

Parallel A* search algorithm

Group 1

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Outline

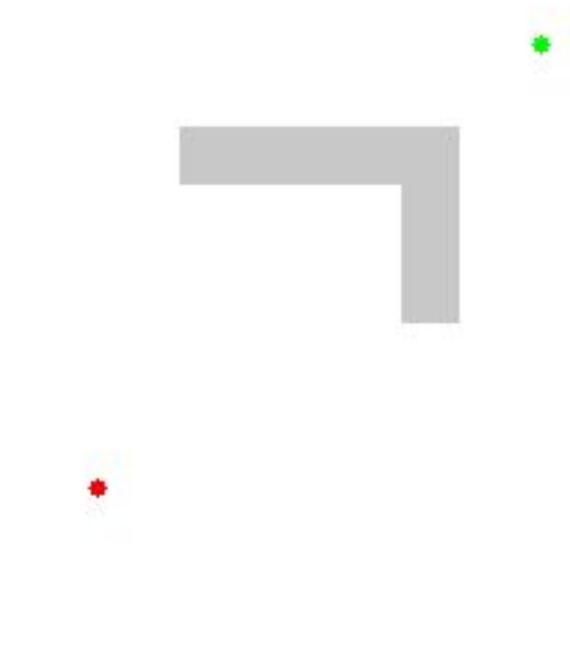
- ❏ A* search algorithm
- ❏ Experimental Setup
- ❏ Centralized Parallel A*
- ❏ Analyzing CPA
- ❏ Decentralized Parallel A*

A* search algorithm

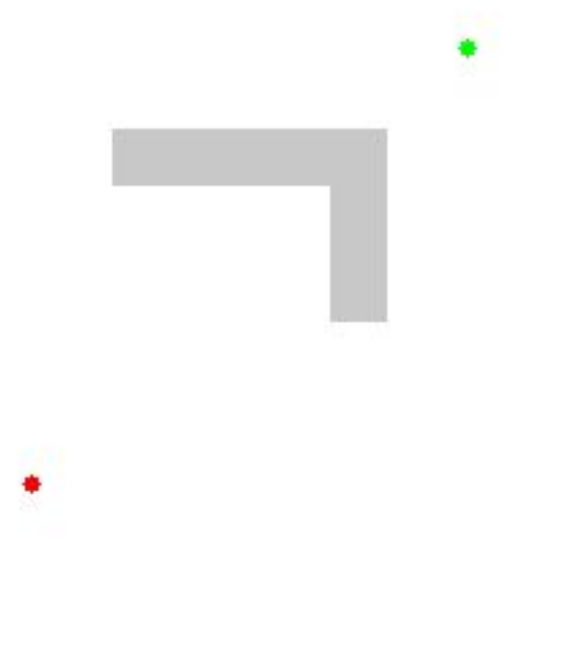
- Informed search algorithm
- Best first search
- Evaluation : $f(n) = g(n) + h(n)$
- $g(n)$: cost function, the actual cost from start node to node n
- $h(n)$: heuristic function
 - admissible \Rightarrow never overestimate
 - ex : In shortest path problem \Rightarrow straight line distance

A* search algorithm (Cont.)

- BFS



- A*



Experimental Setup

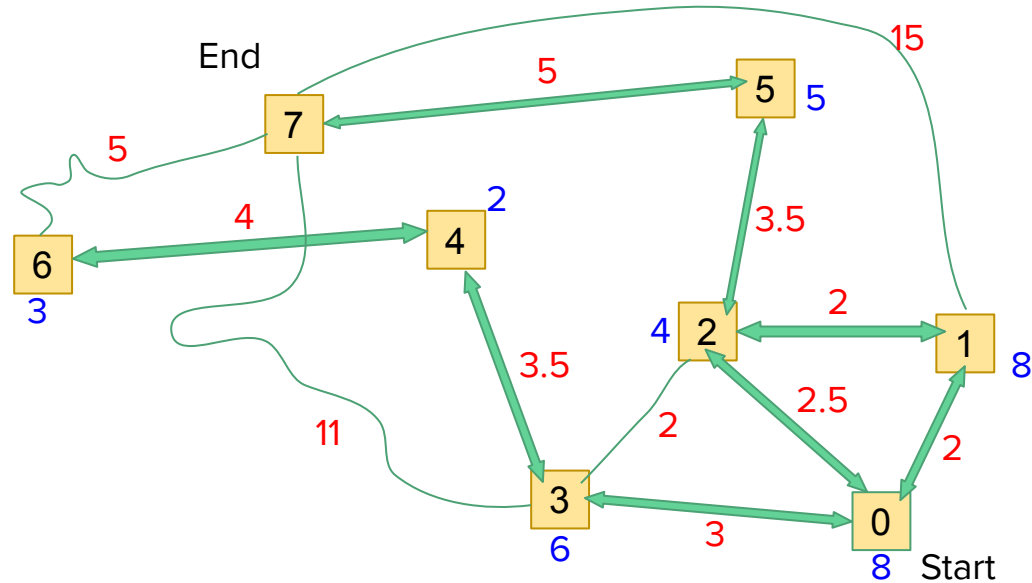
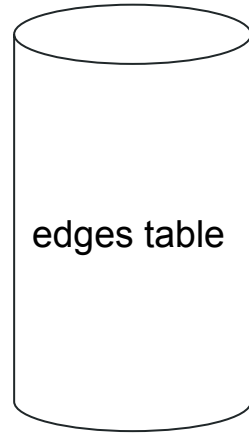
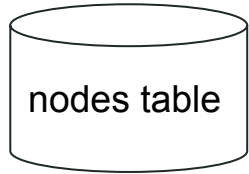
- Graph / Map Construction
 - Real road data from Hsinchu using OpenStreetMap
- Heuristic
 - The straight distance between each node and the target node



Centralized Parallel A*

- node
 - ID
 - g
 - h
 - previous
 - open
 - closed
 - Lock
- edge
 - start
 - end
 - distance
- Lock
 - Lock1 (OPEN/CLOSED)
 - Lock2(Incumbent cost)
- Node Set
 - OPEN
 - CLOSED

Centralized Parallel A* – An example

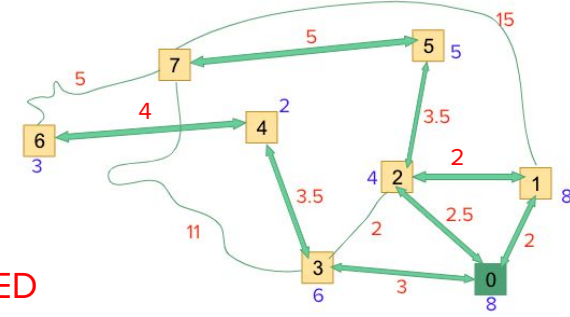


Centralized Parallel A*

Start



CLOSED



thread 0



thread 1



thread 2



thread 3

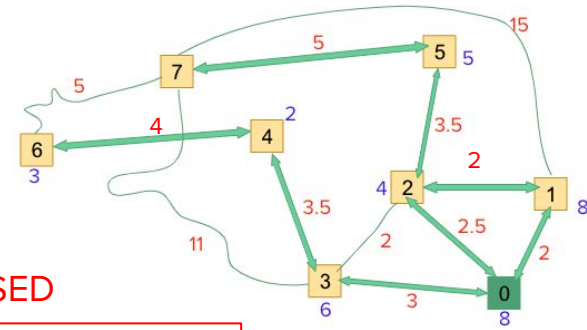
incumbent_cost = ∞

Centralized Parallel A*

OPEN



CLOSED



0



thread 0

wait



thread 1

wait



thread 2

wait



thread 3

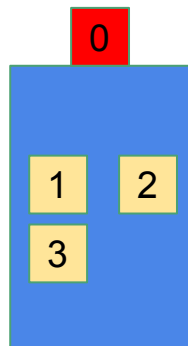
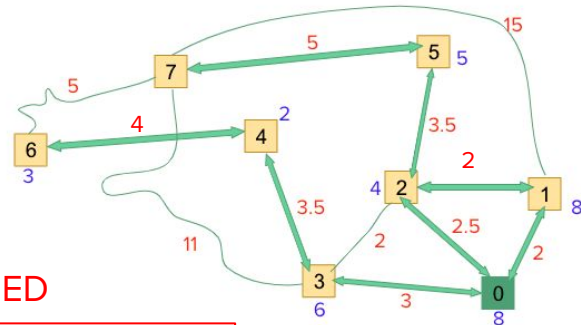
incumbent_cost = ∞

