Introduction:

Carbon Cap-and-Trade is currently a leading proposal for controlling carbon emissions. In the US, the State of California instituted the world's first Carbon Cap-and-Trade policy in 2013[1]. Since then, 40+ national governments have enacted similar policies[2]. Under Cap-and-Trade, any entity emitting carbon above their allowance ('credits') must pay for someone else to sequester carbon ('offsets'). As a result of this structure, estimating expected future carbon sequestration is a major topic in the academic and commercial fields developing around carbon sequestration.

California's Cap-and-Trade policy allows carbon offsets to be purchased in US states outside of California, extending the policy's effects to other states, including Wisconsin. Forested land receiving regular precipitation has the greatest carbon sequestration potential, thus states with large forests and sufficient rainfall are likely candidates to sell carbon offsets. In Wisconsin, almost 50% of the state's area is forested, and in most years the region receives sufficient rainfall for rapid forest growth[3]. As a result, Wisconsin currently relies on forestry and forest products industry to provide 64,000 jobs and $24.5 billion in state GDP annually[4]. However, in recent years, several wood mills in Northern Wisconsin have closed, putting thousands of workers out of a job and disrupting forest economies in the state[5].

Climate change is also altering Wisconsin forests; precipitation patterns have changed dramatically over the past two decades while drought, extreme wind, and extreme rain have all

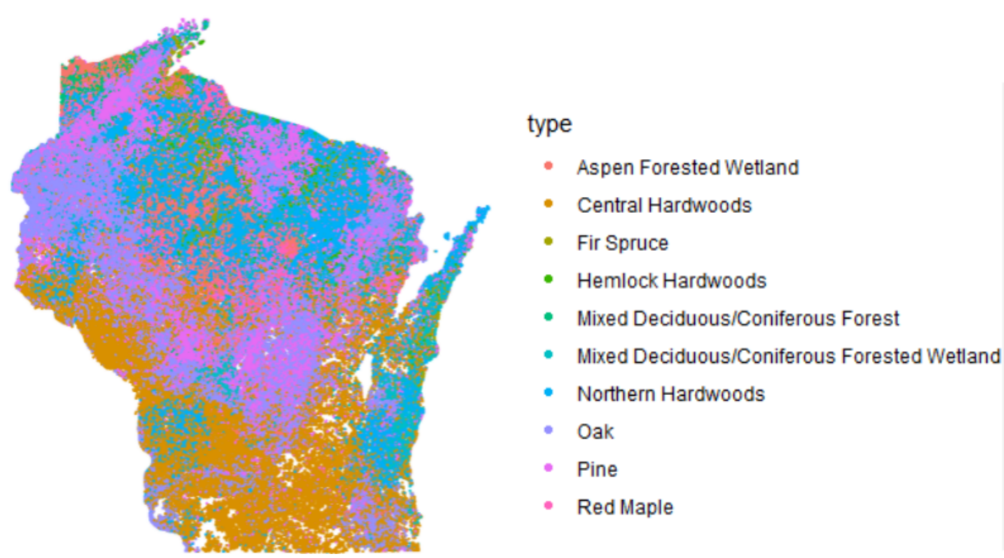
[1] Becker, Rachel. "California Says It Will Review Cap-and-Trade Amid Growing Criticism." KQED. February 18, 2021. Link.
[2] Plumer, B., Popovich, N. "These Countries Have Prices on Carbon. Are They Working?" New York Times. April 2, 2019. Link.
[3] Wisconsin Department of Natural Resources. "Forest Planning : Chapter 4." Accessed November 2, 2021. Link.
[4] Wisconsin DNR. "Forestry and the Wisconsin Economy." 2021. link.
[5] Mentzer, Rob. "Paper Mill Closures Drove a Bust for Northwoods Loggers and Some are Leaving the Industry." Wisconsin Public Radio. October 19, 2021. link.

become more common. Some tree species are better adapted to these extreme conditions, and are expected to weather these changes better than other species[6].

