$\square$

下面证[1] $(1) \Leftrightarrow (4)$，即 $A \subseteq B \Leftrightarrow A - B \subseteq \sim A$。

证明:

$$
\begin{aligned}
A - B \subseteq \sim A &\iff \forall x(x \in A - B \to x \in \sim A) && \text{(子集关系定义)}\\
&\iff \forall x((x \in A \land x \notin B) \to x \in \sim A) && \text{(相对补定义)}\\
&\iff \forall x(\neg(x \in A \land x \notin B) \lor x \in \sim A) && \text{(蕴涵等值式)}\\
&\iff \forall x((\neg x \in A \lor \neg x \notin B) \lor x \in \sim A) && \text{(命题逻辑德·摩根律)}\\
&\iff \forall x((\neg x \in A \lor \neg x \notin B) \lor x \notin A) && \text{(绝对补定义)}\\
&\iff \forall x((\neg x \in A \lor \neg\neg x \in B) \lor \neg x \in A) && \text{($\notin$ 定义)}\\
&\iff \forall x((\neg x \in A \lor x \in B) \lor \neg x \in A) && \text{(命题逻辑双重否定律)}\\
&\iff \forall x(\neg x \in A \lor \neg x \in A \lor x \in B) && \text{(命题逻辑结合律、交换律)}\\
&\iff \forall x(\neg x \in A \lor x \in B) && \text{(命题逻辑幂等律)}\\
&\iff \forall x(x \in A \to x \in B) && \text{(蕴涵等值式)}\\
&\iff A \subseteq B && \text{(子集关系定义)}
\end{aligned}
$$

$\square$

最后证：$(1) \Leftrightarrow (5)$，即 $A \subseteq B \Leftrightarrow A - B \subseteq B$。

证明:

$$
\begin{aligned}
A - B \subseteq B &\iff \forall x(x \in A - B \to x \in B) && \text{(子集关系定义)}\\
&\iff \forall x((x \in A \land x \notin B) \to x \in B) && \text{(相对补定义)}\\
&\iff \forall x(\neg(x \in A \land x \notin B) \lor x \in B) && \text{(蕴涵等值式)}\\
&\iff \forall x((\neg x \in A \lor \neg x \notin B) \lor x \in B) && \text{(命题逻辑德·摩根律)}\\
&\iff \forall x((\neg x \in A \lor \neg\neg x \in B) \lor x \in B) && \text{($\notin$ 定义)}\\
&\iff \forall x((\neg x \in A \lor x \in B) \lor x \in B) && \text{(命题逻辑双重否定律)}\\
&\iff \forall x(\neg x \in A \lor x \in B) && \text{(命题逻辑结合律、幂等律)}\\
&\iff \forall x(x \in A \to x \in B) && \text{(蕴涵等值式)}\\
&\iff A \subseteq B && \text{(子集关系定义)}
\end{aligned}
$$

$\square$

1.29

证明: $\forall x$，

$$
\begin{aligned}
& x \in (\cap\mathscr{A}) \cap (\cap\mathscr{B})\\
\iff & x \in (\cap\mathscr{A}) \land x \in (\cap\mathscr{B}) && \text{(集合交定义)}\\
\iff & \forall z(z \in \mathscr{A} \to x \in z) \land \forall z(z \in \mathscr{B} \to x \in z) && \text{(广义交定义)}\\
\iff & \forall z((z \in \mathscr{A} \to x \in z) \land (z \in \mathscr{B} \to x \in z)) && \text{(量词分配等值式)}\\
\implies & \forall z(z \in \mathscr{A} \to x \in z) && \text{(命题逻辑化简律)}\\
\implies & \forall z((z \in \mathscr{A} \to x \in z) \lor (z \in \mathscr{B} \to x \in z)) && \text{(命题逻辑附加律)}\\
\iff & \forall z((\neg(z \in \mathscr{A}) \lor (x \in z)) \lor (\neg(z \in \mathscr{B}) \lor (x \in z))) && \text{(蕴涵等值式)}
\end{aligned}
$$

[1]感谢北大未名BBS的chouxiaoya网友提供 $(1) \Leftrightarrow (4)$ 和 $(1) \Leftrightarrow (5)$ 的形式化证明。