# STAT W4201 001, Homework 6

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Code is attached here and also posted at https://github.com/BrianWeinstein/advanced-data-analysis. Where relevant, code snippets and output are are included in-line.

# Problem 1: Ramsey 9.14

(a) Draw a matrix of scatterplots of the four variables. Construct it so that the bottom row of plots all have heart on the vertical axis. If you do not have this facility, draw scatterplots of heart versus each of the other variables individually.

A matrix of pairwise scatterplots is shown in Figure 1.

(b) Obtain the least squares fit to the linear regression of heart on bank, walk, and talk.

```
> lmPace <- lm(formula=Heart ~ Bank + Walk + Talk, data=paceData)</pre>
> summary(lmPace)
Call:
lm(formula = Heart ~ Bank + Walk + Talk, data = paceData)
Residuals:
           1Q Median
   Min
                         3Q
-8.4014 -3.0263 0.0602 2.6748 8.4646
Coefficients:
          Estimate Std. Error t value Pr(>|t|)
(Intercept) 3.1787 6.3369 0.502 0.6194
Bank
           0.4052
                    0.1971 2.056 0.0480 *
Walk
           -0.1796 0.2222 -0.808 0.4249
Talk
Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1
Residual standard error: 4.805 on 32 degrees of freedom
Multiple R-squared: 0.2236, Adjusted R-squared: 0.1509
F-statistic: 3.073 on 3 and 32 DF, p-value: 0.04162
```

(c) Plot the residuals versus the fitted values. Is there evidence that the variance of the residuals increases with increasing fitted values or that there are any outliers?

The residual plot is shown in Figure 2. There does not seem to be evidence that the variance of the residuals increases with increasing fitted values, or that there are any extreme outliers.

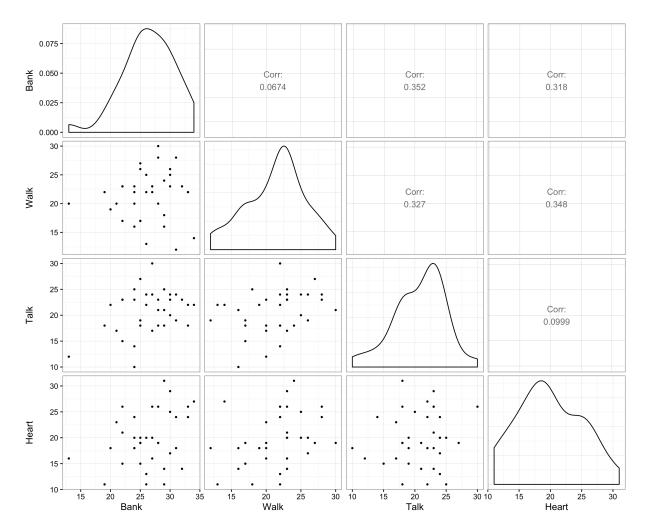


Figure 1: Pairwise scatterplots of the variables in the "Pace of Life and Heart Disease" dataset.

(d) Report a summary of the least squares fit. Write down the estimated equation with standard errors below each estimated coefficient.

Under the parallel lines regression model, the age-adjusted death rate due to heart disease (Heart) increases by 0.4052 for every one unit increase in the bank clerk speed (Bank) (95% confidence interval from 0.0037 to 0.8067). Similarly, Heart increases by .4516 for every one unit increase in the pedestrian walking speed (Walk) (95% confidence interval from 0.0424 to 0.8608). The data provides no evidence that Heart is associated with postal clerk talking speed (Talk) (two sided p-value = 0.4249 for a test that the Talk coefficient is zero).

The estimated equation is:

$$\mu\{\text{Heart}|\text{Bank},\text{Walk},\text{Talk}\} = 3.1787 + \phantom{-}0.4052 \cdot \text{Bank} + \phantom{-}0.4516 \cdot \text{Walk} + \phantom{-}(-0.1796) \cdot \text{Talk} \\ \phantom{\mu}(6.3369) \quad (0.1971) \qquad \qquad (0.2009) \qquad \qquad (0.2222)$$

Problem 2: Ramsey 9.16

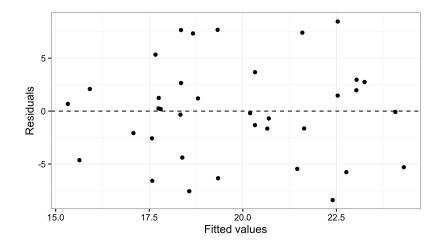


Figure 2: Residual plot for the fitted model from part (b).

