Policy Gradient

Spring, 2021

Outline

Table of Contents

Policy-Based Reinforcement Learning

- In the last lecture we approximated the value or action-value function using parameters θ,

$$V_\theta(s) \approx V^\pi(s)$$
$$Q_\theta(s,a) \approx Q^\pi(s,a)$$

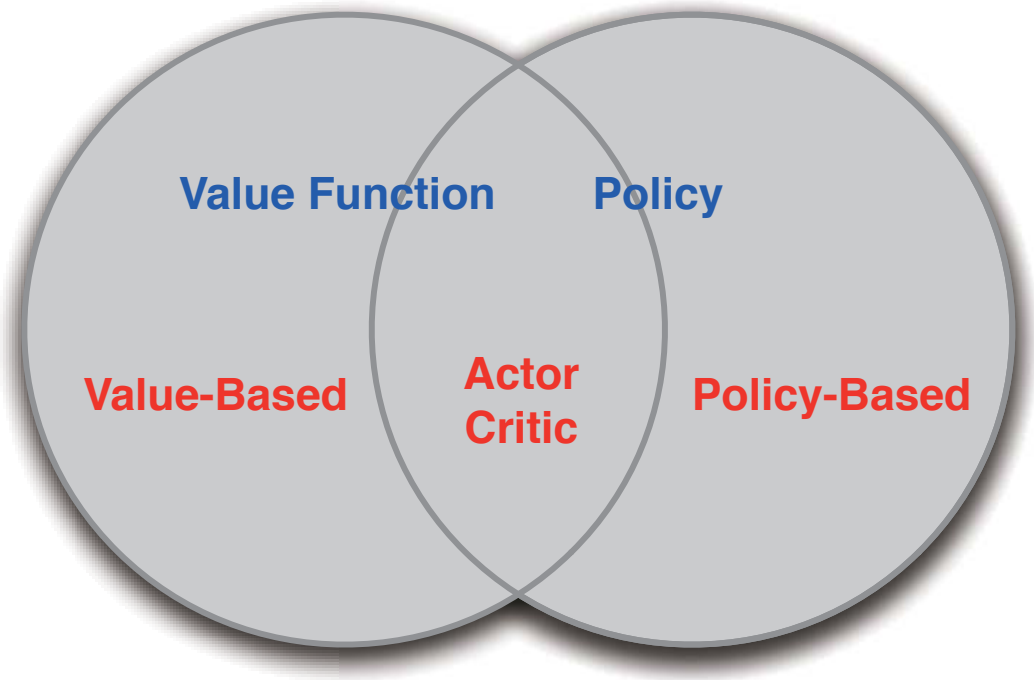
- A policy was generated directly from the value function
  - e.g. using ϵ-greedy
- In this lecture we will directly parametrize the policy

$$\pi_\theta(s,a) = \mathbb{P}[a|s,\theta]$$

- We will focus again on model-free reinforcement learning

Value-Based and Policy-Based RL

- Value Based
  - Learnt Value Function
  - Implicit policy (e.g.ϵ-greedy)
- Policy Based
  - No Value Function
  - Learnt Policy
- Actor-Critic
  - Learnt Value Function
  - Learnt Policy



Advantages of Policy-Based RL

Advantages:

- Better convergence properties
- Effective in high-dimensional or continuous action spaces
- Can learn stochastic policies

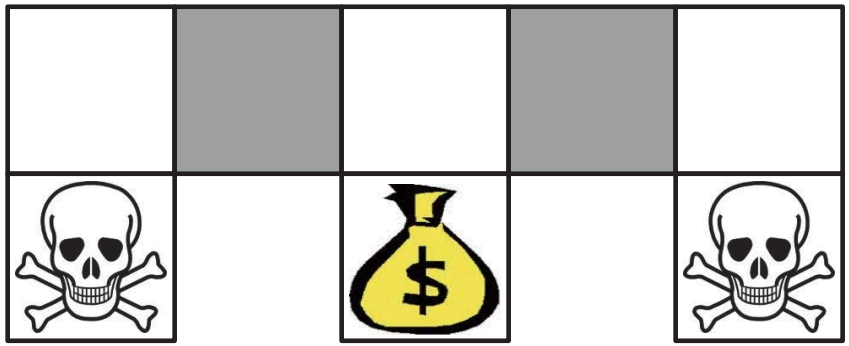
Disadvantages:

- Typically converge to a local rather than global optimum
- Evaluating a policy is typically inefficient and high variance

Example: Rock-Paper-Scissors

- Two-player game of rock-paper-scissors
  - Scissors beats paper
  - Rock beats scissors
  - Paper beats rock
- Consider policies for *iterated* rock-paper-scissors
  - A deterministic policy is easily exploited
  - A uniform random policy is optimal (i.e. Nash equilibrium)

Example: Aliased Gridworld (1)



- The agent cannot differentiate the grey states
- Consider features of the following form (for all N, E, S, W)

$$\phi(s, a) = \mathbf{1}(\text{wall to N, a} = \text{move E})$$

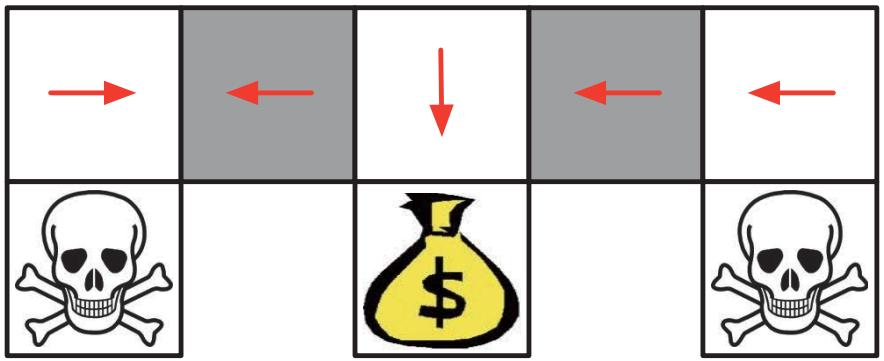
- Compare value-based RL, using an approximate value function

$$Q_\theta(s, a) = f(\phi(s, a), \theta)$$

- To policy-based RL, using a parametrized policy

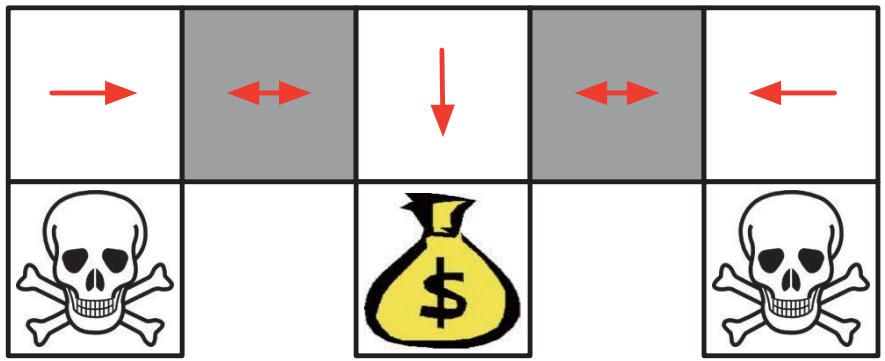
$$\pi_\theta(s, a) = g(\phi(s, a), \theta)$$

Example: Aliased Gridworld (2)



- Under aliasing, an optimal deterministic policy will either
  - move W in both grey states (shown by red arrows)
  - move E in both grey states
- Either way, it can get stuck and never reach the money
- Value-based RL learns a near-deterministic policy
  - e.g. greedy or ϵ-greedy
- So it will traverse the corridor for a long time

Example: Aliased Gridworld (3)



- An optimal stochastic policy will randomly move E or W in grey states

$$\pi_\theta(\text{wall to N and S, move to E}) = 0.5$$
$$\pi_\theta(\text{wall to N and S, move to W}) = 0.5$$

- It will reach the goal state in a few steps with high probability
- Policy-based RL can learn the optimal stochastic policy

Table of Contents

Policy Gradient

Basic Components

	Actor	Env	Reward Function
Video Game			Get 20 scores when killing a monster
Go			The rule of GO

Policy Gradient

Basic Components

You cannot control

	Actor	Env	Reward Function
Video Game			Get 20 scores when killing a monster
Go	AlphaGo Google DeepMind		The rule of GO

Policy Gradient

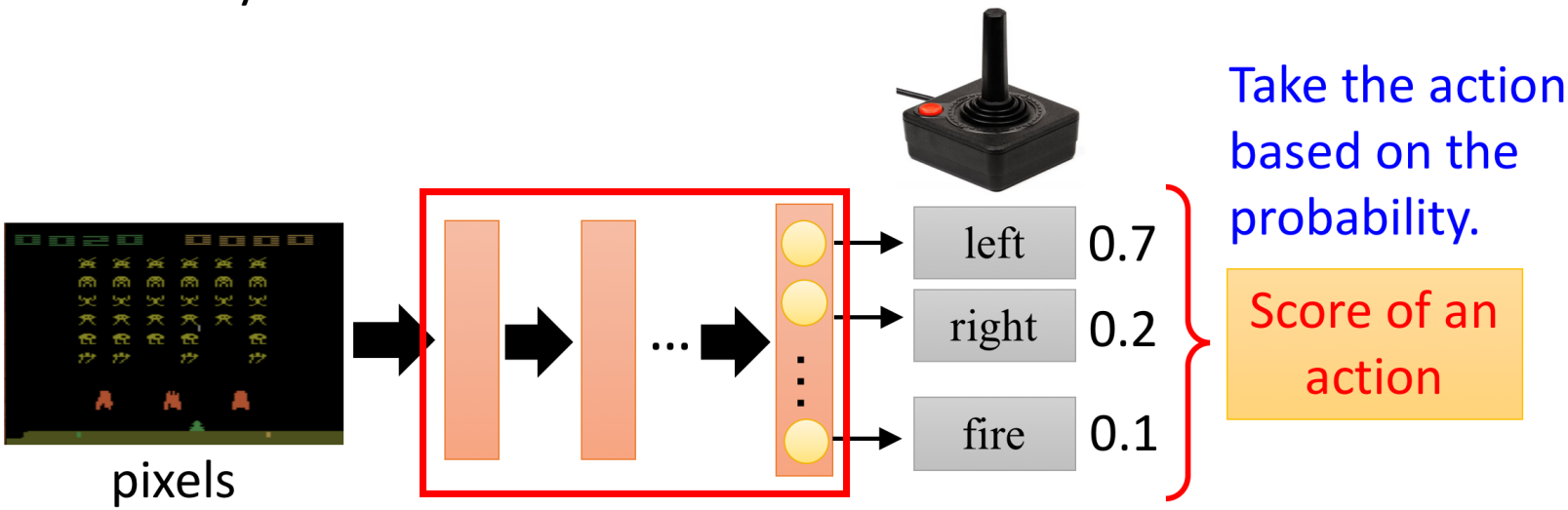
Policy of Actor

- Policy π is a network with parameter θ
  - Input: the observation of machine represented as a vector or a matrix
  - Output: each action corresponds to a neuron in output layer

