数字逻辑第五章

信息科学与工程学院计算机系

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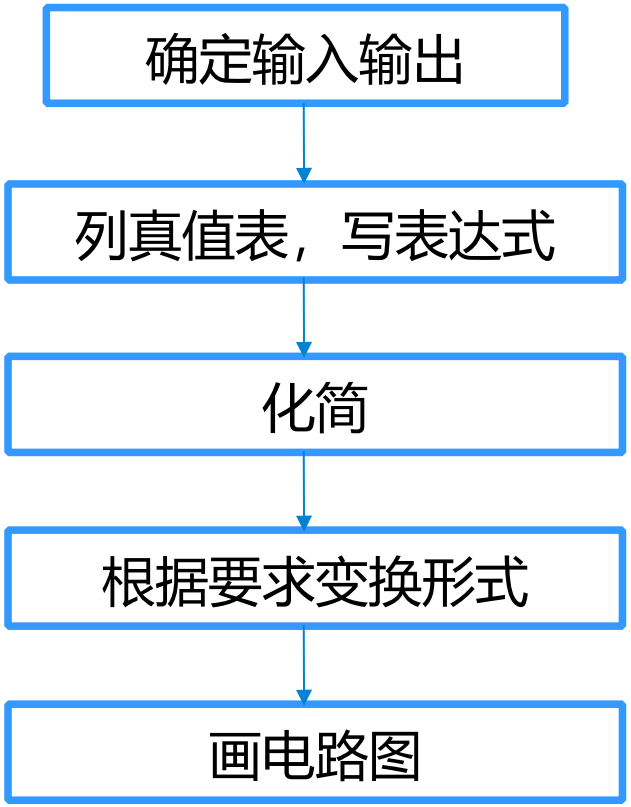
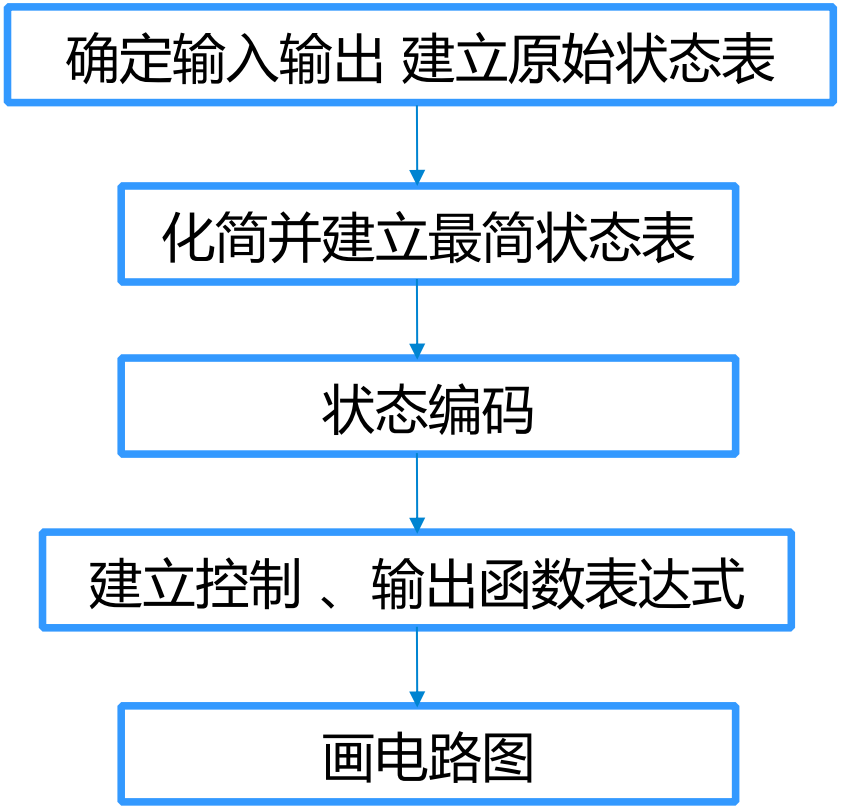
同步时序线路的设计

