STOR 665 HW 7

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4/23/2021

Problem 7

(a)

The data in the ergoStool dataframe comes from an ergometrics experiment. In this experiment nine subjects were asked to sit on four stools, each with a different design type, and rate the difficulty of rising. The experimenter opted to measure 'effort' via the Borg scale. This scale ranges from 6-20 with integer values (i.e. with greater values corresponding to greater perceived effort). Thus, there are 36 observations (i.e. 9 subjects x 4 stool types) and 3 variables. The variable 'effort' is numeric and the remaining two variables are factors with levels corresponding to stool types and particular subjects.

```
#load data
ergoStool <- MEMSS::ergoStool

#examine the data structure
str(ergoStool)
```

```
## 'data.frame':    36 obs. of  3 variables:
##  $ effort : num  12 15 12 10 10 14 13 12 7 14 ...
##  $ Type   : Factor w/ 4 levels "T1","T2","T3",..: 1 2 3 4 1 2 3 4 1 2 ...
##  $ Subject: Factor w/ 9 levels "A","B","C","D",..: 1 1 1 1 2 2 2 2 3 3 ...
```

```
#data summary
summary(ergoStool)
```

```
##      effort      Type      Subject
##  Min.   : 7.00   T1:9   A      : 4
##  1st Qu.: 8.00   T2:9   B      : 4
##  Median :10.00   T3:9   C      : 4
##  Mean   :10.25   T4:9   D      : 4
##  3rd Qu.:12.00          E      : 4
##  Max.   :15.00          F      : 4
##                         (Other):12
```

Inspection of the contingency table below reveals that the factors 'Type' and 'Subject' are completely crossed (i.e. there is at least one observation for each combination of factor levels). Further, this is an unreplicated design.

```
xtabs(~Type + Subject, ergoStool)
```

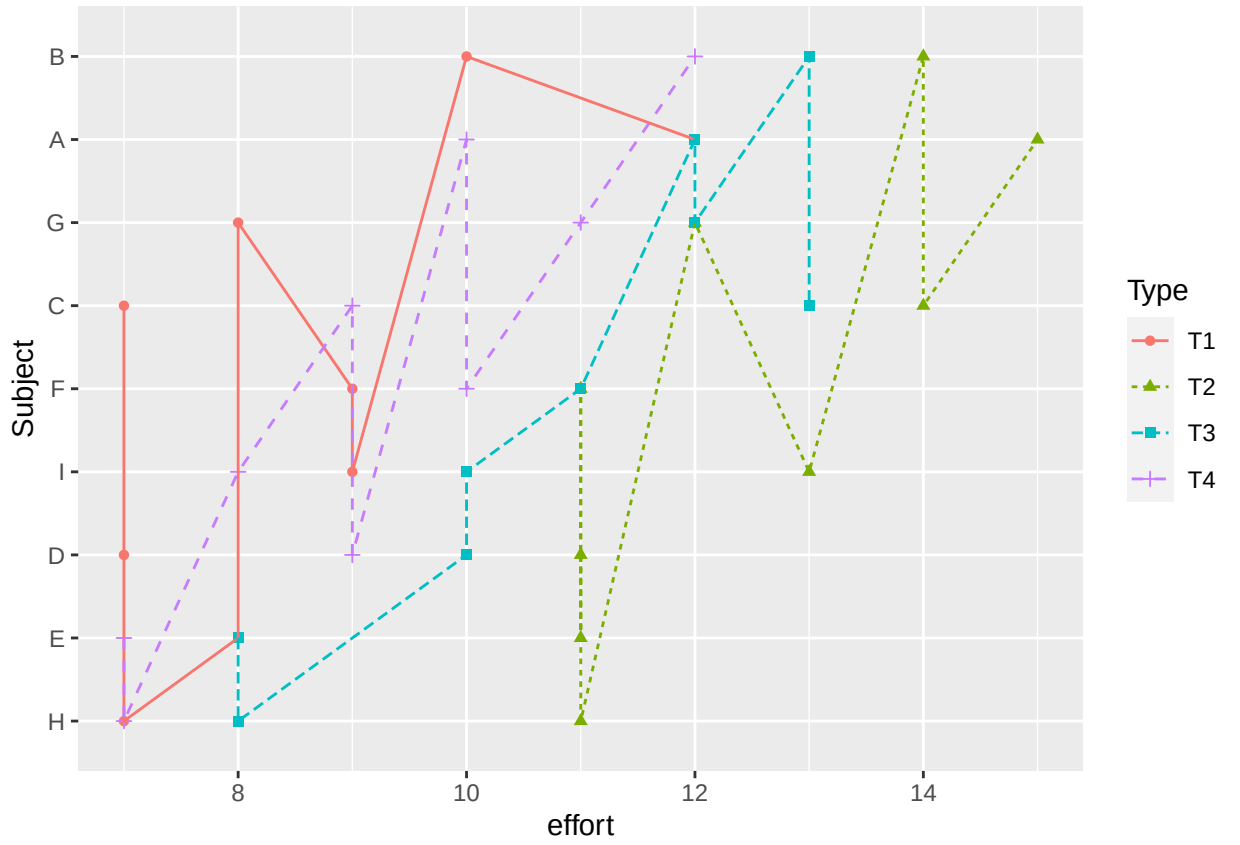
```
##     Subject
## Type A B C D E F G H I
##   T1 1 1 1 1 1 1 1 1 1
##   T2 1 1 1 1 1 1 1 1 1
##   T3 1 1 1 1 1 1 1 1 1
##   T4 1 1 1 1 1 1 1 1 1
```

