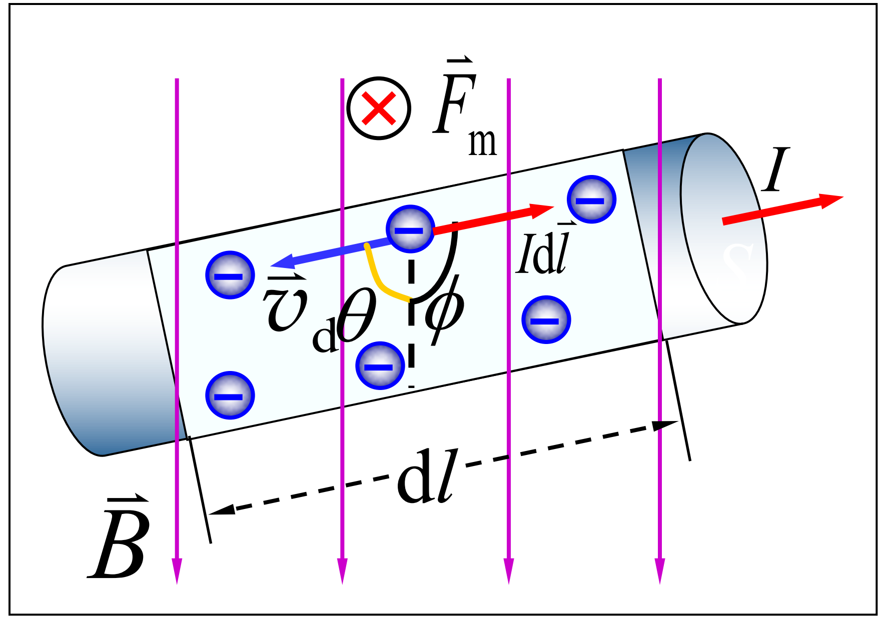
一 安培力

洛伦兹力

$$\vec{F}_{\mathrm{m}} = -e\vec{v}_{\mathrm{d}} \times \vec{B}$$

$$F_{\mathrm{m}} = ev_{\mathrm{d}}B\sin\theta$$

$$\mathrm{d}F = nev_{\mathrm{d}}S\mathrm{d}lB\sin\theta$$



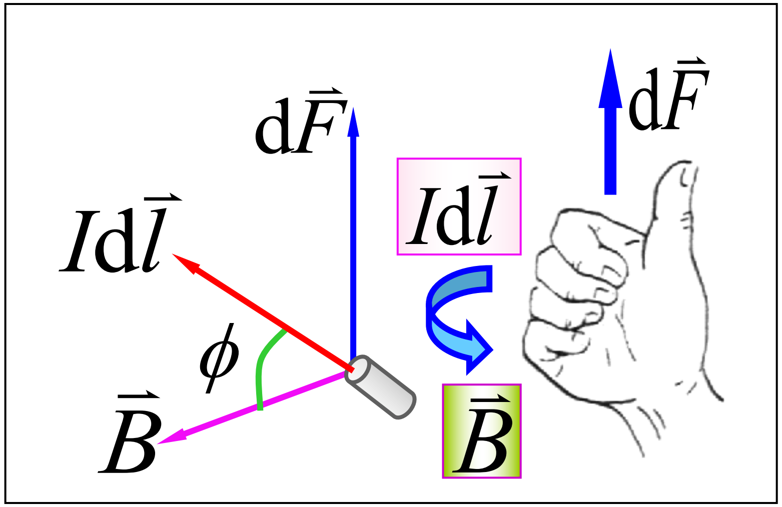
$$I = nev_{\mathrm{d}}S \qquad \mathrm{d}F = I\mathrm{d}lB\sin\theta = I\mathrm{d}lB\sin\phi$$

安培力 $\boxed{\mathrm{d}\vec{F} = I\mathrm{d}\vec{l} \times \vec{B}}$

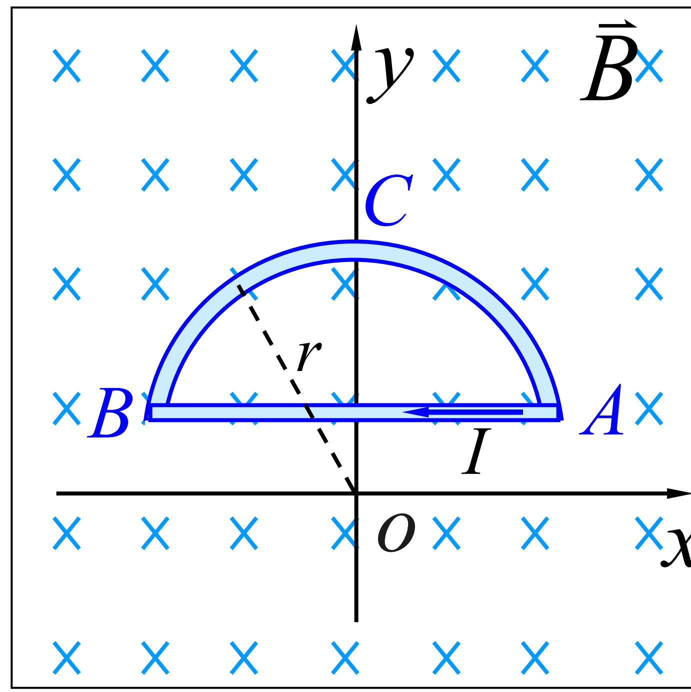
◆ 有限长载流导线所受的安培力

$$\boxed{\mathrm{d}\vec{F} = I\mathrm{d}\vec{l} \times \vec{B}}$$

$$\vec{F} = \int_l \mathrm{d}\vec{F} = \int_l I\mathrm{d}\vec{l} \times \vec{B}$$