Carbon farming in Wisconsin forests has the potential to create alternate livelihoods to logging, while removing carbon from the atmosphere and benefiting ecosystems. But in order to place Wisconsin carbon offsets on the California carbon market, organizations certifying and selling carbon offsets must first arrive at a basis for pricing, and must do so even as climate conditions change rapidly, and vary from year to year in more extreme fashion.

This paper outlines one possible approach to this problem of estimating carbon offsets. We use Bayesian multiple regression with built-in heteroskedasticity to estimate future carbon sequestration by 10 forest types in 16 ecological regions of Wisconsin, leveraging 21 years of carbon capture data. Data for our model come from several public sources, including annual Net Primary Production estimates from the University of Montana Numerical Terradynamic Simulation Group, and land-cover and ecological region data from the Wisconsin Department of Natural Resources (see 'Data' section). Since precipitation patterns in Wisconsin began to change dramatically around the year 2000, and post-2000 precipitation patterns are expected to continue in the near

Figure A: **Sample of Species Distribution Data**

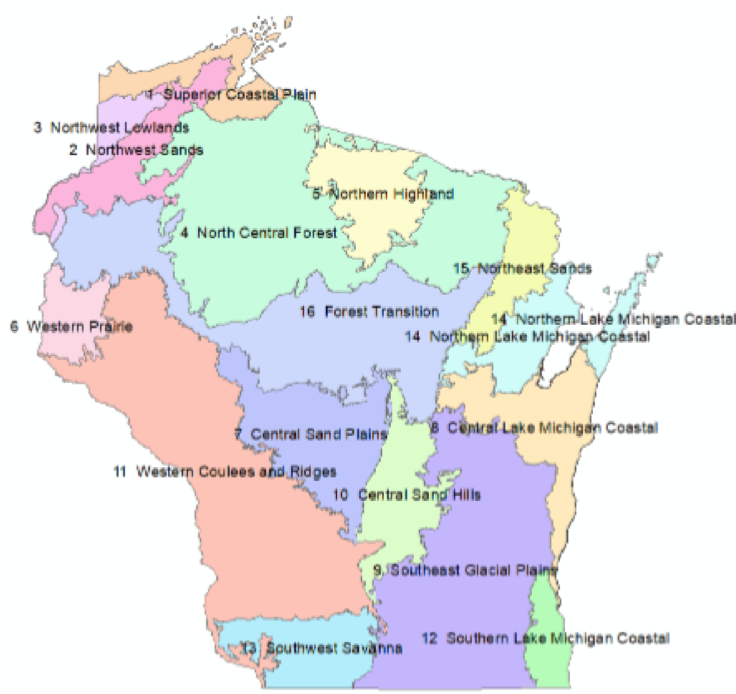


[6] Janowiak, M. et al. United States Forest Service. (2014). Forest Ecosystem Vulnerability Assessment and Synthesis for Northern Wisconsin and Western Upper Michigan. link.

future[7], we use carbon sequestration estimates from years 2000 through 2021 as the basis for estimating carbon sequestration in coming decades. To arrive at estimates of carbon sequestration, we fit a Bayesian model using Markov Chain Monte Carlo simulation, and create posterior distributions and 90% credible intervals for expected future carbon sequestration for each forest type-region combination present in our aggregated data. We use the *Stan* software package (through *RStan*), as well as R packages from the *tidyverse* collection, plus *raster, sp, sf,* and *RGDAL* to process and analyze our data.

After training a Bayesian model on our data, we find that our model achieves fairly accurate results, as measured using a 80-20 training-testing split. While our posterior distributions encompass only as much variability as our data contain, we suggest training this model on a greater amount of data and/or training on a subset of data reflecting expected future climate conditions. Posterior distributions from our model both (1) provide inference on past carbon sequestration dynamics and (2) can be used to predict future carbon sequestration, suggesting a possible framework for awarding carbon prices to Wisconsin forests based on limited information. However, we also recognize a number of limitations in our model.

