

年级、专业 _____ 姓名 _____ 学号 _____ 名单序号 (勿换行) _____

选课时间 (打√): 周一 56、周三 56 节 ☐ _____ 周一 78、周三 78 节 ☐ _____

实验时间 2019 年 4 月 日 使用软件版本 MATLAB _____

实验报告: 实验 2 数据建模——插值与拟合

1. 编写一个自定义函数实现 Lagrangian 插值。

```
1 function y=lagrange1(x0,y0,x);
2 n=length(x0);m=length(x);
3 for i=1:m
4     z=x(i);
5     s=0.0;
6     for k=1:n
7         p=1.0;
8         for j=1:n
9             if j~=k
10                p=p*(z-x0(j))/(x0(k)-x0(j));
11            end
12        end
13        s=p*y0(k)+s;
14    end
15    y(i)=s;
16 end
```

```
% 1
x=linspace(1,10,10)
```

```
x = 1×10
     1     2     3     4     5     6     7     8     9    10
```

```
y=sin(x)
```

```
y = 1×10
     0.8415     0.9093     0.1411    -0.7568    -0.9589    -0.2794     0.6570     0.9894
```

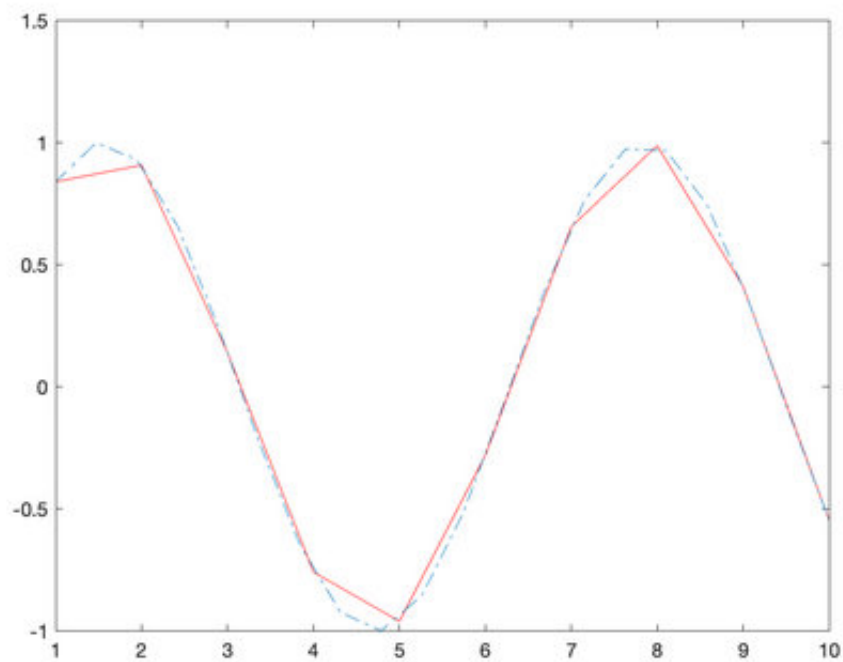
```
x2=linspace(1,10,20)
```

```
x2 = 1×20
     1.0000     1.4737     1.9474     2.4211     2.8947     3.3684     3.8421     4.3158
```

```
y2=lagrange1(x,y,x2)
```

```
y2 = 1×20
     0.8415     1.0015     0.9303     0.6587     0.2442    -0.2246    -0.6445    -0.9225
```

```
plot(x,y,"-r",x2,y2,"-.")
```



2. 在下面的函数中选择两个，在 n 个结点上 (n 不要太大，如 5~11) 分别用 Lagrangian、分段线性、三次样条插值三种插值方法，计算 m 个插值点的函数值 (m 要适中，如 50~100)。通过数值和图形输出，将三种插值结果与精确值进行比较。适当增加 n ，再做比较，由此做初步分析。

