Statistics

Introduction to R for Public Health Researchers

Statistics

Now we are going to cover how to perform a variety of basic statistical tests in R.

- Correlation
- T-tests/Rank-sum tests
- Linear Regression
- Logistic Regression
- Proportion tests
- Chi-squared
- Fisher's Exact Test

Note: We will be glossing over the statistical theory and “formulas” for these tests. There are plenty of resources online for learning more about these tests, as well as dedicated Biostatistics series at the School of Public Health

Correlation

`cor()` performs correlation in R

```
cor(x, y = NULL, use = "everything",
    method = c("pearson", "kendall", "spearman"))
```

Like other functions, if there are NAs, you get NA as the result. But if you specify use only the complete observations, then it will give you correlation on the non-missing data.

```
library(readr)
circ = read_csv("http://johnmuschelli.com/intro_to_r/data/Charm_City_Circulato
cor(circ$orangeAverage, circ$purpleAverage, use="complete.obs")
```

```
[1] 0.9195356
```

Correlation

You can also get the correlation between matrix columns

```
library(dplyr)
avgs = circ %>% select(ends_with("Average"))
avgs_cor = cor(avgs, use = "complete.obs")
signif(avgs_cor,3)
```

```
              orangeAverage purpleAverage greenAverage bannerAverage
orangeAverage         1.000         0.908        0.840         0.545
purpleAverage         0.908         1.000        0.867         0.521
greenAverage          0.840         0.867        1.000         0.453
bannerAverage         0.545         0.521        0.453         1.000
```

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Correlation

You can also get the correlation between matrix columns

Or between columns of two matrices/dfs, column by column.

```
op = avgs %>% select(orangeAverage, purpleAverage)
gb = avgs %>% select(greenAverage, bannerAverage)
signif(cor(op, gb, use = "complete.obs"), 3)
```

```
              greenAverage bannerAverage
orangeAverage        0.840         0.545
purpleAverage        0.867         0.521
```

Correlation

You can also use `cor.test()` to test for whether correlation is significant (ie non-zero). Note that linear regression may be better, especially if you want to regress out other confounders.

```
ct = cor.test(circ$orangeAverage, circ$purpleAverage,
              use = "complete.obs")
ct
```

```

	Pearson's product-moment correlation

data:  circ$orangeAverage and circ$purpleAverage
t = 73.656, df = 991, p-value < 2.2e-16
alternative hypothesis: true correlation is not equal to 0
95 percent confidence interval:
 0.9093438 0.9286245
sample estimates:
      cor
0.9195356
```

Correlation

For many of these testing result objects, you can extract specific slots/results as numbers, as the `ct` object is just a list.

```
# str(ct)
names(ct)
```

```
[1] "statistic"   "parameter"   "p.value"     "estimate"    "null.value"
[6] "alternative" "method"      "data.name"   "conf.int"
```

```
ct$statistic
```

```
       t 
73.65553 
```

```
ct$p.value
```

```
[1] 0
```

Broom package

The `broom` package has a `tidy` function that puts most objects into `data.frames` so that they are easily manipulated:

```
library(broom)
tidy_ct = tidy(ct)
tidy_ct
```

```
# A tibble: 1 x 8
  estimate statistic p.value parameter conf.low conf.high method
     <dbl>     <dbl>   <dbl>     <int>    <dbl>     <dbl> <chr>
1    0.920      73.7       0       991    0.909     0.929 Pears~
# ... with 1 more variable: alternative <chr>
```


