STAT S4240 002, Homework 2

Brian Weinstein (bmw2148)

July 23, 2015

Problem 1: PCA

(a) Column means

```
> apply(rawData, 2, mean)
       x1        x2        x3        x4        x5
 6.049104 -8.277221  4.665532  7.914270 62.138753
```

Row means

```
> apply(rawData, 1, mean)
  [1]  -0.1277116  20.8162864  -8.8984358  25.5999204  -9.7472153
  [6]  64.0626702  22.0392371  23.3914888  31.7598224 -13.8680290
...
 [91]   1.2105932  21.2145724  -8.4896595  19.0639963  20.9767512
 [96]   3.5962333  22.3461063   0.7145014   6.3080005  64.8829556
```

The nonzero column means indicate that each variable isn't centered. In this context the row means are just the average of the coordinates for each observation.

(b) Empirical covariance matrix

```
          x1         x2        x3        x4        x5
x1  72.96417  -83.90858  53.23708  120.1162  568.4105
x2 -83.90858  110.89101 -63.89570 -115.9430 -817.3388
x3  53.23708  -63.89570  39.60282   83.7386  445.2511
x4 120.11620 -115.94304  83.73860  232.1333  683.5587
x5 568.41046 -817.33884 445.25112  683.5587 6288.8569
```

The diagonal values tell us the variance of the variable indicated in the column (or equivalently, the row). The off-diagonal elements indicate the covariance between the two variables that intersect at that element.

(c) The eigenvalues and eigenvectors of the empirical covariance matrix `sig`:

```
> eigen(sig)
$values
[1] 6.557348e+03 1.868951e+02 2.038354e-01 9.775594e-04 9.373658e-05

$vectors
            [,1]       [,2]         [,3]        [,4]        [,5]
[1,]  0.09009603 -0.3247102 -0.383470773  0.82286709  0.24957150
[2,] -0.12797842  0.1364755  0.227047683 -0.11412319  0.94890526
[3,]  0.07028767 -0.1941349  0.894987159  0.37278501 -0.13191135
[4,]  0.11077853 -0.9008231 -0.019718518 -0.40719485  0.10024632
[5,]  0.97892389  0.1636064  0.002946326 -0.07133967  0.09921159
```

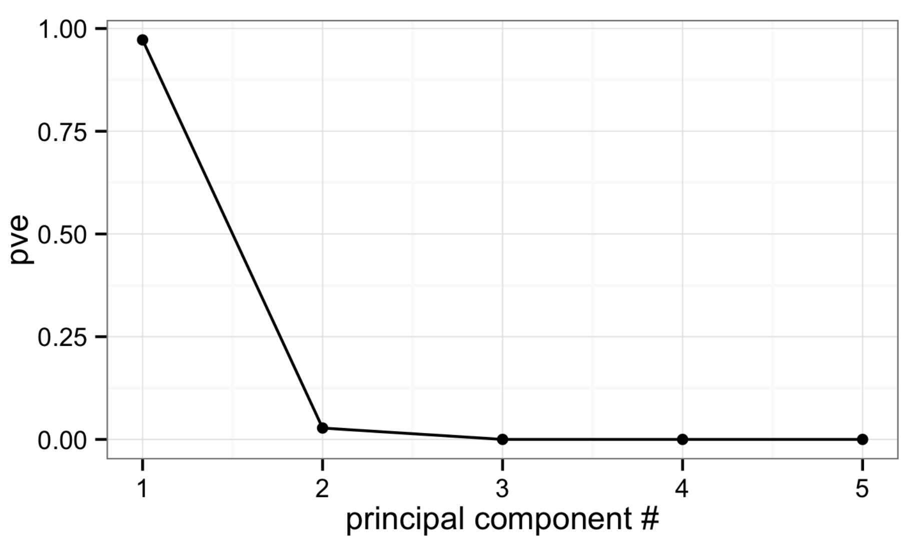
where the eigenvectors are represented as columns in the `$vectors` component.

Since it's a symmetric matrix, `sig` has the same left eigenvectors as right eigenvectors.