$$(1) y = \sqrt{1-x^2}, -1 \leq x \leq 1 \quad (1)$$

$$(2) y = \sin^6 x, -\pi \leq x \leq \pi \quad (2)$$

$$(3) y = e^{-x^2}, -2 \leq x \leq 2 \quad (3)$$

% 2

```
a=-1;  
b=1;  
x=linspace(a,b,10);  
x2=linspace(a,b,50);  
y=f1(x)
```

y = 1×10

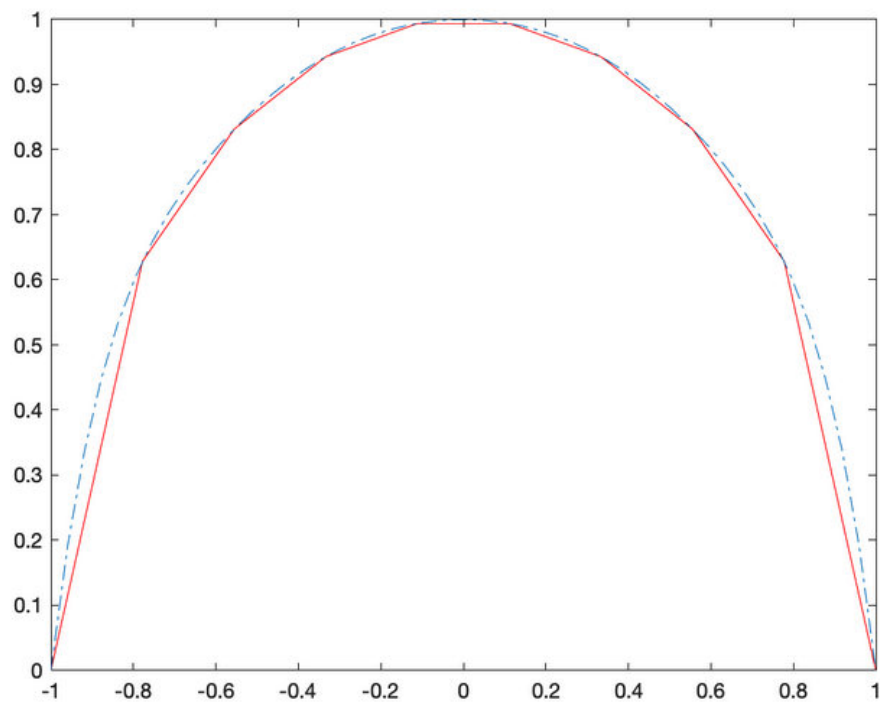
0 0.6285 0.8315 0.9428 0.9938 0.9938 ...

```
y1=lagrange1(x,y,x2)
```