Centralized Parallel A*

OPEN



CLOSED



thread 0



thread 1



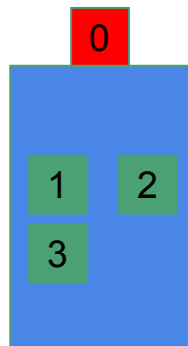
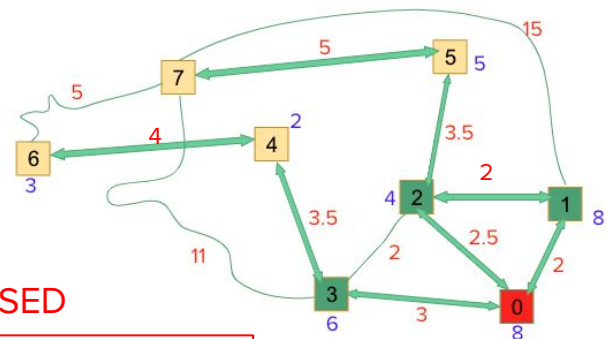
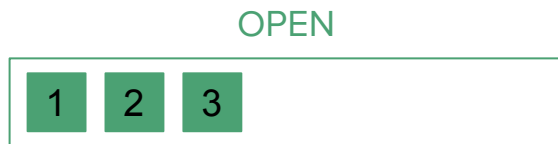
thread 2



thread 3

incumbent_cost = ∞

Centralized Parallel A*



thread 0

wait

thread 1

wait

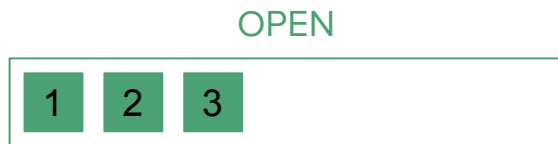
thread 2

wait

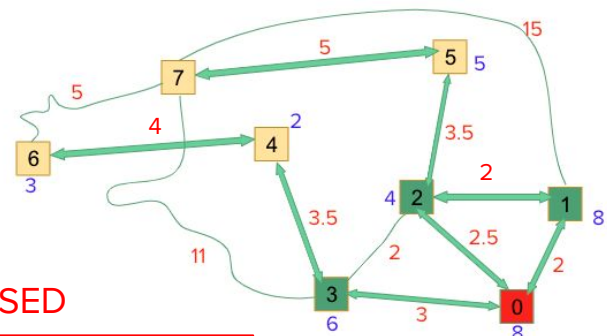
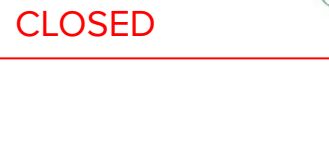
thread 3

$$\text{incumbent_cost} = \infty$$

Centralized Parallel A*



finding best



Lock1



thread 0



thread 1



thread 2



thread 3

$$\text{incumbent_cost} = \infty$$

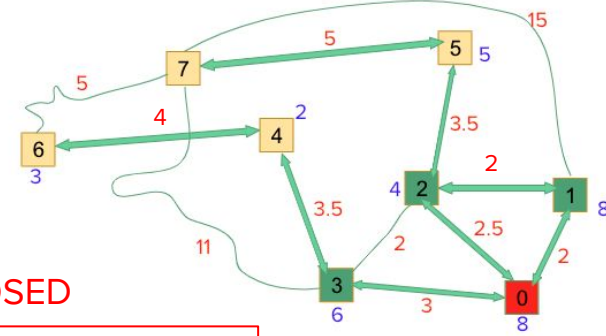
Centralized Parallel A*

OPEN



found

CLOSED



wait

thread 0

wait

thread 1

wait

thread 2

wait

thread 3

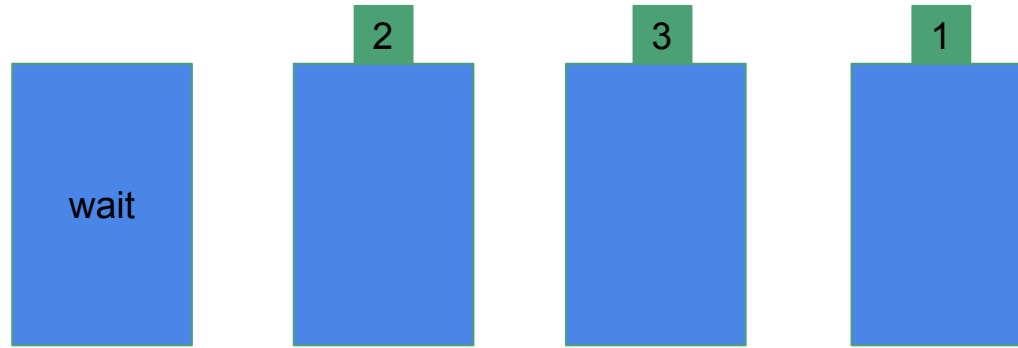
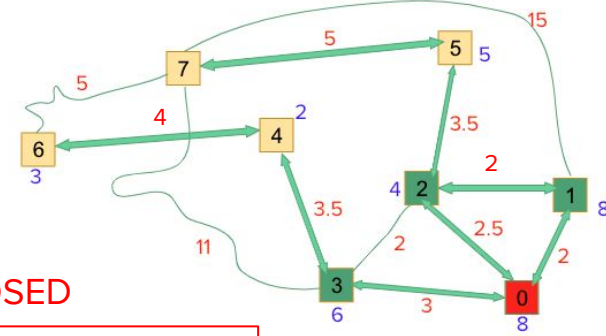
incumbent_cost = ∞

Centralized Parallel A*

OPEN



CLOSED



thread 0

thread 1

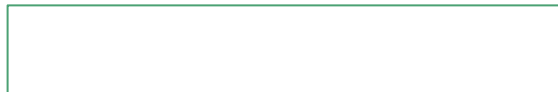
thread 2

thread 3

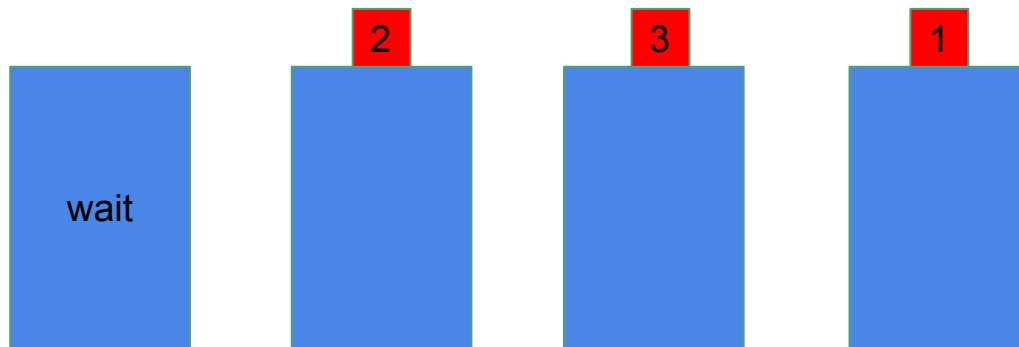
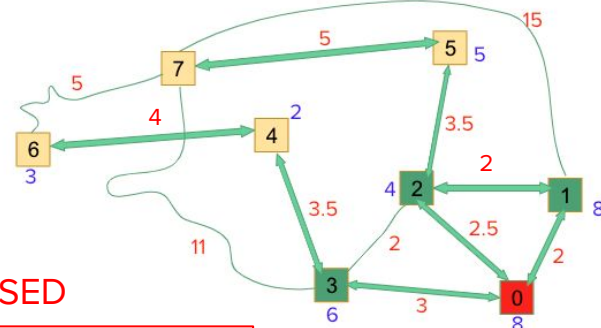
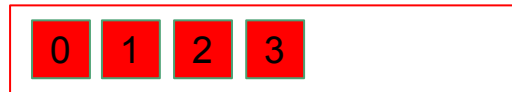
incumbent_cost = ∞

Centralized Parallel A*

OPEN



CLOSED



thread 0

thread 1

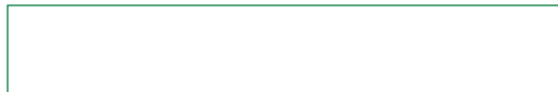
thread 2

thread 3

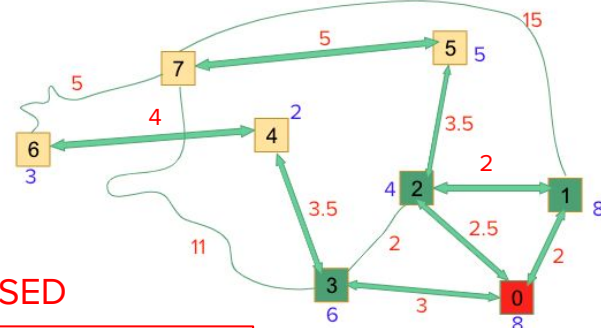
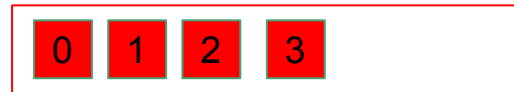
incumbent_cost = ∞

Centralized Parallel A*

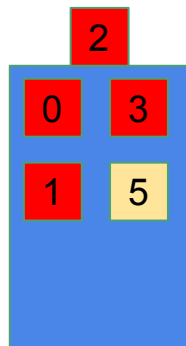
OPEN



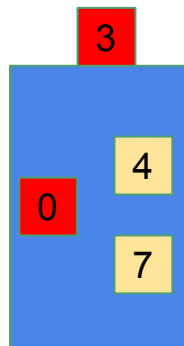
CLOSED



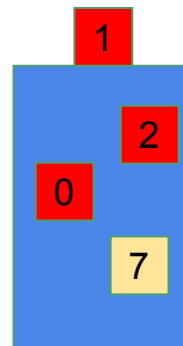
thread 0



thread 1



thread 2



thread 3

Node 1,2,3
Ambiguous
But no problem
=> finally closed

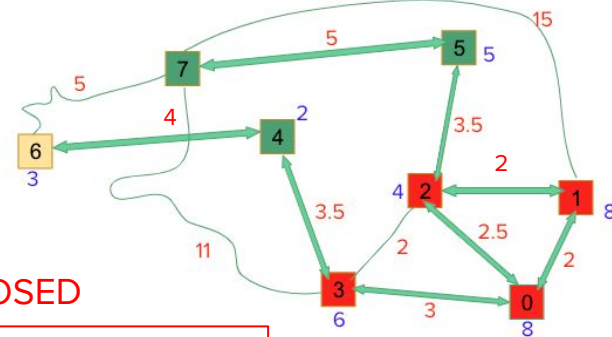
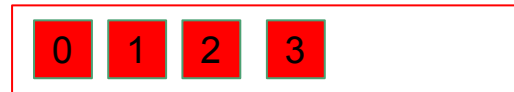
incumbent_cost = ∞

Centralized Parallel A*

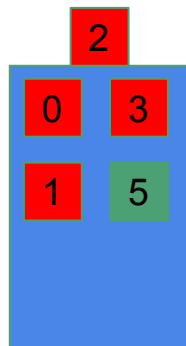
OPEN



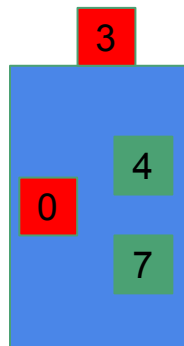
CLOSED



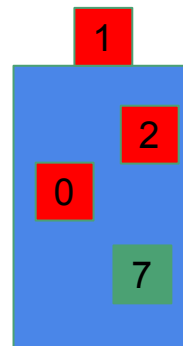
thread 0



thread 1



thread 2



thread 3

node 7 is dangerous.
lock it for safety.

