            fill_n(in, n, INF);
            for (int i = 0; i < SZ(E); ++i)
                if (E[i].u != E[i].v && E[i].w < in[E[i].v])
                    pe[E[i].v] = i, in[E[i].v] = E[i].w;
            for (int u = 0; u < n; ++u) // no solution
                if (u != root && in[u] == INF) return -INF;
            int cntnode = 0;
            fill_n(id, n, -1), fill_n(vis, n, -1);
            for (int u = 0; u < n; ++u) {
                if (u != root) ans += in[u];
                int v = u;
                while (vis[v] != u && !~id[v] && v != root)
                    vis[v] = u, v = E[pe[v]].u;
                if (v != root && !~id[v]) {
                    for (int x = E[pe[v]].u; x != v;
                        x = E[pe[x]].u)
                        id[x] = cntnode;
                    id[v] = cntnode++;
                }
            }
            if (!cntnode) break; // no cycle
            for (int u = 0; u < n; ++u)
                if (!~id[u]) id[u] = cntnode++;
            for (int i = 0; i < SZ(E); ++i) {
                int v = E[i].v;
                E[i].u = id[E[i].u], E[i].v = id[E[i].v];
                if (E[i].u != E[i].v) E[i].w -= in[v];
            }
            n = cntnode, root = id[root];
        }
        return ans;
    }
};

```

## 2.10 Vizing's theorem

```

namespace vizing { // returns edge coloring in adjacent
// matrix G. 1 - based
int C[kN][kN], G[kN][kN];
void clear(int N) {
    for (int i = 0; i <= N; i++) {
        for (int j = 0; j <= N; j++) C[i][j] = G[i][j] = 0;
    }
}

```

```

}
void solve(vector<pair<int, int>> &E, int N, int M) {
    int X[kN] = {}, a;
    auto update = [&](int u) {
        for (X[u] = 1; C[u][X[u]]; X[u]++)
            ;
    };
    auto color = [&](int u, int v, int c) {
        int p = G[u][v];
        G[u][v] = G[v][u] = c;
        C[u][c] = v, C[v][c] = u;
        C[u][p] = C[v][p] = 0;
        if (p) X[u] = X[v] = p;
        else update(u), update(v);
        return p;
    };
    auto flip = [&](int u, int c1, int c2) {
        int p = C[u][c1];
        swap(C[u][c1], C[u][c2]);
        if (p) G[u][p] = G[p][u] = c2;
        if (!C[u][c1]) X[u] = c1;
        if (!C[u][c2]) X[u] = c2;
        return p;
    };
    for (int i = 1; i <= N; i++) X[i] = 1;
    for (int t = 0; t < E.size(); t++) {
        int u = E[t].first, v0 = E[t].second, v = v0,
            c0 = X[u], c = c0, d;
        vector<pair<int, int>> L;
        int vst[kN] = {};
        while (!G[u][v0]) {
            L.emplace_back(v, d = X[v]);
            if (!C[v][c])
                for (a = (int)L.size() - 1; a >= 0; a--)
                    c = color(u, L[a].first, c);
            else if (!C[u][d])
                for (a = (int)L.size() - 1; a >= 0; a--)
                    color(u, L[a].first, L[a].second);
            else if (vst[d]) break;
            else vst[d] = 1, v = C[u][d];
        }
        if (!G[u][v0]) {
            for (; v; v = flip(v, c, d), swap(c, d))
                ;
            if (C[u][c0]) {
                for (a = (int)L.size() - 2;
                    a >= 0 && L[a].second != c; a--)
                    ;
                for (; a >= 0; a--)
                    color(u, L[a].first, L[a].second);
            } else t--;
        }
    }
}
} // namespace vizing

```

## 2.11 Minimum Clique Cover\*

```

struct Clique_Cover { // 0-base, O(n2^n)
    int co[1 << N], n, E[N];
    int dp[1 << N];
    void init(int _n) {
        n = _n, fill_n(dp, 1 << n, 0);
        fill_n(E, n, 0), fill_n(co, 1 << n, 0);
    }
    void add_edge(int u, int v) {
        E[u] |= 1 << v, E[v] |= 1 << u;
    }
    int solve() {
        for (int i = 0; i < n; ++i)
            co[1 << i] = E[i] | (1 << i);
        co[0] = (1 << n) - 1;
        dp[0] = (n & 1) * 2 - 1;
        for (int i = 1; i < (1 << n); ++i) {
            int t = i & -i;
            dp[i] = -dp[i ^ t];
            co[i] = co[i ^ t] & co[t];
        }
        for (int i = 0; i < (1 << n); ++i)
            co[i] = (co[i] & i) == i;
    }
};

```

```

fwt(co, 1 << n);
for (int ans = 1; ans < n; ++ans) {
    int sum = 0;
    for (int i = 0; i < (1 << n); ++i)
        sum += (dp[i] * co[i]);
    if (sum) return ans;
}
return n;
}
};

```

## 2.12 NumberofMaximalClique\*

```

struct BronKerbosch { // 1-base
    int n, a[N], g[N][N];
    int S, all[N][N], some[N][N], none[N][N];
    void init(int _n) {
        n = _n;
        for (int i = 1; i <= n; ++i)
            for (int j = 1; j <= n; ++j) g[i][j] = 0;
    }
    void add_edge(int u, int v) {
        g[u][v] = g[v][u] = 1;
    }
    void dfs(int d, int an, int sn, int nn) {
        if (S > 1000) return; // pruning
        if (sn == 0 && nn == 0) ++S;
        int u = some[d][0];
        for (int i = 0; i < sn; ++i) {
            int v = some[d][i];
            if (g[u][v]) continue;
            int tsu = 0, tnn = 0;
            copy_n(all[d], an, all[d + 1]);
            all[d + 1][an] = v;
            for (int j = 0; j < sn; ++j)
                if (g[v][some[d][j]])
                    some[d + 1][tsu++] = some[d][j];
            for (int j = 0; j < nn; ++j)
                if (g[v][none[d][j]])
                    none[d + 1][tnn++] = none[d][j];
            dfs(d + 1, an + 1, tsu, tnn);
            some[d][i] = 0, none[d][nn++] = v;
        }
    }
    int solve() {
        iota(some[0], some[0] + n, 1);
        S = 0, dfs(0, 0, n, 0);
        return S;
    }
};

```

## 3 Data Structure

### 3.1 Leftist Tree

```

struct node {
    ll v, data, sz, sum;
    node *l, *r;
    node(ll k)
        : v(0), data(k), sz(1), l(0), r(0), sum(k) {}
};
ll sz(node *p) { return p ? p->sz : 0; }
ll V(node *p) { return p ? p->v : -1; }
ll sum(node *p) { return p ? p->sum : 0; }
node *merge(node *a, node *b) {
    if (!a || !b) return a ? a : b;
    if (a->data < b->data) swap(a, b);
    a->r = merge(a->r, b);
    if (V(a->r) > V(a->l)) swap(a->r, a->l);
    a->v = V(a->r) + 1, a->sz = sz(a->l) + sz(a->r) + 1;
    a->sum = sum(a->l) + sum(a->r) + a->data;
    return a;
}
void pop(node *&o) {
    node *tmp = o;
    o = merge(o->l, o->r);
}

```

```
delete tmp;
}
```

### 3.2 Heavy light Decomposition

```
struct Heavy_light_Decomposition { // 1-base
    int n, ulink[10005], deep[10005], mxson[10005],
        w[10005], pa[10005];
    int t, pl[10005], data[10005], dt[10005], bln[10005],
        edge[10005], et;
    vector<pii> G[10005];
    void init(int _n) {
        n = _n, t = 0, et = 1;
        for (int i = 1; i <= n; ++i)
            G[i].clear(), mxson[i] = 0;
    }
    void add_edge(int a, int b, int w) {
        G[a].pb(pii(b, et)), G[b].pb(pii(a, et)),
            edge[et++] = w;
    }
    void dfs(int u, int f, int d) {
        w[u] = 1, pa[u] = f, deep[u] = d++;
        for (auto &i : G[u])
            if (i.X != f) {
                dfs(i.X, u, d), w[u] += w[i.X];
                if (w[mxson[u]] < w[i.X]) mxson[u] = i.X;
                else bln[i.Y] = u, dt[u] = edge[i.Y];
            }
    }
    void cut(int u, int link) {
        data[pl[u] = t++] = dt[u], ulink[u] = link;
        if (!mxson[u]) return;
        cut(mxson[u], link);
        for (auto i : G[u])
            if (i.X != pa[u] && i.X != mxson[u])
                cut(i.X, i.X);
    }
    void build() { dfs(1, 1, 1), cut(1, 1), /*build*/; }
    int query(int a, int b) {
        int ta = ulink[a], tb = ulink[b], re = 0;
        while (ta != tb)
            if (deep[ta] < deep[tb])
                /*query*/, tb = ulink[b = pa[tb]];
            else /*query*/, ta = ulink[a = pa[ta]];
        if (a == b) return re;
        if (pl[a] > pl[b]) swap(a, b);
        /*query*/
        return re;
    }
};
```

### 3.3 Centroid Decomposition\*

```
struct Cent_Dec { // 1-base
    vector<pii> G[N];
    pii info[N]; // store info. of itself
    pii upinfo[N]; // store info. of climbing up
    int n, pa[N], layer[N], sz[N], done[N];
    ll dis[___lg(N) + 1][N];
    void init(int _n) {
        n = _n, layer[0] = -1;
        fill_n(pa + 1, n, 0), fill_n(done + 1, n, 0);
        for (int i = 1; i <= n; ++i) G[i].clear();
    }
    void add_edge(int a, int b, int w) {
        G[a].pb(pii(b, w)), G[b].pb(pii(a, w));
    }
    void get_cent(
        int u, int f, int &mx, int &c, int num) {
        int mxsz = 0;
        sz[u] = 1;
        for (pii e : G[u])
            if (!done[e.X] && e.X != f) {
                get_cent(e.X, u, mx, c, num);
                sz[u] += sz[e.X], mxsz = max(mxsz, sz[e.X]);
            }
        if (mx > max(mxsz, num - sz[u]))
            mx = max(mxsz, num - sz[u]), c = u;
    }
};
```

```
void dfs(int u, int f, ll d, int org) {
    // if required, add self info or climbing info
    dis[layer[org]][u] = d;
    for (pii e : G[u])
        if (!done[e.X] && e.X != f)
            dfs(e.X, u, d + e.Y, org);
}
int cut(int u, int f, int num) {
    int mx = 1e9, c = 0, lc;
    get_cent(u, f, mx, c, num);
    done[c] = 1, pa[c] = f, layer[c] = layer[f] + 1;
    for (pii e : G[c])
        if (!done[e.X]) {
            if (sz[e.X] > sz[c])
                lc = cut(e.X, c, num - sz[c]);
            else lc = cut(e.X, c, sz[e.X]);
            upinfo[lc] = pii(), dfs(e.X, c, e.Y, c);
        }
    return done[c] = 0, c;
}
void build() { cut(1, 0, n); }
void modify(int u) {
    for (int a = u, ly = layer[a]; a;
        a = pa[a], --ly) {
        info[a].X += dis[ly][u], ++info[a].Y;
        if (pa[a])
            upinfo[a].X += dis[ly - 1][u], ++upinfo[a].Y;
    }
}
ll query(int u) {
    ll rt = 0;
    for (int a = u, ly = layer[a]; a;
        a = pa[a], --ly) {
        rt += info[a].X + info[a].Y * dis[ly][u];
        if (pa[a])
            rt -= upinfo[a].X + upinfo[a].Y * dis[ly - 1][u];
    }
    return rt;
}
};
```

### 3.4 Link cut tree\*

```
struct Splay { // xor-sum
    static Splay nil;
    Splay *ch[2], *f;
    int val, sum, rev, size;
    Splay(int _val = 0)
        : val(_val), sum(_val), rev(0), size(1) {
        f = ch[0] = ch[1] = &nil;
    }
    bool isr() {
        return f->ch[0] != this && f->ch[1] != this;
    }
    int dir() { return f->ch[0] == this ? 0 : 1; }
    void setCh(Splay *c, int d) {
        ch[d] = c;
        if (c != &nil) c->f = this;
        pull();
    }
    void push() {
        if (!rev) return;
        swap(ch[0], ch[1]);
        if (ch[0] != &nil) ch[0]->rev ^= 1;
        if (ch[1] != &nil) ch[1]->rev ^= 1;
        rev = 0;
    }
    void pull() {
        // take care of the nil!
        size = ch[0]->size + ch[1]->size + 1;
        sum = ch[0]->sum ^ ch[1]->sum ^ val;
        if (ch[0] != &nil) ch[0]->f = this;
        if (ch[1] != &nil) ch[1]->f = this;
    }
} Splay::nil;
Splay *nil = &Splay::nil;
void rotate(Splay *x) {
    Splay *p = x->f;
    int d = x->dir();
```



```

if (!p->isr()) p->f->setCh(x, p->dir());
else x->f = p->f;
p->setCh(x->ch[!d], d);
x->setCh(p, !d);
p->pull(), x->pull();
}
void splay(Splay *x) {
    vector<Splay *> splayVec;
    for (Splay *q = x;; q = q->f) {
        splayVec.pb(q);
        if (q->isr()) break;
    }
    reverse(ALL(splayVec));
    for (auto it : splayVec) it->push();
    while (!x->isr()) {
        if (x->f->isr()) rotate(x);
        else if (x->dir() == x->f->dir())
            rotate(x->f), rotate(x);
        else rotate(x), rotate(x);
    }
}
Splay *access(Splay *x) {
    Splay *q = nil;
    for (; x != nil; x = x->f)
        splay(x), x->setCh(q, 1), q = x;
    return q;
}
void root_path(Splay *x) { access(x), splay(x); }
void chroot(Splay *x) {
    root_path(x), x->rev ^= 1;
    x->push(), x->pull();
}
void split(Splay *x, Splay *y) {
    chroot(x), root_path(y);
}
void link(Splay *x, Splay *y) {
    root_path(x), chroot(y);
    x->setCh(y, 1);
}
void cut(Splay *x, Splay *y) {
    split(x, y);
    if (y->size != 5) return;
    y->push();
    y->ch[0] = y->ch[0]->f = nil;
}
Splay *get_root(Splay *x) {
    for (root_path(x); x->ch[0] != nil; x = x->ch[0])
        x->push();
    splay(x);
    return x;
}
bool conn(Splay *x, Splay *y) {
    return get_root(x) == get_root(y);
}
Splay *lca(Splay *x, Splay *y) {
    access(x), root_path(y);
    if (y->f == nil) return y;
    return y->f;
}
void change(Splay *x, int val) {
    splay(x), x->val = val, x->pull();
}
int query(Splay *x, Splay *y) {
    split(x, y);
    return y->sum;
}
}

```

### 3.5 KDTree

```

namespace kdt {
int root, lc[maxn], rc[maxn], xl[maxn], xr[maxn],
    yl[maxn], yr[maxn];
point p[maxn];
int build(int l, int r, int dep = 0) {
    if (l == r) return -1;
    function<bool(const point &, const point &)> f =
        [dep](const point &a, const point &b) {
            if (dep & 1) return a.x < b.x;
            else return a.y < b.y;
        };
}

```