Policy Gradient

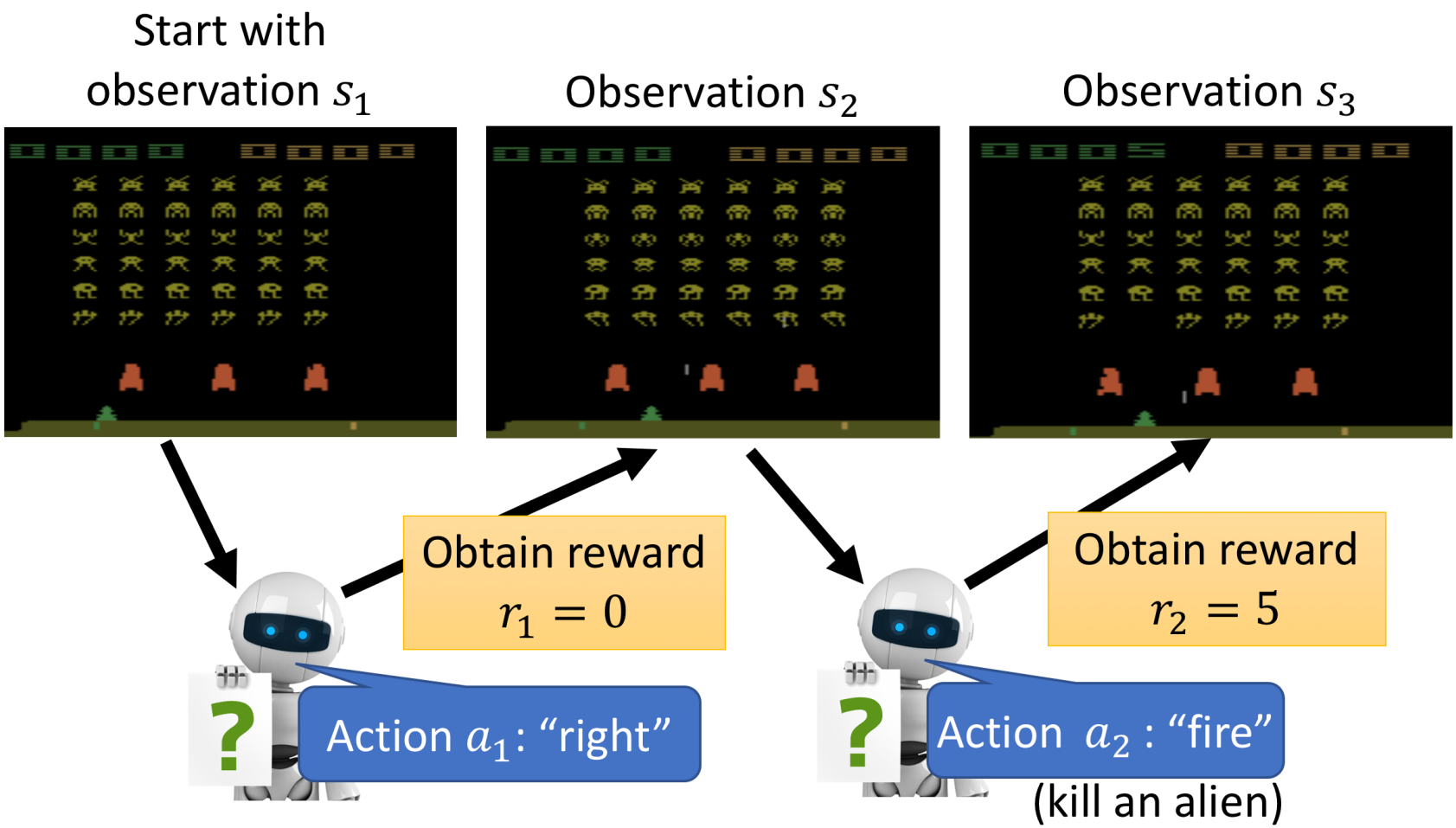
Policy of Actor

- Policy π is a network with parameter θ
  - Input: the observation of machine represented as a vector or a matrix
  - Output: each action corresponds to a neuron in output layer



Policy Gradient

Example: Playing Video Game

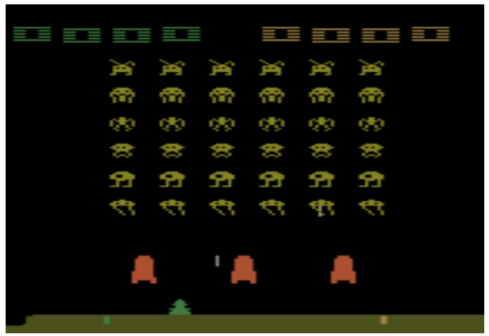


Policy Gradient

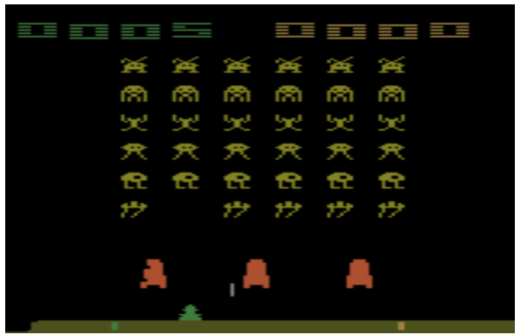
Example: Playing Video Game

Start with observation s_1

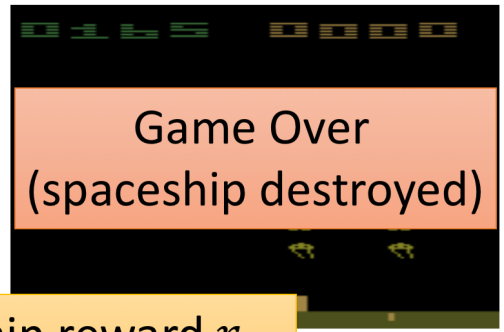
Observation s_2



Observation s_3



After many turns



Obtain reward r_T

Action a_T

This is an ***episode***.

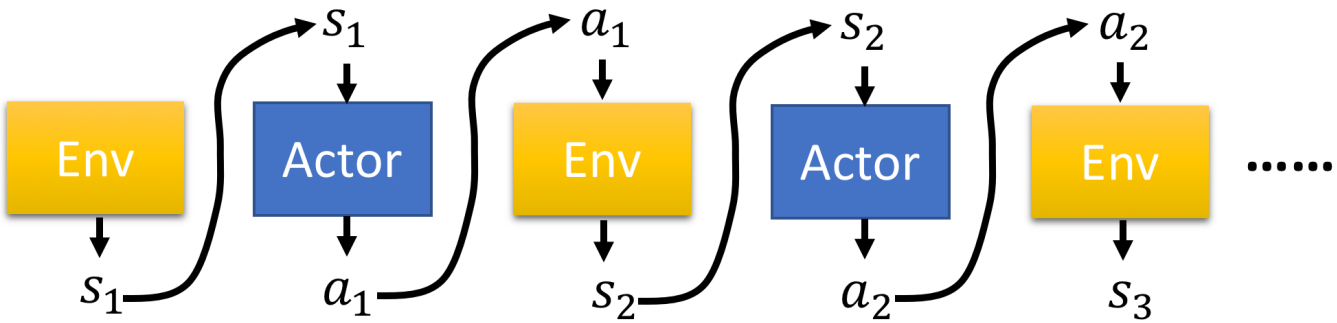
Total reward:

$$R = \sum_{t=1}^{T} r_t$$

We want the total reward be maximized.

Policy Gradient

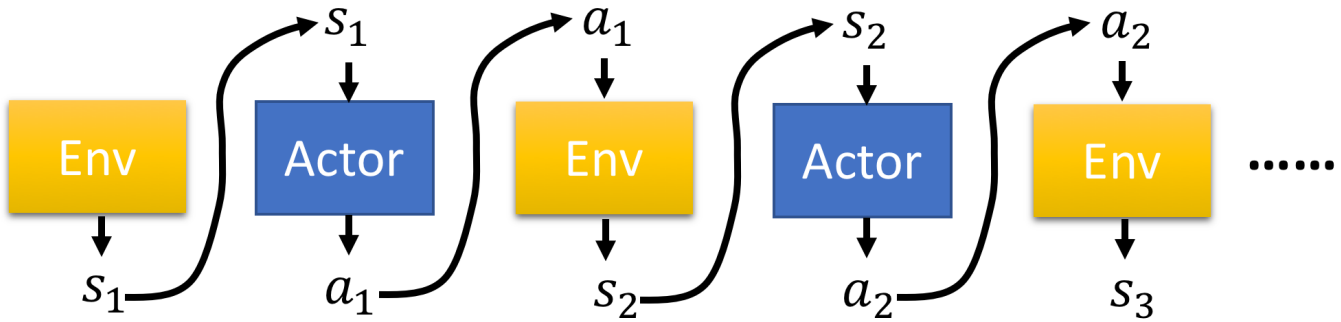
Actor, Environment, Reward



Trajectory $\tau = \{s_1, a_1, s_2, a_2, \cdots, s_T, a_T\}$

Policy Gradient

Actor, Environment, Reward



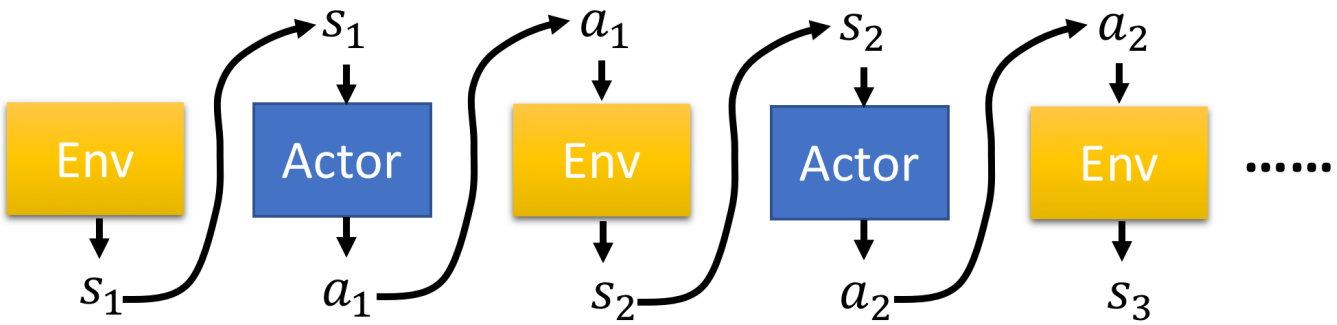
Trajectory $\tau = \{s_1, a_1, s_2, a_2, \cdots, s_T, a_T\}$

$$p_\theta(\tau)$$

$$= p(s_1)p_\theta(a_1|s_1)p(s_2|s_1, a_1)p_\theta(a_2|s_2)p(s_3|s_2, a_2)\cdots$$

