NO.

Date

注：1. f在P_0的全微分唯一表示为 $df|_{(x_0,y_0)} = f_x'(x_0,y_0)\Delta x + f_y'(x_0,y_0)\Delta y$

2. 自变量的微分等于自变量的增量，即

$dx=\Delta x$ ， $dy=\Delta y$

$df(x_0,y_0) = f_x'(x_0,y_0)dx + f_y'(x_0,y_0)dy$

3. 若f在区域D上每一点(x,y)可微，则称f在区域D上可微，且f在D上的全微分为 $df(x,y) = f_x'(x,y)dx + f_y'(x,y)dy$.

例6 考察函数 $f(x,y)=\begin{cases}\frac{xy}{\sqrt{x^2+y^2}}, & x^2+y^2\neq 0 \\ 0 & x^2+y^2=0\end{cases}$ 在$(0,0)$可微性.

分析：若 $\boxed{\frac{\Delta z - f_x'(x_0,y_0)\Delta x - f_y'(x_0,y_0)\Delta y}{\rho} \to 0 \quad (\rho\to 0)}$，则f在$(x_0,y_0)$可微

解：按偏导数定义 $f_x'(0,0)=\lim\limits_{\Delta x\to 0}\frac{f(\Delta x,0)-f(0,0)}{\Delta x} = \lim\limits_{\Delta x\to 0}\frac{0-0}{\Delta x} = 0$

同理 $f_y'(0,0)=0$

$$\lim_{\rho\to 0}\frac{\Delta z - f_x'(0,0)\Delta x - f_y'(0,0)\Delta y}{\rho} = \lim_{\substack{\Delta x\to 0\\ \Delta y\to 0}}\frac{\Delta x\cdot\Delta y}{\left(\sqrt{(\Delta x)^2+(\Delta y)^2}\right)^2}$$

取 $\Delta y = k\Delta x$ $\quad\lim\limits_{\substack{\Delta x\to 0\\ \Delta y\to 0}}\frac{\Delta x\cdot k\Delta x}{\left(\sqrt{(\Delta x)^2+(k\Delta x)^2}\right)^2} = \lim\limits_{\Delta x\to 0}\frac{k}{1+k^2} = \frac{k}{1+k^2}$

由极限唯一性知，$\lim\limits_{\substack{\Delta x\to 0\\ \Delta y\to 0}}\frac{\Delta x\cdot\Delta y}{\left(\sqrt{(\Delta x)^2+(\Delta y)^2}\right)^2}$ 不存在，从而f在$(0,0)$不可微

注：偏导数存在 $\not\longrightarrow$ 函数可微

(3) 可微充分条件

定理：若函数$f(x,y)$在点(x_0,y_0)的某邻域内偏导数存在，且$f_x'(x,y)$, $f_y'(x,y)$在(x_0,y_0)处连续，则函数f在(x_0,y_0)可微.