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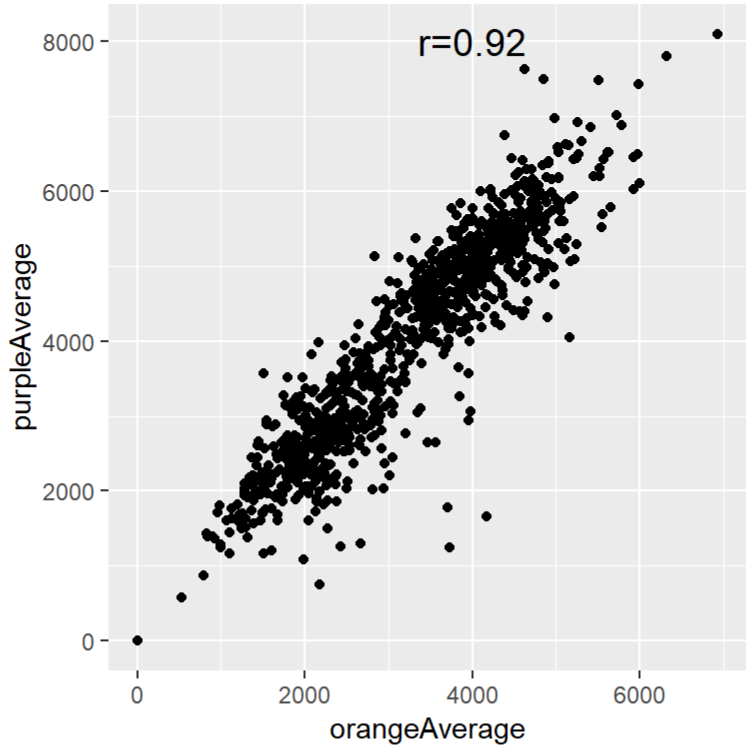
Correlation

Note that you can add the correlation to a plot, via the `annotate`

```
library(ggplot2)
txt = paste0("r=", signif(ct$estimate,3))
q = qplot(data = circ, x = orangeAverage, y = purpleAverage)
q + annotate("text", x = 4000, y = 8000, label = txt, size = 5)
```



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T-tests

The T-test is performed using the `t.test()` function, which essentially tests for the difference in means of a variable between two groups.

In this syntax, x and y are the column of data for each group.

```
tt = t.test(circ$orangeAverage, circ$purpleAverage)
tt
```

```

    Welch Two Sample t-test

data:  circ$orangeAverage and circ$purpleAverage
t = -17.076, df = 1984, p-value < 2.2e-16
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -1096.7602  -870.7867
sample estimates:
mean of x mean of y 
 3033.161  4016.935 
```

T-tests

Using `t.test` treats the data as independent. Realistically, this data should be treated as a paired t-test. The `paired = TRUE` argument to do a paired test

```
t.test(circ$orangeAverage, circ$purpleAverage, paired = TRUE)
```

```
    Paired t-test

data:  circ$orangeAverage and circ$purpleAverage
t = -42.075, df = 992, p-value < 2.2e-16
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -799.783 -728.505
sample estimates:
mean of the differences
               -764.144
```

T-tests

`t.test` saves a lot of information: the difference in means `estimate`, confidence interval for the difference `conf.int`, the p-value `p.value`, etc.

```
names(tt)
```

```
 [1] "statistic"   "parameter"   "p.value"     "conf.int"    "estimate"
 [6] "null.value"  "stderr"      "alternative" "method"      "data.name"
```

T-tests

```
tidy(tt)
```

```
# A tibble: 1 x 10
  estimate estimate1 estimate2 statistic  p.value parameter conf.low
     <dbl>     <dbl>     <dbl>     <dbl>    <dbl>     <dbl>    <dbl>
1    -984.     3033.     4017.     -17.1 4.20e-61     1984.   -1097.
# ... with 3 more variables: conf.high <dbl>, method <chr>,
#   alternative <chr>
```

T-tests

You can also use the ‘formula’ notation. In this syntax, it is `y ~ x`, where `x` is a factor with 2 levels or a binary variable and `y` is a vector of the same length.

```
library(tidyr)
long = circ %>%
  select(date, orangeAverage, purpleAverage) %>%
  gather(key = line, value = avg, -date)
tt = t.test(avg ~ line, data = long)
tidy(tt)
```

```
# A tibble: 1 x 10
  estimate estimate1 estimate2 statistic  p.value parameter conf.low
     <dbl>     <dbl>     <dbl>     <dbl>    <dbl>     <dbl>    <dbl>
1    -984.     3033.     4017.     -17.1 4.20e-61     1984.   -1097.
# ... with 3 more variables: conf.high <dbl>, method <chr>,
#   alternative <chr>
```


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Wilcoxon Rank-Sum Tests

Nonparametric analog to t-test (testing medians):

```
tidy(wilcox.test(avg ~ line, data = long))
```

```
# A tibble: 1 x 4
  statistic  p.value method                                      alternative
      <dbl>    <dbl> <chr>                                       <chr>
1   336714. 4.56e-58 Wilcoxon rank sum test with continuity co~ two.sided
```

Lab Part 1

[Website]

Linear Regression

Now we will briefly cover linear regression. I will use a little notation here so some of the commands are easier to put in the proper context. $y_i = \alpha + \beta x_i + \varepsilon_i$ where:

- y_i is the outcome for person i
- α is the intercept
- β is the slope
- x_i is the predictor for person i
- ε_i is the residual variation for person i

Linear Regression

The `R` version of the regression model is:

```
y ~ x
```

where:

- y is your outcome
- x is/are your predictor(s)

Linear Regression

For a linear regression, when the predictor is binary this is the same as a t-test:

```
fit = lm(avg ~ line, data = long)
fit
```

```
Call:
lm(formula = avg ~ line, data = long)

Coefficients:
      (Intercept)  linepurpleAverage  
           3033.2              983.8  
```

'(Intercept)' is α

'linepurpleAverage' is β

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Linear Regression

The `summary` command gets all the additional information (p-values, t-statistics, r-square) that you usually want from a regression.

```
sfit = summary(fit)
print(sfit)
```

```

Call:
lm(formula = avg ~ line, data = long)

Residuals:
    Min      1Q  Median      3Q     Max 
-4016.9 -1121.2    64.3  1060.8  4072.6 

Coefficients:
                  Estimate Std. Error t value Pr(>|t|)    
(Intercept)        3033.16      38.99   77.79   <2e-16 ***
linepurpleAverage   983.77      57.09   17.23   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1314 on 2127 degrees of freedom
  (163 observations deleted due to missingness)
Multiple R-squared:  0.1225,	Adjusted R-squared:  0.1221 
F-statistic: 296.9 on 1 and 2127 DF,  p-value: < 2.2e-16
```

Linear Regression

The coefficients from a `summary` are the coefficients, standard errors, t-statistcs, and p-values for all the estimates.

```
names(sfit)
```

```
 [1] "call"          "terms"         "residuals"     "coefficients"
 [5] "aliased"       "sigma"         "df"            "r.squared"
 [9] "adj.r.squared" "fstatistic"    "cov.unscaled"  "na.action"
```

```
sfit$coef
```

```
                    Estimate Std. Error  t value     Pr(>|t|)
(Intercept)        3033.1611   38.98983 77.79365 0.000000e+00
linepurpleAverage   983.7735   57.09059 17.23180 2.163655e-62
```

Linear Regression

We can `tidy` linear models as well and it gives us all of this in one::

```
tidy(fit)
```

```
# A tibble: 2 x 5
  term                  estimate std.error statistic  p.value
  <chr>                    <dbl>     <dbl>     <dbl>    <dbl>
1 (Intercept)              3033.      39.0      77.8 0.
2 linepurpleAverage         984.      57.1      17.2 2.16e-62
```

Using Cars Data

```r
http_data_dir = "http://johnmuschelli.com/intro_to_r/data/"
cars = read_csv(
  paste0(http_data_dir, "kaggleCarAuction.csv"),
  col_types = cols(VehBCost = col_double()))
head(cars)
```

```
# A tibble: 6 x 34
  RefId IsBadBuy PurchDate Auction VehYear VehicleAge Make  Model Trim 
  <dbl>    <dbl> <chr>     <chr>     <dbl>      <dbl> <chr> <chr> <chr>
1     1        0 12/7/2009 ADESA      2006          3 MAZDA MAZD~ i    
2     2        0 12/7/2009 ADESA      2004          5 DODGE 1500~ ST   
3     3        0 12/7/2009 ADESA      2005          4 DODGE STRA~ SXT  
4     4        0 12/7/2009 ADESA      2004          5 DODGE NEON  SXT  
5     5        0 12/7/2009 ADESA      2005          4 FORD  FOCUS ZX3  
6     6        0 12/7/2009 ADESA      2004          5 MITS~ GALA~ ES   
# ... with 25 more variables: SubModel <chr>, Color <chr>,
#   Transmission <chr>, WheelTypeID <chr>, WheelType <chr>, VehOdo <dbl>,
#   Nationality <chr>, Size <chr>, TopThreeAmericanName <chr>,
#   MMRAcquisitionAuctionAveragePrice <chr>,
#   MMRAcquisitionAuctionCleanPrice <chr>,
#   MMRAcquisitionRetailAveragePrice <chr>,
#   MMRAcquisitonRetailCleanPrice <chr>,
#   MMRCurrentAuctionAveragePrice <chr>,
#   MMRCurrentAuctionCleanPrice <chr>, MMRCurrentRetailAveragePrice <chr>,
#   MMRCurrentRetailCleanPrice <chr>, PRIMEUNIT <chr>, AUCGUART <chr>,
#   BYRNO <dbl>, VNZIP1 <dbl>, VNST <chr>, VehBCost <dbl>,
#   IsOnlineSale <dbl>, WarrantyCost <dbl>
```