Figure B: Ecological Zones of Wisconsin



[7] ibid.

Data:

To investigate carbon sequestration dynamics in Wisconsin, we sought out datasets that: (1) divide the state into regions with underlying similarities (2) spatially estimate plant communities (3) spatially estimate carbon capture by photosynthesis.

The Wisconsin Department of Natural Resources (WI DNR) divides the state into sixteen ecological zones based on traits such as soil, water features, and plant/animal communities (see Figure B: Ecological Zones of Wisconsin above)[8]. We used the WI DNR's system of ecological zones to aggregate our data into regions sharing underlying characteristics beyond the scope of our analysis, specifically: terrain, geology, water dynamics, and local climate.

WI DNR also maintains a land-cover database for the state of Wisconsin called 'WiscLand2' (available since September, 2016)[9]. This project, developed jointly between the DNR and the University of Wisconsin-Madison, combines satellite imagery from Landsat satellites 5, 7, and 8 (data from 2010-2014); topographic, soil, and groundwater data; and ground truth species data to create a classification algorithm for land cover[10]. For the purposes of our analysis, we took WiscLand2's estimations at face value, even though members of the project have identified uncertainty in classification, especially in distinguishing between different members of a genetic family (e.g. white spruce versus blue spruce). For this analysis, we limited our scope to 10 common forest types (see Figure A: Sample of Species Distribution above), with the intent of analyzing forest types that are likely candidates for carbon farming.

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[8] Wisconsin Department of Natural Resources. "Ecological Landscapes of Wisconsin". Accessed November 1, 2021. Link.
[9] Wisconsin Department of Natural Resources. "Land Cover Data (WiscLand)". Accessed November 1, 2021. Link.
[10] Lacy, Jim. "WiscLand2 Project Complete, Data Now Available." Wisconsin Geospatial News. September 23, 2016. Link.

Carbon capture data for this analysis come from the University of Montana Numerical Terradynamic Simulation Group (NTSG) which uses Landsat Surface Reflectance data—essentially digital photographs taken by US scientific satellites—to estimate annual Net Primary Productivity (NPP)[11]. NPP measures net carbon sequestration: the amount of carbon taken in by plants during photosynthesis, minus carbon emitted by plants during photorespiration and respiration. We used Google Earth Engine to access NPP data for Wisconsin from 2000-2020 (inclusive), for a total of 21 years of estimated carbon capture data[12].

NPP and land cover data were initially at 30m x 30m resolution, which proved intractable for vectorized computation, and were unnecessarily fine-grained for our question of interest: namely, how landscape-scale carbon capture varies between forest types and regions in Wisconsin. To make computation more manageable, we aggregated both forest type and NPP data to 120m x 120m resolution. For the (continuous) carbon capture data, we aggregated to a coarser resolution by taking the mean of the twelve 30m x 30m pixels contained in each 120m x 120 aggregated pixel. To aggregate the (categorical) land cover data, we used modal assignment, where the assigned aggregated value for each 120m x 120m pixel was the most common value of the twelve pre-aggregated 30m x 30m pixels, with random assignment in the case of a tie.

After limiting our scope to the 10 forest types listed above and reducing the resolution of our raster data, our dataset consisted of approximately 78.5 million discrete observations. To condense this large dataset into a scale suitable for Markov Chain Monte Carlo (MCMC) methods on personal computers, we then aggregated the carbon capture data for each region-type-year pairing. Since we observed long tails in the distributions of NPP values for

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[11] University of Montana Numerical Terradynamic Simulation Group. "Landsat Productivity." 2018. Link.
[12] Earth Engine Data Catalogue. "Landsat Net Primary Productivity CONUS". Accessed November 1, 2021. Link.

many region-type combinations, we decided to perform aggregation using median (rather than mean) NPP for each region-type-year combination. We wanted to arrive at a ‘best guess’ for carbon sequestration for each region-type-year, and were concerned that using the mean to aggregate values would overestimate the location of the bulk of observed data (if curious, please see Supplemental Materials, Figure 1: Distribution of MNPP, Median vs Mean).

