

STAT W4201 001, Homework 3

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Code is attached here and also posted at <https://github.com/BrianWeinstein/advanced-data-analysis>. Where relevant, code snippets and output are included in-line.

Problem 1: Ramsey 4.30

A boxplot of SPF estimates for each subject is shown in Figure 1, where the SPF estimate is calculated for each subject as the number minutes that a person was able to withstand sunlight before applying sunscreen divided by the number of minutes they were able to withstand after applying sunscreen. The dataset contains paired observations, and the quantity in question has a nearly normal distribution with no outliers, making it a strong candidate for the one-sample paired t-test.

	PreTreatment	Sunscreen	spfEstimate
1	30	120	4.000000
2	45	240	5.333333
3	180	480	2.666667
4	15	150	10.000000
5	200	480	2.400000
6	20	270	13.500000
7	15	300	20.000000
8	10	180	18.000000
9	20	300	15.000000
10	20	240	12.000000
11	60	480	8.000000
12	60	300	5.000000
13	120	480	4.000000

The SPF Estimate has a mean of 9.223, and the standard error on the mean is $SD(sp\hat{f}Estimate)/\sqrt{13} = 1.664$. Using a t distribution with $(13 - 1) = 12$ degrees of freedom, a 95% confidence interval for the mean is

$$9.223 \pm t_{12}(1 - (0.05/2)) \times 1.664$$

$$9.223 \pm (2.179)(1.664)$$

$$9.223 \pm 3.626$$

$$\Rightarrow 95\% \text{ CI for the mean SPF Estimate: } [5.598, 12.848]$$

A potentially confounding variable could be the strength of sunlight during the measurement period for each subjects' data points. For a given participant, if the sunlight strength wasn't constant for both measurements, it likely would have had an impact on the duration for which they were able to withstand the sunlight.

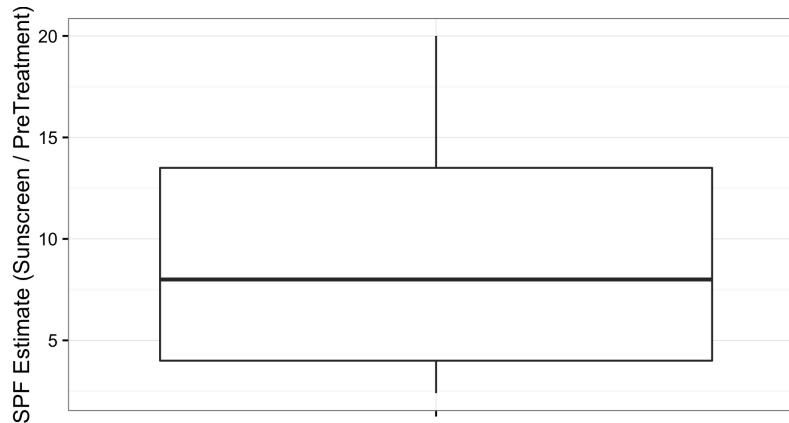


Figure 1: SPF estimates for 13 subjects.

Problem 2: Ramsey 4.32**Problem 3: Ramsey 5.19**

- (a) The pooled estimate of the variance s_p^2 is given by

$$\begin{aligned}
 s_p^2 &= \frac{\sum_{i=1}^I (n_i - 1) s_i^2}{\sum_{i=1}^I (n_i - 1)} \\
 &= \frac{(127 - 1)(0.4979)^2 + (44 - 1)(0.4235)^2 + (24 - 1)(0.3955)^2 + \dots}{(127 - 1) + (44 - 1) + (24 - 1) + \dots} \\
 &\quad \frac{\dots + (41 - 1)(0.3183)^2 + (18 - 1)(0.3111)^2 + (16 - 1)(0.4649)^2 + \dots}{\dots + (41 - 1) + (18 - 1) + (16 - 1) + \dots} \\
 &\quad \frac{\dots + (11 - 1)(0.2963)^2 + (7 - 1)(0.3242)^2 + (6 - 1)(0.5842)^2}{\dots + (11 - 1) + (7 - 1) + (6 - 1)} \\
 &= 0.1919.
 \end{aligned}$$

- (b) To construct an ANOVA table to test for species differences, we first compute SS_W , the sum of squared residuals of the “within” group.

$$\begin{aligned}
 SS_W &= \sum_{i=1}^9 \left[\sum_{j=1}^{n_i} (Y_{ij} - \bar{Y}_i)^2 \right] = \sum_{i=1}^9 [(n_i - 1) \text{SE}(Y_i)] \\
 &= (127 - 1)(0.4979)^2 + (44 - 1)(0.4235)^2 + \dots + (6 - 1)(0.5842)^2 \\
 &= 122.8658.
 \end{aligned}$$

Given the sample standard deviation of all 294 observations as one group is $SD_{Total} = 0.4962$, the total sum of squared residuals SS_{Total} is

Should
SS total
be -9 (-1)
instead
of -1 ?

$$\begin{aligned}
SS_{Total} &= \sum_{i=1}^9 \left[\sum_{j=1}^{n_i} (Y_{ij} - \bar{Y})^2 \right] = (n_1 + n_2 + \cdots + n_9 - 1)(SD_{Total}) \\
&= (127 + 44 + \cdots + 6 - 1)(0.4962) = (293)(0.4962) \\
&= 145.3866.
\end{aligned}$$

Therefore, the sum of squared residuals of the “between” group, SS_B is:

$$SS_B = SS_{Total} - SS_W = 145.3866 - 122.8658 = 22.5208.$$

The Total, Within, and Between degrees of freedom are $df_{Total} = (n - 1) = 293$, $df_W = (n - I) = (294 - 9) = 285$, and $df_B = df_{Total} - df_W = 8$, respectively.