```

int m = (l + r) >> 1;
nth_element(p + l, p + m, p + r, f);
xl[m] = xr[m] = p[m].x;
yl[m] = yr[m] = p[m].y;
lc[m] = build(l, m, dep + 1);
if (~lc[m]) {
    xl[m] = min(xl[m], xl[lc[m]]);
    xr[m] = max(xr[m], xr[lc[m]]);
    yl[m] = min(yl[m], yl[lc[m]]);
    yr[m] = max(yr[m], yr[lc[m]]);
}
rc[m] = build(m + 1, r, dep + 1);
if (~rc[m]) {
    xl[m] = min(xl[m], xl[rc[m]]);
    xr[m] = max(xr[m], xr[rc[m]]);
    yl[m] = min(yl[m], yl[rc[m]]);
    yr[m] = max(yr[m], yr[rc[m]]);
}
return m;
}
bool bound(const point &q, int o, long long d) {
    double ds = sqrt(d + 1.0);
    if (q.x < xl[o] - ds || q.x > xr[o] + ds ||
        q.y < yl[o] - ds || q.y > yr[o] + ds)
        return false;
    return true;
}
long long dist(const point &a, const point &b) {
    return (a.x - b.x) * 1ll * (a.x - b.x) +
        (a.y - b.y) * 1ll * (a.y - b.y);
}
void dfs(
    const point &q, long long &d, int o, int dep = 0) {
    if (!bound(q, o, d)) return;
    long long cd = dist(p[o], q);
    if (cd != 0) d = min(d, cd);
    if ((dep & 1) && q.x < p[o].x ||
        !(dep & 1) && q.y < p[o].y) {
        if (~lc[o]) dfs(q, d, lc[o], dep + 1);
        if (~rc[o]) dfs(q, d, rc[o], dep + 1);
    } else {
        if (~rc[o]) dfs(q, d, rc[o], dep + 1);
        if (~lc[o]) dfs(q, d, lc[o], dep + 1);
    }
}
void init(const vector<point> &v) {
    for (int i = 0; i < v.size(); ++i) p[i] = v[i];
    root = build(0, v.size());
}
long long nearest(const point &q) {
    long long res = 1e18;
    dfs(q, res, root);
    return res;
}
} // namespace kdt

```

## 4 Flow/Matching

### 4.1 Kuhn Munkres

```

struct KM { // 0-base
    int w[MAXN][MAXN], hl[MAXN], hr[MAXN], slk[MAXN], n;
    int fl[MAXN], fr[MAXN], pre[MAXN], qu[MAXN], ql, qr;
    bool vl[MAXN], vr[MAXN];
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i)
            for (int j = 0; j < n; ++j) w[i][j] = -INF;
    }
    void add_edge(int a, int b, int wei) {
        w[a][b] = wei;
    }
    bool Check(int x) {
        if (vl[x] = 1, ~fl[x])
            return vr[qu[qr++]] = fl[x] = 1;
        while (~x) swap(x, fr[fl[x] = pre[x]]);
        return 0;
    }
}

```

```

void Bfs(int s) {
    fill(slk, slk + n, INF);
    fill(vl, vl + n, 0), fill(vr, vr + n, 0);
    ql = qr = 0, qu[qr++] = s, vr[s] = 1;
    while (1) {
        int d;
        while (ql < qr)
            for (int x = 0; y = qu[ql++]; x < n; ++x)
                if (!vl[x] &&
                    slk[x] >= (d = hl[x] + hr[y] - w[x][y]))
                    if (pre[x] = y, d) slk[x] = d;
                    else if (!Check(x)) return;
        d = INF;
        for (int x = 0; x < n; ++x)
            if (!vl[x] && d > slk[x]) d = slk[x];
        for (int x = 0; x < n; ++x) {
            if (vl[x]) hl[x] += d;
            else slk[x] -= d;
            if (vr[x]) hr[x] -= d;
        }
        for (int x = 0; x < n; ++x)
            if (!vl[x] && !slk[x] && !Check(x)) return;
    }
}

int Solve() {
    fill(fl, fl + n, -1), fill(fr, fr + n, -1),
    fill(hr, hr + n, 0);
    for (int i = 0; i < n; ++i)
        hl[i] = *max_element(w[i], w[i] + n);
    for (int i = 0; i < n; ++i) Bfs(i);
    int res = 0;
    for (int i = 0; i < n; ++i) res += w[i][fl[i]];
    return res;
}
};

```

## 4.2 MincostMaxflow

```

struct MCMF { // 0-base
    struct edge {
        ll from, to, cap, flow, cost, rev;
    } * past[MAXN];
    vector<edge> G[MAXN];
    bitset<MAXN> inq;
    ll dis[MAXN], up[MAXN], s, t, mx, n;
    bool BellmanFord(ll &flow, ll &cost) {
        fill(dis, dis + n, INF);
        queue<ll> q;
        q.push(s), inq.reset(), inq[s] = 1;
        up[s] = mx - flow, past[s] = 0, dis[s] = 0;
        while (!q.empty()) {
            ll u = q.front();
            q.pop(), inq[u] = 0;
            if (!up[u]) continue;
            for (auto &e : G[u])
                if (e.flow != e.cap &&
                    dis[e.to] > dis[u] + e.cost) {
                    dis[e.to] = dis[u] + e.cost, past[e.to] = &e;
                    up[e.to] = min(up[u], e.cap - e.flow);
                    if (!inq[e.to]) inq[e.to] = 1, q.push(e.to);
                }
        }
        if (dis[t] == INF) return 0;
        flow += up[t], cost += up[t] * dis[t];
        for (ll i = t; past[i]; i = past[i]->from) {
            auto &e = *past[i];
            e.flow += up[t], G[e.to][e.rev].flow -= up[t];
        }
        return 1;
    }
    ll MinCostMaxFlow(ll _s, ll _t, ll &cost) {
        s = _s, t = _t, cost = 0;
        ll flow = 0;
        while (BellmanFord(flow, cost))
            ;
        return flow;
    }
    void init(ll _n, ll _mx) {
        n = _n, mx = _mx;
        for (int i = 0; i < n; ++i) G[i].clear();
    }
};

```

```

}
void add_edge(ll a, ll b, ll cap, ll cost) {
    G[a].pb(edge{a, b, cap, 0, cost, G[b].size()});
    G[b].pb(edge{b, a, 0, 0, -cost, G[a].size() - 1});
}
};

```

## 4.3 Maximum Simple Graph Matching\*

```

struct GenMatch { // 1-base
    int V, pr[N];
    bool el[N][N], inq[N], inp[N], inb[N];
    int st, ed, nb, bk[N], djs[N], ans;
    void init(int _V) {
        V = _V;
        for (int i = 0; i <= V; ++i) {
            for (int j = 0; j <= V; ++j) el[i][j] = 0;
            pr[i] = bk[i] = djs[i] = 0;
            inq[i] = inp[i] = inb[i] = 0;
        }
    }
    void add_edge(int u, int v) {
        el[u][v] = el[v][u] = 1;
    }
    int lca(int u, int v) {
        fill_n(inp, V + 1, 0);
        while (1)
            if (u = djs[u], inp[u] = true, u == st) break;
            else u = bk[pr[u]];
        while (1)
            if (v = djs[v], inp[v]) return v;
            else v = bk[pr[v]];
        return v;
    }
    void upd(int u) {
        for (int v; djs[u] != nb;) {
            v = pr[u], inb[djs[u]] = inb[djs[v]] = true;
            u = bk[v];
            if (djs[u] != nb) bk[u] = v;
        }
    }
    void blo(int u, int v, queue<int> &qe) {
        nb = lca(u, v), fill_n(inb, V + 1, 0);
        upd(u), upd(v);
        if (djs[u] != nb) bk[u] = v;
        if (djs[v] != nb) bk[v] = u;
        for (int tu = 1; tu <= V; ++tu)
            if (inb[djs[tu]])
                if (djs[tu] = nb, !inq[tu])
                    qe.push(tu), inq[tu] = 1;
    }
    void flow() {
        fill_n(inq + 1, V, 0), fill_n(bk + 1, V, 0);
        iota(djs + 1, djs + V + 1, 1);
        queue<int> qe;
        qe.push(st), inq[st] = 1, ed = 0;
        while (!qe.empty()) {
            int u = qe.front();
            qe.pop();
            for (int v = 1; v <= V; ++v)
                if (el[u][v] && djs[u] != djs[v] &&
                    pr[u] != v) {
                    if ((v == st) ||
                        (pr[v] > 0 && bk[pr[v]] > 0))
                        blo(u, v, qe);
                    else if (!bk[v]) {
                        if (bk[v] = u, pr[v] > 0) {
                            if (!inq[pr[v]]) qe.push(pr[v]);
                        } else return ed = v, void();
                    }
                }
        }
    }
    void aug() {
        for (int u = ed, v, w; u > 0;)
            v = bk[u], w = pr[v], pr[v] = u, pr[u] = v,
            u = w;
    }
    int solve() {
        fill_n(pr, V + 1, 0), ans = 0;
    }
};

```



```

    for (int u = 1; u <= V; ++u)
        if (!pr[u])
            if (st = u, flow(), ed > 0) aug(), ++ans;
    return ans;
}
};

```

#### 4.4 Minimum Weight Matching (Clique version)\*

```

struct Graph { // 0-base (Perfect Match), n is even
    int n, match[N], onstk[N], stk[N], tp;
    ll edge[N][N], dis[N];
    void init(int _n) {
        n = _n, tp = 0;
        for (int i = 0; i < n; ++i) fill_n(edge[i], n, 0);
    }
    void add_edge(int u, int v, ll w) {
        edge[u][v] = edge[v][u] = w;
    }
    bool SPFA(int u) {
        stk[tp++] = u, onstk[u] = 1;
        for (int v = 0; v < n; ++v)
            if (!onstk[v] && match[u] != v) {
                int m = match[v];
                if (dis[m] >
                    dis[u] - edge[v][m] + edge[u][v]) {
                    dis[m] = dis[u] - edge[v][m] + edge[u][v];
                    onstk[v] = 1, stk[tp++] = v;
                    if (onstk[m] || SPFA(m)) return 1;
                    --tp, onstk[v] = 0;
                }
            }
        onstk[u] = 0, --tp;
        return 0;
    }
    ll solve() { // find a match
        for (int i = 0; i < n; ++i) match[i] = i ^ 1;
        while (1) {
            int found = 0;
            fill_n(dis, n, 0);
            fill_n(onstk, n, 0);
            for (int i = 0; i < n; ++i)
                if (tp = 0, !onstk[i] && SPFA(i))
                    for (found = 1; tp >= 2;) {
                        int u = stk[--tp];
                        int v = stk[--tp];
                        match[u] = v, match[v] = u;
                    }
            if (!found) break;
        }
        ll ret = 0;
        for (int i = 0; i < n; ++i)
            ret += edge[i][match[i]];
        return ret >> 1;
    }
};

```

#### 4.5 SW-mincut

```

// global min cut
struct SW { // O(V^3)
    static const int MXN = 514;
    int n, vst[MXN], del[MXN];
    int edge[MXN][MXN], wei[MXN];
    void init(int _n) {
        n = _n, MEM(edge, 0), MEM(del, 0);
    }
    void addEdge(int u, int v, int w) {
        edge[u][v] += w, edge[v][u] += w;
    }
    void search(int &s, int &t) {
        MEM(vst, 0), MEM(wei, 0), s = t = -1;
        while (1) {
            int mx = -1, cur = 0;
            for (int i = 0; i < n; ++i)
                if (!del[i] && !vst[i] && mx < wei[i])

```

```

                    cur = i, mx = wei[i];
            if (mx == -1) break;
            vst[cur] = 1, s = t, t = cur;
            for (int i = 0; i < n; ++i)
                if (!vst[i] && !del[i]) wei[i] += edge[cur][i];
        }
    }
    int solve() {
        int res = INF;
        for (int i = 0, x, y; i < n - 1; ++i) {
            search(x, y), res = min(res, wei[y]), del[y] = 1;
            for (int j = 0; j < n; ++j)
                edge[x][j] = (edge[j][x] += edge[y][j]);
        }
        return res;
    }
};

```

#### 4.6 BoundedFlow(Dinic\*)

```

struct BoundedFlow { // 0-base
    struct edge {
        int to, cap, flow, rev;
    };
    vector<edge> G[N];
    int n, s, t, dis[N], cur[N], cnt[N];
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n + 2; ++i)
            G[i].clear(), cnt[i] = 0;
    }
    void add_edge(int u, int v, int lcap, int rcap) {
        cnt[u] -= lcap, cnt[v] += lcap;
        G[u].pb(edge{v, rcap, lcap, SZ(G[v])});
        G[v].pb(edge{u, 0, 0, SZ(G[u]) - 1});
    }
    void add_edge(int u, int v, int cap) {
        G[u].pb(edge{v, cap, 0, SZ(G[v])});
        G[v].pb(edge{u, 0, 0, SZ(G[u]) - 1});
    }
    int dfs(int u, int cap) {
        if (u == t || !cap) return cap;
        for (int &i = cur[u]; i < SZ(G[u]); ++i) {
            edge &e = G[u][i];
            if (dis[e.to] == dis[u] + 1 && e.cap != e.flow) {
                int df = dfs(e.to, min(e.cap - e.flow, cap));
                if (df) {
                    e.flow += df, G[e.to][e.rev].flow -= df;
                    return df;
                }
            }
        }
        dis[u] = -1;
        return 0;
    }
    bool bfs() {
        fill_n(dis, n + 3, -1);
        queue<int> q;
        q.push(s), dis[s] = 0;
        while (!q.empty()) {
            int u = q.front();
            q.pop();
            for (edge &e : G[u])
                if (!dis[e.to] && e.flow != e.cap)
                    q.push(e.to), dis[e.to] = dis[u] + 1;
        }
        return dis[t] != -1;
    }
    int maxflow(int _s, int _t) {
        s = _s, t = _t;
        int flow = 0, df;
        while (bfs()) {
            fill_n(cur, n + 3, 0);
            while ((df = dfs(s, INF))) flow += df;
        }
        return flow;
    }
    bool solve() {
        int sum = 0;
        for (int i = 0; i < n; ++i)

```

```

    if (cnt[i] > 0)
        add_edge(n + 1, i, cnt[i]), sum += cnt[i];
    else if (cnt[i] < 0) add_edge(i, n + 2, -cnt[i]);
    if (sum != maxflow(n + 1, n + 2)) sum = -1;
    for (int i = 0; i < n; ++i)
        if (cnt[i] > 0)
            G[n + 1].pop_back(), G[i].pop_back();
        else if (cnt[i] < 0)
            G[i].pop_back(), G[n + 2].pop_back();
    return sum != -1;
}
int solve(int _s, int _t) {
    add_edge(_t, _s, INF);
    if (!solve()) return -1; // invalid flow
    int x = G[_t].back().flow;
    return G[_t].pop_back(), G[_s].pop_back(), x;
}
};

```

## 4.7 Gomory Hu tree

