P94. 4. 令 $u=x+y$, $v=\frac{y}{x}$, 则 $x=\frac{u}{1+v}$, $y=\frac{uv}{1+v}$,

$$f(u,v)=f(x+y,\frac{y}{x})=x^2-y^2=\left(\frac{u}{1+v}\right)^2-\left(\frac{uv}{1+v}\right)^2=\frac{u^2(1-v)}{1+v},$$

$$f(x,y)=\frac{x^2(1-y)}{1+y}$$

7. (1) $$\lim_{\substack{(x,y)\to(0,0)\\ y=kx}} f(x,y)=\lim_{\substack{(x,y)\to(0,0)\\ y=kx}}\frac{x^4y^4}{(x^4+y^2)^3}=\lim_{x\to 0}\frac{x^4\cdot k^4x^4}{(x^4+k^2x^2)^3}=\lim_{x\to 0}\frac{k^4x^2}{(x^2+k^2)^3}=0$$

$$\lim_{\substack{(x,y)\to(0,0)\\ x=0}} f(x,y)=\lim_{\substack{(x,y)\to(0,0)\\ x=0}}\frac{x^4y^4}{(x^4+y^2)^3}=\lim_{y\to 0}\frac{0^4\cdot y^4}{(0^4+y^2)^3}=0$$

∴ 当 (x,y) 沿任何直线趋于原点时, $f(x,y)$ 趋于 0.

(2) $$\lim_{\substack{(x,y)\to(0,0)\\ y=kx^2}} f(x,y)=\lim_{\substack{(x,y)\to(0,0)\\ y=kx^2}}\frac{x^4y^4}{(x^4+y^2)^3}=\lim_{x\to 0}\frac{x^4\cdot(kx^2)^4}{(x^4+k^2x^4)^3}=\frac{k^4}{(1+k^2)^3}$$ 与 k 有关,

∴ $\lim\limits_{\substack{x\to 0\\ y\to 0}} f(x,y)$ 不存在.

8. (3) $$\lim_{\substack{x\to 0\\ y\to 0}}\frac{\sqrt{(1+4x^2)(1+6y^2)}-1}{2x^2+3y^2}=\lim_{\substack{x\to 0\\ y\to 0}}\left[\frac{(1+4x^2)(1+6y^2)-1}{2x^2+3y^2}\cdot\frac{1}{\sqrt{(1+4x^2)(1+6y^2)}+1}\right]$$

$$=\frac{1}{2}\lim_{\substack{x\to 0\\ y\to 0}}\frac{(1+4x^2)(1+6y^2)-1}{2x^2+3y^2}=\frac{1}{2}\lim_{\substack{x\to 0\\ y\to 0}}\frac{4x^2+6y^2+24x^2y^2}{2x^2+3y^2}=\frac{1}{2}\lim_{\substack{x\to 0\\ y\to 0}}\left[2+\frac{24x^2y^2}{2x^2+3y^2}\right]$$

$$=\frac{1}{2}(2+0)=1.$$ （注: $\lim\limits_{\substack{x\to 0\\ y\to 0}}\frac{24x^2y^2}{2x^2+3y^2}=\lim\limits_{\substack{x\to 0\\ y\to 0}}\left[\underbrace{\frac{24x^2}{2x^2+3y^2}}_{有界}\underbrace{y^2}_{0}\right]=0$ ）

(4) ∵ $$0\le |x^2y^2\ln(x^2+y^2)|\le \left|\left(\frac{x^2+y^2}{2}\right)^2\ln(x^2+y^2)\right|=\frac{1}{4}|(x^2+y^2)^2\ln(x^2+y^2)|$$

$$\lim_{\substack{x\to 0\\ y\to 0}}(x^2+y^2)^2\ln(x^2+y^2)=\lim_{t\to 0^+}t^2\ln t=\lim_{t\to 0^+}\frac{\ln t}{t^{-2}}\overset{洛}{=}\lim_{t\to 0^+}\frac{\frac{1}{t}}{-2t^{-3}}=\lim_{t\to 0^+}\frac{t^2}{-2}=0$$