The Mean Square for the Within and Between groups is the [(Sum of Squares) / df], and the F-statistic is defined as the [(Between Mean Square) / (Within Mean Square)].

Thus the ANOVA table to test for species differences is:

Source of Variation	Sum of Squares	d.f.	Mean Square	F-Statistic	p-Value
Between Groups	22.5208	8	2.8151	6.5300	8.0551×10^{-8}
Within Groups	122.8658	285	0.4311		
Total	145.3866	293			

Where the p-value for the F-Statistic is given by the CDF of an F distribution with 8 and 285 degrees of freedom.

```
> # p-value of F-statistic
> pf(q=6.5300, df1=8, df2=285, lower.tail=FALSE)
[1] 8.055137e-08
```

(c) The ANOVA method for calculating SS_B yields the same answer as the formula:

$$SS_B = \sum_{i=1}^I (n_i \bar{Y}_i^2) - n \bar{Y}^2 = \sum_{i=1}^9 (n_i \bar{Y}_i^2) - n \bar{Y}^2.$$

The overall mean $\bar{Y} = 7.4746$ is computed as a weighted average of the group means and $n = 294$, thus

$$\begin{aligned}
SS_B &= \sum_{i=1}^9 (n_i \bar{Y}_i^2) - n \bar{Y}^2 \\
&= (127)(7.347)^2 + (44)(7.368)^2 + (24)(7.418)^2 + (41)(7.487)^2 + (18)(7.563)^2 + \cdots \\
&\quad \cdots + (16)(7.568)^2 + (11)(8.214)^2 + (7)(8.272)^2 + (6)(8.297)^2 - (294)(7.4746)^2 \\
&= 17.45402
\end{aligned}$$

17.45402
!=
22.5208
: maybe
my SS
total calc
is incor-
rect?

(d) If the first 6 species have one common mean and the last 3 have another common mean, we find that

finish 3d

Problem 4:

Consider the Bumpuss data in Chapter 2, compute the power of the two-sided two sample t -test of size 0.05 (i.e., reject the null hypothesis if the absolute value the t -statistic is greater than or equal to 2), under the alternative that $\mu_x - \mu_y = \bar{x} - \bar{y} = 0.01$ and $\sigma = s_p = 0.0214$.

Problem 5:

Show that the two-sided two sample t -test is equivalent to the anova F -test, if the number of groups is two.

For $I = 2$ groups, the F -statistic is given by

$$F\text{-statistic} = \frac{SS_B / [(n-1) - (n-I)]}{SS_W / (n-I)},$$

where n_1 and n_2 are the sizes of samples 1 and 2, respectively, $n = n_1 + n_2$ is the total sample size, SS_B is the “between groups” sum of squared residuals, and SS_W is the “within groups” sum of squared residuals.

Simplifying, we find

$$F\text{-statistic} = \frac{SS_B / (I-1)}{SS_W / (n-I)} = \frac{SS_B / (2-1)}{SS_W / (n-2)} = \frac{SS_B / 1}{SS_W / (n-2)}.$$

If the observations from group 1 are $\sim N(\mu_1, \sigma^2)$ and the observations from group 2 are $\sim N(\mu_2, \sigma^2)$, we know that

$$F\text{-statistic} \sim F_{1, n-2}, \text{ which is equivalent to } t_{n-2}^2.$$

i.e., an F distribution with a numerator degrees of freedom of 1 and a denominator degrees of freedom of $n-2$ is equivalent to the square of a t distribution with $n-2$ degrees of freedom.

Finish
problem
5

Problem 6:

Consider X_1, \dots, X_{10} are i.i.d. $N(0, \sigma^2)$, Y_1, \dots, Y_{10} are i.i.d. $N(\mu, \sigma^2)$ and hypothesis testing:

$$H_0 : \mu = 0$$

$$H_A : \mu \neq 0.$$

Compute the power of a two sided two sample t -test of size 0.05 when $\sigma^2 = 1$ and $\mu = 0.1, 0.5, 1, \text{ and } 2$. Plot the power as a function of μ . Then, increase the sample size in each group to 20 and draw the power function in the same plot as that of the sample size 10.

Problem 7:

Under the setting of the previous problem, show that, under the null hypothesis, the p -value follows the uniform distribution on the interval $[0, 1]$ and perform simulations to confirm it.

Todo list

Should SS total be -9 (-I) instead of -1 ?	2
17.45402 != 22.5208 : maybe my SS total calc is incorrect?	3
finish 3d	3
Finish problem 5	4