例 1 如图一通有电流 I 的闭合回路放在磁感应强度为 $\vec{B}$ 的均匀磁场中，回路平面与磁感强度 $\vec{B}$ 垂直．回路由直导线 AB 和半径为 r 的圆弧导线 BCA 组成，电流为顺时针方向，求磁场作用于闭合导线的力.



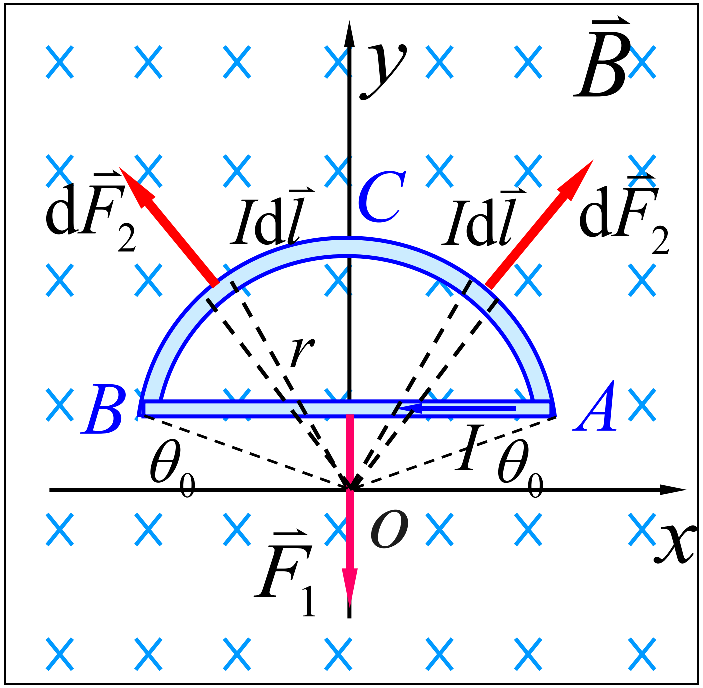
解 $\vec{F}_1 = -I\,\overline{AB}B\vec{j}$

根据对称性分析 $F_{2x} = 0$

$$\vec{F}_2 = F_{2y}\vec{j}$$

$$F_2 = \int \mathrm{d}F_{2y} = \int \mathrm{d}F_2 \sin\theta$$

$$= \int BI\mathrm{d}l \sin\theta$$



因 $\mathrm{d}l = r\mathrm{d}\theta$

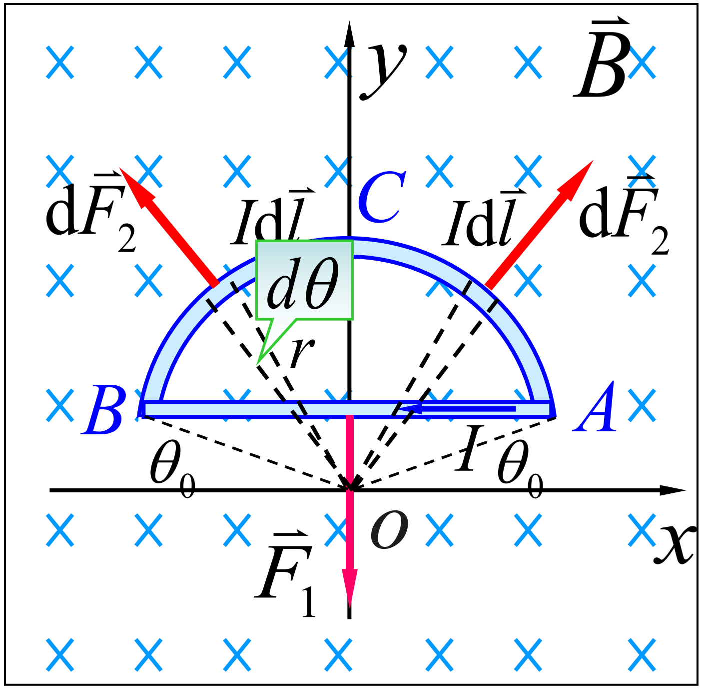
$$F_2 = BIr\int_{\theta_0}^{\pi-\theta_0} \sin\theta\,\mathrm{d}\theta$$