Policy Gradient

Actor, Environment, Reward



Trajectory $\tau = \{s_1, a_1, s_2, a_2, \cdots, s_T, a_T\}$

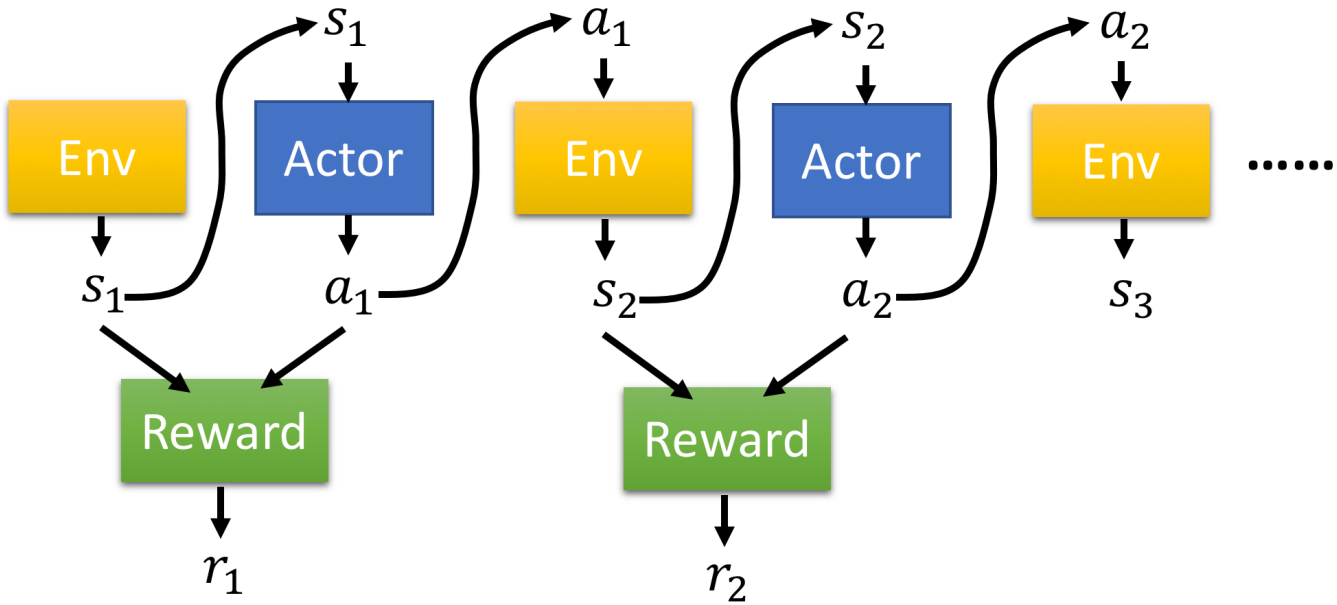
$$p_\theta(\tau)$$

$$= p(s_1)p_\theta(a_1|s_1)p(s_2|s_1,a_1)p_\theta(a_2|s_2)p(s_3|s_2,a_2)\cdots$$

$$= p(s_1)\prod_{t=1}^{T} p_\theta(a_t|s_t)p(s_{t+1}|s_t,a_t)$$

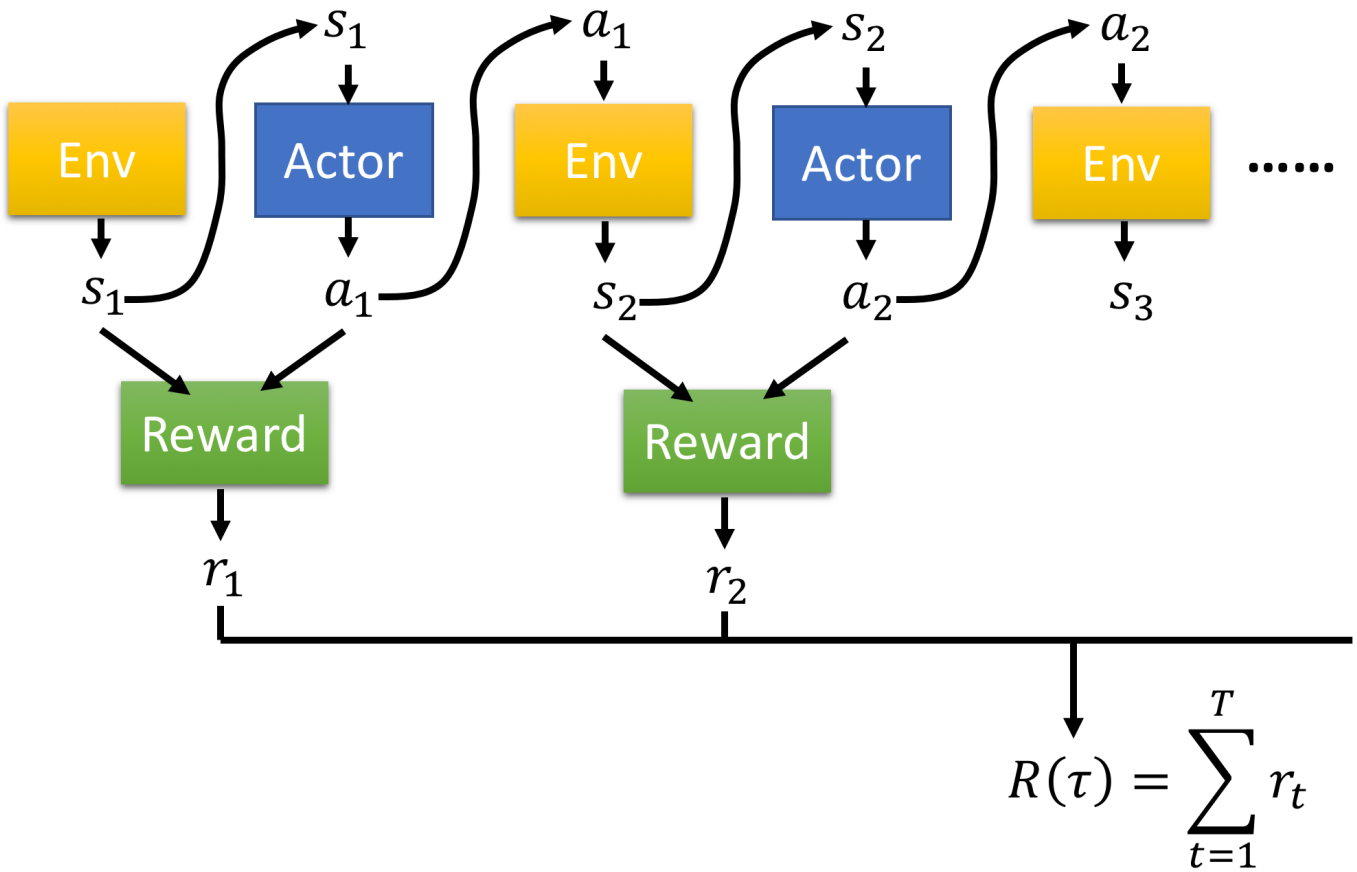
Policy Gradient

Actor, Environment, Reward

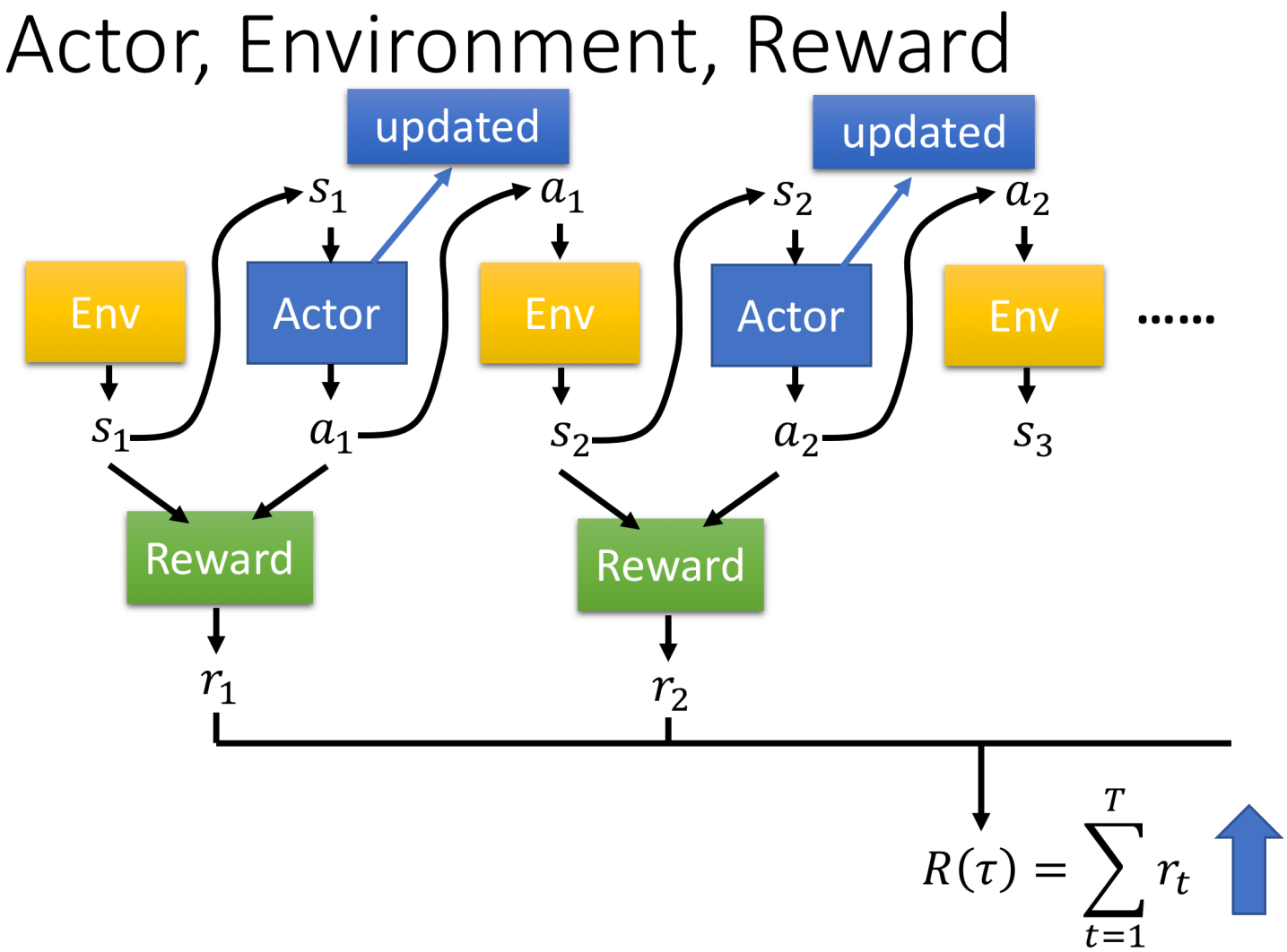


Policy Gradient

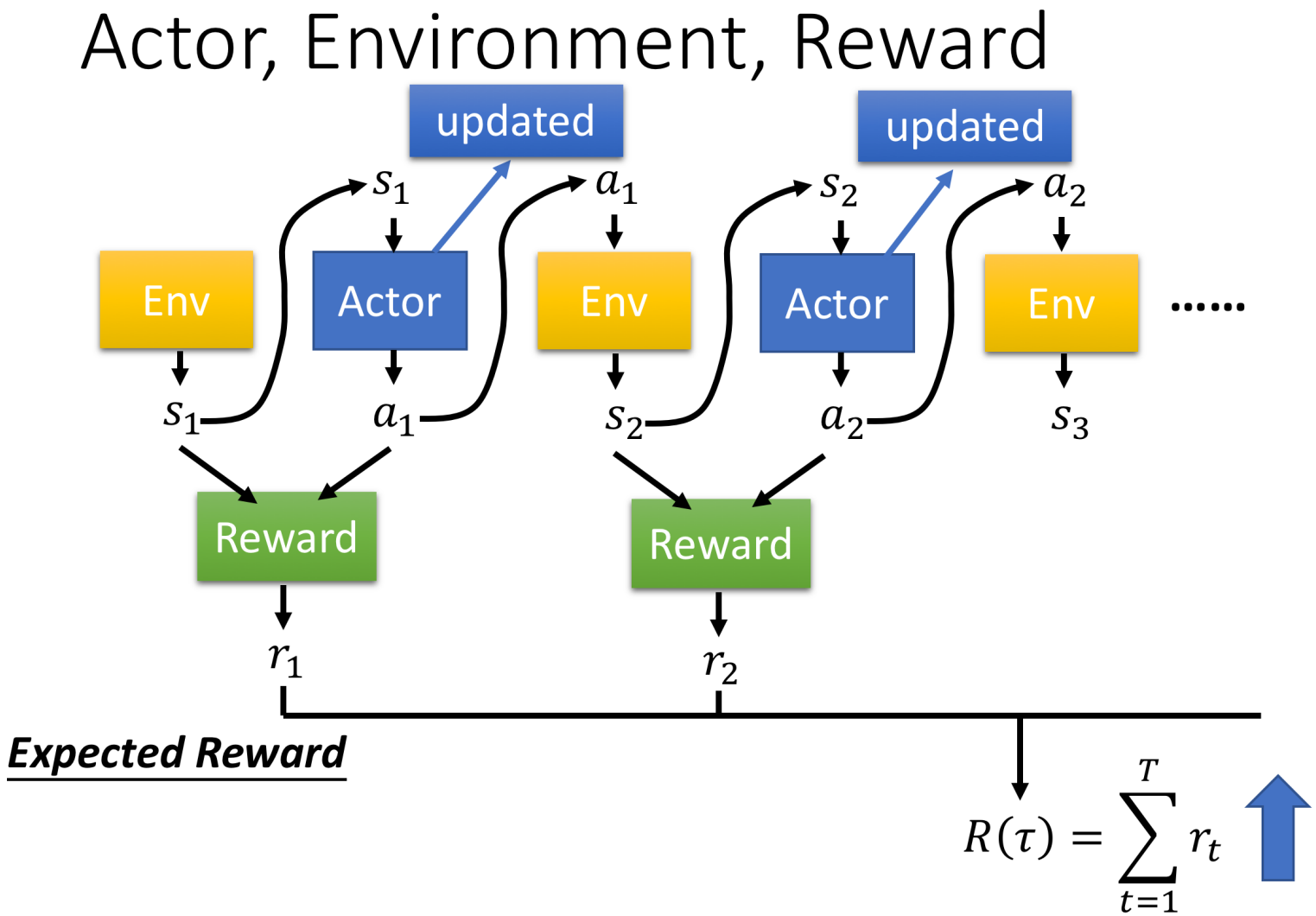
Actor, Environment, Reward



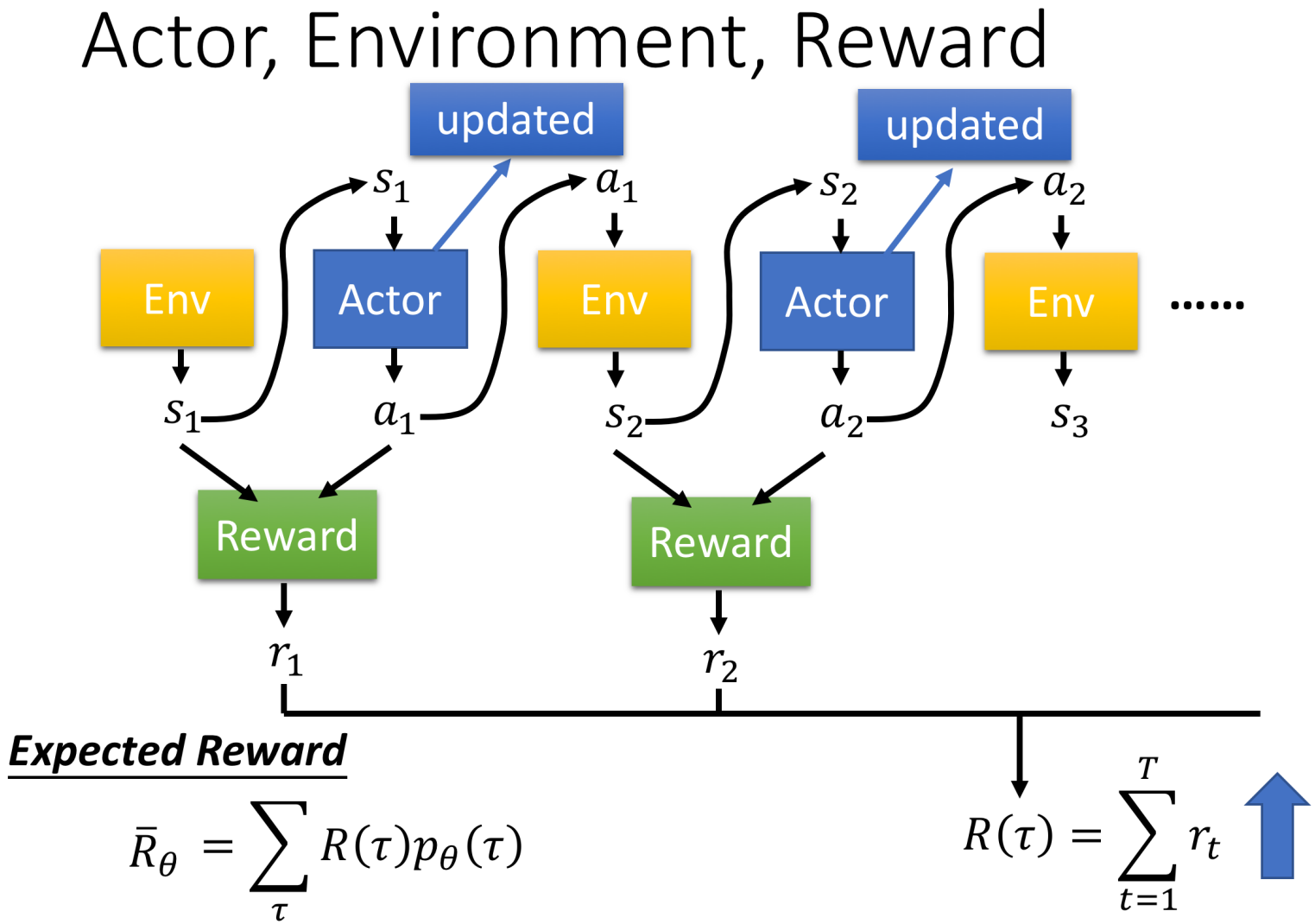
Policy Gradient



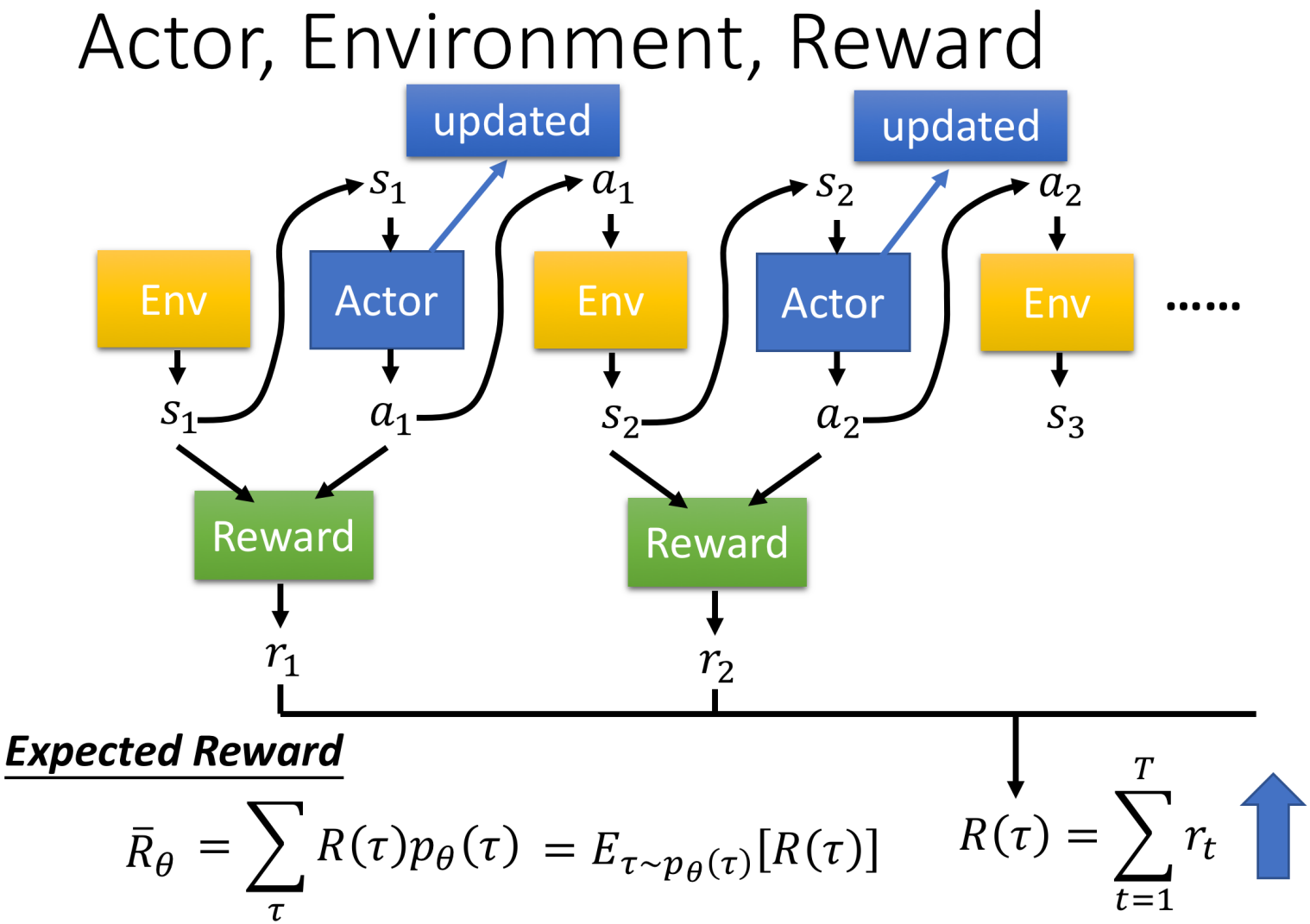
Policy Gradient



Policy Gradient



Policy Gradient



Policy Gradient

Policy Gradient $\quad \bar{R}_\theta = \sum_\tau R(\tau) p_\theta(\tau) \quad \nabla \bar{R}_\theta = ?$

Policy Gradient

Policy Gradient

$$\bar{R}_\theta = \sum_\tau R(\tau) p_\theta(\tau) \qquad \nabla \bar{R}_\theta = ?$$

$$\nabla \bar{R}_\theta = \sum_\tau R(\tau) \nabla p_\theta(\tau)$$

$R(\tau)$ do not have to be differentiable

Policy Gradient

Policy Gradient

$$\bar{R}_\theta = \sum_\tau R(\tau) p_\theta(\tau) \qquad \nabla \bar{R}_\theta = ?$$

$$\nabla \bar{R}_\theta = \sum_\tau R(\tau) \nabla p_\theta(\tau)$$

$R(\tau)$ do not have to be differentiable

It can even be a black box.

Policy Gradient

Policy Gradient

$$\bar{R}_\theta = \sum_\tau R(\tau) p_\theta(\tau) \qquad \nabla \bar{R}_\theta = ?$$

$$\nabla \bar{R}_\theta = \sum_\tau R(\tau) \nabla p_\theta(\tau) = \sum_\tau R(\tau) p_\theta(\tau) \frac{\nabla p_\theta(\tau)}{p_\theta(\tau)}$$

$R(\tau)$ do not have to be differentiable

It can even be a black box.

Policy Gradient

Policy Gradient

$$\bar{R}_\theta = \sum_\tau R(\tau) p_\theta(\tau) \qquad \nabla \bar{R}_\theta = ?$$

$$\nabla \bar{R}_\theta = \sum_\tau R(\tau) \nabla p_\theta(\tau) = \sum_\tau R(\tau) p_\theta(\tau) \frac{\nabla p_\theta(\tau)}{p_\theta(\tau)}$$

$R(\tau)$ do not have to be differentiable