组合线路设计方法

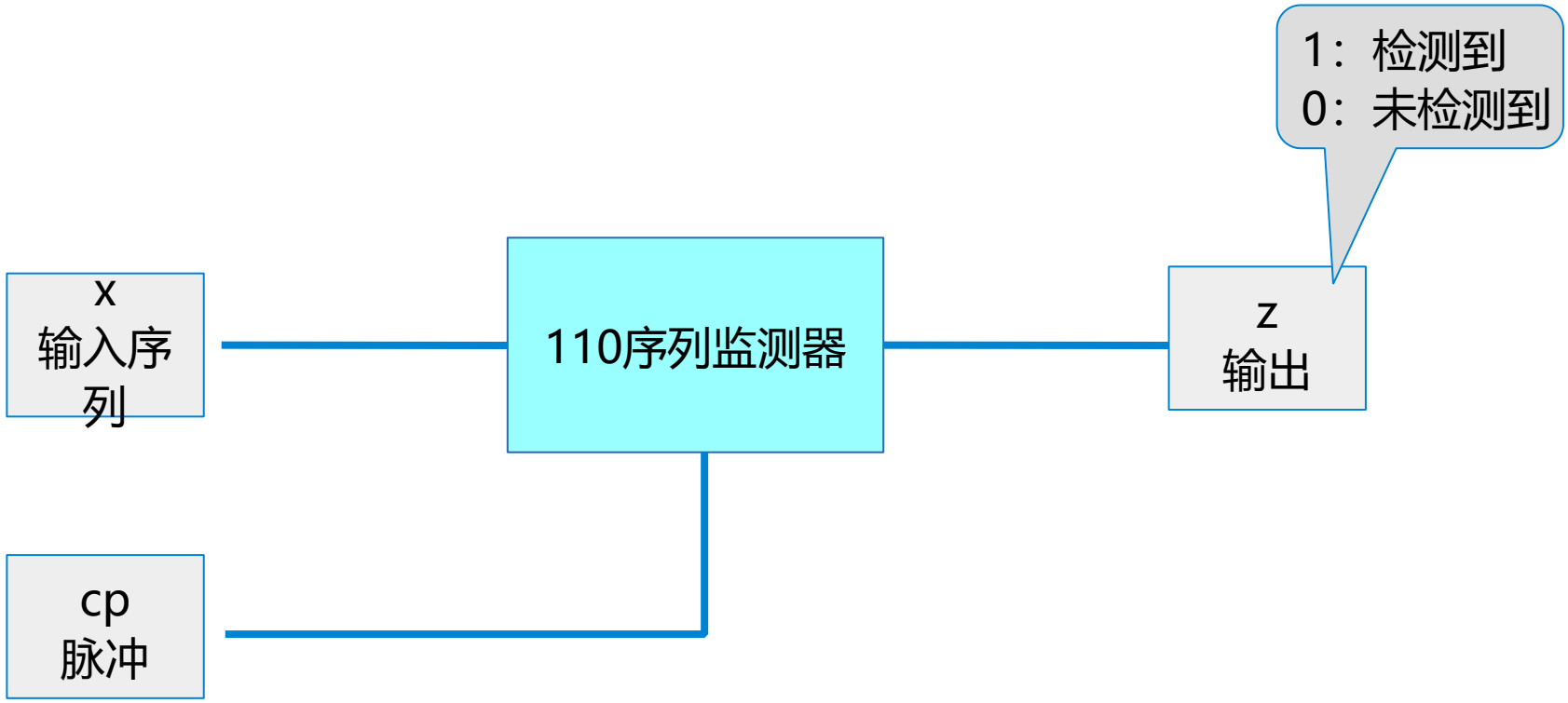


时序线路设计方法



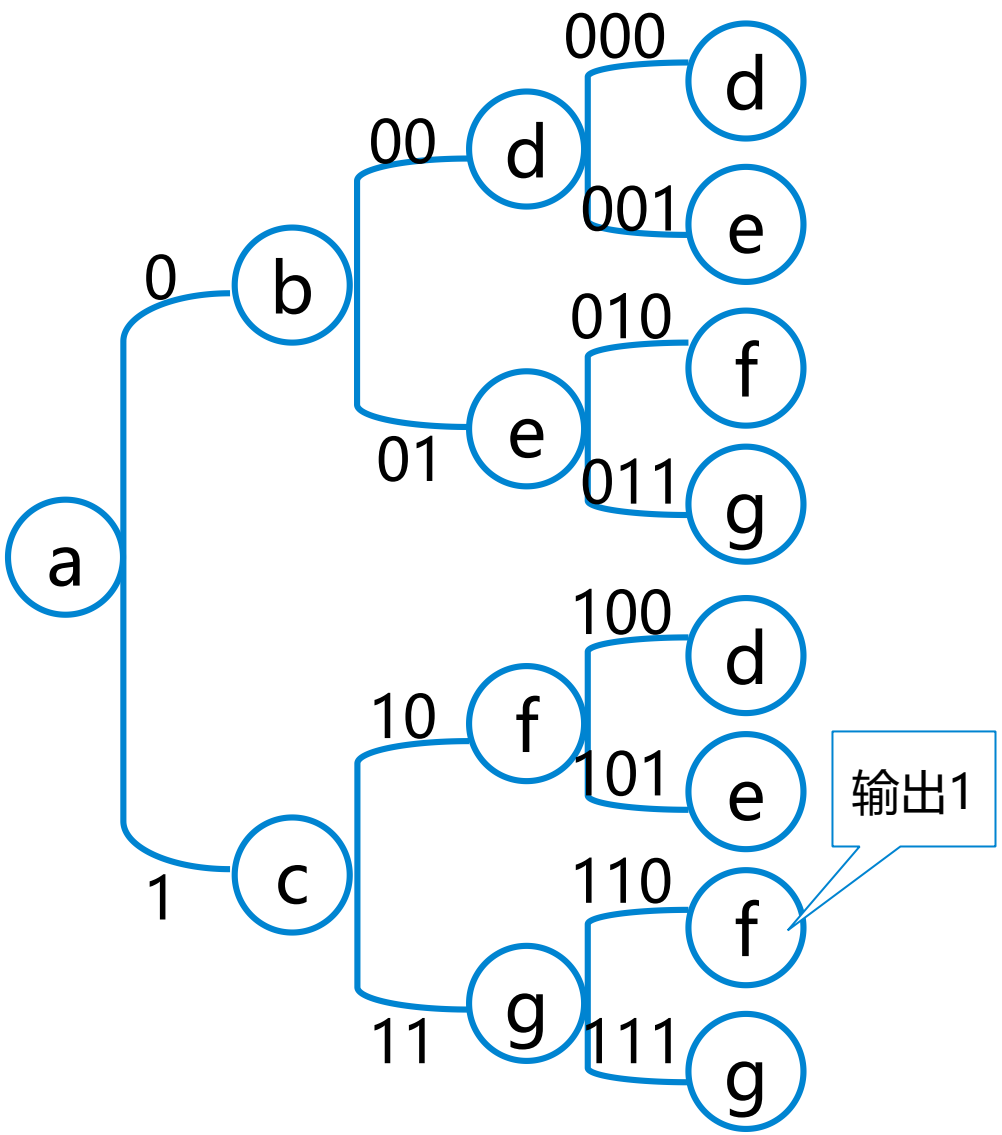
引例

- 题目：用与非门和JK触发器设计一个同步时序线路，检测输入为连续的110
- 一、确定输入、输出， 建立原始状态表



原始状态表

- 设初始状态为a



原始状态表

s \ x	0	1
a	b	c
b	d	e
c	f	g
d	d	e
e	f	g
f	d	e
g	f	g

化简，建立最简状态表

s \ x	0	1
a	b,0	c,0
b	d,0	e,0
c	f,0	g,0
d	d,0	e,0
e	f,0	g,0
f	d,0	e,0
g	f,1	g,0

q_1

q_2

s \ x	0	1
a	q_1,0	q_2,0
q_1	q_1,0	q_2,0
q_2	q_1,0	g,0
g	q_1,1	g,0

化简，建立最简状态表

- 得到最终的最简状态表

s \ x	0	1
a	q_1,0	q_2,0
q_1	q_1,0	q_2,0
q_2	q_1,0	g,0
g	q_1,1	g,0

s_1

s_2

s_3

s \ x	0	1
s_1	s_1,0	s_2,0
s_2	s_1,0	s_3,0
s_3	s_1,1	s_3,0

状态编码

- 三个状态，需要？位编码

	S_1	S_2	S_3
Y_2	0	1	1
Y_1	0	0	1

最终得到状态表

s \ x	0	1
00	00,0	10,0
10	00,0	11,0
11	00,1	11,0

确定输出及控制函数

- 根据右侧的状态激励表，可以得到状态转移表，最终得到控制及输出函数真值表

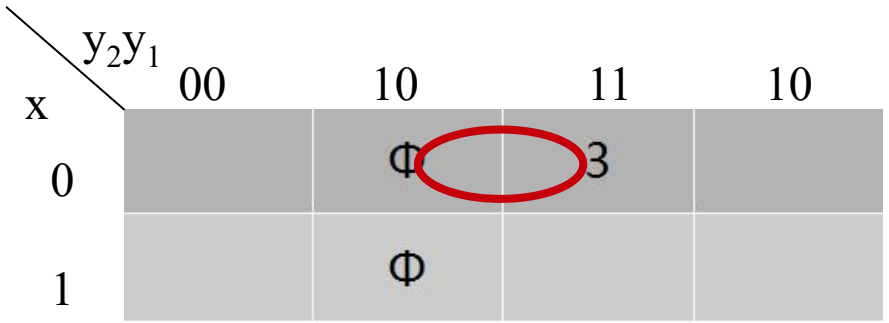
x	y_2	y_1	y_2^{n+1}	y_1^{n+1}	J_2K_2	J_1K_1	z
0	0	0	0	0	0Φ	0Φ	0
0	1	0	0	0	Φ1	0Φ	0
0	1	1	0	0	Φ1	Φ1	1
1	0	0	1	0	1Φ	0Φ	0
1	1	0	1	1	Φ0	1Φ	0
1	1	1	1	1	Φ0	Φ0	0
0	0	1	Φ	Φ	ΦΦ	ΦΦ	Φ
1	0	1	Φ	Φ	ΦΦ	ΦΦ	Φ

激励表（JK触发器）

Q	Q^{n+1}	J	K
0	0	0	Φ
0	1	1	Φ
1	0	Φ	1
1	1	Φ	0

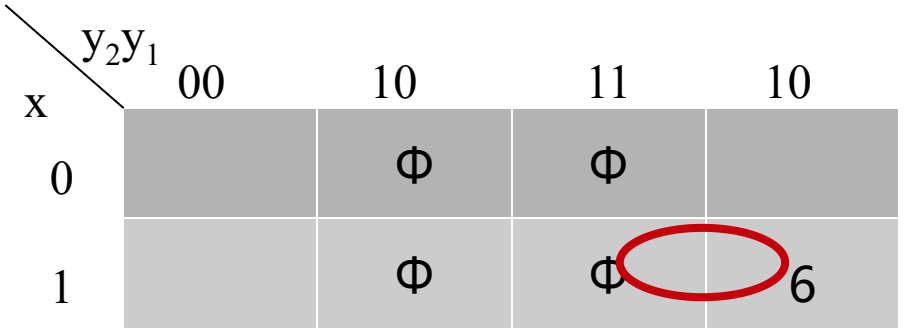
确定输出及控制函数

- 根据控制及输出函数真值表，得到　　使用卡诺图化简JK、Z和输入之间的关系:
  - z=∑(3)+∑Φ(1,5)
  - J1=∑(6)+∑Φ(1,3,5,7)
  - K1=∑(3)+∑Φ(0,1,2,4,5,6)
  - J2=∑(4)+∑Φ(1,2,3,5,6,7)
  - K2=∑(2,3)+∑Φ(0,1,4,5)

