2.11

(1) $R_1 \cup R_2 = \{\langle a,b\rangle, \langle a,c\rangle, \langle b,d\rangle, \langle c,c\rangle, \langle c,d\rangle, \langle d,b\rangle, \langle d,d\rangle\}$；

$R_1 \cap R_2 = \{\langle b,d\rangle\}$；

$R_1 \oplus R_2 = (R_1 \cup R_2) - (R_1 \cap R_2)\{\langle a,b\rangle, \langle a,c\rangle, \langle c,c\rangle, \langle c,d\rangle, \langle d,b\rangle, \langle d,d\rangle\}$；

(2) $\operatorname{dom} R_1 = \{a,b,c\}$；

$\operatorname{dom} R_2 = \{a,b,d\}$；

$\operatorname{dom}(R_1 \cup R_2) = \operatorname{dom} R_1 \cup \operatorname{dom} R_2 = \{a,b,c,d\}$；

(3) $\operatorname{ran} R_1 = \{b,c,d\}$；

$\operatorname{ran} R_2 = \{b,c,d\}$；

$\operatorname{ran} R_1 \cap \operatorname{ran} R_2 = \{b,c,d\}$；

(4) $R_1 \restriction A = \{\langle a,b\rangle, \langle c,c\rangle, \langle c,d\rangle\}$；

$R_1 \restriction \{c\} = \{\langle c,c\rangle, \langle c,d\rangle\}$；

$(R_1 \cup R_2) \restriction A = \{\langle a,b\rangle, \langle a,c\rangle, \langle c,c\rangle, \langle c,d\rangle\}$；

$R_2 \restriction A = \{\langle a,c\rangle\}$；

(5) $R_1[A] = \{b,c,d\}$；

$R_2[A] = \{c\}$；

$(R_1 \cap R_2)[A] = \varnothing$；

(6) $R_1 \circ R_2 = \{\langle a,c\rangle, \langle a,d\rangle, \langle d,d\rangle\}$；

$R_2 \circ R_1 = \{\langle a,d\rangle, \langle b,b\rangle, \langle b,d\rangle, \langle c,b\rangle, \langle c,d\rangle\}$；

$R_1 \circ R_1 = \{\langle a,d\rangle, \langle c,c\rangle, \langle c,d\rangle\}$。

2.12

(1) $R^{-1} = \{\langle\{\varnothing,\{\varnothing\}\},\varnothing\rangle, \langle\varnothing,\{\varnothing\}\rangle, \langle\varnothing,\varnothing\rangle\}$；

(2) $R \circ R = \{\langle\varnothing,\varnothing\rangle, \langle\varnothing,\{\varnothing,\{\varnothing\}\}\rangle, \langle\{\varnothing\},\varnothing\rangle, \langle\{\varnothing\},\{\varnothing,\{\varnothing\}\}\rangle\}$；

(3) $R \restriction \varnothing = \varnothing$；

$R \restriction \{\varnothing\} = \{\langle\varnothing,\{\varnothing,\{\varnothing\}\}\rangle, \langle\varnothing,\varnothing\rangle\}$；

$R \restriction \{\{\varnothing\}\} = \{\langle\{\varnothing\},\varnothing\rangle\}$；

$R \restriction \{\varnothing,\{\varnothing\}\} = R = \{\langle\varnothing,\{\varnothing,\{\varnothing\}\}\rangle, \langle\{\varnothing\},\varnothing\rangle, \langle\varnothing,\varnothing\rangle\}$；

(4) $R[\varnothing] = \varnothing$；

$R[\{\varnothing\}] = \{\{\varnothing,\{\varnothing\}\},\varnothing\}$；

$R[\{\{\varnothing\}\}] = \{\varnothing\}$；

$R[\{\varnothing,\{\varnothing\}\}] = \operatorname{ran} R = \{\{\varnothing,\{\varnothing\}\},\varnothing\}$；

(5) $\operatorname{dom} R = \{\varnothing,\{\varnothing\}\}$；

$\operatorname{ran} R = \{\{\varnothing,\{\varnothing\}\},\varnothing\}$；

$\operatorname{fld} R = \operatorname{dom} R \cup \operatorname{ran} R = \{\varnothing,\{\varnothing\},\{\varnothing,\{\varnothing\}\}\}$。

2.13

(1)

证明: 由 R 是二元关系易知，$R \cup R^{-1}$ 也是二元关系。

由引理 1.3 知，$R \subseteq R \cup R^{-1}$，即 $R \cup R^{-1}$ 包含 R。

而对任意 $\langle x,y\rangle$，有：

$$
\begin{aligned}
& \langle x,y\rangle \in R \cup R^{-1} \\
\Longleftrightarrow\ & \langle x,y\rangle \in R \lor \langle x,y\rangle \in R^{-1} && \text{(集合并定义)} \\
\Longleftrightarrow\ & \langle y,x\rangle \in R^{-1} \lor \langle y,x\rangle \in R && \text{(逆关系定义)}
\end{aligned}
$$