由两边夹定理知 $\lim\limits_{\substack{x\to 0\\ y\to 0}}|x^2y^2\ln(x^2+y^2)|=0$, 从而 $\lim\limits_{\substack{x\to 0\\ y\to 0}}x^2y^2\ln(x^2+y^2)=0$.

$$\lim_{\substack{x\to 0\\ y\to 0}}(x^2+y^2)^{x^2y^2}=\lim_{\substack{x\to 0\\ y\to 0}}e^{x^2y^2\ln(x^2+y^2)}=e^{\lim\limits_{\substack{x\to 0\\ y\to 0}}[x^2y^2\ln(x^2+y^2)]}=e^0=1.$$

8. (5) $\because 0 \le |(x+y)\ln(x^2+y^2)| \le (|x|+|y|)\cdot|\ln(x^2+y^2)| \le \sqrt{2(x^2+y^2)}\cdot|\ln(x^2+y^2)|$

又 $\lim\limits_{\substack{x\to 0\\ y\to 0}} \sqrt{2(x^2+y^2)}\ln(x^2+y^2) = \lim\limits_{t\to 0^+} \sqrt{2}\, t^{\frac{1}{2}}\ln t = \sqrt{2}\lim\limits_{t\to 0^+}\frac{\ln t}{t^{-\frac{1}{2}}} \overset{洛}{=} \sqrt{2}\lim\limits_{t\to 0^+}\frac{\frac{1}{t}}{-\frac{1}{2}t^{-\frac{3}{2}}} = -2\sqrt{2}\lim\limits_{t\to 0^+} t^{\frac{1}{2}} = 0$

$\therefore \lim\limits_{\substack{x\to 0\\ y\to 0}} \sqrt{2(x^2+y^2)}|\ln(x^2+y^2)| = 0$, $\therefore \lim\limits_{\substack{x\to 0\\ y\to 0}} |(x+y)\ln(x^2+y^2)| = 0$, $\therefore \lim\limits_{\substack{x\to 0\\ y\to 0}} (x+y)\ln(x^2+y^2) = 0$.

(6) $\because \lim\limits_{\substack{x\to 1\\ y\to 2}} (2x-y) = 0$, $\therefore \lim\limits_{\substack{x\to 1\\ y\to 2}} \frac{1}{2x-y} = \infty$.

(7) $\because 0 \le \frac{x^2}{x^4+y^4} \le \frac{x^2}{x^4} = \frac{1}{x^2}$, $0 \le \frac{y^2}{x^4+y^4} \le \frac{y^2}{y^4} = \frac{1}{y^2}$, 又 $\lim\limits_{\substack{x\to +\infty\\ y\to +\infty}} \frac{1}{x^2} = 0$, $\lim\limits_{\substack{x\to +\infty\\ y\to +\infty}} \frac{1}{y^2} = 0$,

$\therefore \lim\limits_{\substack{x\to +\infty\\ y\to +\infty}} \frac{x^2}{x^4+y^4} = 0$, $\lim\limits_{\substack{x\to +\infty\\ y\to +\infty}} \frac{y^2}{x^4+y^4} = 0$

$$\therefore \lim_{\substack{x\to +\infty\\ y\to +\infty}} \frac{x^2+y^2}{x^4+y^4} = \lim_{\substack{x\to +\infty\\ y\to +\infty}} \left(\frac{x^2}{x^4+y^4} + \frac{y^2}{x^4+y^4}\right) = \lim_{\substack{x\to +\infty\\ y\to +\infty}} \frac{x^2}{x^4+y^4} + \lim_{\substack{x\to +\infty\\ y\to +\infty}} \frac{y^2}{x^4+y^4} = 0+0 = 0.$$

(8) $\lim\limits_{\substack{x\to +\infty\\ y\to 0}} \left(1+\frac{1}{x}\right)^{\frac{x^2}{x+y}} = \lim\limits_{\substack{x\to +\infty\\ y\to 0}} \left[\left(1+\frac{1}{x}\right)^x\right]^{\frac{x}{x+y}} = e^{\lim\limits_{\substack{x\to +\infty\\ y\to 0}} \frac{x}{x+y}} = e^{\lim\limits_{\substack{x\to +\infty\\ y\to 0}} \frac{1}{1+\frac{y}{x}}} = e$.