$$\vec{F}_2 = BI(2r\cos\theta_0)\vec{j}$$

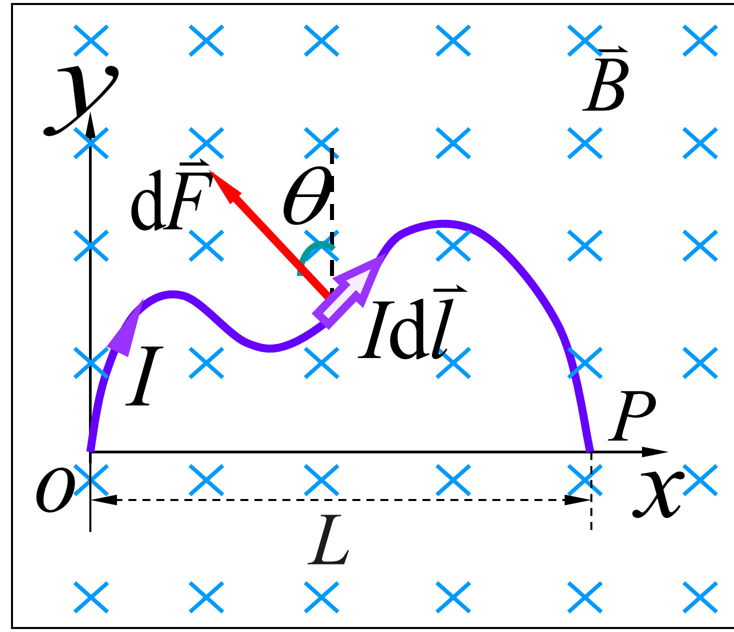
$$= BI\,\overline{AB}\,\vec{j}$$

由于 $\vec{F}_1 = -BI\,\overline{AB}\,\vec{j}$

故 $\vec{F} = \vec{F}_1 + \vec{F}_2 = 0$



例 2 求如图不规则的平面载流导线在均匀磁场中所受的力，已知 $\vec{B}$ 和 I.



解 取一段电流元 $I\mathrm{d}\vec{l}$

$$\mathrm{d}\vec{F} = I\mathrm{d}\vec{l} \times \vec{B}$$

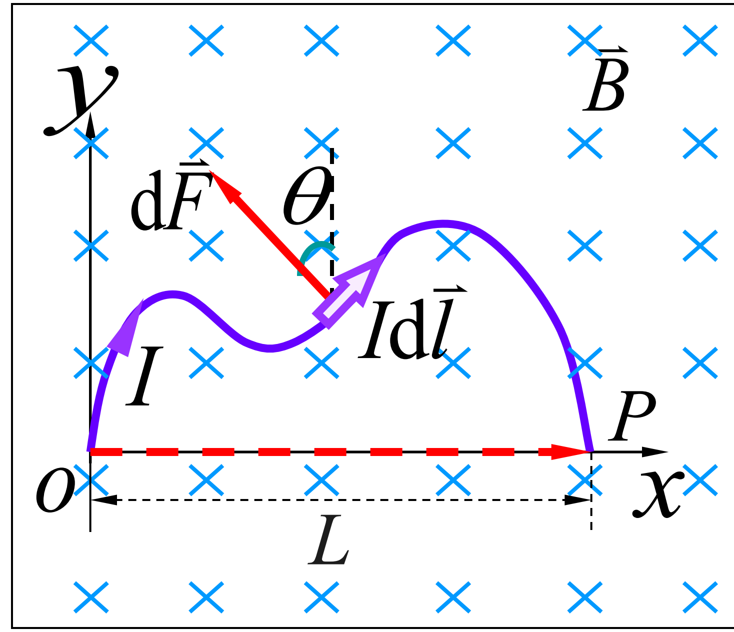
$$\mathrm{d}F_{\mathrm{x}} = \mathrm{d}F\sin\theta = BI\mathrm{d}l\sin\theta$$

$$\mathrm{d}F_{\mathrm{y}} = \mathrm{d}F\cos\theta = BI\mathrm{d}l\cos\theta$$

$$F_x = \int \mathrm{d}F_x = BI\int_0^0 \mathrm{d}y = 0$$

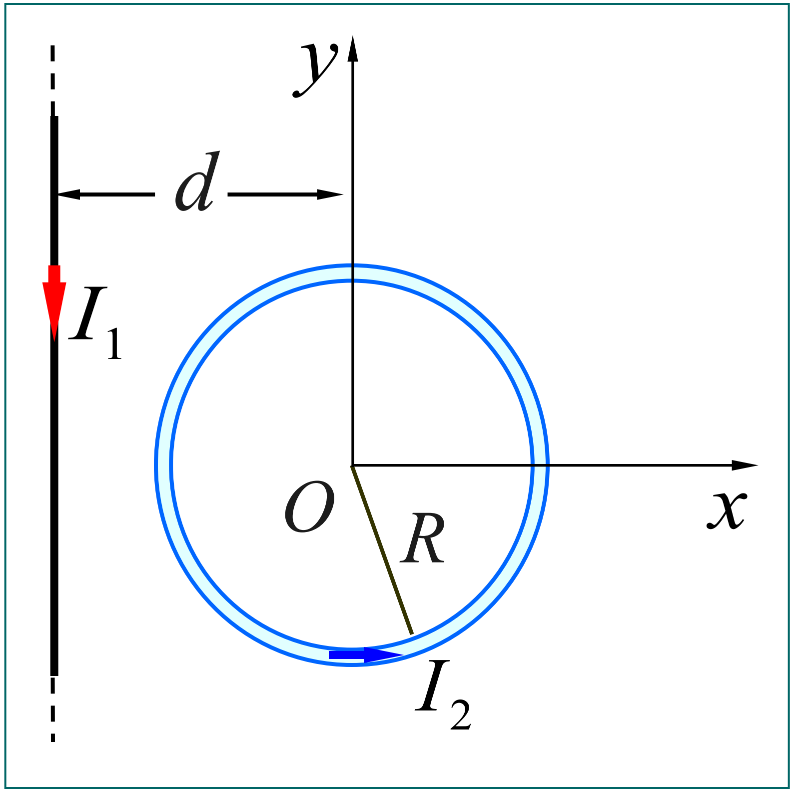
$$F_y = \int \mathrm{d}F_y = BI\int_0^l \mathrm{d}x = BIl$$

