Multi-Channel Current Measurement System Arduino-Based Oscilloscope Enhancement

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Abstract

This documentation presents the mathematical framework for a multi-channel current measurement system designed to enhance a laptop's measurement capabilities using Arduino. The system employs multiple instrumentation amplifiers (AD620) with different gains connected through a multiplexer (74HC4051) to achieve wide dynamic range current measurements. The analysis focuses on current sensitivity optimization and explores both linear and non-linear scaling approaches.

1 Introduction

1.1 Project Overview

The project aims to transform an Arduino Uno R3 into a precision measurement instrument capable of measuring both voltage and current across multiple ranges. By employing a multi-stage amplification approach with instrumentation amplifiers (AD620) of different gains and a multiplexer for channel selection, the system achieves enhanced dynamic range and resolution compared to standard Arduino ADC capabilities.

1.2 System Architecture

- **Current Sensing**: Circuit current I_c passes through a shunt resistor R_{mes}
- **Amplification Stage**: Multiple AD620 instrumentation amps with gains A_{dn} (where n represents channel number)
- **Signal Conditioning**: Output voltage $V_{on} = A_{dn} \times I_c \times R_{\text{mes}}$
- **Multiplexing**: 74HC4051 multiplexer selects between channels under Arduino control
- **ADC Conversion**: Arduino's 10-bit ADC digitizes the selected channel's output

2 Fundamental Definitions and Relationships

2.1 System Parameters

- I_c: Circuit current being measured (constant across all channels)
- V_{on}: Output voltage of channel n (varies due to different gains A_{dn})
- K_{cn}: Current sensitivity for channel n (amperes per ADC step)
- K_p: Voltage per ADC step (constant: $\frac{5}{1024}$ V/step for 5V reference)
- R_{mes}: Measurement shunt resistance (constant)
- A_{dn}: Gain of instrumentation amplifier for channel n

2.2 ADC Voltage Reference and Step Size

The Arduino's 10-bit ADC with 5V reference provides:

$$K_p = \frac{V_{\text{ref}}}{2^{10}} = \frac{5}{1024} \approx 0.0048828 \text{ V/step}$$

This represents the fundamental voltage resolution of the measurement system.

2.3 ADC Conversion Relationship

The digital output from the ADC relates to the input voltage by:

$$\text{Num} = \frac{V_o}{K_p}$$

where **Num** is the integer value stored in the ADC register, representing the digitized voltage in discrete steps.

3 Current Measurement Theory

3.1 Basic Current-Voltage Relationship

The fundamental relationship between circuit current and measured voltage is:

$$I_c = \frac{V_o}{A_d R_{\text{mes}}}$$

This equation forms the basis for converting measured voltages back to current values, considering the amplifier gain and shunt resistance.

3.2 Current Sensitivity Derivation

To understand the system's current measurement resolution, we analyze the sensitivity:

$$\Delta I_c = \frac{\Delta V_o}{A_d R_{\text{mes}}}$$
$$\Delta V_o = K_p \cdot \Delta \text{Num}$$
$$\Delta I_c = \frac{K_p}{A_d R_{\text{mes}}} \cdot \Delta \text{Num}$$

Thus, the current sensitivity is defined as:

$$K_c = \frac{\Delta I_c}{\Delta \text{Num}} = \frac{K_p}{A_d R_{\text{mes}}} \text{ A/step}$$

For practical applications where currents are typically in microamperes:

$$K_c = \frac{K_p \times 10^6}{A_d R_{\text{mes}}} \ \mu\text{A/step}$$

3.3 Multi-Channel Formulation

For a system with n channels, each with different amplifier gains A_{dn}:

$$I_c = \frac{V_{on}}{R_{\text{mes}} A_{dn}}$$
$$K_{cn} = \frac{K_p}{A_{dn} R_{\text{mes}}}$$
$$A_{dn} = \frac{K_p}{K_{cn} R_{\text{mes}}}$$

These relationships allow us to design each channel's gain based on desired current sensitivity or vice versa.