(b)

The requested plot is output by the code-chunk below. Note, there is a minor irrelevant discrepancy between this plot and the corresponding plot in the package tutorial (i.e. the paths connecting points with the same Type level are not the same).

```
#determine the average effort for each level of the factor Subject
ergoStool %>%
  group_by(Subject) %>%
  summarize(avg_effort=mean(effort)) -> avg_effort_df

ergoStool %>%
  mutate(avg_effort=rep(avg_effort_df$avg_effort, each=4, times=1)) %>%
  #re-order the levels of Subject by avg_effort
  mutate(Subject=fct_reorder(Subject, avg_effort)) %>%
  ggplot(aes(x=effort, y=Subject, shape=Type, color=Type)) +
  geom_point() +
  geom_line(aes(x=effort, y=Subject, group=Type, color=Type, linetype=Type))
```



(c)

A linear mixed model with random effects for Type and Subject is fit below. The standard deviations for the estimates corresponding to Type, Subject and residual variability are 1.332, 1.695, and 1.100, as discerned from the summary output.

```
library(lme4)

#fit a model with random effects for Type and Subject via REML
summary(lmm_ergo <- lmer(effort ~ 1 + (1|Type) + (1|Subject), ergoStool))
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: effort ~ 1 + (1 | Type) + (1 | Subject)
##    Data: ergoStool
##
## REML criterion at convergence: 134.3
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -1.87089 -0.70269  0.08069  0.68483  1.65288
##
## Random effects:
##  Groups   Name        Variance Std.Dev.
##  Subject  (Intercept) 1.775    1.332
##  Type     (Intercept) 2.873    1.695
```

```
##  Residual             1.211    1.100
## Number of obs: 36, groups:  Subject, 9; Type, 4
##
## Fixed effects:
##             Estimate Std. Error t value
## (Intercept)  10.2500     0.9742   10.52
```

(d)

The model from part (c) is refit using maximum likelihood estimation. The standard deviations for the estimates corresponding to Type, Subject and residual variability are now 1.305, 1.505, and 1.101 (i.e. a decrease, relative to the model in part (c) except for the residual standard deviation).

```
summary(lmm_ergo2 <- update(lmm_ergo, REML=FALSE))
```

```
## Linear mixed model fit by maximum likelihood  ['lmerMod']
## Formula: effort ~ 1 + (1 | Type) + (1 | Subject)
##    Data: ergoStool
##
##      AIC      BIC   logLik deviance df.resid
##    144.0    150.4    -68.0    136.0       32
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -1.88436 -0.68813  0.06718  0.67820  1.65934
##
## Random effects:
##  Groups   Name        Variance Std.Dev.
##  Subject  (Intercept) 1.704    1.305
##  Type     (Intercept) 2.265    1.505
##  Residual             1.213    1.101
## Number of obs: 36, groups:  Subject, 9; Type, 4
##
## Fixed effects:
##             Estimate Std. Error t value
## (Intercept)  10.2500     0.8883   11.54
```

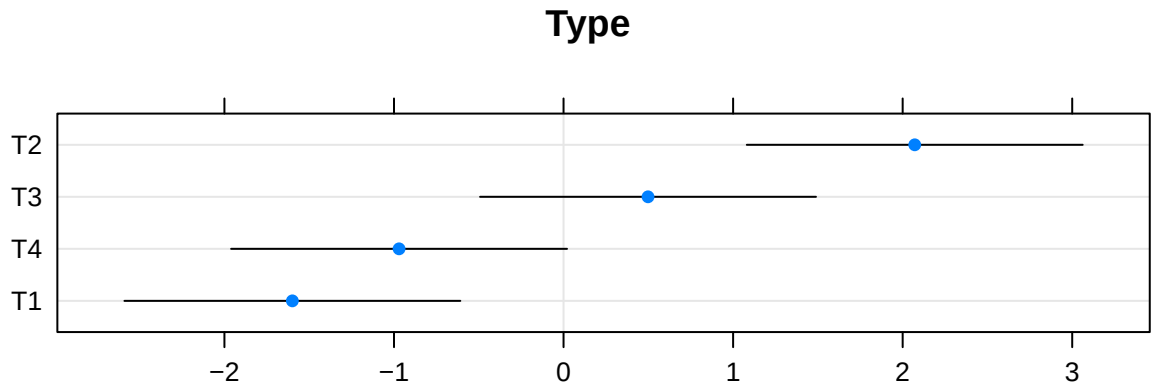
(e)