It can even be a black box.

$$= \sum_\tau R(\tau) p_\theta(\tau) \nabla log p_\theta(\tau)$$

$$\nabla f(x) = f(x) \nabla log f(x)$$

Policy Gradient

Policy Gradient

$$\bar{R}_\theta = \sum_\tau R(\tau) p_\theta(\tau) \qquad \nabla \bar{R}_\theta = ?$$

$$\nabla \bar{R}_\theta = \sum_\tau R(\tau) \nabla p_\theta(\tau) = \sum_\tau R(\tau) p_\theta(\tau) \frac{\nabla p_\theta(\tau)}{p_\theta(\tau)}$$

$R(\tau)$ do not have to be differentiable

It can even be a black box.

$$= \sum_\tau R(\tau) p_\theta(\tau) \nabla log p_\theta(\tau)$$

$$= E_{\tau \sim p_\theta(\tau)} [R(\tau) \nabla log p_\theta(\tau)]$$

$$\nabla f(x) = f(x) \nabla log f(x)$$

Policy Gradient

Policy Gradient

$$\bar{R}_\theta = \sum_\tau R(\tau) p_\theta(\tau) \qquad \nabla \bar{R}_\theta = ?$$

$$\nabla \bar{R}_\theta = \sum_\tau R(\tau) \nabla p_\theta(\tau) = \sum_\tau R(\tau) p_\theta(\tau) \frac{\nabla p_\theta(\tau)}{p_\theta(\tau)}$$

$R(\tau)$ do not have to be differentiable

It can even be a black box.

$$= \sum_\tau R(\tau) p_\theta(\tau) \nabla log p_\theta(\tau)$$

$$\nabla f(x) = f(x) \nabla log f(x)$$

$$= E_{\tau \sim p_\theta(\tau)} [R(\tau) \nabla log p_\theta(\tau)] \approx \frac{1}{N} \sum_{n=1}^{N} R(\tau^n) \nabla log p_\theta(\tau^n)$$

Policy Gradient

Policy Gradient

$$\bar{R}_\theta = \sum_\tau R(\tau) p_\theta(\tau) \qquad \nabla \bar{R}_\theta = ?$$

$$\nabla \bar{R}_\theta = \sum_\tau R(\tau) \nabla p_\theta(\tau) = \sum_\tau R(\tau) p_\theta(\tau) \frac{\nabla p_\theta(\tau)}{p_\theta(\tau)}$$

$R(\tau)$ do not have to be differentiable

It can even be a black box.

$$= \sum_\tau R(\tau) p_\theta(\tau) \nabla log p_\theta(\tau)$$

$$\nabla f(x) = f(x) \nabla log f(x)$$

$$= E_{\tau \sim p_\theta(\tau)} [R(\tau) \nabla log p_\theta(\tau)] \approx \frac{1}{N} \sum_{n=1}^{N} R(\tau^n) \nabla log p_\theta(\tau^n)$$

$$= \frac{1}{N} \sum_{n=1}^{N} \sum_{t=1}^{T_n} R(\tau^n) \nabla log p_\theta(a_t^n | s_t^n)$$

