HOMEWORK 5

ALGORITHMS AND DATA STRUCTURES (CH08-320201)

Spring 2018

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Due on Monday, Mar 12, 2018, 23:55.

Problem 1: Quicksort *(8+5+2=15 points)*

(a) *Implement* a modified version of the Quicksort algorithm, where the sequence is always split into three subsequences by simultaneously using the first two elements as pivots.

(b) Derive the best-case and worst-case running time for the modified Quicksort in (a).

(c) *Implement* a modified version of the Randomized Quicksort algorithm, where the sequence is always split into three subsequences by simultaneously using two random elements as pivots.

Problem 2: Randomized Quicksort *(6+4=10 points))*

To formally complete the proof of the expected time complexity $E[T(n)]$ for the Randomized Quicksort algorithm when applied to an input sequence of length n, provide the following steps:

(a) Show by induction that

$$\sum_{k=2}^{n-1} k \lg k \leq \frac{1}{2} n^2 \lg n - \frac{1}{8} n^2$$

(b) Show by induction that

$$E[T(n)] \geq cn \lg n$$

for a constant $c > 0$.

Problem 3: Decision Trees. *(4 points)*

Show that $\lg n! = \Theta(n \lg n)$ without using Stirling's formula.

Remarks

Solutions have to be handed in via Moodle by the due date. For late submissions you need to get in contact with the TAs directly. You need to upload one zip-file that contains a PDF-file for the theoretical parts and source files (no executables or object files) for the programming assignments. The source files need to include a makefile. Programming assignments need to be handed in as C, C++, or Python code. **Please write your own code.** It is ok to take snippets from online resources, but they need to be clearly marked.