Binary outcomes

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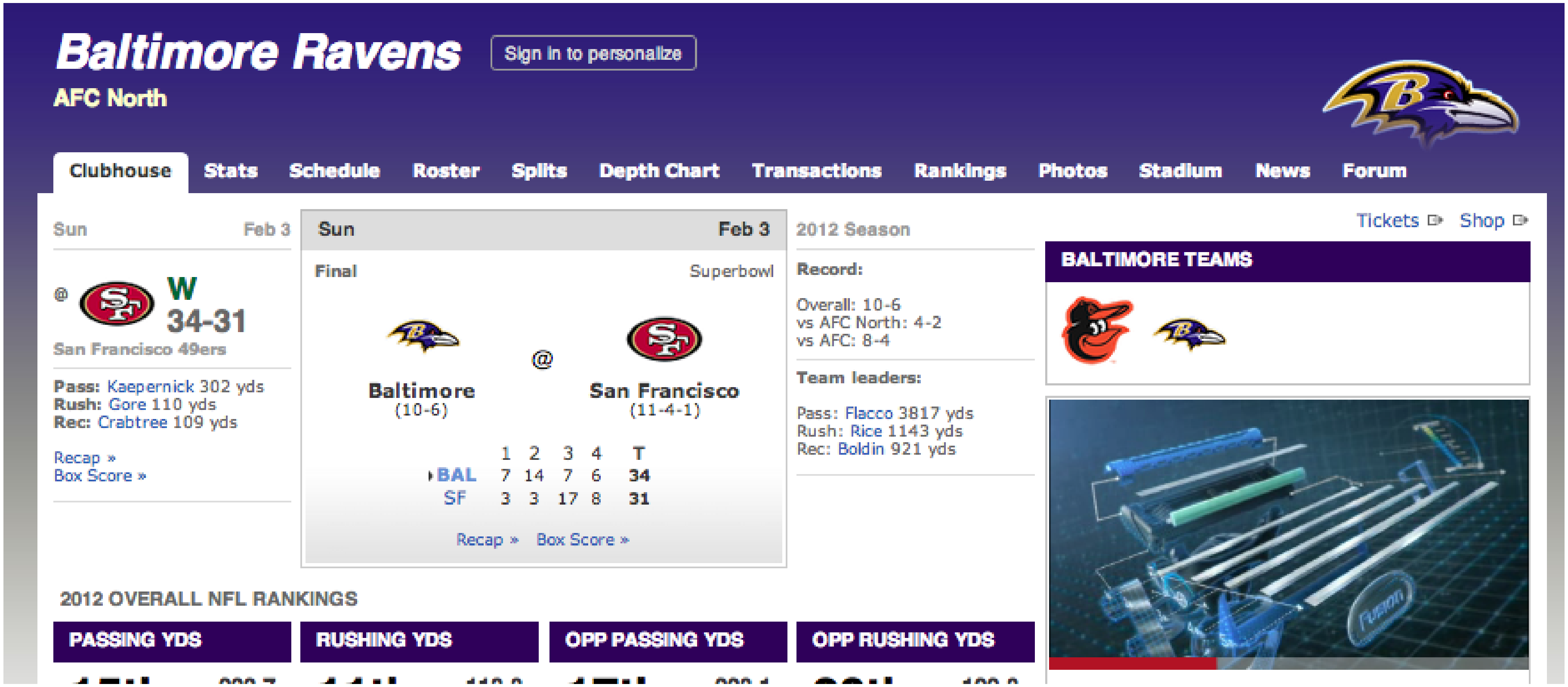
Key ideas

- Frequently we care about outcomes that have two values
  - Alive/dead
  - Win/loss
  - Success/Failure
  - etc
- Called binary outcomes or 0/1 outcomes
- Linear regression (like we've seen) may not be the best

Example: Baltimore Ravens



http://espn.go.com/nfl/team/_/name/bal/baltimore-ravens

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Ravens Data

```r
download.file("https://dl.dropboxusercontent.com/u/7710864/data/ravensData.rda",
              destfile="./data/ravensData.rda",method="curl")
load("./data/ravensData.rda")
head(ravensData)
```

```
  ravenWinNum ravenWin ravenScore opponentScore
1           1        W         24             9
2           1        W         38            35
3           1        W         28            13
4           1        W         34            31
5           1        W         44            13
6           0        L         23            24
```