(d) The loadings are the eigenvectors (see part c). The scores are:

```
> data%*%t(evecs)
             [,1]         [,2]        [,3]         [,4]        [,5]
  [1,] -25.9233299  -50.96254603 -4.06557021  -8.91350845 -10.7755359
  [2,]  13.3064897   13.56908728  6.16049505   0.82185440   4.8621981
  [3,] -37.6872799  -93.30323983 -1.02562352 -18.15040050 -16.3848300
...
 [98,] -27.0931525  -38.88284377 -8.26671502  -5.26069573 -10.6527232
 [99,] -13.1627026  -31.46409161 -1.20277265  -5.83681744  -5.6197025
[100,]  85.7232563  184.73133084 10.16179165  33.54024954  36.1560703
```

(e) Proportion of variance explained



We only need one principal component. PC #1 accounts for 97% of the variance on its own, and including any additional PCs introduces more complexity than it's worth.

(f) The scores for the new observations:

```
            [,1]       [,2]      [,3]        [,4]        [,5]
[1,] -16.855658 -66.790142  6.776900 -15.3066224  -8.5649268
[2,]   4.861635  26.789815 -3.710359   6.3835809   2.7072847
[3,]   2.536648   1.355434  1.590414  -0.2530266   0.8012362
[4,] -30.863147 -57.879087 -5.570018  -9.7920953 -12.6610309
[5,] -11.787862 -14.507653 -3.653059  -1.8812777  -4.6755677
```

(g) Coordinates of the projections in the original space:

```
            [,1]       [,2]     [,3]    [,4]     [,5]
[1,] -10.871807 -38.241645 4.665532 7.91427 62.13875
[2,]  37.567018  66.568511 4.665532 7.91427 62.13875
[3,]  16.460691  25.729053 4.665532 7.91427 62.13875
[4,]   5.978194 -41.185647 4.665532 7.91427 62.13875
[5,]   1.857621  -6.945432 4.665532 7.91427 62.13875
```

Euclidean distance from the original data points.

```
[1] 81.49181
[1] 88.13304
[1] 36.01572
[1] 79.53924
[1] 19.04255
```

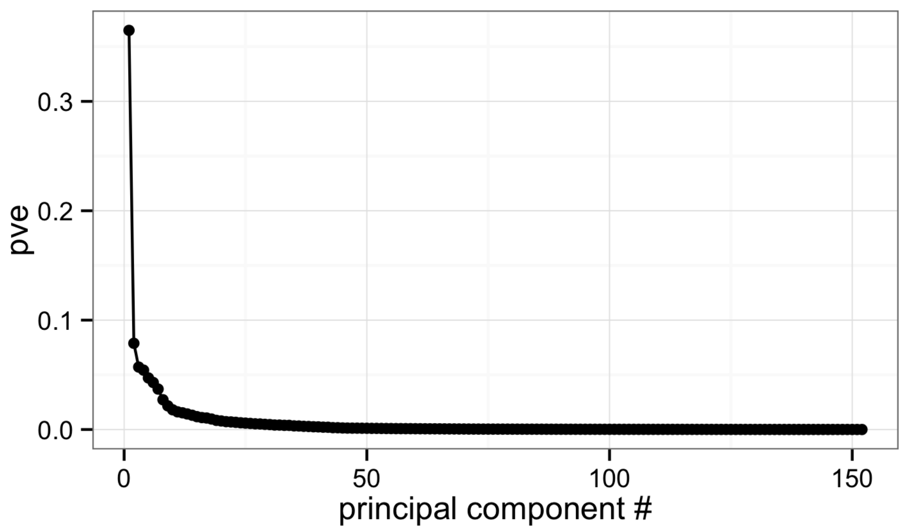
(h) The error vectors are more or less orthogonal to the direction of the first principal component. This is because the error vectors are defined as the direction from the original points to their *orthogonal projections* onto the reduced-dimension space, which is primarily captured by the first PC.