4 Channel Range Analysis

4.1 Current Range per Channel

The measurable current range for each channel depends on the output voltage swing and amplifier gain:

$$\text{Channel range} = \frac{V_{on_{\max}} K_{cn}}{K_p} - \frac{V_{on_{\min}} K_{cn}}{K_p} = \frac{\Delta V_{on} K_{cn}}{K_p}$$

Substituting the sensitivity expression:

$$K_{cn} = \frac{I_c K_p}{V_{on}} = \frac{K_p}{A_d R_{\text{mes}}}$$

We obtain the fundamental channel range expression:

$$\text{Channel range} = \frac{\Delta V_{on}}{A_{dn} R_{\text{mes}}} = \Delta V_{on} \frac{I_c}{V_{on}}$$

4.2 Channel Span in ADC Steps

The number of ADC steps available for each channel's range is:

$$\text{Channel span (in ADC steps)} = \frac{\Delta V_{on} K_{cn}}{K_p}$$

This determines the digital resolution available for current measurements in each channel.
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5 Precision Analysis with Offset Voltage Constraints

5.1 Accuracy and Precision Definitions

The **precision** at the minimum measurable current is defined as the ratio of resolution to minimum current:

$$P_n = \frac{K_{cn}}{I_{c_{\min_n}}}$$

The **accuracy** at minimum current is:

$$\text{Acc}_n = 1 - P_n = 1 - \frac{K_{cn}}{I_{c_{\min_n}}}$$

5.2 Offset Voltage Constraint

The operational amplifier's input offset voltage V_{os} imposes a fundamental limit on the minimum measurable input voltage:

$$V_{in_{\min_n}} = I_{c_{\min_n}} \cdot R_{\text{mes}} \geq \frac{V_{os}}{P_{in_n}}$$

where P_{in_n} is the **input precision** representing the fractional error due to offset voltage at the minimum current:

$$P_{in_n} = \frac{V_{os}}{I_{c_{\min_n}} \cdot R_{\text{mes}}}$$

5.3 Fundamental Sensitivity Relationships

From the ADC conversion:

$$K_{cn} = \frac{K_p}{A_{dn} R_{\text{mes}}}$$

The output voltage at minimum current is:

$$V_{on_{\min}} = A_{dn} \cdot I_{c_{\min_n}} \cdot R_{\text{mes}}$$

Substitute $A_{dn} R_{\text{mes}}$ from the voltage equation into the sensitivity:

$$A_{dn} R_{\text{mes}} = \frac{V_{on_{\min}}}{I_{c_{\min_n}}}$$

$$K_{cn} = \frac{K_p}{A_{dn} R_{\text{mes}}} = \frac{K_p}{\frac{V_{on_{\min}}}{I_{c_{\min_n}}}} = \frac{K_p \cdot I_{c_{\min_n}}}{V_{on_{\min}}}$$

5.4 Complete System Constraints

The amplifier gain can be expressed in terms of both input and output parameters:

$$A_{dn} = \frac{V_{on_{\min}}}{I_{c_{\min_n}} \cdot R_{\text{mes}}} = \frac{V_{on_{\min}} \cdot P_{in_n}}{V_{os}}$$

The output precision is determined by ADC characteristics:

$$P_n = \frac{K_p}{V_{on_{\min}}}$$

While the input precision is constrained by op-amp limitations:

$$P_{in_n} = \frac{V_{os}}{I_{c_{\min_n}} \cdot R_{\text{mes}}}$$

5.5 Geometric Scaling Implications

For power law scaling with ratio r, the input precision follows:

$$P_{in_n} = P_{in_1} \cdot r^{-(n-1)}$$

and the amplifier gains follow:

$$A_{dn} = A_{d1} \cdot r^{-(n-1)}$$

This reveals that both input precision requirements and amplifier gains decrease geometrically with channel number. The above equations hold if there is nor overlap between the channels.

5.6 Final Results

$$\boxed{\text{Acc}_n = 1 - \frac{K_p}{V_{on_{\min}}}}$$

$$\boxed{P_n = \frac{K_p}{V_{on_{\min}}}}$$

$$\boxed{P_{in_n} = \frac{V_{os}}{I_{c_{\min_n}} \cdot R_{\text{mes}}}}$$

$$\boxed{A_{dn} = \frac{V_{on_{\min}}}{I_{c_{\min_n}} \cdot R_{\text{mes}}}}$$

The output accuracy and precision are **constant across all channels** and depend only on ADC resolution and minimum output voltage. However, the input precision and required amplifier gains vary significantly across channels due to offset voltage constraints. “‘

6 Linear Cumulative Approach

6.1 Concept and Definitions

The linear cumulative approach represents the simplest multi-channel design strategy, where each channel's current range increases linearly with channel number.

