习题2.6

同样存在 $x_2\in(x_0,+\infty)$, s.t. $f(x_2)=a$

$f(x)$ 在 $[x_1,x_2]$ 应用罗尔定理得，$\exists\xi\in(x_1,x_2)\subset(a,+\infty)$, s.t. $f'(\xi)=0$

15. (1) $y'=(x^4-2x^2-5)'=4x^3-4x$

$=4x(x^2-1)$

$\underset{-1}{-}\ \underset{0}{+}\ \underset{1}{-}\ +$

令 $y'=0$ 得 $x_1=0$, $x_2=-1$, $x_3=1$

当 $x<-1$ 时，$y'<0$，∴ y 单减

当 $-1<x<0$ 时，$y'>0$，∴ y 单增

当 $0<x<1$ 时，$y'<0$，∴ y 单减

当 $x>1$ 时，$y'>0$，∴ y 单增

∴单调增加区间：$(-1,0)\cup(1,+\infty)$

单调减少区间：$(-\infty,-1)\cup(0,1)$

16. (1) 证：令 $f(x)=1+x\ln(x+\sqrt{1+x^2})-\sqrt{1+x^2}$, $f(0)=0$

$$f'(x)=\ln(x+\sqrt{1+x^2})+x\cdot\frac{1+\frac{x}{\sqrt{1+x^2}}}{x+\sqrt{1+x^2}}-\frac{2x}{2\sqrt{1+x^2}}$$

$$=\ln(x+\sqrt{1+x^2})+\frac{x}{\sqrt{1+x^2}}-\frac{x}{\sqrt{1+x^2}}$$

$$=\ln(x+\sqrt{1+x^2})$$

$f'(0)=0$

$$f''(x)=\frac{1+\frac{x}{\sqrt{1+x^2}}}{x+\sqrt{1+x^2}}=\frac{1}{\sqrt{1+x^2}}$$

当 $x>0$ 时，$f''(x)>0$ ∴ $f'(x)$ 严格单增

∴ $f'(x)>f'(0)=0$ ∴ $f(x)$ 严格单增

∴ $f(x)>f(0)=0$ ∴ $1+x\ln(x+\sqrt{1+x^2})>\sqrt{1+x^2}$

习题2.7

1. (12) $\lim\limits_{x\to\frac{\pi}{2}}(\cos x)^{\frac{\pi}{2}-x}$

$$=\lim_{x\to\frac{\pi}{2}}e^{\ln(\cos x)^{\frac{\pi}{2}-x}}$$

$$=e^{\lim\limits_{x\to\frac{\pi}{2}}(\frac{\pi}{2}-x)\ln(\cos x)}$$

其中 $\lim\limits_{x\to\frac{\pi}{2}}(\frac{\pi}{2}-x)\ln\cos x$

$$=\lim_{x\to\frac{\pi}{2}}\frac{\ln\cos x}{\frac{1}{\frac{\pi}{2}-x}}=\lim_{x\to\frac{\pi}{2}}\frac{\frac{-\sin x}{\cos x}}{\frac{1}{(\frac{\pi}{2}-x)^2}}$$

$$=\lim_{x\to\frac{\pi}{2}}\frac{-(\frac{\pi}{2}-x)^2}{\cos x}=\lim_{x\to\frac{\pi}{2}}\frac{+2(\frac{\pi}{2}-x)}{-\sin x}=0$$

∴原式 $=e^0=1$

4. $0\neq a=\lim\limits_{x\to0}\dfrac{\frac{2}{3}(\cos x-\cos2x)}{x^k}$

$$=\lim_{x\to0}\frac{\frac{2}{3}(-\sin x+2\sin2x)}{kx^{k-1}}$$

$$=\lim_{x\to0}\frac{\frac{2}{3}(-\cos x+4\cos2x)}{k(k-1)x^{k-2}}$$

∴当 $k=2$ 时，上式 $=1$

∴当 $x\to0$ 时，$\frac{2}{3}(\cos x-\cos2x)$ 是 x 的2阶无穷小

5. $4=\lim\limits_{x\to+\infty}\left(\dfrac{x+c}{x-c}\right)^x=\lim\limits_{x\to+\infty}e^{x\ln(\frac{x+c}{x-c})}$