(a) Draw a coded scatterplot of proportion of pollen removed versus duration of visit; use different symbols or letters as the plotting codes for queens and workers. Does it appear that the relationship between proportion removed and duration is a straight line?

A scatterplot of proportion of pollen removed versus duration of visit, by bee type is shown in Figure 3. It does not appear that the relationship between proportion removed and duration is a straight line.

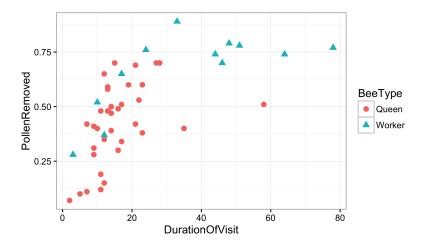


Figure 3: Scatterplot of pollen removed versus duration of visit, by bee type.

(b) The logit transformation is often useful for proportions between 0 and 1. If p is the proportion then the logit is  $\log[p/(1-p)]$ . This is the log of the ratio of the amount of pollen removed to the amount not removed. Draw a coded scatterplot of the logit versus duration.

A scatterplot of the logit of proportion of pollen removed versus duration of visit, by bee type is shown in Figure 4.

(c) Draw a coded scatterplot of the logit versus log duration. From the three plots, which transformations appear to be worthy of pursuing with a regression model?

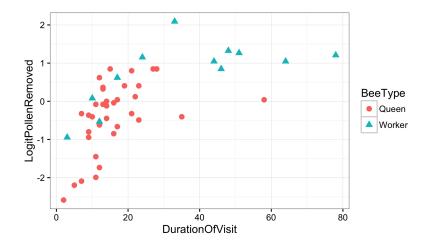


Figure 4: Scatterplot of the logit of pollen removed versus duration of visit, by bee type.

A scatterplot of the logit of proportion of pollen removed versus the log duration of visit, by bee type is shown in Figure 5. The Logit(PollenRemoved) vs Log(DurationOfVisit) transformations produce the most linear relationship, and is worthy of pursuing with a regression model.

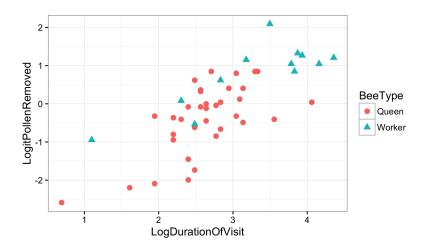


Figure 5: Scatterplot of the logit of pollen removed versus the log duration of visit, by bee type.

(d) Fit the multiple linear regression of the logit of the proportion of pollen removed on (i) log duration, (ii) an indicator variable for whether the bee is a queen or a worker, and (iii) a product term for the interaction of the first two explanatory variables. By examining the p-value of the interaction term, determine whether there is any evidence that the proportion of pollen depends on duration of visit differently for queens than for workers.

The large p-value (0.342) of the interaction term (LogDurationOfVisit:BeeTypeWorker) indicates there is little evidence to suggest that the proportion of pollen removed depends on the duration of visit differently for queens than it does for workers.

(e) Refit the multiple regression but without the interaction term. Is there evidence that, after accounting for the amount of time on the flower, queens tend to remove a smaller proportion of pollen than workers? Why is the p-value for the significance of the indicator variable so different in this model than in the one with the interaction term?

After accounting for the amount of time on the flower, there is moderate evidence that queen bees tend to remove a smaller proportion of pollen than worker bees. On average, worker bees remove 0.5697 more pollen (on the logit scale) than queen bees after accounting for time on the flower (95% confidence interval 0.0932 to 1.0462; two sided p-value = 0.0202 for a test that the BeeTypeWorker coefficient is zero).

In this model, the indicator variable BeeTypeWorker has a different meaning than in the model that included the interaction term. Without the interaction term, the coefficient on BeeTypeWorker measures how much more pollen a worker bee extracts from a flower than a queen bee. When the interaction term is included, though, the coefficient on BeeTypeWorker still measures how much more pollen a worker bee extracts from a flower than a queen bee, but must account for the fact that difference can now depend on the duration. Since there's little evidence to suggest that duration has an impact on the proportion of pollen removed, the inclusion of that interaction term decreases the strength of the model. The p-value for the significance of BeeTypeWorker is thus lower in this model than it is in the model that includes the interaction term.

