$$
\begin{aligned}
&= \varnothing \cap B \cap B && (\pi_1 \text{ 是划分}) \\
&= \varnothing && (\text{零律})
\end{aligned}
$$

(3)

$$
\begin{aligned}
A \cap B &= \cup\pi_1 \cap B && (\pi_1 \text{ 是 } A \text{ 的划分}) \\
&= \left(\bigcup_{i=1}^{n} A_i\right) \cap B && (\text{广义并定义}) \\
&= \bigcup_{i=1}^{n} (A_i \cap B) && (\text{分配律}) \\
&= \bigcup_{k=1}^{m} (A_{i_k} \cap B) && (A_i \cap B \text{ 中有 } m \text{ 个非空的}) \\
&= \bigcup_{k=1}^{m} B_{i_k} && (\text{定义}) \\
&= \cup\pi_2 && (\text{广义并定义})
\end{aligned}
$$

综上所述，π_2 满足划分的全部条件。因此 π_2 是 $A \cap B$ 的一个划分。 □

2.37

证明：由模运算性质立即得证。 □

$$A/R = \{\{1,6,11,16\},\{2,7,12,17\},\{3,8,13,18\},\{4,9,14,19\},\{5,10,15,20\}\}\text{。}$$

2.38

证明：(1) 由 $\mathscr{A}$ 的定义知，$\varnothing \notin \mathscr{A}$。

(2) $\forall A_{i_1} \cap B_{j_1}, A_{i_2} \cap B_{j_2} \in \mathscr{A}$，

$$
\begin{aligned}
& A_{i_1} \cap B_{j_1} \cap A_{i_2} \cap B_{j_2} \neq \varnothing \\
\Longleftrightarrow\ & \exists x(x \in A_{i_1} \cap B_{j_1} \cap A_{i_2} \cap B_{j_2}) && (\varnothing \text{ 定义}) \\
\Longleftrightarrow\ & \exists x(x \in A_{i_1} \wedge x \in B_{j_1} \wedge x \in A_{i_2} \wedge x \in B_{j_2}) && (\text{交集定义}) \\
\Longleftrightarrow\ & \exists x(x \in A_{i_1} \wedge x \in A_{i_2} \wedge x \in B_{j_1} \wedge x \in B_{j_2}) && (\text{命题逻辑交换律}) \\
\Longrightarrow\ & \exists x(x \in A_{i_1} \wedge x \in A_{i_2}) \wedge \exists x(x \in B_{j_1} \wedge x \in B_{j_2}) && (\text{一阶谓词推理定律}) \\
\Longleftrightarrow\ & \exists x(x \in A_{i_1} \cap A_{i_2}) \wedge \exists x(x \in B_{j_1} \cap B_{j_2}) && (\text{交集定义}) \\
\Longleftrightarrow\ & A_{i_1} \cap A_{i_2} \neq \varnothing \wedge B_{j_1} \cap B_{j_2} \neq \varnothing && (\varnothing \text{ 定义}) \\
\Longrightarrow\ & A_{i_1} = A_{i_2} \wedge B_{j_1} = B_{j_2} && (\pi_1, \pi_2 \text{ 是划分}) \\
\Longrightarrow\ & A_{i_1} \cap B_{j_1} = A_{i_2} \cap B_{j_2} && (\text{交集定义、外延原则})
\end{aligned}
$$

可见，对任意 $A_{i_1} \cap B_{j_1}, A_{i_2} \cap B_{j_2} \in R$，若 $A_{i_1} \cap B_{j_1} \neq A_{i_2} \cap B_{j_2}$，则 $A_{i_1} \cap B_{j_1} \cap A_{i_2} \cap B_{j_2} = \varnothing$。

(3)

$$
\begin{aligned}
A &= A \cap A && (\text{幂等律}) \\
&= (\cup\pi_1) \cap (\cup\pi_2) && (\pi_1, \pi_2 \text{ 是划分}) \\
&= \left(\bigcup_{i=1}^{m} A_i\right) \cap \left(\bigcup_{j=1}^{n} B_j\right) && (\pi_1, \pi_2 \text{ 定义}) \\
&= \bigcup_{\substack{1 \le i \le m \\ 1 \le j \le n}} (A_i \cap B_j) && (\text{分配律}) \\
&= \cup\mathscr{A} && (\mathscr{A} \text{ 定义})
\end{aligned}
$$