Finally, we removed 19 zone-type pairings which consisted of $n < 150$ (120m x 120m plots), out of concern that these categories would not provide reliable estimates of forest-type and zone effects. After cleaning and aggregating, our data consists of 2646 observations of median NPP (denoted ‘MNPP’ below).

Methods:

To model carbon sequestration dynamics based on zone and forest type, we constructed a Bayesian model with the intent to: (1) incorporate prior uncertainty about parameters of interest (2) perform partially-pooled analysis of (a) region, (b) forest type, and (c) year effect on MNPP, and (3) combine prior uncertainty with observed data to arrive at posterior mean estimates and credible intervals for future carbon sequestration for each zone-type combination observed in our data.

Exploring our data before modeling, we first consider each zone-type combination separately. We observe that for each zone-type combination, MNPP appears to regress to a mean (‘baseline’) which remains fairly constant over time. From year to year, MNPP seems to deviate from its zone-type baseline in a fairly random way, while the sign of year effect (±) appears to be shared across zones and types. Additionally, MNPP of different forest types in a given region maintains a fairly consistent relationship over time (see Supplemental Materials Figure 2: MNPP versus Year).

Aggregating across years, the spread of MNPP differs by region and type, without a consistent relationship to the number of observations (Supplemental Figure 3: Log($\boldsymbol{\sigma}_{\mathrm{MNPP}}$) vs Log(count)). Additionally, exploratory modeling using ordinary least squares (OLS) suggests that observed variation in both MNPP and SD are best explained by taking into account both zone and type (Supplemental Table 1: Exploratory OLS R^2 values).

In short, we find evidence that zone and forest type together influence *both* baseline NPP and year-to-year variability in NPP, suggesting a mixed effects model, where both (fixed) baseline and (random) spread are affected by region and zone. For simplicity's sake, we model baseline NPP (μ_{NPP}) as a linear combination of effects, and log standard deviation of median NPP (σ_{NPP}) using decomposition of variance:

$$\boldsymbol{\mu_{NPP}} \sim \tau + X \begin{bmatrix} \Delta_{type} \\ \Delta_{zone} \\ \Delta_{year} \end{bmatrix} \qquad \tau := grand\ mean\ of\ MNPP\ values = 7212$$

$$\boldsymbol{\sigma_{NPP}} \sim \sigma_0 * \sigma_{type} * \sigma_{zone} \ \leftrightarrow \ \log(\sigma_{NPP}) \sim \log(\sigma_0) + \log(\sigma_{type}) + \log(\sigma_{zone})$$

$$\log(\sigma_0) := grand\ mean\ of \log(\sigma_{NPP})\ values = \ 6.854$$

We note that, since each predictor is treated as a factor, *X* is a sparse matrix with 1's indicating each observation's type, zone, and year membership, while each effect parameter represents a vector of effect estimates for each (e.g. Δ_{type} is a vector of 10 type effects).

We treat each effect type as independent and distributed according to their own prior distribution. To elicit prior hyperparameters, we use OLS regression on MNPP data, according to the model above, to identify ballpark prior standard deviation for year, type, and zone effects. We note that MCMC simulation methods minimize a different cost function from OLS methods (log probability vs. sum of squared error), but lacking additional subject-area expertise to use for prior elicitation, we use this approach as a 'best guess' as to the variability of zone, type, and year effects.

Our full model combines the above models for baseline and spread of MNPP with prior distributions for each effect type and a normal distribution generating observed values (at right):

Fitting this model using *RStan*, with 4 chains and 5000 iterations per chain, all $\hat{R}$ values are between 0.999 and 1.002, indicating that the chains converge on effect estimates. Effective *n* values are between 1440 and 25946, with lowest values (<1650) associated with estimation of $log(\sigma_{type})$ for 7 of the 10 types, and $log(\sigma_{zone})$ for 4 of the 16 zones. This is not unexpected, considering that our aggregated data contains less than 150 observations for zones with relatively few forest types present, suggesting that some effect estimates should have relatively low effective *n* values.