In the code-chunk below, the 95% prediction intervals corresponding to the random effects of the model from part (d) (i.e. the model fit via MLE) are generated. Based upon this plot, it is clear that stool type 1 (i.e. T1) outperforms the other types w.r.t. effort minimization.

```
## Substitute the name of your fitted model for fm in the call to ranef)
dotplot(ranef(lmm_ergo2, which = "Type", postVar = TRUE), aspect = 0.2, strip = FALSE)
```

```
## Warning in ranef.merMod(lmm_ergo2, which = "Type", postVar = TRUE): 'postVar' is
## deprecated: please use 'condVar' instead
```

```
## $Type
```



(f)

The significance of the random effect Type is assessed by comparing the model fit in Part (d) with a reduced model with only Subject as the random effect. Inspection of the anova output indicates a p-value of approximately zero. Thus, we reject the null-hypothesis $H_0 : \sigma_2 = 0$. This suggests that the more complex model, lmm_ergo2, fits the observed data better.

```
summary(lmm_ergo3 <- lmer(effort ~ 1 + (1|Subject), ergoStool, REML=FALSE))
```

```
## Linear mixed model fit by maximum likelihood  ['lmerMod']
## Formula: effort ~ 1 + (1 | Subject)
##    Data: ergoStool
##
##      AIC      BIC   logLik deviance df.resid
##    164.2    168.9    -79.1    158.2       33
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -1.71903 -0.72603  0.01465  0.75533  1.90786
##
## Random effects:
##  Groups   Name        Variance Std.Dev.
##  Subject  (Intercept) 0.8264   0.9091
##  Residual             4.0833   2.0207
## Number of obs: 36, groups:  Subject, 9
```

```
##
## Fixed effects:
##             Estimate Std. Error t value
## (Intercept)   10.250      0.453   22.62
```

```
anova(lmm_ergo3, lmm_ergo2)
```

```
## Data: ergoStool
## Models:
## lmm_ergo3: effort ~ 1 + (1 | Subject)
## lmm_ergo2: effort ~ 1 + (1 | Type) + (1 | Subject)
##           npar    AIC    BIC  logLik deviance  Chisq Df Pr(>Chisq)
## lmm_ergo3    3 164.15 168.90 -79.075   158.15
## lmm_ergo2    4 144.02 150.36 -68.011   136.02 22.128  1  2.551e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

(g)

The model in question is fit below (i.e. 'Type' is treated as fixed and 'Subject' as random) via MLE. For the fixed-effect parameter estimates note the following: relative to the model fit in part (d) the intercept (TypeT1) has decreased. Further, while TypeT2, TypeT3 and TypeT4 are positive, their values are smaller than what is suggested by the mean effort for the corresponding stool types.

```
lmm_ergo4 <- lmer(effort ~ 1 + Type + (1|Subject), ergoStool, REML = 0)
summary(lmm_ergo4)
```

```
## Linear mixed model fit by maximum likelihood  ['lmerMod']
## Formula: effort ~ 1 + Type + (1 | Subject)
##    Data: ergoStool
##
##      AIC      BIC   logLik deviance df.resid
##    134.1    143.6    -61.1    122.1       30
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -1.91131 -0.68218  0.06134  0.74352  1.73038
##
## Random effects:
##  Groups   Name        Variance Std.Dev.
##  Subject  (Intercept) 1.578    1.256
##  Residual             1.076    1.037
## Number of obs: 36, groups:  Subject, 9
##
## Fixed effects:
##             Estimate Std. Error t value
## (Intercept)   8.5556     0.5431  15.754
## TypeT2        3.8889     0.4890   7.952
## TypeT3        2.2222     0.4890   4.544
## TypeT4        0.6667     0.4890   1.363
##
## Correlation of Fixed Effects:
```

```
##        (Intr) TypeT2 TypeT3
## TypeT2 -0.450
## TypeT3 -0.450  0.500
## TypeT4 -0.450  0.500  0.500
```

```
ergoStool %>% group_by(Type) %>% summarise(mean_effort=mean(effort))
```

```
## # A tibble: 4 x 2
##   Type  mean_effort
##   <fct>       <dbl>
## 1 T1           8.56
## 2 T2          12.4
## 3 T3          10.8
## 4 T4           9.22
```