Problem 2: PCA with Yale Faces B

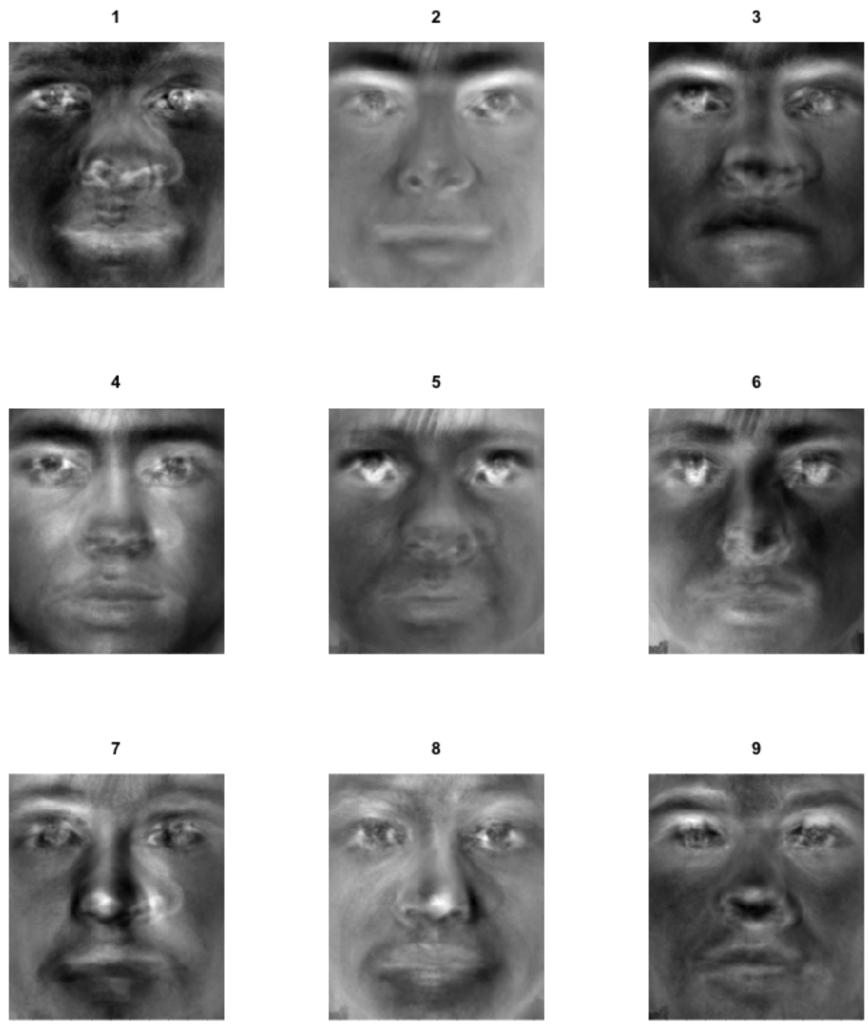
(a) The matrix is 152 rows by 32,256 columns. Each row of the matrix is one photograph (38 subjects with 4 views each), and each column is a pixel in each image (originally 192 x 168).

(b) Mean face:

(c) Proportion of variance explained



(d) The first 9 eigenfaces are the (constructed) images that capture the most variable aspects of the faces in the dataset. The first image amplifies the most variable pixels in the original images, the second image amplifies the most variable pixels in a direction orthogonal to the first principal component, the third the most variable pixels in a direction orthogonal to the first two principal components, etc.



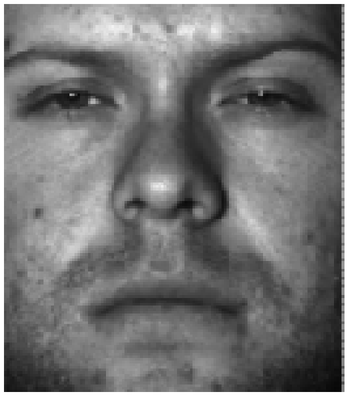
(e) Reconstructing `yaleB05_P00A+010E+00.pgm` using 24 eigenfaces:

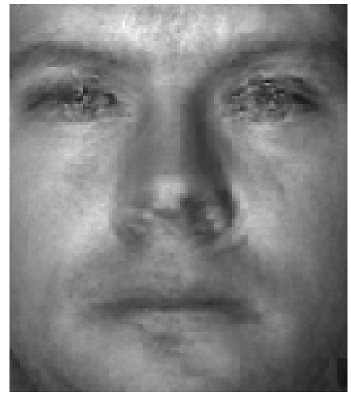
Reconstructing `yaleB05_P00A+010E+00.pgm` using 120 eigenfaces, in steps of 5:

After incorporating 14 or 15 eigenfaces, the person is relatively recognizable

(f) After removing the images of Subject 1, re-running PCA on the reduced dataset, and then reconstructing a photo of Subject 1 using the principal components; the reconstructed photo (right) has similar features to the original (left), but it's definitely not a recognizable image.

It doesn't look much like the original image because we excluded all photos of Subject 1 before computing the principal components. As such, the features present in the subject's original photos weren't incorporated into either the "mean face" or into the principal components.





Problem 3: James 3.7.3

(a) asdfasdf

Problem 4: James 3.7.4

(a) asdfasdf

Problem 5: question

Todo list