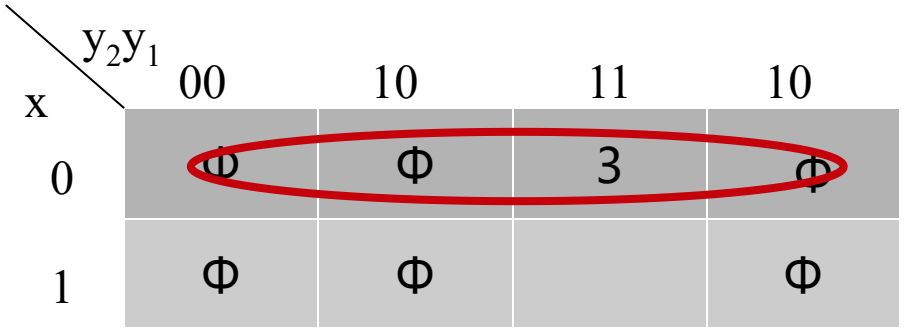


$$z = \bar{x}\, y_1$$

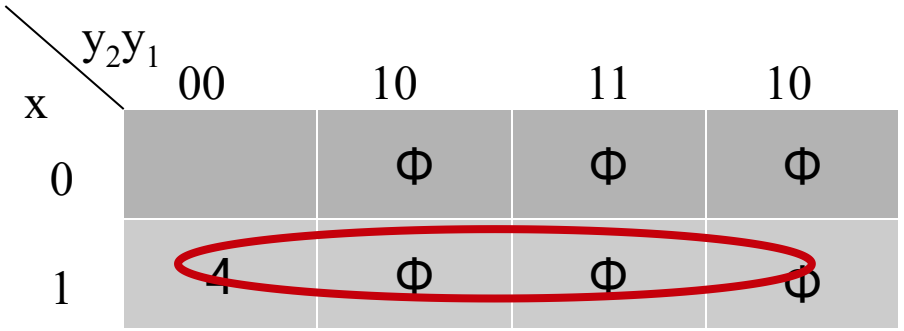
通过化简确定输入输出函数



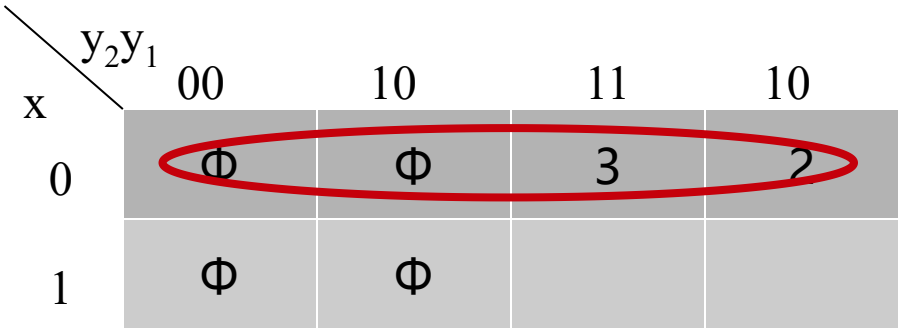
$$J_1 = x\,y_2$$



$$K_1 = \overline{x}$$

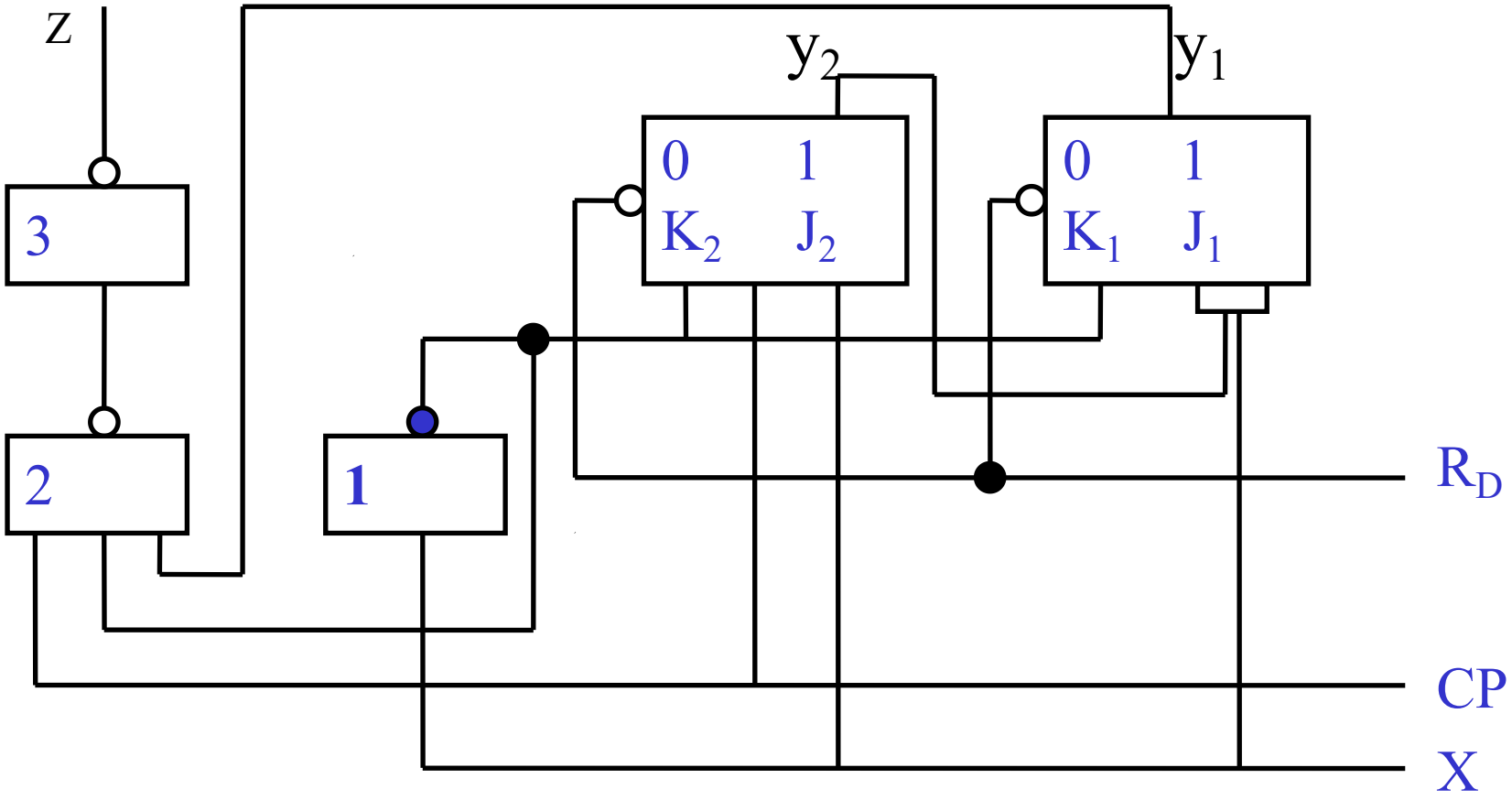


$$J_2 = x$$

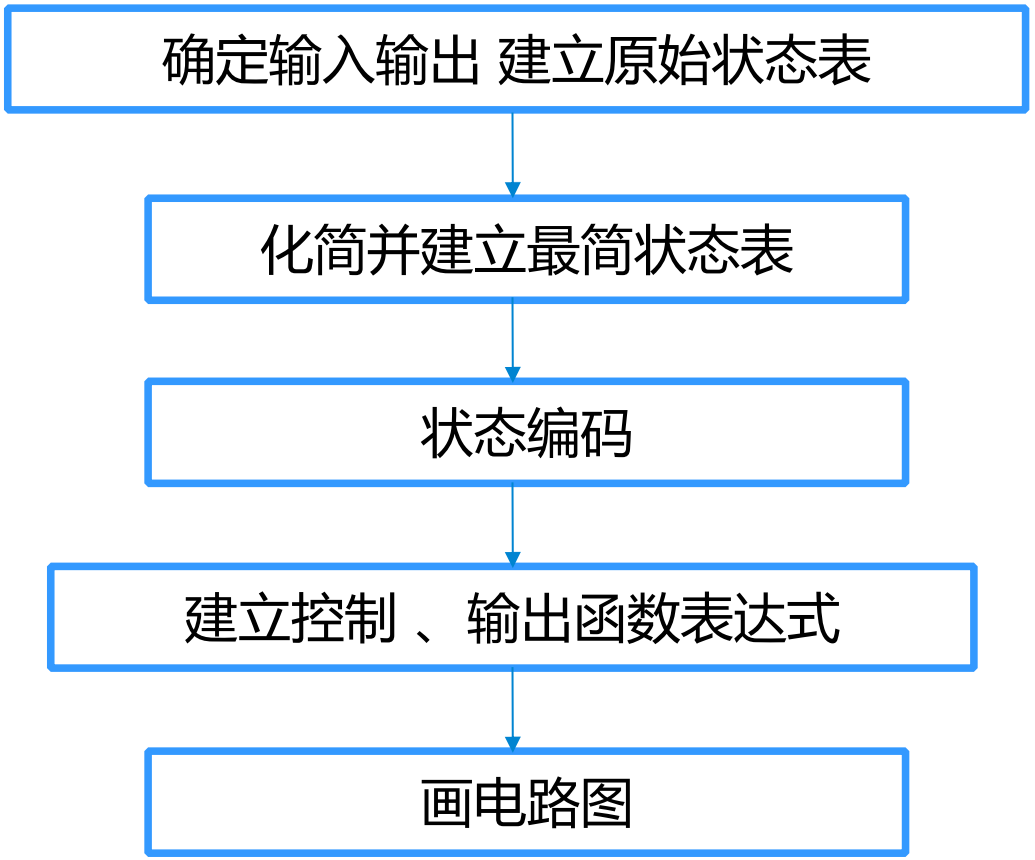


$$K_2 = \overline{x}$$

画逻辑电路图

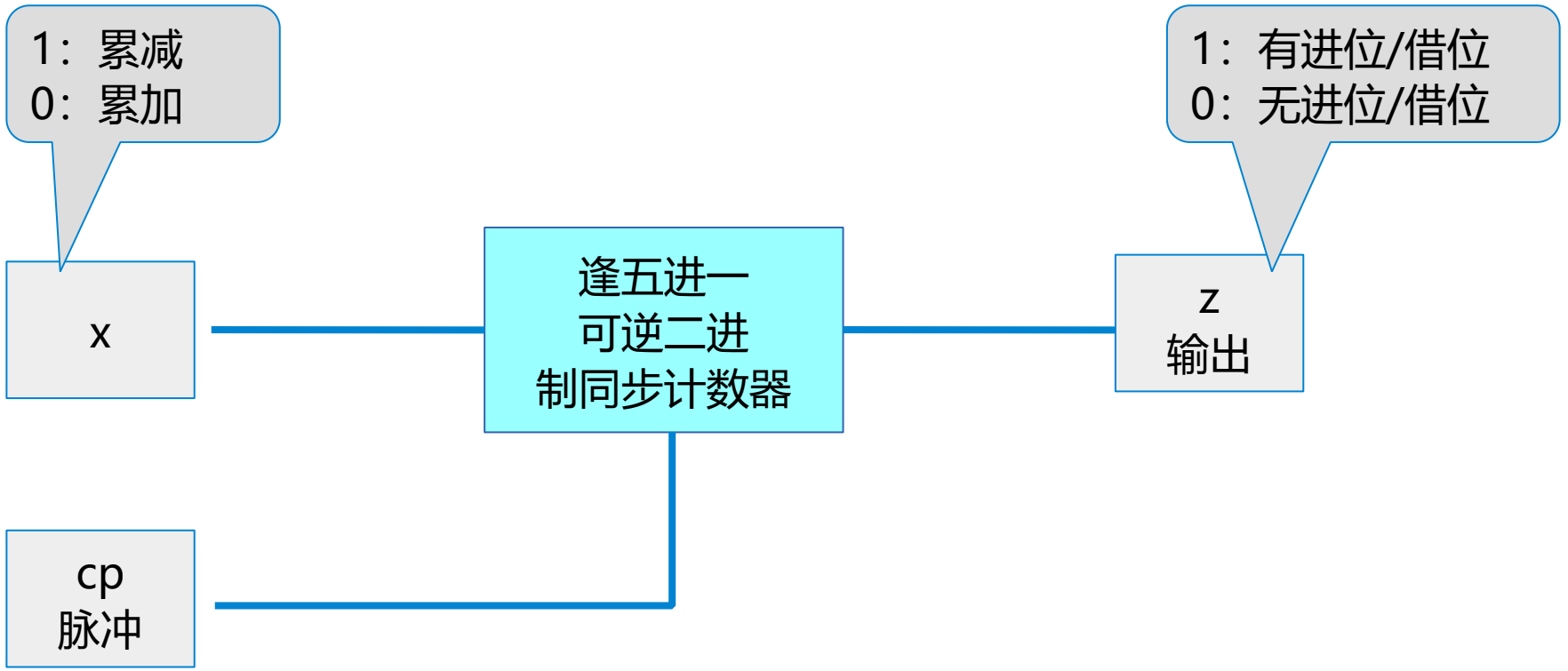


时序线路设计步骤总结



构成原始状态表的方法

- 例1：建立逢五进一可逆二进制同步计数器



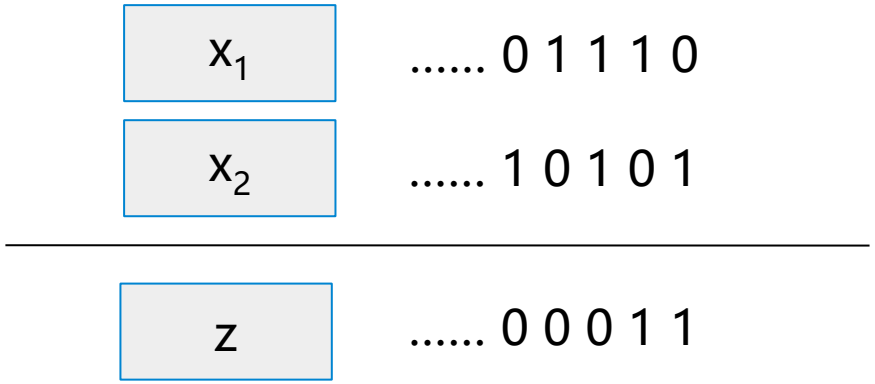
构成原始状态表的方法

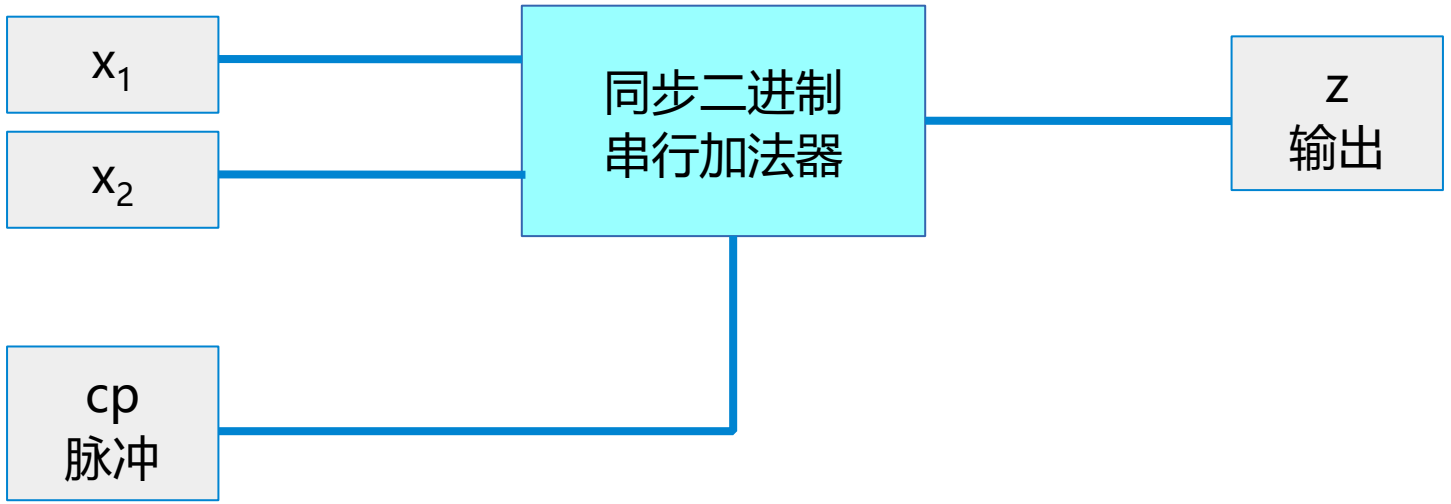
- 例1：建立逢五进一可逆二进制同步计数器

s \ x	0	1
a	b,0	e,0
b	c,0	a,1
c	d,0	b,0
d	e,0	c,0
e	a,1	d,0

构成原始状态表的方法

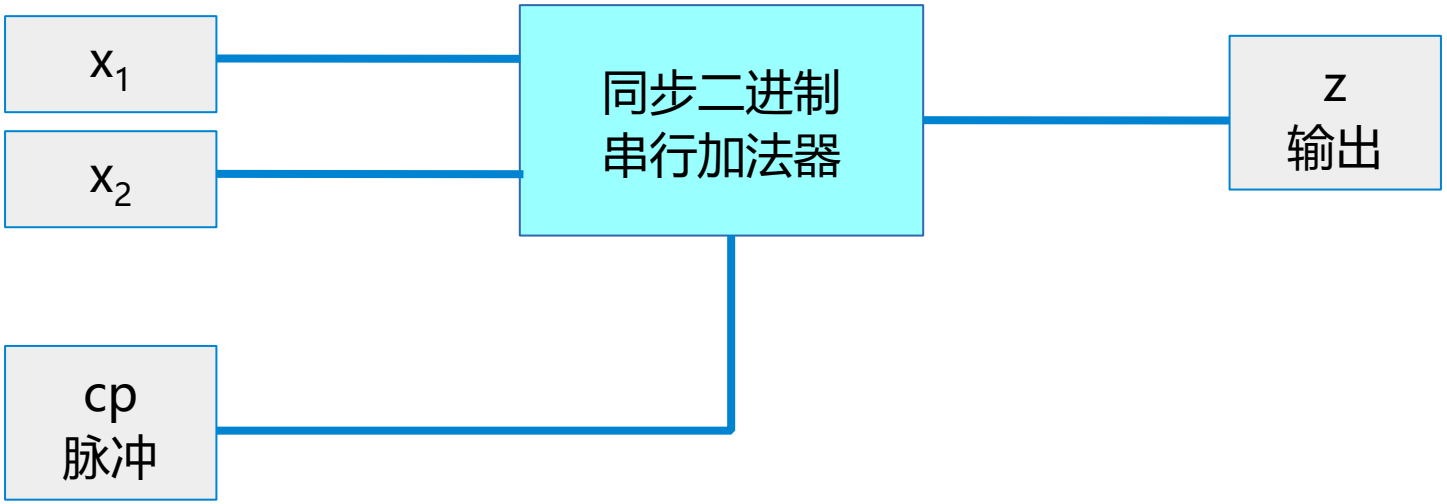
- 例2:同步二进制串行加法器





构成原始状态表的方法

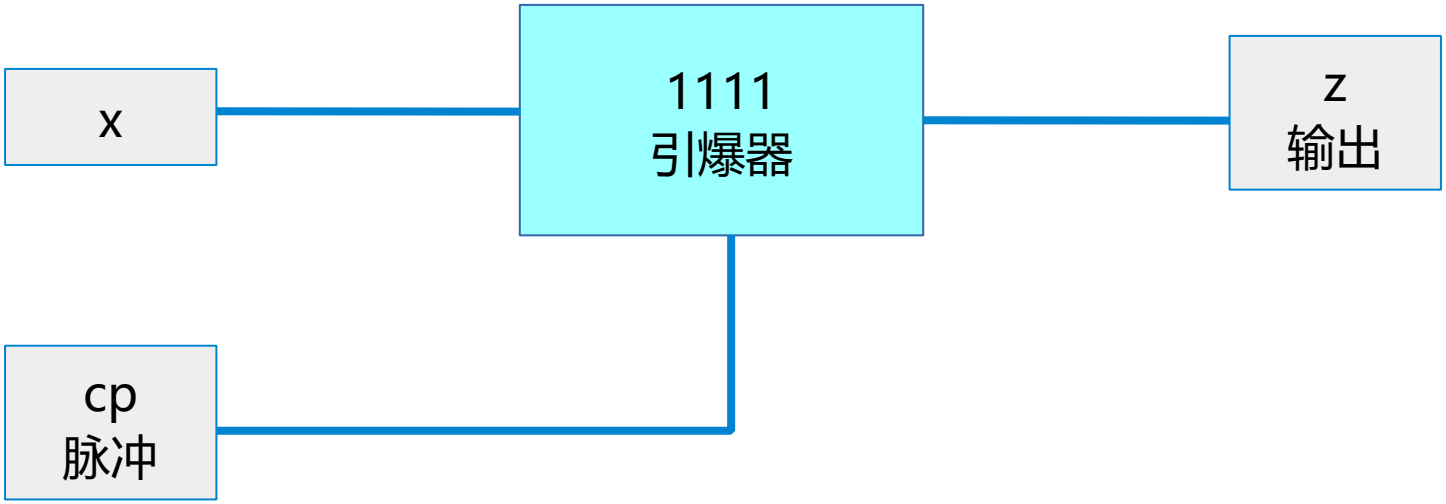
- 例2:同步二进制串行加法器



s \ x_1x_2	00	01	10	11
a	a,0	a,1	a,1	b,0
b	a,1	b,0	b,0	b,1

构成原始状态表的方法