(9) $\lim\limits_{\substack{x\to 0\\ y\to 0}} \frac{\sin(x^3+y^3)}{x^2+y^2} = \lim\limits_{\substack{x\to 0\\ y\to 0}} \left[\frac{\sin(x^3+y^3)}{x^3+y^3}\cdot\frac{x^3+y^3}{x^2+y^2}\right] = \lim\limits_{\substack{x\to 0\\ y\to 0}} \frac{x^3+y^3}{x^2+y^2} = \lim\limits_{\substack{x\to 0\\ y\to 0}} \left[\frac{x^2}{x^2+y^2}\cdot x + \frac{y^2}{x^2+y^2}\cdot y\right]$

$= 0+0 = 0$.

(注 $\because \frac{x^2}{x^2+y^2}$ 有界, $\lim\limits_{\substack{x\to 0\\ y\to 0}} x = 0$, $\therefore \lim\limits_{\substack{x\to 0\\ y\to 0}} \left(\frac{x^2}{x^2+y^2}\cdot x\right) = 0$).

(10) $x>0$ 且 $y>0$ 时, $0 \le (x^2+y^2)e^{-(x+y)} = \frac{x^2+y^2}{e^{x+y}} \le \frac{(x+y)^2}{e^{x+y}}$

$$\lim_{\substack{x\to +\infty\\ y\to +\infty}} \frac{(x+y)^2}{e^{x+y}} = \lim_{t\to +\infty} \frac{t^2}{e^t} \overset{洛}{=} \lim_{t\to +\infty} \frac{2t}{e^t} \overset{洛}{=} \lim_{t\to +\infty} \frac{2}{e^t} = 0$$

$\therefore \lim\limits_{\substack{x\to +\infty\\ y\to +\infty}} (x^2+y^2)e^{-(x+y)} = 0$.

P108. 1. (1) $z=(1+xy)^y$, $\frac{\partial z}{\partial x}=y(1+xy)^{y-1}\cdot y=y^2(1+xy)^{y-1}$

$$\frac{\partial z}{\partial y}=\frac{\partial}{\partial y}[(1+xy)^y]=\frac{\partial}{\partial y}[e^{y\ln(1+xy)}]=e^{y\ln(1+xy)}\cdot\left[\ln(1+xy)+y\cdot\frac{1}{1+xy}\cdot x\right]=(1+xy)^y\cdot\left[\ln(1+xy)+\frac{xy}{1+xy}\right].$$

(5) $u=\left(\frac{x}{y}\right)^z$, $\frac{\partial u}{\partial x}=z\left(\frac{x}{y}\right)^{z-1}\cdot\frac{1}{y}=\frac{z}{y}\left(\frac{x}{y}\right)^{z-1}$.

$$\frac{\partial u}{\partial y}=z\left(\frac{x}{y}\right)^{z-1}\cdot\left(-\frac{x}{y^2}\right)=-\frac{zx}{y^2}\left(\frac{x}{y}\right)^{z-1},\quad \frac{\partial u}{\partial z}=\left(\frac{x}{y}\right)^z\ln\frac{x}{y}.$$

3. $$f'_x(0,1)=\lim_{x\to0}\frac{f(x,1)-f(0,1)}{x-0}=\lim_{x\to0}\frac{\frac{1}{2x}\sin(x^2)-0}{x}=\lim_{x\to0}\frac{\sin(x^2)}{2x^2}=\frac{1}{2}.$$

$$f'_y(0,1)=\lim_{y\to1}\frac{f(0,y)-f(0,1)}{y-1}=\lim_{y\to1}\frac{0-0}{y-1}=0.$$

6. $$f'_x(0,0)=\lim_{x\to0}\frac{f(x,0)-f(0,0)}{x-0}=\lim_{x\to0}\frac{0-0}{x}=0$$

$$f'_y(0,0)=\lim_{y\to0}\frac{f(0,y)-f(0,0)}{y-0}=\lim_{y\to0}\frac{y\sin\frac{1}{y^2}-0}{y}=\lim_{y\to0}\sin\frac{1}{y^2}\text{ 不存在}.$$

