MAT137 PS6 1

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Claim 1. *Let* $a, b \in \mathbb{R}$ *with* $a < b$ *and let* f *be a function defined on* $[a, b]$*. If* f *is continuous on* $[a, b]$*, then* f *is bounded on* $[a, b]$*.*

Proof. Let f be an arbitrary function. Let $a, b \in \mathbb{R}$ and assume $a < b$. Also, assume f is defined on $[a, b]$ $\wedge f$ is continuous on $[a, b]$. By Extreme Value Theorem, f has a maximum and a minimum on $[a, b]$, that is,

$$\exists M \in [a, b] \text{ s.t. } \forall x \in [a, b], f(M) \geq f(x) \text{ (the maximum)} \tag{1}$$
$$\exists m \in [a, b] \text{ s.t. } \forall x \in [a, b], f(m) \leq f(x) \text{ (the minimum).} \tag{2}$$

Clearly, (1) satisfies f having an supremum on $[a, b]$ (with the supremum being $f(M)$) and (2) satisfies f having an infimum on $[a, b]$ (with the infimum being $f(m)$). Therefore, by definition, f is bounded on the interval $[a, b]$.

□