```

struct Gomory_Hu_tree { // 0-base
    MaxFlow Dinic;
    int n;
    vector<pii> G[MAXN];
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i) G[i].clear();
    }
    void solve(vector<int> &v) {
        if (v.size() <= 1) return;
        int s = rand() % SZ(v);
        swap(v.back(), v[s]), s = v.back();
        int t = v[rand() % (SZ(v) - 1)];
        vector<int> L, R;
        int x = (Dinic.reset(), Dinic.maxflow(s, t));
        G[s].pb(pii(t, x)), G[t].pb(pii(s, x));
        for (int i : v)
            if (~Dinic.dis[i]) L.pb(i);
            else R.pb(i);
        solve(L), solve(R);
    }
    void build() {
        vector<int> v(n);
        for (int i = 0; i < n; ++i) v[i] = i;
        solve(v);
    }
} ght; // test by BZOJ 4519
MaxFlow &Dinic = ght.Dinic;

```

## 4.8 Flow Models

- Maximum/Minimum flow with lower bound / Circulation problem
  - Construct super source  $S$  and sink  $T$ .
  - For each edge  $(x, y, l, u)$ , connect  $x \rightarrow y$  with capacity  $u - l$ .
  - For each vertex  $v$ , denote by  $in(v)$  the difference between the sum of incoming lower bounds and the sum of outgoing lower bounds.
  - If  $in(v) > 0$ , connect  $S \rightarrow v$  with capacity  $in(v)$ , otherwise, connect  $v \rightarrow T$  with capacity  $-in(v)$ .
    - To maximize, connect  $t \rightarrow s$  with capacity  $\infty$  (skip this in circulation problem), and let  $f$  be the maximum flow from  $S$  to  $T$ . If  $f \neq \sum_{v \in V, in(v) > 0} in(v)$ , there's no solution. Otherwise, the maximum flow from  $s$  to  $t$  is the answer.
    - To minimize, let  $f$  be the maximum flow from  $S$  to  $T$ . Connect  $t \rightarrow s$  with capacity  $\infty$  and let the flow from  $S$  to  $T$  be  $f'$ . If  $f + f' \neq \sum_{v \in V, in(v) > 0} in(v)$ , there's no solution. Otherwise,  $f'$  is the answer.
  - The solution of each edge  $e$  is  $l_e + f_e$ , where  $f_e$  corresponds to the flow of edge  $e$  on the graph.
- Construct minimum vertex cover from maximum matching  $M$  on bipartite graph  $(X, Y)$ 
  - Redirect every edge:  $y \rightarrow x$  if  $(x, y) \in M$ ,  $x \rightarrow y$  otherwise.
  - DFS from unmatched vertices in  $X$ .
  - $x \in X$  is chosen iff  $x$  is unvisited.
  - $y \in Y$  is chosen iff  $y$  is visited.
- Minimum cost cyclic flow
  - Construct super source  $S$  and sink  $T$
  - For each edge  $(x, y, c)$ , connect  $x \rightarrow y$  with  $(cost, cap) = (c, 1)$  if  $c > 0$ , otherwise connect  $y \rightarrow x$  with  $(cost, cap) = (-c, 1)$

- For each edge with  $c < 0$ , sum these cost as  $K$ , then increase  $d(y)$  by 1, decrease  $d(x)$  by 1
- For each vertex  $v$  with  $d(v) > 0$ , connect  $S \rightarrow v$  with  $(cost, cap) = (0, d(v))$
- For each vertex  $v$  with  $d(v) < 0$ , connect  $v \rightarrow T$  with  $(cost, cap) = (0, -d(v))$
- Flow from  $S$  to  $T$ , the answer is the cost of the flow  $C + K$

### • Maximum density induced subgraph

- Binary search on answer, suppose we're checking answer  $T$
- Construct a max flow model, let  $K$  be the sum of all weights
- Connect source  $s \rightarrow v$ ,  $v \in G$  with capacity  $K$
- For each edge  $(u, v, w)$  in  $G$ , connect  $u \rightarrow v$  and  $v \rightarrow u$  with capacity  $w$
- For  $v \in G$ , connect it with sink  $v \rightarrow t$  with capacity  $K + 2T - (\sum_{e \in E(v)} w(e)) - 2w(v)$
- $T$  is a valid answer if the maximum flow  $f < K|V|$

### • Minimum weight edge cover

- For each  $v \in V$  create a copy  $v'$ , and connect  $u' \rightarrow v'$  with weight  $w(u, v)$ .
- Connect  $v \rightarrow v'$  with weight  $2\mu(v)$ , where  $\mu(v)$  is the cost of the cheapest edge incident to  $v$ .
- Find the minimum weight perfect matching on  $G'$ .

### • Project selection problem

- If  $p_v > 0$ , create edge  $(s, v)$  with capacity  $p_v$ ; otherwise, create edge  $(v, t)$  with capacity  $-p_v$ .
- Create edge  $(u, v)$  with capacity  $w$  with  $w$  being the cost of choosing  $u$  without choosing  $v$ .
- The mincut is equivalent to the maximum profit of a subset of projects.

### • 0/1 quadratic programming

$$\sum_x c_x x + \sum_y c_y \bar{y} + \sum_{xy} c_{xy} x \bar{y} + \sum_{xyx'y'} c_{xyx'y'} (x \bar{y} + x' \bar{y}')$$

can be minimized by the mincut of the following graph:

- Create edge  $(x, t)$  with capacity  $c_x$  and create edge  $(s, y)$  with capacity  $c_y$ .
- Create edge  $(x, y)$  with capacity  $c_{xy}$ .
- Create edge  $(x, y)$  and edge  $(x', y')$  with capacity  $c_{xyx'y'}$ .

## 5 String

### 5.1 KMP

```

int F[MAXN];
vector<int> match(string A, string B) {
    vector<int> ans;
    F[0] = -1, F[1] = 0;
    for (int i = 1, j = 0; i < SZ(B); F[++i] = ++j) {
        if (B[i] == B[j]) F[i] = F[j]; // optimize
        while (j != -1 && B[i] != B[j]) j = F[j];
    }
    for (int i = 0, j = 0; i < SZ(A); ++i) {
        while (j != -1 && A[i] != B[j]) j = F[j];
        if (++j == SZ(B)) ans.pb(i + 1 - j), j = F[j];
    }
    return ans;
}

```

### 5.2 Z-value

```

const int MAXn = 1e5 + 5;
int z[MAXn];
void make_z(string s) {
    int l = 0, r = 0;
    for (int i = 1; i < s.size(); ++i) {
        for (z[i] = max(0, min(r - i + 1, z[i - l]));
            i + z[i] < s.size() && s[i + z[i]] == s[z[i]];
            z[i]++);
        if (i + z[i] - 1 > r) l = i, r = i + z[i] - 1;
    }
}

```

### 5.3 Manacher\*

```
int z[MAXN];
int Manacher(string tmp) {
    string s = "&";
    int l = 0, r = 0, x, ans;
    for (char c : tmp) s.pb(c), s.pb('%');
    ans = 0, x = 0;
    for (int i = 1; i < SZ(s); ++i) {
        z[i] = r > i ? min(z[2 * l - i], r - i) : 1;
        while (s[i + z[i]] == s[i - z[i]]) ++z[i];
        if (z[i] + i > r) r = z[i] + i, l = i;
    }
    for (int i = 1; i < SZ(s); ++i)
        if (s[i] == '%') x = max(x, z[i]);
    ans = x / 2 * 2, x = 0;
    for (int i = 1; i < SZ(s); ++i)
        if (s[i] != '%') x = max(x, z[i]);
    return max(ans, (x - 1) / 2 * 2 + 1);
}
```

### 5.4 Suffix Array

```
struct suffix_array {
    int box[MAXN], tp[MAXN], m;
    bool not_equ(int a, int b, int k, int n) {
        return ra[a] != ra[b] || a + k >= n ||
            b + k >= n || ra[a + k] != ra[b + k];
    }
    void radix(int *key, int *it, int *ot, int n) {
        fill_n(box, m, 0);
        for (int i = 0; i < n; ++i) ++box[key[i]];
        partial_sum(box, box + m, box);
        for (int i = n - 1; i >= 0; --i)
            ot[--box[key[it[i]]]] = it[i];
    }
    void make_sa(string s, int n) {
        int k = 1;
        for (int i = 0; i < n; ++i) ra[i] = s[i];
        do {
            iota(tp, tp + k, n - k), iota(sa + k, sa + n, 0);
            radix(ra + k, sa + k, tp + k, n - k);
            radix(ra, tp, sa, n);
            tp[sa[0]] = 0, m = 1;
            for (int i = 1; i < n; ++i) {
                m += not_equ(sa[i], sa[i - 1], k, n);
                tp[sa[i]] = m - 1;
            }
            copy_n(tp, n, ra);
            k *= 2;
        } while (k < n && m != n);
    }
    void make_he(string s, int n) {
        for (int j = 0, k = 0; j < n; ++j) {
            if (ra[j])
                for (; s[j + k] == s[sa[ra[j] - 1] + k]; ++k)
                    he[ra[j]] = k, k = max(0, k - 1);
        }
    }
    int sa[MAXN], ra[MAXN], he[MAXN];
    void build(string s) {
        FILL(sa, 0), FILL(ra, 0), FILL(he, 0);
        FILL(box, 0), FILL(tp, 0), m = 256;
        make_sa(s, s.size());
        make_he(s, s.size());
    }
};
```

### 5.5 SAIS\*

```
class SAIS {
public:
    int *SA, *H;
    // zero based, string content MUST > 0
    // result height H[i] is LCP(SA[i - 1], SA[i])
    // string, length, |sigma|

```

```
void build(int *s, int n, int m = 128) {
    copy_n(s, n, _s);
    _h[0] = _s[n++] = 0;
    sais(_s, _sa, _p, _q, _t, _c, n, m);
    mkhei(n);
    SA = _sa + 1;
    H = _h + 1;
}

private:
    bool _t[N * 2];
    int _s[N * 2], _c[N * 2], x[N], _p[N], _q[N * 2],
        r[N], _sa[N * 2], _h[N];
    void mkhei(int n) {
        for (int i = 0; i < n; i++) r[_sa[i]] = i;
        for (int i = 0; i < n; i++)
            if (r[i]) {
                int ans = i > 0 ? max(_h[r[i - 1]] - 1, 0) : 0;
                while (_s[i + ans] == _s[_sa[r[i] - 1] + ans])
                    ans++;
                _h[r[i]] = ans;
            }
    }
    void sais(int *s, int *sa, int *p, int *q, bool *t,
        int *c, int n, int z) {
        bool uniq = t[n - 1] = 1, neq;
        int nn = 0, nmzx = -1, *nsa = sa + n, *ns = s + n,
            lst = -1;

#define MAGIC(XD) \
        fill_n(sa, n, 0); \
        copy_n(c, z, x); \
        XD; \
        copy_n(c, z - 1, x + 1); \
        for (int i = 0; i < n; i++) \
            if (sa[i] && !t[sa[i] - 1]) \
                sa[x[s[sa[i] - 1]]++] = sa[i] - 1; \
        copy_n(c, z, x); \
        for (int i = n - 1; i >= 0; i--) \
            if (sa[i] && t[sa[i] - 1]) \
                sa[--x[s[sa[i] - 1]]] = sa[i] - 1; \

        fill_n(c, z, 0);
        for (int i = 0; i < n; i++) uniq &= ++c[s[i]] < 2;
        partial_sum(c, c + z, c);
        if (uniq) {
            for (int i = 0; i < n; i++) sa[--c[s[i]]] = i;
            return;
        }
        for (int i = n - 2; i >= 0; i--)
            t[i] = (s[i] == s[i + 1] ? t[i + 1] :
                : s[i] < s[i + 1]);
        MAGIC(for (int i = 1; i <= n - 1;
            i++) if (t[i] && !t[i - 1])
                sa[--x[s[i]]] = p[q[i] = nn++] = i);
        for (int i = 0; i < n; i++)
            if (sa[i] && t[sa[i]] && !t[sa[i] - 1]) {
                neq = (lst < 0) ||
                    !equal(s + lst,
                        s + lst + p[q[sa[i]] + 1] - sa[i],
                        s + sa[i]);
                ns[q[lst = sa[i]]] = nmzx += neq;
            }
        sais(ns, nsa, p + nn, q + n, t + n, c + z, nn,
            nmzx + 1);
        MAGIC(for (int i = nn - 1; i >= 0; i--)
            sa[--x[s[p[nsa[i]]]]] = p[nsa[i]]);
    }
} sa;
```

### 5.6 Aho-Corasick Automatan

```
const int len = 400000, sigma = 26;
struct AC_Automatan {
    int nx[len][sigma], fl[len], cnt[len], pri[len], top;
    int newnode() {
        fill(nx[top], nx[top] + sigma, -1);
        return top++;
    }
    void init() { top = 1, newnode(); }
```

```

int input(
    string &s) { // return the end_node of string
    int X = 1;
    for (char c : s) {
        if (!~nx[X][c - 'a']) nx[X][c - 'a'] = newNode();
        X = nx[X][c - 'a'];
    }
    return X;
}

void make_fl() {
    queue<int> q;
    q.push(1), fl[1] = 0;
    for (int t = 0; !q.empty(); ) {
        int R = q.front();
        q.pop(), pri[t++] = R;
        for (int i = 0; i < sigma; ++i)
            if (~nx[R][i]) {
                int X = nx[R][i], Z = fl[R];
                for (; Z && !~nx[Z][i];) Z = fl[Z];
                fl[X] = Z ? nx[Z][i] : 1, q.push(X);
            }
    }
}

void get_v(string &s) {
    int X = 1;
    fill(cnt, cnt + top, 0);
    for (char c : s) {
        while (X && !~nx[X][c - 'a']) X = fl[X];
        X = X ? nx[X][c - 'a'] : 1, ++cnt[X];
    }
    for (int i = top - 2; i > 0; --i)
        cnt[fl[pri[i]]] += cnt[pri[i]];
}
};

```

## 5.7 Smallest Rotation

```

string mcp(string s) {
    int n = SZ(s), i = 0, j = 1;
    s += s;
    while (i < n && j < n) {
        int k = 0;
        while (k < n && s[i + k] == s[j + k]) ++k;
        if (s[i + k] <= s[j + k]) j += k + 1;
        else i += k + 1;
        if (i == j) ++j;
    }
    int ans = i < n ? i : j;
    return s.substr(ans, n);
}

```

## 5.8 De Bruijn sequence\*

```

constexpr int MAXC = 10, MAXN = 1e5 + 10;
struct DBSeq {
    int C, N, K, L, buf[MAXC * MAXN]; // K <= C^N
    void dfs(int *out, int t, int p, int &ptr) {
        if (ptr >= L) return;
        if (t > N) {
            if (N % p) return;
            for (int i = 1; i <= p && ptr < L; ++i)
                out[ptr++] = buf[i];
        } else {
            buf[t] = buf[t - p], dfs(out, t + 1, p, ptr);
            for (int j = buf[t - p] + 1; j < C; ++j)
                buf[t] = j, dfs(out, t + 1, t, ptr);
        }
    }
    void solve(int _c, int _n, int _k, int *out) {
        int p = 0;
        C = _c, N = _n, K = _k, L = N + K - 1;
        dfs(out, 1, 1, p);
        if (p < L) fill(out + p, out + L, 0);
    }
} dbs;

```

## 5.9 SAM

```

const int MAXM = 1000010;
struct SAM {
    int tot, root, lst, mom[MAXM], mx[MAXM];
    int acc[MAXM], nxt[MAXM][33];
    int newNode() {
        int res = ++tot;
        fill(nxt[res], nxt[res] + 33, 0);
        mom[res] = mx[res] = acc[res] = 0;
        return res;
    }
    void init() {
        tot = 0;
        root = newNode();
        mom[root] = 0, mx[root] = 0;
        lst = root;
    }
    void push(int c) {
        int p = lst;
        int np = newNode();
        mx[np] = mx[p] + 1;
        for (; p && nxt[p][c] == 0; p = mom[p])
            nxt[p][c] = np;
        if (p == 0) mom[np] = root;
        else {
            int q = nxt[p][c];
            if (mx[p] + 1 == mx[q]) mom[np] = q;
            else {
                int nq = newNode();
                mx[nq] = mx[p] + 1;
                for (int i = 0; i < 33; ++i)
                    nxt[nq][i] = nxt[q][i];
                mom[nq] = mom[q];
                mom[q] = nq;
                mom[np] = nq;
                for (; p && nxt[p][c] == q; p = mom[p])
                    nxt[p][c] = nq;
            }
        }
        lst = np;
    }
    void push(char *str) {
        for (int i = 0; str[i]; ++i)
            push(str[i] - 'a' + 1);
    }
} sam;

```

## 5.10 PalTree

```

struct palindromic_tree { // Check by APIO 2014
    // palindrome
    struct node {
        int next[26], fail, len;
        int cnt, num; // cnt: appear times, num: number of
        // pal. suf.
        node(int l = 0) : fail(0), len(l), cnt(0), num(0) {
            for (int i = 0; i < 26; ++i) next[i] = 0;
        }
    };
    vector<node> St;
    vector<char> s;
    int last, n;
    palindromic_tree() : St(2), last(1), n(0) {
        St[0].fail = 1, St[1].len = -1, s.pb(-1);
    }
    inline void clear() {
        St.clear(), s.clear(), last = 1, n = 0;
        St.pb(0), St.pb(-1);
        St[0].fail = 1, s.pb(-1);
    }
    inline int get_fail(int x) {
        while (s[n - St[x].len - 1] != s[n])
            x = St[x].fail;
        return x;
    }
    inline void add(int c) {
        s.push_back(c - 'a'), ++n;
        int cur = get_fail(last);
    }
};

```

```

    if (!St[cur].next[c]) {
        int now = SZ(St);
        St.pb(St[cur].len + 2);
        St[now].fail =
            St[get_fail(St[cur].fail)].next[c];
        St[cur].next[c] = now;
        St[now].num = St[St[now].fail].num + 1;
    }
    last = St[cur].next[c], ++St[last].cnt;
}
inline void count() { // counting cnt
    auto i = St.rbegin();
    for (; i != St.rend(); ++i) {
        St[i->fail].cnt += i->cnt;
    }
}
inline int size() { // The number of diff. pal.
    return SZ(St) - 2;
}
};

```

## 5.11 cyclicLCS