7. (1) $\because \lim\limits_{\substack{(x,y)\to(0,0)\\ y=x}}f(x,y)=\lim\limits_{\substack{(x,y)\to(0,0)\\ y=x}}\frac{x^3y}{x^6+y^6}=\lim\limits_{x\to0}\frac{x^3\cdot x}{x^6+x^6}=\lim\limits_{x\to0}\frac{1}{2x^2}=\infty$, $\therefore f(x,y)$在$(0,0)$处不连续.

(2) $\because f'_x(0,0)=\lim\limits_{x\to0}\frac{f(x,0)-f(0,0)}{x-0}=\lim\limits_{x\to0}\frac{0-0}{x}=0$, $f'_y(0,0)=\lim\limits_{y\to0}\frac{f(0,y)-f(0,0)}{y-0}=\lim\limits_{y\to0}\frac{0-0}{y}=0$,

$\therefore f(x,y)$在$(0,0)$处的两个偏导数都存在.

(3) 当$x^2+y^2\neq0$时, $f'_x(x,y)=\frac{3x^2y(x^6+y^6)-x^3y\cdot6x^5}{(x^6+y^6)^2}=\frac{3x^2y(y^6-x^6)}{(x^6+y^6)^2}$

$$\because \lim_{\substack{x\to0\\ y\to0\\ y=2x}}f'_x(x,y)=\lim_{\substack{x\to0\\ y\to0\\ y=2x}}\frac{3x^2y(y^6-x^6)}{(x^6+y^6)^2}=\lim_{x\to0}\frac{3x^2\cdot2x(2^6x^6-x^6)}{(x^6+2^6x^6)^2}=\lim_{x\to0}\frac{6(2^6-1)x^9}{(1+2^6)^2\cdot x^{12}}=\lim_{x\to0}\frac{6(2^6-1)}{(1+2^6)^2\cdot x^3}=\infty$$

$\therefore$ $\cancel{f_x(x,}$ $\lim\limits_{\substack{x\to0\\ y\to0}}f'_x(x,y)$不存在, $\therefore f'_x(x,y)$在$(0,0)$处不连续.

类似地, 可证明$f'_y(x,y)$在$(0,0)$处不连续. (当$x^2+y^2\neq0$时, $f'_y(x,y)=\frac{x^3(x^6-5y^6)}{(x^6+y^6)^2}$ ……)

10. (2) $$dz=\frac{\partial z}{\partial x}dx+\frac{\partial z}{\partial y}dy=\left[\ln(xy)+x\cdot\frac{1}{xy}\cdot y\right]dx+\left[x\cdot\frac{1}{xy}\cdot x\right]dy=[\ln(xy)+1]dx+\frac{x}{y}dy$$

$$dz\Big|_{M(-1,-1)}=[\ln(xy)+1]\Big|_{M(-1,-1)}dx+\frac{x}{y}\Big|_{M(-1,-1)}dy=dx+dy.$$

P109. 11. $f_x'(0,0)=\lim\limits_{x\to 0}\dfrac{f(x,0)-f(0,0)}{x-0}=\lim\limits_{x\to 0}\dfrac{0-0}{x}=0$

$f_y'(0,0)=\lim\limits_{y\to 0}\dfrac{f(0,y)-f(0,0)}{y-0}=\lim\limits_{y\to 0}\dfrac{0-0}{y}=0$

$\because \lim\limits_{\substack{x\to 0\\ y\to 0}}\dfrac{f(x,y)-f(0,0)-f_x'(0,0)x-f_y'(0,0)y}{\sqrt{x^2+y^2}}=\lim\limits_{\substack{x\to 0\\ y\to 0}}\dfrac{xy\sin\frac{1}{x^2+y^2}-0-0-0}{\sqrt{x^2+y^2}}$

$=\lim\limits_{\substack{x\to 0\\ y\to 0}}\left[x\cdot\dfrac{y}{\sqrt{x^2+y^2}}\cdot\sin\dfrac{1}{x^2+y^2}\right]=0$ （因为 $\lim\limits_{\substack{x\to 0\\ y\to 0}}x=0$，$\dfrac{y}{\sqrt{x^2+y^2}}\cdot\sin\dfrac{1}{x^2+y^2}$ 有界）

$\therefore f(x,y)$ 在 $(0,0)$ 处可微.

