STOR 664 HW 2

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Problem 20

```r
library(data.table)
dmark <- fread('http://rls.sites.oasis.unc.edu/faculty/rs/source/Data/dmark.dat')
#pull in data from website
head(dmark)
```

```
##    V1    V2    V3
## 1:  1 1.153 1.529
## 2:  2 1.152 1.530
## 3:  3 1.161 1.490
## 4:  4 1.161 1.483
## 5:  5 1.159 1.457
## 6:  6 1.154 1.460
```

```r
colnames(dmark)[] <- c("week", "mark", "pound")
#give variables descriptive names
colnames(dmark)
```

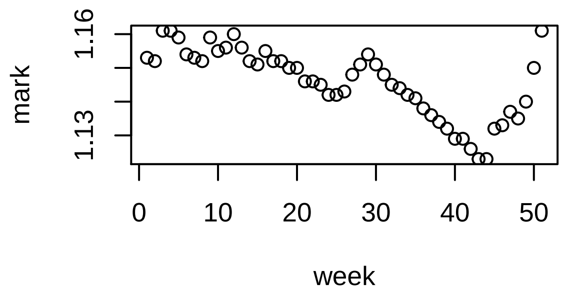
```
## [1] "week"  "mark"  "pound"
```

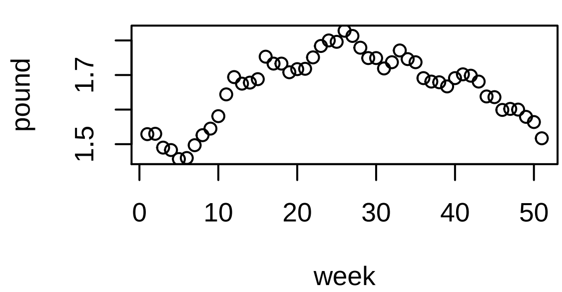
```r
attach(dmark)
#column names of mydat recognized independently
```

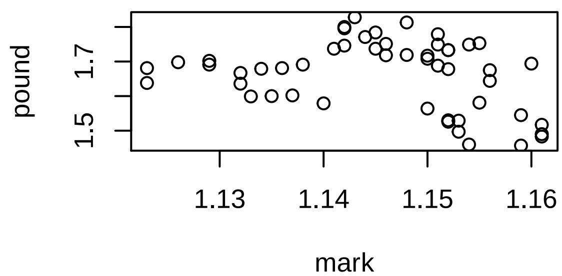
(a)

Note, there is strong visual evidence of autocorrelation in the time series.

```r
par(mfrow=c(2, 2))
plot(mark~week)
plot(pound~week)
plot(pound~mark)
```







(b)

Assume the individaul weekly observaions are independent. The linear regression equation $\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$ is computed via the code below. A point estimate for $\hat{\beta}_1$ is approximately -2.9 with a 90% confidence interval of $[-5.12, -0.69]$. Consider the following hypotheses: $H_0 : \beta_1 = 0$ vs $H_1 : \beta_1 \neq 0$ with $\alpha = 0.1$. Observe that 0 is not an element of $[-5.12, -0.69]$, the 90% confidence interval for β_1. Thus, H_0 is rejected; there is evidence to suggest that $\beta_1 \neq 0$.

```r
mp_lm <- lm(pound~mark)
summary(mp_lm)
```