```

#define L 0
#define LU 1
#define U 2
const int mov[3][2] = {0, -1, -1, -1, -1, 0};
int al, bl;
char a[MAXL * 2], b[MAXL * 2]; // 0-indexed
int dp[MAXL * 2][MAXL];
char pred[MAXL * 2][MAXL];
inline int lcs_length(int r) {
    int i = r + al, j = bl, l = 0;
    while (i > r) {
        char dir = pred[i][j];
        if (dir == LU) l++;
        i += mov[dir][0];
        j += mov[dir][1];
    }
    return l;
}
inline void reroot(int r) { // r = new base row
    int i = r, j = 1;
    while (j <= bl && pred[i][j] != LU) j++;
    if (j > bl) return;
    pred[i][j] = L;
    while (i < 2 * al && j <= bl) {
        if (pred[i + 1][j] == U) {
            i++;
            pred[i][j] = L;
        } else if (j < bl && pred[i + 1][j + 1] == LU) {
            i++;
            j++;
            pred[i][j] = L;
        } else {
            j++;
        }
    }
}
int cyclic_lcs() {
    // a, b, al, bl should be properly filled
    // note: a WILL be altered in process
    // -- concatenated after itself
    char tmp[MAXL];
    if (al > bl) {
        swap(al, bl);
        strcpy(tmp, a);
        strcpy(a, b);
        strcpy(b, tmp);
    }
    strcpy(tmp, a);
    strcat(a, tmp);
    // basic lcs
    for (int i = 0; i <= 2 * al; i++) {
        dp[i][0] = 0;
        pred[i][0] = U;
    }
    for (int j = 0; j <= bl; j++) {
        dp[0][j] = 0;
        pred[0][j] = L;
    }
}

```

```

}
for (int i = 1; i <= 2 * al; i++) {
    for (int j = 1; j <= bl; j++) {
        if (a[i - 1] == b[j - 1])
            dp[i][j] = dp[i - 1][j - 1] + 1;
        else dp[i][j] = max(dp[i - 1][j], dp[i][j - 1]);
        if (dp[i][j - 1] == dp[i][j]) pred[i][j] = L;
        else if (a[i - 1] == b[j - 1]) pred[i][j] = LU;
        else pred[i][j] = U;
    }
}
// do cyclic lcs
int clcs = 0;
for (int i = 0; i < al; i++) {
    clcs = max(clcs, lcs_length(i));
    reroot(i + 1);
}
// recover a
a[al] = '\0';
return clcs;
}

```

## 6 Math

### 6.1 ax+by=gcd\*

```

pll exgcd(pll a, pll b) {
    if (b == 0) return pll(1, 0);
    else {
        pll p = a / b;
        pll q = exgcd(b, a % b);
        return pll(q.Y, q.X - q.Y * p);
    }
}

```

### 6.2 floor and ceil

```

int floor(int a, int b) {
    return a / b - (a % b && a < 0 ^ b < 0);
}
int ceil(int a, int b) {
    return a / b + (a % b && a < 0 ^ b > 0);
}

```

### 6.3 Miller Rabin\*

```

// n < 4,759,123,141      3 : 2, 7, 61
// n < 1,122,004,669,633  4 : 2, 13, 23, 1662803
// n < 3,474,749,660,383  6 : pirms <= 13
// n < 2^64               7 :
// 2, 325, 9375, 28178, 450775, 9780504, 1795265022
bool Miller_Rabin(pll a, pll n) {
    if ((a = a % n) == 0) return 1;
    if ((n & 1) ^ 1) return n == 2;
    pll tmp = (n - 1) / ((n - 1) & (1 - n));
    pll t = __lg(((n - 1) & (1 - n))), x = 1;
    for (; tmp; tmp >>= 1, a = mul(a, a, n))
        if (tmp & 1) x = mul(x, a, n);
    if (x == 1 || x == n - 1) return 1;
    while (--t)
        if ((x = mul(x, x, n)) == n - 1) return 1;
    return 0;
}

```

### 6.4 Fraction

```

struct fraction {
    ll n, d;
    fraction(const ll &n=0, const ll &d=1) : n(_n), d(_d) {
        ll t = __gcd(n, d);
        n /= t, d /= t;
        if (d < 0) n = -n, d = -d;
    }
}

```

```

}
fraction operator-()const{
    return fraction(-n,d);
}
fraction operator+(const fraction &b)const{
    return fraction(n*b.d+b.n*d,d*b.d);
}
fraction operator-(const fraction &b)const{
    return fraction(n*b.d-b.n*d,d*b.d);
}
fraction operator*(const fraction &b)const{
    return fraction(n*b.n,d*b.d);
}
fraction operator/(const fraction &b)const{
    return fraction(n*b.d,d*b.n);
}
}
void print(){
    cout << n;
    if(d!=1) cout << "/" << d;
}
};

```

## 6.5 Simultaneous Equations

```

struct matrix { //m variables, n equations
    int n, m;
    fraction M[MAXN][MAXN + 1], sol[MAXN];
    int solve() { //-1: inconsistent, >= 0: rank
        for (int i = 0; i < n; ++i) {
            int piv = 0;
            while (piv < m && !M[i][piv].n) ++piv;
            if (piv == m) continue;
            for (int j = 0; j < n; ++j) {
                if (i == j) continue;
                fraction tmp = -M[j][piv] / M[i][piv];
                for (int k = 0; k <= m; ++k) M[j][k] = tmp * M[i][k] + M[j][k];
            }
        }
        int rank = 0;
        for (int i = 0; i < n; ++i) {
            int piv = 0;
            while (piv < m && !M[i][piv].n) ++piv;
            if (piv == m && M[i][m].n) return -1;
            else if (piv < m) ++rank, sol[piv] = M[i][m] / M[i][piv];
        }
        return rank;
    }
};

```

## 6.6 Pollard Rho

```

// does not work when n is prime
ll f(ll x, ll mod){ return add(mul(x,x,mod),1,mod); }
ll pollard_rho(ll n){
    if(!(n&1)) return 2;
    while(1){
        ll y=2,x=rand()%(n-1)+1,res=1;
        for(int sz=2;res==1;y=x,sz*=2)
            for(int i=0;i<sz&&res!=1;++i)
                x=f(x,n),res=__gcd(abs(x-y),n);
        if(res!=0&&res!=n) return res;
    }
}

```

## 6.7 Simplex Algorithm

```

const int MAXN = 111;
const int MAXM = 111;
const double eps = 1E-10;
double a[MAXN][MAXM], b[MAXN], c[MAXM], d[MAXN][MAXM];
double x[MAXM];
int ix[MAXN + MAXM]; // !!! array ALL indexed from 0
// max{cx} subject to {Ax<=b,x>=0}
// n: constraints, m: vars !!!

```

```

// x[] is the optimal solution vector
// usage :
// value = simplex(a, b, c, N, M);
double simplex(double a[MAXN][MAXM], double b[MAXN],
    double c[MAXM], int n, int m){
    ++m;
    int r = n, s = m - 1;
    memset(d, 0, sizeof(d));
    for (int i = 0; i < n + m; ++i) ix[i] = i;
    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < m - 1; ++j) d[i][j] = -a[i][j];
        d[i][m - 1] = 1;
        d[i][m] = b[i];
        if (d[r][m] > d[i][m]) r = i;
    }
    for (int j = 0; j < m - 1; ++j) d[n][j] = c[j];
    d[n + 1][m - 1] = -1;
    for (double dd; ) {
        if (r < n) {
            int t = ix[s]; ix[s] = ix[r + m]; ix[r + m] = t;
            d[r][s] = 1.0 / d[r][s];
            for (int j = 0; j <= m; ++j)
                if (j != s) d[r][j] *= -d[r][s];
            for (int i = 0; i <= n + 1; ++i) if (i != r) {
                for (int j = 0; j <= m; ++j) if (j != s)
                    d[i][j] += d[r][j] * d[i][s];
                d[i][s] *= d[r][s];
            }
        }
        r = -1; s = -1;
        for (int j = 0; j < m; ++j)
            if (s < 0 || ix[s] > ix[j]) {
                if (d[n + 1][j] > eps ||
                    (d[n + 1][j] > -eps && d[n][j] > eps))
                    s = j;
            }
        if (s < 0) break;
        for (int i = 0; i < n; ++i) if (d[i][s] < -eps) {
            if (r < 0 ||
                (dd = d[r][m] / d[r][s] - d[i][m] / d[i][s]) < -eps ||
                (dd < eps && ix[r + m] > ix[i + m]))
                r = i;
        }
        if (r < 0) return -1; // not bounded
    }
    if (d[n + 1][m] < -eps) return -1; // not executable
    double ans = 0;
    for(int i=0; i<m; i++) x[i] = 0;
    for (int i = m; i < n + m; ++i) { // the missing
        enumerated x[i] = 0
        if (ix[i] < m - 1){
            ans += d[i - m][m] * c[ix[i]];
            x[ix[i]] = d[i - m][m];
        }
    }
    return ans;
}

```

### 6.7.1 Construction

Standard form: maximize  $c^T x$  subject to  $Ax \leq b$  and  $x \geq 0$ .  
 Dual LP: minimize  $b^T y$  subject to  $A^T y \geq c$  and  $y \geq 0$ .  
 $\bar{x}$  and  $\bar{y}$  are optimal if and only if for all  $i \in [1, n]$ , either  $\bar{x}_i = 0$  or  $\sum_{j=1}^m A_{ji} \bar{y}_j = c_i$  holds and for all  $i \in [1, m]$  either  $\bar{y}_i = 0$  or  $\sum_{j=1}^n A_{ij} \bar{x}_j = b_j$  holds.

1. In case of minimization, let  $c'_i = -c_i$
2.  $\sum_{1 \leq i \leq n} A_{ji} x_i \geq b_j \rightarrow \sum_{1 \leq i \leq n} -A_{ji} x_i \leq -b_j$
3.  $\sum_{1 \leq i \leq n} A_{ji} x_i = b_j$

$$\bullet \begin{cases} \sum_{1 \leq i \leq n} A_{ji} x_i \leq b_j \\ \sum_{1 \leq i \leq n} A_{ji} x_i \geq b_j \end{cases}$$

4. If  $x_i$  has no lower bound, replace  $x_i$  with  $x_i - x'_i$

## 6.8 Schreier-Sims Algorithm\*

```

namespace schreier {
int n;
vector<vector<vector<int>>> bkts, binv;

```



```

vector<vector<int>> lk;
vector<int> operator*(const vector<int> &a, const
vector<int> &b) {
vector<int> res(SZ(a));
for (int i = 0; i < SZ(a); ++i) res[i] = b[a[i]];
return res;
}
vector<int> inv(const vector<int> &a) {
vector<int> res(SZ(a));
for (int i = 0; i < SZ(a); ++i) res[a[i]] = i;
return res;
}
int filter(const vector<int> &g, bool add = true) {
n = SZ(bkts);
vector<int> p = g;
for (int i = 0; i < n; ++i) {
assert(p[i] >= 0 && p[i] < SZ(lk[i]));
if (lk[i][p[i]] == -1) {
if (add) {
bkts[i].pb(p);
binv[i].pb(inv(p));
lk[i][p[i]] = SZ(bkts[i]) - 1;
}
return i;
}
}
p = p * binv[i][lk[i][p[i]]];
}
return -1;
}
bool inside(const vector<int> &g) { return filter(g,
false) == -1; }
void solve(const vector<vector<int>> &gen, int _n) {
n = _n;
bkts.clear(), bkts.resize(n);
binv.clear(), binv.resize(n);
lk.clear(), lk.resize(n);
vector<int> iden(n);
iota(iden.begin(), iden.end(), 0);
for (int i = 0; i < n; ++i) {
lk[i].resize(n, -1);
bkts[i].pb(iden);
binv[i].pb(iden);
lk[i][i] = 0;
}
for (int i = 0; i < SZ(gen); ++i) filter(gen[i]);
queue<pair<pii, pii>> upd;
for (int i = 0; i < n; ++i)
for (int j = i; j < n; ++j)
for (int k = 0; k < SZ(bkts[i]); ++k)
for (int l = 0; l < SZ(bkts[j]); ++l)
upd.emplace(pii(i, k), pii(j, l));
while (!upd.empty()) {
auto a = upd.front().X;
auto b = upd.front().Y;
upd.pop();
int res = filter(bkts[a.X][a.Y] * bkts[b.X][b.Y]);
if (res == -1) continue;
pii pr = pii(res, SZ(bkts[res]) - 1);
for (int i = 0; i < n; ++i)
for (int j = 0; j < SZ(bkts[i]); ++j) {
if (i <= res) upd.emplace(pii(i, j), pr);
if (res <= i) upd.emplace(pr, pii(i, j));
}
}
}
long long size() {
long long res = 1;
for (int i = 0; i < n; ++i) res = res * SZ(bkts[i]);
return res;
}
}

```

## 6.9 chineseRemainder

```

LL solve(LL x1, LL m1, LL x2, LL m2) {
LL g = __gcd(m1, m2);
if((x2 - x1) % g) return -1; // no sol

```

```

m1 /= g; m2 /= g;
pair<LL,LL> p = gcd(m1, m2);
LL lcm = m1 * m2 * g;
LL res = p.first * (x2 - x1) * m1 + x1;
return (res % lcm + lcm) % lcm;
}

```

## 6.10 QuadraticResidue

```

int Jacobi(int a, int m) {
int s = 1;
for (; m > 1; ) {
a %= m;
if (a == 0) return 0;
const int r = __builtin_ctz(a);
if ((r & 1) && ((m + 2) & 4)) s = -s;
a >>= r;
if (a & m & 2) s = -s;
swap(a, m);
}
return s;
}
int QuadraticResidue(int a, int p) {
if (p == 2) return a & 1;
const int jc = Jacobi(a, p);
if (jc == 0) return 0;
if (jc == -1) return -1;
int b, d;
for (; ; ) {
b = rand() % p;
d = (1LL * b * b + p - a) % p;
if (Jacobi(d, p) == -1) break;
}
int f0 = b, f1 = 1, g0 = 1, g1 = 0, tmp;
for (int e = (1LL + p) >> 1; e >>= 1; ) {
if (e & 1) {
tmp = (1LL * g0 * f0 + 1LL * d * (1LL * g1 * f1 %
p)) % p;
g1 = (1LL * g0 * f1 + 1LL * g1 * f0) % p;
g0 = tmp;
}
tmp = (1LL * f0 * f0 + 1LL * d * (1LL * f1 * f1 %
p)) % p;
f1 = (2LL * f0 * f1) % p;
f0 = tmp;
}
return g0;
}

```

## 6.11 PiCount

```

int64_t PrimeCount(int64_t n) {
if (n <= 1) return 0;
const int v = sqrt(n);
vector<int> smalls(v + 1);
for (int i = 2; i <= v; ++i) smalls[i] = (i + 1) / 2;
int s = (v + 1) / 2;
vector<int> roughs(s);
for (int i = 0; i < s; ++i) roughs[i] = 2 * i + 1;
vector<int64_t> larges(s);
for (int i = 0; i < s; ++i) larges[i] = (n / (2 * i +
1) + 1) / 2;
vector<bool> skip(v + 1);
int pc = 0;
for (int p = 3; p <= v; ++p) {
if (smalls[p] > smalls[p - 1]) {
int q = p * p;
pc++;
if (1LL * q * q > n) break;
skip[p] = true;
for (int i = q; i <= v; i += 2 * p) skip[i] =
true;
int ns = 0;
for (int k = 0; k < s; ++k) {
int i = roughs[k];
if (skip[i]) continue;
int64_t d = 1LL * i * p;

```

```

    larges[ns] = larges[k] - (d <= v ? larges[
        smalls[d] - pc] : smalls[n / d]) + pc;
    roughs[ns++] = i;
}
s = ns;
for (int j = v / p; j >= p; --j) {
    int c = smalls[j] - pc;
    for (int i = j * p, e = min(i + p, v + 1); i <
        e; ++i) smalls[i] -= c;
}
}
for (int k = 1; k < s; ++k) {
    const int64_t m = n / roughs[k];
    int64_t s = larges[k] - (pc + k - 1);
    for (int l = 1; l < k; ++l) {
        int p = roughs[l];
        if (1LL * p * p > m) break;
        s -= smalls[m / p] - (pc + l - 1);
    }
    larges[0] -= s;
}
return larges[0];
}

```

## 6.12 Primes