12. $\because \lim\limits_{\substack{x\to 0\\ y\to 0}}f(x,y)=\lim\limits_{\substack{x\to 0\\ y\to 0}}\dfrac{x^2y}{x^2+y^2}=\lim\limits_{\substack{x\to 0\\ y\to 0}}\left[\dfrac{x^2}{x^2+y^2}\cdot y\right]=0=f(0,0)$ （注：$\dfrac{x^2}{x^2+y^2}$ 有界，$\lim\limits_{\substack{x\to 0\\ y\to 0}}y=0$）

$\therefore f(x,y)$ 在 $(0,0)$ 处连续.

$f_x'(0,0)=\lim\limits_{x\to 0}\dfrac{f(x,0)-f(0,0)}{x-0}=\lim\limits_{x\to 0}\dfrac{0-0}{x}=0$

$f_y'(0,0)=\lim\limits_{y\to 0}\dfrac{f(0,y)-f(0,0)}{y-0}=\lim\limits_{y\to 0}\dfrac{0-0}{y}=0$

$\lim\limits_{\substack{x\to 0\\ y\to 0}}\dfrac{f(x,y)-f(0,0)-f_x'(0,0)x-f_y'(0,0)y}{\sqrt{x^2+y^2}}=\lim\limits_{\substack{x\to 0\\ y\to 0}}\dfrac{\frac{x^2y}{x^2+y^2}-0-0-0}{\sqrt{x^2+y^2}}$

$=\lim\limits_{\substack{x\to 0\\ y\to 0}}\dfrac{x^2y}{(x^2+y^2)^{\frac{3}{2}}}$

而 $\lim\limits_{\substack{x\to 0\\ y\to 0\\ y=kx}}\dfrac{x^2y}{(x^2+y^2)^{\frac{3}{2}}}=\lim\limits_{x\to 0}\dfrac{x^2\cdot kx}{(x^2+k^2x^2)^{\frac{3}{2}}}=\dfrac{k}{(1+k^2)^{\frac{3}{2}}}$ 与 k 有关

$\therefore \lim\limits_{\substack{x\to 0\\ y\to 0}}\dfrac{x^2y}{(x^2+y^2)^{\frac{3}{2}}}$ 不存在

$\therefore f(x,y)$ 在 $(0,0)$ 处不可微.

P114 1. (1) $\frac{\partial z}{\partial x}\Big|_{P_0} = -\sin(x+y)\Big|_{(0,\frac{\pi}{2})} = -1$, $\frac{\partial z}{\partial y}\Big|_{P_0} = -\sin(x+y)\Big|_{(0,\frac{\pi}{2})} = -1$, $\|l\|=5$,

$e_l = (\frac{3}{5}, -\frac{4}{5})$

$$\frac{\partial z}{\partial l}\Big|_{P_0} = \frac{\partial z}{\partial x}\Big|_{P_0}\cdot\frac{3}{5} + \frac{\partial z}{\partial y}\Big|_{P_0}\cdot(-\frac{4}{5}) = -1\cdot\frac{3}{5} + (-1)\cdot(-\frac{4}{5}) = \frac{1}{5}.$$

(2) $\frac{\partial u}{\partial x}\Big|_{P_0} = (y^2-yz)\Big|_{(1,1,2)} = -1$, $\frac{\partial u}{\partial y}\Big|_{P_0} = (2xy-xz)\Big|_{(1,1,2)} = 0$

$\frac{\partial u}{\partial z}\Big|_{P_0} = (3z^2-xy)\Big|_{(1,1,2)} = 11$, $\|l\| = \sqrt{\cos^2\frac{\pi}{3}+\cos^2\frac{\pi}{4}+\cos^2\frac{\pi}{3}} = 1$.