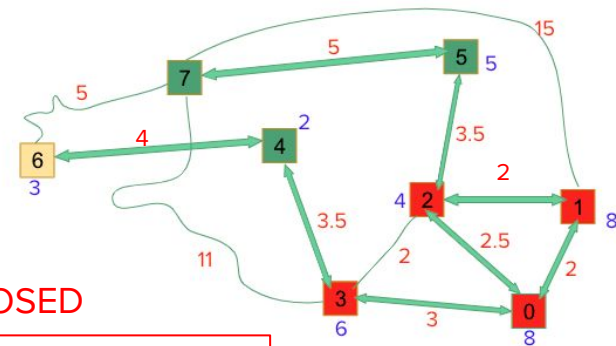
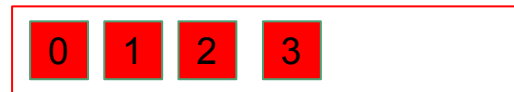
incumbent_cost = ∞

Centralized Parallel A*

OPEN



CLOSED



Lock1

wait

wait

wait

wait

thread 0

thread 1

thread 2

thread 3

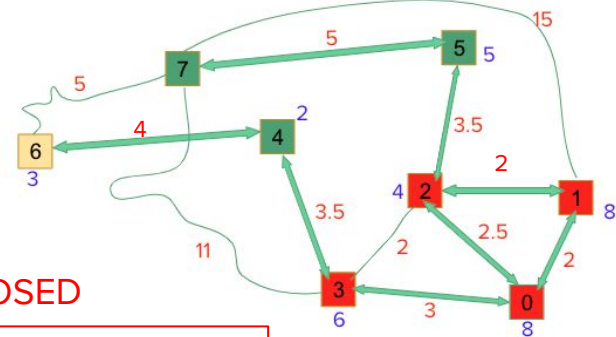
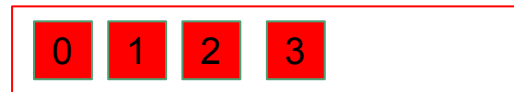
incumbent_cost = ∞

Centralized Parallel A*

OPEN



CLOSED



finding best

Lock1

wait

wait

wait

wait

thread 0

thread 1

thread 2

thread 3

incumbent_cost = ∞

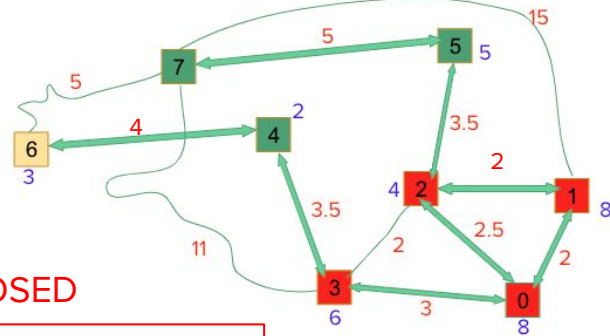
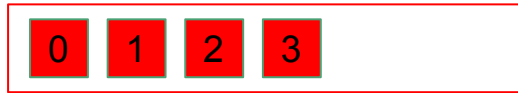
Centralized Parallel A*

OPEN



found

CLOSED



wait

wait

wait

wait

thread 0

thread 1

thread 2

thread 3

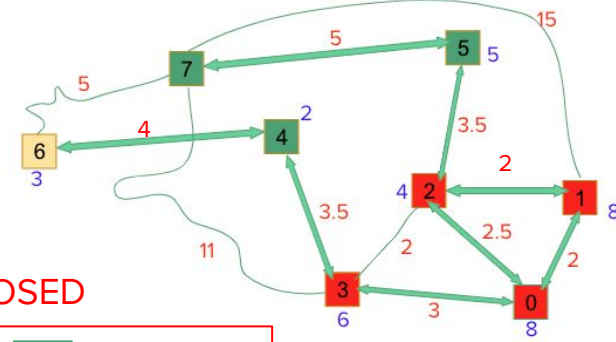
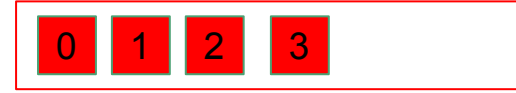
$$\text{incumbent_cost} = \infty$$

Centralized Parallel A*

OPEN



CLOSED



thread 0



thread 1



thread 2

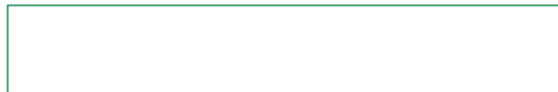


thread 3

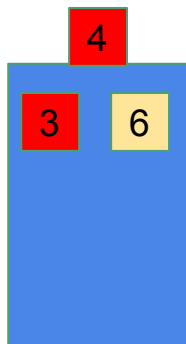
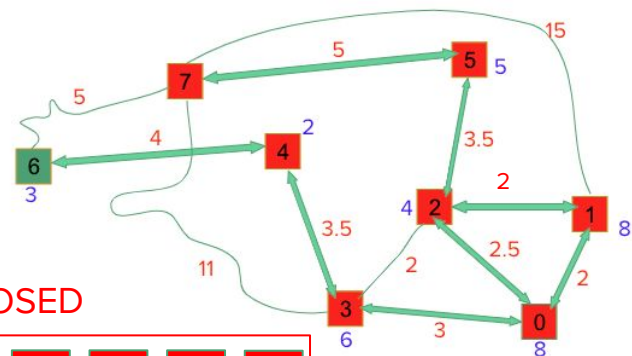
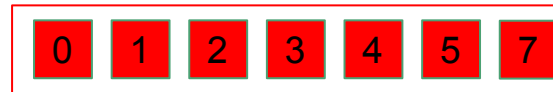
incumbent_cost = ∞

Centralized Parallel A*

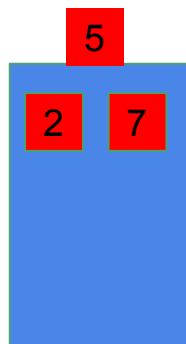
OPEN



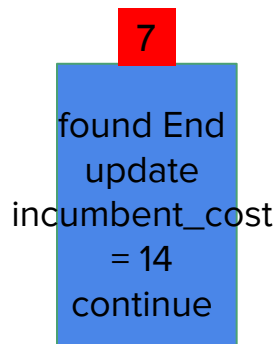
CLOSED



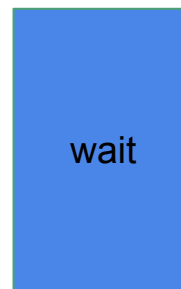
thread 0



thread 1



thread 2

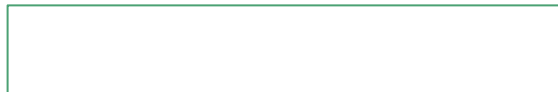


thread 3

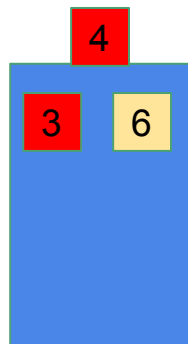
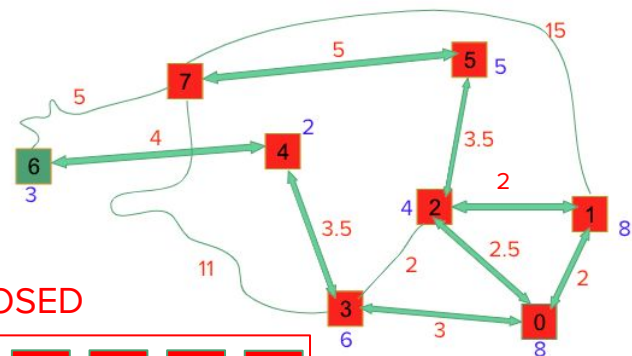
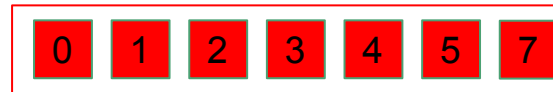
incumbent_cost = ∞