- 例3:引爆条件 ‘1111’



构成原始状态表的方法

- 例3:引爆条件 ‘1111’

s \ x	0	1
a	a,0	b,0
b	Φ,Φ	c,0
c	Φ,Φ	d,0
d	Φ,Φ	Φ,1

s \ x	0	1
a	a,0	b,0
b	a,0	c,0
c	a,0	d,0
d	a,0	Φ,1

考虑输入为0恢复初始状态

完全定义状态表的化简

- 一、化简原理：找出等价状态并将它们合并
- 等价状态：1、必要条件：在同样的输入作用下，有相同的输出
  2、同样的输入条件下，相应的次态彼此等价
- 等价次态：1、对应的次态相同
  2、次态为两个现态本身或交错
  3、两个次态为状态对封闭链中的一对
  4、两个次态的某一后续状态对可以合并

等价状态具有可传递性：AB等价，AC等价 = >BC等价，则A、B、C为等价类

- 等价类：彼此等价的状态的集合
- 最大等价类：一个等价类不包含在任何其他等价类中

化简原始状态表 = >寻找最大等价类

完全定义状态表的化简

- 例1

s \ x	0	1
a	c,1	b,0
b	c,1	e,0
c	b,1	e,0
d	d,1	b,1
e	d,1	b,1

q_1

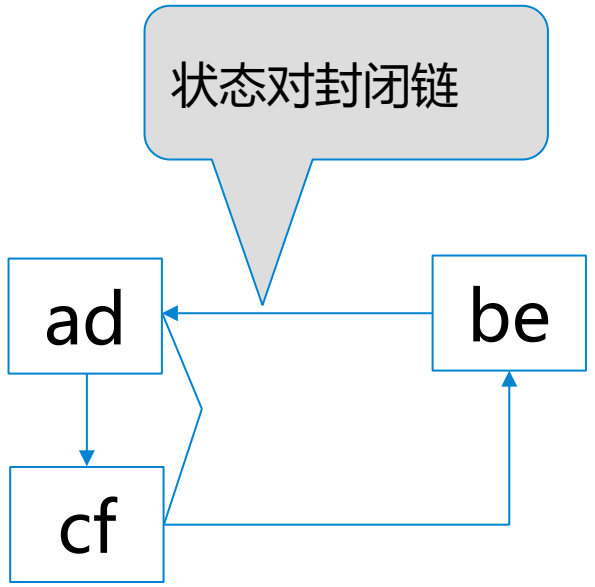
q_2: 2)次态为两个现态本身或交错

q_3:1)对应的次态相同

完全定义状态表的化简

- 例2

s \ x	0	1
a	e,0	d,0
b	a,1	f,0
c	c,0	a,1
d	b,0	a,0
e	d,1	c,0
f	c,0	d,1
g	h,1	g,1
h	c,1	b,1



q_1=(a，d)
q_2=(b，e)　q_3=(c，f)
q_4=（g）　q_5=（h）

完全定义状态表的化简

- k次划分法：先找出输出相同的状态集合，为一次划分，再找第二次输入下输出也相同的集合，依此类推，直到k + 1次不能再划分，找到最大等价类集合。

