Carbon Dating and Golf Trajectory

Brian Zhu

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1 Carbon Dating

1.1 Analytical Derivation the relation between half-life time $T_{\frac{1}{2}}$ and decay constant τ

The general relationship of the relationship between the half-life $T_{1/2}$ and the decay constant τ is:

$$N(t) = N_0 e^{-\frac{t}{\tau}}$$

At $t = T_{1/2}$, $N(T_{1/2}) = \frac{N_0}{2}$. Substituting this into the equation above would give:

$$\frac{N_0}{2} = N_0 e^{-\frac{T_{1/2}}{\tau}}$$

Solving for τ, we get:

$$\tau = \frac{T_{1/2}}{\ln 2}$$

With $T_{1/2} = 5700$ years, you can calculate the decay constant τ.

1.2 Numerical Calculation of relationship between $\mathbf{T}_{\frac{1}{2}}$ and τ

1.2.1 Basics of Euler Method

Euler method solve ordinary differential equations by approximating the solution step by step. Given a first-order differential equation $\frac{dN}{dt} = f(N, t)$, where $N(t)$ is the unknown function, the Euler method estimates the value of $N(t)$ at each time step t by using the slope (or derivative) of the function. The general update formula for the Euler method is:

$$N(t + \Delta t) = N(t) + \frac{dN}{dt} \cdot \Delta t$$

Where:

- Δt is the time step.
- $\frac{dN}{dt}$ is the derivative of N at time t.

1.2.2 Codes for Euler Method in Carbon Dating

I defined the following function using Euler method:

```
def nu_sol(N0, tau, delta_t, duration):
    num_steps = int(duration / delta_t) + 1
    time_points = np.linspace(0, duration, num_steps)
    N = np.zeros(num_steps)
    N[0] = N0

    for i in range(1, num_steps):
        a = -N[i-1] / tau
        N[i] = N[i-1] + a * delta_t
return time_points, N
```

I first calculate the number of time steps and create an array of time points. Then I use a loop that calculate the rate of decay at the current time step and Update the amount of Carbon-14 using Euler's method. The function will return the time points and the numerical solution for $N(t)$.

1.2.3 Results

I plot the curve of Carbon-14 decay in ancient artifacts using Euler method. The steps are 10 an 100 years respectively. Then I plot the Analytical solution in the same graph and we can see they are very close, almost overlapping each other completely.

Then I increase the time-step width to 1,000 years and re-plot. We can clearly see that the result is not as satisfying as before. There is a significant difference between analytical and numerical sulotions.

We can easily see that at year 20000 analytical result is approximately $9.0e - 14$ while the numerical result is $7.4e - 14$, giving to a approximately 22% difference. This result is unacceptable.

We can see that the the smaller the time-step the more accurate the result from Euler method would be. This is just as what I have expected, as the smaller the time-step the more similar it will be to the real world.

Figure 1: Carbon-14 Decay in Ancient Artifacts Time Step 10, 100 years

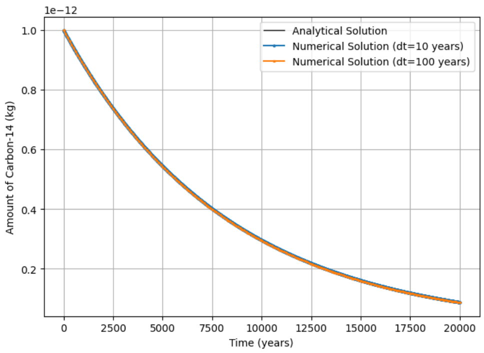
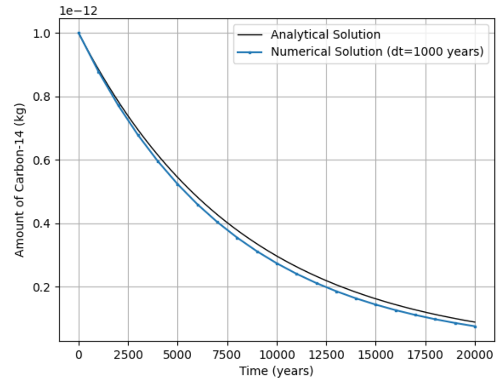


Figure 2: Carbon-14 Decay in Ancient Artifacts Time Step 1000 years



2 Golf Trajectory

2.1 Method

To find out the trajectory of golf ball, I take dimple, drag and spin into consideration, and get the final displacement X_{max} and v_{final}. In ideal situation, the only force acted on the golf ball is gravity. In dimpled spin, everything is taken into consideration, including dimple, drag and spin. Thus, I draw the trajectory of the golf ball when shoot out at 45°, 30°, 15°, 9°.