```

/* 12721 13331 14341 75577 123457 222557 556679 999983
   1097774749 1076767633 100102021 999997771
   1001010013 1000512343 987654361 999991231 999888733
   98789101 987777733 999991921 1010101333 1010102101
   1000000000039 100000000000037 2305843009213693951
   4611686018427387847 9223372036854775783
   18446744073709551557 */

```

## 6.13 Theorem

### 6.13.1 Kirchhoff's Theorem

Denote  $L$  be a  $n \times n$  matrix as the Laplacian matrix of graph  $G$ , where  $L_{ii} = d(i)$ ,  $L_{ij} = -c$  where  $c$  is the number of edge  $(i, j)$  in  $G$ .

- The number of undirected spanning in  $G$  is  $|\det(\tilde{L}_{11})|$ .
- The number of directed spanning tree rooted at  $r$  in  $G$  is  $|\det(\tilde{L}_{rr})|$ .

### 6.13.2 Tutte's Matrix

Let  $D$  be a  $n \times n$  matrix, where  $d_{ij} = x_{ij}$  ( $x_{ij}$  is chosen uniformly at random) if  $i < j$  and  $(i, j) \in E$ , otherwise  $d_{ij} = -d_{ji}$ .  $\frac{\text{rank}(D)}{2}$  is the maximum matching on  $G$ .

### 6.13.3 Cayley's Formula

- Given a degree sequence  $d_1, d_2, \dots, d_n$  for each *labeled* vertices, there are  $\frac{(n-2)!}{(d_1-1)!(d_2-1)! \dots (d_n-1)!}$  spanning trees.
- Let  $T_{n,k}$  be the number of *labeled* forests on  $n$  vertices with  $k$  components, such that vertex  $1, 2, \dots, k$  belong to different components. Then  $T_{n,k} = kn^{n-k-1}$ .

### 6.13.4 Erdős-Gallai theorem

A sequence of nonnegative integers  $d_1 \geq \dots \geq d_n$  can be represented as the degree sequence of a finite simple graph on  $n$  vertices if and only

if  $d_1 + \dots + d_n$  is even and  $\sum_{i=1}^k d_i \leq k(k-1) + \sum_{i=k+1}^n \min(d_i, k)$  holds for every  $1 \leq k \leq n$ .

### 6.13.5 Gale-Ryser theorem

A pair of sequences of nonnegative integers  $a_1 \geq \dots \geq a_n$  and  $b_1, \dots, b_n$  is bigraphic if and only if  $\sum_{i=1}^n a_i = \sum_{i=1}^n b_i$  and  $\sum_{i=1}^k a_i \leq \sum_{i=1}^n \min(b_i, k)$  holds for every  $1 \leq k \leq n$ .

### 6.13.6 Fulkerson-Chen-Anstee theorem

A sequence  $(a_1, b_1), \dots, (a_n, b_n)$  of nonnegative integer pairs with  $a_1 \geq \dots \geq a_n$  is digraphic if and only if  $\sum_{i=1}^n a_i = \sum_{i=1}^n b_i$  and  $\sum_{i=1}^k a_i \leq \sum_{i=k+1}^n \min(b_i, k-1) + \sum_{i=k+1}^n \min(b_i, k)$  holds for every  $1 \leq k \leq n$ .

## 6.14 Euclidean Algorithms

- $m = \lfloor \frac{a+b}{c} \rfloor$
- Time complexity:  $O(\log n)$

$$f(a, b, c, n) = \sum_{i=0}^n \lfloor \frac{ai+b}{c} \rfloor$$

$$= \begin{cases} \lfloor \frac{a}{c} \rfloor \cdot \frac{n(n+1)}{2} + \lfloor \frac{b}{c} \rfloor \cdot (n+1) \\ + f(a \bmod c, b \bmod c, c, n), & a \geq c \vee b \geq c \\ 0, & n < 0 \vee a = 0 \\ nm - f(c, c-b-1, a, m-1), & \text{otherwise} \end{cases}$$

$$g(a, b, c, n) = \sum_{i=0}^n i \lfloor \frac{ai+b}{c} \rfloor$$

$$= \begin{cases} \lfloor \frac{a}{c} \rfloor \cdot \frac{n(n+1)(2n+1)}{6} + \lfloor \frac{b}{c} \rfloor \cdot \frac{n(n+1)}{2} \\ + g(a \bmod c, b \bmod c, c, n), & a \geq c \vee b \geq c \\ 0, & n < 0 \vee a = 0 \\ \frac{1}{2} \cdot (n(n+1)m - f(c, c-b-1, a, m-1)) \\ - h(c, c-b-1, a, m-1), & \text{otherwise} \end{cases}$$

$$h(a, b, c, n) = \sum_{i=0}^n i \lfloor \frac{ai+b}{c} \rfloor^2$$

$$= \begin{cases} \lfloor \frac{a}{c} \rfloor^2 \cdot \frac{n(n+1)(2n+1)}{6} + \lfloor \frac{b}{c} \rfloor^2 \cdot (n+1) \\ + \lfloor \frac{a}{c} \rfloor \cdot \lfloor \frac{b}{c} \rfloor \cdot n(n+1) \\ + h(a \bmod c, b \bmod c, c, n) \\ + 2 \lfloor \frac{a}{c} \rfloor \cdot g(a \bmod c, b \bmod c, c, n) \\ + 2 \lfloor \frac{b}{c} \rfloor \cdot f(a \bmod c, b \bmod c, c, n), & a \geq c \vee b \geq c \\ 0, & n < 0 \vee a = 0 \\ nm(m+1) - 2g(c, c-b-1, a, m-1) \\ - 2f(c, c-b-1, a, m-1) - f(a, b, c, n), & \text{otherwise} \end{cases}$$

## 6.15 General Purpose Numbers

### 6.15.1 Bernoulli numbers

$$B_0 = 1, B_1^\pm = \pm \frac{1}{2}, B_2 = \frac{1}{6}, B_3 = 0$$

$$\sum_{j=0}^m \binom{m+1}{j} B_j = 0, \text{ EGF is } B(x) = \frac{x}{e^x - 1} = \sum_{n=0}^{\infty} B_n \frac{x^n}{n!}.$$

$$S_m(n) = \sum_{k=1}^n k^m = \frac{1}{m+1} \sum_{k=0}^m \binom{m+1}{k} B_k^+ n^{m+1-k}$$

### 6.15.2 Stirling numbers of the second kind

Partitions of  $n$  distinct elements into exactly  $k$  groups.

$$S(n, k) = S(n-1, k-1) + kS(n-1, k), S(n, 1) = S(n, n) = 1$$

$$S(n, k) = \frac{1}{k!} \sum_{i=0}^k (-1)^{k-i} \binom{k}{i} i^n$$

## 7 Polynomial

### 7.1 Fast Fourier Transform

```

template<int MAXN>
struct FFT {
    using val_t = complex<double>;
    const double PI = acos(-1);
    val_t w[MAXN];
    FFT() {
        for (int i = 0; i < MAXN; ++i) {
            double arg = 2 * PI * i / MAXN;
            w[i] = val_t(cos(arg), sin(arg));
        }
    }
    void bitrev(val_t *a, int n); // see NTT

```

```
void trans(val_t *a, int n, bool inv = false); // see
NTT;
// remember to replace LL with val_t
};
```

## 7.2 Number Theory Transform

```
//(2^16)+1, 65537, 3
//7*17*(2^23)+1, 998244353, 3
//1255*(2^20)+1, 1315962881, 3
//51*(2^25)+1, 1711276033, 29
template<int MAXN, LL P, LL RT> //MAXN must be 2^k
struct NTT {
    LL w[MAXN];
    LL mpow(LL a, LL n);
    LL minv(LL a) { return mpow(a, P - 2); }
    NTT() {
        LL dw = mpow(RT, (P - 1) / MAXN);
        w[0] = 1;
        for (int i = 1; i < MAXN; ++i) w[i] = w[i - 1] * dw
            % P;
    }
    void bitrev(LL *a, int n) {
        int i = 0;
        for (int j = 1; j < n - 1; ++j) {
            for (int k = n >> 1; (i ^= k) < k; k >>= 1);
            if (j < i) swap(a[i], a[j]);
        }
    }
    void operator()(LL *a, int n, bool inv = false) { //0
        <= a[i] < P
        bitrev(a, n);
        for (int L = 2; L <= n; L <= 1) {
            int dx = MAXN / L, dl = L >> 1;
            for (int i = 0; i < n; i += L) {
                for (int j = i, x = 0; j < i + dl; ++j, x += dx)
                {
                    LL tmp = a[j + dl] * w[x] % P;
                    if ((a[j + dl] = a[j] - tmp) < 0) a[j + dl]
                        += P;
                    if ((a[j] += tmp) >= P) a[j] -= P;
                }
            }
        }
        if (inv) {
            reverse(a + 1, a + n);
            LL invn = minv(n);
            for (int i = 0; i < n; ++i) a[i] = a[i] * invn %
                P;
        }
    }
};
```

## 7.3 Fast Walsh Transform\*

```
/* x: a[j], y: a[j + (L >> 1)]
or: (y += x * op), and: (x += y * op)
xor: (x, y = (x + y) * op, (x - y) * op)
invop: or, and, xor = -1, -1, 1/2 */
void fwt(int *a, int n, int op) { //or
    for (int L = 2; L <= n; L <= 1)
        for (int i = 0; i < n; i += L)
            for (int j = i; j < i + (L >> 1); ++j)
                a[j + (L >> 1)] += a[j] * op;
}
const int N = 21;
int f[N][1 << N], g[N][1 << N], h[N][1 << N], ct[1 << N]
];
void subset_convolution(int *a, int *b, int *c, int L)
{
    // c_k = \sum_{i+j=k, i&j=0} a_i * b_j
    int n = 1 << L;
    for (int i = 1; i < n; ++i)
        ct[i] = ct[i & (i - 1)] + 1;
    for (int i = 0; i < n; ++i)
        f[ct[i]][i] = a[i], g[ct[i]][i] = b[i];
    for (int i = 0; i < L; ++i)
        fwt(f[i], n, 1), fwt(g[i], n, 1);
```

```
for (int i = 0; i <= L; ++i)
    for (int j = 0; j <= i; ++j)
        for (int x = 0; x < n; ++x)
            h[i][x] += f[j][x] * g[i - j][x];
for (int i = 0; i <= L; ++i)
    fwt(h[i], n, -1);
for (int i = 0; i < n; ++i)
    c[i] = h[ct[i]][i];
}
```

## 7.4 Polynomial Operation

```
#define fi(s, n) for (int i = (int)(s); i < (int)(n);
++i)
template<int MAXN, LL P, LL RT> // MAXN = 2^k
struct Poly : vector<LL> { // coefficients in [0, P)
    using vector<LL>::vector;
    static NTT<MAXN, P, RT> ntt;
    int n() const { return (int)size(); } // n() >= 1
    Poly(const Poly &p, int _n) : vector<LL>(_n) {
        copy_n(p.data(), min(p.n(), _n), data());
    }
    Poly& irev() { return reverse(data(), data() + n()),
        *this; }
    Poly& isz(int _n) { return resize(_n), *this; }
    Poly& iadd(const Poly &rhs) { // n() == rhs.n()
        fi(0, n()) if ((*this)[i] += rhs[i]) >= P (*this)
            [i] -= P;
        return *this;
    }
    Poly& imul(LL k) {
        fi(0, n()) (*this)[i] = (*this)[i] * k % P;
        return *this;
    }
    Poly Mul(const Poly &rhs) const {
        int _n = 1;
        while (_n < n() + rhs.n() - 1) _n <= 1;
        Poly X(*this, _n), Y(rhs, _n);
        ntt(X.data(), _n), ntt(Y.data(), _n);
        fi(0, _n) X[i] = X[i] * Y[i] % P;
        ntt(X.data(), _n, true);
        return X.isz(n() + rhs.n() - 1);
    }
    Poly Inv() const { // (*this)[0] != 0
        if (n() == 1) return {ntt.minv((*this)[0])};
        int _n = 1;
        while (_n < n() * 2) _n <= 1;
        Poly Xi = Poly(*this, (n() + 1) / 2).Inv().isz(_n);
        Poly Y(*this, _n);
        ntt(Xi.data(), _n), ntt(Y.data(), _n);
        fi(0, _n) {
            Xi[i] *= (2 - Xi[i] * Y[i]) % P;
            if ((Xi[i] %= P) < 0) Xi[i] += P;
        }
        ntt(Xi.data(), _n, true);
        return Xi.isz(n());
    }
    Poly Sqrt() const { // Jacobi((*this)[0], P) = 1
        if (n() == 1) return {QuadraticResidue((*this)[0],
            P)};
        Poly X = Poly(*this, (n() + 1) / 2).Sqrt().isz(n());
        ;
        return X.iadd(Mul(X.Inv()).isz(n())).imul(P / 2 +
            1);
    }
    pair<Poly, Poly> DivMod(const Poly &rhs) const { // (
        rhs.back() != 0
        if (n() < rhs.n()) return {{0}, *this};
        const int _n = n() - rhs.n() + 1;
        Poly X(rhs); X.irev().isz(_n);
        Poly Y(*this); Y.irev().isz(_n);
        Poly Q = Y.Mul(X.Inv()).isz(_n).irev();
        X = rhs.Mul(Q), Y = *this;
        fi(0, n()) if ((Y[i] -= X[i]) < 0) Y[i] += P;
        return {Q, Y.isz(max(1, rhs.n() - 1))};
    }
    Poly Dx() const {
        Poly ret(n() - 1);
        fi(0, ret.n()) ret[i] = (i + 1) * (*this)[i + 1] %
            P;
    }
```

```

    return ret.isz(max(1, ret.n()));
}
Poly Sx() const {
    Poly ret(n() + 1);
    fi(0, n()) ret[i + 1] = ntt.minv(i + 1) * (*this)[i]
        % P;
    return ret;
}
Poly _tmul(int nn, const Poly &rhs) const {
    Poly Y = Mul(rhs).isz(n() + nn - 1);
    return Poly(Y.data() + n() - 1, Y.data() + Y.n());
}
vector<LL> _eval(const vector<LL> &x, const vector<
    Poly> &up) const {
    const int _n = (int)x.size();
    if (!_n) return {};
    vector<Poly> down(_n * 2);
    down[1] = DivMod(up[1]).second;
    fi(2, _n * 2) down[i] = down[i / 2].DivMod(up[i]).
        second;
    /* down[1] = Poly(up[1]).irev().isz(n()).Inv().irev
        ()._tmul(_n, *this);
    fi(2, _n * 2) down[i] = up[i ^ 1]._tmul(up[i].n() -
        1, down[i / 2]); */
    vector<LL> y(_n);
    fi(0, _n) y[i] = down[_n + i][0];
    return y;
}
static vector<Poly> _tree1(const vector<LL> &x) {
    const int _n = (int)x.size();
    vector<Poly> up(_n * 2);
    fi(0, _n) up[_n + i] = {(x[i] ? P - x[i] : 0), 1};
    for (int i = _n - 1; i > 0; --i) up[i] = up[i * 2].
        Mul(up[i * 2 + 1]);
    return up;
}
vector<LL> Eval(const vector<LL> &x) const {
    auto up = _tree1(x); return _eval(x, up);
}
static Poly Interpolate(const vector<LL> &x, const
    vector<LL> &y) {
    const int _n = (int)x.size();
    vector<Poly> up = _tree1(x), down(_n * 2);
    vector<LL> z = up[1].Dx()._eval(x, up);
    fi(0, _n) z[i] = y[i] * ntt.minv(z[i]) % P;
    fi(0, _n) down[_n + i] = {z[i]};
    for (int i = _n - 1; i > 0; --i) down[i] = down[i *
        2].Mul(up[i * 2 + 1]).iadd(down[i * 2 + 1].Mul
        (up[i * 2]));
    return down[1];
}
Poly Ln() const { // (*this)[0] == 1
    return Dx().Mul(Inv()).Sx().isz(n());
}
Poly Exp() const { // (*this)[0] == 0
    if (n() == 1) return {1};
    Poly X = Poly(*this, (n() + 1) / 2).Exp().isz(n());
    Poly Y = X.Ln(); Y[0] = P - 1;
    fi(0, n()) if ((Y[i] = (*this)[i] - Y[i]) < 0) Y[i]
        += P;
    return X.Mul(Y).isz(n());
}
Poly Pow(const string &K) const {
    int nz = 0;
    while (nz < n() && !(*this)[nz]) ++nz;
    LL nk = 0, nk2 = 0;
    for (char c : K) {
        nk = (nk * 10 + c - '0') % P;
        nk2 = nk2 * 10 + c - '0';
        if (nk2 * nz >= n()) return Poly(n());
        nk2 %= P - 1;
    }
    if (!nk && !nk2) return Poly(Poly {1}, n());
    Poly X(data() + nz, data() + nz + n() - nz * nk2);
    LL x0 = X[0];
    return X.imul(ntt.minv(x0)).Ln().imul(nk).Exp()
        .imul(ntt.mpow(x0, nk2)).irev().isz(n()).irev();
}
static LL LinearRecursion(const vector<LL> &a, const
    vector<LL> &coef, LL n) { // a_n = \sum c_j a_{n-
        j}
    const int k = (int)a.size();

```