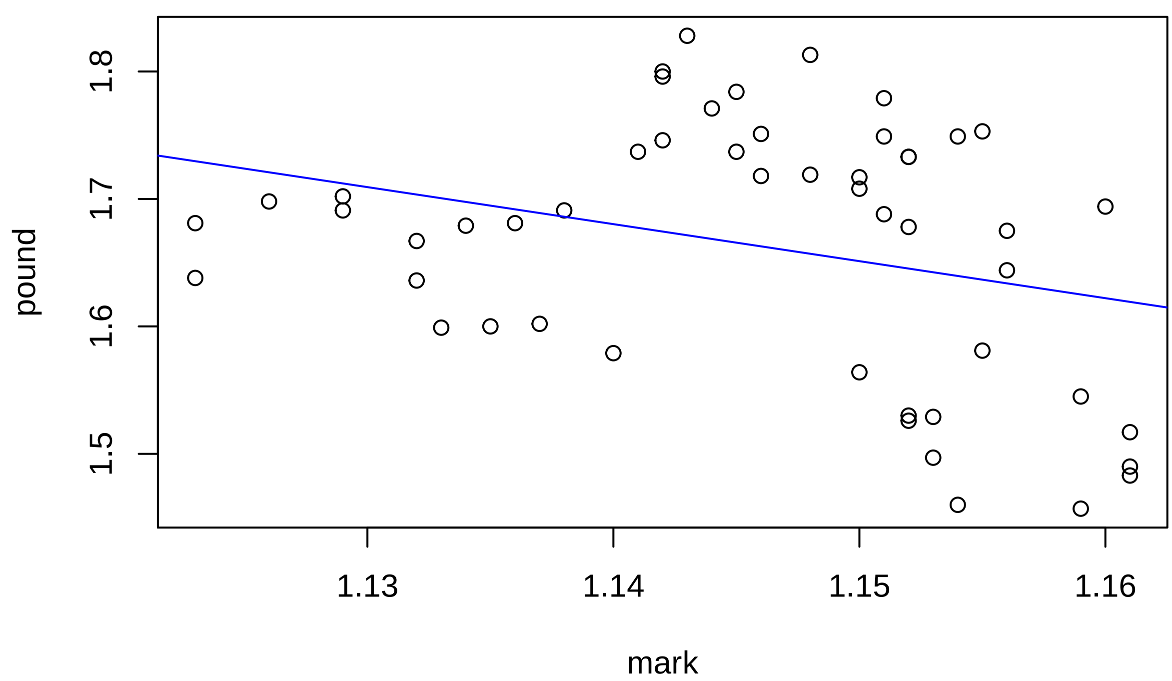
```
##
## Call:
## lm(formula = pound ~ mark)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.179567 -0.089384  0.004972  0.079741  0.156491
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)    4.991      1.516   3.292  0.00185 **
## mark          -2.904      1.323  -2.195  0.03293 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.09665 on 49 degrees of freedom
```

```
## Multiple R-squared:  0.08952,    Adjusted R-squared:  0.07094
## F-statistic: 4.818 on 1 and 49 DF,  p-value: 0.03293
```

```
confint(mp_lm, level=0.90)
```

```
##                     5 %       95 %
## (Intercept)  2.449006  7.5321114
## mark        -5.121743 -0.6858684
```

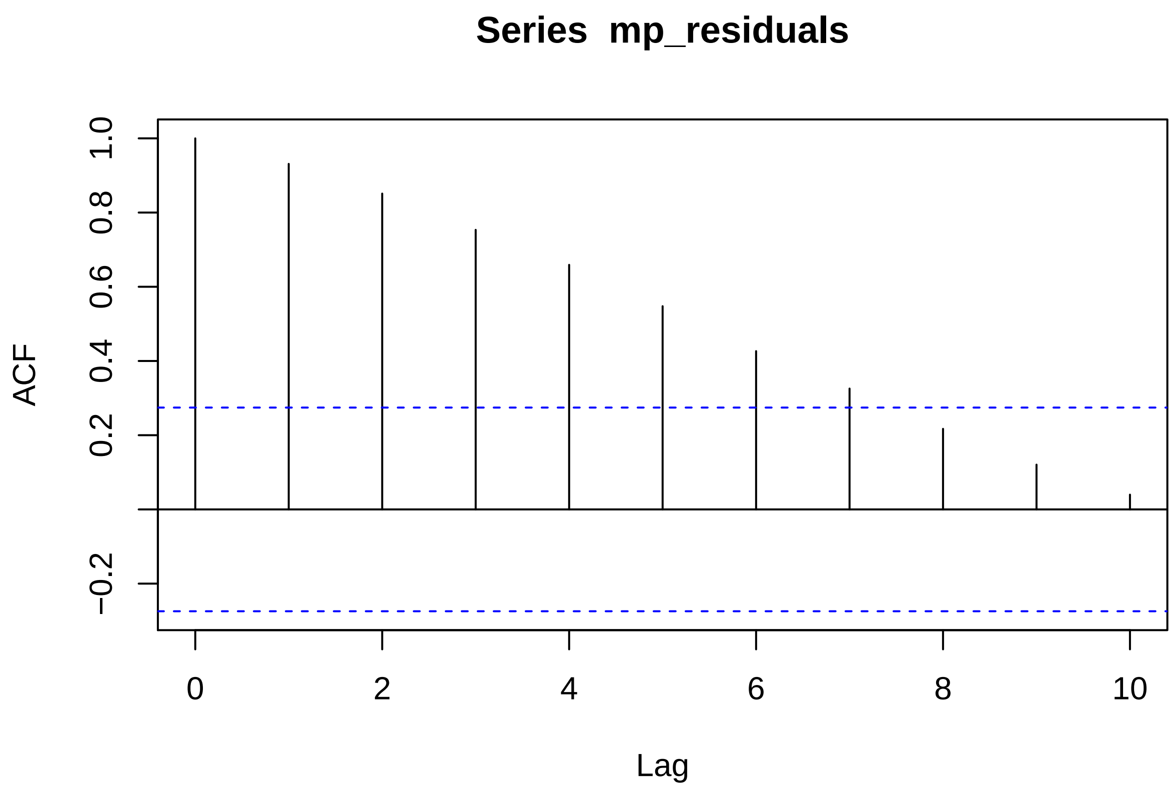
```
plot(pound~mark)
abline(mp_lm, col="blue")
```