其中 $\lim\limits_{x\to+\infty}x\ln\dfrac{x+c}{x-c}=\lim\limits_{x\to+\infty}\dfrac{\ln\frac{x+c}{x-c}}{\frac{1}{x}}$

$$=\lim_{x\to+\infty}\frac{\frac{x-c}{x+c}\cdot\frac{x-c-(x+c)}{(x-c)^2}}{-\frac{1}{x^2}}=\lim_{x\to+\infty}\frac{+2cx^2}{x^2-c^2}$$

$=2c$

∴ $4=e^{2c}$ ∴ $c=\ln2$

习题2.7

6. $0=\lim\limits_{x\to0}\dfrac{\sin3x+ax+bx^3}{x^3}=\lim\limits_{x\to0}\dfrac{3\cos3x+a+3bx^2}{3x^2}$

∴ $\lim\limits_{x\to0}(3\cos3x+a+3bx^2)=0$

∴ $a=-3$

$0=\lim\limits_{x\to0}\dfrac{3\cos3x-3+3bx^2}{3x^2}=\lim\limits_{x\to0}\dfrac{-9\sin3x+6bx}{6x}$

$=\lim\limits_{x\to0}\dfrac{-27\cos3x+6b}{6}=\dfrac{-27+6b}{6}$

∴ $b=\dfrac{9}{2}$

7. 证: $\lim\limits_{h\to0}\dfrac{f(a+2h)-2f(a+h)+f(a)}{h^2}$

$=\lim\limits_{h\to0}\dfrac{2f'(a+2h)-2f'(a+h)}{2h}$

$=\lim\limits_{h\to0}\dfrac{f'(a+2h)-f'(a)+f'(a)-f'(a+h)}{h}$

$=\lim\limits_{h\to0}\left[\dfrac{f'(a+2h)-f'(a)}{2h}\cdot2+\dfrac{f'(a)-f'(a+h)}{-h}\cdot(-1)\right]$

$=2f''(a)-f''(a)=f''(a)$

8. (1) 若$f(x)$在$x=0$处连续，则$\lim\limits_{x\to0}f(x)=f(0)$

即 $\lim\limits_{x\to0}\dfrac{g(x)-\cos x}{x}=a$

∴ $a=\lim\limits_{x\to0}\dfrac{g(x)-\cos x}{x}=\lim\limits_{x\to0}\dfrac{g'(x)+\sin x}{1}=g'(0)$

(2) 当$x\neq0$时，$f'(x)=\left(\dfrac{g(x)-\cos x}{x}\right)'=\dfrac{(g'(x)+\sin x)x-(g(x)-\cos x)}{x^2}$

$f'(0)=\lim\limits_{x\to0}\dfrac{f(x)-f(0)}{x-0}=\lim\limits_{x\to0}\dfrac{\frac{g(x)-\cos x}{x}-a}{x}$

$=\lim\limits_{x\to0}\dfrac{g(x)-\cos x-g'(0)x}{x^2}$

$=\lim\limits_{x\to0}\dfrac{g'(x)+\sin x-g'(0)}{2x}$

$=\lim\limits_{x\to0}\dfrac{g''(x)+\cos x}{2}=\dfrac{g''(0)+1}{2}$

∴ $f'(x)=\begin{cases}\dfrac{(g'(x)+\sin x)x-g(x)+\cos x}{x^2}, & x\neq0\\[2mm] \dfrac{g''(0)+1}{2}, & x=0\end{cases}$

(3) $\lim\limits_{x\to0}f'(x)=\lim\limits_{x\to0}\dfrac{(g'(x)+\sin x)x-g(x)+\cos x}{x^2}$

$=\lim\limits_{x\to0}\dfrac{(g''(x)+\cos x)x+g'(x)+\sin x-g'(x)-\sin x}{2x}$

$=\lim\limits_{x\to0}\dfrac{g''(x)+\cos x}{2}$

$=\dfrac{g''(0)+1}{2}=f'(0)$

∴ $f'(x)$在$x=0$处连续.

9. ∵ $\lim\limits_{x\to0}\dfrac{f(x)}{x}=0$

∴ $\lim\limits_{x\to0}f(x)=0$ 又连续 ∴ $f(0)=0$

∴ $\lim\limits_{x\to0}\dfrac{f(x)}{x^2}=\lim\limits_{x\to0}\dfrac{f'(x)}{2x}$

而 $f''(0)=\lim\limits_{x\to0}\dfrac{f'(x)-f'(0)}{x-0}=\lim\limits_{x\to0}\dfrac{f'(x)}{x}=4$

其中 $f'(0)=\lim\limits_{x\to0}\dfrac{f(x)-f(0)}{x-0}=\lim\limits_{x\to0}\dfrac{f(x)}{x}=0$

故 $\lim\limits_{x\to0}\dfrac{f(x)}{x^2}=\lim\limits_{x\to0}\dfrac{f'(x)}{x}\cdot\dfrac{1}{2}=2$.