$e_l = (\cos\frac{\pi}{3}, \cos\frac{\pi}{4}, \cos\frac{\pi}{3}) = (\frac{1}{2}, \frac{\sqrt{2}}{2}, \frac{1}{2})$

$$\frac{\partial u}{\partial l}\Big|_{P_0} = \frac{\partial u}{\partial x}\Big|_{P_0}\cdot\frac{1}{2} + \frac{\partial u}{\partial y}\Big|_{P_0}\cdot\frac{\sqrt{2}}{2} + \frac{\partial u}{\partial z}\Big|_{P_0}\cdot\frac{1}{2} = -1\cdot\frac{1}{2} + 0\cdot\frac{\sqrt{2}}{2} + 11\cdot\frac{1}{2} = 5.$$

3. (1) $\frac{\partial f}{\partial x}\Big|_{(0,0)} = [e^x(\cos y+\sin y)]\Big|_{(0,0)} = 1$, $\frac{\partial f}{\partial y}\Big|_{(0,0)} = [e^x(-\sin y+\cos y)]\Big|_{(0,0)} = 1$

$f(x,y)$ 在 $(0,0)$ 处增加最快的方向是 $\text{grad} f\Big|_{(0,0)} = (\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y})\Big|_{(0,0)} = (1,1)$.

4. (2) $\frac{\partial f}{\partial x}\Big|_{(-1,1,3)} = \frac{1}{y+z}\Big|_{(-1,1,3)} = \frac{1}{4}$, $\frac{\partial f}{\partial y}\Big|_{(-1,1,3)} = -\frac{x-z}{(y+z)^2}\Big|_{(-1,1,3)} = \frac{1}{4}$

$$\frac{\partial f}{\partial z}\Big|_{(-1,1,3)} = \frac{-(y+z)-(x-z)}{(y+z)^2}\Big|_{(-1,1,3)} = 0$$

$f(x,y,z)$ 在 $(-1,1,3)$ 处减小最快的方向是 $-\text{grad} f\Big|_{(-1,1,3)} = -(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z})\Big|_{(-1,1,3)}$

$= -(\frac{1}{4}, \frac{1}{4}, 0)$.

P123. 1. (2) 法一. $\frac{du}{dt}=\frac{\partial u}{\partial x}\cdot\frac{dx}{dt}+\frac{\partial u}{\partial y}\cdot\frac{dy}{dt}+\frac{\partial u}{\partial t}=e^{x-2y}\cdot\cos t+e^{x-2y}\cdot(-2)\cdot 3t^2-\frac{1}{t^2}$

$=e^{\sin t-2t^3}\cos t-6t^2 e^{\sin t-2t^3}-\frac{1}{t^2}=(\cos t-6t^2)e^{\sin t-2t^3}-\frac{1}{t^2}$

法二. $u=e^{x-2y}+\frac{1}{t}=e^{\sin t-2t^3}+\frac{1}{t}$

$\frac{du}{dt}=e^{\sin t-2t^3}\cdot(\cos t-6t^2)+\frac{1}{t}$.

2. 法一. $\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}=e^u\sin v\cdot y+e^u\cos v\cdot 1=[y\sin(x-y)+\cos(x-y)]e^{xy}$,

$\frac{\partial z}{\partial y}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y}=e^u\sin v\cdot x+e^u\cos v\cdot(-1)=[x\sin(x-y)-\cos(x-y)]e^{xy}$.

法二. $z=e^u\sin v=e^{xy}\sin(x-y)$

$\frac{\partial z}{\partial x}=e^{xy}\cdot y\cdot\sin(x-y)+e^{xy}\cdot\cos(x-y)\cdot 1=[y\sin(x-y)+\cos(x-y)]e^{xy}$,

$\frac{\partial z}{\partial y}=e^{xy}\cdot x\cdot\sin(x-y)+e^{xy}\cdot\cos(x-y)\cdot(-1)=[x\sin(x-y)-\cos(x-y)]e^{xy}$.