Linear regression

$$RW_i = b_0 + b_1 RS_i + e_i$$

RW_i - 1 if a Ravens win, 0 if not

RS_i - Number of points Ravens scored

b_0 - probability of a Ravens win if they score 0 points

b_1 - increase in probability of a Ravens win for each additional point

e_i - variation due to everything we didn't measure

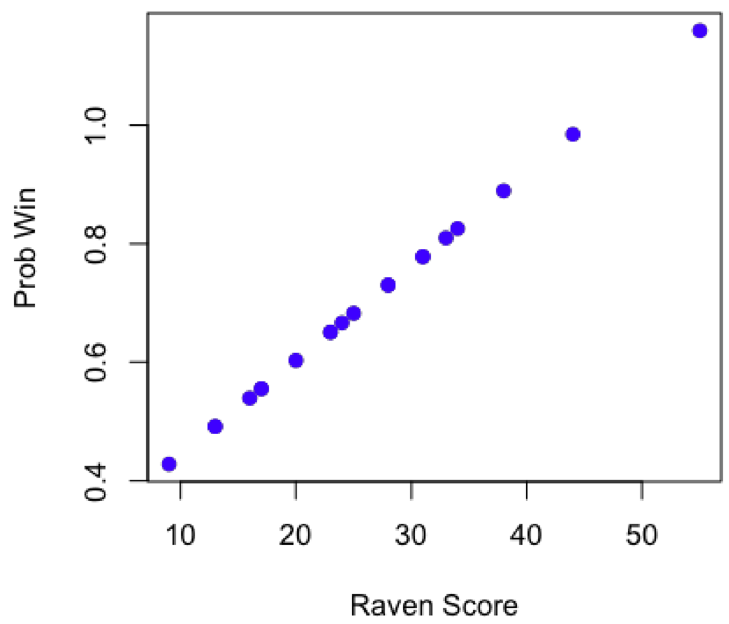
Linear regression in R

```
lmRavens <- lm(ravensData$ravenWinNum ~ ravensData$ravenScore)
summary(lmRavens)
```

```
Call:
lm(formula = ravensData$ravenWinNum ~ ravensData$ravenScore)

Residuals:
   Min      1Q Median      3Q     Max
-0.730 -0.508  0.182  0.322  0.572

Coefficients:
                      Estimate Std. Error t value Pr(>|t|)
(Intercept)            0.28503    0.25664    1.11    0.281
ravensData$ravenScore  0.01590    0.00906    1.76    0.096 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.446 on 18 degrees of freedom
Multiple R-squared:  0.146, Adjusted R-squared:  0.0987
F-statistic: 3.08 on 1 and 18 DF,  p-value: 0.0963
```

Linear regression

```
plot(ravensData$ravenScore,lmRavens$fitted,pch=19,col="blue",ylab="Prob Win",xlab="Raven Score")
```



Odds

Binary Outcome 0/1

$$RW_i$$

Probability (0,1)

$$\mathrm{Pr}(\mathrm{RW_i}|\mathrm{RS_i}, \mathrm{b_0}, \mathrm{b_1})$$

Odds $(0, \infty)$

$$\frac{\mathrm{Pr}(\mathrm{RW_i}|\mathrm{RS_i}, \mathrm{b_0}, \mathrm{b_1})}{1 - \mathrm{Pr}(\mathrm{RW_i}|\mathrm{RS_i}, \mathrm{b_0}, \mathrm{b_1})}$$

Log odds $(-\infty, \infty)$

$$\log\left(\frac{\mathrm{Pr}(\mathrm{RW_i}|\mathrm{RS_i}, \mathrm{b_0}, \mathrm{b_1})}{1 - \mathrm{Pr}(\mathrm{RW_i}|\mathrm{RS_i}, \mathrm{b_0}, \mathrm{b_1})}\right)$$