$\lim\limits_{x\to0}\left(1+\dfrac{f(x)}{x}\right)^{\frac{1}{x}}=\lim\limits_{x\to0}\left[\left(1+\dfrac{f(x)}{x}\right)^{\frac{x}{f(x)}}\right]^{\frac{f(x)}{x^2}}$

$=e^2$

习题2.8

P136 1.(1) $f(x)=x^5-x^2+2x-1 \quad x=1,\ f(1)=1$

$f'(x)=5x^4-2x+2 \qquad f'(1)=5$

$f''(x)=20x^3-2 \qquad f''(1)=18$

$f'''(x)=60x^2 \qquad f'''(1)=60$

$f^{(4)}(x)=120x,\ f^{(4)}(1)=120;\quad f^{(5)}(x)=120,\ f^{(5)}(1)=120$

$\therefore f(x)=1+5(x-1)+9(x-1)^2+10(x-1)^3+5(x-1)^4+(x-1)^5$

2(1) $f(x)=\frac{1}{1-x}$

$\left(\frac{1}{1-x}\right)^{(n)}=\frac{n!\ \cancel{(n+1)!}}{(1-x)^{n+1}} \qquad f^{(n)}(0)=n!\ \cancel{(n+1)!} \quad f(0)=1$

$\frac{1}{1-x}=1+x+\cancel{x}^3+\cdots+x^n+\frac{x^{n+1}}{(1-\theta x)^{n+2}},\ 0<\theta<1$

4(2) $\lim\limits_{x\to\infty}\left[x-x^2\ln\left(1+\frac{1}{x}\right)\right]$

$=\lim\limits_{x\to\infty}\left\{x-x^2\left[\frac{1}{x}+\frac{1}{2}\frac{1}{x^2}+o\left(\frac{1}{x^2}\right)\right]\right\}$

$=\lim\limits_{x\to\infty}\left(-\frac{1}{2}-\frac{o\left(\frac{1}{x^2}\right)}{\frac{1}{x^2}}\right)=-\frac{1}{2}$

(3) $\lim\limits_{x\to 0}\frac{\frac{x^2}{2}+1-\sqrt{1+x^2}}{x^2\sin x^2}$

$=\lim\limits_{x\to 0}\frac{\frac{x^2}{2}+1-\left[1+\frac{1}{2}x^2-\frac{1}{8}x^4+o(x^4)\right]}{x^4}$

$=\lim\limits_{x\to 0}\frac{\frac{1}{8}x^4+o(x^4)}{x^4}=\frac{1}{8}$

(4) $\lim\limits_{x\to 0}\frac{\sqrt{1+2\tan x}-e^x+x^2}{\arcsin x-\sin x}$

$\cancel{=\lim\limits_{x\to 0}\frac{1+\frac{1}{2}2\tan x+\frac{\frac{1}{2}\cdot\left(-\frac{1}{2}\right)}{2}4\tan^2x+o(x^2)-e^x+x^2}{\arcsin x-\sin x}}$

$=\lim\limits_{x\to 0}\frac{1+x-\frac{1}{2}x^2+\frac{5}{6}x^3+o(x^3)-\left(1+x+\frac{1}{2}x^2+\frac{1}{6}x^3+o(x^3)\right)+x^2}{x+\frac{1}{6}x^3+o(x^3)-\left(x-\frac{1}{6}x^3+o(x^3)\right)}$

$=\lim\limits_{x\to 0}\frac{\frac{2}{3}x^3+o(x^3)}{\frac{1}{3}x^3+o(x^3)}=2$

5. $0=\lim\limits_{x\to 0}\frac{x-(a+b\cos x)\sin x}{x^5}$

$=\lim\limits_{x\to 0}\frac{x-\left[a+b\left(1-\frac{1}{2}x^2+\frac{1}{24}x^4+o(x^4)\right)\right]\sin x}{x^5}$

$=\lim\limits_{x\to 0}\frac{x-\left[a+b\left(1-\frac{1}{2}x^2+\frac{1}{24}x^4+o(x^5)\right)\right]\left[x-\frac{1}{6}x^3+\frac{1}{120}x^5+o(x^6)\right]}{x^5}$