Centralized Parallel A*

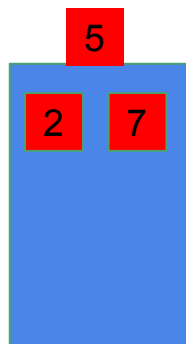
OPEN



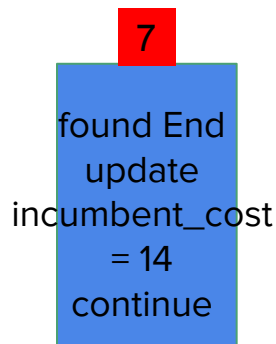
CLOSED



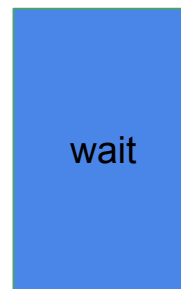
thread 0



thread 1



thread 2



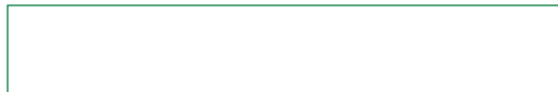
thread 3

Lock2

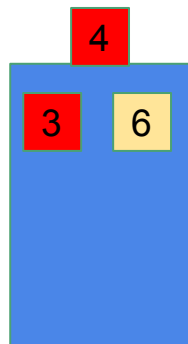
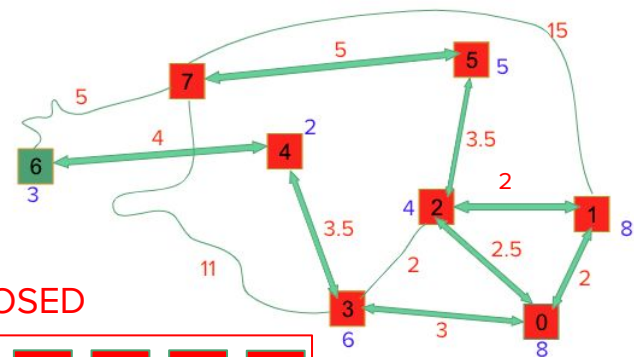
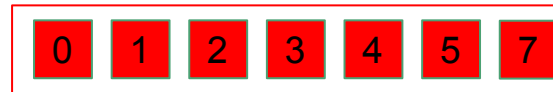
incumbent_cost = ∞

Centralized Parallel A*

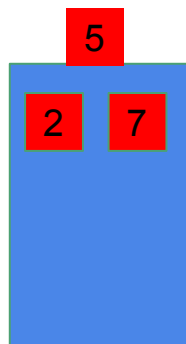
OPEN



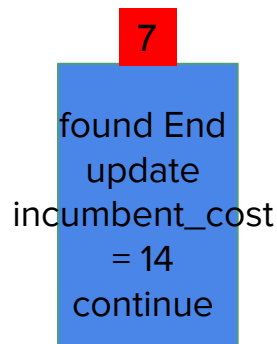
CLOSED



thread 0



thread 1



thread 2



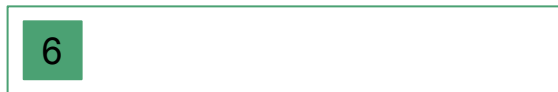
thread 3

Lock2

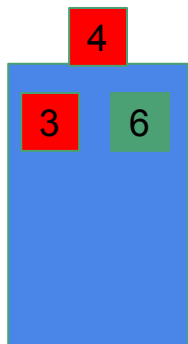
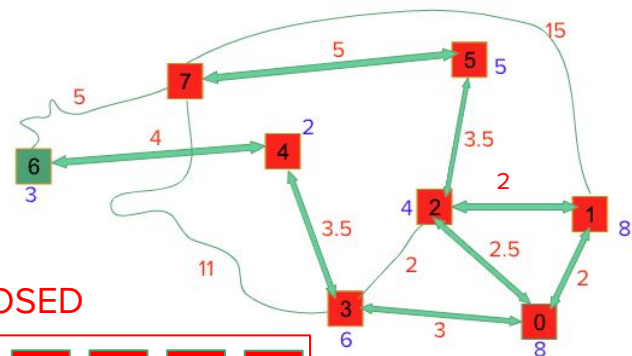
incumbent_cost = 14

Centralized Parallel A*

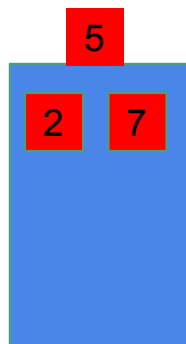
OPEN



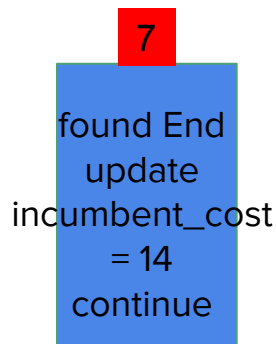
CLOSED



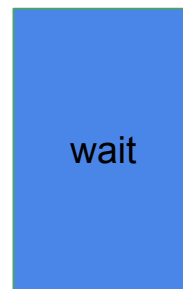
thread 0



thread 1



thread 2

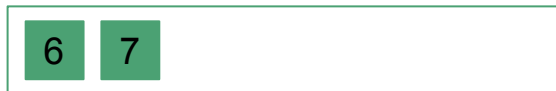


thread 3

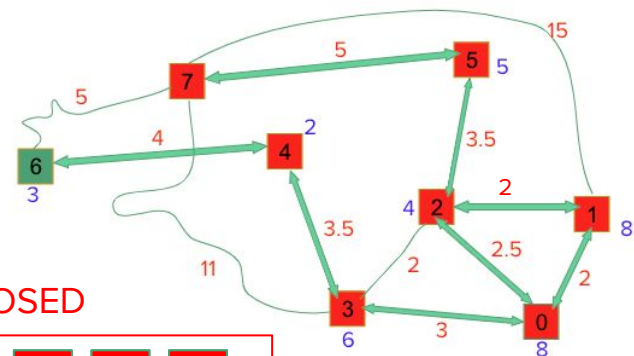
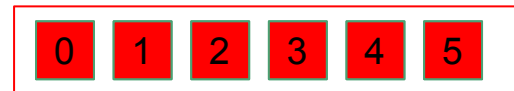
incumbent_cost = 14

Centralized Parallel A*

OPEN



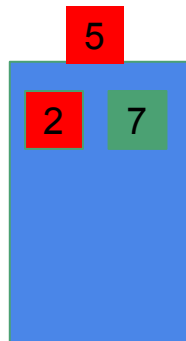
CLOSED



$g(\text{node } 5) + \text{edge}(5,7) < g(\text{node } 7)$
Put node 7 to OPEN



thread 0



thread 1



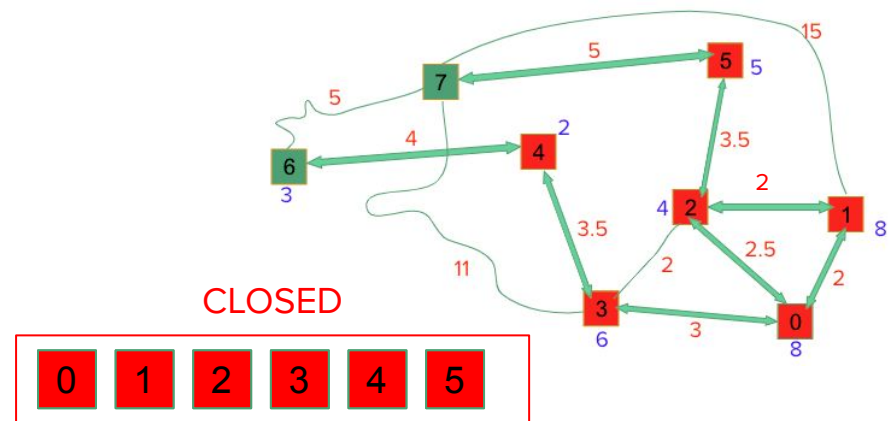
thread 2



thread 3

incumbent_cost = 14

Centralized Parallel A*



finding best

Lock 1



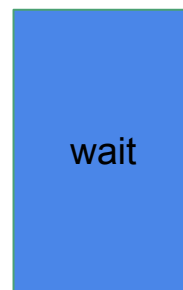
thread 0



thread 1



thread 2



thread 3

incumbent_cost = 14

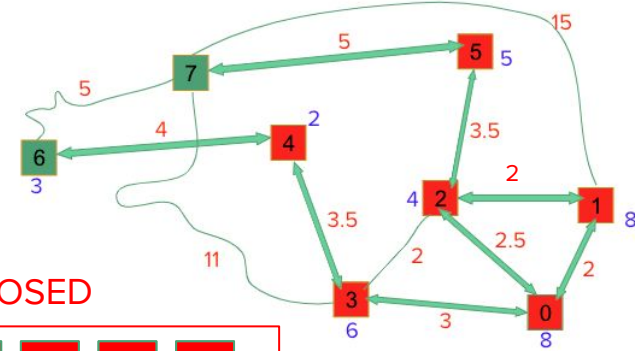
Centralized Parallel A*

OPEN



found

CLOSED



wait

wait

wait

wait

thread 0

thread 1

thread 2

thread 3

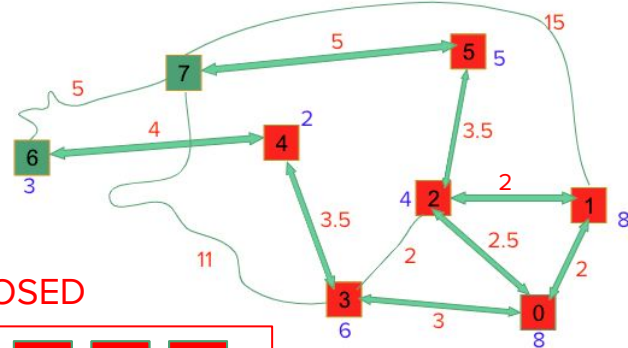
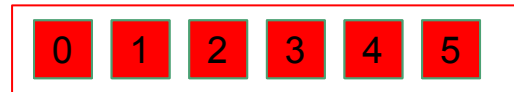
incumbent_cost = 14