# Problem 3: Ramsey 9.18

(a) Construct a scatterplot of average wing size against latitude, in which the four groups defined by continent and sex are coded differently. Do these suggest that the wing sizes of the NA flies have evolved toward the same cline as in EU?

A scatterplot of average wing size vs latitude, by continent and sex is shown in Figure 6. From the scatterplot, it appears that the wing sizes of the NA flies have evolved toward a similar cline as in the EU.

(b) Construct a multiple linear regression model with wing size as the response, with latitude as a linear explanatory variable, and with indicator variables to distinguish the sexes and continents. As there are four groups, you will want to have three indicator variables: the continent indicator, the sex indicator, and the product of the two. Construct the model in such a way that one parameter measures the difference between the slopes of the wing size

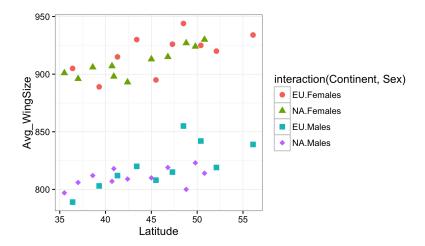


Figure 6: Scatterplot of average wing size vs latitude, by continent and sex.

versus latitude regressions of NA and EU for males, one measures the difference between the NA-EU slope difference for females and that for males, one measures the difference between the intercepts of the regressions of NA and EU for males, and one measures the difference between the NA-EU intercepts' difference for females and that for males.

The separate lines linear regression model is shown below:

```
> lmWing <- lm(formula=Avg_WingSize ~ Latitude + Sex * Continent + Latitude * Sex * Continent,
                    data=wingDataLong)
> summary(lmWing)$coefficients
                                 Estimate Std. Error
                                                       t value
                                                                  Pr(>|t|)
(Intercept)
                              706.9255278 28.0447861 25.2070215 1.387507e-23
Latitude
                                SexFemales
                              129.2648702 39.6613169
                                                     3.2592178 2.539391e-03
ContinentNA
                               72.9386446 40.1512790 1.8165958 7.810402e-02
                              -89.8325160 56.7824834 -1.5820463 1.228981e-01
SexFemales:ContinentNA
Latitude:SexFemales
                               -0.6770556
                                          0.8549550 -0.7919196 4.338982e-01
Latitude:ContinentNA
                               -1.7544085
                                          0.8943897 -1.9615706 5.804243e-02
Latitude:SexFemales:ContinentNA
                                2.0653489
                                          1.2648580 1.6328702 1.117246e-01
```

The coefficient estimate on:

- Latitude:ContinentNA measures the difference between the slopes of the wing size versus latitude regressions of NA and EU for males,
- Latitude:SexFemales:ContinentNA measures the difference between the NA-EU slope difference for females and that for males,
- ContinentNA measures the difference between the intercepts of the regressions of NA and EU for males, and
- SexFemales: ContinentNA measures the difference between the NA-EU intercepts' difference for females and that for males.

#### Problem 4: Ramsey 9.20

(a) Find a model for describing the mean of either winning time or winning speed as a function of year, whichever works better.

Scatterplots of Speed vs Year (see Figure 1) or Time vs Year reveal a likely quadratic component to both regressions. The regression of Speed on Year and Year<sup>2</sup> has a slightly

lower R-squared coefficient than the regression of Time on Year and Year<sup>2</sup>.

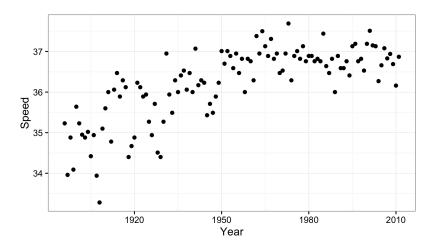


Figure 7: Scatterplot of Speed vs Year.

```
> lmDerbySpeed <- lm(formula=Speed ~ Year + Year2, data=derbyData)
> summary(lmDerbySpeed) # higher R-squared
Call:
lm(formula = Speed ~ Year + Year2, data = derbyData)
Residuals:
              10
                                30
    Min
                   Median
                                        Max
-1.77397 -0.29516 0.02933 0.31150 1.15734
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -1.048e+03 1.912e+02 -5.481 2.61e-07 ***
            1.090e+00 1.957e-01 5.569 1.75e-07 ***
Year
           -2.740e-04 5.010e-05 -5.468 2.76e-07 ***
Year2
Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1
Residual standard error: 0.5411 on 113 degrees of freedom
Multiple R-squared: 0.6442, Adjusted R-squared: 0.6379
F-statistic: 102.3 on 2 and 113 DF, p-value: < 2.2e-16
```

(b) Quantify the amount (in seconds or miles per hour) by which the mean winning time or speed on fast tracks exceeds the mean on slow tracks (using the two-category variable Conditions), after accounting for the effect of year.