$=\lim\limits_{x\to 0}\frac{x-ax-bx+\frac{1}{6}(4b+a)x^3-\frac{a+16b}{120}x^5+o(x^5)}{x^5}$

$\therefore\begin{cases}1-a-b=0\\ a+4b=0\end{cases} \qquad \therefore\begin{cases}a=\frac{4}{3}\\ b=-\frac{1}{3}\end{cases}$

6. $\because \lim\limits_{x\to x_0}\frac{f(x)}{(x-x_0)^2}=\mu\neq 0.\quad\therefore \lim\limits_{x\to x_0}\frac{f(x)}{x-x_0}=0$

$\therefore \lim\limits_{x\to x_0}f(x)=0$ 　f有三阶导数 $\therefore f(x_0)=0$

且 $f'(x_0)=\lim\limits_{x\to x_0}\frac{f(x)-f(x_0)}{x-x_0}=\lim\limits_{x\to x_0}\frac{f(x)}{x-x_0}=0$

$\mu=\lim\limits_{x\to x_0}\frac{f(x)}{(x-x_0)^2}=\lim\limits_{x\to x_0}\frac{f'(x)}{2(x-x_0)}=\lim\limits_{x\to x_0}\frac{f''(x)}{2}$

$=\frac{1}{2}f''(x_0) \qquad \therefore f''(x_0)=2\mu.$

$\therefore f(x)=\mu(x-x_0)^2+o\left[(x-x_0)^2\right]$

$\lim\limits_{x\to x_0}\frac{f(x)}{(x-x_0)^2}=\lim\limits_{x\to x_0}\frac{f'(x)}{2(x-x_0)}=\lim\limits_{x\to x_0}\frac{f''(x)}{2}=\frac{f''(x_0)}{2}$

7. $\because f$ 二阶可导 且 $\lim\limits_{x\to 0}\frac{f(x)}{x^2}=0$

$\therefore \lim\limits_{x\to 0}f(x)=0 \qquad \therefore f(0)=0$

$\cancel{f'(0)=\lim\limits_{x\to 0}\frac{f(x)-f(0)}{x-0}=\lim\limits_{x\to 0}\frac{f(x)}{x}}=$

$0=\lim\limits_{x\to 0}\frac{f(x)}{x^2}=\lim\limits_{x\to 0}\frac{f'(x)}{2x}\Rightarrow \lim\limits_{x\to 0}f'(x)=0$

$\therefore f'(0)=0$

$$f''(0)=\lim_{x\to 0}\frac{f'(x)-f'(0)}{x-0}=\lim_{x\to 0}\frac{f'(x)}{x}$$

$$=\lim_{x\to 0}\frac{f(x)}{\frac{1}{2}x^2}=2\lim_{x\to 0}\frac{f(x)}{x^2}=0$$

$f(1)=0=f(0)$ $f(x)$在$[0,1]$上应用Rolle定理，则$\exists\xi_1\in(0,1)$，s.t. $f'(\xi_1)=0$

$f'(x)$在$[0,\xi_1]$上应用Rolle定理，则$\exists\xi_2\in(0,\xi_1)\subset(0,1)$ s.t. $f''(\xi_2)=0$

而$f''(0)=0$ $\therefore f''(x)$在$[0,\xi_2]$上应用Rolle定理，则$\exists\xi\in(0,\xi_2)\subset(0,1)$ s.t. $f''(\xi)=0$

法2：由法1方法可得 $f(0)=0$, $f'(0)=0$, $f''(0)=0$

$\therefore f(x)=\frac{f'''(\xi)}{3!}x^3$ ξ介于0与x之间

而$f(1)=0$ $\therefore \frac{f'''(\xi)}{3!}=0$ $\therefore f'''(\xi)=0$

且ξ介于0与1之间.

