

Program 2: Towers of Hanoi

Values of $f(n)$ for the values of n from 1 to 20:

N	F(n)
1	1
2	3
3	7
4	15
5	31
6	63
7	127
8	255
9	511
10	1023
11	2047
12	4095
13	8191
14	16383
15	32767
16	65535
17	131071
18	262143
19	524287
20	1048575

Let $f(n)$ = num of moves

$$f(n) = \begin{cases} 1, & n=1 \\ 2(f(n-1)) + 1, & n \geq 2 \end{cases}$$

$$f(n) = 1 + 2f(n-1)$$

$$f(n) = 1 + 2 + 4[1 + 2f(n-3)]$$

$$= 1 + 2 + 4 + 8f(n-3)$$

$$= 1 + 2 + 4 + 8 + 16f(n-4)$$

$$= 1 + 2^1 + 2^2 + 2^3 + 2^4 f(n-4)$$

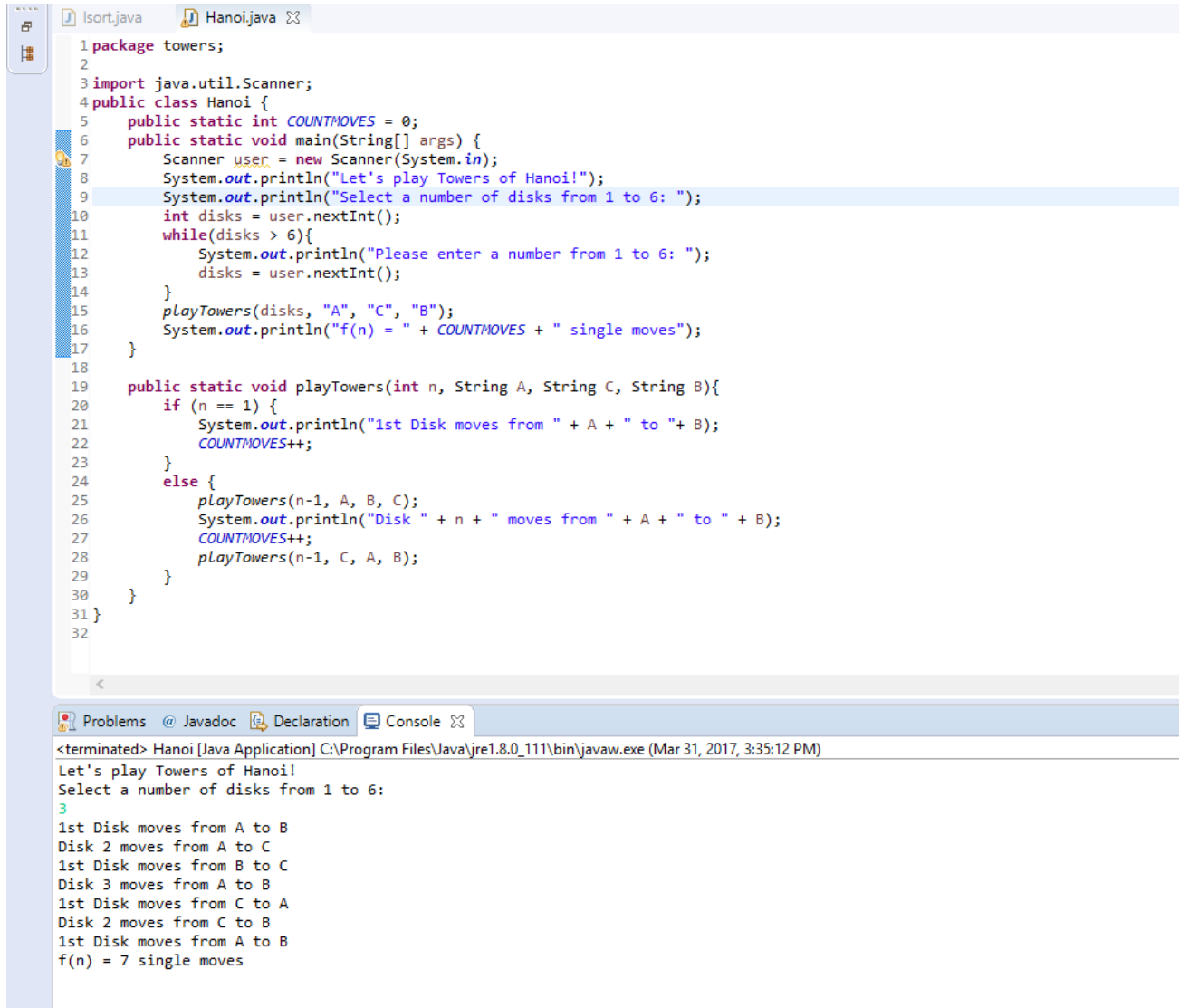
$$f(n) = 1 + 2 + 2^2 + \dots + 2^{n-1} \quad \text{Geo}$$

$$\frac{2^n - 1}{2 - 1} = 2^n - 1$$

$$f(n) = 2^n - 1$$

Sample outputs:

- When $n = 3$



The screenshot shows an IDE with two tabs: 'Isort.java' and 'Hanoi.java'. The 'Hanoi.java' tab is active, displaying the following Java code:

```
1 package towers;
2
3 import java.util.Scanner;
4 public class Hanoi {
5     public static int COUNTMOVES = 0;
6     public static void main(String[] args) {
7         Scanner user = new Scanner(System.in);
8         System.out.println("Let's play Towers of Hanoi!");
9         System.out.println("Select a number of disks from 1 to 6: ");
10        int disks = user.nextInt();
11        while(disks > 6){
12            System.out.println("Please enter a number from 1 to 6: ");
13            disks = user.nextInt();
14        }
15        playTowers(disks, "A", "C", "B");
16        System.out.println("f(n) = " + COUNTMOVES + " single moves");
17    }
18
19    public static void playTowers(int n, String A, String C, String B){
20        if (n == 1) {
21            System.out.println("1st Disk moves from " + A + " to " + B);
22            COUNTMOVES++;
23        }
24        else {
25            playTowers(n-1, A, B, C);
26            System.out.println("Disk " + n + " moves from " + A + " to " + B);
27            COUNTMOVES++;
28            playTowers(n-1, C, A, B);
29        }
30    }
31 }
32
```

Below the code editor, the 'Console' tab is active, showing the program's execution output:

```
<terminated> Hanoi [Java Application] C:\Program Files\Java\jre1.8.0_111\bin\javaw.exe (Mar 31, 2017, 3:35:12 PM)
Let's play Towers of Hanoi!
Select a number of disks from 1 to 6:
3
1st Disk moves from A to B
Disk 2 moves from A to C
1st Disk moves from B to C
Disk 3 moves from A to B
1st Disk moves from C to A
Disk 2 moves from C to B
1st Disk moves from A to B
f(n) = 7 single moves
```

- When $n = 5$

```

1 package towers;
2
3 import java.util.Scanner;
4 public class Hanoi {
5     public static int COUNTMOVES = 0;
6     public static void main(String[] args) {
7         Scanner user = new Scanner(System.in);
8         System.out.println("Let's play Towers of Hanoi!");
9         System.out.println("Select a number of disks from 1 to 6: ");
10        int disks = user.nextInt();
11        while(disks > 6){
12            System.out.println("Please enter a number from 1 to 6: ");
13            disks = user.nextInt();
14        }
15        playTowers(disks, "A", "C", "B");
16        System.out.println("f(n) = " + COUNTMOVES + " single moves");
17    }
18
19    public static void playTowers(int n, String A, String C, String B){

```

```

<terminated> Hanoi [Java Application] C:\Program Files\Java\jre1.8.0_111\bin\javaw.exe (Mar 31, 2017, 3:35:36 PM)
Let's play Towers of Hanoi!
Select a number of disks from 1 to 6:
5
1st Disk moves from A to B
Disk 2 moves from A to C
1st Disk moves from B to C
Disk 3 moves from A to B
1st Disk moves from C to A
Disk 2 moves from C to B
1st Disk moves from A to B
Disk 4 moves from A to C
1st Disk moves from B to C
Disk 2 moves from B to A
1st Disk moves from C to A
Disk 3 moves from B to C
1st Disk moves from A to B
Disk 2 moves from A to C
1st Disk moves from B to C
Disk 5 moves from A to B
1st Disk moves from C to A
Disk 2 moves from C to B
1st Disk moves from A to B
Disk 3 moves from C to A
1st Disk moves from B to C
Disk 2 moves from B to A
1st Disk moves from C to A
Disk 4 moves from C to B
1st Disk moves from A to B
Disk 2 moves from A to C
1st Disk moves from B to C
Disk 3 moves from A to B
1st Disk moves from C to A
Disk 2 moves from C to B
1st Disk moves from A to B
f(n) = 31 single moves

```