- 等价次态：
  - a）对应的次态相同
  - b）次态为两个现态本身或交错
  - c）两个次态为状态对封闭链中的一对
  - d）两个次态的某一后续状态对可以合并

完全定义状态表的化简

s \ x	0	1
a	c,0	b,1
b	f,0	a,1
c	d,0	g,0
d	d,1	e,0
e	c,0	e,1
f	d,0	g,0
g	c,1	d,0

- 1、一次划分
  - q_1={ a_{21}, b_{21} ,e_{21} }
  - q_2={ c_{33} , f_{33} }
  - q_3={ d_{31} , g_{23}}
- 2、二次划分(把一次划分中下标不同的分出来)
  - q_1={ a_{21}, b_{21} ,e_{21} }
  - q_2={ c_{33} , f_{33} }={ c_{34} , f_{34} }
  - q_3={ d_{31} }
  - q_4={ g_{23} }
- 3、所有下标都相同，划分结束。

完全定义状态表的化简

s \ x	0	1
a	e,0	d,0
b	a,1	f,0
c	f,0	a,1
d	b,0	a,0
e	d,1	c,0
f	c,0	d,1
g	h,1	g,1
h	c,1	b,1

- 1、一次划分
  - $q_1=\{ a_{21}, d_{21} \}$
  - $q_2=\{ b_{13} , e_{13} \}$
  - $q_3=\{ c_{31} , f_{31}\}$
  - $q_4=\{ g_{44} , h_{32}\}$
- 2、二次划分
  - $q_1=\{ a_{21}, d_{21} \}$
  - $q_2=\{ b_{13} , e_{13} \}$
  - $q_3=\{ c_{31} , f_{31}\}$
  - $q_4=\{ g_{54}\}$
  - $q_5=\{ h_{32}\}$
- 3、划分结束