Policy Gradient

Policy Gradient

$$\theta \leftarrow \theta + \eta \nabla \bar{R}_\theta$$

$$\nabla \bar{R}_\theta = \frac{1}{N} \sum_{n=1}^{N} \sum_{t=1}^{T_n} R(\tau^n) \nabla log p_\theta(a_t^n | s_t^n)$$

Policy Gradient

Policy Gradient

Given policy π_θ

$$
\begin{array}{lll}
\tau^1: & (s_1^1, a_1^1) & R(\tau^1) \\
 & (s_2^1, a_2^1) & R(\tau^1) \\
 & \vdots & \vdots \\
\tau^2: & (s_1^2, a_1^2) & R(\tau^2) \\
 & (s_2^2, a_2^2) & R(\tau^2) \\
 & \vdots & \vdots
\end{array}
$$

$$\theta \leftarrow \theta + \eta \nabla \bar{R}_\theta$$

$$\nabla \bar{R}_\theta = \frac{1}{N} \sum_{n=1}^{N} \sum_{t=1}^{T_n} R(\tau^n) \nabla log p_\theta(a_t^n | s_t^n)$$

Policy Gradient

Policy Gradient

Given policy π_θ

τ^1: (s_1^1, a_1^1) $R(\tau^1)$

(s_2^1, a_2^1) $R(\tau^1)$

$\vdots$ $\vdots$

τ^2: (s_1^2, a_1^2) $R(\tau^2)$

(s_2^2, a_2^2) $R(\tau^2)$

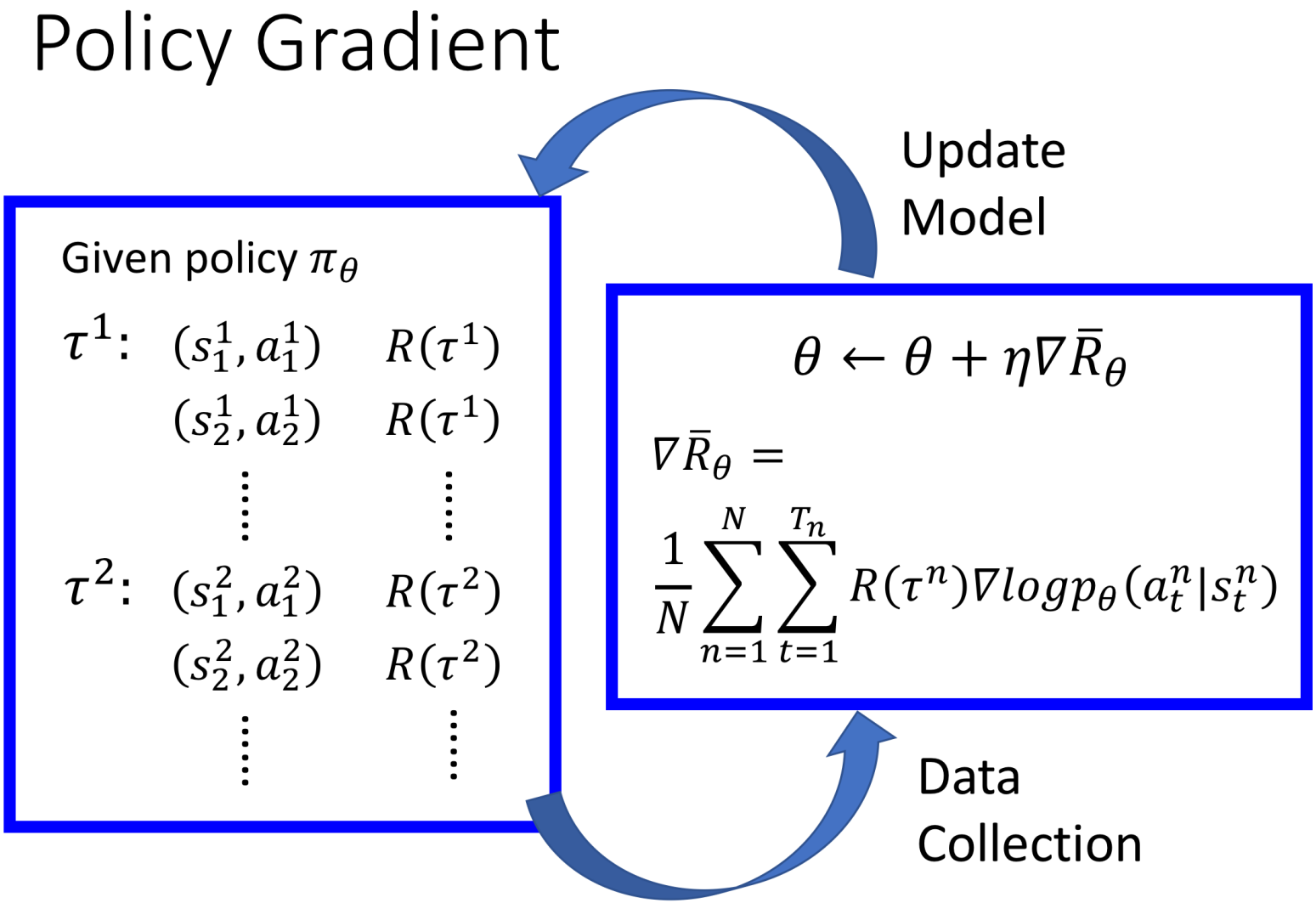
$\vdots$ $\vdots$

$$\theta \leftarrow \theta + \eta \nabla \bar{R}_\theta$$

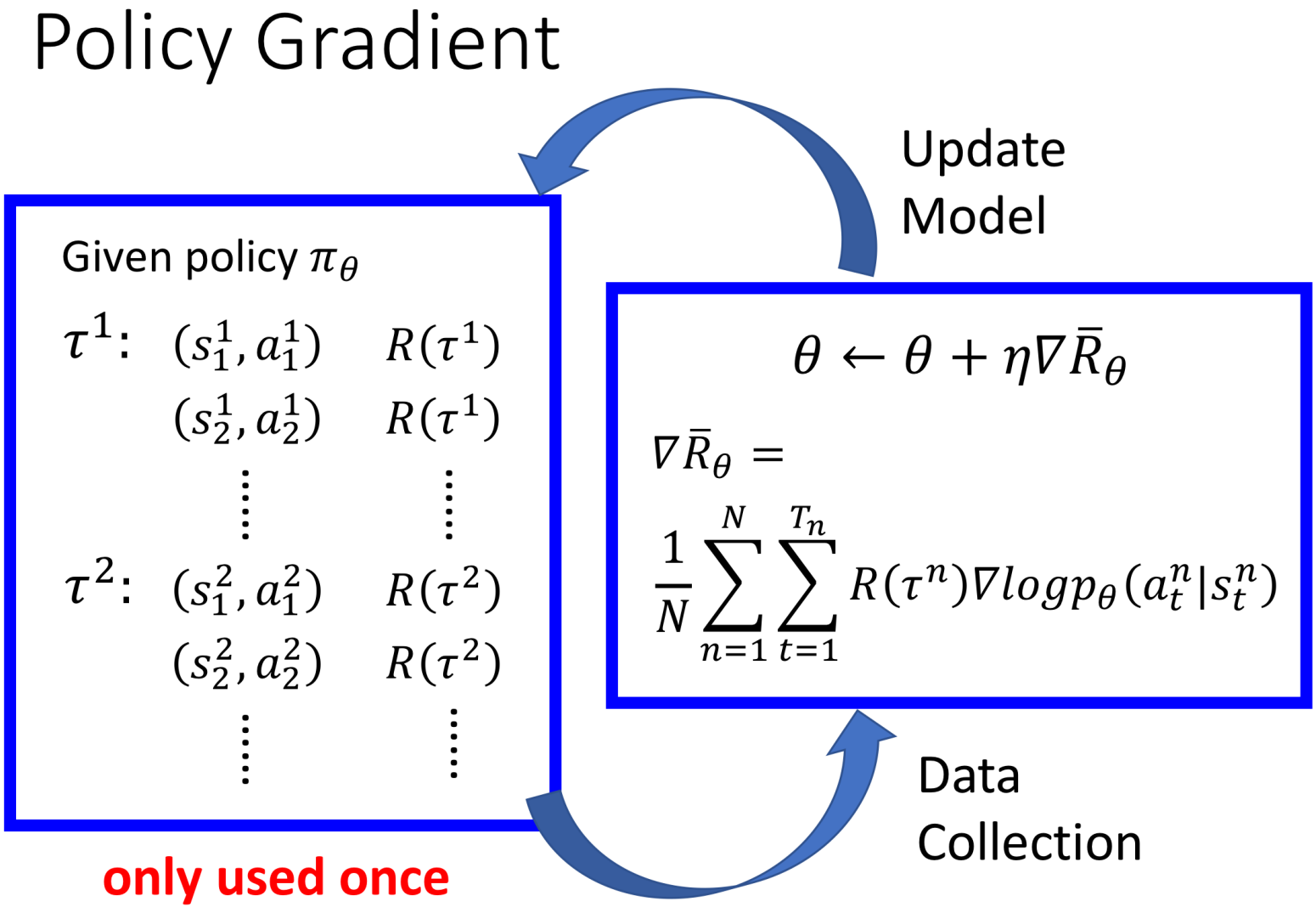
$$\nabla \bar{R}_\theta = \frac{1}{N} \sum_{n=1}^{N} \sum_{t=1}^{T_n} R(\tau^n) \nabla log p_\theta(a_t^n | s_t^n)$$

Data Collection

Policy Gradient



Policy Gradient



Monte-Carlo Policy Gradient (REINFORCE)

- Update parameters by stochastic gradient ascent
- Using policy gradient theorem
- Using return v_t as an unbiased sample of $Q^{\pi_\theta}(s_t, a_t)$

$$\Delta\theta_t = \alpha \nabla_\theta \log \pi_\theta(s_t, a_t) v_t$$

function REINFORCE

Initialise θ arbitrarily

for each episode $\{s_1, a_1, r_2, ..., s_{T-1}, a_{T-1}, r_T\} \sim \pi_\theta$ **do**

for $t = 1$ to $T - 1$ **do**

$\theta \leftarrow \theta + \alpha \nabla_\theta \log \pi_\theta(s_t, a_t) v_t$

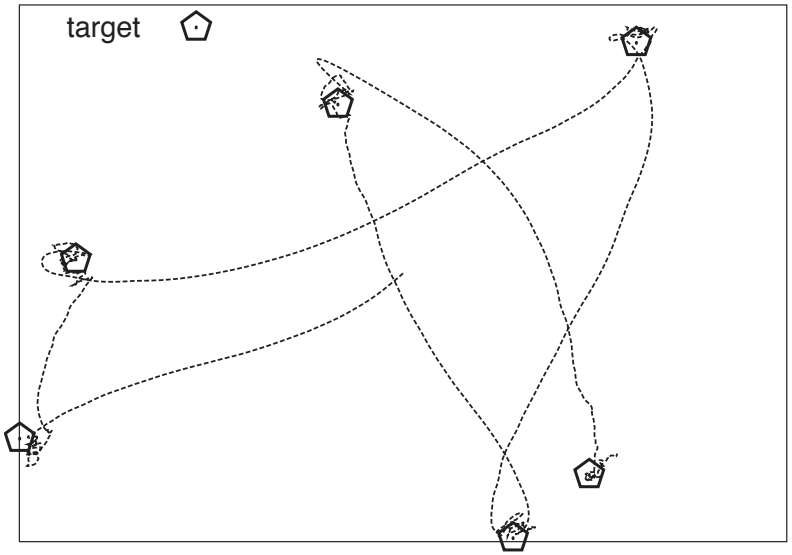
end for

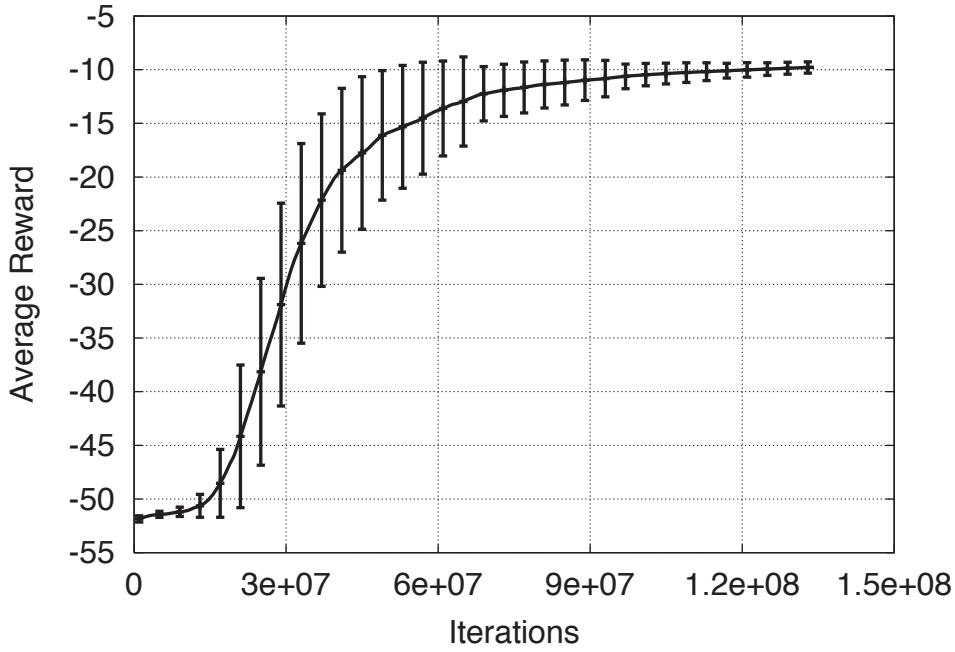
end for

return θ

end function

Puck World Example





- Continuous actions exert small force on puck
- Puck is rewarded for getting close to target
- Target location is reset every 30 seconds
- Policy is training using variant of Monte-Carlo policy gradient

