

Problem 2

b) The input to the algorithm is an unsorted array of integers $A[a_1 \dots a_n]$

Pseudo code:

for $i = a_1$ to a_{n-1}

$\text{min} = i$

 for $j = i+1$ to n

 if $A[j] \leq A[\text{min}]$

$\text{min} = j$

 Swap $A[i]$ with $A[\text{min}]$

Loop invariant:

Before the start of each loop, the following holds

$A[\text{min}] \leq A[i \dots j-1]$ (sorted side of the array)

Proof of loop invariant:

Start of program

At the beginning of the program, the min is set to the first element in the array, and $j = i+1$ since the algorithm just ^{$i = [0, i]$} sorted and the min is the first element and the only element in the sub array. it is hence ~~smaller~~ equal to what is in the sub array $A[\min] = A[i \dots j-1]$ and hence loop invariant is true.

Maintenance (Running of code):

Before passing the 2nd for loop; we assume that $\min =$ ~~smallest~~ ^{largest} element in sub array. In the 2nd for loop, there exist 2 possible cases either $A[j] < A[\min]$ or does not

case 1: If $A[\min] < A[j]$ Then min still represents the ~~smallest~~ ^{largest} element in the sorted sub array meaning the loop invariant holds.

case 2: If $A[j] < A[\min]$ then ~~the~~ $\min = j$ and a swap function is run to place the element in its new position in the sorted sub array where it will be the biggest element in the sub array. meaning that $A[\min]$ now represents a new element that is // the sorted array. The loop invariant holds.

Termination of program

At the termination of the inner loop, min indexes an element ~~less~~^{larger} than or equal to all elements in the array $A[i \dots n]$ ~~because~~ as the array will be sorted an min will now index the largest elements making the loop invariant true.