

STAT W4201 001, Homework 19

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Code is attached here and also posted at <https://github.com/BrianWeinstein/advanced-data-analysis>. Where relevant, code snippets and output are included in-line.

Problem 1: Ramsey 20.12

Duchenne Muscular Dystrophy (DMD) is a genetically transmitted disease, passed from a mother to her children. Doctors must rely on some kind of test to detect the presence of the disease. The data in Display 20.15 are levels of two enzymes in the blood, creatine kinase (CK) and hemopexin (H), for 38 known DMD carriers and 82 women who are not carriers. It is desired to use these data to obtain an equation for indicating whether a woman is a likely carrier.

- (a) *Make a scatterplot of H versus $\log(CK)$; use one plotting symbol to represent the controls on the plot and another to represent the carriers. Does it appear from the plot that these enzymes might be useful predictors of whether a woman is a carrier?*

A coded scatterplot of H vs $\log(CK)$ is shown in Figure 1. Based on the scatterplot, it does appear that these enzymes might be useful predictors of whether a woman is a carrier — visually, at least, it looks like the carriers have higher levels of CK, and slightly higher levels of H .

```
> mdData <- Sleuth3::ex2012  
> mdData$Group <- relevel(mdData$Group, ref = "Control")
```

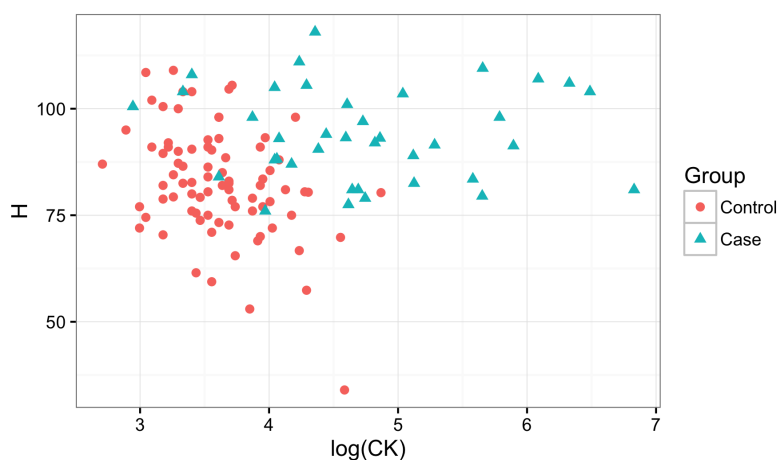


Figure 1: A coded scatterplot of H vs $\log(CK)$.

- (b) Fit the logistic regression of carrier on CK and CK -squared. Does the CK -squared term significantly differ from 0? Next fit the logistic regression of carrier on $\log(CK)$ and $[\log(CK)]^2$. Does the squared term significantly differ from 0? Which scale (untransformed or log-transformed) seems more appropriate for CK ?

The logistic regression of carrier on CK and CK -squared is shown below. The CK -squared term does not significantly differ from 0 (two-sided p-value 0.1219).

```
> glm_1b1 <- glm(formula = Group ~ CK + I(CK^2),
+                 data = mdData, family = binomial)
> summary(glm_1b1)$coefficients
```

	Estimate	Std. Error	z value	Pr(> z)
(Intercept)	-4.17746146864	0.72637612139	-5.751100	0.000000008866482
CK	0.05797905485	0.01299217478	4.462614	0.000008096600851
I(CK^2)	-0.00005054336	0.00003267841	-1.546690	0.121938045015287

The logistic regression of carrier on $\log(CK)$ and $[\log(CK)]^2$ is shown below. The $[\log(CK)]^2$ term does not significantly differ from 0 (two-sided p-value 0.1737).

```
> glm_1b2 <- glm(formula = Group ~ log(CK) + I(log(CK)^2),
+                 data = mdData, family = binomial)
> summary(glm_1b2)$coefficients
```

	Estimate	Std. Error	z value	Pr(> z)
(Intercept)	9.735313	16.297521	0.5973493	0.5502742
log(CK)	-8.516251	8.358066	-1.0189261	0.3082381
I(log(CK)^2)	1.445731	1.062746	1.3603736	0.1737117

The log-transformed scale is more appropriate for CK , since it ranges over many orders of magnitude on the untransformed scale (from 15 to 925). A coded scatterplot of H vs CK , shown in Figure 2, further illustrates the need for the transformation.