```

    assert((int)coef.size() == k + 1);
    Poly C(k + 1), W(Poly {1}, k), M = {0, 1};
    fi(1, k + 1) C[k - i] = coef[i] ? P - coef[i] : 0;
    C[k] = 1;
    while (n) {
        if (n % 2) W = W.Mul(M).DivMod(C).second;
        n /= 2, M = M.Mul(M).DivMod(C).second;
    }
    LL ret = 0;
    fi(0, k) ret = (ret + W[i] * a[i]) % P;
    return ret;
}
};
#undef fi
using Poly_t = Poly<131072 * 2, 998244353, 3>;
template<> decltype(Poly_t::ntt) Poly_t::ntt = {};

```

## 7.5 Newton's Method

Given  $F(x)$  where

$$F(x) = \sum_{i=0}^{\infty} \alpha_i (x - \beta)^i$$

for  $\beta$  being some constant. Polynomial  $P$  such that  $F(P) = 0$  can be found iteratively. Denote by  $Q_k$  the polynomial such that  $F(Q_k) = 0 \pmod{x^{2^k}}$ , then

$$Q_{k+1} = Q_k - \frac{F(Q_k)}{F'(Q_k)} \pmod{x^{2^{k+1}}}$$

## 8 Geometry

### 8.1 Default Code

```

typedef pair<double, double> pdd;
typedef pair<pdd, pdd> Line;
struct Cir { pdd O; double R; };
const double eps = 1e-8;
pdd operator+(const pdd &a, const pdd &b)
{ return pdd(a.X + b.X, a.Y + b.Y); }
pdd operator-(const pdd &a, const pdd &b)
{ return pdd(a.X - b.X, a.Y - b.Y); }
pdd operator*(const pdd &a, const double &b)
{ return pdd(a.X * b, a.Y * b); }
pdd operator/(const pdd &a, const double &b)
{ return pdd(a.X / b, a.Y / b); }
double dot(const pdd &a, const pdd &b)
{ return a.X * b.X + a.Y * b.Y; }
double cross(const pdd &a, const pdd &b)
{ return a.X * b.Y - a.Y * b.X; }
double abs2(const pdd &a)
{ return dot(a, a); }
double abs(const pdd &a)
{ return sqrt(dot(a, a)); }
int sign(const double &a)
{ return fabs(a) < eps ? 0 : a > 0 ? 1 : -1; }
int ori(const pdd &a, const pdd &b, const pdd &c)
{ return sign(cross(b - a, c - a)); }
bool collinearity(const pdd &p1, const pdd &p2, const
    pdd &p3)
{ return fabs(cross(p1 - p3, p2 - p3)) < eps; }
bool btw(const pdd &p1, const pdd &p2, const pdd &p3) {
    if (!collinearity(p1, p2, p3)) return 0;
    return dot(p1 - p3, p2 - p3) < eps;
}
bool seg_intersect(const pdd &p1, const pdd &p2, const
    pdd &p3, const pdd &p4) {
    int a123 = ori(p1, p2, p3);
    int a124 = ori(p1, p2, p4);
    int a341 = ori(p3, p4, p1);
    int a342 = ori(p3, p4, p2);
    if (a123 == 0 && a124 == 0)
        return btw(p1, p2, p3) || btw(p1, p2, p4) ||
            btw(p3, p4, p1) || btw(p3, p4, p2);
    return a123 * a124 <= 0 && a341 * a342 <= 0;
}

```

```
pdd intersect(const pdd &p1, const pdd &p2, const pdd &
    p3, const pdd &p4) {
    double a123 = cross(p2 - p1, p3 - p1);
    double a124 = cross(p2 - p1, p4 - p1);
    return (p4 * a123 - p3 * a124) / (a123 - a124);
}
pdd perp(const pdd &p1)
{ return pdd(-p1.Y, p1.X);}
pdd foot(const pdd &p1, const pdd &p2, const pdd &p3)
{ return intersect(p1, p2, p3, p3 + perp(p2 - p1));}
```

## 8.2 Convex hull\*

```
void hull(vector<p11> &dots) {
    sort(dots.begin(), dots.end());
    vector<p11> ans(1, dots[0]);
    for (int ct = 0; ct < 2; ++ct, reverse(ALL(dots)))
        for (int i = 1, t = SZ(ans); i < SZ(dots); ans.pb(
            dots[i++]))
            while (SZ(ans) > t && ori(ans[SZ(ans) - 2], ans.
                back(), dots[i]) <= 0)
                ans.pop_back();
    ans.pop_back(), ans.swap(dots);
}
```

## 8.3 External bisector

```
pdd external_bisector(pdd p1,pdd p2,pdd p3){//213
    pdd l1=p2-p1,l2=p3-p1;
    L2=L2*abs(L1)/abs(L2);
    return L1+L2;
}
```

## 8.4 Heart

```
pdd excenter(pdd p0,pdd p1,pdd p2,double &radius){
    p1=p1-p0,p2=p2-p0;
    double x1=p1.X,y1=p1.Y,x2=p2.X,y2=p2.Y;
    double m=2.*(x1*y2-y1*x2);
    center.X=(x1*x1*y2-x2*x2*y1+y1*y2*(y1-y2))/m;
    center.Y=(x1*x2*(x2-x1)-y1*y1*x2+x1*y2*y2)/m;
    return radius=abs(center),center+p0;
}

pdd incenter(pdd p1,pdd p2,pdd p3,double &radius){
    double a=abs(p2-p1),b=abs(p3-p1),c=abs(p3-p2);
    double s=(a+b+c)/2,area=sqrt(s*(s-a)*(s-b)*(s-c));
    pdd l1=external_bisector(p1,p2,p3),l2=
        external_bisector(p2,p1,p3);
    return radius=area/s,intersect(p1,p1+l1,p2,p2+l2),
}

pdd escenter(pdd p1,pdd p2,pdd p3){//213
    pdd l1=external_bisector(p1,p2,p3),l2=
        external_bisector(p2,p2+p2-p1,p3);
    return intersect(p1,p1+l1,p2,p2+l2);
}

pdd barycenter(pdd p1,pdd p2,pdd p3){
    return (p1+p2+p3)/3;
}

pdd orthocenter(pdd p1,pdd p2,pdd p3){
    pdd l1=p3-p2,l2=p3-p1;
    swap(l1.X,l1.Y),l1.X*=-1;
    swap(l2.X,l2.Y),l2.X*=-1;
    return intersect(p1,p1+l1,p2,p2+l2);
}
```

## 8.5 Minimum Enclosing Circle\*

```
pdd Minimum_Enclosing_Circle(vector<pdd> dots, double &
    r) {
    pdd cent;
    random_shuffle(ALL(dots));
    cent = dots[0], r = 0;
    for (int i = 1; i < SZ(dots); ++i)
        if (abs(dots[i] - cent) > r) {
            cent = dots[i], r = 0;
            for (int j = 0; j < i; ++j)
                if (abs(dots[j] - cent) > r) {
                    cent = (dots[i] + dots[j]) / 2;
                    r = abs(dots[i] - cent);
                    for (int k = 0; k < j; ++k)
                        if (abs(dots[k] - cent) > r)
                            cent = excenter(dots[i], dots[j], dots[k]
                                ], r);
                }
        }
    return cent;
}
```

## 8.6 Polar Angle Sort\*

```
pdd center;//sort base
int Quadrant(pdd a) {
    if(a.X > 0 && a.Y >= 0) return 1;
    if(a.X <= 0 && a.Y > 0) return 2;
    if(a.X < 0 && a.Y <= 0) return 3;
    if(a.X >= 0 && a.Y < 0) return 4;
}
bool cmp(p11 a, p11 b) {
    a = a - center, b = b - center;
    if (Quadrant(a) != Quadrant(b))
        return Quadrant(a) < Quadrant(b);
    if (cross(b, a) == 0) return abs2(a) < abs2(b);
    return cross(a, b) > 0;
}
bool cmp(pdd a, pdd b) {
    a = a - center, b = b - center;
    if(fabs(atan2(a.Y, a.X) - atan2(b.Y, b.X)) > eps)
        return atan2(a.Y, a.X) < atan2(b.Y, b.X);
    return abs(a) < abs(b);
}
```

## 8.7 Intersection of two circles\*

```
bool CCinter(Cir &a, Cir &b, pdd &p1, pdd &p2) {
    pdd o1 = a.o, o2 = b.o;
    double r1 = a.R, r2 = b.R, d2 = abs2(o1 - o2), d =
        sqrt(d2);
    if(d < max(r1, r2) - min(r1, r2) || d > r1 + r2)
        return 0;
    pdd u = (o1 + o2) * 0.5 + (o1 - o2) * ((r2 * r2 - r1
        * r1) / (2 * d2));
    double A = sqrt((r1 + r2 + d) * (r1 - r2 + d) * (r1 +
        r2 - d) * (-r1 + r2 + d));
    pdd v = pdd(o1.Y - o2.Y, -o1.X + o2.X) * A / (2 * d2)
        ;
    p1 = u + v, p2 = u - v;
    return 1;
}
```

## 8.8 Intersection of polygon and circle

```
// Divides into multiple triangle, and sum up
// test by HDU2892
const double PI=acos(-1);
double _area(pdd pa, pdd pb, double r){
    if(abs(pa)<abs(pb)) swap(pa, pb);
    if(abs(pb)<eps) return 0;
    double S, h, theta;
    double a=abs(pb),b=abs(pa),c=abs(pb-pa);
    double cosB = dot(pb,pb-pa) / a / c, B = acos(cosB);
    double cosC = dot(pa,pb) / a / b, C = acos(cosC);
    if(a > r){
        S = (C/2)*r*r;
    }
```

```

    h = a*b*sin(C)/c;
    if (h < r && B < PI/2) S -= (acos(h/r)*r*r - h*sqrt(
        (r*r-h*h)));
}
else if(b > r){
    theta = PI - B - asin(sin(B)/r*a);
    S = .5*a*r*sin(theta) + (C-theta)/2*r*r;
}
else S = .5*sin(C)*a*b;
return S;
}
double area_poly_circle(const vector<pdd> poly,const
    pdd &O,const double r){
    double S=0;
    for(int i=0;i<SZ(poly);++i)
        S+=_area(poly[i]-O,poly[(i+1)%SZ(poly)]-O,r)*ori(0,
            poly[i],poly[(i+1)%SZ(poly)]);
    return fabs(S);
}

```

## 8.9 Intersection of line and circle

```

vector<pdd> line_interCircle(const pdd &p1,const pdd &
    p2,const pdd &c,const double r){
    pdd ft=foot(p1,p2,c),vec=p2-p1;
    double dis=abs(c-ft);
    if(fabs(dis-r)<eps) return vector<pdd>{ft};
    if(dis>r) return {};
    vec=vec*sqrt(r*r-dis*dis)/abs(vec);
    return vector<pdd>{ft+vec,ft-vec};
}

```

## 8.10 point in circle

```

// return p4 is strictly in circumcircle of tri(p1,p2,
    p3)
long long sqr(long long x) { return x * x; }
bool in_cc(const p1l& p1, const p1l& p2, const p1l& p3,
    const p1l& p4) {
    long long u11 = p1.X - p4.X; long long u12 = p1.Y -
        p4.Y;
    long long u21 = p2.X - p4.X; long long u22 = p2.Y -
        p4.Y;
    long long u31 = p3.X - p4.X; long long u32 = p3.Y -
        p4.Y;
    long long u13 = sqr(p1.X) - sqr(p4.X) + sqr(p1.Y) -
        sqr(p4.Y);
    long long u23 = sqr(p2.X) - sqr(p4.X) + sqr(p2.Y) -
        sqr(p4.Y);
    long long u33 = sqr(p3.X) - sqr(p4.X) + sqr(p3.Y) -
        sqr(p4.Y);
    __int128 det = (__int128)-u13 * u22 * u31 + (
        __int128)u12 * u23 * u31 + (__int128)u13 * u21
        * u32 - (__int128)u11 * u23 * u32 - (__int128)
        u12 * u21 * u33 + (__int128)u11 * u22 * u33;
    return det > eps;
}

```

## 8.11 Half plane intersection

```

bool isin( Line l0, Line l1, Line l2 ){
    // Check inter(l1, l2) in l0
    pdd p = intersect(l1.X,l1.Y,l2.X,l2.Y);
    return cross(l0.Y - l0.X,p - l0.X) > eps;
}
/* If no solution, check: 1. ret.size() < 3
 * Or more precisely, 2. interPnt(ret[0], ret[1])
 * in all the lines. (use (l.Y - l.X) ^ (p - l.X) > 0
 */
/* --- Line.X --- Line.Y --- */
vector<Line> halfPlaneInter(vector<Line> lines){
    int sz = lines.size();
    vector<double> ata(sz),ord(sz);
    for(int i=0; i<sz; ++i) {
        ord[i] = i;
        pdd d = lines[i].Y - lines[i].X;

```

```

        ata[i] = atan2(d.Y, d.X);
    }
    sort(ord.begin(), ord.end(), [&](int i,int j){
        if( fabs(ata[i] - ata[j]) < eps )
            return (cross(lines[i].Y-lines[i].X,
                lines[j].Y-lines[j].X)<0;
        return ata[i] < ata[j];
    });
    vector<Line> fin;
    for (int i=0; i<sz; ++i)
        if (!i || fabs(ata[ord[i]] - ata[ord[i-1]]) > eps)
            fin.pb(lines[ord[i]]);
    deque<Line> dq;
    for (int i=0; i<SZ(fin); i++){
        while(SZ(dq)>=2&&!isin(fin[i],dq[SZ(dq)-2],dq.back(
            )))
            dq.pop_back();
        while(SZ(dq)>=2&&!isin(fin[i],dq[0],dq[1]))
            dq.pop_front();
        dq.push_back(fin[i]);
    }
    while(SZ(dq)>=3&&!isin(dq[0],dq[SZ(dq)-2],dq.back()))
        dq.pop_back();
    while(SZ(dq)>=3&&!isin(dq.back(), dq[0], dq[1]))
        dq.pop_front();
    vector<Line> res(ALL(dq));
    return res;
}

```

## 8.12 CircleCover\*

