

(2)

证明:

$$\forall x, y$$

$$\langle x, y \rangle \in (A - B) \times (C - D)$$

$$\iff x \in A \wedge x \notin B \wedge y \in C \wedge y \notin D$$

(卡氏积定义、相对补定义)

$$\iff x \in A \wedge y \in C \wedge x \notin B \wedge y \notin D$$

(命题逻辑交换律)

$$\implies x \in A \wedge y \in C \wedge x \notin B$$

(命题逻辑化简律)

$$\implies (x \in A \wedge y \in C \wedge x \notin B) \vee (x \in A \wedge y \in C \wedge y \notin D)$$

(命题逻辑附加律)

$$\iff (x \in A \wedge y \in C) \wedge (x \notin B \vee y \notin D)$$

(命题逻辑分配律)

$$\iff (x \in A \wedge y \in C) \wedge (\neg(x \in B) \vee \neg(y \in D))$$

(\notin 定义)

$$\iff (x \in A \wedge y \in C) \wedge \neg(x \in B \wedge y \in D)$$

(命题逻辑德·摩根律)

$$\iff (\langle x, y \rangle \in A \times C) \wedge \neg(\langle x, y \rangle \in B \times D)$$

(卡氏积定义)

$$\iff (\langle x, y \rangle \in A \times C) \wedge (\langle x, y \rangle \notin B \times D)$$

(\notin 定义)

$$\iff \langle x, y \rangle \in (A \times C) - (B \times D)$$

(相对补定义)

$$\text{故有: } (A - B) \times (C - D) \subseteq (A \times C) - (B \times D).$$

□

2.7

(1)

证明:

$$\forall x, y$$

$$\langle x, y \rangle \in (A - B) \times C$$

$$\iff x \in (A - B) \wedge y \in C$$

(卡氏积定义)

$$\iff x \in A \wedge x \notin B \wedge y \in C$$

(相对补定义)

$$\iff x \in A \wedge \neg x \in B \wedge y \in C$$

(\notin 定义)

$$\iff (x \in A \wedge \neg x \in B \wedge y \in C) \vee 0$$

(命题逻辑同一律)

$$\iff (x \in A \wedge \neg x \in B \wedge y \in C) \vee (x \in A \wedge 0)$$

(命题逻辑零律)

$$\iff (x \in A \wedge \neg x \in B \wedge y \in C) \vee (x \in A \wedge \neg y \in C \wedge y \in C)$$

(命题逻辑矛盾律)

$$\iff (x \in A \wedge y \in C) \wedge (\neg x \in B \vee \neg y \in C)$$

(命题逻辑分配律)

$$\iff (x \in A \wedge y \in C) \wedge \neg(x \in B \wedge y \in C)$$

(命题逻辑德·摩根律)

$$\iff (\langle x, y \rangle \in A \times C) \wedge \neg(\langle x, y \rangle \in B \times C)$$

(卡氏积定义)

$$\iff (\langle x, y \rangle \in A \times C) \wedge (\langle x, y \rangle \notin B \times C)$$

(\notin 定义)

$$\iff \langle x, y \rangle \in (A \times C) - (B \times C)$$

(相对补定义)

$$\text{故有: } (A - B) \times C = (A \times C) - (B \times C).$$

□

(2)

证明:

$$(A \oplus B) \times C = ((A - B) \cup (B - A)) \times C$$

(对称差性质)

$$= ((A - B) \times C) \cup ((B - A) \times C)$$

(卡氏积性质)

$$= ((A \times C) - (B \times C)) \cup ((B \times C) - (A \times C))$$

(第 (1) 小题结论)