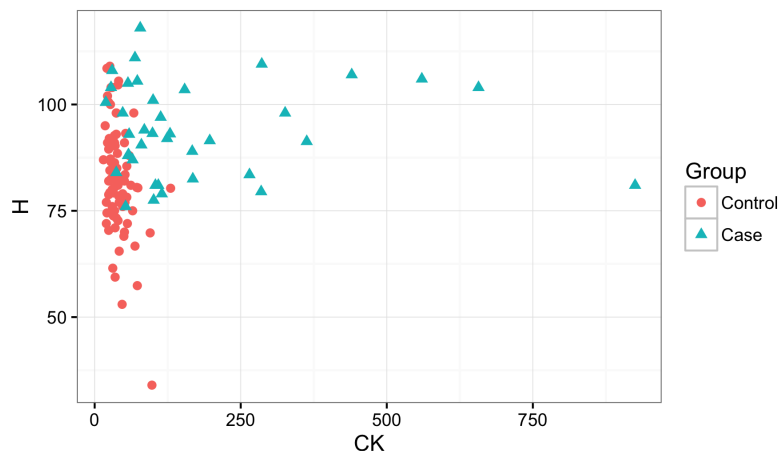


Figure 2: A coded scatterplot of H vs CK .

- (c) Fit the logistic regression of carrier on $\log(CK)$ and H . Report the coefficients and standard errors.

The coefficients and standard errors in the logistic regression of carrier on $\log(CK)$ and H is shown below.

```
> glm_1c <- glm(formula = Group ~ log(CK) + H,
+               data = mdData, family = binomial)
> summary(glm_1c)$coefficients
              Estimate Std. Error   z value    Pr(>|z|)
(Intercept) -28.9134030  5.80016937  -4.984924 0.000000619862
log(CK)       4.0204252  0.82909534   4.849171 0.000001239784
H             0.1365189  0.03654202   3.735943 0.000187013358
```

- (d) Carry out a drop-in-deviance test for the hypothesis that neither $\log(CK)$ nor H are useful predictors of whether a woman is a carrier.

We first fit a reduced model that includes only an intercept term.

```
> glm_1d <- glm(formula = Group ~ 1,
+               data = mdData, family = binomial)
> summary(glm_1d)$coefficients
              Estimate Std. Error   z value    Pr(>|z|)
(Intercept) -0.7691331  0.1962419  -3.919311 0.00008880244
```

We then compare the model from part (c) to this reduced model using a drop-in-deviance test (likelihood ratio test), testing the hypothesis that neither $\log(CK)$ nor H are useful predictors of whether a woman is a carrier.

```
> anova(glm_1c, glm_1d, test="LRT")
Analysis of Deviance Table

Model 1: Group ~ log(CK) + H
Model 2: Group ~ 1
    Resid. Df Resid. Dev Df Deviance  Pr(>Chi)
1         117         61.992
2         119        149.840 -2   -87.847 < 2.2e-16 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1 1
```

There is overwhelming evidence that either (1) one of the variables or (2) both of the variables are useful predictors of whether a woman is a carrier of DMD (two-sided p-value 2.2×10^{-16} from a drop-in-deviance test).

- (e) Typical values of CK and H are 80 and 85. Suppose that a suspected carrier has values of 300 and 100. What are the odds that she is a carrier relative to the odds that a woman with typical values (80 and 85) is a carrier?

The odds of a woman being a carrier with values of 300 and 100 is 1575 times higher than the odds of a woman with values of 80 and 85.

```
> # calculate odds and probability of having DMD at CK=80, H=85
> odds1 <- exp(predict(glm_1c, data.frame(CK=80, H=85)))[[1]] ; odds1
[1] 1.361127
> 1 / (1 + exp(-odds1))
[1] 0.7959428
> # calculate odds and probability of having DMD at CK=300, H=100
> odds2 <- exp(predict(glm_1c, data.frame(CK=300, H=100)))[[1]] ; odds2
[1] 2143.332
> 1 / (1 + exp(-odds2))
[1] 1
> # calculate the odds ratio
> odds2/odds1
[1] 1574.675
```

Problem 2: Ramsey 21.16

Twenty tanks of rainbow trout embryos were exposed to one of five doses of Aflatoxicol for one hour. The data in Display 21.20 (from George Bailey and Jerry Hendricks) represent the numbers of fish in each tank and the numbers of these that had liver tumors after one year. Describe the relationship between dose of Aflatoxicol and odds of liver tumor. It is also of interest to determine the dose at which 50% of the fish will get liver tumors. (Note: Tank effects are to be expected, meaning that tanks given the same dose may have slightly different π s. Thus, one should suspect extra-binomial variation.)

Problem 3:

Suppose that a population of individuals is partitioned into sub-populations or groups, G_1 and G_2 . It may be helpful to think of G_1 in an epidemiological context as the carriers of a particular virus, comprising $100\pi_1\%$ of the population, and G_2 as the non-carriers. Measurements Z made on individuals have the following distributions in the two groups:

$$\begin{aligned} G_1 &: Z \sim N(\mu_1, \Sigma) \\ G_2 &: Z \sim N(\mu_2, \Sigma) \end{aligned}$$

Let z be an observation made on an individual drawn at random from the combined population. The prior odds that the individual belongs to G_1 are $\pi_1/(1 - \pi_1)$. Show that the posterior odds given z are

$$\frac{\pi_1}{1 - \pi_1} \exp(\alpha + \beta^T z)$$

and give the form of α and β .