To find the quantity of interest, we perform a regression of Speed on Year, Year<sup>2</sup>, and Conditions.

After accounting for the effect of year, there is overwhelming evidence that the mean winning speed on fast tracks is greater than the mean on slow tracks. On average, the winning speed on fast tracks is 0.9861 miles/hour greater than the winning speed on slow tracks (95% confidence interval 0.7902 to 1.1820 miles/hour; two sided p-value =  $3.76 \times 10^{-17}$  for a test that the ConditionsSlow coefficient is zero).

(c) After accounting for the effects of year and track conditions, is there any evidence that the mean winning time or speed depends on number of horses in the race (Starters)? Is there any evidence of an interactive effect of Starters and Conditions; that is, does the effect of number of horses on the response depend on whether the track was fast or slow? Describe the effect of number of horses on mean winning time or speed.

To find the quantities of interest, we perform a regression of Speed on Year, Year<sup>2</sup>, Conditions, Starters, and the interaction between Conditions and Starters.

```
> lmDerbyC <- lm(formula=Speed ~ Year + Year2 + Conditions * Starters, data=derbyData)
> summary(lmDerbyC)$coefficients
                             Estimate
                                        Std. Error
                                                      t value
                                                                  Pr(>|t|)
(Intercept)
                        -1.027755e+03 1.418196e+02 -7.2469146 6.234836e-11
                         1.071109e+00 1.450714e-01 7.3833237 3.148597e-11
Year
Year2
                        -2.692808e-04 3.708723e-05 -7.2607403 5.818773e-11
ConditionsSlow
                        -1.175438e+00 2.541940e-01 -4.6241775 1.031213e-05
                        -2.495694e-02 1.084907e-02 -2.3003754 2.331292e-02
Starters
ConditionsSlow:Starters 1.621581e-02 1.826900e-02 0.8876137 3.766852e-01
```

After accounting for the effects of year and track conditions, there is moderate evidence that the mean winning speed depends on the number of horses in the race. On average, for each additional horse in a race, the winning speed decreases by 0.0250 miles/hour (95% confidence interval 0.0035 to 0.0465 miles/hour; two sided p-value = 0.0233 for a test that the ConditionsSlow coefficient is zero).

There is no evidence that the effect of number of horses on speed depends on whether the track was fast or slow. The interaction term between Conditions and Starters has a two sided p-value of 0.3766 for a test that the ConditionsSlow:Starters coefficient is zero.

# Problem 5: Ramsey 10.19

Carry out a lack-of-fit F-test for the regression of number of flowers on light intensity and an indicator variable for time, using the data in Display 9.2 (data file case0901).

(a) Fit the regression of flowers on light and an indicator variable for time = 24, and obtain the analysis of variance table.

(b) Fit the same regression except with light treated as a factor (using 5 indicator variables to distinguish the 6 groups), and with the interaction of these two factors, and obtain the analysis of variance table.

(c) Perform an extra-sum-of-squares F-test comparing the full model in part (b) to the reduced model in part (a). (Note: The full model contains 12 parameters, which is equivalent to the model in which a separate mean exists for each of the 12 groups. No pattern is implied in this model. See Displays 9.7 and 9.8 for help.

Using the results from the full model in part (b) and the reduced model in part (a), the data provides no evidence that the association between the average number of flowers per plant (Flowers) and timing (Time24) differs with light intensity (Intensity) (p-value = 0.8894; extra sum of squares F-test).

```
> # compute the extra sum of squares
> esos <- anovaA$'Sum Sq'[3] - anovaB$'Sum Sq'[4]
>
> # define the numerator and denominator degrees of freedom
> dfn <- anovaA$Df[3] - anovaB$Df[4]
> dfd <- summary(lmMeadowB)$df[2]
>
> # compute the f stat
> fstat <- (esos / (length(lmMeadowB$coefficients) - length(lmMeadowA$coefficients))) /
+ (summary(lmMeadowB)$sigma)^2
> fstat
[1] 0.4376736
>
> # compute the pvalue
> pval <- 1 - pf(q=fstat, df1=dfn, df2=dfd); pval
[1] 0.8893576</pre>
```