```

const int N = 1021;
struct CircleCover {
    int C;
    Cir c[N];
    bool g[N][N], overlap[N][N];
    // Area[i] : area covered by at least i circles
    double Area[ N ];
    void init(int _C){ C = _C; }
    struct Teve {
        pdd p; double ang; int add;
        Teve() {}
        Teve(pdd _a, double _b, int _c):p(_a), ang(_b), add(
            _c){}
        bool operator<(const Teve &a)const {
            return ang < a.ang; }
    }eve[N * 2];
    // strict: x = 0, otherwise x = -1
    bool disjuct(Cir &a, Cir &b, int x)
    {return sign(abs(a.O - b.O) - a.R - b.R) > x;}
    bool contain(Cir &a, Cir &b, int x)
    {return sign(a.R - b.R - abs(a.O - b.O)) > x;}
    bool contain(int i, int j) {
        /* c[j] is non-strictly in c[i]. */
        return (sign(c[i].R - c[j].R) > 0 || (sign(c[i].R -
            c[j].R) == 0 && i < j)) && contain(c[i], c[j],
            -1);
    }
    void solve(){
        fill_n(Area, C + 2, 0);
        for(int i = 0; i < C; ++i)
            for(int j = 0; j < C; ++j)
                overlap[i][j] = contain(i, j);
        for(int i = 0; i < C; ++i)
            for(int j = 0; j < C; ++j)
                g[i][j] = !(overlap[i][j] || overlap[j][i] ||
                    disjuct(c[i], c[j], -1));
        for(int i = 0; i < C; ++i){
            int E = 0, cnt = 1;
            for(int j = 0; j < C; ++j)
                if(j != i && overlap[j][i])
                    ++cnt;
            for(int j = 0; j < C; ++j)
                if(i != j && g[i][j]) {
                    pdd aa, bb;
                    CCinter(c[i], c[j], aa, bb);
                    double A = atan2(aa.Y - c[i].O.Y, aa.X - c[i]
                        ].O.X);
                    double B = atan2(bb.Y - c[i].O.Y, bb.X - c[i]
                        ].O.X);

```



```

    eve[E++] = Teve(bb, B, 1), eve[E++] = Teve(aa
        , A, -1);
    if(B > A) ++cnt;
}
if(E == 0) Area[cnt] += pi * c[i].R * c[i].R;
else{
    sort(eve, eve + E);
    eve[E] = eve[0];
    for(int j = 0; j < E; ++j){
        cnt += eve[j].add;
        Area[cnt] += cross(eve[j].p, eve[j + 1].p) *
            .5;
        double theta = eve[j + 1].ang - eve[j].ang;
        if (theta < 0) theta += 2. * pi;
        Area[cnt] += (theta - sin(theta)) * c[i].R *
            c[i].R * .5;
    }
}
}
};

```

### 8.13 3Dpoint\*

```

struct Point {
    double x, y, z;
    Point(double _x = 0, double _y = 0, double _z = 0): x
        (_x), y(_y), z(_z){}
    Point(pdd p) { x = p.X, y = p.Y, z = abs2(p); }
};
Point operator-(const Point &p1, const Point &p2)
{ return Point(p1.x - p2.x, p1.y - p2.y, p1.z - p2.z);}
Point cross(const Point &p1, const Point &p2)
{ return Point(p1.y * p2.z - p1.z * p2.y, p1.z * p2.x -
    p1.x * p2.z, p1.x * p2.y - p1.y * p2.x);}
double dot(const Point &p1, const Point &p2)
{ return p1.x * p2.x + p1.y * p2.y + p1.z * p2.z;}
double abs(const Point &a)
{ return sqrt(dot(a, a));}
Point cross3(const Point &a, const Point &b, const
    Point &c)
{ return cross(b - a, c - a);}
double area(Point a, Point b, Point c)
{ return abs(cross3(a, b, c));}
double volume(Point a, Point b, Point c, Point d)
{return dot(cross3(a, b, c), d - a);}
pdd proj(Point a, Point b, Point c, Point u) {
    // proj. u to the plane of a, b, and c
    Point e1 = b - a;
    Point e2 = c - a;
    e1 = e1 / abs(e1);
    e2 = e2 - e1 * dot(e2, e1);
    e2 = e2 / abs(e2);
    Point p = u - a;
    return pdd(dot(p, e1), dot(p, e2));
}

```

### 8.14 Convexhull3D\*

```

struct CH3D {
    struct face{int a, b, c; bool ok;} F[8 * N];
    double dblcmp(Point &p, face &f)
    {return dot(cross3(P[f.a], P[f.b], P[f.c]), p - P[f.a
        ]);}
    int g[N][N], num, n;
    Point P[N];
    void deal(int p, int a, int b) {
        int f = g[a][b];
        face add;
        if (F[f].ok) {
            if (dblcmp(P[p], F[f]) > eps) dfs(p, f);
        }
        else
            add.a = b, add.b = a, add.c = p, add.ok = 1, g[
                p][b] = g[a][p] = g[b][a] = num, F[num++] =
                add;
    }
    void dfs(int p, int now) {

```

```

        F[now].ok = 0;
        deal(p, F[now].b, F[now].a), deal(p, F[now].c, F[
            now].b), deal(p, F[now].a, F[now].c);
    }
    bool same(int s, int t){
        Point &a = P[F[s].a];
        Point &b = P[F[s].b];
        Point &c = P[F[s].c];
        return fabs(volume(a, b, c, P[F[t].a])) < eps &&
            fabs(volume(a, b, c, P[F[t].b])) < eps && fabs(
                volume(a, b, c, P[F[t].c])) < eps;
    }
    void init(int _n){n = _n, num = 0;}
    void solve() {
        face add;
        num = 0;
        if(n < 4) return;
        if([&](){
            for (int i = 1; i < n; ++i)
                if (abs(P[0] - P[i]) > eps)
                    return swap(P[1], P[i]), 0;
            return 1;
        }()) || [&](){
            for (int i = 2; i < n; ++i)
                if (abs(cross3(P[i], P[0], P[1])) > eps)
                    return swap(P[2], P[i]), 0;
            return 1;
        }()) || [&](){
            for (int i = 3; i < n; ++i)
                if (fabs(dot(cross(P[0] - P[1], P[1] - P[2]), P
                    [0] - P[i])) > eps)
                    return swap(P[3], P[i]), 0;
            return 1;
        }())return;
        for (int i = 0; i < 4; ++i) {
            add.a = (i + 1) % 4, add.b = (i + 2) % 4, add.c =
                (i + 3) % 4, add.ok = true;
            if (dblcmp(P[i], add) > 0) swap(add.b, add.c);
            g[add.a][add.b] = g[add.b][add.c] = g[add.c][add.
                a] = num;
            F[num++] = add;
        }
        for (int i = 4; i < n; ++i)
            for (int j = 0; j < num; ++j)
                if (F[j].ok && dblcmp(P[i], F[j]) > eps) {
                    dfs(i, j);
                    break;
                }
        for (int tmp = num, i = (num = 0); i < tmp; ++i)
            if (F[i].ok) F[num++] = F[i];
    }
    double get_area() {
        double res = 0.0;
        if (n == 3)
            return abs(cross3(P[0], P[1], P[2])) / 2.0;
        for (int i = 0; i < num; ++i)
            res += area(P[F[i].a], P[F[i].b], P[F[i].c]);
        return res / 2.0;
    }
    double get_volume() {
        double res = 0.0;
        for (int i = 0; i < num; ++i)
            res += volume(Point(0, 0, 0), P[F[i].a], P[F[i].b
                ], P[F[i].c]);
        return fabs(res / 6.0);
    }
    int triangle() {return num;}
    int polygon() {
        int res = 0;
        for (int i = 0, flag = 1; i < num; ++i, res += flag
            , flag = 1)
            for (int j = 0; j < i && flag; ++j)
                flag &= !same(i, j);
        return res;
    }
    Point getcent(){
        Point ans(0, 0, 0), temp = P[F[0].a];
        double v = 0.0, t2;
        for (int i = 0; i < num; ++i)
            if (F[i].ok == true) {
                Point p1 = P[F[i].a], p2 = P[F[i].b], p3 = P[F[
                    i].c];

```

```

    t2 = volume(temp, p1, p2, p3) / 6.0;
    if (t2 > 0)
        ans.x += (p1.x + p2.x + p3.x + temp.x) * t2,
        ans.y += (p1.y + p2.y + p3.y + temp.y) *
        t2, ans.z += (p1.z + p2.z + p3.z + temp.z
        ) * t2, v += t2;
    }
    ans.x /= (4 * v), ans.y /= (4 * v), ans.z /= (4 * v
    );
    return ans;
}
double pointmindis(Point p) {
    double rt = 99999999;
    for(int i = 0; i < num; ++i)
        if(F[i].ok == true) {
            Point p1 = P[F[i].a], p2 = P[F[i].b], p3 = P[F[
            i].c];
            double a = (p2.y - p1.y) * (p3.z - p1.z) - (p2.
            z - p1.z) * (p3.y - p1.y);
            double b = (p2.z - p1.z) * (p3.x - p1.x) - (p2.
            x - p1.x) * (p3.z - p1.z);
            double c = (p2.x - p1.x) * (p3.y - p1.y) - (p2.
            y - p1.y) * (p3.x - p1.x);
            double d = 0 - (a * p1.x + b * p1.y + c * p1.z)
            ;
            double temp = fabs(a * p.x + b * p.y + c * p.z
            + d) / sqrt(a * a + b * b + c * c);
            rt = min(rt, temp);
        }
    return rt;
}
};

```

## 8.15 DelaunayTriangulation\*

```

/* Delaunay Triangulation:
Given a sets of points on 2D plane, find a
triangulation such that no points will strictly
inside circumcircle of any triangle.
find : return a triangle contain given point
add_point : add a point into triangulation
A Triangle is in triangulation iff. its has_chd is 0.
Region of triangle u: iterate each u.edge[i].tri,
each points are u.p[(i+1)%3], u.p[(i+2)%3]
Voronoi diagram: for each triangle in triangulation,
the bisector of all its edges will split the region.
nearest point will belong to the triangle containing it
*/
const ll inf = MAXC * MAXC * 100; // Lower_bound
unknown
struct Tri;
struct Edge {
    Tri* tri; int side;
    Edge(): tri(0), side(0){}
    Edge(Tri* _tri, int _side): tri(_tri), side(_side)
    {}
};
struct Tri {
    pll p[3];
    Edge edge[3];
    Tri* chd[3];
    Tri() {}
    Tri(const pll& p0, const pll& p1, const pll& p2) {
        p[0] = p0; p[1] = p1; p[2] = p2;
        chd[0] = chd[1] = chd[2] = 0;
    }
    bool has_chd() const { return chd[0] != 0; }
    int num_chd() const {
        return !!chd[0] + !!chd[1] + !!chd[2];
    }
    bool contains(pll const& q) const {
        for (int i = 0; i < 3; ++i)
            if (ori(p[i], p[(i + 1) % 3], q) < 0)
                return 0;
        return 1;
    }
} pool[N * 10], *tris;
void edge(Edge a, Edge b) {
    if(a.tri) a.tri -> edge[a.side] = b;
    if(b.tri) b.tri -> edge[b.side] = a;
}

```

```

}
struct Trig { // Triangulation
    Trig() {
        the_root = // Tri should at least contain all
        points
        new(tris++) Tri(pll(-inf, -inf), pll(inf +
        inf, -inf), pll(-inf, inf + inf));
    }
    Tri* find(pll p) { return find(the_root, p); }
    void add_point(const pll &p) { add_point(find(
        the_root, p), p); }
    Tri* the_root;
    static Tri* find(Tri* root, const pll &p) {
        while (1) {
            if (!root -> has_chd())
                return root;
            for (int i = 0; i < 3 && root -> chd[i]; ++
            i)
                if (root -> chd[i] -> contains(p)) {
                    root = root -> chd[i];
                    break;
                }
        }
        assert(0); // "point not found"
    }
    void add_point(Tri* root, pll const& p) {
        Tri* t[3];
        /* split it into three triangles */
        for (int i = 0; i < 3; ++i)
            t[i] = new(tris++) Tri(root -> p[i], root
            -> p[(i + 1) % 3], p);
        for (int i = 0; i < 3; ++i)
            edge(Edge(t[i], 0), Edge(t[(i + 1) % 3], 1)
            );
        for (int i = 0; i < 3; ++i)
            edge(Edge(t[i], 2), root -> edge[(i + 2) %
            3]);
        for (int i = 0; i < 3; ++i)
            root -> chd[i] = t[i];
        for (int i = 0; i < 3; ++i)
            flip(t[i], 2);
    }
    void flip(Tri* tri, int pi) {
        Tri* trj = tri -> edge[pi].tri;
        int pj = tri -> edge[pi].side;
        if (!trj) return;
        if (!in_cc(tri -> p[0], tri -> p[1], tri -> p
        [2], trj -> p[pj])) return;
        /* flip edge between tri, trj */
        Tri* trk = new(tris++) Tri(tri -> p[(pi + 1) %
        3], trj -> p[pj], tri -> p[pi]);
        Tri* trl = new(tris++) Tri(trj -> p[(pj + 1) %
        3], tri -> p[pi], trj -> p[pj]);
        edge(Edge(trk, 0), Edge(trl, 0));
        edge(Edge(trk, 1), tri -> edge[(pi + 2) % 3]);
        edge(Edge(trk, 2), trj -> edge[(pj + 1) % 3]);
        edge(Edge(trl, 1), trj -> edge[(pj + 2) % 3]);
        edge(Edge(trl, 2), tri -> edge[(pi + 1) % 3]);
        tri -> chd[0] = trk; tri -> chd[1] = trl; tri
        -> chd[2] = 0;
        trj -> chd[0] = trk; trj -> chd[1] = trl; trj
        -> chd[2] = 0;
        flip(trk, 1); flip(trk, 2);
        flip(trl, 1); flip(trl, 2);
    }
};
vector<Tri*> triang; // vector of all triangle
set<Tri*> vst;
void go(Tri* now) { // store all tri into triang
    if (vst.find(now) != vst.end())
        return;
    vst.insert(now);
    if (!now -> has_chd())
        return triang.push_back(now);
    for (int i = 0; i < now -> num_chd(); ++i)
        go(now -> chd[i]);
}
void build(int n, pll* ps) { // build triangulation
    tris = pool; triang.clear(); vst.clear();
    random_shuffle(ps, ps + n);
    Trig tri; // the triangulation structure
    for (int i = 0; i < n; ++i)

```

```

    tri.add_point(ps[i]);
    go(tri.the_root);
}

```

## 8.16 Triangulation Voronoi\*

```

vector<Line> ls[N];
pll arr[N];
Line make_line(pdd p, Line l) {
    pdd d = l.Y - l.X; d = perp(d);
    pdd m = (l.X + l.Y) / 2;
    l = Line(m, m + d);
    if (ori(l.X, l.Y, p) < 0)
        l = Line(m + d, m);
    return l;
}
double calc_area(int id) {
    // use to calculate the area of point "strictly in
    // the convex hull"
    vector<Line> hpi = halfPlaneInter(ls[id]);
    vector<pdd> ps;
    for (int i = 0; i < SZ(hpi); ++i)
        ps.pb(intersect(hpi[i].X, hpi[i].Y, hpi[(i + 1) % SZ(hpi)].X, hpi[(i + 1) % SZ(hpi)].Y));
    double rt = 0;
    for (int i = 0; i < SZ(ps); ++i)
        rt += cross(ps[i], ps[(i + 1) % SZ(ps)]);
    return fabs(rt) / 2;
}
void solve(int n, pii *oarr) {
    map<pll, int> mp;
    for (int i = 0; i < n; ++i)
        arr[i] = pll(oarr[i].X, oarr[i].Y), mp[arr[i]] = i;
    build(n, arr); // Triangulation
    for (auto *t : triang) {
        vector<int> p;
        for (int i = 0; i < 3; ++i)
            if (mp.find(t -> p[i]) != mp.end())
                p.pb(mp[t -> p[i]]);
        for (int i = 0; i < SZ(p); ++i)
            for (int j = i + 1; j < SZ(p); ++j) {
                Line l(oarr[p[i]], oarr[p[j]]);
                ls[p[i]].pb(make_line(oarr[p[i]], l));
                ls[p[j]].pb(make_line(oarr[p[j]], l));
            }
    }
}

```

## 8.17 Tangent line of two circles

```

vector<Line> go( const Cir& c1 , const Cir& c2 , int
    sign1 ){
    // sign1 = 1 for outer tang, -1 for inter tang
    vector<Line> ret;
    double d_sq = norm2( c1.O - c2.O );
    if( d_sq < eps ) return ret;
    double d = sqrt( d_sq );
    Pt v = ( c2.O - c1.O ) / d;
    double c = ( c1.R - sign1 * c2.R ) / d;
    if( c * c > 1 ) return ret;
    double h = sqrt( max( 0.0 , 1.0 - c * c ) );
    for( int sign2 = 1 ; sign2 >= -1 ; sign2 -= 2 ){
        Pt n = { v.X * c - sign2 * h * v.Y ,
            v.Y * c + sign2 * h * v.X };
        Pt p1 = c1.O + n * c1.R;
        Pt p2 = c2.O + n * ( c2.R * sign1 );
        if( fabs( p1.X - p2.X ) < eps and
            fabs( p1.Y - p2.Y ) < eps )
            p2 = p1 + perp( c2.O - c1.O );
        ret.push_back( { p1 , p2 } );
    }
    return ret;
}

```

## 8.18 minMaxEnclosingRectangle