Figure 3: Bayesian Model

1 - MNPP location and spread models:

$$\boldsymbol{\mu_{NPP}} \sim \tau + X \begin{bmatrix} \Delta_{type} \\ \Delta_{zone} \\ \Delta_{year} \end{bmatrix} \qquad \tau := grand\ mean\ of\ MNPP\ values = 7212$$

$$\boldsymbol{\sigma_{NPP}} \sim \sigma_0 * \sigma_{type} * \sigma_{zone} \leftrightarrow \log(\sigma_{NPP}) \sim \log(\sigma_0) + \log(\sigma_{type}) + \log(\sigma_{zone})$$

$$\log(\sigma_0) := grand\ mean\ of\ \log(\sigma_{NPP})\ values = 6.854$$

2 - prior distributions:

$$\Delta_{type} \sim N\left(0, 420^2\right) \leftrightarrow \frac{\Delta_{type}}{420} \sim N(0, 1)$$

$$\Delta_{zone} \sim N\left(0, 371^2\right) \leftrightarrow \frac{\Delta_{zone}}{370} \sim N(0, 1)$$

$$\Delta_{year} \sim N\left(0, 807^2\right) \leftrightarrow \frac{\Delta_{year}}{807} \sim N(0, 1)$$

$$\log(\sigma_{type}) \sim N(0, 1)$$

$$log(\sigma_{zone}) \sim N(0, 1)$$

3 - observed value generating distribution:

$$Median\ NPP \sim N(\mu_{NPP}, \sigma^2_{NPP})$$

Posterior Summaries:

After fitting our model, we analyze the resulting posterior distributions. We note that after starting with non-specific priors, our model appears to converge on fairly narrow estimates of all effects (see Supplemental Figure 4: Prior vs Posterior 'Type' Effect Distributions). One of our initial questions was whether any one forest type would appear to be a clear ‘winner’ in carbon sequestration across the state, and we do not see evidence for this in posterior distributions for Type effect. We do note that Mixed Deciduous/Coniferous Forested Wetlands is the only forest type with a large negative effect estimate, reflecting poor performance in all zones. For other forest types, substantially above or below average carbon sequestration appears likeliest to be a result of zone-specific conditions, suggesting that zone contains useful information about MNPP. For example, Zone 12 exhibits high MNPP values across species.