We can verify this result using the anova function.

```
> anova(lmMeadowB, lmMeadowA)
Analysis of Variance Table

Model 1: Flowers ~ factor(Intensity) * Time24
Model 2: Flowers ~ Intensity + Time24
  Res.Df RSS Df Sum of Sq F Pr(>F)
1    12 655.92
2    21 871.24 -9  -215.31 0.4377 0.8894
```

### Problem 6: Ramsey 10.28

Analyze the data to describe the effect of El Nino on (a) the number of tropical storms, (b) the number of hurricanes, and (c) the storm index after accounting for the effects of West African wetness and for any time trends, if appropriate.

A matrix of pairwise scatterplots is shown in Figure 8.

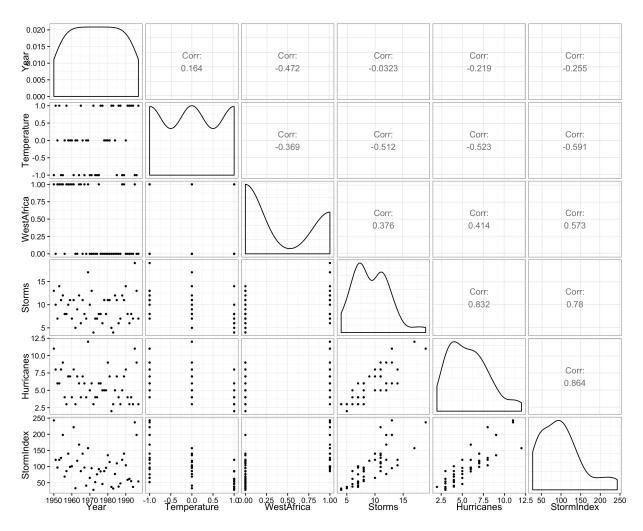


Figure 8: Pairwise scatterplots of the relevant variables in the "El Nino and Hurricanes" dataset.

## (a) the number of tropical storms

We first test a model that describes the effect of Temperature (an ordinal-equivalent variable of ElNino), WestAfrica, and their interaction, on the number of tropical storms (Storms).

```
Temperature * WestAfrica, data=ninoData)
> lmNinoA1 <- lm(formula=Storms ~</pre>
> summary(lmNinoA1)$coefficients
                          Estimate Std. Error
                                                  t value
                                                              Pr(>|t|)
(Intercept)
                         8.9270633
                                    0.5276111 16.9197788 7.045836e-21
Temperature
                        -1.9731286
                                    0.6629759 -2.9761693 4.728831e-03
WestAfrica
                         1.5486123
                                    0.9026445
                                               1.7156392 9.326290e-02
                                   1.0873067
Temperature: WestAfrica 0.7677232
                                               0.7060778 4.838618e-01
```

A plot of the residuals vs fitted values reveals a horn-shaped pattern, and a plot of the residuals vs year indicates the presence of serial correlation (here, a likely (quadratic) association between Year and Storms). Both plots are shown in Figure 9.

Incorporating a quadratic association between Storms and Year, another model is shown

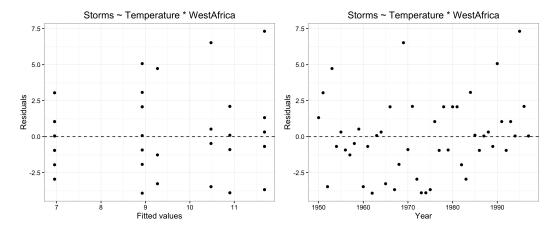


Figure 9: Model A1: (Left) A plot of residuals vs fitted values. (Right) A plot of residuals vs Year.

below.

```
> lmNinoA2 <- lm(formula=Storms ~ Temperature * WestAfrica + Year + I(Year^2), data=ninoData)
> summary(lmNinoA2)$coefficients
                             Estimate
                                        Std. Error
                                                      t value
                                                                  Pr(>|t|)
(Intercept)
                         1.935330e+04 9.125963e+03
                                                    2.120686 0.0398956685
Temperature
                        -2.500296e+00 6.560643e-01 -3.811053 0.0004457422
WestAfrica
                         1.797312e+00 1.021008e+00
                                                    1.760332 0.0856326798
                        -1.965294e+01 9.245902e+00 -2.125583 0.0394617401
Year
I(Year<sup>2</sup>)
                         4.991337e-03 2.341799e-03
                                                    2.131411 0.0389508103
Temperature:WestAfrica
                        1.618917e+00 1.088058e+00
                                                    1.487895 0.1442487103
```

The plots of residuals vs fitted values and of residuals vs year shown in Figure 10 indicates that the inclusion of the Year terms has resolved both the horn-shaped pattern and much of the serial correlation issue.

