NO.

Date

同理可得 $f'_y(0,0)=0$

$\Delta z - f'_x(0,0)\Delta x - f'_y(0,0)\Delta y = |\Delta x \cdot \Delta y|$

$$0 \le \frac{|\Delta x \cdot \Delta y|}{\rho} = \frac{|\Delta x \cdot \Delta y|}{\sqrt{(\Delta x)^2+(\Delta y)^2}} \le \frac{\frac{1}{2}[(\Delta x)^2+(\Delta y)^2]}{\sqrt{(\Delta x)^2+(\Delta y)^2}} = \frac{1}{2}\sqrt{(\Delta x)^2+(\Delta y)^2} \to 0 \quad (\rho \to 0)$$

$$\therefore \lim_{\rho\to 0} \frac{\Delta z - f'_x(0,0)\Delta x - f'_y(0,0)\Delta y}{\rho} = 0$$

$\therefore f(x,y)=|xy|$ 在 $(0,0)$ 可微.

$$f'_x(x,y) = \lim_{\Delta x\to 0} \frac{f(x+\Delta x, y)-f(x,y)}{\Delta x} = \lim_{\Delta x\to 0} \frac{|(x+\Delta x)y|-|xy|}{\Delta x}$$

当 $x=0$, $y\neq 0$ 时, $f'_x(0,y) = \lim\limits_{\Delta x\to 0} \frac{|\Delta x|}{\Delta x}\cdot|y|$ 不存在.

$\therefore f'_x(x,y)$ 在 $(0,0)$ 不连续

同样 当 $x\neq 0$, $y=0$ 时 $f'_y(x,0)$ 也不存在, 从而 $f'_y(x,y)$ 在 $(0,0)$ 不连续.

总结: 偏导数连续 $\longrightarrow$ 函数可微 $\longrightarrow$ 函数可微 / 偏导数存在.

(4) 全微分应用

a. 近似计算

$\Delta z \approx dz$ $\qquad |\Delta x|$, $|\Delta y|$ 充分小

或 $f(x_0+\Delta x, y_0+\Delta y) \approx f(x_0,y_0) + f'_x(x_0,y_0)\Delta x + f'_y(x_0,y_0)\cdot\Delta y$

b. 误差估计.

$z=f(x,y)$, x, y 绝对误差分别为 δ_x, δ_y 即 $|\Delta x|\le\delta_x$, $|\Delta y|\le\delta_y$

则 z 的误差 $|\Delta z| \approx |dz| = \left|\frac{\partial z}{\partial x}\Delta x + \frac{\partial z}{\partial y}\Delta y\right|$

$\le \left|\frac{\partial z}{\partial x}\right|\delta_x + \left|\frac{\partial z}{\partial y}\right|\delta_y$

z 的绝对误差: $\delta_z = \left|\frac{\partial z}{\partial x}\right|\delta_x + \left|\frac{\partial z}{\partial y}\right|\delta_y$ 相对误差: $\frac{\delta_z}{|z|}$