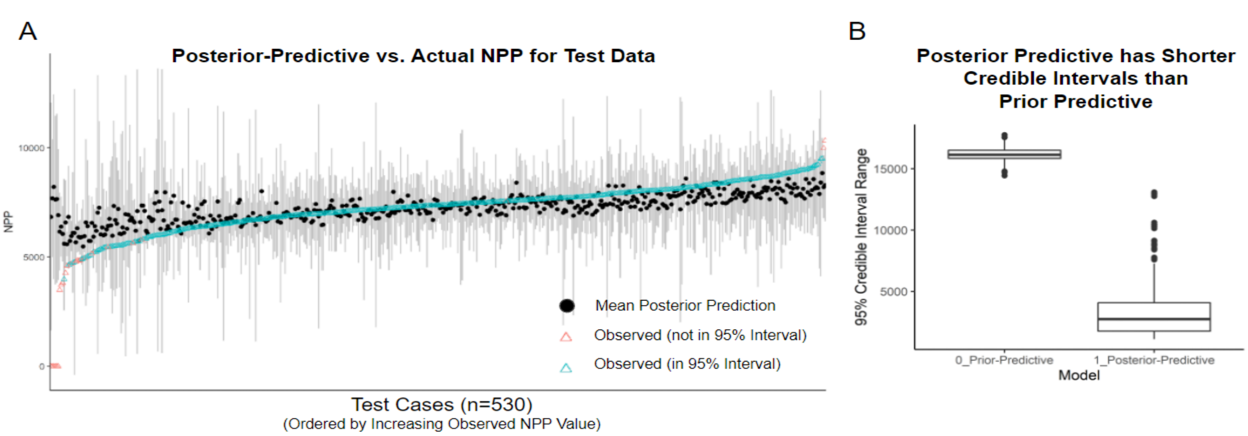
Posterior Predictive Distribution

Our posterior data takes the form of a 3d array, with a predictive MNPP value associated with every combination of year, type, and zone. Aggregating all 21 years of data, our posterior distribution reflects the posterior expected value and variation of MNPP for the given zone-type combination over 21 years of observed data (**Figure 5** below). We feel that this is our most significant model result, as these posterior distributions can be used to create carbon prices for individual plots of land based on zone and forest types present. However, specific pricing would likely depend on consumer and regulatory tolerance for variability, as well as greater knowledge about future climate conditions.

Figure 5: The posterior predictive distribution given a certain type and zone. The x-axis represents the different zones while the y-axis represents median NPP values. Each color represents a different zone while each faceted graph represents a different forest type.

Testing Posterior Predictions

To test the validity of our model, we partition our aggregated data into two segments: one including 80% of the data, the other 20%. We use the larger subset to elicit prior standard deviations and fit a Bayesian model as described above (5000 iterations, 4 chains). The smaller subset of data was used to test the validity of the model. Of the 530 test cases in the smaller dataset, 95.3% (505/530) of them were predicted to lie in the 95% credible interval of the posterior-predictive (**Figure 6**). Of the 25 instances where observed median NPP values were larger or smaller than expected by our posterior, 15 (60%) were related to observations from years 2012 and 2013 (see Supplemental Table 2 for more detail), the severest drought period observed in the state of Wisconsin over the last 20 years. We interpret this as evidence that a model trained on exclusively non-drought years will perform poorly in predicting MNPP during a drought. We also found that while our prior predictive featured all but 4 test cases in its 95% credible interval, 95% prior predictive credible intervals were quite large, while posterior predictive distributions were much smaller, especially for certain zone-type combinations (**Figure 7**).



<u>Figure 7:</u> Fairly accurate posterior predictions of median NPP in 80-20 training-testing split. A) Comparing observed median NPP values (triangles) in the test set to the posterior prediction means (black circles) and 95% credible intervals (grey bars). B) Boxplots comparing the range of 95% credible intervals between the prior predictive (left) and posterior predictive (right).

<u>Results</u>:

Regional Latitude Does Not Substantially Alter Median NPP Emission of Different Species

To assess whether certain types of forest had different median NPP values for regions in southern vs. northern Wisconsin, we analyzed our posterior distribution across all 21 years represented in our dataset. Regions were ranked from southernmost to northernmost Wisconsin based on median latitude coordinates of the pixelated data points available for a region. While slight differences were observed for the median NPP across regions for each forest type, posterior 95% credible intervals always overlapped for a species across regions (**Supplemental Figure 5**). However, we observed that for some forest species, such as "Mixed Deciduous/Coniferous Forested Wetlands", the NPP distribution was more variable in southern regions compared to northern regions (**Figure 8-A**). We considered whether these differences could be due to a greater proportion of northern regions being covered by such forest species [i.e. data availability bias] (**Figure 8-B**), but found that for the species "Mixed Deciduous / Coniferous Forested Wetlands" no clear linear relationship with region median latitude was found. This suggests that south vs. north region-specific variability in median NPP for "Mixed Deciduous / Coniferous Forested Wetlands" and other species could be linked to biological factors such as soil and climate differences between the two ends of the state, or technical factors such as inaccurate remotely sensed estimation of these forest types stemming from their proximity to water. Similar trends were observed for Pine forest types (**Supplemental Figure 6**).