- ΔI_{c_b}: Base current range (minimum measurable by first channel)
- $n_{\max}$: Maximum number of channels
- ΔI_c: Total current range across all channels

6.2 Mathematical Formulation

In the linear cumulative model, each channel measures from zero to its maximum current:

$$\Delta I_{c_n} = n \cdot \frac{\Delta I_c}{n_{\max}}$$

The current sensitivity for each channel becomes:

$$K_{c_n} = \frac{K_p \cdot \Delta I_{c_n}}{\Delta V_{o_n}} = \frac{K_p \cdot n \cdot \Delta I_c}{n_{\max} \cdot \Delta V_{o_n}}$$

6.3 Sensitivity Analysis with Standard Constraints

Applying practical design constraints:

- Target $K_{c_1} = 1$ μA/step for high-resolution small-current measurement
- Maximum $\Delta V_{o_n} = 3$ V (typical op-amp output swing)
- $n_{\max} = 5$ channels
- $\Delta I_c = 200$ mA total current range
- $K_p = 0.0048828$ V/step

6.3.1 Derivative Analysis

Applying practical design constraints:

- Target $K_{c_1} = 1$ μA/step for high-resolution small-current measurement
- Maximum $\Delta V_{o_n} = 3$ V (typical op-amp output swing)
- $n_{\max} = 5$ channels
- $\Delta I_c = 200$ mA total current range
- $K_p = 0.0048828$ V/step

Sensitivity to Voltage Swing:

$$\frac{\partial K_{c_n}}{\partial(\Delta V_{o_n})} = -\frac{K_p \cdot n \cdot \Delta I_c}{n_{\max} \cdot (\Delta V_{o_n})^2}$$

For Channel 1:

$$\frac{\partial K_{c_1}}{\partial(\Delta V_{o_1})} \approx -21.70 \ \mu\text{A/step/V}$$

Key Insight: Each 1V increase in output voltage swing improves Channel 1 resolution by approximately 21.7 µA/step. This demonstrates that higher voltage headroom directly enhances measurement precision, though practical op-amp limitations typically restrict ΔV_{o_n} to 3-5V ranges.

Sensitivity to Channel Count:

$$\frac{\partial K_{c_n}}{\partial n_{\max}} = -\frac{K_p \cdot n \cdot \Delta I_c}{n_{\max}^2 \cdot \Delta V_{o_n}}$$

For Channel 1:

$$\frac{\partial K_{c_1}}{\partial n_{\max}} \approx -13.02\ \mu\text{A/step/channel}$$

Key Insight: Increasing the total number of channels by one improves Channel 1 resolution by approximately 13 µA/step. This occurs because adding channels redistributes the total current range, reducing each channel's individual range and thereby improving sensitivity. The resolution of higher-numbered channels (K_{c_n} for $n > 1$) is determined by their relationship to Channel 1: $K_{c_n} = n \cdot K_{c_1}$.

Voltage Requirement vs Channel Count:

$$\frac{\partial(\Delta V_{o_n})}{\partial n_{\max}} = -\frac{K_p \cdot n \cdot \Delta I_c}{n_{\max}^2 \cdot K_{c_n}}$$

For Channel 1 with 1µA/step target:

$$\frac{\partial(\Delta V_{o_1})}{\partial n_{\max}} \approx -39.06\ \text{V/channel}$$

Key Insight: To maintain 1µA/step resolution while adding channels, the required voltage swing decreases by approximately 39V per additional channel. This counterintuitive result occurs because more channels allow smaller individual current ranges, reducing the voltage needed to maintain high resolution. However, this highlights the impracticality of achieving 1µA/step with the linear approach, as it would require impossibly large voltage swings.

6.3.2 Channel Interdependence Principle

Fundamental Relationship: In the linear cumulative approach, all channel sensitivities are mathematically linked:

$$K_{c_n} = n \cdot K_{c_1}$$

This means that improving Channel 1 resolution automatically degrades higher-channel resolution by the same factor. The design cannot independently optimize sensitivity across channels - enhancing small-current measurement capability necessarily compromises large-current resolution.

Practical Implication: System designers must choose between:

- **High small-current resolution**: Many channels with excellent Channel 1 performance but poor higher-channel sensitivity
- **Balanced performance**: Fewer channels with moderate resolution across all ranges

This interdependence motivates the need for non-linear scaling approaches that can break this mathematical coupling.