Centralized Parallel A*

OPEN



CLOSED



thread 0



thread 1



thread 2

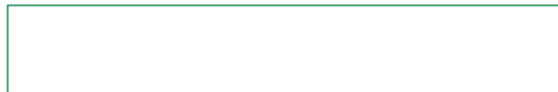


thread 3

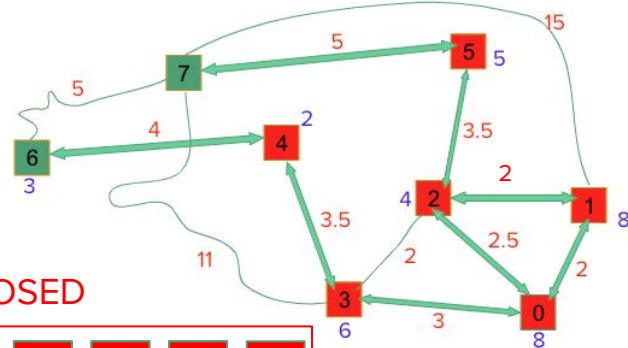
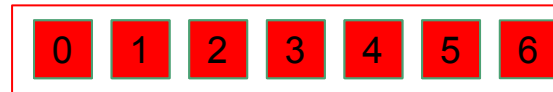
incumbent_cost = 14

Centralized Parallel A*

OPEN



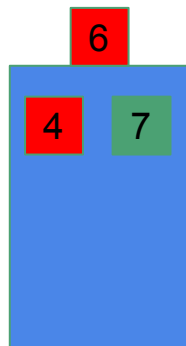
CLOSED



$g(\text{End}) = 11 < \text{incumbent_cost}$
remain OPEN



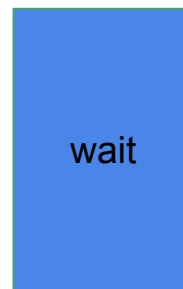
thread 0



thread 1



thread 2

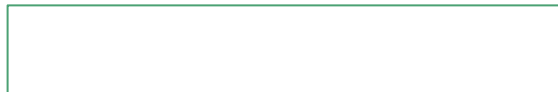


thread 3

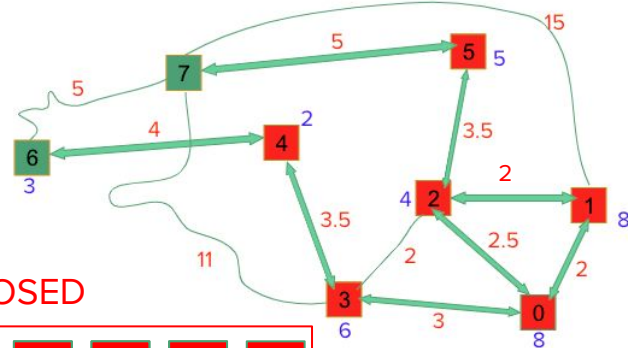
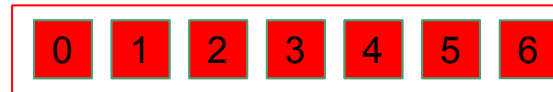
incumbent_cost = 14

Centralized Parallel A*

OPEN



CLOSED



$g(\text{End}) = 11 < \text{incumbent_cost}$
remain OPEN



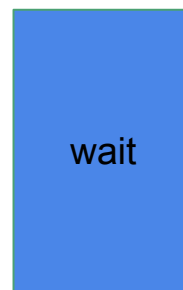
thread 0



thread 1



thread 2



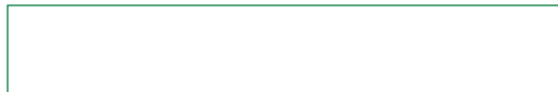
thread 3

Lock2

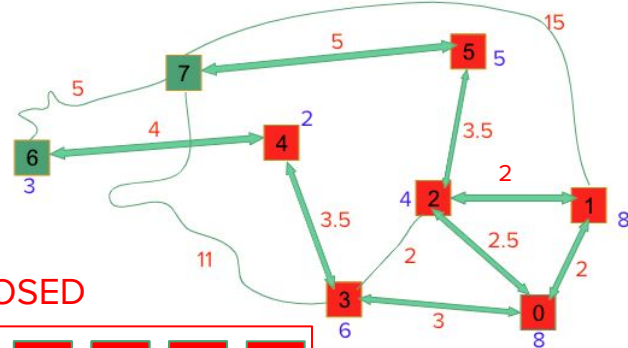
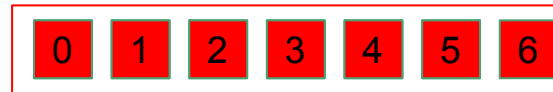
incumbent_cost = 14

Centralized Parallel A*

OPEN



CLOSED



$g(\text{End}) = 11 < \text{incumbent_cost}$
remain OPEN



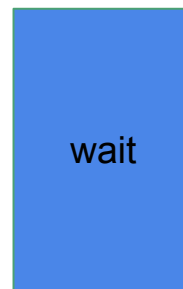
thread 0



thread 1



thread 2



thread 3

Lock2

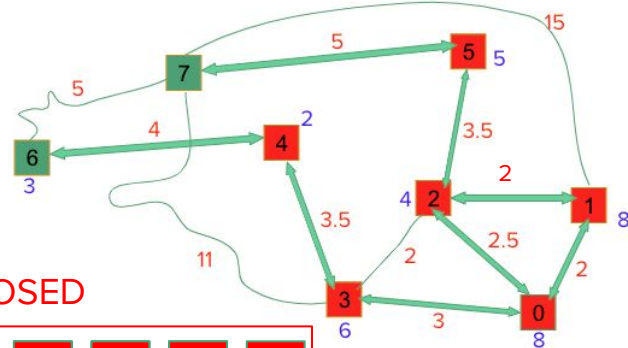
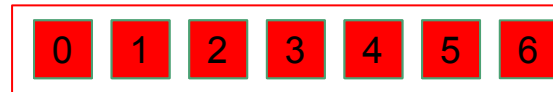
incumbent_cost = 11

Centralized Parallel A*

OPEN



CLOSED



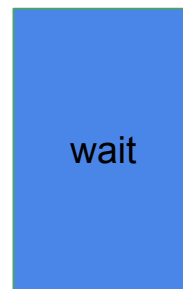
thread 0



thread 1



thread 2



thread 3

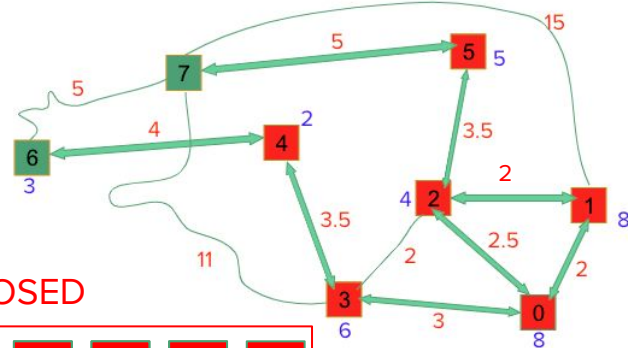
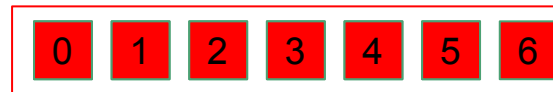
incumbent_cost = 11