Policy Gradient

Tip 1: Add a Baseline

$$\theta \leftarrow \theta + \eta \nabla \bar{R}_\theta$$

$$\nabla \bar{R}_\theta \approx \frac{1}{N} \sum_{n=1}^{N} \sum_{t=1}^{T_n} R(\tau^n) \nabla log p_\theta(a_t^n | s_t^n)$$

Policy Gradient

Tip 1: Add a Baseline

$\theta \leftarrow \theta + \eta \nabla \bar{R}_\theta$

It is possible that $R(\tau^n)$ is always positive.

$$\nabla \bar{R}_\theta \approx \frac{1}{N} \sum_{n=1}^{N} \sum_{t=1}^{T_n} R(\tau^n) \nabla log p_\theta(a_t^n | s_t^n)$$

Policy Gradient

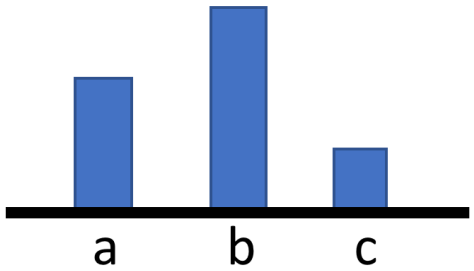
Tip 1: Add a Baseline

$$\theta \leftarrow \theta + \eta \nabla \bar{R}_\theta$$

It is possible that $R(\tau^n)$ is always positive.

$$\nabla \bar{R}_\theta \approx \frac{1}{N} \sum_{n=1}^{N} \sum_{t=1}^{T_n} R(\tau^n) \nabla log p_\theta(a_t^n | s_t^n)$$

Ideal case



Policy Gradient

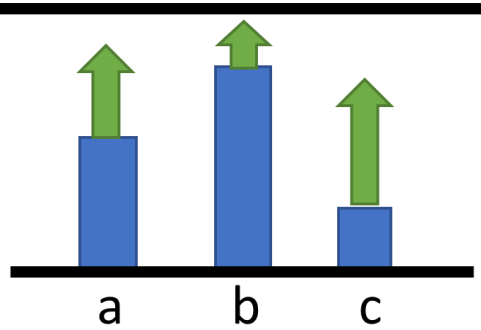
Tip 1: Add a Baseline

$$\theta \leftarrow \theta + \eta \nabla \bar{R}_\theta$$

It is possible that $R(\tau^n)$ is always positive.

$$\nabla \bar{R}_\theta \approx \frac{1}{N} \sum_{n=1}^{N} \sum_{t=1}^{T_n} R(\tau^n) \nabla log p_\theta(a_t^n | s_t^n)$$

Ideal case



Policy Gradient

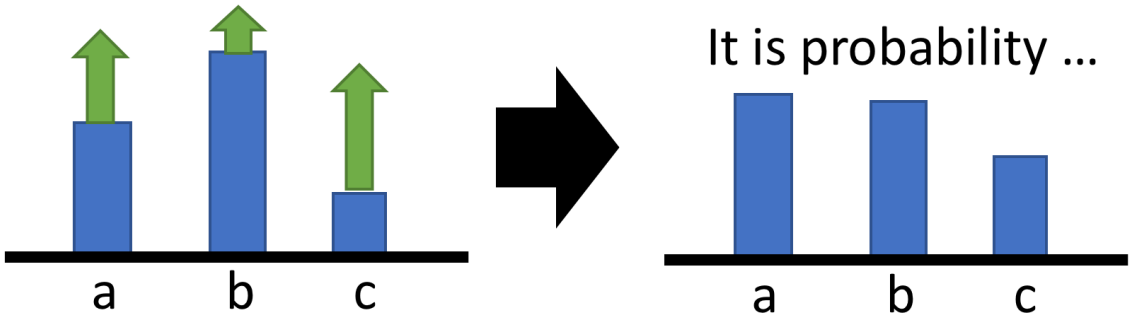
Tip 1: Add a Baseline

$$\theta \leftarrow \theta + \eta \nabla \bar{R}_\theta$$

It is possible that $R(\tau^n)$ is always positive.

$$\nabla \bar{R}_\theta \approx \frac{1}{N} \sum_{n=1}^{N} \sum_{t=1}^{T_n} R(\tau^n) \nabla log p_\theta(a_t^n | s_t^n)$$

Ideal case



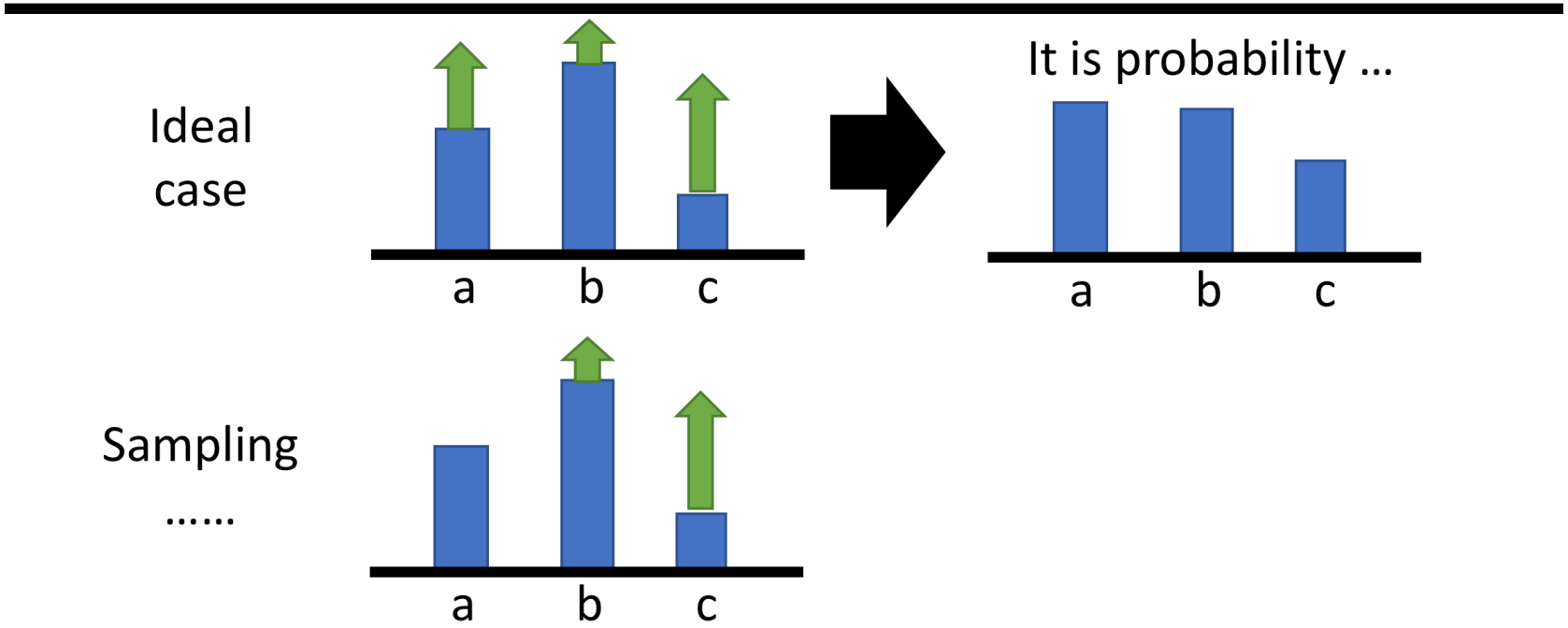
Policy Gradient

Tip 1: Add a Baseline

$$\theta \leftarrow \theta + \eta \nabla \bar{R}_\theta$$

It is possible that $R(\tau^n)$ is always positive.

$$\nabla \bar{R}_\theta \approx \frac{1}{N} \sum_{n=1}^{N} \sum_{t=1}^{T_n} R(\tau^n) \nabla log p_\theta(a_t^n | s_t^n)$$



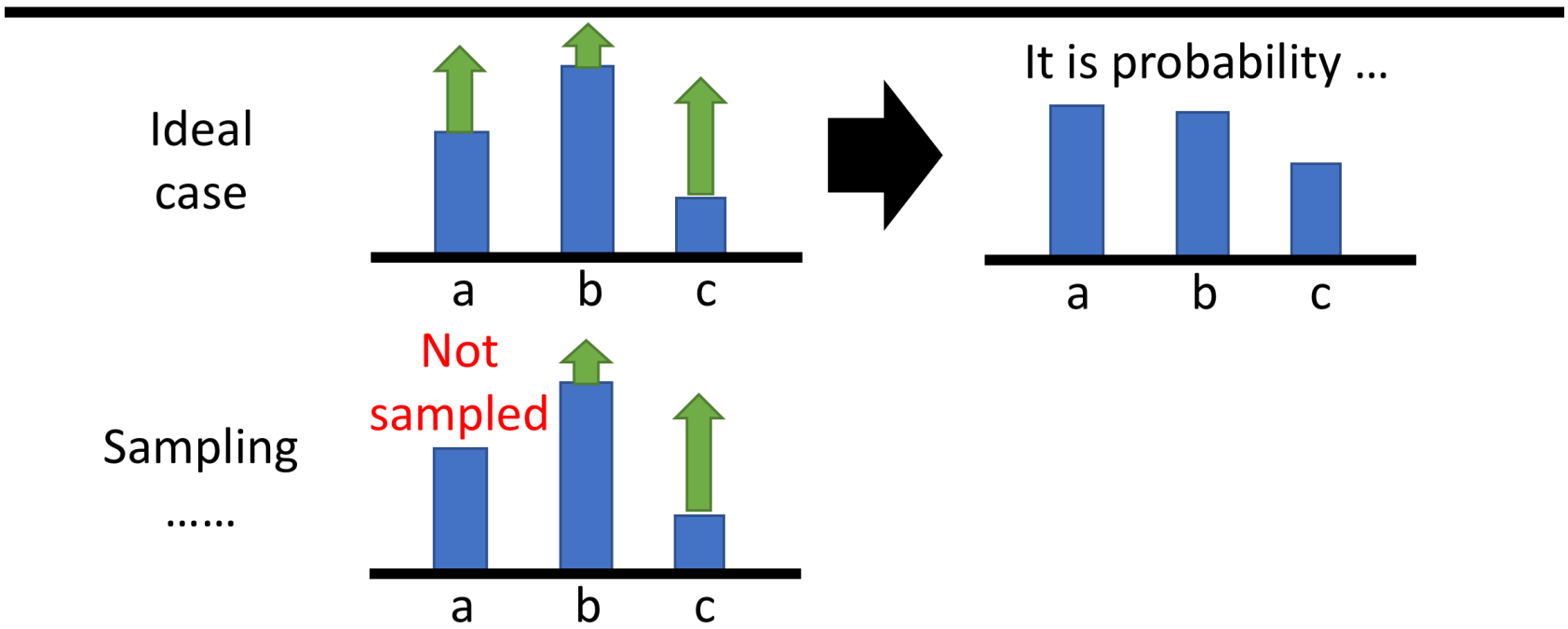
Policy Gradient

Tip 1: Add a Baseline

$$\theta \leftarrow \theta + \eta \nabla \bar{R}_\theta$$

It is possible that $R(\tau^n)$ is always positive.