6.4 Empirical Verification

Case 1: 2-Channel System

- Channel 1: $\Delta I_{c_1} = 0.1$ A, $K_{c_1} = 162.76\ \mu$A/step
- Channel 2: $\Delta I_{c_2} = 0.2$ A, $K_{c_2} = 325.52\ \mu$A/step

Case 2: 5-Channel System

- Channel 1: $\Delta I_{c_1} = 0.04$ A, $K_{c_1} = 65.10\ \mu$A/step
- Channel 2: $\Delta I_{c_2} = 0.08$ A, $K_{c_2} = 130.21\ \mu$A/step
- Channel 3: $\Delta I_{c_3} = 0.12$ A, $K_{c_3} = 195.31\ \mu$A/step

6.5 Design Implications and Limitations

6.5.1 Dual Relationship Discovery

Analysis reveals a fundamental dual relationship:

Effect 1: Within Fixed System

$$\frac{K_{c_n}}{K_{c_1}} = \frac{\Delta I_{c_n}}{\Delta I_{c_1}} = n$$

Higher-numbered channels have proportionally worse resolution, creating a **positive correlation** between channel number and sensitivity.

Effect 2: Across System Configurations

$$K_{c_1} \propto \frac{1}{n_{\max}}$$

Increasing the total number of channels improves Channel 1 resolution, demonstrating a **negative correlation**.

6.5.2 Fundamental Design Compromise

The linear approach creates an unavoidable trade-off:

- **Many channels**: Excellent Channel 1 resolution but poor higher-channel performance
- **Few channels**: Balanced resolution but limited small-current measurement capability
- **Physical constraint**: Achieving 1μA/step sensitivity requires impractically large voltage swings.

6.5.3 Linear Scaling Performance Table

Channel	ΔI_{c_n} (mA)	K_{c_n} (μA/step)	Precision ($\times 10^{-3}$)
1	2.0	4.88	4.883
2	4.0	9.76	4.883
3	6.0	14.65	4.883

Table 1: Linear Scaling with 3 Channels and 2 mA Base Range

7 Power Law Scaling Approach

7.1 Concept and Motivation

The power law scaling approach addresses the fundamental limitations of linear cumulative design by implementing decade-range scaling, which aligns with practical measurement requirements and human perceptual sensitivity to current variations.

7.2 Mathematical Formulation

The channel current ranges follow a geometric progression:

$$\Delta I_{cn} = \Delta I_{c1} \cdot 10^{(n-1)}$$

where $\Delta I_{c1} = 2$ mA represents the base current range.

7.3 Fundamental Sensitivity Relationship

The intrinsic coupling between current range and sensitivity remains:

$$K_{cn} = \frac{K_p \cdot \Delta I_{cn}}{\Delta V_{on}} = C \cdot \Delta I_{cn}$$

where $C = \frac{K_p}{\Delta V_{on}}$ is constant. This demonstrates the unavoidable positive correlation: any increase in ΔI_{cn} directly proportionally increases K_{cn}.

7.4 Design Implementation

7.4.1 Parameter Selection

- Base range: $\Delta I_{c1} = 2$ mA
- Voltage swing: $\Delta V_{on} = 2$ V
- Voltage per step: $K_p = \frac{5}{1024} \approx 0.0048828$ V/step

7.4.2 Channel Sensitivity Calculation

Using the relationship $K_{cn} = \dfrac{\Delta I_{cn} \cdot K_p}{\Delta V_{on}}$:

$$\text{Channel 1: } \Delta I_{c1} = 2 \text{ mA} \Rightarrow K_{c1} = \frac{0.002 \times 0.0048828}{2} = 4.88\ \mu\text{A/step}$$
$$\text{Channel 2: } \Delta I_{c2} = 20 \text{ mA} \Rightarrow K_{c2} = \frac{0.02 \times 0.0048828}{2} = 48.83\ \mu\text{A/step}$$
$$\text{Channel 3: } \Delta I_{c3} = 200 \text{ mA} \Rightarrow K_{c3} = \frac{0.2 \times 0.0048828}{2} = 488.3\ \mu\text{A/step}$$

7.4.3 Power law Scaling Performance Table

With $\Delta V_{on} = 2$V, $K_p = 0.0048828$ V/step, $\Delta I_{c1} = 2$mA:

Channel	$\mathbf{\Delta I_{cn}}$ (mA)	$\mathbf{K_{cn}}$	**Precision** $(\times 10^{-3})$
1	2	4.88	4.883 FS
2	20	48.83	4.883 FS
3	200	488.3	4.883 FS

Table 2: Power Law Scaling Performance

7.5 Key Advantages over Linear Approach

7.5.1 Decade Range Optimization

The power law provides distinct measurement decades:

- **Channel 1**: Microamp-to-milliamp precision measurements
- **Channel 2**: Standard milliamp-range signals
- **Channel 3**: Higher current power measurements
- **Channel 4**: Rough current monitoring and fault detection

Each channel serves a unique measurement domain without functional overlap.