$$\vec{F} = \vec{F}_y = BIl\vec{j}$$



结论 任意平面载流导线在均匀磁场中所受的力，与其始点和终点相同的载流直导线所受的磁场力相同.

例 3 半径为 R 载有电流 I_2 的导体圆环与电流为 I_1 的长直导线 放在同一平面内（如图），直导线与圆心相距为 d，且 $R<d$ 两者间绝缘，**求** 作用在圆电流上的磁场力.



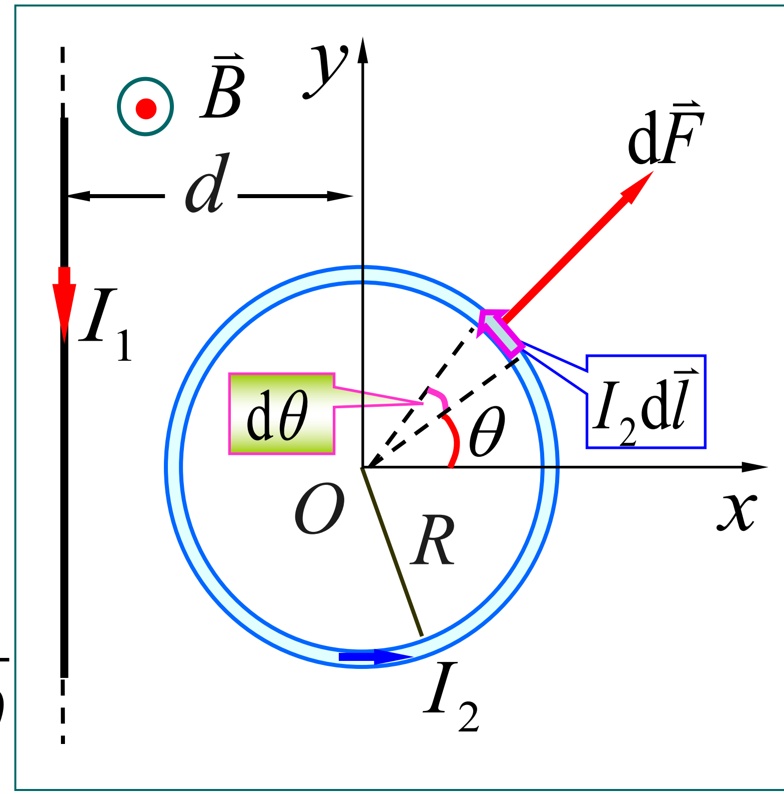
解 $$B=\frac{\mu_0}{2\pi}\frac{I_1}{d+R\cos\theta}$$

$$\mathrm{d}l=R\mathrm{d}\theta$$

$$\mathrm{d}F=BI_2\mathrm{d}l$$

$$=\frac{\mu_0 I_1 I_2}{2\pi}\frac{\mathrm{d}l}{d+R\cos\theta}$$

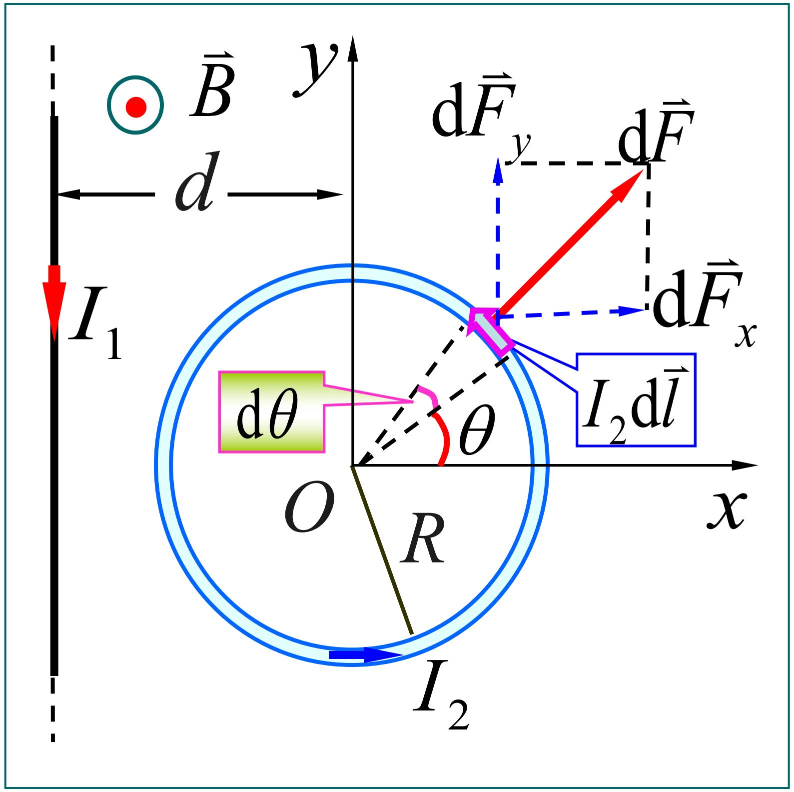
$$\mathrm{d}F=\frac{\mu_0 I_1 I_2}{2\pi}\frac{R\mathrm{d}\theta}{d+R\cos\theta}$$