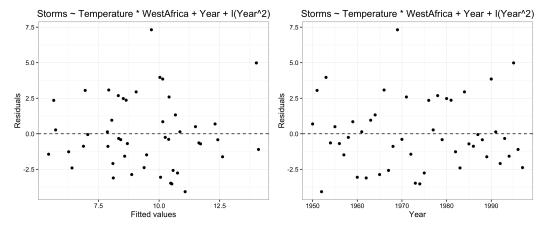


Figure 10: Model A2: (Left) A plot of residuals vs fitted values. (Right) A plot of residuals vs Year.

After accounting for the effect of year: There is overwhelming evidence that the mean number of storms is associated with temperature (two sided p-value = 0.000446 for a test that the Temperature coefficient is zero). On average, cold El Nino years (Temperature==-1)

have 2.5 additional storms/year than neutral El Nino years (Temperature==0), which have 2.5 additional storms/year than warm El Nino years (Temperature==1) (95% confidence interval 1.1763 to 3.8243 additional storms/year). There is suggestive, but inconclusive evidence that wet years in West Africa (WestAfrica==1) are associated with 1.797 additional storms per year (two sided p-value = 0.0856 for a test that the WestAfrica coefficient is zero; 95% confidence interval -0.2632 to 3.8578 additional storms/year). There is no evidence that the effect of temperature on the number of storms depends on whether it was a wet or dry year in West Africa (two sided p-value = 0.1442 for a test that the Temperature:WestAfrica coefficient is zero).

#### (b) the number of hurricanes

We first fit a model that describes the effect of Temperature, WestAfrica, and their interaction, on the number of hurricanes (Hurricanes).

```
> lmNinoB1 <- lm(formula=Hurricanes ~ Temperature * WestAfrica, data=ninoData)
> summary(lmNinoB1)$coefficients
                         Estimate Std. Error
                                                t value
                                                            Pr(>|t|)
                        5.2552783
                                   0.3811964 13.786275 1.417450e-17
(Intercept)
Temperature
                        -1.0940499
                                   0.4789968 -2.284044 2.725047e-02
WestAfrica
                        1.1771541
                                   0.6521561 1.805019 7.791770e-02
Temperature: WestAfrica -0.3654096
                                   0.7855737 -0.465150 6.441181e-01
```

A plot of the residuals vs fitted values reveals no issues with the model, and a plot of the residuals vs year does not reveal any serial correlation, as it did in part (a). Both plots are shown in Figure 11.

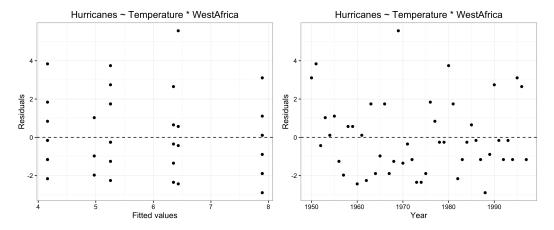


Figure 11: Model B1: (Left) A plot of residuals vs fitted values. (Right) A plot of residuals vs Year.

There is moderate evidence that the mean number of hurricanes is associated with temperature (two sided p-value = 0.0273 for a test that the Temperature coefficient is zero). On average, cold El Nino years (Temperature==-1) have 1.0940 additional hurricanes/year than neutral El Nino years (Temperature==0), which have 1.0940 additional hurricanes/year than warm El Nino years (Temperature==1) (95% confidence interval 0.1287 to 2.0594 additional hurricanes/year). There is suggestive, but inconclusive evidence that wet years in West Africa (WestAfrica==1) are associated with 1.1772 additional hurricanes per year (two sided p-value = 0.0779 for a test that the WestAfrica coefficient is zero; 95% confidence interval -0.1372 to 2.4915 additional hurricanes/year).