7.5.2 Harmonious Resolution Degradation

The power law achieves optimal resolution distribution:

$$\frac{K_{c(n+1)}}{K_{cn}} = \frac{\Delta I_{c(n+1)}}{\Delta I_{cn}} = 10$$

Sensitivity degrades at exactly the same rate that measurement range increases, matching the precision requirements at each current level.

7.5.3 Practical Implementation Benefits

- **Simple channel selection**: Clear thresholds between measurement ranges
- **Efficient auto-ranging**: Natural progression from highest to lowest resolution
- **No wasted channels**: Each channel optimized for specific current domain

7.6 Fundamental Mathematical Insight

The positive correlation between ΔI_{cn} and K_{cn} cannot be eliminated:

$$K_{cn} \propto \Delta I_{cn}$$

Linear Scaling Limitations	**Power Law Advantages**
Mediocre resolution across all ranges	Optimized resolution per range
Substantial range overlap between channels	Distinct decade-range separation
Arbitrary channel selection criteria	Clear range-based channel selection
Fights against $\Delta I_{cn} \propto K_{cn}$ relationship	Strategically leverages the fundamental correlation

Table 3: Scaling Approach Comparison

8 Gain Progression Analysis

8.1 Ideal Geometric Scaling

For a multi-channel current measurement system with perfect geometric scaling and no overlap between channels, the amplifier gains follow a predictable progression.

8.1.1 Mathematical Foundation

The minimum current for each channel follows geometric progression:

$$I_{c_{\min_n}} = I_{c_{\min_1}} \cdot r^{n-1}$$

The amplifier gain for channel n is given by:

$$A_{dn} = \frac{V_{on_{\min}}}{I_{c_{\min_n}} \cdot R_{\text{mes}}}$$

8.1.2 Proof of Constant Gain Ratio

The ratio of consecutive amplifier gains is:

$$\frac{A_{dn}}{A_{d(n+1)}} = \frac{\frac{V_{on_{\min}}}{I_{c_{\min_n}} \cdot R_{\text{mes}}}}{\frac{V_{on_{\min}}}{I_{c_{\min_{n+1}}} \cdot R_{\text{mes}}}} = \frac{I_{c_{\min_{n+1}}}}{I_{c_{\min_n}}}$$

Substituting the geometric progression:

$$\frac{A_{dn}}{A_{d(n+1)}} = \frac{I_{c_{\min_1}} \cdot r^n}{I_{c_{\min_1}} \cdot r^{n-1}} = r$$

For ideal geometric scaling without overlap:

$$\boxed{\frac{A_{dn}}{A_{d(n+1)}} = r}$$

The ratio between consecutive amplifier gains is constant and equal to the range ratio r.

8.2 Effect of Overlap Factor

In practical systems, channel overlap is necessary to ensure continuous current measurement coverage. The overlap factor k (where $0 < k < 1$) modifies the ideal gain progression.

8.2.1 Overlap Definition

With overlap, the minimum current of channel n is:

$$I_{c_{\min_n}} = k \cdot I_{c_{\max_{n-1}}}$$

where $I_{c_{\max_{n-1}}} = I_{c_{\min_{n-1}}} + \Delta I_{c_{n-1}}$

For geometric scaling of ranges:

$$\Delta I_{c_n} = \Delta I_{c_1} \cdot r^{n-1}$$

8.2.2 Modified Gain Ratio

The gain ratio with overlap becomes:

$$\frac{A_{dn}}{A_{d(n+1)}} = \frac{I_{c_{\min_{n+1}}}}{I_{c_{\min_n}}} = \frac{k \cdot (I_{c_{\min_n}} + \Delta I_{c_1} \cdot r^{n-1})}{I_{c_{\min_n}}}$$

Simplifying:

$$\frac{A_{dn}}{A_{d(n+1)}} = k \cdot \left(1 + \frac{\Delta I_{c_1} \cdot r^{n-1}}{I_{c_{\min_n}}}\right)$$

8.2.3 Asymptotic Behavior

For large n, when $I_{c_{\min_n}} \gg \Delta I_{c_1} \cdot r^{n-1}$, the gain ratio approaches:

$$\lim_{n\to\infty} \frac{A_{dn}}{A_{d(n+1)}} = k$$

In multi-channel systems with overlap, the asymptotic gain ratio between consecutive channels approaches the overlap factor k.