(c)

Inspection of the first 10 autocorrelation coeffecients are computed from the residuals via the acf(.) command. Inspection of these, together with the heuristic $\frac{2}{\sqrt{n}} = \frac{2}{\sqrt{51}} \approx 0.28$, suggests that the first 8 autocorrelations are statistically significant. There is clearly evidence that autocorrelation is present. A more precise test, such as the Durbin-Watson could be performed to confirm this heuristic argument.

```
#the residuals of the linear model in question
mp_residuals <- mp_lm$residuals
#the first 10 serial correlations are computed with
#approximate 95% error bounds if the true time series is independent.
mp_ac <- acf(mp_residuals, lag.max=10)
```




```
mp_ac
```

```
##
## Autocorrelations of series 'mp_residuals', by lag
##
##     0     1     2     3     4     5     6     7     8     9    10
## 1.000 0.931 0.851 0.753 0.659 0.548 0.427 0.326 0.217 0.121 0.040
```

```
#the heursitic used to determine statistical significance of the autocorrelations
heuristic<-2/sqrt(51)
#the indices of the autocorrelations that are statistically significant.
which(abs(mp_ac$acf)>heuristic)
```

```
## [1] 1 2 3 4 5 6 7 8
```

```
#Code to compute the Durbin Watson test statistic
dw_num <- rep(0,50)
for(i in 2:51){
  dw_num[i] <- (mp_residuals[i]-mp_residuals[i-1])^2
}

D=sum(dw_num)/sum(mp_residuals^2)
D
```

```
## [1] 0.08659338
```

(d) As autocorrelation is present, the standard deviation of the least squared estimates must be corrected. In particular, the corrected standard deviation of $\hat{\beta}_1$ is computed below for K=8. Note, this corrected value is about 3.77, in contrast to the original value of 1.32. With this new value, observe that the test statistic for the previously considerd hypotheses is $t = -\frac{2.195}{3.77} \approx -0.771$. The p-value associated with this test statistic, with respect to a t_{n-2} distribution, is greater than the 0.1 threshold. Thus, in contrast to the previous conclusion, the evidence does not support the rejection of H_0. In otherwords, there is not statistically significant evidence to suggest that $\beta_1 \neq 0$.

```
rse <- sum((mp_lm$residuals)^2)/49
beta1_se=sqrt(rse/sum((mark-mean(mark))^2))
beta1_se
```

```
## [1] 1.322917
```

```
#the denominator of r_x(k) for all k=1,..,8
k_denom <- vector()
  for(i in 1:51){
    k_denom[i] <- (mark[i]-mean(mark))^2
  }

#dummy vector to be used in the for loop below
rxk_terms <-list(vector(), vector(), vector(), vector(), vector(), vector(), vector(), vector())

#the lag
K=8

#this for loop generates the vectors that are stored in rxk_terms. Each one contains the terms of the s
#the numerator of the ratio that defines r_x(k)
for(i in 1:K){
  for(j in 1:(51-i)){
    rxk_terms[[i]][j] <- (mark[j]-mean(mark))*(mark[j+i]-mean(mark))
  }
}

#dummy vector to be used in the for loop below
a <- vector()

#this for loop fills the dummy vector a with the r_x(k) values where k=1,...,8; where the k'th entry co
for(i in 1:K){
  a[i] <-as.numeric(lapply(rxk_terms, sum)[i])/sum(k_denom)
}

#create vector of terms of the sum in the second term of
#the scaling factor for the corrected variance slope estimate
b <- vector()
for(i in 1:8){
  b[i] <- (mp_ac$acf[i])*a[i]
}

#the slope estimate standard deviation corrected for
#the presence of autocorrelation in the residuals.
beta1_se_corrected <- sqrt(((beta1_se)^2)*(1+2*sum(b)))
beta1_se_corrected
```

```
## [1] 3.765597
```

```
#t statistic with new standard error
t=(mp_lm$coefficients[2])/beta1_se_corrected
t
```

```
##       mark
## -0.7711408
```

```
qt(.05, 49)
```

```
## [1] -1.676551
```