8. 证：$x_0=\frac{1}{n}[x_1+x_2+\cdots+x_n]$

$$f(x)=f(x_0)+f'(x_0)(x-x_0)+\frac{f''(\xi)}{2!}(x-x_0)^2$$

$$\geq f(x_0)+f'(x_0)(x-x_0)\quad(\because f''(x)\geq 0)$$

$\therefore f(x_i)\geq f(x_0)+f'(x_0)(x_i-x_0)$ $i=1,2\cdots n$

$\therefore f(x_1)+\cdots+f(x_n)\geq nf(x_0)+f'(x_0)[x_1+\cdots+x_n-nx_0]$

即 $f(x_1)+\cdots+f(x_n)\geq nf(x_0)$

$\therefore f\left(\frac{x_1+\cdots+x_n}{n}\right)\leq\frac{f(x_1)+\cdots+f(x_n)}{n}$

习题2.9

P144. 1(1) $y=2x^3-6x^2-18x+7$

$y'=6x^2-12x-18=6(x^2-2x-3)=6(x-3)(x+1)$

得驻点 $x=-1$，$x=3$

x	$(-\infty,-1)$	-1	$(1,3)$	3	$(3,+\infty)$
y'	+		−		+
y	↗		↘		↗

由极值第一充分判别法得 $f(-1)=17$ 极大值

$f(3)=-47$ 极小值.

3. $f(x)=a\sin x+\frac{1}{3}\sin 3x$ 可导 且在$x=\frac{\pi}{3}$处取得极值，由费马定理得 $f'(\frac{\pi}{3})=0$

即 $a\cos\frac{\pi}{3}+\cos\pi=0$ $\therefore a=2$

$f''(x)=-2\sin x-3\sin 3x$

$f''(\frac{\pi}{3})=-\sqrt{3}<0$ $\therefore f(\frac{\pi}{3})=\sqrt{3}$是极大值

4. 当$x<0$时，$f'(x)=1$

当$x>0$时 $f'(x)=\ln x+1$

当$x=0$时 $\lim_{x\to 0^-}\frac{x-0}{x-0}=1$ $f'_-(0)=1$

$\lim_{x\to 0^+}\frac{x\ln x-0}{x-0}=-\infty$

$\therefore f(x)$在$x=0$处不可导.

$$\therefore f'(x)=\begin{cases}1 & x<0\\ \ln x+1 & x>0\end{cases}$$

$x>0$，令$f'(x)=0$ 得驻点 $x=\frac{1}{e}$

$f''(x)=\frac{1}{x}$ $f''(\frac{1}{e})=e>0$

$\therefore f(\frac{1}{e})=-\frac{1}{e}$ 是极小值.

习题2.9

5(1) $y = x + 2\sqrt{x}$ $[0,4]$

$y' = 1 + \frac{1}{\sqrt{x}}$

令$y'=0$，无驻点。不可导点 $x=0$

$y|_{x=0} = 0$ $y|_{x=4} = 8$

$\therefore y_{max} = 8$ $y_{min} = 0$

6 证：令 $f(x) = x^p + (1-x)^p \in C[0,1]$，$x \in [0,1]$，$p>1$

$f'(x) = p x^{p-1} + p(1-x)^{p-1} \cdot (-1)$

令 $f'(x)=0$ 得 $x = \frac{1}{2}$ 无不可导点。

$f(\frac{1}{2}) = \frac{1}{2^{p-1}}$ $f(0)=1$，$f(1)=1$

$\therefore f_{max} = 1$. $f_{min} = \frac{1}{2^{p-1}}$

$\therefore \frac{1}{2^{p-1}} \le x^p + (1-x)^p \le 1$

7. 证：令 $f(x) = e^x(1-x)$，$x<1$

$f'(x) = -e^x x$ 令$f'(x)=0$ 得驻点 $x=0$，唯一驻点。

$\lim\limits_{x\to-\infty} e^x(1-x) = \lim\limits_{x\to-\infty} \frac{1-x}{e^{-x}} = \lim\limits_{x\to-\infty} \frac{-1}{-e^{-x}} = 0$

当 $x<0$ 时，$f'(x)>0$ $\therefore f(x)$单增

当 $0<x<1$ 时，$f'(x)<0$ $\therefore f(x)$单减

$\therefore f(0)=1$ 是极大值，也是最大值

$\therefore e^x(1-x) \le 1$ $\therefore e^x \le \frac{1}{1-x}$ $(x<1)$

8. 解：令 $y = x^{\frac{1}{x}}$ $x>0$.

$y' = (e^{\frac{1}{x}\ln x})' = x^{\frac{1}{x}}(\frac{1-\ln x}{x^2})$

令$y'=0$ 得驻点 $x=e$，唯一驻点。

当 $x>e$ 时，$y'<0$；当 $0<x<e$ 时 $y'>0$

$y(e) = \sqrt{e}$ 是极大值，也是最大值。而 $2<e<3$

且 $\sqrt{2} = \sqrt[6]{8} < \sqrt[3]{3} = \sqrt[6]{9}$

$\therefore$ 数列最大项为 $\sqrt[3]{3}$.

