其中：$\Delta x = x - x_0$，　$\Delta y = y - y_0$，　$\rho = \sqrt{(\Delta x)^2 + (\Delta y)^2}$

$$\lim_{\rho\to 0} \frac{f(P) - f(P_0)}{\rho} = \lim_{\rho\to 0} \frac{f_x'(P_0)\Delta x + f_y'(P_0)\Delta y + o(\rho)}{\rho}$$

$$= f_x'(P_0)\cos\alpha + f_y'(P_0)\cos\beta$$

$\therefore \frac{\partial f}{\partial l}\Big|_{P_0}$ 存在 且 $\frac{\partial f}{\partial l}\Big|_{P_0} = f_x'(P_0)\cos\alpha + f_y'(P_0)\cos\beta$.

(3) 推广.

$u = f(x, y, z)$ 在 $P_0(x_0, y_0, z_0)$ 可微，则 $\frac{\partial f}{\partial l}\Big|_{P_0}$ 存在 且

$$\frac{\partial f}{\partial l}\Big|_{P_0} = f_x'(P_0)\cos\alpha + f_y'(P_0)\cos\beta + f_z'(P_0)\cos\gamma$$

α, β, γ 是射线 l 的方向角.

例1. 设 $f(x, y, z) = x + y^2 + z^3$，求 f 在 $P_0(1,1,1)$ 沿方向 $l(2,-2,1)$ 的方向导数.

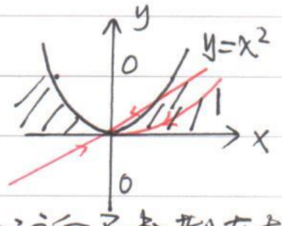
解：$\because f_x' = 1$，$f_y' = 2y$，$f_z' = 3z^2$ 连续

$\therefore f$ 可微 故 $\frac{\partial f}{\partial l}\Big|_{P_0} = f_x'(P_0)\cos\alpha + f_y'(P_0)\cos\beta + f_z'(P_0)\cos\gamma$

$f_x'(P_0) = 1$，$f_y'(P_0) = 2$　$f_z'(P_0) = 3$

$\cos\alpha = \frac{2}{\sqrt{2^2+(-2)^2+1^2}} = \frac{2}{3}$　$\cos\beta = \frac{-2}{3}$　$\cos\gamma = \frac{1}{3}$

从而 $\frac{\partial f}{\partial l}\Big|_{P_0} = 1\times\frac{2}{3} + 2\times(-\frac{2}{3}) + 3\times\frac{1}{3} = \frac{1}{3}$.



例2. 设 $f(x, y) = \begin{cases} 1, & 0<y<x^2, -\infty<x<+\infty \\ 0, & 其余部分 \end{cases}$

在(0,0)不连续(当然也不可微)，但在(0,0)沿任何方向的方向导数都存在.

解：$\lim\limits_{\substack{(x,y)\to(0,0) \\ y=kx}} f(x,y) = 0$　　$\lim\limits_{\substack{(x,y)\to(0,0) \\ y=\frac{1}{2}x^2}} f(x,y) = 1$