$$\mathrm{d}F_{\mathrm{x}}=\mathrm{d}F\cos\theta=\frac{\mu_0 I_1 I_2}{2\pi}\frac{R\cos\theta\mathrm{d}\theta}{d+R\cos\theta}$$

$$F_{\mathrm{x}}=\int_0^{2\pi}\mathrm{d}F_{\mathrm{x}}$$

$$=\mu_0 I_1 I_2\left(1-\frac{d}{\sqrt{d^2-R^2}}\right)$$

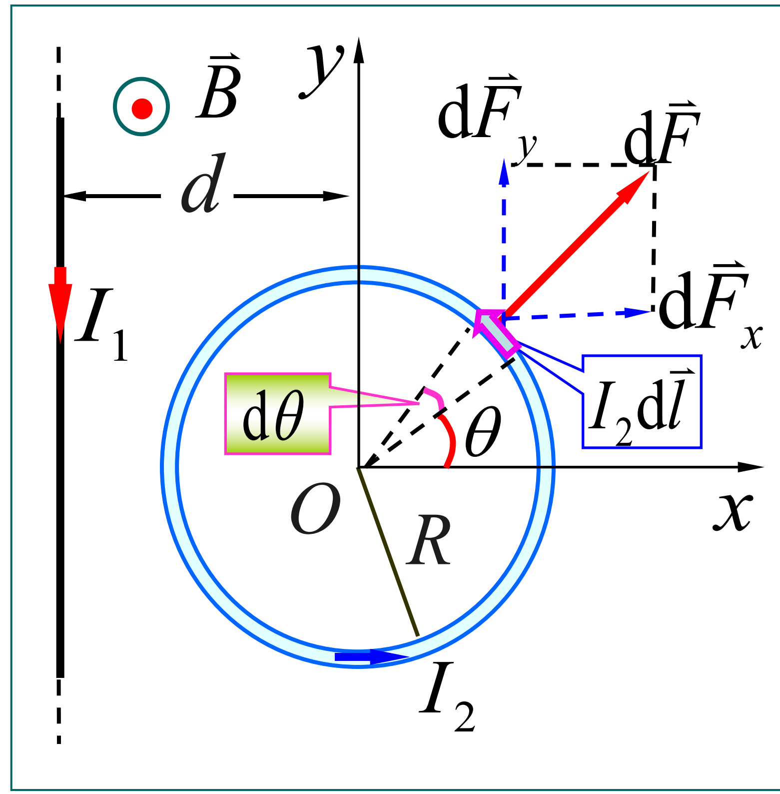


$$\mathrm{d}F_{y} = \mathrm{d}F\sin\theta = \frac{\mu_0 I_1 I_2}{2\pi}\frac{R\sin\theta\mathrm{d}\theta}{d + R\cos\theta}$$

