或 $\frac{\partial z}{\partial x}\Big|_{\substack{x=x_0\\y=y_0}}$ 或 $f_x'(x_0, y_0)$

同理定义 $f_y'(x_0, y_0) = \lim\limits_{\Delta y\to 0} \frac{\Delta_y z}{\Delta y} = \lim\limits_{\Delta y\to 0} \frac{f(x_0, y_0+\Delta y) - f(x_0, y_0)}{\Delta y}$

若 $z=f(x,y)$ 在区域D内每一点 (x,y) 都有关于x的偏导数，则称此为 $z=f(x,y)$ 关于x的偏导函数，简称偏导数，记为 $\frac{\partial z}{\partial x}$ 或 $\frac{\partial f}{\partial x}$ 或 z_x' 或 $f_x'(x,y)$

(2) 几何意义：$f_x'(x_0, y_0) = \frac{d}{dx} f(x, y_0)\Big|_{x=x_0}$ 表示曲线 $\begin{cases} z=f(x,y) \\ y=y_0 \end{cases}$

在 $M(x_0, y_0, f(x_0, y_0))$ 处切线MT对x轴斜率.

(3) 偏导数之计算

例1. $z = x^2 + 3xy + y^2$ 在 $(1, 2)$ 处偏导数

解：$\frac{\partial z}{\partial x} = 2x + 3y$ $\qquad$ $\frac{\partial z}{\partial y} = 3x + 2y$

$\frac{\partial z}{\partial x}\Big|_{(1,2)} = 8$ $\qquad$ $\frac{\partial z}{\partial y}\Big|_{(1,2)} = 7$

例2. $z = x^y$ $(x>0$ 且 $x\neq 1)$ 证明：$\frac{x}{y}\frac{\partial z}{\partial x} + \frac{1}{\ln x}\frac{\partial z}{\partial y} = 2z$

证：$\frac{\partial z}{\partial x} = yx^{y-1}$ $\qquad$ $\frac{\partial z}{\partial y} = x^y \ln x$

$$左 = \frac{x}{y}\cdot yx^{y-1} + \frac{1}{\ln x}\cdot x^y \cdot \ln x = 2x^y = 2z.$$

例3. $z = (xy+1)^y$ 求 $\frac{\partial z}{\partial y}$

两边取对数，$\ln z = y\ln(xy+1)$

对y求导，$\frac{1}{z}\cdot\frac{\partial z}{\partial y} = \ln(xy+1) + y\cdot\frac{x}{xy+1}$

$$\therefore \frac{\partial z}{\partial y} = (xy+1)^y\left[\ln(xy+1) + \frac{xy}{xy+1}\right]$$