8.3 Practical Implications

8.3.1 Amplifier Selection Constraints

The gain progression imposes practical constraints on amplifier selection:

- **Maximum gain ratio**: The ratio $\frac{A_{d1}}{A_{dN}}$ must be within the amplifier's achievable range
- **Gain steps**: Consecutive gains must be realizable with available amplifier configurations
- **Overall range**: For N channels, the total gain range is $A_{d1}/A_{dN} = r^{N-1}$ in ideal case

8.3.2 Design Trade-offs

Overlap	**Gain Progression**	**Practical Considerations**
High ($k \approx 0.9$)	Smooth, small steps	Easy implementation, similar channels
Medium ($k \approx 0.7$)	Moderate steps	Good balance, distinct ranges
Low ($k \approx 0.5$)	Large steps	Challenging, may exceed practical limits

Table 4: Overlap Factor Impact on Gain Progression

8.3.3 Example Calculation

Consider a system with parameters:

$$r = 10 \quad \text{(decade scaling)}$$
$$k = 0.8 \quad \text{(20\% overlap)}$$
$$A_{d1} = 1000 \quad \text{(Channel 1 gain)}$$

The gain progression would be:

$$A_{d2} = \frac{A_{d1}}{k \cdot (1 + \frac{\Delta I_{c_1}}{I_{c_{\min_1}}})} \approx 125$$

$$A_{d3} = \frac{A_{d2}}{k \cdot (1 + \frac{\Delta I_{c_1} \cdot r}{I_{c_{\min_2}}})} \approx 15.6$$

$$A_{d4} = \frac{A_{d3}}{k \cdot (1 + \frac{\Delta I_{c_1} \cdot r^2}{I_{c_{\min_3}}})} \approx 2.0$$

8.4 Design Guidelines

8.4.1 Gain Realizability Check

For a practical design, verify:

$$A_{dn}^{\min} \leq A_{dn} \leq A_{dn}^{\max} \quad \text{for all } n$$

where $A_{dn}^{\min}$ and $A_{dn}^{\max}$ are the amplifier's minimum and maximum achievable gains.

8.4.2 Optimal Overlap Selection

The optimal overlap factor balances:

- Measurement continuity (favors higher k)
- Gain step realizability (favors moderate k)
- Channel distinctness (favors lower k)

For most practical systems, $k = 0.7$ to 0.9 provides a good compromise.

In well-designed multi-channel systems, the overlap factor k should be chosen such that the resulting gain progression is realizable with available amplifier technology while maintaining adequate measurement continuity.

9 Unified Gain Progression Equation

9.1 Derivation from Fundamental Relationships

Starting from the gain ratio expression derived previously:

$$\frac{A_{dn}}{A_{d(n+1)}} = k \left(1 + \frac{\Delta I_{c_n}}{I_{c_{\min_n}}}\right)$$

Recall that the minimum current is constrained by the operational amplifier's offset voltage:

$$I_{c_{\min_n}} = \frac{V_{os}}{P_{in_n} \cdot R_{\text{mes}}}$$

Substituting this expression into the gain ratio equation:

$$\frac{A_{dn}}{A_{d(n+1)}} = k \left(1 + \frac{\Delta I_{c_n}}{\frac{V_{os}}{P_{in_n} \cdot R_{\text{mes}}}}\right)$$

For geometric scaling, the current range follows:

$$\Delta I_{c_n} = \Delta I_{c_1} \cdot r^{n-1}$$

Substituting and simplifying:

$$\frac{A_{dn}}{A_{d(n+1)}} = k \left(1 + \frac{\Delta I_{c_1} \cdot r^{n-1} \cdot P_{in_n} \cdot R_{\text{mes}}}{V_{os}}\right)$$