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Linear vs. logistic regression

Linear

$$RW_i = b_0 + b_1 RS_i + e_i$$

or

$$E[RW_i | RS_i, b_0, b_1] = b_0 + b_1 RS_i$$

Logistic

$$\Pr(RW_i | RS_i, b_0, b_1) = \frac{\exp(b_0 + b_1 RS_i)}{1 + \exp(b_0 + b_1 RS_i)}$$

or

$$\log\left(\frac{\Pr(RW_i | RS_i, b_0, b_1)}{1 - \Pr(RW_i | RS_i, b_0, b_1)}\right) = b_0 + b_1 RS_i$$

Interpreting Logistic Regression

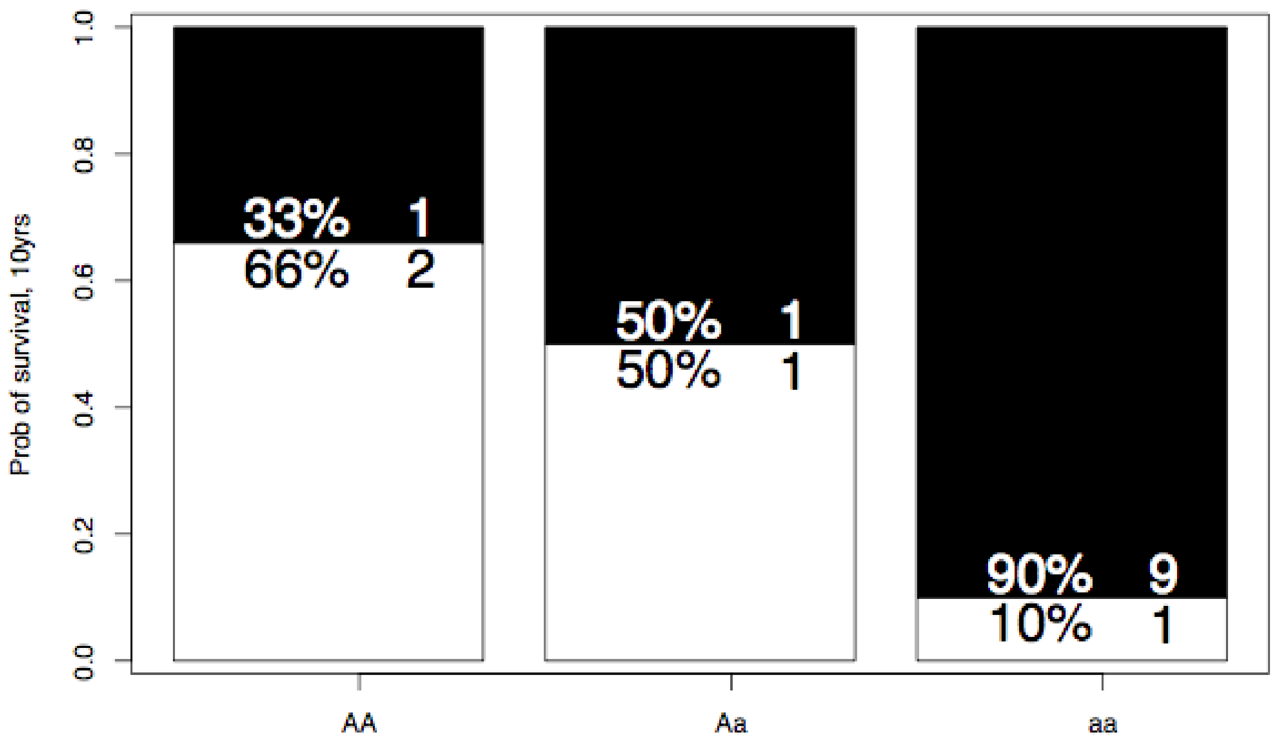
$$\log\left(\frac{\mathrm{Pr(RW_i|RS_i, b_0, b_1)}}{1-\mathrm{Pr(RW_i|RS_i, b_0, b_1)}}\right) = b_0 + b_1 RS_i$$

b_0 - Log odds of a Ravens win if they score zero points

b_1 - Log odds ratio of win probability for each point scored (compared to zero points)