5. $z=\sin y+f(\sin x-\sin y)$, $\cancel{\frac{\partial z}{\partial y}=\cos}$ 记 $f'=f'(\sin x-\sin y)$

$\frac{\partial z}{\partial x}=f'\cdot\cos x$, $\frac{\partial z}{\partial y}=\cos y+f'\cdot(-\cos y)=(1-f')\cos y$

$\frac{\partial z}{\partial x}\cdot\sec x+\frac{\partial z}{\partial y}\sec y=f'\cdot\cos x\cdot\sec x+(1-f')\cos y\cdot\sec y=f'+(1-f')=1$.

6. $F_x'(x,t)=f'(x+2t)\cdot 1+f'(3x-2t)\cdot 3$, $F_x'(0,0)=f'(0)\cdot 1+f'(0)\cdot 3=4f'(0)$.

$F_t'(x,t)=f'(x+2t)\cdot 2+f'(3x-2t)\cdot(-2t)$, $F_t'(0,0)=f'(0)\cdot 2+f'(0)\cdot(-2)=0$.

8. (2) $dz=df(xy,\frac{x}{y})=f_1'd(xy)+f_2'd(\frac{x}{y})=f_1'[ydx+xdy]+f_2'\cdot\frac{ydx-xdy}{y^2}$

$=(yf_1'+\frac{f_2'}{y})dx+(xf_1'-\frac{x}{y^2}f_2')dy$

$\frac{\partial z}{\partial x}=yf_1'+\frac{f_2'}{y}$, $\frac{\partial z}{\partial y}=xf_1'-\frac{x}{y^2}f_2'$

P123. 4. (1) $\frac{\partial z}{\partial x}=f_1'+2xf_2'$, $\frac{\partial z}{\partial y}=f_1'+2yf_2'$.

(2) $\frac{\partial z}{\partial x}=f_1'\cdot\frac{1}{y}+f_2'\cdot(-\frac{y}{x^2})=\frac{1}{y}f_1'-\frac{y}{x^2}f_2'$, $\frac{\partial z}{\partial y}=f_1'\cdot(-\frac{x}{y^2})+f_2'\cdot\frac{1}{x}=-\frac{x}{y^2}f_1'+\frac{1}{x}f_2'$.

(3) $\frac{\partial u}{\partial x}=yf'(xy)g(yz)$, $\frac{\partial u}{\partial y}=xf'(xy)g(yz)+f(xy)\cdot zg'(yz)=xf'(xy)g(yz)+zf(xy)g'(yz)$

$\frac{\partial u}{\partial z}=f(xy)\cdot yg'(yz)=yf(xy)g'(yz)$

注：可写成 $f'(xy)=f'$, $\cancel{g'(xy)=g'}$ $g'(yz)=g'$, $f(xy)=f$, $\cancel{g(yz)=g}$ $g(yz)=g$.

(4) $\frac{\partial u}{\partial x}=f_1'+f_2'\cdot(-2x)+f_3'\cdot z=f_1'-2xf_2'+zf_3'$.

$\frac{\partial u}{\partial y}=f_1'\cdot(-2y)+f_2'=-2yf_1'+f_2'$.

$\frac{\partial u}{\partial z}=xf_3'$.

P124. 9. (2) $z=x^{2y}$, $\frac{\partial z}{\partial x}=2yx^{2y-1}$, $\frac{\partial z}{\partial y}=x^{2y}\ln x\cdot 2=2x^{2y}\ln x$

$$\frac{\partial^2 z}{\partial x^2}=\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)=\frac{\partial}{\partial x}(2yx^{2y-1})=2y(2y-1)x^{2y-2}$$

$$\frac{\partial^2 z}{\partial y\partial x}=\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)=\frac{\partial}{\partial y}(2yx^{2y-1})=2x^{2y-1}+2y\cdot x^{2y-1}\ln x\cdot 2=2(1+2y\ln x)x^{2y-1}$$

$$\frac{\partial^2 z}{\partial x\partial y}=\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)=\frac{\partial}{\partial x}(2x^{2y}\ln x)=2\cdot 2yx^{2y-1}\cdot\ln x+2x^{2y}\cdot\frac{1}{x}=2(2y\ln x+1)x^{2y-1}$$

$$\frac{\partial^2 z}{\partial y^2}=\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)=\frac{\partial}{\partial y}(2x^{2y}\ln x)=2\ln x\cdot x^{2y}\ln x\cdot 2=4x^{2y}\ln^2 x.$$