Centralized Parallel A*

OPEN



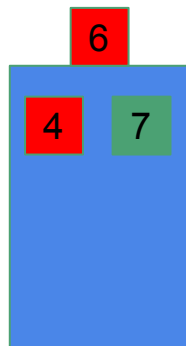
CLOSED



$g(6) + \text{edge}(6,7) > g(7)$
continue



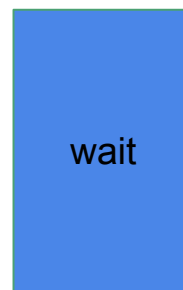
thread 0



thread 1



thread 2



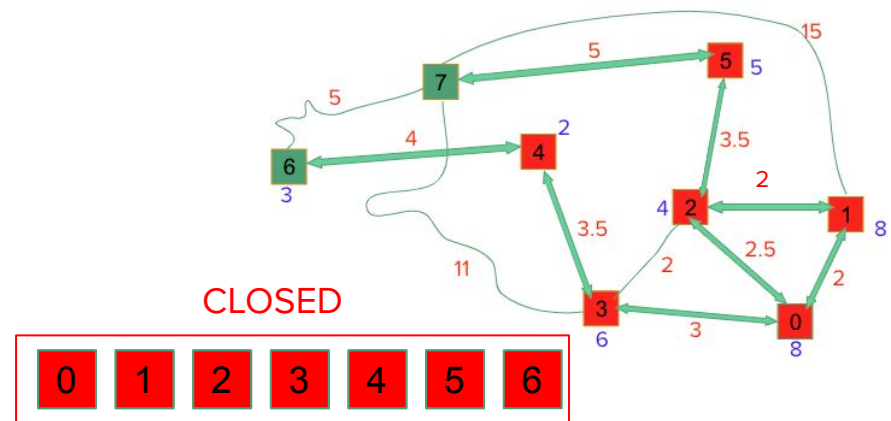
thread 3

incumbent_cost = 11

Centralized Parallel A*



finding best



Lock1



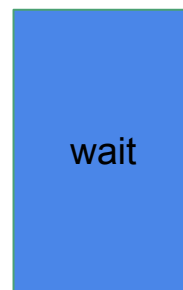
thread 0



thread 1



thread 2



thread 3

incumbent_cost = 11

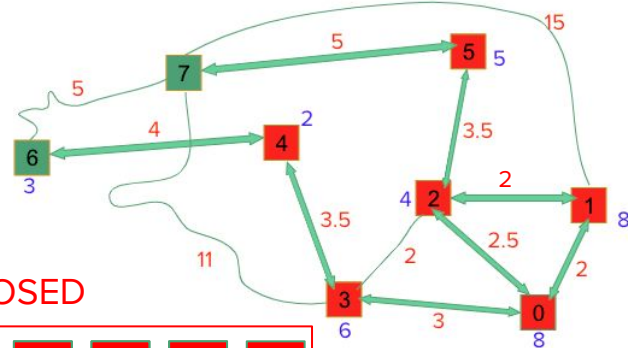
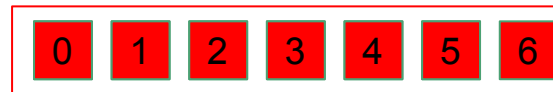
Centralized Parallel A*

OPEN



found

CLOSED



wait

wait

wait

wait

thread 0

thread 1

thread 2

thread 3

incumbent_cost = 11

Centralized Parallel A*

OPEN



found

7



thread 0

wait



thread 1

wait



thread 2

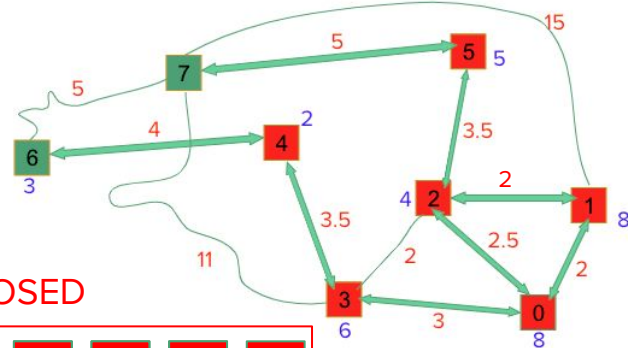
wait



thread 3

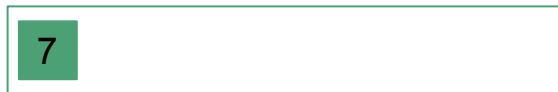
incumbent_cost = 11

CLOSED



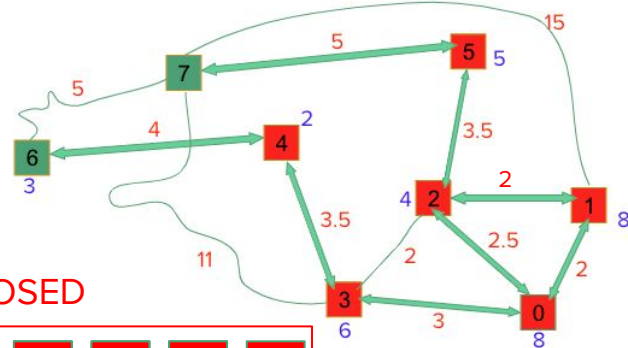
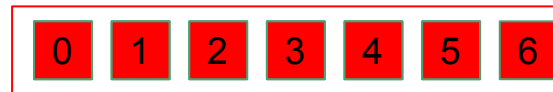
Centralized Parallel A*

OPEN



found

CLOSED



Terminated

wait

wait

wait

thread 0

thread 1

thread 2

thread 3

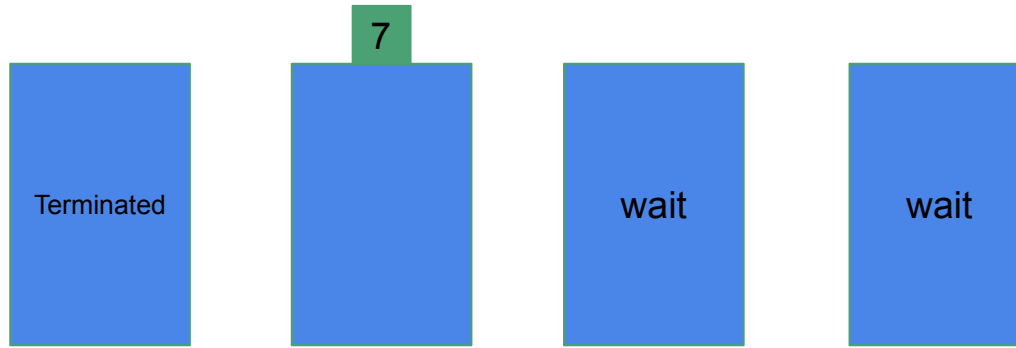
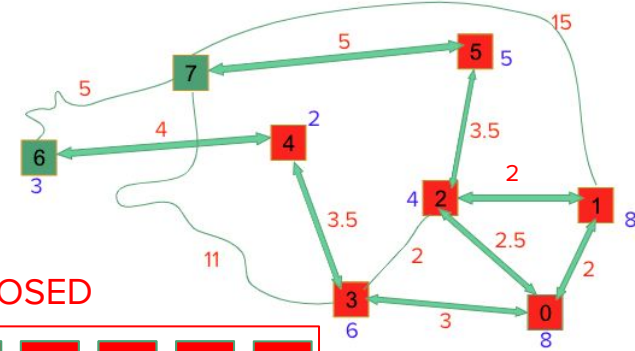
incumbent_cost = 11

Centralized Parallel A*

OPEN



CLOSED



thread 0

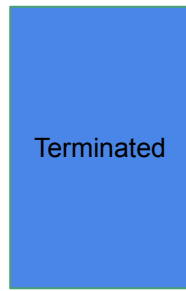
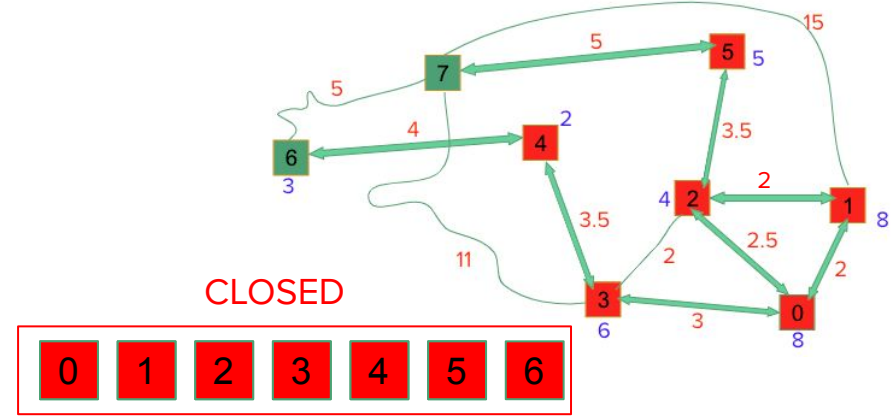
thread 1

thread 2

thread 3

incumbent_cost = 11

Centralized Parallel A*



thread 0



thread 1



thread 2



thread 3

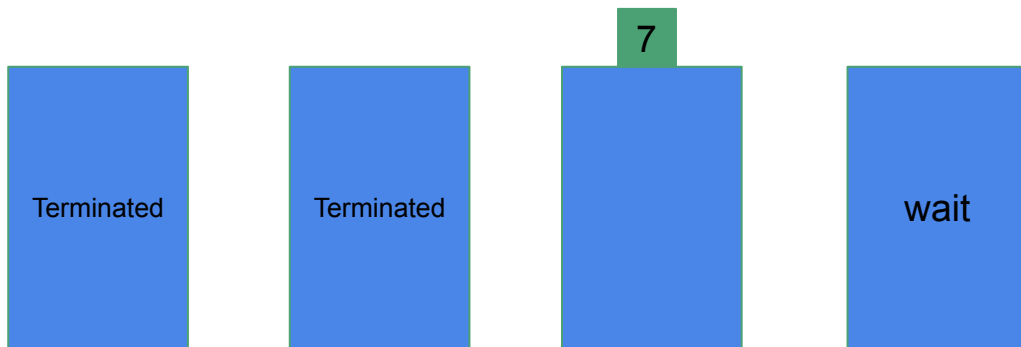
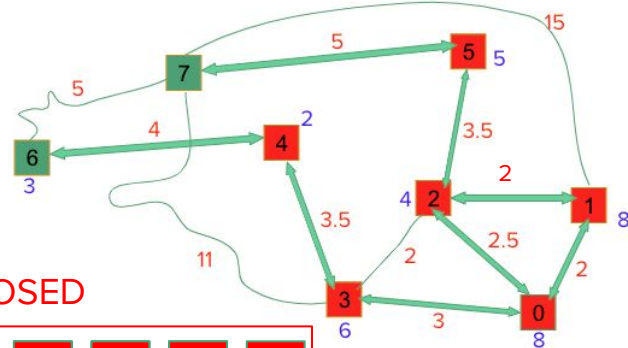
incumbent_cost = 11