There is no evidence that the effect of temperature on the number of hurricanes depends on whether it was a wet or dry year in West Africa (two sided p-value = 0.6441 for a test that the Temperature: WestAfrica coefficient is zero).

#### (c) the storm index

We first test a model that describes the effect of Temperature, WestAfrica, and their interaction, on the storm index (StormIndex).

```
> lmNinoC1 <- lm(formula=StormIndex ~ Temperature * WestAfrica, data=ninoData)
> summary(lmNinoC1)$coefficients
                                                t value
                                                            Pr(>|t|)
                        Estimate Std. Error
(Intercept)
                        82.85413
                                    7.581316 10.9287267 4.024245e-14
Temperature
                        -24.94626
                                    9.526391 -2.6186472 1.206303e-02
WestAfrica
                        43.91344
                                   12.970221 3.3857125 1.503487e-03
Temperature: WestAfrica -10.79428
                                  15.623658 -0.6908935 4.932615e-01
```

A plot of the residuals vs fitted values reveals a moderate horn-shaped pattern, and a plot of the residuals vs year indicates the presence of serial correlation (here, a likely (quadratic) association between Year and StormIndex). Both plots are shown in Figure 12.

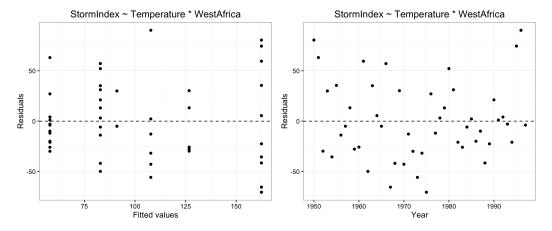


Figure 12: Model C1: (Left) A plot of residuals vs fitted values. (Right) A plot of residuals vs Year.

Incorporating a quadratic association between StormIndex and Year, another model is shown below.

```
> lmNinoC2 <- lm(formula=StormIndex ~ Temperature * WestAfrica + Year + I(Year^2), data=ninoData)
> summary(lmNinoC2)$coefficients
                                        Std. Error
                                                       t value
                                                                  Pr(>|t|)
                             Estimate
(Intercept)
                        3.880408e+05 1.288285e+05
                                                    3.0120726 0.004381156
Temperature
                        -3.145612e+01 9.261465e+00 -3.3964518 0.001503348
WestAfrica
                        3.180147e+01 1.441326e+01
                                                   2.2064029 0.032877636
Year
                        -3.930155e+02 1.305217e+02 -3.0111130 0.004392527
I(Year<sup>2</sup>)
                         9.953118e-02 3.305848e-02 3.0107607 0.004396709
Temperature: WestAfrica -5.193898e+00 1.535979e+01 -0.3381489 0.736935479
```

The plots of residuals vs fitted values and of residuals vs year shown in Figure 13 indicates that the inclusion of the Year terms has resolved both the horn-shaped pattern and much of the serial correlation issue.

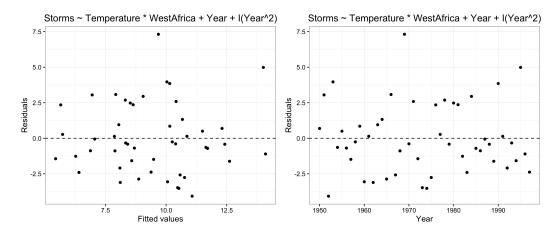


Figure 13: Model C2: (Left) A plot of residuals vs fitted values. (Right) A plot of residuals vs Year.

After accounting for the effect of year: There is overwhelming evidence that the mean storm index is associated with temperature (two sided p-value = 0.00150 for a test that the Temperature coefficient is zero). On average, cold El Nino years (Temperature==-1) have a storm index of 31.46 points higher than neutral El Nino years (Temperature==0), which have a storm index of 31.46 points higher than warm El Nino years (Temperature==1) (95% confidence interval 12.76 to 50.15 points higher). There is moderate evidence that wet years in West Africa (WestAfrica==1) are associated with a storm index 31.80 points higher than dry years (two sided p-value = 0.03288 for a test that the WestAfrica coefficient is zero; 95% confidence interval 2.714 to 60.89 points). There is no evidence that the effect of temperature on storm index depends on whether it was a wet or dry year in West Africa (two sided p-value = 0.73694 for a test that the Temperature:WestAfrica coefficient is zero).