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A.S HW I

Problem 1:

a) $f(n) = 3n$; $g(n) = n^3$

$f(n) \in \Theta(g(n))$:

Θ notation: $0 \leq c_1 g(n) \leq f(n) \leq c_2 g(n)$ for all n greater than n_0

$c_1(n^3) \leq 3n \leq c_2(n^3)$

$2n^3 \leq 3n \leq 4n^3$

$2 \leq \frac{3}{n^2} \leq 4$

$\lim_{n\to\infty} = 0$ (Meaning that as n grows it will eventually be smaller than 2 making this equation false.)

$\boxed{f(n) \notin \Theta(g(n))}$

$- f \in O(g)$

O notation: $0 \leq f(n) \leq c g(n) \; \forall N \geq N_0$

$3n \leq C(n^3)$

$3n \leq 4n^3$

$3 \leq 4n^2$

$\lim_{n\to\infty} = +\infty$ True $\boxed{f \in O(g)}$

$-f \in o(g)$ implies that $\lim_{n \to \infty} \frac{f(n)}{g(n)} = 0$

$\frac{3n}{n^2} = \frac{3}{n}$

$\lim_{n \to \infty} \frac{3}{n} = 0$ true $\boxed{f \in o(g)}$

$f \in \Omega(g)$ $\quad 0 \leq c g(n) \leq f(n)$

$2n^2 \leq 3n$

$3 \leq \frac{3}{n}$

$\lim_{n \to \infty} \frac{3}{n} = 0$ false $\boxed{f \notin \Omega(g)}$

$f \in \omega(g)$ implies $\lim_{n \to \infty} \frac{f(n)}{g(n)} = \infty$

$\frac{3n}{n^2}$

$\frac{3}{n}$

$\lim_{n \to \infty} \frac{3}{n} = 0$ false $\boxed{f \notin \omega(g)}$

a) $g \in \Theta(f)$

$c_1(3n) \leqslant n^3 \leqslant c_2(3n)$

$3n \leqslant n^3 \leqslant 2(3n)$

$3n \leqslant n^3 \leqslant 6n$

$1 \leqslant \frac{n^2}{3} \leqslant 2$

$\lim_{n\to\infty} \frac{n^2}{3} = +\infty$ false $\boxed{g \notin \Theta(f)}$

$g \in O(f)$

$n^3 \leqslant c(3n)$

$n^3 \leqslant 6n$

$n^2 \leqslant 6$ false $\boxed{g \notin O(f)}$

$g \in o(f)$

$\frac{n^3}{3n}$

$\frac{n^2}{3}$ $\lim_{n\to\infty} \frac{n^2}{3} = +\infty$ $\boxed{g \notin o(f)}$

~~a~~ $g \in \Omega(f)$

$c(3n) \geq n^3$

~~cn~~ $3n \geq n^3$

$3 \geq n^2$

~~n~~ $\lim_{n\to\infty} n^2 = \infty$ true $\boxed{g \in \Omega(f)}$

$g \in \omega(f)$

$\frac{n^3}{3n}$

$\frac{n^2}{3}$ $\lim_{n\to\infty} \frac{n^2}{3} = \infty$ $\boxed{g \in \omega(f)}$

b) $f(n) = 7n^{0,7} + 2n^{0,2} + 13 \log n$

$g(n) = \sqrt{n}$

$f \in \Theta g = C_1 \sqrt{n} \le 7n^{0,7} + 2n^{0,2} + 13 \log n \le C_2 \sqrt{n}$

given $\sqrt{n} = n^{0,5}$ | $6\sqrt{n} \le 7n^{0,7} + 2n^{0,2} + 13 \log n \le 8\sqrt{n}$