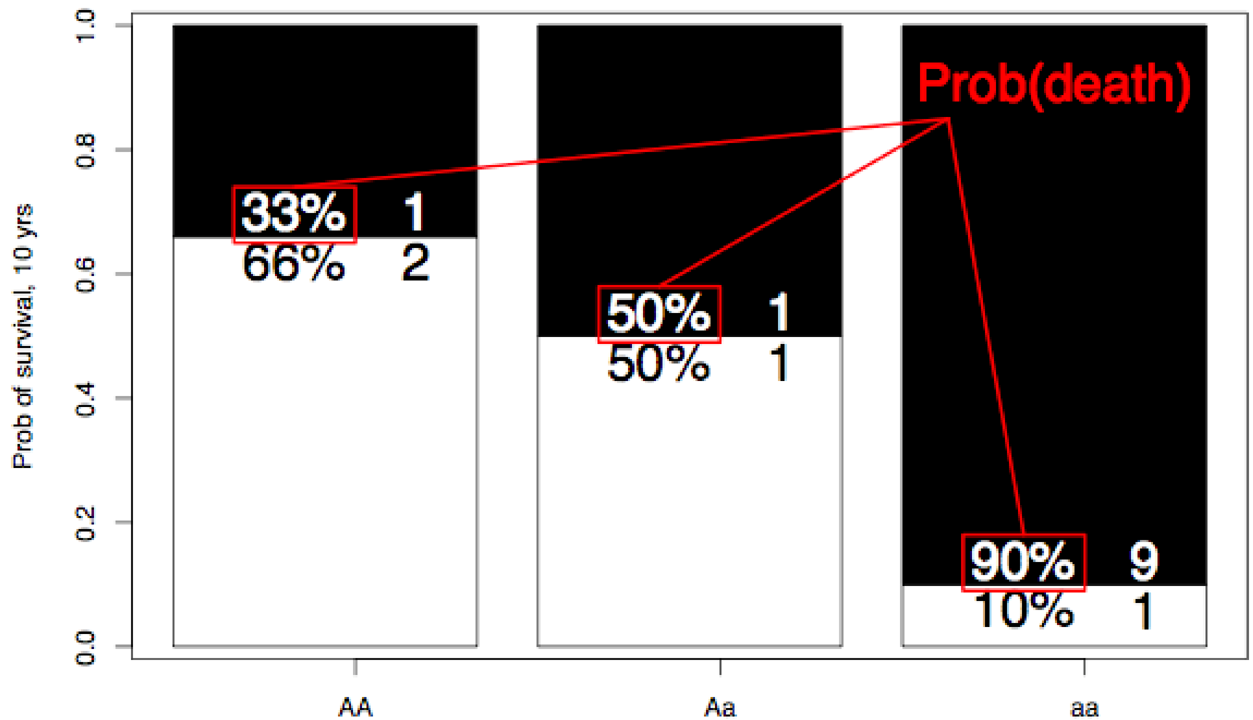
$\exp(b_1)$ - Odds ratio of win probability for each point scored (compared to zero points)

Explaining Odds



[via Ken Rice]