完全定义状态表的化简

- 隐含表法：是一种直角三角形表格，表中每一个小格表示一个状态对的等价或不等价关系。
- 第一步：做隐含表，然后顺序比较。

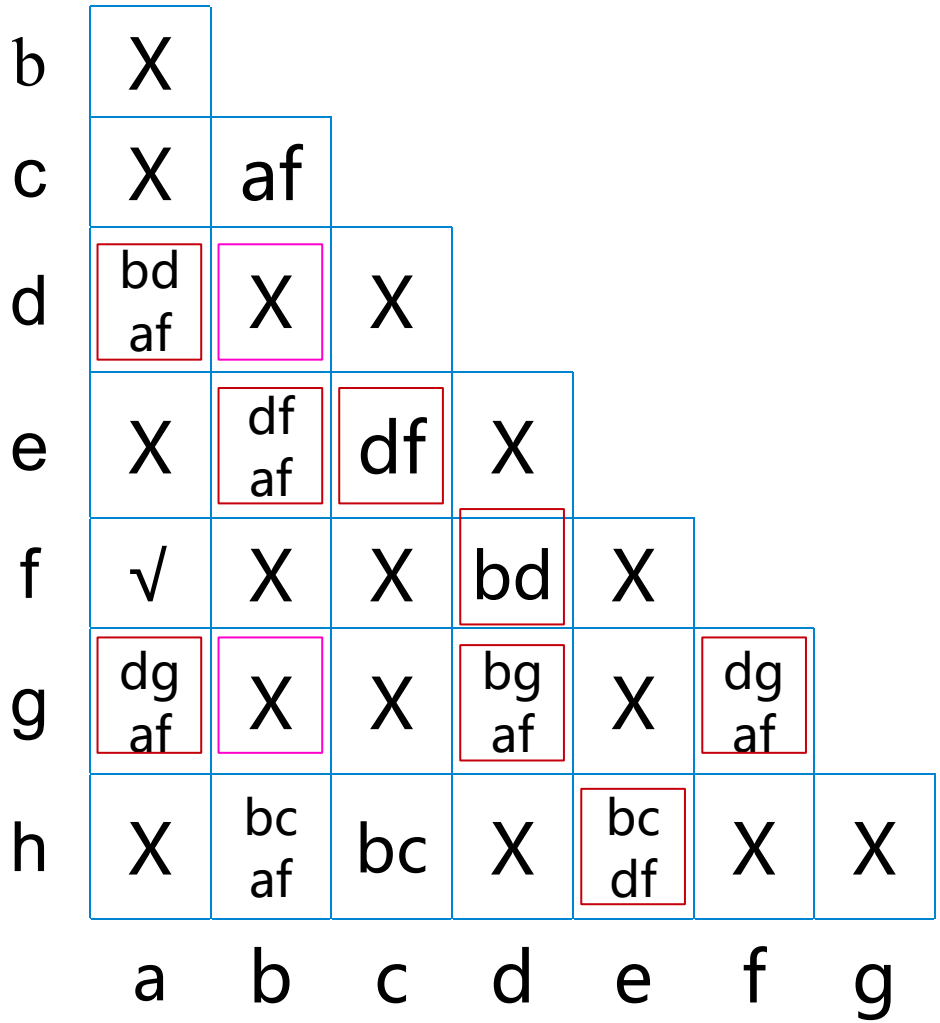
x_1x_2 / s	00	01	11	10
a	d,0	d,0	f,0	a,0
b	c,1	d,0	e,1	f,0
c	c,1	d,0	e,1	a,0
d	d,0	b,0	a,0	f,0
e	c,1	f,0	e,1	a,0
f	d,0	d,0	a,0	f,0
g	g,0	g,0	a,0	a,0
h	b,1	d,0	e,1	a,0

	a	b	c	d	e	f	g
b	X						
c	X	af					
d	bd af	X	X				
e	X	df af	df	X			
f	√	X	X	bd	X		
g	dg af	X	X	bg af	X	dg af	
h	X	bc af	bc	X	bc df	X	X

完全定义状态表的化简

- 第二步：关连比较。继续检查填有隐含条件的那些方格。若检查发现所填的隐含条件肯定不能满足，就在该方格内打“×”

s \ x_1x_2	00	01	11	10
a	d,0	d,0	f,0	a,0
b	c,1	d,0	e,1	f,0
c	c,1	d,0	e,1	a,0
d	d,0	b,0	a,0	f,0
e	c,1	f,0	e,1	a,0
f	d,0	d,0	a,0	f,0
g	g,0	g,0	a,0	a,0
h	b,1	d,0	e,1	a,0



完全定义状态表的化简

- 第三步：寻找最大等价类

未打“×”的方格，都代表一个等价状态对

由此得到全部等价对：

[a，f]、[b，h]、

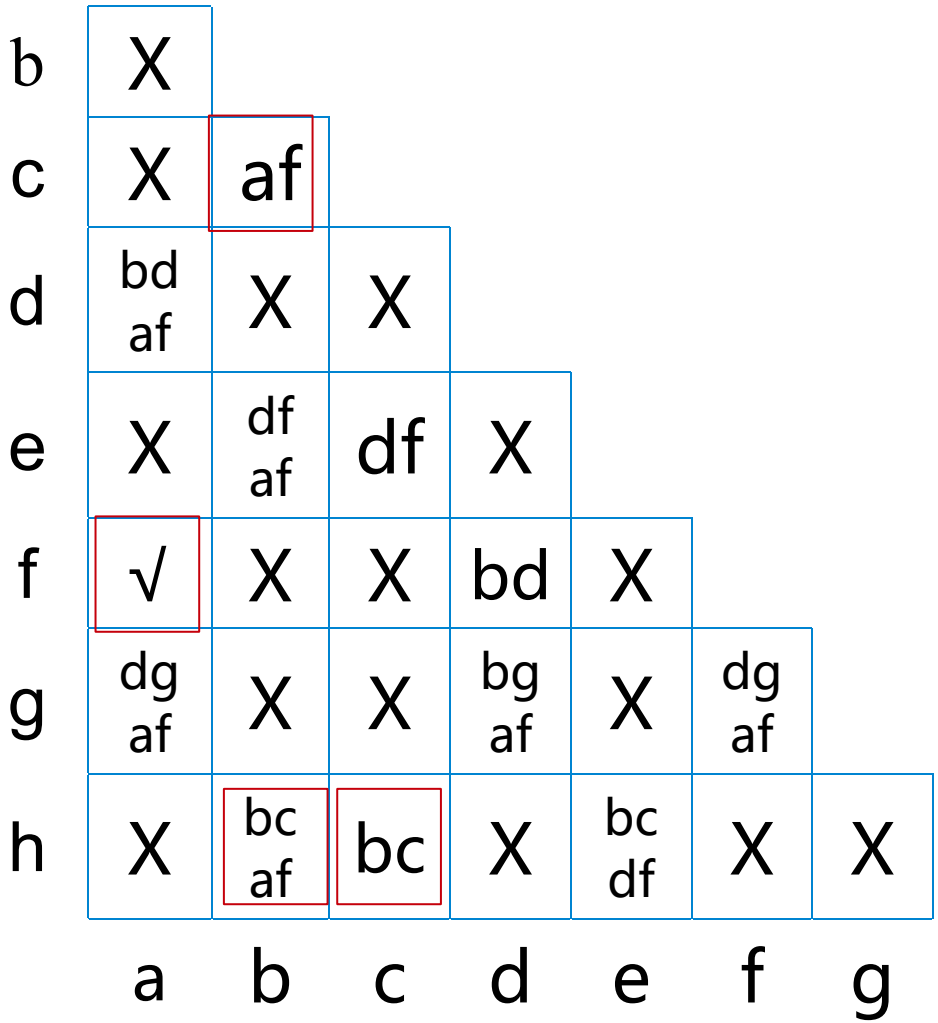
[b，c]、[c，h]

部最大等价类：

[a，f]、[b，c，h]、

[d]、[e]、[g]

