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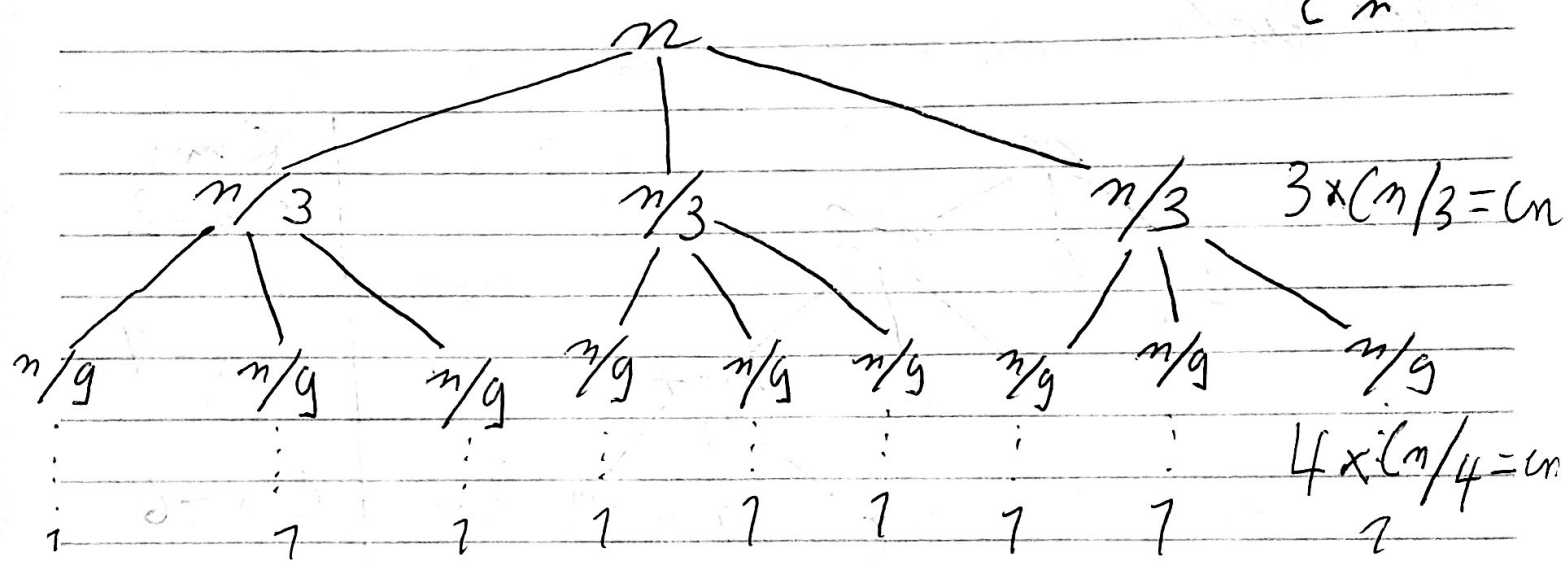
AaDS

Problem 1:

b) Best case:




This happens if the list is divided into 3 equal sublists with every call

$n \cdot c = cn$

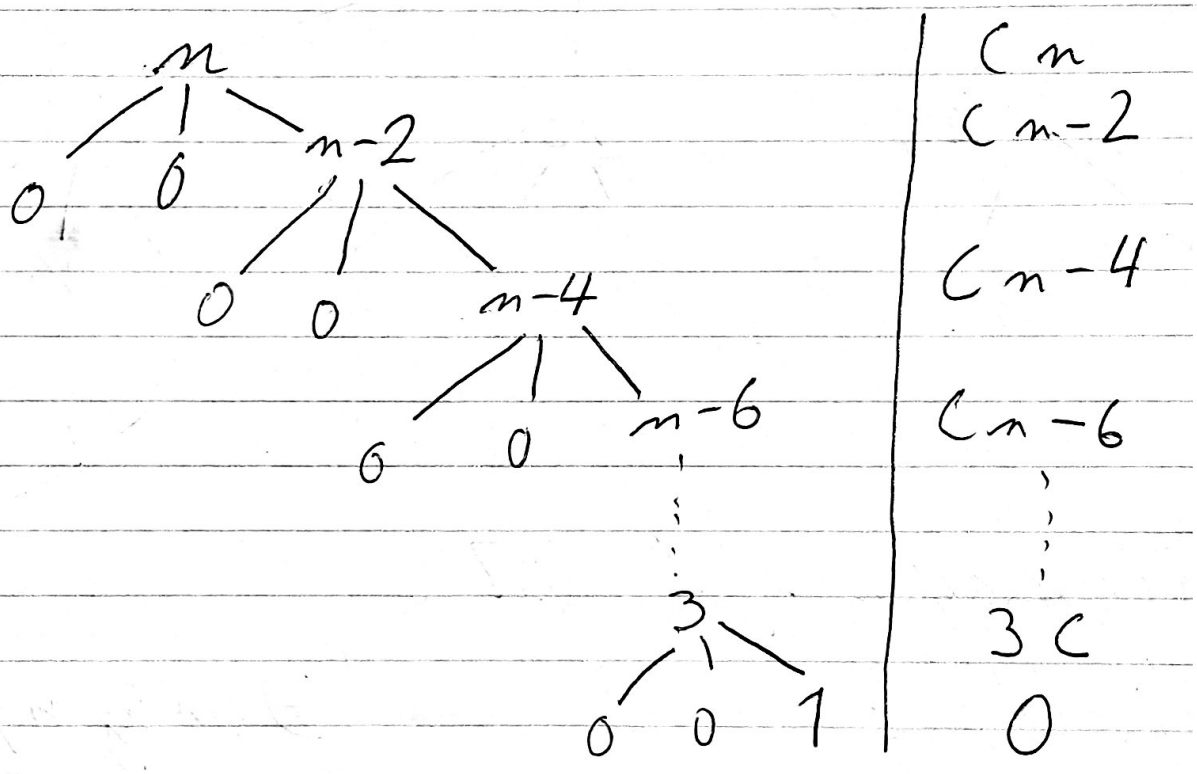
$\Theta(n \lg n)$

Worst case:

The pivot divides the list into 3 sublists

2 of size 0 and 1 size $(n-2)$

This happens if elements are the largest or smallest in the list.




Sum of partitions:

$$Cn + C(n-2) + C(n-4) + \ldots + 3C =$$

$$C(n + (n-2) + (n-4) + \ldots)$$

$$C((n+1)(n/3) - 2)$$

$$\Theta(n^2)$$

Problem 4:

Show that $\lg n! = \Theta(n \lg n)$

Argument using first principles:

We know that $\lg(a \times b) = \log(a) + \log(b)$

∴ So, $\log(n!) = \lg(n) + \lg(n-1) + \ldots + \lg(1)$

$\lg(n) + \lg(n-1) + \ldots + (\lg(2) + \lg(1)$

∴ $\log(n) + \log(n-1) + \ldots + \log(n/2)$

looking at top $n/2$ terms

∴ $n/2 * \log(n) = 1)$

$= n/2 *$

∴ $n/2\,(\lg(n) - 1)$

$= n/2\,[\lg(n) - 1]$

$\Theta(n \lg(n))$