Probability of Death



via Ken Rice

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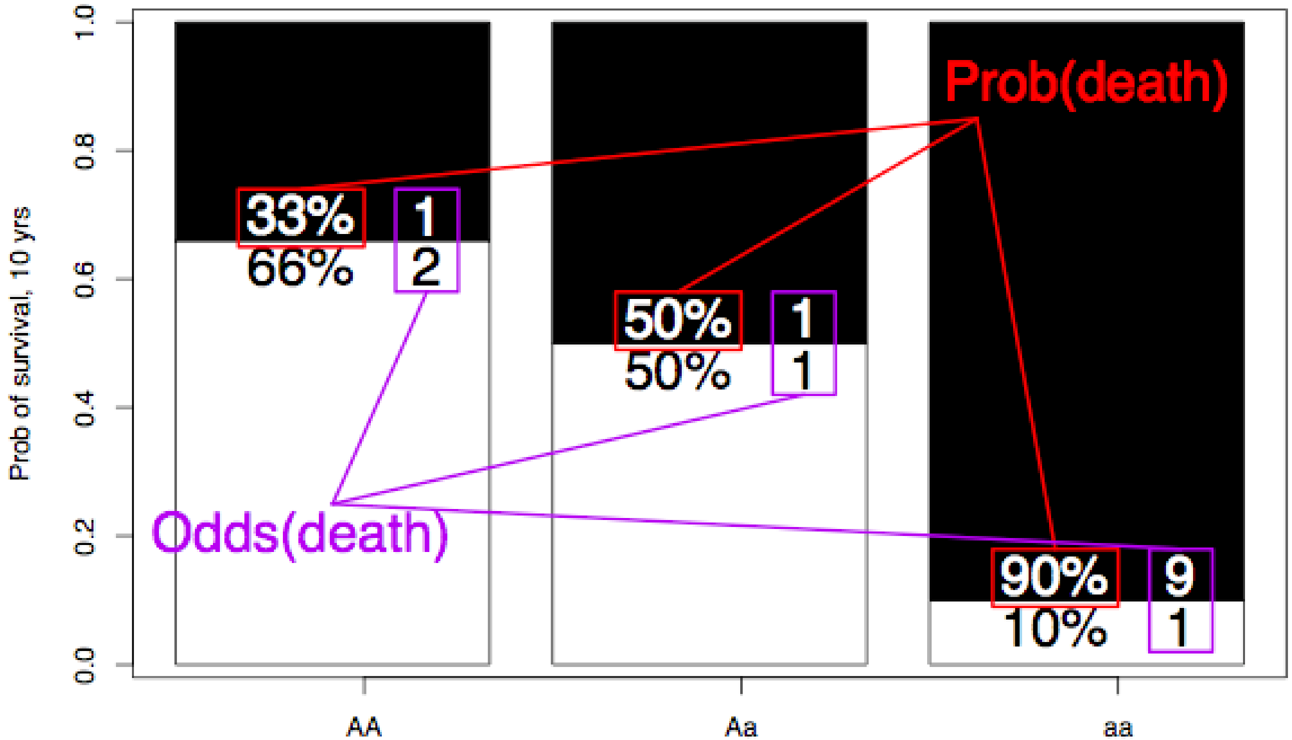
Odds of Death