- 第四步 状态合并，得最简状态表



完全定义状态表的化简

- 隐含表法总结

  - 1、构作隐含表①等价√②不等价×③条件
  - 2、顺序比较追踪，找出所有等价状态
  - 3、形成最大等价类集合
  - 4、构成最简状态表

状态编码

- 确定需要几位二进制码

$$k = [\log_2 N]$$ 向上取整

- 次佳编码法：
  - 1)次态相同，现态相邻
  - 2)现态相同，次态相邻
  - 3)输出相同，现态相邻

优先顺序1>2>3

状态编码

- 例:为下列最简状态表进行编码

s \ x	0	1
a	c,0	d,0
b	c,0	a,0
c	b,0	d,0
d	a,1	b,1

- $\log_2 4=2$ 所以需两位二进制码
  - 1)次态相同，现态相邻

    ab 相邻，ac 相邻
  - 2)现态相同，次态相邻

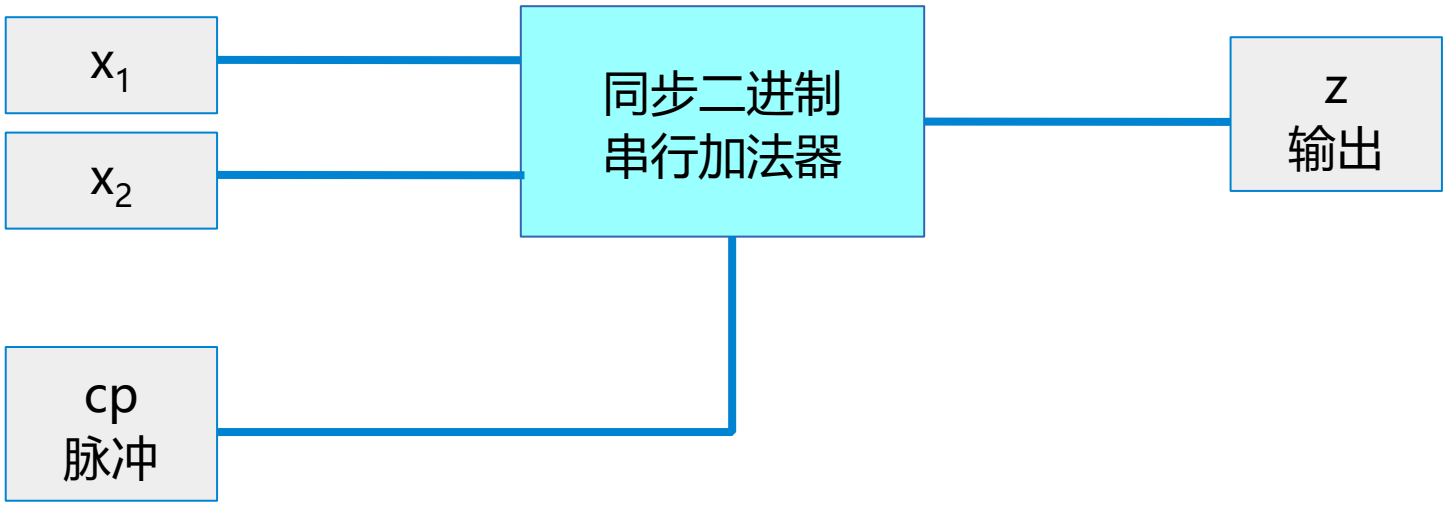
    cd，ca，bd，ab 相邻
  - 3)输出相同，现态相邻

    abc 相 邻

同步时序线路的设计举例

例1:用JK触发器及与非门设计一个同步二进制串行加法器

- 一、确定输入输出，建立原始状态表



s \ x_1x_2	00	01	10	11
a	a,0	a,1	a,1	b,0
b	a,1	b,0	b,0	b,1

同步时序线路的设计举例

- 二、化简 略
- 三、状态编码 a=0 b=1

s \ x_1x_2	00	01	10	11
a	0,0	0,1	0,1	1,0
b	0,1	1,0	1,0	1,1

同步时序线路的设计举例

- 四、列控制、输出函数表达式

Q	Q^{n+1}	J	K
0	0	0	Φ
0	1	1	Φ
1	0	Φ	1
1	1	Φ	0

x_1	x_2	y	y^{n+1}	J	K	z
0	0	0	0	0	Φ	0
0	0	1	0	Φ	1	1
0	1	0	0	0	Φ	1
0	1	1	1	Φ	0	0
1	0	0	0	0	Φ	1
1	0	1	1	Φ	0	0
1	1	0	1	1	Φ	0
1	1	1	1	Φ	0	1

同步时序线路的设计举例

- 四、列控制、输出函数表达式

$$z = x_1\overline{x_2}y + \overline{x_1}x_2\overline{y} + x_1\overline{x_2}\overline{y} + x_1x_2y$$

$$J = x_1y$$

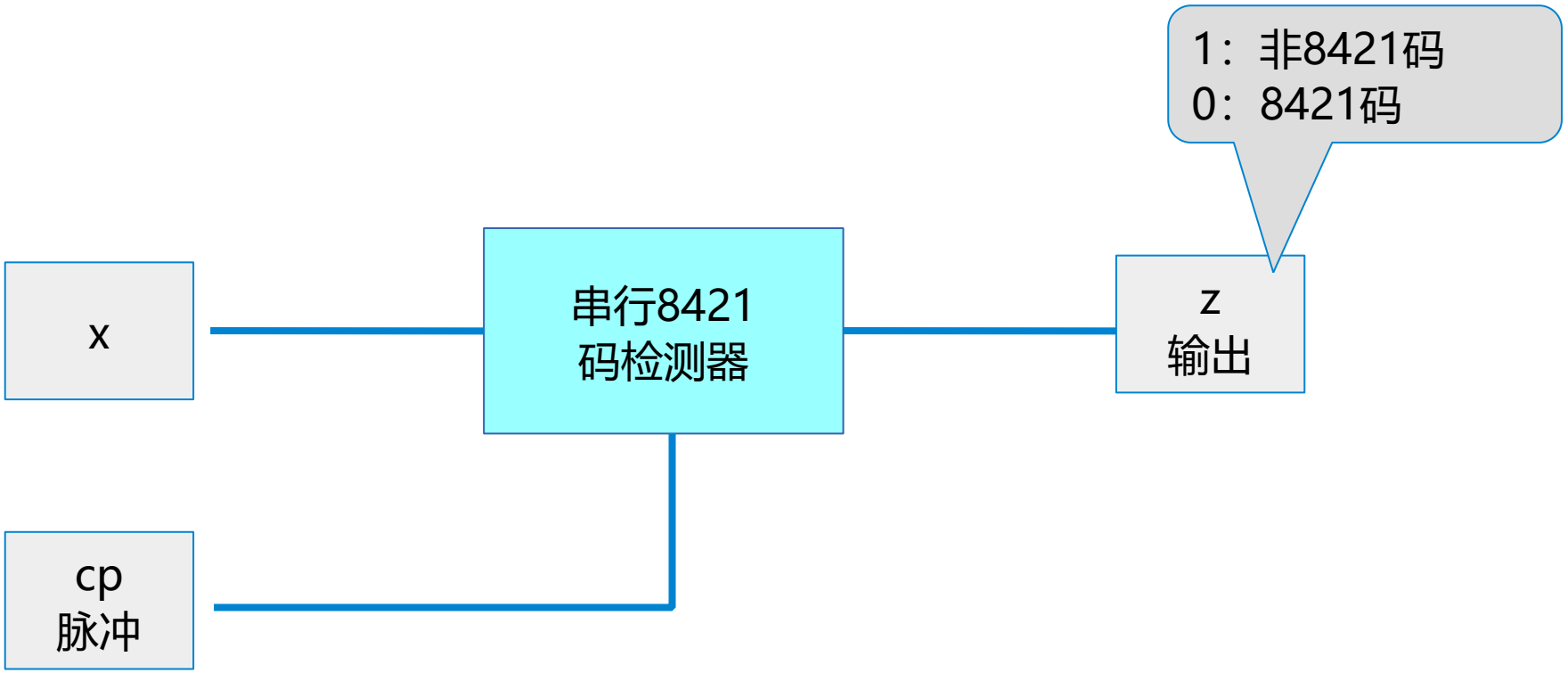
$$K = \overline{x_1}\overline{y}$$

x_1	x_2	y	y^{n+1}	J	K	z
0	0	0	0	0	Φ	0
0	0	1	0	Φ	1	1
0	1	0	0	0	Φ	1
0	1	1	1	Φ	0	0
1	0	0	0	0	Φ	1
1	0	1	1	Φ	0	0
1	1	0	1	1	Φ	0
1	1	1	1	Φ	0	1