$$F_{y} = \int_0^{2\pi}\mathrm{d}F_{y} = 0$$

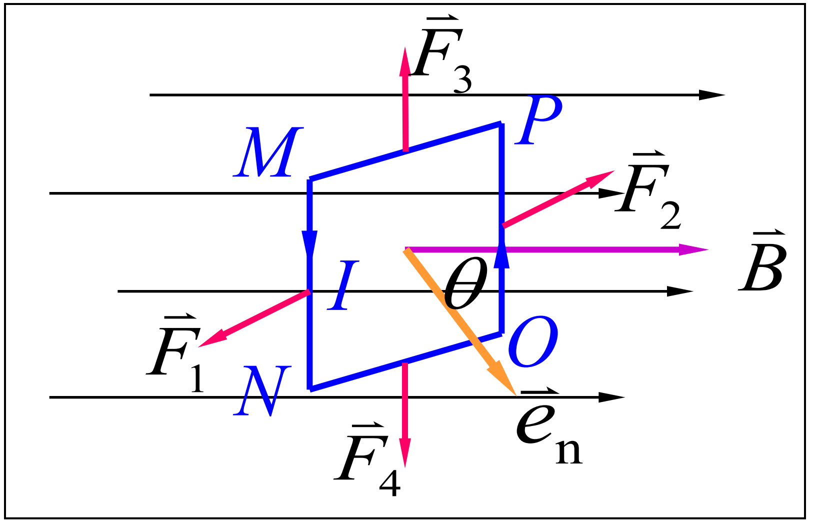
$$\vec{F} = F_{x}\vec{i}$$

$$= \mu_0 I_1 I_2 (1 - \frac{d}{\sqrt{d^2 - R^2}})\vec{i}$$



二 磁场作用于载流线圈的磁力矩

如图 均匀磁场中有一矩形载流线圈 $MNOP$



$$MN = l_2 \quad NO = l_1$$

$$F_1 = BIl_2 \qquad \vec{F}_1 = -\vec{F}_2$$

$$\vec{F} = \sum_{i=1}^{4} \vec{F}_i = 0$$

$$F_3 = BIl_1 \sin(\pi - \phi) \qquad \vec{F}_3 = -\vec{F}_4$$

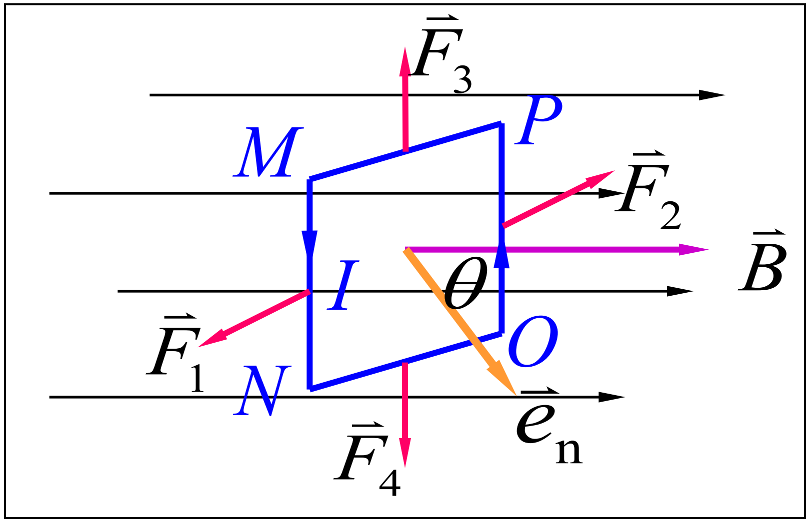
7-8 载流导线在磁场中所受的力

$$MN = l_2 \quad NO = l_1$$

$$M = F_1 l_1 \sin\theta = BIl_2 l_1 \sin\theta$$

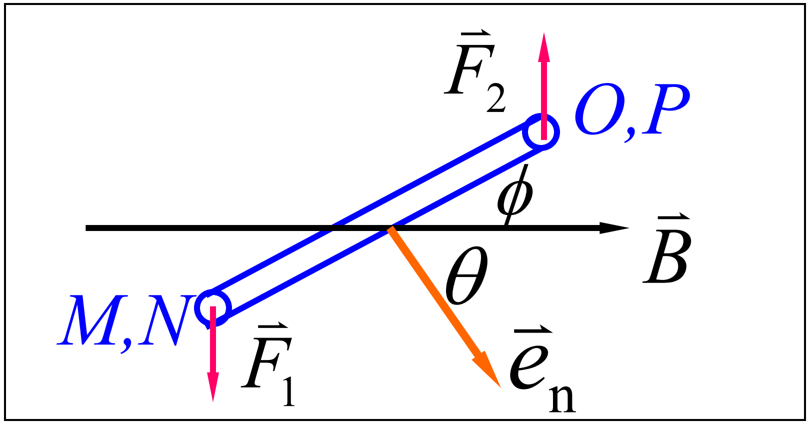
$$M = BIS\sin\theta$$

$$\vec{M} = \boxed{IS\vec{e}_n} \times \vec{B} = \boxed{\vec{m}} \times \vec{B}$$



线圈有N匝时

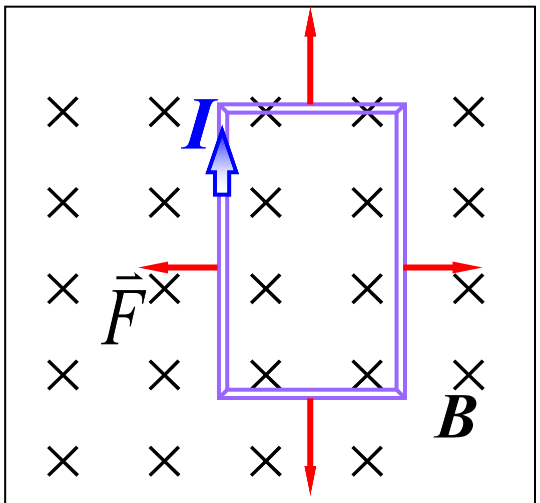
$$\vec{M} = NIS\vec{e}_n \times \vec{B}$$