```

pdd solve(vector<pll> &dots){
    vector<pll> hull;
    const double INF=1e18, qi=acos(-1)/2*3;
    cv.dots=dots;
    hull=cv.hull();
    double Max=0, Min=INF, deg;
    ll n=hull.size();
    hull.pb(hull[0]);
    for(int i=0, u=1, r=1, l; i<n; ++i){
        pll nw=hull[i+1]-hull[i];
        while(cross(nw, hull[u+1]-hull[i])>cross(nw, hull[u]-hull[i]))
            u=(u+1)%n;
        while(dot(nw, hull[r+1]-hull[i])>dot(nw, hull[r]-hull[i]))
            r=(r+1)%n;
        if(!i) l=(r+1)%n;
        while(dot(nw, hull[l+1]-hull[i])<dot(nw, hull[l]-hull[i]))
            l=(l+1)%n;
        Min=min(Min, (double)(dot(nw, hull[r]-hull[i])-dot(nw, hull[l]-hull[i]))*cross(nw, hull[u]-hull[i])/abs2(nw));
        deg=acos((double)dot(hull[r]-hull[l], hull[u]-hull[i])/abs(hull[r]-hull[l])/abs(hull[u]-hull[i]));
        deg=(qi-deg)/2;
        Max=max(Max, (double)abs(hull[r]-hull[l])*abs(hull[u]-hull[i])*sin(deg)*sin(deg));
    }
    return pdd(Min, Max);
}

```

## 8.19 minDistOfTwoConvex

```

// p, q is convex
double TwoConvexHullMinDist(Point P[], Point Q[], int n, int m) {
    int YMinP = 0, YMaxQ = 0;
    double tmp, ans = 999999999;
    for (i = 0; i < n; ++i) if(P[i].y < P[YMinP].y) YMinP = i;
    for (i = 0; i < m; ++i) if(Q[i].y > Q[YMaxQ].y) YMaxQ = i;
    P[n] = P[0], Q[m] = Q[0];
    for (int i = 0; i < n; ++i) {
        while (tmp = Cross(Q[YMaxQ + 1] - P[YMinP + 1], P[YMinP] - P[YMinP + 1]) > Cross(Q[YMaxQ] - P[YMinP + 1], P[YMinP] - P[YMinP + 1])) YMaxQ = (YMaxQ + 1) % m;
        if (tmp < 0) ans = min(ans, PointToSegDist(P[YMinP], P[YMinP + 1], Q[YMaxQ]));
        else ans = min(ans, TwoSegMinDist(P[YMinP], P[YMinP + 1], Q[YMaxQ], Q[YMaxQ + 1]));
        YMinP = (YMinP + 1) % n;
    }
    return ans;
}

```

## 8.20 Minkowski Sum\*

```

vector<pll> Minkowski(vector<pll> A, vector<pll> B) {
    hull(A), hull(B);
    vector<pll> C(1, A[0] + B[0]), s1, s2;
    for(int i = 0; i < SZ(A); ++i)
        s1.pb(A[(i + 1) % SZ(A)] - A[i]);
    for(int i = 0; i < SZ(B); ++i)
        s2.pb(B[(i + 1) % SZ(B)] - B[i]);
    for(int p1 = 0, p2 = 0; p1 < SZ(A) || p2 < SZ(B);)
        if (p2 >= SZ(B) || (p1 < SZ(A) && cross(s1[p1], s2[p2]) >= 0))
            C.pb(C.back() + s1[p1++]);
        else
            C.pb(C.back() + s2[p2++]);
    return hull(C), C;
}

```

## 8.21 RotatingSweepLine

```
void rotatingSweepLine(vector<pii> &ps) {
    int n = SZ(ps);
    vector<int> id(n), pos(n);
    vector<pii> line(n * (n - 1) / 2);
    int m = 0;
    for (int i = 0; i < n; ++i)
        for (int j = i + 1; j < n; ++j)
            line[m++] = pii(i, j);
    sort(ALL(line), [&](const pii &a, const pii &b) ->
        bool {
            if (ps[a.X].X == ps[a.Y].X)
                return 0;
            if (ps[b.X].X == ps[b.Y].X)
                return 1;
            return (double)(ps[a.X].Y - ps[a.Y].Y) / (ps[a.X].X - ps[a.Y].X) < (double)(ps[b.X].Y - ps[b.Y].Y) / (ps[b.X].X - ps[b.Y].X);
        });
    iota(id, id + n, 0);
    sort(ALL(id), [&](const int &a, const int &b) { return ps[a] < ps[b]; });
    for (int i = 0; i < n; ++i) pos[id[i]] = i;
    for (int i = 0; i < m; ++i) {
        auto l = line[i];
        // meow
        tie(pos[l.X], pos[l.Y], id[pos[l.X]], id[pos[l.Y]]) = make_tuple(pos[l.Y], pos[l.X], l.Y, l.X);
    }
}
```

## 9 Else

### 9.1 Mo's Alogrithm(With modification)

```
struct QUERY{//BLOCK=N^{2/3}
    int L,R,id,LBId,RBId,T;
    QUERY(int l,int r,int id,int lb,int rb,int t):
        L(l),R(r),id(id),LBId(lb),RBId(rb),T(t){}
    bool operator<(const QUERY &b)const{
        if(LBId!=b.LBId) return LBId<b.LBId;
        if(RBId!=b.RBId) return RBId<b.RBId;
        return T<b.T;
    }
};
vector<QUERY> query;
int cur_ans,arr[MAXN],ans[MAXN];
void addTime(int L,int R,int T){}
void subTime(int L,int R,int T){}
void add(int x){}
void sub(int x){}
void solve(){
    sort(ALL(query));
    int L=0,R=0,T=-1;
    for(auto q:query){
        while(T<q.T) addTime(L,R,++T);
        while(T>q.T) subTime(L,R,T--);
        while(R<q.R) add(arr[++R]);
        while(L>q.L) add(arr[--L]);
        while(R>q.R) sub(arr[R--]);
        while(L<q.L) sub(arr[L++]);
        ans[q.id]=cur_ans;
    }
}
```

### 9.2 Mo's Alogrithm On Tree

```
const int MAXN=40005;
vector<int> G[MAXN];//1-base
int n,B,arr[MAXN],ans[100005],cur_ans;
int in[MAXN],out[MAXN],dfn[MAXN*2],dft;
int deep[MAXN],sp[___lg(MAXN*2)+1][MAXN*2],bln[MAXN],spt;
bitset<MAXN> inset;
```

```
struct QUERY{
    int L,R,Lid,id,lca;
    QUERY(int l,int r,int _id):L(l),R(r),lca(0),id(_id){}
    bool operator<(const QUERY &b){
        if(Lid!=b.Lid) return Lid<b.Lid;
        return R<b.R;
    }
};
vector<QUERY> query;
void dfs(int u,int f,int d){
    deep[u]=d,sp[0][spt]=u,bln[u]=spt++;
    dfn[dft]=u,in[u]=dft++;
    for(int v:G[u])
        if(v!=f)
            dfs(v,u,d+1),sp[0][spt]=u,bln[u]=spt++;
    dfn[dft]=u,out[u]=dft++;
}
int lca(int u,int v){
    if(bln[u]>bln[v]) swap(u,v);
    int t=___lg(bln[v]-bln[u]+1);
    int a=sp[t][bln[u]],b=sp[t][bln[v]-(1<<t)+1];
    if(deep[a]<deep[b]) return a;
    return b;
}
void sub(int x){}
void add(int x){}
void flip(int x){
    if(inset[x]) sub(arr[x]);
    else add(arr[x]);
    inset[x]=~inset[x];
}
void solve(){
    B=sqrt(2*n),dft=spt=cur_ans=0,dfs(1,1,0);
    for(int i=1,x=2;x<2*n;++i,x<=1)
        for(int j=0;j+x<=2*n;++j)
            if(deep[sp[i-1][j]]<deep[sp[i-1][j+x/2]])
                sp[i][j]=sp[i-1][j];
            else sp[i][j]=sp[i-1][j+x/2];
    for(auto &q:query){
        int c=lca(q.L,q.R);
        if(c==q.L||c==q.R)
            q.L=out[c==q.L?q.R:q.L],q.R=out[c];
        else if(out[q.L]<in[q.R])
            q.lca=c,q.L=out[q.L],q.R=in[q.R];
        else q.lca=c,c=in[q.L],q.L=out[q.R],q.R=c;
        q.Lid=q.L/B;
    }
    sort(ALL(query));
    int L=0,R=-1;
    for(auto q:query){
        while(R<q.R) flip(dfn[++R]);
        while(L>q.L) flip(dfn[--L]);
        while(R>q.R) flip(dfn[R--]);
        while(L<q.L) flip(dfn[L++]);
        if(q.lca) add(arr[q.lca]);
        ans[q.id]=cur_ans;
        if(q.lca) sub(arr[q.lca]);
    }
}
```

### 9.3 DynamicConvexTrick\*

```
// only works for integer coordinates!!
struct Line {
    mutable ll a, b, p;
    bool operator<(const Line &rhs) const { return a < rhs.a; }
    bool operator<(ll x) const { return p < x; }
};
struct DynamicHull : multiset<Line, less<>> {
    static const ll kInf = 1e18;
    ll Div(ll a, ll b) { return a / b - ((a ^ b) < 0 && a % b); }
    bool isect(iterator x, iterator y) {
        if (y == end()) { x -> p = kInf; return 0; }
        if (x -> a == y -> a) x -> p = x -> b > y -> b ? kInf : -kInf;
        else x -> p = Div(y -> b - x -> b, x -> a - y -> a);
        return x -> p >= y -> p;
    }
};
```

```

}
void addline(ll a, ll b) {
    auto z = insert({a, b, 0}), y = z++, x = y;
    while (isect(y, z)) z = erase(z);
    if (x != begin() && isect(--x, y)) isect(x, y =
        erase(y));
    while ((y = x) != begin() && (--x) -> p >= y ->
        p) isect(x, erase(y));
}
ll query(ll x) {
    auto l = *lower_bound(x);
    return l.a * x + l.b;
}
};

```

## 9.4 DLX\*

```

#define TRAV(i, link, start) for (int i = link[start];
    i != start; i = link[i])
template<bool A, bool B = !A> // A: Exact
struct DLX {
    int lt[NN], rg[NN], up[NN], dn[NN], cl[NN], rw[NN],
        bt[NN], s[NN], head, sz, ans;
    int columns;
    bool vis[NN];
    void remove(int c) {
        if (A) lt[rg[c]] = lt[c], rg[lt[c]] = rg[c];
        TRAV(i, dn, c) {
            if (A) {
                TRAV(j, rg, i)
                up[dn[j]] = up[j], dn[up[j]] = dn[j], --s[cl[
                    j]];
            } else {
                lt[rg[i]] = lt[i], rg[lt[i]] = rg[i];
            }
        }
    }
    void restore(int c) {
        TRAV(i, up, c) {
            if (A) {
                TRAV(j, lt, i)
                ++s[cl[j]], up[dn[j]] = j, dn[up[j]] = j;
            } else {
                lt[rg[i]] = rg[lt[i]] = i;
            }
        }
        if (A) lt[rg[c]] = c, rg[lt[c]] = c;
    }
    void init(int c) {
        columns = c;
        for (int i = 0; i < c; ++i) {
            up[i] = dn[i] = bt[i] = i;
            lt[i] = i == 0 ? c : i - 1;
            rg[i] = i == c - 1 ? c : i + 1;
            s[i] = 0;
        }
        rg[c] = 0, lt[c] = c - 1;
        up[c] = dn[c] = -1;
        head = c, sz = c + 1;
    }
    void insert(int r, const vector<int> &col) {
        if (col.empty()) return;
        int f = sz;
        for (int i = 0; i < (int)col.size(); ++i) {
            int c = col[i], v = sz++;
            dn[bt[c]] = v;
            up[v] = bt[c], bt[c] = v;
            rg[v] = (i + 1 == (int)col.size() ? f : v + 1);
            rw[v] = r, cl[v] = c;
            ++s[c];
            if (i > 0) lt[v] = v - 1;
        }
        lt[f] = sz - 1;
    }
    int h() {
        int ret = 0;
        memset(vis, 0, sizeof(bool) * sz);
        TRAV(x, rg, head) {
            if (vis[x]) continue;
            vis[x] = true, ++ret;

```

```

            TRAV(i, dn, x) TRAV(j, rg, i) vis[cl[j]] = true;
        }
        return ret;
    }
    void dfs(int dep) {
        if (dep + (A ? 0 : h()) >= ans) return;
        if (rg[head] == head) return ans = dep, void();
        if (dn[rg[head]] == rg[head]) return;
        int w = rg[head];
        TRAV(x, rg, head) if (s[x] < s[w]) w = x;
        if (A) remove(w);
        TRAV(i, dn, w) {
            if (B) remove(i);
            TRAV(j, rg, i) remove(A ? cl[j] : j);
            dfs(dep + 1);
            TRAV(j, lt, i) restore(A ? cl[j] : j);
            if (B) restore(i);
        }
        if (A) restore(w);
    }
    int solve() {
        for (int i = 0; i < columns; ++i)
            dn[bt[i]] = i, up[i] = bt[i];
        ans = 1e9, dfs(0);
        return ans;
    }
};

```

## 9.5 Matroid Intersection

Start from  $S = \emptyset$ . In each iteration, let

- $Y_1 = \{x \notin S \mid S \cup \{x\} \in I_1\}$
- $Y_2 = \{x \notin S \mid S \cup \{x\} \in I_2\}$

If there exists  $x \in Y_1 \cap Y_2$ , insert  $x$  into  $S$ . Otherwise for each  $x \in S, y \notin S$ , create edges

- $x \rightarrow y$  if  $S - \{x\} \cup \{y\} \in I_1$ .
- $y \rightarrow x$  if  $S - \{x\} \cup \{y\} \in I_2$ .

Find a *shortest* path (with BFS) starting from a vertex in  $Y_1$  and ending at a vertex in  $Y_2$  which doesn't pass through any other vertices in  $Y_2$ , and alternate the path. The size of  $S$  will be incremented by 1 in each iteration. For the weighted case, assign weight  $w(x)$  to vertex  $x$  if  $x \in S$  and  $-w(x)$  if  $x \notin S$ . Find the path with the minimum number of edges among all minimum length paths and alternate it.

## 9.6 AdaptiveSimpson

```

using F_t = function<double(double)>;
pdd simpson(const F_t &f, double l, double r,
    double fl, double fr, double fm = nan("")) {
    if (isnan(fm)) fm = f((l + r) / 2);
    return {fm, (r - l) / 6 * (fl + 4 * fm + fr)};
}
double simpson_ada(const F_t &f, double l, double r,
    double fl, double fm, double fr, double eps) {
    double m = (l + r) / 2;
    s = simpson(f, l, r, fl, fr, fm).second;
    auto [flm, sl] = simpson(f, l, m, fl, fm);
    auto [fmr, sr] = simpson(f, m, r, fm, fr);
    double delta = sl + sr - s;
    if (abs(delta) <= 15 * eps)
        return sl + sr + delta / 15;
    return simpson_ada(f, l, m, fl, flm, fm, eps / 2) +
        simpson_ada(f, m, r, fm, fmr, fr, eps / 2);
}
double simpson_ada(const F_t &f, double l, double r) {
    return simpson_ada(
        f, l, r, f(l), f((l + r) / 2), f(r), 1e-9 / 7122);
}
double simpson_ada2(const F_t &f, double l, double r) {
    double h = (r - l) / 7122, s = 0;
    for (int i = 0; i < 7122; ++i, l += h)
        s += simpson_ada(f, l, l + h);
    return s;
}

```