y1 = 1×50

0 0.1920 0.3376 0.4488 0.5349 0.6028 ...

```
plot(x,y,"-r",x2,y1,"-b.")
```

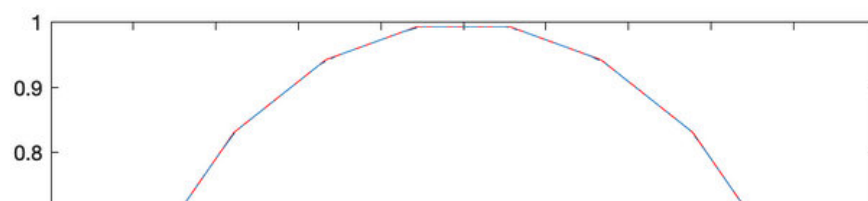


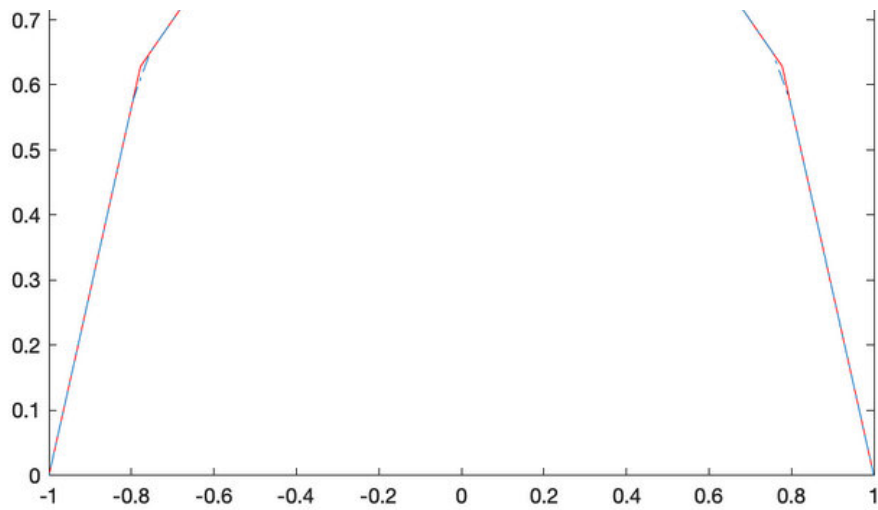
```
y2=interp1(x,y,x2,"linear")
```

y2 = 1×50

0 0.1154 0.2309 0.3463 0.4618 0.5772 ...

```
plot(x,y,"-r",x2,y2,"-b.")
```

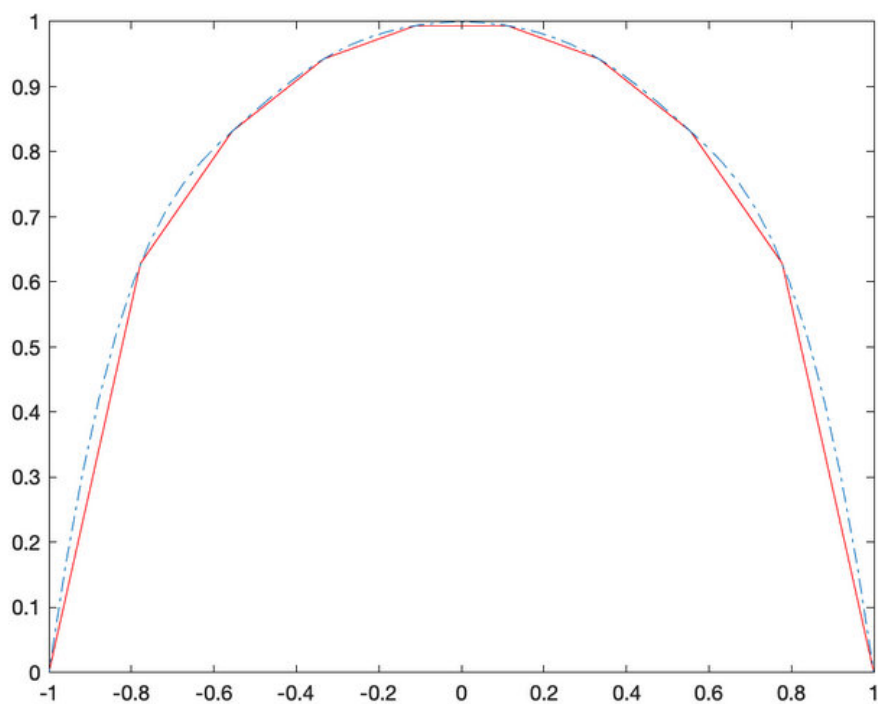




```
y3=interp1(x,y,x2,"spline")
```

```
y3 = 1×50
      0      0.1661      0.3065      0.4237      0.5203      0.5988 ...
```

```
plot(x,y,"-r",x2,y3,"-.")
```



```
clear;
a=-pi;
b=pi;
x=linspace(a,b,10);
x2=linspace(a,b,50);
y=f2(x)
```

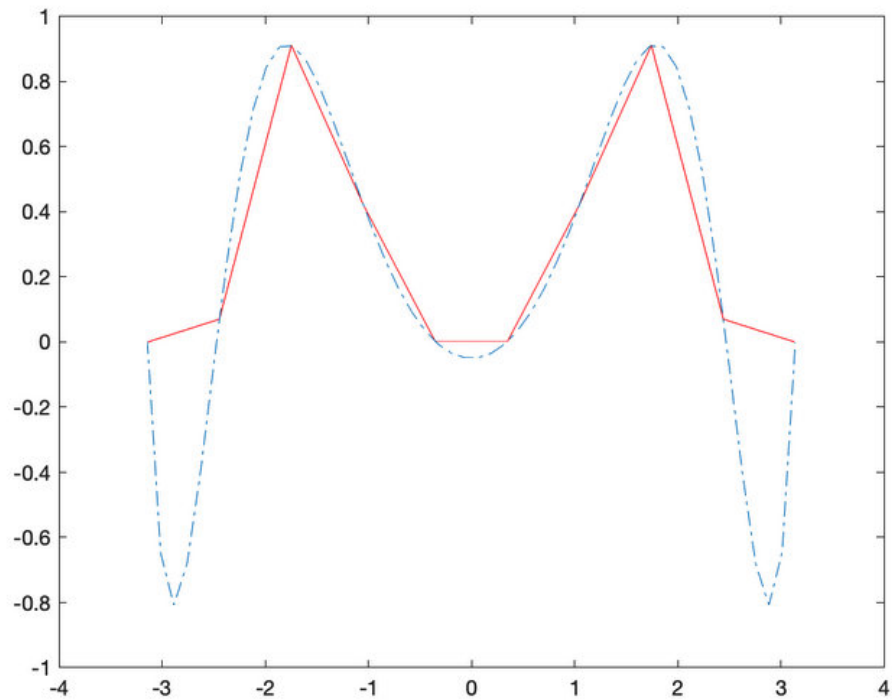
```
y = 1×10
      0      0.0000      0.0705      0.0122      0.4210      0.0016      0.0016 ...
```

0.0000 0.0705 0.9122 0.4219 0.0010 0.0010 ...

```
y1=lagrange1(x,y,x2)
```

```
y1 = 1×50  
0.0000 -0.6461 -0.8080 -0.6806 -0.4039 -0.0749 ...
```