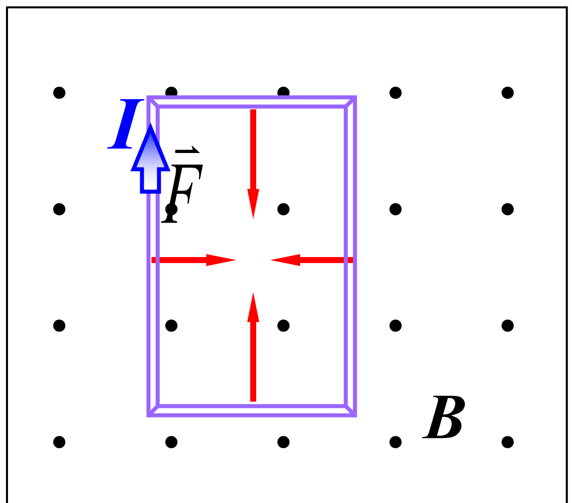
讨 论

（1）$\vec{e}_n$ 与 $\vec{B}$ 同向　（2）方向相反　（3）方向垂直

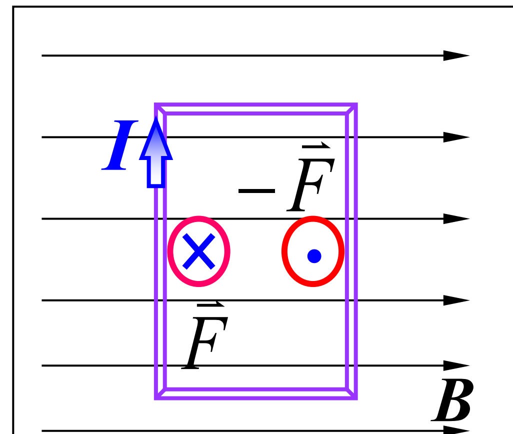
稳定平衡	不稳定平衡	力矩最大



$$\theta = 0^\circ, M = 0$$



$$\theta = \pi, M = 0$$



$$\theta = \frac{\pi}{2}, M = M_{max}$$

➤ **结论：均匀**磁场中，任意形状**刚**性闭合**平面**通电线圈所受的力和力矩为

$$\boxed{\vec{F}=0,\qquad \overrightarrow{M}=\vec{m}\times\vec{B}}$$

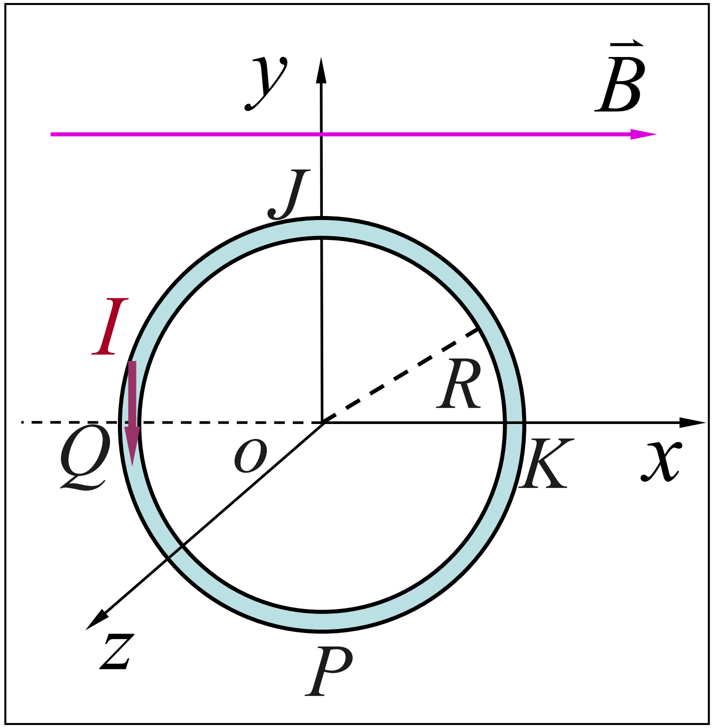
$$\vec{m}\,/\!/\,\vec{B},\quad \vec{M}=0\quad \begin{cases}\theta=0 & \text{稳定平衡}\\ \theta=\pi & \text{非稳定平衡}\end{cases}$$

$$\vec{m}\perp\vec{B},\quad M=M_{\max}=mB,\quad \theta=\pi/2$$

➤ 磁矩 $\boxed{\vec{m}=NIS\vec{e}_{\mathrm{n}}}$　　$\boxed{\vec{e}_{\mathrm{n}}\text{与 }I\text{ 成右螺旋}}$

例4 如图半径为0.20 m，电流为20 A，可绕轴旋转的圆形载流线圈放在均匀磁场中，磁感应强度的大小为0.08 T，方向沿 x 轴正向.

问线圈受力情况怎样? 线圈所受的磁力矩又为多少?