via Ken Rice

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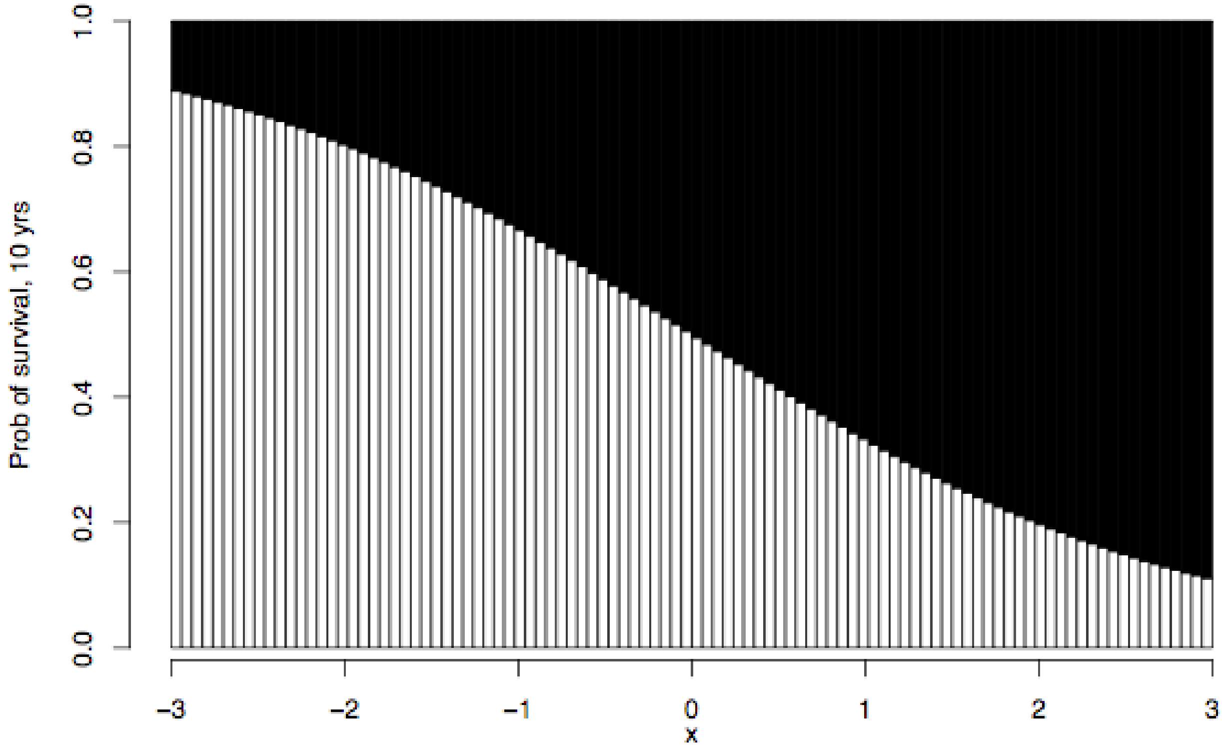
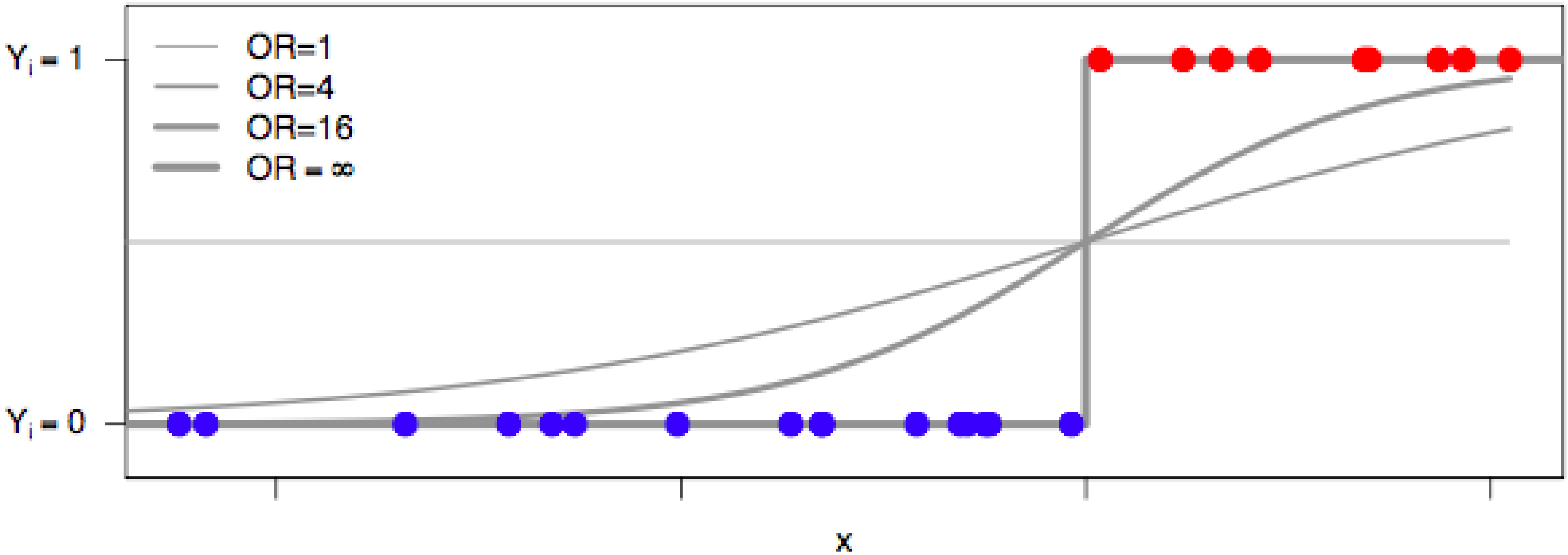
Odds Ratio = 1, Continuous Covariate