$$\nabla \bar{R}_\theta \approx \frac{1}{N} \sum_{n=1}^{N} \sum_{t=1}^{T_n} R(\tau^n) \nabla log p_\theta(a_t^n | s_t^n)$$



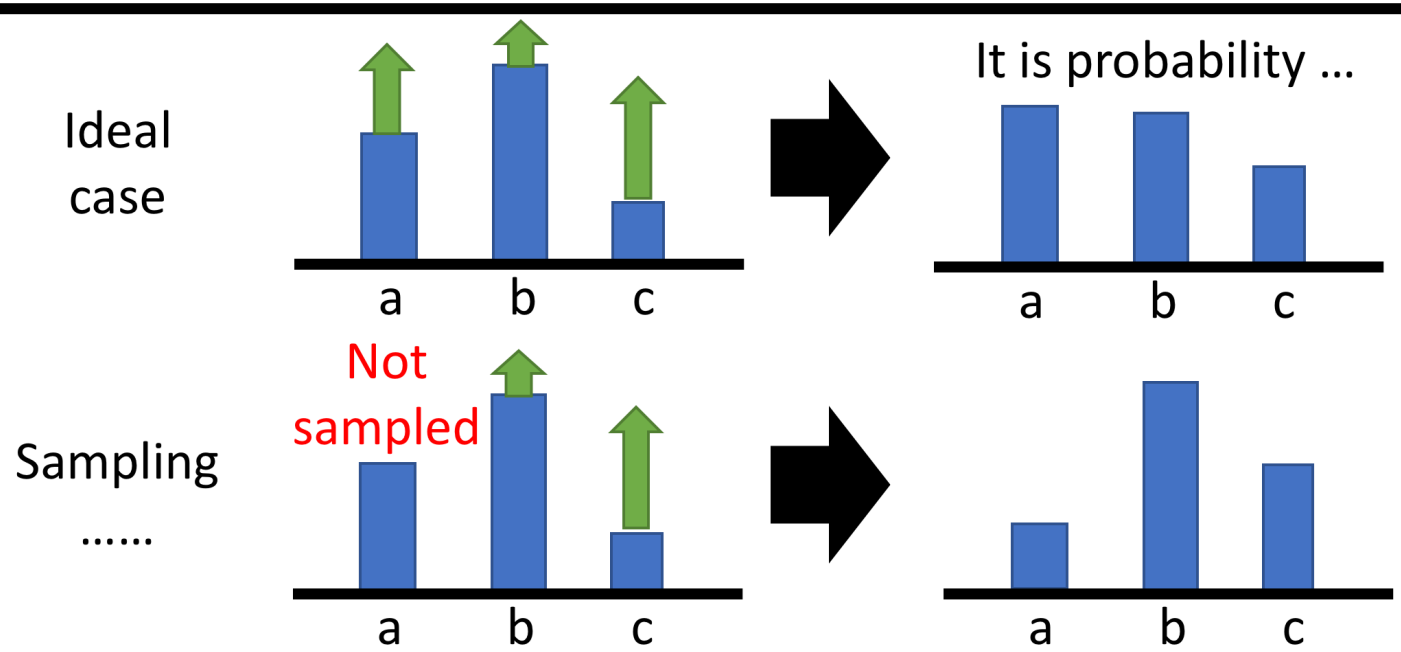
Policy Gradient

Tip 1: Add a Baseline

$$\theta \leftarrow \theta + \eta \nabla \bar{R}_\theta$$

It is possible that $R(\tau^n)$ is always positive.

$$\nabla \bar{R}_\theta \approx \frac{1}{N} \sum_{n=1}^{N} \sum_{t=1}^{T_n} R(\tau^n) \nabla log p_\theta(a_t^n | s_t^n)$$



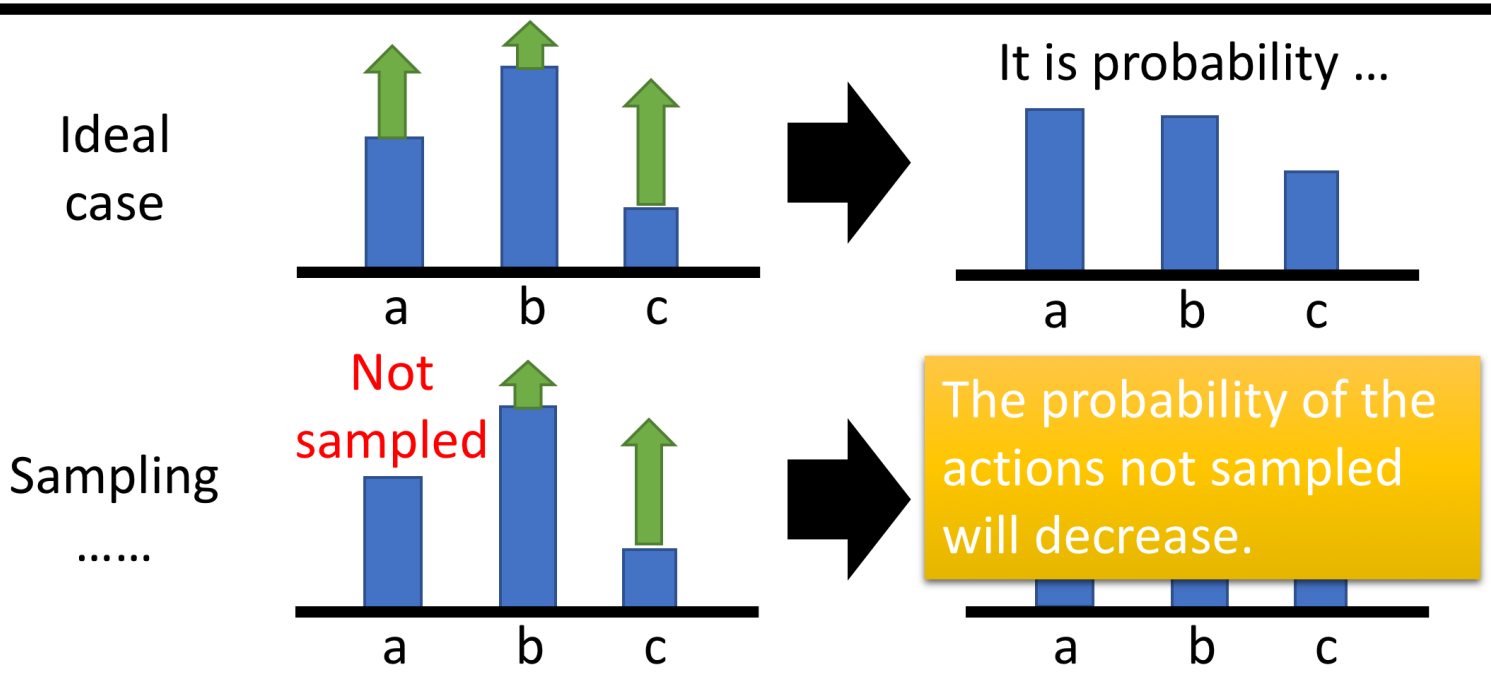
Policy Gradient

Tip 1: Add a Baseline

$\theta \leftarrow \theta + \eta \nabla \bar{R}_\theta$

It is possible that $R(\tau^n)$ is always positive.

$$\nabla \bar{R}_\theta \approx \frac{1}{N} \sum_{n=1}^{N} \sum_{t=1}^{T_n} R(\tau^n) \nabla log p_\theta(a_t^n | s_t^n)$$



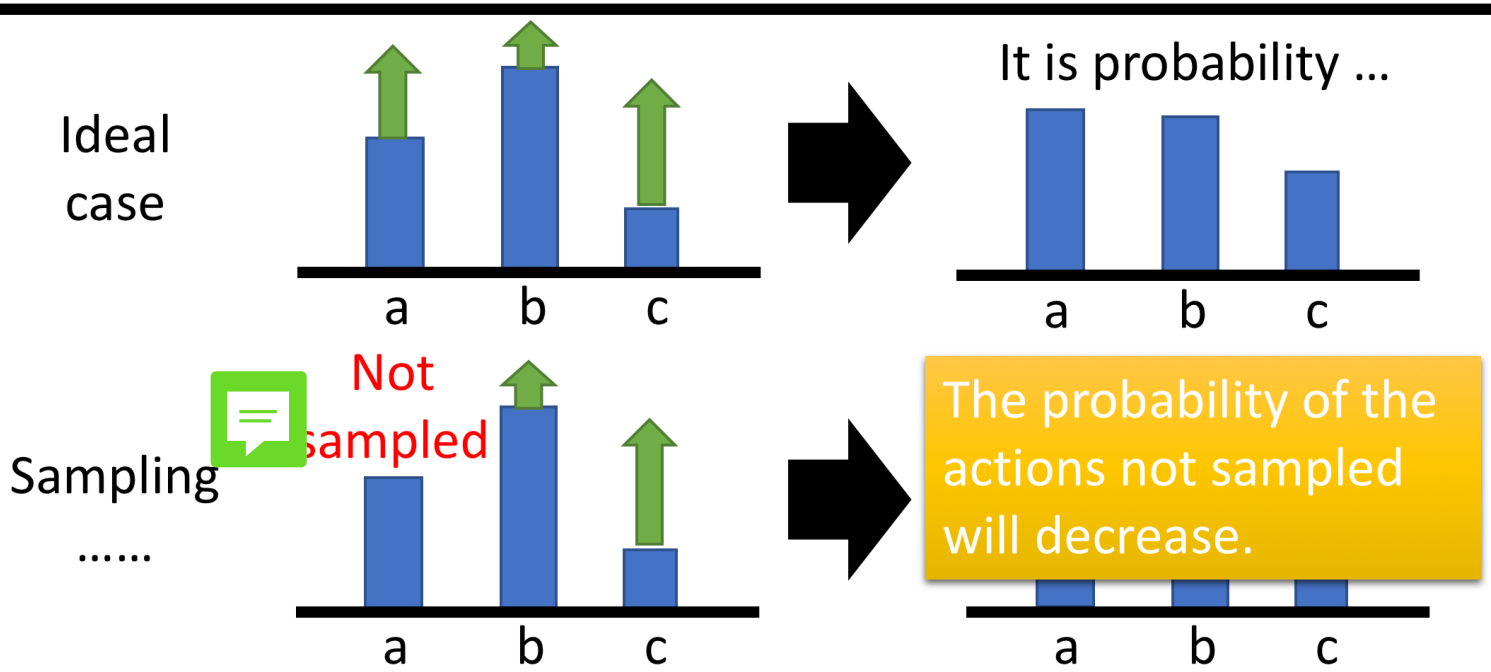
Policy Gradient

Tip 1: Add a Baseline

$\theta \leftarrow \theta + \eta \nabla \bar{R}_\theta$

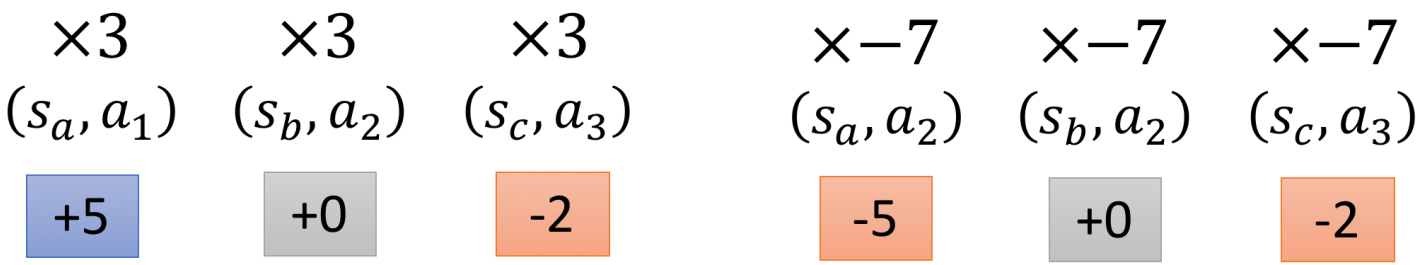
It is possible that $R(\tau^n)$ is always positive.