Linear Regression

We'll look at vehicle odometer value by vehicle age:

```
fit = lm(VehOdo~VehicleAge, data = cars)
print(fit)
```

```
Call:
lm(formula = VehOdo ~ VehicleAge, data = cars)

Coefficients:
(Intercept)   VehicleAge
      60127         2723
```

Linear Regression

Note that you can have more than 1 predictor in regression models.The interpretation for each slope is change in the predictor corresponding to a one-unit change in the outcome, holding all other predictors constant.

```
fit2 = lm(VehOdo ~ IsBadBuy + VehicleAge, data = cars)
summary(fit2)
```

```
Call:
lm(formula = VehOdo ~ IsBadBuy + VehicleAge, data = cars)

Residuals:
   Min     1Q Median     3Q    Max 
-70856  -9490   1390  10311  41193 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 60141.77     134.75  446.33   <2e-16 ***
IsBadBuy     1329.00     157.84    8.42   <2e-16 ***
VehicleAge   2680.33      30.27   88.53   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 13810 on 72980 degrees of freedom
Multiple R-squared:  0.1031,	Adjusted R-squared:  0.1031 
F-statistic:  4196 on 2 and 72980 DF,  p-value: < 2.2e-16
```


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Linear Regression: Interactions

The `*` does interactions:

```
fit3 = lm(VehOdo ~ IsBadBuy * VehicleAge, data = cars)
summary(fit3)
```

```

Call:
lm(formula = VehOdo ~ IsBadBuy * VehicleAge, data = cars)

Residuals:
   Min     1Q Median     3Q    Max 
-70857  -9490   1391  10311  41193 

Coefficients:
                    Estimate Std. Error t value Pr(>|t|)    
(Intercept)         60139.70     143.23 419.872  < 2e-16 ***
IsBadBuy             1347.28     456.23   2.953  0.00315 ** 
VehicleAge           2680.84      32.54  82.380  < 2e-16 ***
IsBadBuy:VehicleAge    -3.79      88.74  -0.043  0.96594    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 13810 on 72979 degrees of freedom
Multiple R-squared:  0.1031,	Adjusted R-squared:  0.1031 
F-statistic:  2798 on 3 and 72979 DF,  p-value: < 2.2e-16
```

Linear Regression: Interactions

You can take out main effects with minus

```
fit4 = lm(VehOdo ~ IsBadBuy * VehicleAge -IsBadBuy , data = cars)
summary(fit4)
```

```

Call:
lm(formula = VehOdo ~ IsBadBuy * VehicleAge - IsBadBuy, data = cars)

Residuals:
   Min     1Q Median     3Q    Max 
-70822  -9493   1389  10311  41172 

Coefficients:
                    Estimate Std. Error t value Pr(>|t|)    
(Intercept)         60272.49     136.00 443.184  < 2e-16 ***
VehicleAge           2652.94      31.14  85.186  < 2e-16 ***
IsBadBuy:VehicleAge   242.08      30.70   7.885 3.19e-15 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 13810 on 72980 degrees of freedom
Multiple R-squared:  0.103,	Adjusted R-squared:  0.103 
F-statistic:  4192 on 2 and 72980 DF,  p-value: < 2.2e-16
```

Linear Regression

Factors get special treatment in regression models - lowest level of the factor is the comparison group, and all other factors are relative to its values.

```
fit3 = lm(VehOdo ~ factor(TopThreeAmericanName), data = cars)
summary(fit3)
```

```
Call:
lm(formula = VehOdo ~ factor(TopThreeAmericanName), data = cars)

Residuals:
   Min     1Q Median     3Q    Max 
-71947  -9634   1532  10472  45936 

Coefficients:
                                  Estimate Std. Error t value Pr(>|t|)    
(Intercept)                       68248.48      92.98 733.984  < 2e-16 ***
factor(TopThreeAmericanName)FORD   8523.49     158.35  53.828  < 2e-16 ***
factor(TopThreeAmericanName)GM     4952.18     128.99  38.393  < 2e-16 ***
factor(TopThreeAmericanName)NULL  -2004.68    6361.60  -0.315 0.752670    
factor(TopThreeAmericanName)OTHER   584.87     159.92   3.657 0.000255 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 14220 on 72978 degrees of freedom
Multiple R-squared:  0.04822,	Adjusted R-squared:  0.04817 
F-statistic: 924.3 on 4 and 72978 DF,  p-value: < 2.2e-16
```