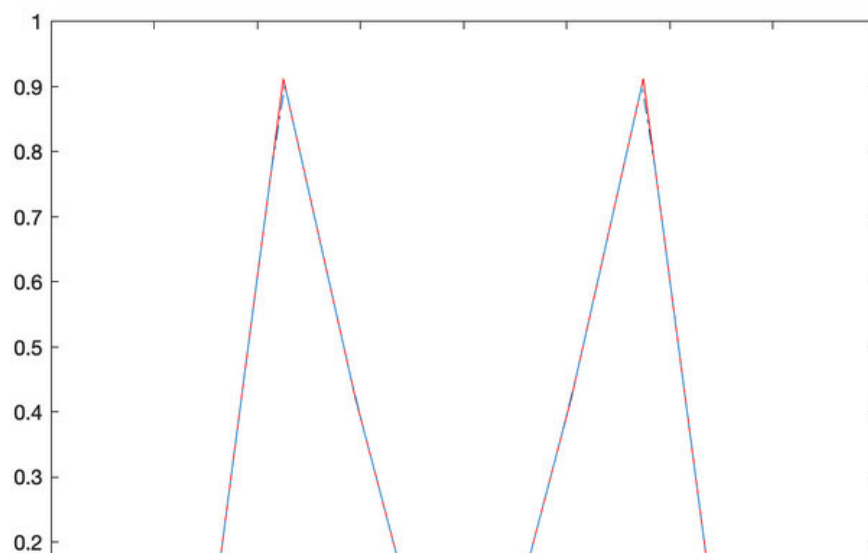
```
plot(x,y,"-r",x2,y1,"-b.")
```

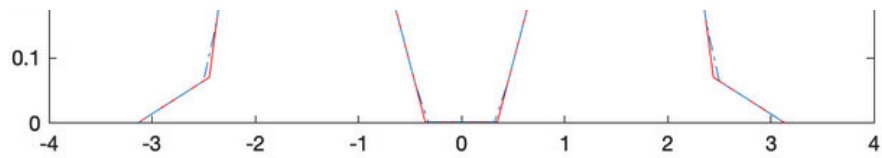


```
y2=interp1(x,y,x2,"linear")
```

```
y2 = 1×50  
0.0000 0.0130 0.0259 0.0389 0.0518 0.0648 ...
```

```
plot(x,y,"-r",x2,y2,"-b.")
```

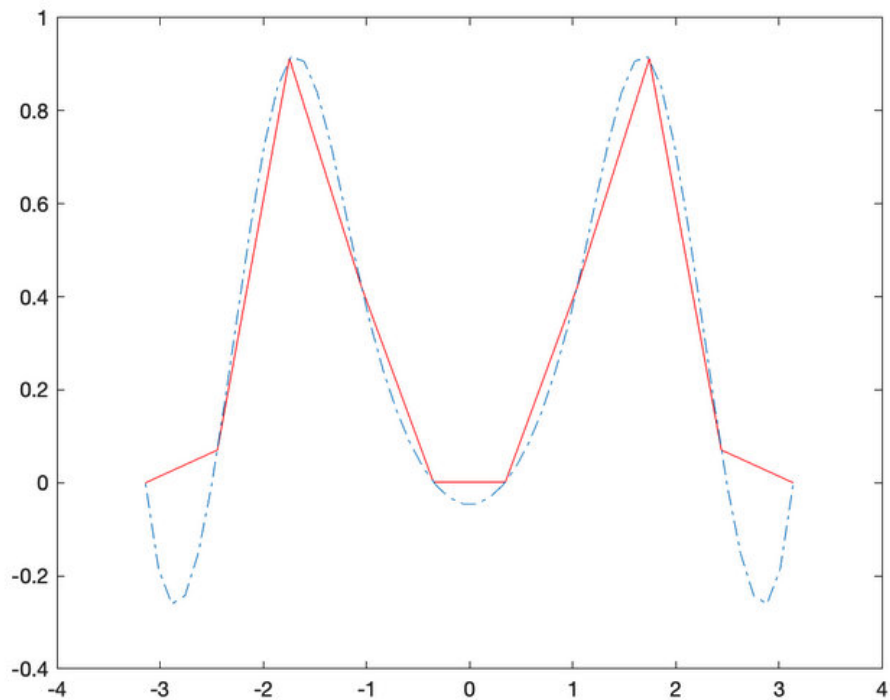




```
y3=interp1(x,y,x2,"spline")
```

```
y3 = 1×50
    0.0000    -0.1860    -0.2604    -0.2424    -0.1512    -0.0062 ...
```

```
plot(x,y,"-r",x2,y3,"-b")
```



```
clear;
```

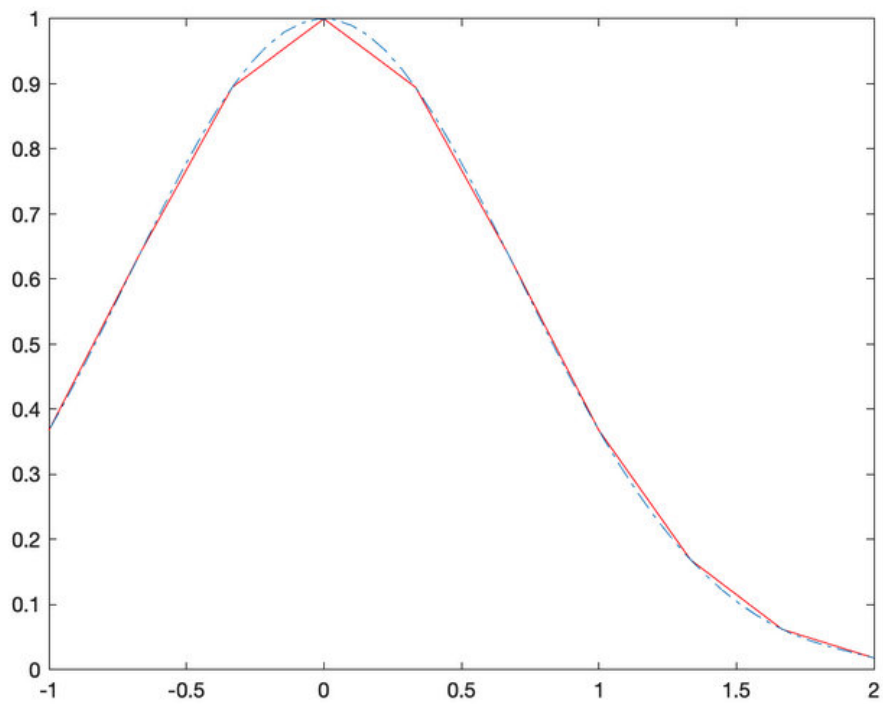
```
a=-1;
b=2;
x=linspace(a,b,10);
x2=linspace(a,b,50);
y=f3(x)
```

```
y = 1×10
    0.3679    0.6412    0.8948    1.0000    0.8948    0.6412 ...
```

```
y1=lagrange1(x,y,x2)
```

```
y1 = 1×50
    0.3679    0.4142    0.4629    0.5136    0.5655    0.6179 ...
```