$$\nabla \bar{R}_\theta \approx \frac{1}{N} \sum_{n=1}^{N} \sum_{t=1}^{T_n} (R(\tau^n) - b) \nabla log p_\theta(a_t^n | s_t^n) \qquad b \approx E[R(\tau)]$$



Policy Gradient

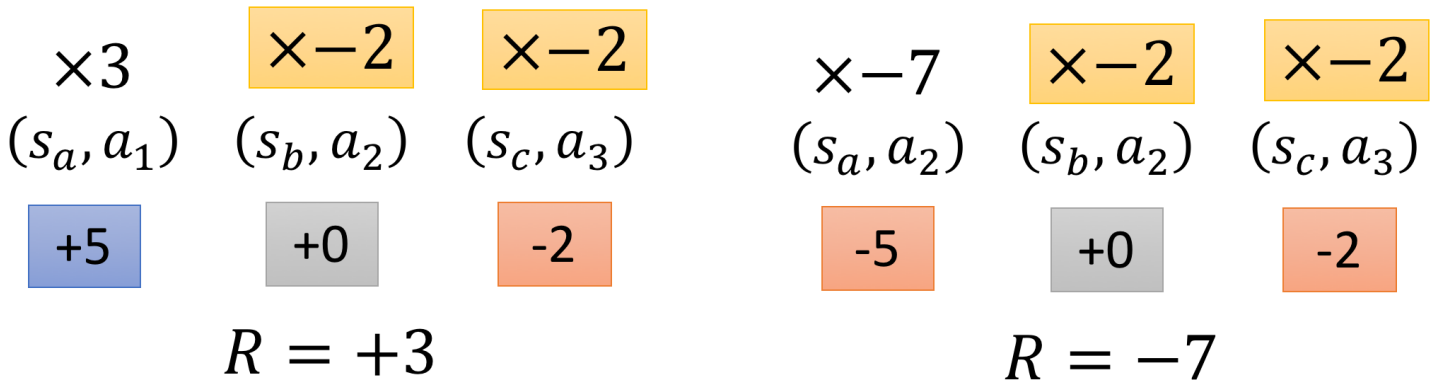
Tip 2: Assign Suitable Credit



$$\nabla \bar{R}_\theta \approx \frac{1}{N} \sum_{n=1}^{N} \sum_{t=1}^{T_n} (R(\tau^n) - b) \nabla log p_\theta(a_t^n | s_t^n)$$

Policy Gradient

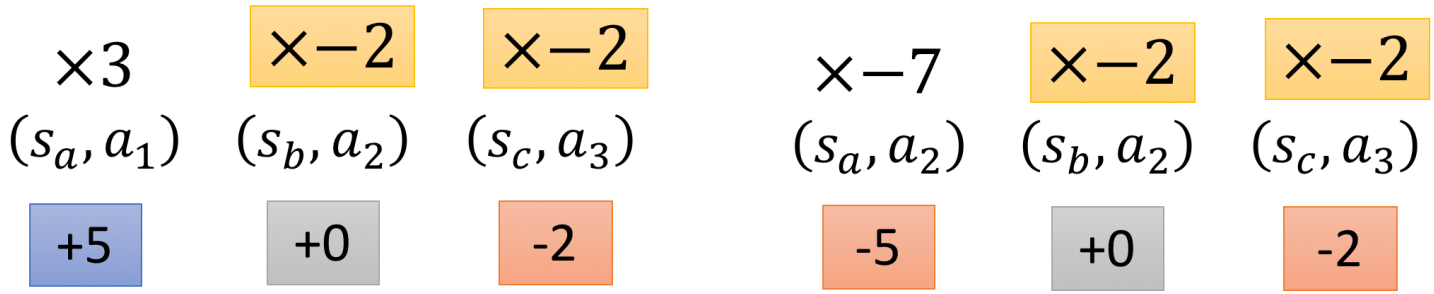
Tip 2: Assign Suitable Credit



$$\nabla \bar{R}_\theta \approx \frac{1}{N} \sum_{n=1}^{N} \sum_{t=1}^{T_n} (R(\tau^n) - b) \nabla log p_\theta(a_t^n | s_t^n)$$

Policy Gradient

Tip 2: Assign Suitable Credit



$$\nabla \bar{R}_\theta \approx \frac{1}{N} \sum_{n=1}^{N} \sum_{t=1}^{T_n} (R(\tau^n) - b) \nabla log p_\theta(a_t^n | s_t^n)$$

$$R(\tau^n) \rightarrow \sum_{t'=t}^{T_n} r_{t'}^n$$

Policy Gradient

Tip 2: Assign Suitable Credit

$$\nabla \bar{R}_\theta \approx \frac{1}{N} \sum_{n=1}^{N} \sum_{t=1}^{T_n} (R(\tau^n) - b) \nabla log p_\theta(a_t^n | s_t^n)$$

$$\sum_{t'=t}^{T_n} r_{t'}^n$$

Add discount factor

Policy Gradient

Tip 2: Assign Suitable Credit

$$\nabla \bar{R}_\theta \approx \frac{1}{N} \sum_{n=1}^{N} \sum_{t=1}^{T_n} (R(\tau^n) - b) \nabla log p_\theta(a_t^n | s_t^n)$$

$$\sum_{t'=t}^{T_n} r_{t'}^n \rightarrow \sum_{t'=t}^{T_n} \gamma^{t'-t} r_{t'}^n$$

Add discount factor $\gamma < 1$

Policy Gradient

Tip 2: Assign Suitable Credit

Can be state-dependent

$$\nabla \bar{R}_\theta \approx \frac{1}{N}\sum_{n=1}^{N}\sum_{t=1}^{T_n}(R(\tau^n) - b)\nabla log p_\theta(a_t^n|s_t^n)$$

$$\sum_{t'=t}^{T_n} r_{t'}^n \rightarrow \sum_{t'=t}^{T_n} \gamma^{t'-t} r_{t'}^n$$

Add discount factor $\gamma < 1$

Policy Gradient

Tip 2: Assign Suitable Credit

Advantage Function $A^{\theta}(s_t, a_t)$

Can be state-dependent

$$\nabla \bar{R}_\theta \approx \frac{1}{N} \sum_{n=1}^{N} \sum_{t=1}^{T_n} (R(\tau^n) - b) \nabla log p_\theta(a_t^n | s_t^n)$$

$$\sum_{t'=t}^{T_n} r_{t'}^n \rightarrow \sum_{t'=t}^{T_n} \gamma^{t'-t} r_{t'}^n$$

Add discount factor $\gamma < 1$

Policy Gradient

Tip 2: Assign Suitable Credit

Advantage Function $A^{\theta}(s_t, a_t)$

How good it is if we take a_t other than other actions at s_t.

Estimated by "*critic*" (later)

Can be state-dependent

$$\nabla \bar{R}_\theta \approx \frac{1}{N}\sum_{n=1}^{N}\sum_{t=1}^{T_n}(R(\tau^n) - b)\nabla log p_\theta(a_t^n|s_t^n)$$

$$\sum_{t'=t}^{T_n} r_{t'}^n \rightarrow \sum_{t'=t}^{T_n} \gamma^{t'-t} r_{t'}^n$$

Add discount factor $\gamma < 1$

Table of Contents

Policy Objective Functions

- Goal: given policy $\pi_\theta(s, a)$ with parameters θ, find best θ
- But how do we measure the quality of a policy π_θ?
- In episodic environment we can use the start value

$$J_1(\theta) = V^{\pi_\theta}(s_1) = \mathbb{E}[v_1]$$

- In continuing environment we can use the average value

$$J_{avV}(\theta) = \sum_s d^{\pi_\theta}(s) V^{\pi_\theta}(s)$$

- Or the average reward per time-step

$$J_{avR}(\theta) = \sum_s d^{\pi_\theta}(s) \sum_a \pi_\theta(s, a) \mathcal{R}_s^a$$

where $d^{\pi_\theta}(s)$ is stationary distribution of Markov chain for π_θ

Policy Optimization

- Policy based reinforcement learning is an optimization problem
- Find θ that maximizes $J(\theta)$
- Some approaches do not use gradient
  - Hill climbing
  - Simplex / amoeba / Nelder Mead
  - Genetic algorithms
- Greater efficiency often possible using gradient
  - Gradient descent
  - Conjugate gradient
  - Quasi-newton
- We focus on gradient descent, many extensions possible
- And on methods that exploit sequential structure

Score Function

- We now compute the policy gradient *analytically*
- Assume policy π_θ is differentiable whenever it is non-zero
- and we know the gradient $\nabla_\theta \pi_\theta(s, a)$
- Likelihood ratios exploit the following identity

$$\begin{aligned}\nabla_\theta \pi_\theta(s, a) &= \pi_\theta(s, a) \frac{\nabla_\theta \pi_\theta(s, a)}{\pi_\theta(s, a)} \\ &= \pi_\theta(s, a) \nabla_\theta \log \pi_\theta(s, a)\end{aligned}$$

- The score function is $\nabla_\theta \log \pi_\theta(s, a)$

Softmax Policy

- We will use a softmax policy as a running example
- Weight actions using linear combination of features $\phi(s,a)^T\theta$
- Probability of action is proportional to exponentiated weight

$$\pi_\theta(s,a) \propto e^{\phi(s,a)^T\theta}$$

- The score function is

$$\nabla_\theta \log \pi_\theta(s,a) = \phi(s,a) - \mathbb{E}_{\pi_\theta}[\phi(s,\cdot)]$$

Gaussian Policy

- In continuous action spaces, a Guassian policy is natural
- Mean is a linear combination of state features $\mu(s) = \phi(s)^T\theta$
- Variance may be fixed σ^2, or can also parametrized
- Policy is Guassian, $a \sim \mathcal{N}(\mu(s), \sigma^2)$
- The score function is

$$\nabla_\theta \log \pi_\theta(s, a) = \frac{(a - \mu(s))\phi(s)}{\sigma^2}$$

One-Step MDPs

- Consider a simple class of one-step MDPs
  - Starting in state $s \sim d(s)$
  - Terminating after one time-step with reward $r = \mathcal{R}_{s,a}$
- Use likelihood ratios to compute the policy gradient

$$\begin{aligned} J(\theta) &= \mathbb{E}_{\pi_\theta}[r] \\ &= \sum_{s \in \mathcal{S}} d(s) \sum_{a \in \mathcal{A}} \pi_\theta(s,a) \mathcal{R}_{s,a} \\ \nabla_\theta J(\theta) &= \sum_{s \in \mathcal{S}} d(s) \sum_{a \in \mathcal{A}} \pi_\theta(s,a) \nabla_\theta \log \pi_\theta(s,a) \mathcal{R}_{s,a} \\ &= \mathbb{E}_{\pi_\theta}[\nabla_\theta \log \pi_\theta(s,a) r] \end{aligned}$$

Policy Gradient Theorem

- The policy gradient theorem generalizes the likelihood ratio approach to multi-step MDPs
- Replaces instantaneous reward r with long-term value $Q^{\pi}(s, a)$
- Policy gradient theorem applies to start state objective, average reward and average value objective

Theorem

For any differentiable policy $\pi_\theta(s, a)$, for any of the policy objective functions $J = J_1, J_{avR}$, or $\frac{1}{1-\gamma} J_{avV}$, the policy gradient is

$$\nabla_\theta J(\theta) = \mathbb{E}_{\pi_\theta}[\nabla_\theta \log \pi_\theta(s, a) Q^{\pi_\theta}(s, a)]$$

Monte-Carlo Policy Gradient (REINFORCE)

- Update parameters by stochastic gradient ascent
- Using policy gradient theorem
- Using return v_t as an unbiased sample of $Q^{\pi_\theta}(s_t, a_t)$

$$\Delta\theta_t = \alpha \nabla_\theta \log \pi_\theta(s_t, a_t) v_t$$

function REINFORCE
  Initialise θ arbitrarily
  for each episode $\{s_1, a_1, r_2, ..., s_{T-1}, a_{T-1}, r_T\} \sim \pi_\theta$ **do**
    for $t = 1$ to $T - 1$ **do**
      $\theta \leftarrow \theta + \alpha \nabla_\theta \log \pi_\theta(s_t, a_t) v_t$
    end for
  end for
  return θ
end function