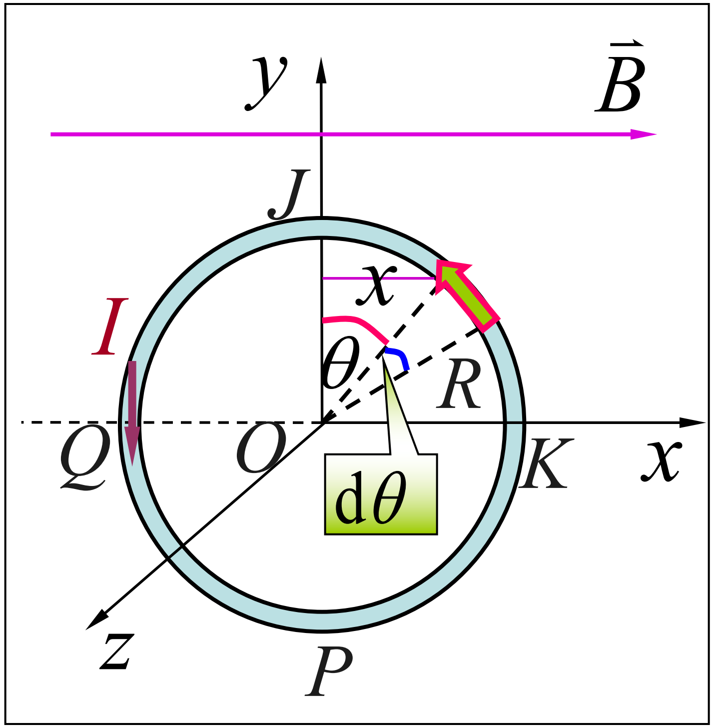
解 （方法一）把线圈分为JQP和PKJ两部分

$$\vec{F}_{\mathrm{JQP}}=BI(2R)\vec{k}$$

$$=0.64\vec{k}\,\mathrm{N}=-\vec{F}_{\mathrm{PKJ}}$$

以Oy为轴，$I\mathrm{d}\vec{l}$所受磁力矩大小

$$\mathrm{d}M=x\mathrm{d}F=I\mathrm{d}lBx\sin\theta$$

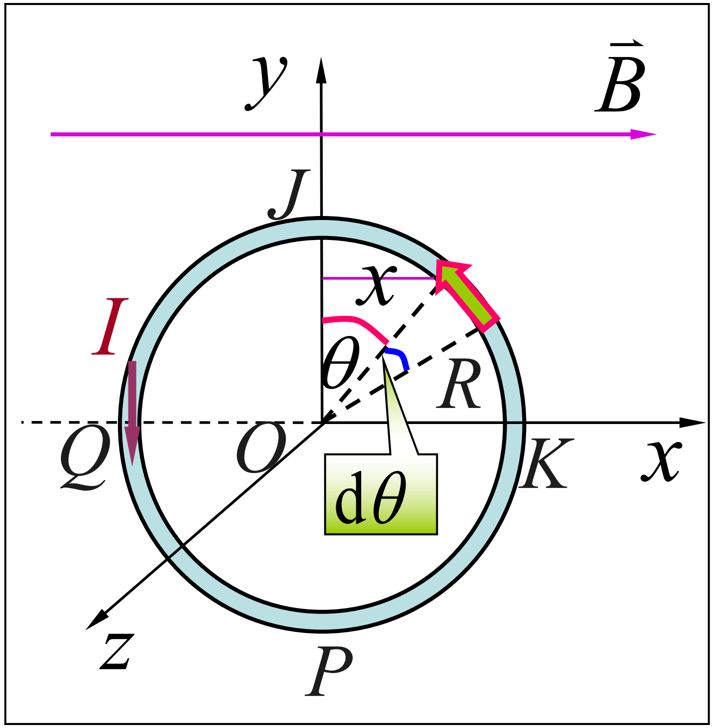


$$x = R\sin\theta, \quad \mathrm{d}l = R\mathrm{d}\theta$$

$$\mathrm{d}M = IBR^2 \sin^2\theta \mathrm{d}\theta$$

$$M = IBR^2 \int_0^{2\pi} \sin^2\theta \mathrm{d}\theta$$

$$M = IB\pi R^2$$

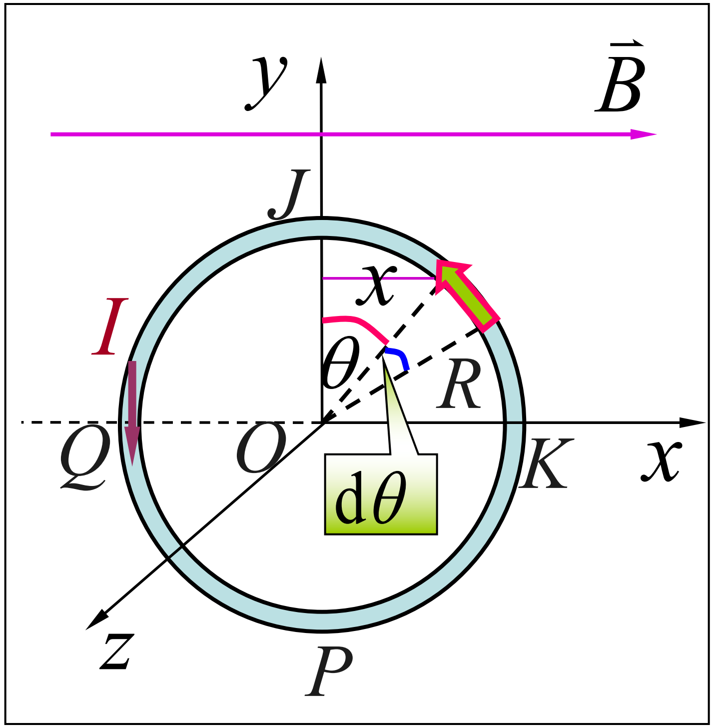


（方法二）

$$\vec{m} = IS\vec{k} = I\pi R^2\vec{k}$$

$$\vec{B} = B\vec{i}$$

$$\vec{M} = \vec{m} \times \vec{B}$$
$$= I\pi R^2 B\vec{k} \times \vec{i}$$
$$= I\pi R^2 B\vec{j}$$



选择进入下一节：

7-4　毕奥-萨伐尔定律

7-5　磁通量　磁场的高斯定理

7-6　安培环路定理

7-7　带电粒子在电场和磁场中的运动

7-8　载流导线在磁场中所受的力

7-9　磁场中的磁介质

END