[Unified Gain Progression] For a multi-channel current measurement system with geometric scaling and offset voltage constraints, the ratio of consecutive amplifier gains is given by:

$$\boxed{\frac{A_{dn}}{A_{d(n+1)}} = k\left(1 + \frac{\Delta I_{c_1} \cdot r^{n-1} \cdot P_{in_n} \cdot R_{\text{mes}}}{V_{os}}\right)}$$

where:

- k is the overlap factor ($0 < k < 1$)
- ΔI_{c_1} is the base current range
- r is the range ratio
- P_{in_n} is the input precision for channel n
- R_{mes} is the shunt resistance
- V_{os} is the operational amplifier offset voltage

9.2 Physical Interpretation

The unified equation reveals several important physical insights:

9.2.1 Offset Voltage Dominance

When the offset voltage constraint dominates (V_{os} large relative to other terms), the equation simplifies to:

$$\frac{A_{dn}}{A_{d(n+1)}} \approx k$$

In this regime, the gain progression is determined primarily by the overlap factor, and the system is offset-limited.

9.2.2 Range Scaling Dominance

When the current range term dominates ($\Delta I_{c_1} \cdot r^{n-1}$ large), the equation becomes:

$$\frac{A_{dn}}{A_{d(n+1)}} \approx k \cdot \frac{\Delta I_{c_1} \cdot r^{n-1} \cdot P_{in_n} \cdot R_{\text{mes}}}{V_{os}}$$

In this regime, the gain progression scales with the geometric expansion of current ranges.

9.2.3 Input Precision Progression

For systems with geometrically decreasing input precision ($P_{in_n} = P_{in_1} \cdot r^{-(n-1)}$), the r^{n-1} and P_{in_n} terms cancel:

$$\frac{A_{dn}}{A_{d(n+1)}} = k\left(1 + \frac{\Delta I_{c_1} \cdot P_{in_1} \cdot R_{\text{mes}}}{V_{os}}\right)$$

Resulting in a constant gain ratio across all channels.

9.3 Design Implications

9.3.1 Shunt Resistance as Design Parameter

The equation highlights the critical role of R_{mes} as a design degree of freedom:

$$\frac{A_{dn}}{A_{d(n+1)}} \propto R_{\text{mes}}$$

Larger shunt values increase the gain ratio between channels, while smaller values reduce it.

9.3.2 Offset Voltage Impact

The inverse relationship with offset voltage:

$$\frac{A_{dn}}{A_{d(n+1)}} \propto \frac{1}{V_{os}}$$

demonstrates that amplifiers with lower offset voltages enable more aggressive gain progressions.

9.3.3 Channel-Dependent Behavior

The gain ratio varies with channel number n through the r^{n-1} term, indicating that the progression is not constant but evolves throughout the channel chain.

9.4 Special Case: Geometric Input Precision

For systems with geometric current scaling, the input precision necessarily follows geometric progression:

$$P_{in_n} = P_{in_1} \cdot r^{-(n-1)}$$

Substituting into the unified equation:

$$\frac{A_{dn}}{A_{d(n+1)}} = k\left(1 + \frac{\Delta I_{c_1} \cdot r^{n-1} \cdot (P_{in_1} \cdot r^{-(n-1)}) \cdot R_{\text{mes}}}{V_{os}}\right)$$

The r^{n-1} and $r^{-(n-1)}$ terms cancel:

$$\frac{A_{dn}}{A_{d(n+1)}} = k\left(1 + \frac{\Delta I_{c_1} \cdot P_{in_1} \cdot R_{\text{mes}}}{V_{os}}\right)$$

For geometrically scaled multi-channel systems, the gain ratio between consecutive channels is constant and given by:

$$\boxed{\frac{A_{dn}}{A_{d(n+1)}} = k\left(1 + \frac{\Delta I_{c_1} \cdot P_{in_1} \cdot R_{\text{mes}}}{V_{os}}\right)}$$

This constant ratio applies to all channel transitions $n \to n+1$.

9.5 Physical Constraint

The condition $P_{in_n} = P_{in_1} \cdot r^{-(n-1)}$ is not a design choice but a mathematical necessity arising from:

- Geometric current scaling: $I_{c_{\text{min}_n}} = I_{c_{\text{min}_1}} \cdot r^{n-1}$
- Offset voltage constraint: $P_{in_n} = \dfrac{V_{os}}{I_{c_{\text{min}_n}} \cdot R_{\text{mes}}}$

Attempting to maintain constant input precision across channels would require constant minimum currents, defeating the purpose of multi-range measurement.