10. $z=f(x+y, xy, \frac{x}{y})$, $\frac{\partial z}{\partial x}=f_1'+yf_2'+\frac{1}{y}f_3'$, $\frac{\partial z}{\partial y}=f_1'+xf_2'-\frac{x}{y^2}f_3'$

$$\frac{\partial^2 z}{\partial x^2}=\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)=\frac{\partial}{\partial x}(f_1'+yf_2'+\frac{1}{y}f_3')=\frac{\partial f_1'}{\partial x}+y\frac{\partial f_2'}{\partial x}+\frac{1}{y}\frac{\partial f_3'}{\partial x}$$

$$=f_{11}''\cdot 1+f_{12}''\cdot y+f_{13}''\cdot\frac{1}{y}+y[f_{21}''\cdot 1+f_{22}''\cdot y+f_{23}''\cdot\frac{1}{y}]+\frac{1}{y}[f_{31}''\cdot 1+f_{32}''\cdot y+f_{33}''\cdot\frac{1}{y}]$$

$$=f_{11}''+yf_{12}''+\frac{1}{y}f_{13}''+yf_{21}''+y^2f_{22}''+f_{23}''+\frac{1}{y}f_{31}''+f_{32}''+\frac{1}{y^2}f_{33}''.$$

$$\frac{\partial^2 z}{\partial x\partial y}=\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)=\frac{\partial}{\partial x}(f_1'+xf_2'-\frac{x}{y^2}f_3')=\frac{\partial f_1'}{\partial x}+f_2'+x\frac{\partial f_2'}{\partial x}-\frac{1}{y^2}f_3'-\frac{x}{y^2}\frac{\partial f_3'}{\partial x}$$

$$=f_{11}''\cdot 1+f_{12}''\cdot y+f_{13}''\cdot\frac{1}{y}+f_2'+x[f_{21}''\cdot 1+f_{22}''\cdot y+f_{23}''\cdot\frac{1}{y}]-\frac{1}{y^2}f_3'-\frac{x}{y^2}[f_{31}''\cdot 1+f_{32}''\cdot y+f_{33}''\cdot\frac{1}{y}]$$

$$=f_2'-\frac{1}{y^2}f_3'+f_{11}''+yf_{12}''+\frac{1}{y}f_{13}''+xf_{21}''+xyf_{22}''+\frac{x}{y}f_{23}''-\frac{x}{y^2}f_{31}''-\frac{x}{y}f_{32}''-\frac{x}{y^3}f_{33}''.$$

12. $(x,y)\neq(0,0)$ 时, $f_x'(x,y)=\frac{2y(x^2+y^2)-2xy\cdot 2x}{(x^2+y^2)^2}=\frac{2y(y^2-x^2)}{(x^2+y^2)^2}$

$$f_x'(0,0)=\lim_{x\to 0}\frac{f(x,0)-f(0,0)}{x-0}=\lim_{x\to 0}\frac{0-0}{x}=0$$

$$\therefore f_x'(x,y)=\begin{cases}\frac{2y(y^2-x^2)}{(x^2+y^2)^2}, & x^2+y^2\neq 0,\\ 0, & x^2+y^2=0.\end{cases}$$

$$f_{xy}''(0,0)=\left.\frac{\partial f_x'(x,y)}{\partial y}\right|_{(0,0)}=\lim_{y\to 0}\frac{f_x'(0,y)-f_x'(0,0)}{y-0}=\lim_{y\to 0}\frac{\frac{2y(y^2-0^2)}{(0^2+y^2)^2}-0}{y}$$

$$=\lim_{y\to 0}\frac{2}{y^2}=\infty.\quad\therefore f_{xy}''(0,0)\text{ 不存在.}$$