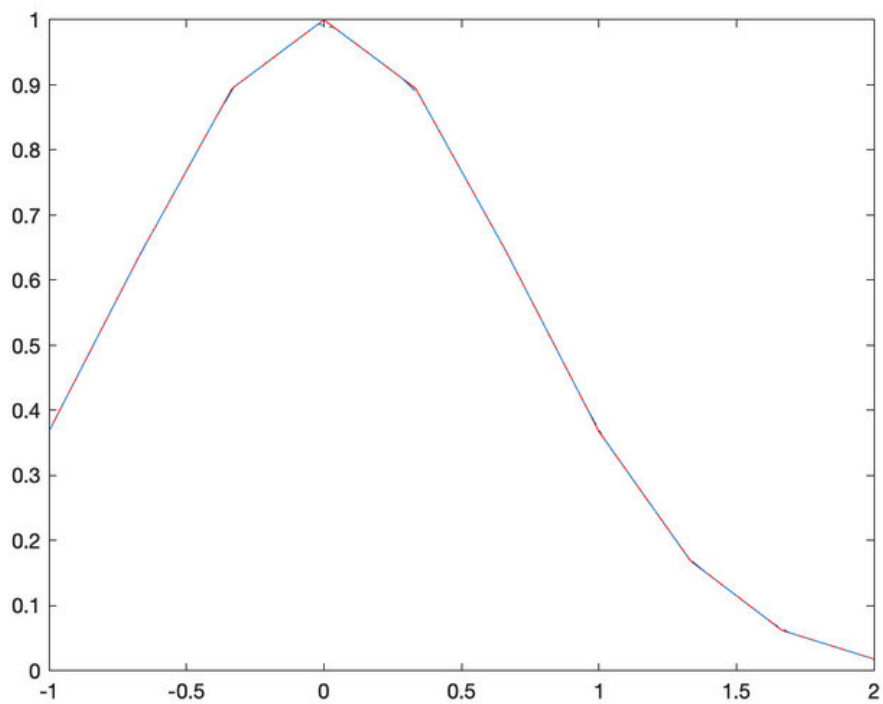
```
plot(x,y,"-r",x2,y1,"-b")
```



```
y2=interp1(x,y,x2,"linear")
```

```
y2 = 1×50
    0.3679    0.4181    0.4683    0.5185    0.5687    0.6189 ...
```

```
plot(x,y,"-r",x2,y2,"-b")
```



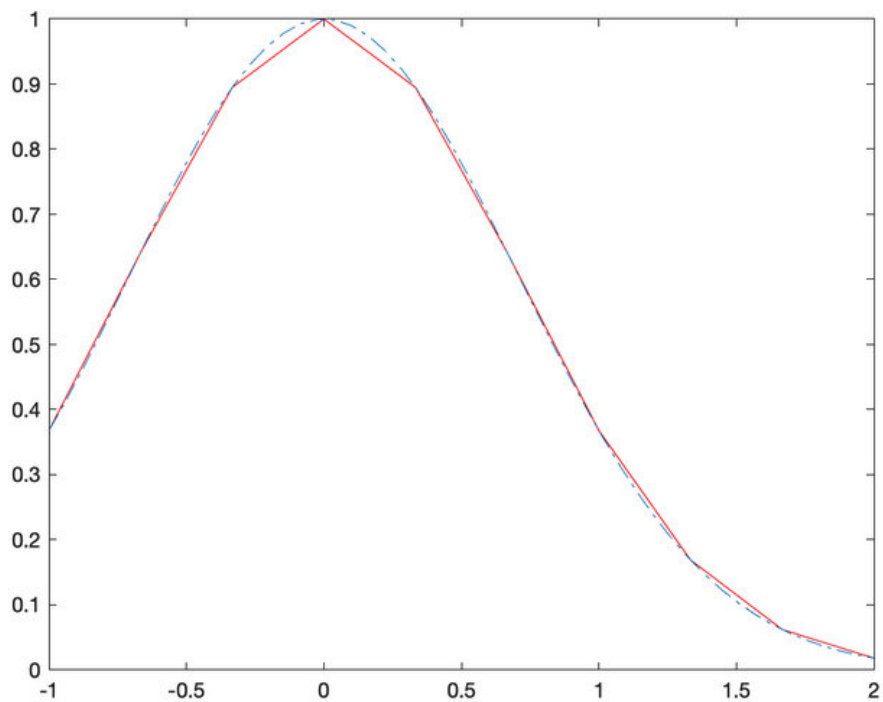
```
y3=interp1(x,y,x2,"spline")
```

```
y3 = 1×50
```