Logistic Regression and GLMs

Generalized Linear Models (GLMs) allow for fitting regressions for non-continous/normal outcomes. The `glm` has similar syntax to the `lm` command. Logistic regression is one example.

```
glmfit = glm(IsBadBuy ~ VehOdo + VehicleAge, data=cars, family = binomial())
summary(glmfit)
```

```

Call:
glm(formula = IsBadBuy ~ VehOdo + VehicleAge, family = binomial(),
    data = cars)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-0.9943  -0.5481  -0.4534  -0.3783   2.6318

Coefficients:
              Estimate Std. Error z value Pr(>|z|)
(Intercept) -3.778e+00  6.381e-02 -59.211   <2e-16 ***
VehOdo       8.341e-06  8.526e-07   9.783   <2e-16 ***
VehicleAge   2.681e-01  6.772e-03  39.589   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 54421  on 72982  degrees of freedom
Residual deviance: 52346  on 72980  degrees of freedom
AIC: 52352

Number of Fisher Scoring iterations: 5
```


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Tidying GLMs

```
tidy(glmfit, conf.int = TRUE)
```

```
# A tibble: 3 x 7
  term          estimate  std.error statistic  p.value  conf.low conf.high
  <chr>            <dbl>      <dbl>     <dbl>    <dbl>     <dbl>     <dbl>
1 (Intercep~ -3.78          6.38e-2    -59.2  0.        -3.90e+0  -3.65e+0
2 VehOdo      0.00000834    8.53e-7      9.78 1.33e-22   6.67e-6   1.00e-5
3 VehicleAge  0.268         6.77e-3     39.6  0.         2.55e-1   2.81e-1
```

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Tidying GLMs

```
tidy(glmfit, conf.int = TRUE, exponentiate = TRUE)
```

```
# A tibble: 3 x 7
  term        estimate   std.error statistic  p.value conf.low conf.high
  <chr>          <dbl>       <dbl>     <dbl>    <dbl>    <dbl>     <dbl>
1 (Intercept)   0.0229 0.0638          -59.2  0.          0.0202    0.0259
2 VehOdo        1.00   0.000000853       9.78 1.33e-22    1.00      1.00
3 VehicleAge    1.31   0.00677          39.6  0.          1.29      1.32
```

Logistic Regression

Note the coefficients are on the original scale, we must exponentiate them for odds ratios:

```
exp(coef(glmfit))
```

```
(Intercept)      VehOdo  VehicleAge
 0.02286316  1.00000834  1.30748911
```

Chi-squared tests

`chisq.test()` performs chi-squared contingency table tests and goodness-of-fit tests.

```
chisq.test(x, y = NULL, correct = TRUE,
           p = rep(1/length(x), length(x)), rescale.p = FALSE,
           simulate.p.value = FALSE, B = 2000)
```

```
tab = table(cars$IsBadBuy, cars$IsOnlineSale)
tab
```

```
       0     1
  0 62375  1632
  1  8763   213
```

Chi-squared tests

You can also pass in a table object (such as `tab` here)

```
cq = chisq.test(tab)
cq
```

```

	Pearson's Chi-squared test with Yates' continuity correction

data:  tab
X-squared = 0.92735, df = 1, p-value = 0.3356
```

```
names(cq)
```

```
[1] "statistic" "parameter" "p.value"   "method"    "data.name" "observed"
[7] "expected"  "residuals" "stdres"
```

```
cq$p.value
```

```
[1] 0.3355516
```

Chi-squared tests

Note that does the same test as `prop.test`, for a 2x2 table (`prop.test` not relevant for greater than 2x2).

```
chisq.test(tab)
```

```
	Pearson's Chi-squared test with Yates' continuity correction

data:  tab
X-squared = 0.92735, df = 1, p-value = 0.3356
```

```
prop.test(tab)
```

```
	2-sample test for equality of proportions with continuity
	correction

data:  tab
X-squared = 0.92735, df = 1, p-value = 0.3356
alternative hypothesis: two.sided
95 percent confidence interval:
 -0.005208049  0.001673519
sample estimates:
   prop 1    prop 2 
0.9745028 0.9762701 
```