$$6 \le 7n^{0,2} + 2n^{-0,3} + 13 \log n \le 8$$

Considering only the highest term $6 \le 7n^{0,2} \le 8$

$$\lim_{n\to\infty} 7n^{0,2} = \infty$$

$\boxed{f \notin \Theta(g)}$

$f \in O(g)$

$$7n^{0,7} + 2n^{0,2} + 13 \log n \le c\,(g(n)) \quad (\sqrt{n})$$

Take the highest Power | $7n^{0,7} \le 8n^{0,5}$

$$7n^{0,2} \le 8$$

$$\lim_{n\to\infty} 7n^{0,2} = +\infty \quad \text{False}$$

$\boxed{f \notin O(g)}$

$f \in o(g)$

$$\frac{7n^{0,7}+2n^{0,2}+13\log n}{n^{0,5}}$$

$$\lim_{n\to\infty} 7n^{0,2}+2n^{-0,3}+\frac{13\log n}{n^{0,5}} = +\infty$$

$$\boxed{f \notin o(g)}$$

$f \in \Omega(g)$

$$c\sqrt{n} \leq 7n^{0,7}+2n^{0,2}+13\log n$$

$$6n^{0,5} \leq 7n^{0,7} \quad | \text{ only take highest Power}$$

$$6 \leq 7n^{0,2}$$

$$\lim_{n\to\infty} 7n^{0,2} = \infty_+ \quad \text{True}$$

$$\boxed{f \in \Omega(g)}$$

$$f \in \omega(g) = \frac{7n^{0,7}+2n^{0,2}+13\log n}{n^{0,5}}$$

$$\lim_{n\to\infty} 7n^{0,2}+2n^{-0,3}+\frac{13\log n}{n^{0,5}} = +\infty \quad \text{True}$$

$$\boxed{f \in \omega(g)}$$

b) $g \in \Theta(f) =$

$$C(7n^{0,7}) \leq n^{0,5} \leq C_2(7n^{0,7})$$

$$7n^{0,7} \leq n^{0,5} \leq 14n^{0,7}$$

$$\lim_{n\to\infty} n^{0,5} = +\infty \quad \text{false} \quad \boxed{g \notin \Theta(f)}$$