Centralized Parallel A*

OPEN



CLOSED



thread 0

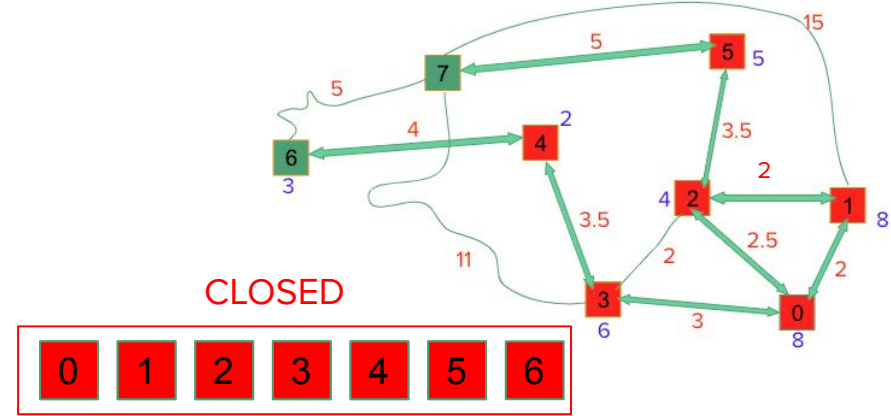
thread 1

thread 2

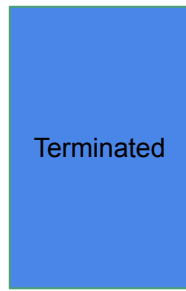
thread 3

incumbent_cost = 11

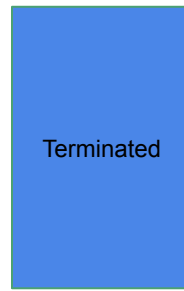
Centralized Parallel A*



thread 0



thread 1



thread 2

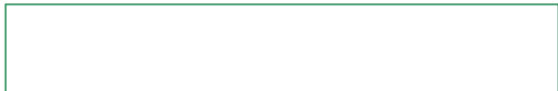


thread 3

incumbent_cost = 11

Centralized Parallel A*

OPEN



CLOSED



7

Terminated

Terminated

Terminated

wait

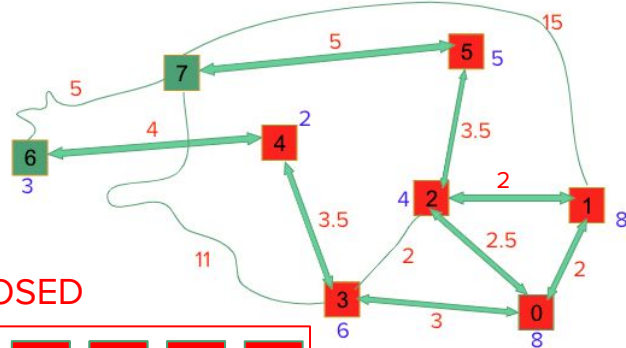
thread 0

thread 1

thread 2

thread 3

incumbent_cost = 11

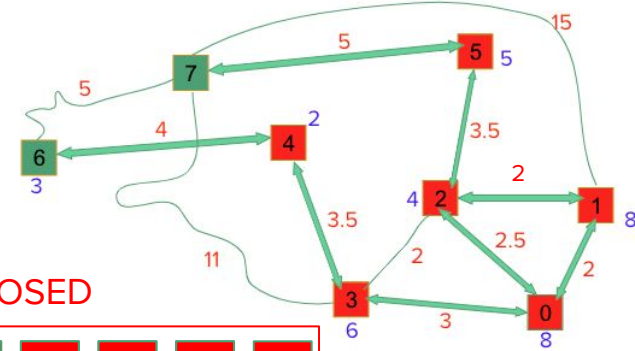


Centralized Parallel A*

OPEN



CLOSED



Terminated

Terminated

Terminated

Terminated

thread 0

thread 1

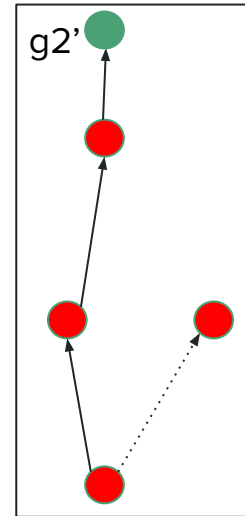
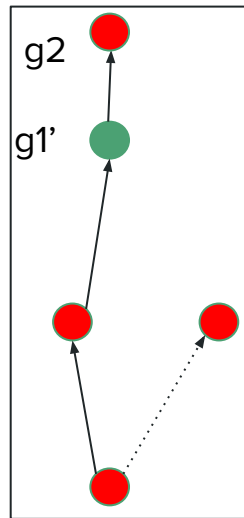
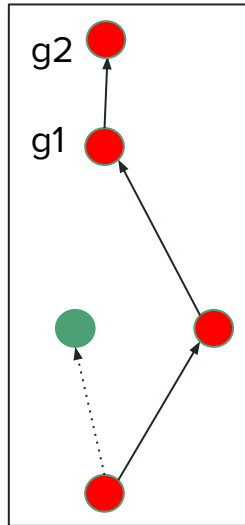
thread 2

thread 3

incumbent_cost = 11

Centralized Parallel A*

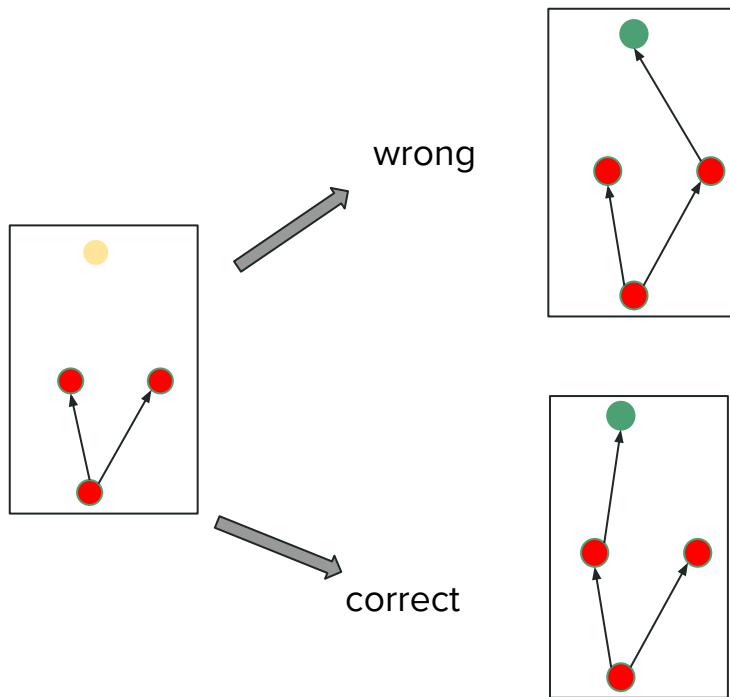
- Case 1



$$\begin{aligned} g_1' &< g_1 \\ g_2' &< g_2 \end{aligned}$$

Centralized Parallel A*

- Case 2



Lock →

Unlock →



Algorithm 2: Simple Parallel A* (SPA*)

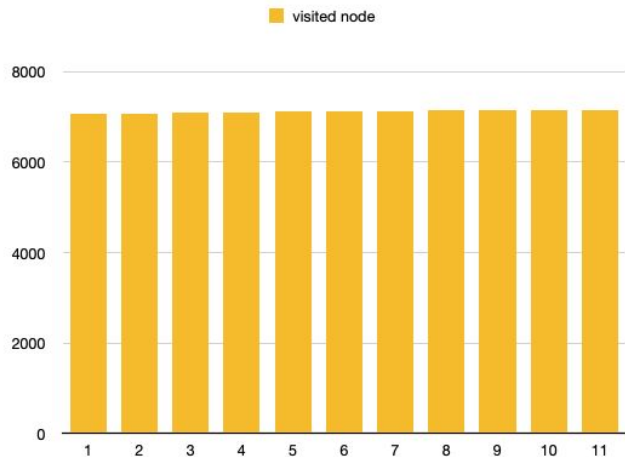
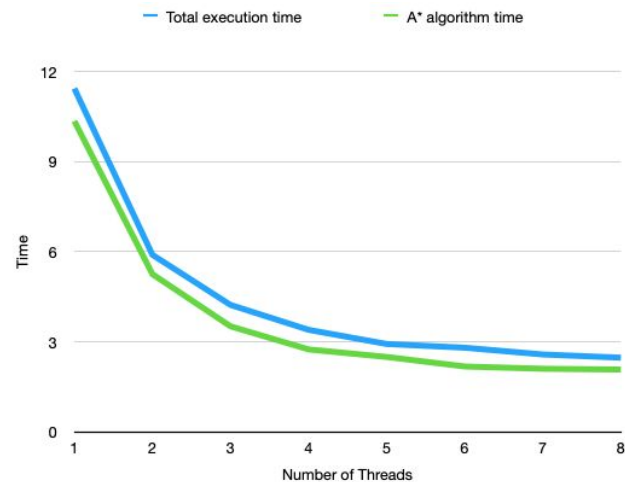
```

1 Initialize  $OPEN_{shared}$  to  $\{s_0\}$ ;
2 Initialize Lock  $l_o, l_i$ ;
3 Initialize  $incumbent.cost = \infty$ ;
4 In parallel, on each thread, execute 5-32;
5 while TerminateDetection() do
6   if  $OPEN_{shared} = \emptyset$  or Smallest  $f(n)$  value of  $n \in OPEN_{shared} \geq incumbent.cost$  then
7     Continue;
8   AcquireLock( $l_o$ );
9   Get and remove from  $OPEN_{shared}$  a node  $n$  with a smallest  $f(n)$ ;
10  ReleaseLock( $l_o$ );
11  Add  $n$  to  $CLOSED_{shared}$ ;
12  if  $n$  is a goal node then
13    AcquireLock( $l_i$ );
14    if path cost from  $s_0$  to  $n < incumbent.cost$  then
15       $incumbent =$  path from  $s_0$  to  $n$ ;
16       $incumbent.cost =$  path cost from  $s_0$  to  $n$ ;
17    ReleaseLock( $l_i$ );
18  for every successor  $n'$  of  $n$  do
19     $g_1 = g(n) + c(n, n')$ ;
20    if  $n' \in CLOSED_{shared}$  then
21      if  $g_1 < g(n')$  then
22        Remove  $n'$  from  $CLOSED_{shared}$  and add it to  $OPEN_{shared}$ ;
23      else
24        Continue;
25    else
26      if  $n' \notin OPEN_{shared}$  then
27        Add  $n'$  to  $OPEN_{shared}$ ;
28      else if  $g_1 \geq g(n')$  then
29        Continue;
30    Set  $g(n') = g_1$ ;
31    Set  $f(n') = g(n') + h(n')$ ;
32    Set parent( $n'$ ) =  $n$ ;
33 if  $incumbent.cost = \infty$  then
34   Return failure (no path exists);
35 else
36   Return solution path from  $s_0$  to  $n$ ;
  