同步时序线路的设计举例

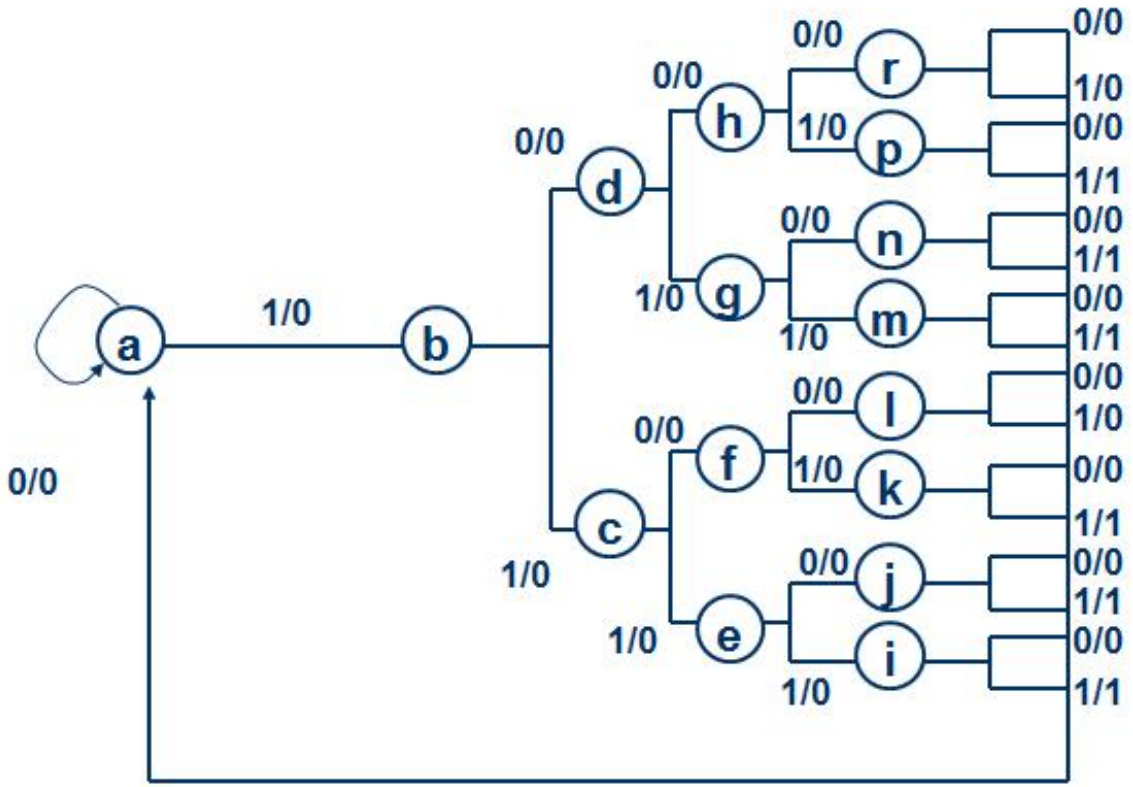
例2用与非门，与或非门及JK触发器，设计串行8421码检测器。

- 一、确定输入输出，建立原始状态表



同步时序线路的设计举例

- 一、确定输入输出，建立原始状态表



s \ x	0	1
a	a,0	b,0
b	d,0	c,0
c	f,0	e,0
d	h,0	g,0
e	j,0	i,0
f	l,0	k,0
g	n,0	m,0
h	r,0	p ,0
i	a,0	a,1
j	a,0	a,1
k	a,0	a,1
l	a,0	a,0
m	a,0	a,1
n	a,0	a,1
p	a,0	a,1
r	a,0	a,0

同步时序线路的设计举例

- 二、状态表化简（K次划分法）

$$q_1 = (a, b, c, d, e, f, g, h, l, r)$$

$$q_2 = (i, j, k, m, n, p)$$

更新下标：

$$q_1 = (a_{11}, b_{11}, c_{11}, d_{11}, f_{12}, g_{22}, h_{12}, l_{11}, r_{11})$$

$$q_2 = (p)$$

第二次划分：

$$q_1 = (a_{11}, b_{11}, c_{11}, d_{11}, l_{11}, r_{11})$$

$$q_2 = (e_{22}, g_{22})$$

$$q_3 = (f_{12}, h_{12})$$

$$q_4 = (p)$$

s \ x	0	1
a	a,0	b,0
b	d,0	c,0
c	f,0	e,0
d	h,0	g,0
e	j,0	i,0
f	l,0	k,0
g	n,0	m,0
h	r,0	p ,0
i	a,0	a,1
j	a,0	a,1
k	a,0	a,1
l	a,0	a,0
m	a,0	a,1
n	a,0	a,1
p	a,0	a,1
r	a,0	a,0

同步时序线路的设计举例

- 更新下标

$$q_1 = (a_{11}, b_{11}, c_{32}, d_{32}, l_{11}, r_{11})$$

$$q_2 = (e_{44}, g_{44})$$

$$q_3 = (f_{14}, h_{14})$$

$$q_4 = (p)$$

第三次划分：

$$q_1 = (a_{11}, b_{11}, l_{11}, r_{11})$$

$$q_2 = (c_{32}, d_{32},)$$

$$q_3 = (e_{44}, g_{44})$$

$$q_4 = (f_{14}, h_{14})$$

$$q_5 = (p)$$

s \ x	0	1
a	a,0	b,0
b	d,0	c,0
c	f,0	e,0
d	h,0	g,0
e	j,0	i,0
f	l,0	k,0
g	n,0	m,0
h	r,0	p ,0
i	a,0	a,1
j	a,0	a,1
k	a,0	a,1
l	a,0	a,0
m	a,0	a,1
n	a,0	a,1
p	a,0	a,1
r	a,0	a,0

同步时序线路的设计举例

- 更新下标

$$q_1 = (a_{11}, b_{22}, l_{11}, r_{11})$$

$$q_2 = (c_{43}, d_{43},)$$

$$q_3 = (e_{55}, g_{55})$$

$$q_4 = (f_{15}, h_{15})$$

$$q_5 = (p)$$

第四次划分：

$$q_1 = (a_{11}, l_{11}, r_{11})$$

$$q_2 = (b_{22})$$

$$q_3 = (c_{43}, d_{43},)$$

$$q_4 = (e_{55}, g_{55})$$

$$q_5 = (f_{15}, h_{15})$$

$$q_6 = (p)$$

s \ x	0	1
a	a,0	b,0
b	d,0	c,0
c	f,0	e,0
d	h,0	g,0
e	j,0	i,0
f	l,0	k,0
g	n,0	m,0
h	r,0	p ,0
i	a,0	a,1
j	a,0	a,1
k	a,0	a,1
l	a,0	a,0
m	a,0	a,1
n	a,0	a,1
p	a,0	a,1
r	a,0	a,0

同步时序线路的设计举例

- 更新下标

$$q_1 = (a_{12}, l_{11}, r_{11})$$

$$q_2 = (b_{33})$$

$$q_3 = (c_{43}, d_{43},)$$

$$q_4 = (e_{55}, g_{55})$$

$$q_5 = (f_{15}, h_{15})$$

$$q_6 = (p)$$

第五次划分：

$$q_1 = (a_{12})$$

$$q_2 = (b_{33})$$

$$q_3 = (c_{43}, d_{43},)$$

$$q_4 = (e_{55}, g_{55})$$

$$q_5 = (f_{15}, h_{15})$$

$$q_6 = (p)$$

$$q_7 = (\, l_{11}, r_{11})$$

s \ x	0	1
a	a,0	b,0
b	d,0	c,0
c	f,0	e,0
d	h,0	g,0
e	j,0	i,0
f	l,0	k,0
g	n,0	m,0
h	r,0	p ,0
i	a,0	a,1
j	a,0	a,1
k	a,0	a,1
l	a,0	a,0
m	a,0	a,1
n	a,0	a,1
p	a,0	a,1
r	a,0	a,0

同步时序线路的设计举例

- 更新下标

$$q_1 = (a_{12})$$

$$q_2 = (b_{33})$$

$$q_3 = (c_{54}, d_{54},)$$

$$q_4 = (e_{66}, g_{66})$$

$$q_5 = (f_{16}, h_{16})$$

$$q_6 = (p)$$

$$q_7 = (\ l_{11}, r_{11})$$

得到 最终结果。

s \ x	0	1
a	a,0	b,0
b	d,0	c,0
c	f,0	e,0
d	h,0	g,0
e	j,0	i,0
f	l,0	k,0
g	n,0	m,0
h	r,0	p ,0
i	a,0	a,1
j	a,0	a,1
k	a,0	a,1
l	a,0	a,0
m	a,0	a,1
n	a,0	a,1
p	a,0	a,1
r	a,0	a,0

同步时序线路的设计举例

- 更新下标

$$q_1 = (a_{12})$$

$$q_2 = (b_{33})$$

$$q_3 = (c_{54}, d_{54},)$$

$$q_4 = (e_{66}, g_{66})$$

$$q_5 = (f_{16}, h_{16})$$

$$q_6 = (p)$$

$$q_7 = (\ l_{11}, r_{11})$$

得到最简状态表。

s \ x	0	1
q_1	q_1,0	q_2,0
q_2	q_3,0	q_3,0
q_3	q_5,0	q_4,0
q_4	q_6,0	q_6,0
q_5	q_7,0	q_6,0
q_6	q_1,0	q_1,1
q_7	q_1,0	q_1,0

同步时序线路的设计举例

- 后续步骤不再赘述。