$g \in O(f)$

$$n^{0,5} \leq C\, 7n^{0,7}$$

$$n^{0,5} \leq 7n^{0,7}$$

$$n^{-0,2} \leq 7$$

$$\lim_{n\to\infty} n^{-0,2} = 0 \quad \text{True}$$

$$\boxed{g \in O(f)}$$

$g \in o(f)$

$$\frac{n^{0,5}}{7n^{0,7}}$$

$$\lim_{n\to\infty} \frac{n^{0,5}}{7n^{0,7}} = 0 \quad \boxed{g \in o(f)}$$

$g \in \Omega(f)$

$c(7n^{0,7}) \geq \cancel{3}\, n^{0,5}$

$7n^{0,7} \geq n^{0,5}$

$7 \geq n^{-0,2}$

$\lim_{n \to \infty} n^{-0,2} = 0$ false

$\boxed{g \notin \Omega(f)}$

$g \in \omega(f)$

$\lim_{n \to \infty} \frac{n^{0,5}}{7n^{0,7}} = 0$ false

$\boxed{g \notin \omega(f)}$

c) $f(n) = n^2/\log(n)$

$g(n) = n \log n$

$f \in \Theta g$

$c_1 n \log n \leq n^2/\log(n) \leq c_2 n \log n$

$n \log n \leq n^2 \log(n)^{-1} \leq 2 n \log n$

$n \leq n^2 \log(n)^{-2} \leq 2n$

$1 \leq \frac{n^2 \log(n)^{-2}}{n} \leq 2$

$\lim_{n\to\infty} \frac{n^2 \log(n)^{-2}}{n^2} = \infty$ false

$f \notin \Theta g$

$f \in O(g)$

$n^2/\log(n) \leq C(n \log n)$

$n^2 \log(n)^{-1} \leq n \log n$

$n \log(n)^{-2} \leq 1$

$\lim_{n\to\infty} = n \log(n)^{-2} = \infty$ false $f \notin O(g)$

$f \in o(g)$

$$\frac{n^2 \log(n)^{-1}}{n \log n}$$

$$\lim_{n \to \infty} n \log(n)^{-2} = \infty$$

$\boxed{f \notin o(g)}$

$f \in \Omega(g) = c \, n \log n \leq n^2 / \log n$

$n \log n \leq n^2 / \log n$

$1 \leq n \log(n)^{-2}$

$n \log(n)^{-2} = \infty$ True $\boxed{f \in \Omega(g)}$

$f \in \omega(g)$

$n \log(n)^{-2} = \infty$ $\boxed{f \in \omega(g)}$

c) $g \in \Theta f$

$$C(n^2 \log(n)^{-1}) \leq n \log n \leq C_2(n^2 \log(n)^{-1})$$

$$n^2 \log(n)^{-1} \leq n \log n \leq 2(n^2 \log(n^{-1}))$$

$$1 \leq \frac{n \log n}{(n^2 \log(n)^{-1})} \leq 2$$

$\lim \; n \log n / n^2 \log(n)^{-1} = 0$ false

$$\boxed{g \notin \Theta(f)}$$

$g \in O(f)$

$$n \log n \leq C(n^2 \log(n)^{-1})$$

$$n \log n \leq n^2 \log(n)^{-1}$$

$$\frac{n \log n}{n^2 \log(n)^{-1}} \leq 1$$ true

$\lim\limits_{n \to \infty} = 0$

$$\boxed{\cancel{g \notin O(f)}}$$

$$\boxed{g \in O(f)}$$

$f \in o(g) = \dfrac{n \log n}{n^2 / \log n} =$

$\lim_{n\to\infty} = 0$ true

$\boxed{f \in o(g)}$

$g \in \Omega(f) = c\,(n^2/\log n) \geq n \log n$

$\cancel{\dfrac{n^2/\log n \geq 1}{n \log n}}$

$n^2 \log n^{-1} \geq n \log n$

$1 \geq \dfrac{n \log n}{n^2 \log n^{-1}}$

$\lim_{n\to\infty} = 0$ false

$\boxed{g \notin \Omega f}$

$g \in \omega f$

$\lim_{n\to\infty} \dfrac{n \log n}{n^2 \log n^{1}} = 0$

$g \notin \omega f$

d) $f(n) = (\log(3n))^3$

$g(n) = 9 \log n$

$f \in \Theta(g)$

$$c_1(9\log n) \le (\log(3n))^3 \le c_2(9\log n)$$

$$9\log n \le (\log(3n))^3 \le 18\log n$$

$$9 \le \frac{(\log(3n))^3}{\log n} \le 18$$

$\lim_{n\to\infty} (\log(3n))^3/\log n = \infty$ false

$\boxed{f \notin \Theta(g)}$

$f \in O(g)$

$$(\log(3n))^3 \le c\,(9\log n)$$

$$(\log(3n))^3 \le 9\log n$$

$$\frac{(\log(3n))^3}{\log n} \le 9$$

$\lim_{n\to\infty} \frac{(\log(3n))^3}{\log n} = \infty$ false

$\boxed{f \notin O(g)}$ $f \in O(g)$

$f \in o(g) =$

$$\lim_{n\to\infty} \frac{(\log(3n))^3}{\log n} = \cancel{\infty}\ 0$$

~~$f \notin o(g)$~~ $\boxed{f \in o(g)}$

$f \in \Omega(g) = \quad C n \log n \leq (\log(3n))^3$

$$1 \leq \frac{(\log(\cancel{n}3n))^3}{n \log n}$$

$$\lim_{n\to\infty} = 0$$

$\boxed{f \notin \Omega(g)}$

$$f \in \omega(g) = \frac{(\log(3n))^3}{n \log n}$$

$$\lim_{n\to\infty} = 0 \qquad f \notin \omega(g)$$

d) $g \in \Theta(f) = c_1(\log(3n))^3 \leq 9\log n \leq c_2(\log(3n))^3$

$8(\log(3n))^3 \leq 9\log n \leq 10(\log(3n))^3$

$8 \leq \dfrac{9\log n}{(\log(3n))^3} \leq 10$

$\lim_{n\to\infty} = 0$ false $g \notin \Theta(f)$

$g \in O(f)$

$9\log n \leq c(\log(3n))^3$

$\dfrac{9\log n}{\log(3n))^3} \leq 1$

$\lim_{n\to\infty} = 0$ true $\boxed{g \in O f}$

$g \in o(f) = \dfrac{9\log n}{(\log(3n))^3}$

$\lim_{n\to\infty} = 0$ true $\boxed{g \in o f}$

$\cancel{f \in \Omega(g)}$

$g \in \Omega(f) = \cancel{cg}$

$$c\,(\log(3n))^3 \leq 9\log n$$

$$1 \leq \frac{9\log n}{(\log(3n))^3}$$

$\lim_{n\to\infty} = 0$

false $\boxed{g \notin \Omega(f)}$

$f \in \omega(g) = \dfrac{9\log n}{(\log(3n))^3}$

$\lim_{n\to\infty} = 0$ false

$\boxed{g \notin \omega f}$