y3 = 1×50

0.3679 0.4132 0.4617 0.5126 0.5649 0.6177 ...

plot(x,y,"-r",x2,y3,"-b")



3. 某天的气温变化如第 3 题数据表，试用最小二乘法找出这一天的气温变化规律。↵

第 3 题数据表（工作表：气温变化）↵

| | | | | | | | | | | | | | |
|----------------------------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| 时刻 $t(\text{h})$ ↵ | 0 ↵ | 1 ↵ | 2 ↵ | 3 ↵ | 4 ↵ | 5 ↵ | 6 ↵ | 7 ↵ | 8 ↵ | 9 ↵ | 10 ↵ | 11 ↵ | 12 ↵ |
| 温度 $T(^{\circ}\text{C})$ ↵ | 15 ↵ | 14 ↵ | 14 ↵ | 14 ↵ | 14 ↵ | 15 ↵ | 16 ↵ | 18 ↵ | 20 ↵ | 22 ↵ | 23 ↵ | 25 ↵ | 28 ↵ |
| 时刻 $t(\text{h})$ ↵ | 13 ↵ | 14 ↵ | 15 ↵ | 16 ↵ | 17 ↵ | 18 ↵ | 19 ↵ | 20 ↵ | 21 ↵ | 22 ↵ | 23 ↵ | 24 ↵ | ↵ |
| 温度 $T(^{\circ}\text{C})$ ↵ | 31 ↵ | 32 ↵ | 31 ↵ | 29 ↵ | 27 ↵ | 25 ↵ | 24 ↵ | 22 ↵ | 20 ↵ | 18 ↵ | 17 ↵ | 16 ↵ | ↵ |


```
% 3
t= linspace(0,24,25)
```

```
t = 1×25
      0      1      2      3      4      5      6      7      8      9     10
```

```
y=[15    14        14        14        14        15        16        18
```

```
      y = 1×25
          15      14      14      14      14      15      16      18      20      22      23
```

```
p = polyfit(t,y,5)
```

```
p = 1×6
      0.0001    -0.0037     0.0450     0.0157    -0.8304     15.0539
```

```
y1 = polyval(p,25)
```

```
y1 = 19.6326
```

4. 用电压 $V=10\text{v}$ 的电池给电容器充电, 电容器上 t 时刻的电压为 $u(t) = V - (V - V_0)e^{-t/\tau}$,

其中 V_0 是电容器的初始电压, τ 是充电常数。试由第 4 题数据表中的一组 $t, U(t)$ 数

据确定 V_0 和 τ 。

第 4 题数据表 (工作表: 电容器)

| | | | | | | | | |
|---------------|-----|-----|-----|-----|-----|---|-----|-----|
| $t(\text{s})$ | 0.5 | 1 | 2 | 3 | 4 | 5 | 7 | 9 |
| $U(t)$ | 6.4 | 6.5 | 7.3 | 8.2 | 8.7 | 9 | 9.4 | 9.6 |

```
% 4
% v(t) = V - (V - V0)e^(-t/T) => ln(V-v) = ln(V - V0) - t/T
% y = ln(V - v), x1 = -1/T, x2 = ln(V-V0)
% y = x1t + x2
% T = -1/x1, V0 = V - e^x2
```

```
V = 10;
t = [0.5,1,2,3,4,5,7,9];
v = [6.4,6.4,7.3,8.2,8.6,8.9,9.4,9.6];
y = log(V - v);
x = polyfit(t,y,1)
```

```
x = 1×2
    -0.2732    1.4690
```

```
T = 1/x(1)
```

```
T = -3.6609
```

```
V0 = V - exp(x(2))
```

```
V0 = 5.6551
```

5. 给定数据见第 5 题数据表：↵

第 5 题数据表（工作表：三次样条）↵

| | | | | | |
|----|---------|---------|---------|---------|----------|
| x↵ | 0.25↵ | 0.30↵ | 0.39↵ | 0.45↵ | 0.53↵↵ |
| y↵ | 0.5000↵ | 0.5477↵ | 0.6245↵ | 0.6708↵ | 0.7280↵↵ |