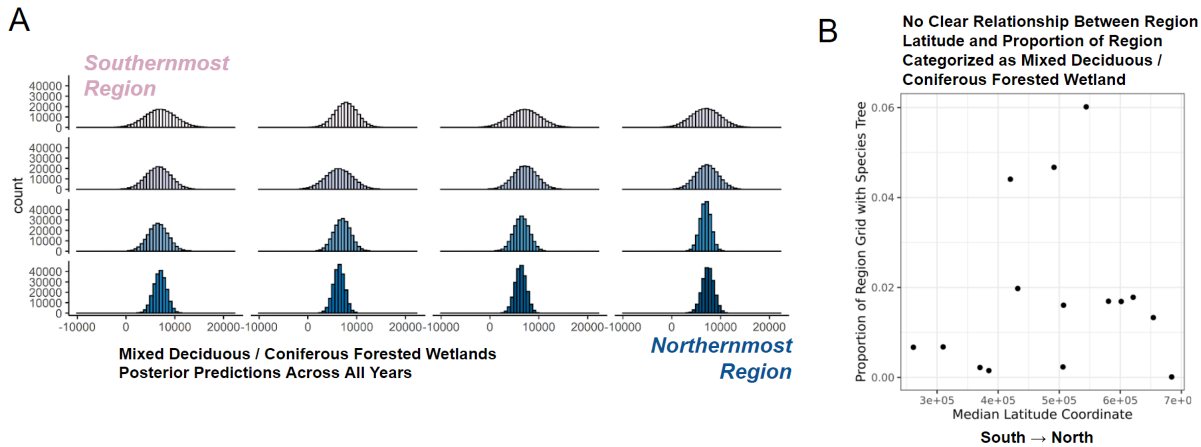


Figure 8-A, 8-B: A) Histograms of median NPP predictions from the posterior distribution for the forest species "Mixed Deciduous / Coniferous Forested Wetland" across all years for each region. B) Each point in the scatterplot represents one of the 16 regions. The x-axis shows the median latitude coordinate for each region (metric used to order from south to north), while the y-axis shows the proportion of each region covered by "Mixed Deciduous / Coniferous Forested Wetland" forest.

Conclusion and Model Critique:

To investigate carbon capture dynamics across different forest types and regions of Wisconsin, we constructed a Bayesian mixed effects model with built-in heteroskedasticity, using an aggregated dataset of 2,646 MNPP values for zone-type-year combinations. When we constructed our model using 80% of the data available, predictions of NPP on the remaining 20% of data revealed that our model could use observed to generate fairly accurate estimates of zone and type effects on both location and spread of MNPP; the model was able to predict most observations in the testing data, with the notable exception of some observations from drought years, suggesting that long-term model accuracy relies on training data containing a variety of possible weather conditions. Analyzing posterior distributions of our model when trained on the full dataset revealed two major findings: (1) for a number of forest types, variability in carbon sequestration was greater in southern zones than in northern zones, independent of number of observations and (2) no single species appeared to sequester substantially more carbon than others across the entire state, though some zones had dramatic effects, and some forest types sequestered more carbon than others in particular zones, suggesting that a potential carbon pricing scheme in Wisconsin would benefit from considering both forest type and zone.

We noted that our model could be improved in a number of ways. For the most part, our data did not exhibit long-term time trends, but we did see evidence of short-term autocorrelation. Allowing model predictions to be at least partially auto-correlated might improve model performance, as would incorporating data about the precipitation, drought, storm, and extreme wind patterns that we believe are likely to drive these short term trends. Additionally, our model examined only *existing* forests, whose locations are a result of a number of unstudied factors, including soil and land management. Any inference drawn from our model about carbon sequestration of forest types should not be applied to *future* forests that might be planted in unsuitable locations. In other words, this model produces estimates of carbon sequestration that can be used as the basis for carbon pricing of existing forests, but without more detailed investigation, it says little about what types of forests should be planted or encouraged in the future. Finally, due to constraints in computing power, as well as our knowledge of modeling with Stan, we aggregated our initially large dataset into a dramatically smaller dataset. Provided additional computational resources and time, we could potentially employ more sophisticated approaches to fit a Bayesian model using a less aggregated version of our dataset.