```

Centralized Parallel A*

- Result

Enviroment : Macbook Pro 2019 13吋 CPU : i5- 8257U 4 Core 8 Thread			
Number of thread	Total execution time	A* algorithm time	visited node
1	11.433	10.347	7074
2	5.896	5.252	7080
3	4.228	3.516	7093
4	3.399	2.747	7104
5	2.928	2.496	7118
6	2.804	2.179	7130
7	2.579	2.103	7135
8	2.472	2.077	7137
9	2.601	2.088	7140
10	2.484	1.966	7147
11	2.555	1.987	7151



Centralized Parallel A*

$$S_{\text{latency}}(s) = \frac{1}{(1 - p) + \frac{p}{s}}$$

- Amdahl's law
 - **p** : The proportion of parallelizable region.
 - **1 - p** : The proportion of non-parallelizable region.
 - **s = 8** : Number of threads.
 - $S_{\text{latency}}(s) = \mathbf{4.98}$: speed up

Number of thread	Total execution time	A* algorithm time
1	11.433	10.347

4.98x ↓


- => **p = 0.91**
- Cost a lot when finding neighbor.

8	2.472	2.077
---	-------	-------

Centralized Parallel A*

- When finding neighbors.
 - Search edges table to get neighbors' ID.
 - Search nodes table to get neighbor 1's g, h, open, closed ...
 - Search nodes table to get neighbor 2's g, h, open, closed ...
 - Search nodes table to get neighbor 3's g, h, open, closed ...
 - ...

nodes table



ID	g	h	...
14567			
...			
14576			
...			
14588			
...			
14590			

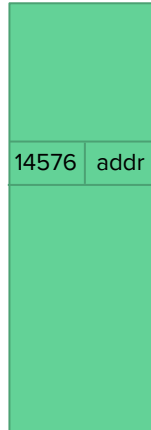
edges table

start	end	
14576	14590	
14576	14567	
14576	14588	

Centralized Parallel A*

- Solution : Use Index

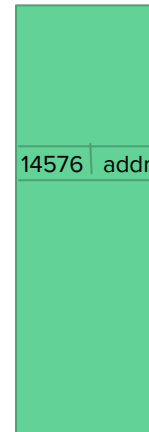
nodeToNodeIdx



nodes table

ID	g	h
14576			

nodeToEdgeIdx



edges table

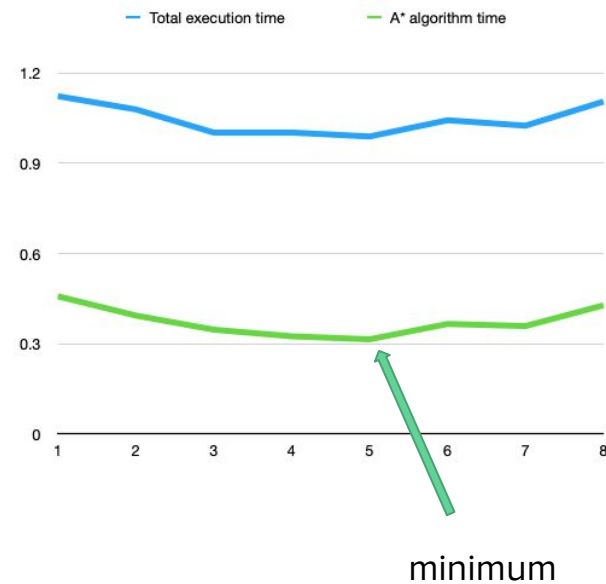
start	end	
14576	14590	
14576	14567	
14576	14588	

Centralized Parallel A*

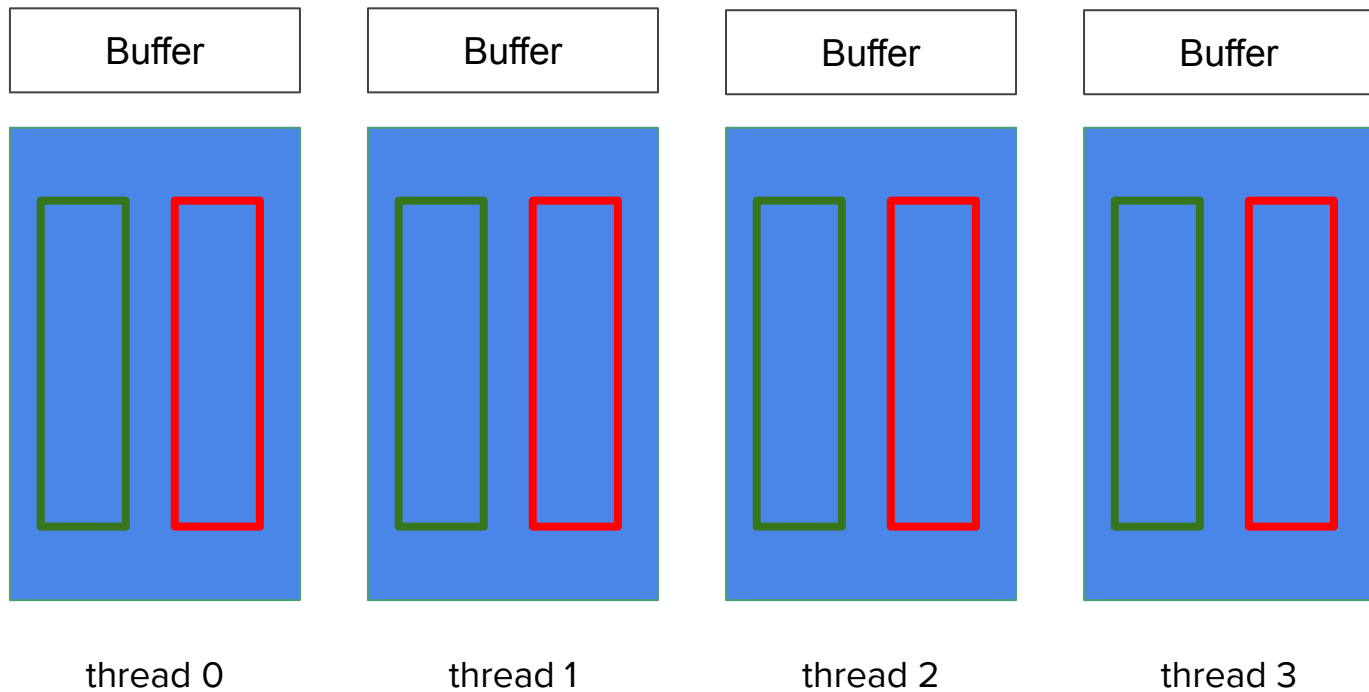
Centralized parallel A* with new structure

Number of thread	Total execution time	A* algorithm time	Visited node
1	1.123	0.458	7074
2	1.079	0.394	7075
3	1.002	0.347	7075
4	1.002	0.325	7074
5	0.989	0.315	7075
6	1.043	0.366	7075
7	1.025	0.359	7076
8	1.105	0.428	7075
9	2.601	2.088	7077
10	2.484	1.966	7077
11	2.555	1.987	7078

Intel i5 8257U is 4 Core 8 Thread



Decentralized Parallel A*



Decentralized Parallel A*

- Each thread has their own OPEN, CLOSED, and BUFFER set
- Faster than CPA
- Constraint: The Graph must not be bidirectional
 - Real world map is bidirectional
 - Oscillating between two nodes

References

- [1] Alex Fukunaga, Adi Botea, Yuu Jinnai, and Akihiro Kishimoto, “A Survey of Parallel A*”
- [2] Ariana Weinstock and Rachel Holladay, “Parallel A* Graph Search”