分别就下列边界条件求三次样条函数 $S(x)$ 并作图。↵

(1) $S'(0.25)=1, S'(0.53)=0.6868$;↵

(2) $S''(0.25)=S''(0.53)=0$.↵



由 Xnip 截图

```
% 5
```

```
x=[0.25 0.2 0.29 0.45 0.53]
```

```
x = 1×5
    0.2500    0.2000    0.2900    0.4500    0.5300
```

```
y=[0.5 0.5477 0.6245 0.6708 0.7280]
```

```
y = 1×5
    0.5000    0.5477    0.6245    0.6708    0.7280
```

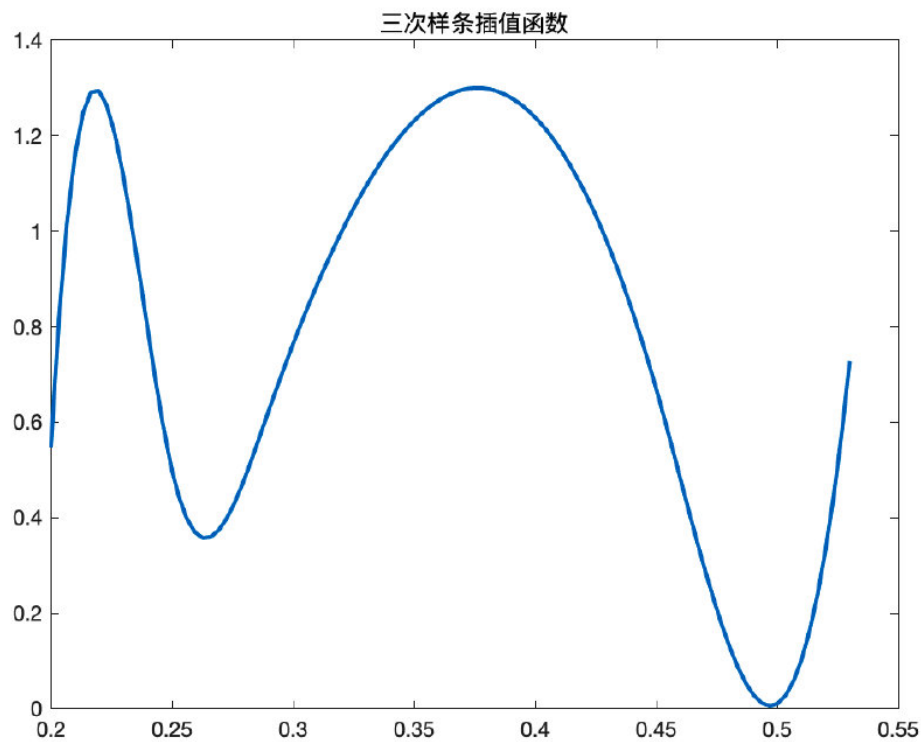
```
pp=csape(x,y,'complete',[1 0.6868])
```

```
pp = 包含以下字段的 struct:
      form: 'pp'
      breaks: [0.2000 0.2500 0.2900 0.4500 0.5300]
      coefs: [4×4 double]
      pieces: 4
      order: 4
      dim: 1
```

pp.coefs

```
ans = 4×4
104 ×
    2.8243   -0.3251    0.0091    0.0001
   -0.8746    0.0985   -0.0022    0.0001
   -0.0154   -0.0064    0.0015    0.0001
    0.4637   -0.0138   -0.0018    0.0001
```

```
fnplt(pp)
title("三次样条插值函数");
```



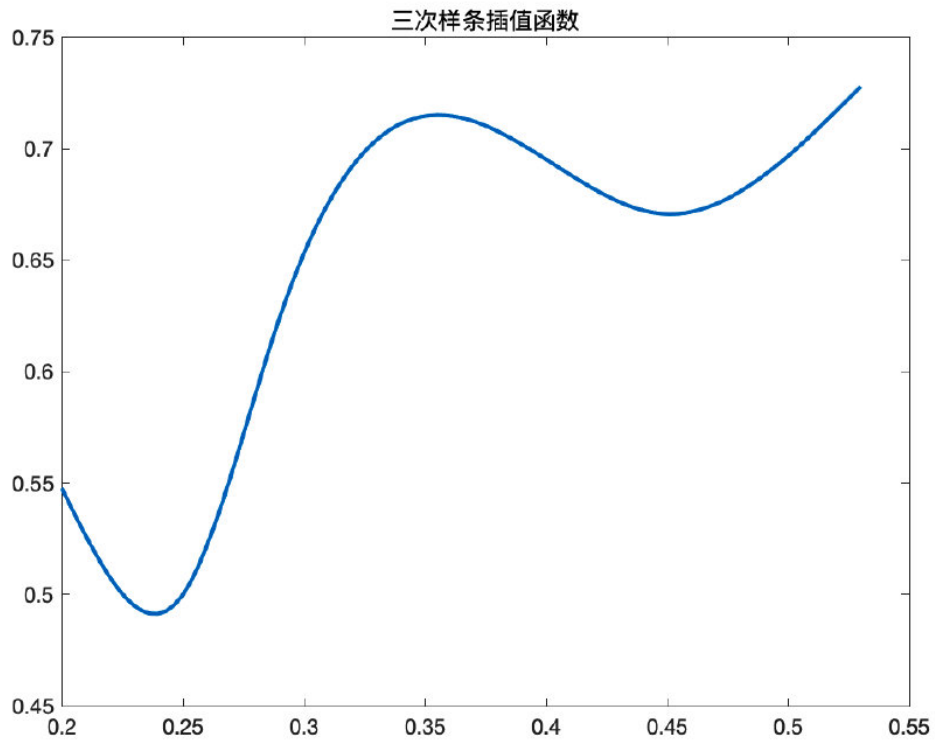
```
pp=csape(x,y,'variational','[0 0]')
```

```
pp = 包含以下字段的 struct:
      form: 'pp'
      breaks: [0.2000 0.2500 0.2900 0.4500 0.5300]
      coefs: [4×4 double]
      pieces: 4
      order: 4
      dim: 1
```