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Fisher's Exact test

`fisher.test()` performs contingency table test using the hypogeometric distribution (used for small sample sizes).

```
fisher.test(x, y = NULL, workspace = 200000, hybrid = FALSE,
            control = list(), or = 1, alternative = "two.sided",
            conf.int = TRUE, conf.level = 0.95,
            simulate.p.value = FALSE, B = 2000)
```

```
fisher.test(tab)
```

```

    Fisher's Exact Test for Count Data

data:  tab
p-value = 0.3324
alternative hypothesis: true odds ratio is not equal to 1
95 percent confidence interval:
 0.8001727 1.0742114
sample estimates:
odds ratio
 0.9289923
```

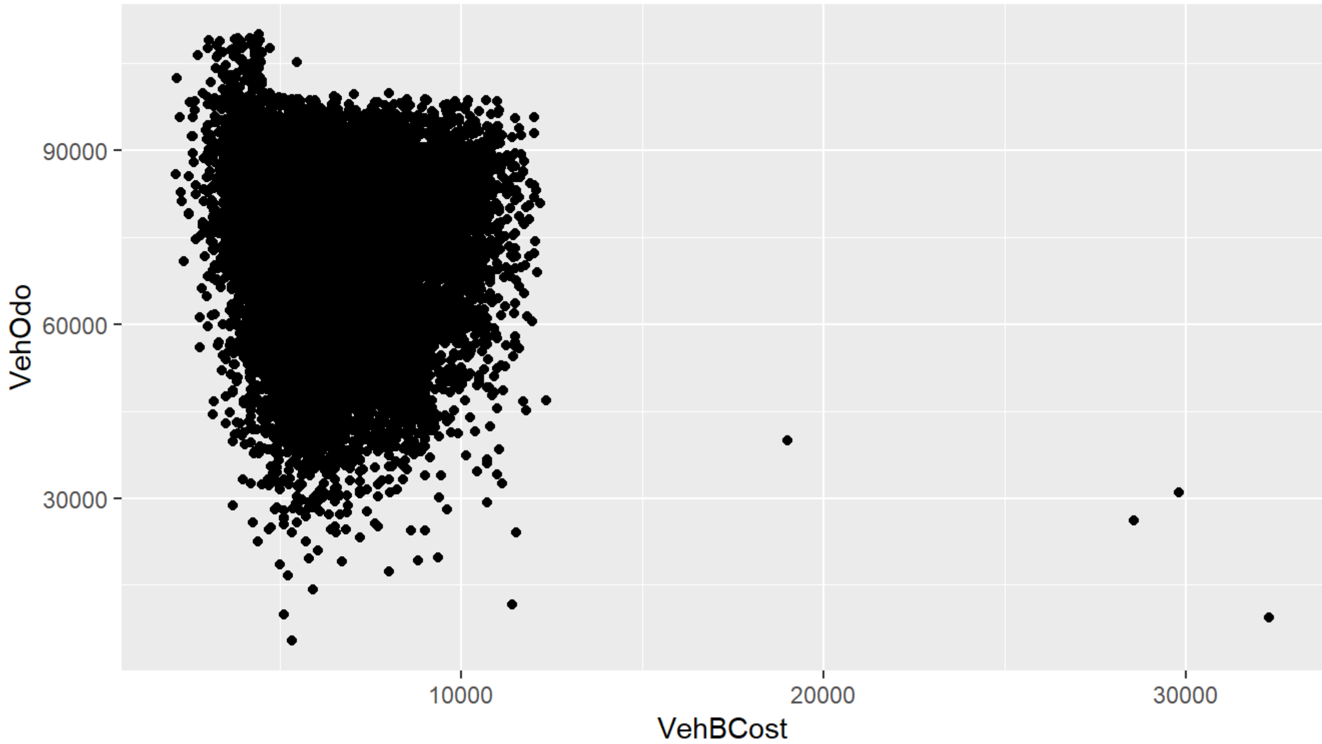
Lab Part 2

Website

Sampling

Also, if you want to only plot a subset of the data (for speed/time or overplotting)

```
samp.cars = sample_n(cars, size = 10000)
samp.cars = sample_frac(cars, size = 0.2)
ggplot(aes(x = VehBCost, y = VehOdo),
       data = samp.cars) + geom_point()
```



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Lab Part 3

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