13. $9x^2 + 4y^2 = 72$ 对x两边求导得

$18x + 8y \cdot y' = 0$ $\therefore y' = -\frac{9x}{4y}$

设(x_0, y_0)为切点，故 $9x_0^2 + 4y_0^2 = 72$.

则直线方程：$y - y_0 = -\frac{9x_0}{4y_0}(x - x_0)$

与x轴.y轴截距分别 $X = \frac{8}{x_0}$，$Y = \frac{18}{y_0}$.

设面积 $S(x,y) = \frac{72}{xy}$，其中 $9x^2+4y^2=72$.

$S(x) = \frac{72}{x \cdot \sqrt{\frac{72-9x^2}{4}}}$

设 $A(x)$

截 ~~...~~ $= x^2(72-9x^2)$

$A'(x) = 144x - 36x^3$ 令$A'(x)=0$ 得 $x=0$(舍)

$x=-2$(舍)，$x = 2$ 唯一驻点

$A''(x) = 144 - 108x^2$ $A''(2) < 0$

$\therefore A(2)$ 是极大值，也是最大值

此时 $S(x)|_{x=2}$ 是最小值.

$\therefore$ 当 $x=2$，$y=3$，切线与坐标轴所围面积最小.

14. 解：设 $f(x) = \ln x - ax$ $a>0$

定义域 $(0, +\infty)$

$\lim\limits_{x\to0^+} f(x) = \lim\limits_{x\to0^+}(\ln x - ax) = -\infty$

$\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty} x[\frac{\ln x}{x} - a] = -\infty$

$f'(x) = \frac{1}{x} - a$ 令$f'(x)=0$ 唯一驻点 $x = \frac{1}{a}$

$f''(x) = -\frac{1}{x^2}$ $f''(\frac{1}{a}) < 0$ $\therefore f(\frac{1}{a}) = \ln\frac{1}{a} - 1$

是极大值，也是最大值.

习题2.9

当 $x<\frac{1}{a}$ 时，$f'(x)>0$． 当 $x>\frac{1}{a}$ 时，$f'(x)<0$

当 $f(\frac{1}{a})=\ln\frac{1}{a}-1=0$ 时，$a=\frac{1}{e}$．

∴(1)当 $0<a<\frac{1}{e}$ 时，此时 $f(\frac{1}{a})>0$，方程有两个根．

(2)当 $a=\frac{1}{e}$ 时，$f(\frac{1}{a})=0$，方程有1个根 $x=e$．

(3)当 $a>\frac{1}{e}$ 时，$f(\frac{1}{a})<0$，方程无实根．

习题2.8 $\lim\limits_{x\to 0}\dfrac{\sqrt{1+2\tan x}-e^x+x^2}{\arcsin x-\sin x}$

$$\sqrt{1+2\tan x}=1+\frac{1}{2}(2\tan x)+\frac{\frac{1}{2}(\frac{1}{2}-1)}{2!}\cdot(2\tan x)^2+\frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{3!}(2\tan x)^3+o(x^3)$$

$$=1+\tan x-\frac{1}{2}\tan^2 x+\frac{1}{2}\tan^3 x+o(x^3)$$

$$\tan x=x+\frac{2}{3!}x^3+o(x^3)=x+\frac{1}{3}x^3+o(x^3)$$

$$\therefore \sqrt{1+2\tan x}=1+(x+\frac{1}{3}x^3)+(-\frac{1}{2})(x+\frac{1}{3}x^3)^2$$
$$\frac{1}{2}(x+\frac{1}{3}x^3)^3+o(x^3)$$
$$=1+x-\frac{1}{2}x^2+\frac{5}{6}x^3+o(x^3)$$

$$\arcsin x=x+\frac{1}{6}x^3+o(x^3)$$

$$原式=\lim_{x\to 0}\frac{1+x-\frac{1}{2}x^2+\frac{5}{6}x^3-(1+x+\frac{1}{2}x^2+\frac{1}{6}x^3)+x^2+o(x^3)}{x+\frac{1}{6}x^3-(x-\frac{1}{6}x^3)+o(x^3)}$$

$$=\lim_{x\to 0}\frac{\frac{2}{3}x^3+o(x^3)}{\frac{1}{3}x^3+o(x^3)}=2.$$