pp.coefs

```
ans = 4×4
    502.7368         0   -2.2108    0.5477
   -914.7533    75.4105    1.5597    0.5000
    100.9861   -34.3599    3.2017    0.6245
   -58.8060    14.1134   -0.0377    0.6708
```

```
fnplt(pp)
title("三次样条插值函数")
```



6. 用模型 $P = ae^{bt}$ 拟合以下数据, 求出拟合系数 a, b . 要求用两种方法: ↵

(1) 直接法 ↵

(2) 通过变换, 化成线性拟合问题 ↵

| | | | | | | | |
|-----|-----|------|-------|-------|-------|-------|---|
| t ↵ | 7 ↵ | 14 ↵ | 21 ↵ | 28 ↵ | 35 ↵ | 42 ↵ | ↵ |
| P ↵ | 8 ↵ | 41 ↵ | 133 ↵ | 250 ↵ | 280 ↵ | 297 ↵ | ↵ |

由 Xnip 截图

```
% 6
```

```
t=[7 14 21 28 35 42 ]
```

```
t = 1×6
```

```
7    14    21    28    35    42
```

```
P =[8 41 133 250 280 297]
```

```
P = 1×6
```

```
8    41    133    250    280    297
```

```
pp=polyfit(t,p,1)
```

```
pp = 1×2
    0.2970    -4.8959
```

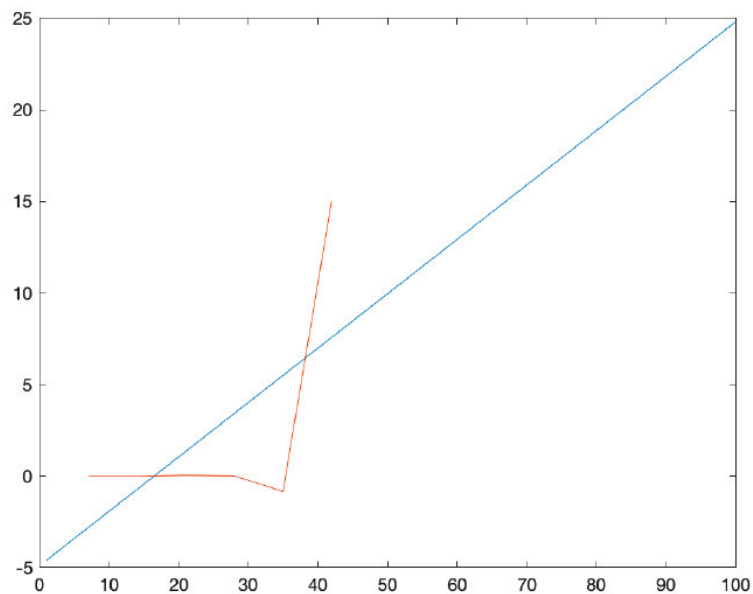
```
x=linspace(1,100,100)
```

```
x = 1×100
    1     2     3     4     5     6     7     8     9    10    11    12    13    14    15    16
```

```
y=polyval(pp,x)
```

```
y = 1×100
   -4.5989   -4.3019   -4.0049   -3.7079   -3.4110   -3.1140   -2.8170   -2.5200   -2.2231   -1.9261
```

```
plot(x,y,t,p)
```



```
pp=polyfit(t,p,4)
```

```
pp = 1×5
    0.0002   -0.0128    0.3680   -4.1779   15.3236
```

```
x=linspace(1,100,100)
```

```
x = 1×100
    1     2     3     4     5     6     7     8     9    10    11    12    13    14    15    16
```

```
y=polyval(pp,x)
```

```
y = 1×100
103 ×
    0.0115    0.0083    0.0058    0.0037    0.0021    0.0009    0.0001   -0.0005   -0.0008   -0.0011
```

```
plot(x,y,t,p)
```