Problem 21

First, the data set is imported and the columns given descriptive names.

```
marathon <- fread('http://rls.sites.oasis.unc.edu/faculty/rs/source/Data/marathon.dat')
#pull in data from website
head(marathon)
```

```
##          V1      V2
## 1: 1.000178  9402.5
## 2: 1.000178  9404.0
## 3: 1.000178  9402.0
## 4: 1.000178  9403.0
## 5: 0.000000 12163.5
## 6: 0.000000 14981.5
```

```
colnames(marathon)[] <- c("length", "count")
#give variables descriptive names
colnames(marathon)
```

```
## [1] "length" "count"
```

```
#column names of mydat recognized independently
```

Next, approximate inverse regression is performed. Note, a zero in the lenght column corresponds to a missing value. I will use inverse regression to predict these unknown length values from the corresponding known count value. Further, I will produce standard errors for these estimates and a 95% confidence interval for the total length of the unknown length entries. The predicted values, along with their corresponding y values and standard errors, are compiled in the data frame named 'results_df'. This data frame is output below.

```
#remove rows with unkown x values from data
marathon_known <- marathon[-c(which(marathon$length==0)),]

#make variable names usable without reference to data set
attach(marathon_known)
```

```r
#perform linear regression on the known data
lm_marathon <- lm(count~length)

#create data vectors for new unknown x and known y values
new_y <- marathon$count[which(marathon$length==0)]
new_x <- vector()

#compute inverse regression estimates of
#the unknown length values using the corresponding known y values
for(i in 1:13){
  new_x[i] <- mean(length)+(new_y[i]-lm_marathon$coefficients[1])/lm_marathon$coefficients[2]
}

#the predicted x values
new_x
```

```
##  [1] 2.185045 2.485169 4.466438 5.127020 2.808511 3.440232 5.157214 2.922842
##  [9] 3.668254 6.194922 1.500499 1.465141 1.058460
```

```r
#compute the prediction standard errors of the length estimates
summary(lm_marathon)
```

```
##
## Call:
## lm(formula = count ~ length)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -9.772 -7.727  1.230  7.063  8.938
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)     3.95      10.33   0.382     0.71
## length       9389.44      11.34 828.238   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 7.652 on 10 degrees of freedom
## Multiple R-squared:      1,  Adjusted R-squared:      1
## F-statistic: 6.86e+05 on 1 and 10 DF,  p-value: < 2.2e-16
```

```r
resid_se=7.652


#compute the sum of the squared known centered length values
length_c <- vector()
for(i in 1:12){
  length_c[i] <- length[i]-mean(length)
}

d=sum(length_c^2)
```

```r
#compute the standard errors of the new length estimates
se_x <- vector()
for(i in 1:13){
  se_x[i] <- (resid_se/lm_marathon$coefficients[2])*sqrt((1/12)+(((new_x[i]-mean(length))^2)/d)+1)
}

#standard errors of the predicted x values
se_x
```

```
##  [1] 0.0017788184 0.0021044290 0.0044005232 0.0051854019 0.0024667237
##  [6] 0.0031937081 0.0052213685 0.0025967770 0.0034598964 0.0064608123
## [11] 0.0011237280 0.0010962006 0.0008722732
```

```r
results_df <- data.frame(new_x=new_x, y=marathon$count[which(marathon$length==0)], se_new_x=se_x)
results_df
```

```
##       new_x       y     se_new_x
## 1  2.185045 12163.5 0.0017788184
## 2  2.485169 14981.5 0.0021044290
## 3  4.466438 33584.5 0.0044005232
## 4  5.127020 39787.0 0.0051854019
## 5  2.808511 18017.5 0.0024667237
## 6  3.440232 23949.0 0.0031937081
## 7  5.157214 40070.5 0.0052213685
## 8  2.922842 19091.0 0.0025967770
## 9  3.668254 26090.0 0.0034598964
## 10 6.194922 49814.0 0.0064608123
## 11 1.500499  5736.0 0.0011237280
## 12 1.465141  5404.0 0.0010962006
## 13 1.058460  1585.5 0.0008722732
```

```r
list_test <- list(c(1, 2, 3), c("a", "b"), c(2, 4, 5))
list_test[1]
```

```
## [[1]]
## [1] 1 2 3
```

```r
list_test[[3]][2]
```

```
## [1] 4
```