via Ken Rice

Different odds ratios



via Ken Rice

Ravens logistic regression

```
logRegRavens <- glm(ravensData$ravenWinNum ~ ravensData$ravenScore,family="binomial")
summary(logRegRavens)
```

```

Call:
glm(formula = ravensData$ravenWinNum ~ ravensData$ravenScore, 
    family = "binomial")

Deviance Residuals: 
   Min      1Q  Median      3Q     Max  
-1.758  -1.100   0.530   0.806   1.495  

Coefficients:
                      Estimate Std. Error z value Pr(>|z|)
(Intercept)            -1.6800     1.5541   -1.08     0.28
ravensData$ravenScore   0.1066     0.0667    1.60     0.11

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 24.435  on 19  degrees of freedom
Residual deviance: 20.895  on 18  degrees of freedom
AIC: 24.89
```

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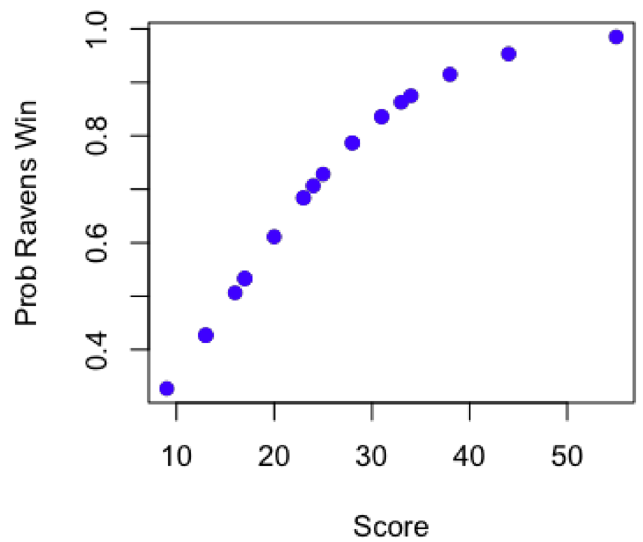
Ravens fitted values

```
plot(ravensData$ravenScore,logRegRavens$fitted,pch=19,col="blue",xlab="Score",ylab="Prob Ravens Win")
```



Odds ratios and confidence intervals

```
exp(logRegRavens$coeff)
```

```
          (Intercept) ravensData$ravenScore
               0.1864                1.1125
```

```
exp(confint(logRegRavens))
```

```
                         2.5 % 97.5 %
(Intercept)           0.005675  3.106
ravensData$ravenScore 0.996230  1.303
```

ANOVA for logistic regression

```
anova(logRegRavens,test="Chisq")
```

```
Analysis of Deviance Table

Model: binomial, link: logit

Response: ravensData$ravenWinNum

Terms added sequentially (first to last)

                      Df Deviance Resid. Df Resid. Dev Pr(>Chi)
NULL                                     19       24.4
ravensData$ravenScore  1     3.54        18       20.9     0.06 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Simpson's paradox

	Treatment A	Treatment B
Small Stones	*Group 1* **93% (81/87)**	*Group 2* 87% (234/270)
Large Stones	*Group 3* **73% (192/263)**	*Group 4* 69% (55/80)
Both	78% (273/350)	**83% (289/350)**

http://en.wikipedia.org/wiki/Simpson's_paradox

Interpreting Odds Ratios

- Not probabilities
- Odds ratio of 1 = no difference in odds
- Log odds ratio of 0 = no difference in odds
- Odds ratio < 0.5 or > 2 commonly a "moderate effect"
- Relative risk $\frac{\Pr(RW_i | RS_i = 10)}{\Pr(RW_i | RS_i = 0)}$ often easier to interpret, harder to estimate
- For small probabilities RR $\approx$ OR but **they are not the same**!

Wikipedia on Odds Ratio

Further resources

- Wikipedia on Logistic Regression
- Logistic regression and glms in R
- Brian Caffo's lecture notes on: Simpson's paradox, Case-control studies
- Open Intro Chapter on Logistic Regression