Table 1: Data for Calculation of Crystal Structures and Lattice Parameters of 1018 Steel

θ degree	45	33	15	9
X_{max_ideal}(m)	499.47	432.53	249.70	154.31
v_{final_ideal}(m/s)	69.99	69.99	69.99	69.99
$X_{max\_dimpled\_spin}$(m)	173.59	261.13	323.99	343.48
$v_{final\_dimpledspin}$(m/s)	38.48	36.00	32.29	30.167

Then I plotted out the trajectory of golf ball under four circumstances-ideal, with drag but without dimple, with drag and dimple, with drag dimple and spin.I draw the trajectory of the golf ball when shoot out at 45°, 30°, 15°, 9°. The figures are shown below.

Figure 3: Trajectory of Golf Ball when shoot out at 45°

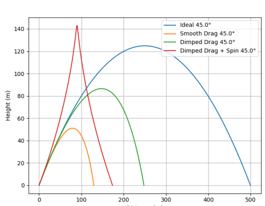


Figure 4: Trajectory of Golf Ball when shoot out at 30°

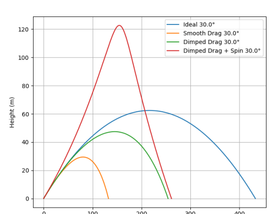


Figure 5: Trajectory of Golf Ball when shoot out at 15°

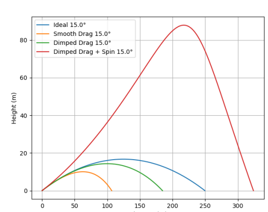
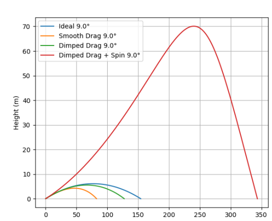
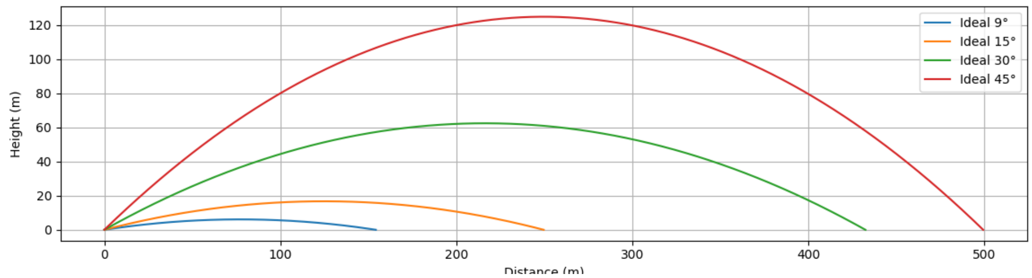


Figure 6: Trajectory of Golf Ball when shoot out at 9°



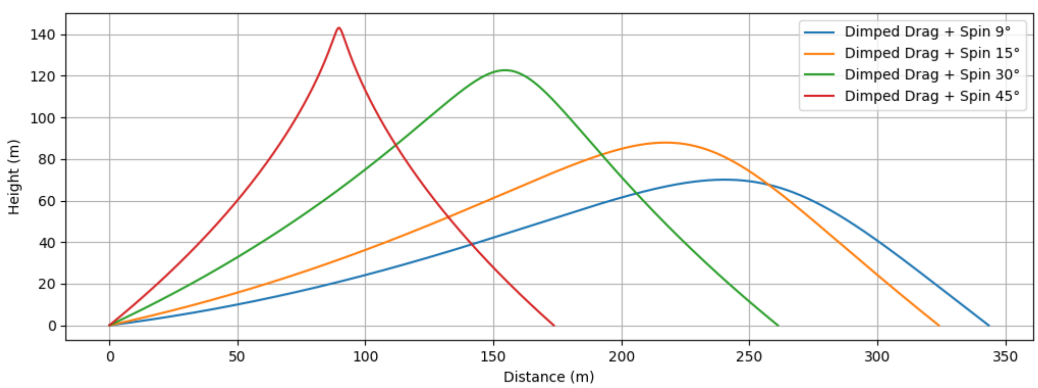
To compare different shooting angles under different circumstances, I draw the trajectory of four different angles on the same plot as seen below. In the trajectory under ideal circumstance, the farthest one is 45°, which is exactly the same as the result physical calculations give. The smaller the angle, the shorter the distance.

Figure 7: Golf Ball Trajectory in different Shooting Angles under Ideal Situation



In the trajectory under circumstances most close to reality, the farthest one is 9°, and the bigger the angle, the shorter the distance. This is very counter intuitive, so I looked it up on the internet and found the ideal shooting angle is about 7° to 9°.

Figure 8: Golf Ball Trajectory in different Shooting Angles under Drag and Spin



2.2 Result Discussion

The maximum distance is almost 350 meters, which is a little bit larger than it is supposed to be. The reason for this may be the lack of consideration on the reduction of spin due to air friction.

The program shows that if θ is 0°, the maximum x would be 350.4m, which is larger than 9°. However, according to physics, it is unreasonable because it would be impossible for a golf cue to exert a backward spin on the ball it hit out at 0°.