P_{132}. 2.(3) $z=e^{2x}(x+2y+y^2)$

由 $\begin{cases}\dfrac{\partial z}{\partial x}=2e^{2x}(x+2y+y^2)+e^{2x}\cdot 1=e^{2x}(2x+4y+2y^2+1)=0\\[2mm] \dfrac{\partial z}{\partial y}=e^{2x}\cdot(2+2y)=0\end{cases}$ 得驻点为$(\frac{1}{2},-1)$

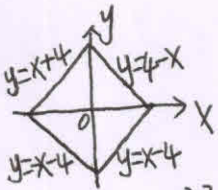
$A=\frac{\partial}{\partial x}(\frac{\partial z}{\partial x})=\frac{\partial}{\partial x}[e^{2x}(2x+4y+2y^2+1)]=2e^{2x}(2x+4y+2y^2+1)+e^{2x}\cdot 2=4e^{2x}(x+2y+y^2+1)$

$B=\frac{\partial}{\partial x}(\frac{\partial z}{\partial y})=\frac{\partial}{\partial x}[e^{2x}\cdot(2+2y)]=4(1+y)e^{2x}$

$C=\frac{\partial}{\partial y}(\frac{\partial z}{\partial y})=\frac{\partial}{\partial y}[e^{2x}(2+2y)]=2e^{2x}$

在$(\frac{1}{2},-1)$处，$A=2e$，$B=0$，$C=2e$，$B^2-AC=-4e^2<0$ 且 $A>0$，

$\therefore(\frac{1}{2},-1)$是极小值点，极小值为 $[e^{2x}(x+2y+y^2)]\Big|_{(\frac{1}{2},-1)}=-\frac{e}{2}$.

3. (2) 

$D=\{(x,y)\mid |x|+|y|\le 4\}$

在D内，由 $\begin{cases}\dfrac{\partial z}{\partial x}=2x-y=0\\[2mm] \dfrac{\partial z}{\partial y}=-x+2y=0\end{cases}$ 得驻点$(0,0)$，$z|_{(0,0)}=(x^2-xy+y^2)|_{(0,0)}=0$.

在∂D上，即$|x|+|y|=4$时，

$y=4-x$ $(0\le x\le 4)$时，$z=x^2-x(4-x)+(4-x)^2=3x^2-12x+16$，由$\frac{dz}{dx}=6x-12=0$得$x=2$，

此时$y=2$，$z|_{(2,2)}=4$，$z|_{(0,4)}=16$，$z|_{(4,0)}=16$.

$y=x+4$ $(-4\le x\le 0)$时，$z=x^2-x(x+4)+(x+4)^2=x^2+4x+16$，由$\frac{dz}{dx}=2x+4=0$得$x=-2$，

此时$y=2$，$z|_{(-2,2)}=12$，$z|_{(-4,0)}=16$，$z|_{(0,4)}=16$.

$y=-x-4$ $(-4\le x\le 0)$时，$z=x^2-x(-x-4)+(-x-4)^2=3x^2+12x+16$，由$\frac{dz}{dx}=6x+12=0$得$x=-2$，

此时$y=-2$，$z|_{(-2,-2)}=4$，$z|_{(-4,0)}=16$，$z|_{(0,4)}=16$

$y=x-4$ $(0\le x\le 4)$时，$z=x^2-x(x-4)+(x-4)^2=x^2-4x+16$，由$\frac{dz}{dx}=2x-4=0$得$x=2$，

此时$y=-2$，$z|_{(2,-2)}=12$，$z|_{(0,-4)}=16$，$z|_{(4,0)}=16$.

综上所述，$z=x^2-xy+x^2$在$D=\{(x,y)\mid |x|+|y|\le 4\}$上的最大值是16，最小值是0.

P132. 5. 不妨设半球面方程为 $x^2+y^2+z^2=a^2\ (z\geq 0)$. 内接长方体的顶点在第一卦限的坐标为 (x,y,z). 则内接长方体体积为 $V=4xyz=4xy\sqrt{a^2-x^2-y^2}$

由 $$\begin{cases}\dfrac{\partial V}{\partial x}=4y\sqrt{a^2-x^2-y^2}+4xy\cdot\dfrac{-2x}{2\sqrt{a^2-x^2-y^2}}=\dfrac{4y(a^2-2x^2-y^2)}{\sqrt{a^2-x^2-y^2}}=0\\ \dfrac{\partial V}{\partial y}=4x\sqrt{a^2-x^2-y^2}+4xy\cdot\dfrac{-2y}{2\sqrt{a^2-x^2-y^2}}=\dfrac{4x(a^2-x^2-2y^2)}{\sqrt{a^2-x^2-y^2}}=0\end{cases}$$

得驻点 $\left(\frac{a}{\sqrt3},\frac{a}{\sqrt3}\right)$. $x=\frac{a}{\sqrt3}, y=\frac{a}{\sqrt3}$ 时 $z=\sqrt{a^2-(\frac{a}{\sqrt3})^2-(\frac{a}{\sqrt3})^2}=\frac{a}{\sqrt3}$.

当长方体的长宽均为 $\frac{2a}{\sqrt3}$, 高为 $\frac{a}{\sqrt3}$ 时体积最大.

P151. 2. 曲面 $z=\sqrt{2+x^2+4y^2}$ 上点 (x,y,z) 到平面 $x-2y+3z=1$ 的距离为

$$d=\frac{|x-2y+3z-1|}{\sqrt{1^2+(-2)^2+3^2}}=\frac{|x-2y+3z-1|}{\sqrt{14}}$$, 其中 $z=\sqrt{2+x^2+4y^2}$

令 $L(x,y,z,\lambda)=(x-2y+3z-1)^2+\lambda(2+x^2+4y^2-z^2)\quad (z\geq 0)$

则由 $$\begin{cases}\dfrac{\partial L}{\partial x}=2(x-2y+3z-1)+2\lambda x=0\\ \dfrac{\partial L}{\partial y}=2(x-2y+3z-1)\cdot(-2)+8\lambda y=0\\ \dfrac{\partial L}{\partial z}=2(x-2y+3z-1)\cdot 3-2\lambda z=0\\ \dfrac{\partial L}{\partial \lambda}=2+x^2+4y^2-z^2=0\end{cases}$$

得 $-4\lambda x=8\lambda y$, $6\lambda x=-2\lambda z$, 即 $x=-2y$, $z=6y$

由 $z=\sqrt{2+x^2+4y^2}$ 得 $6y=\sqrt{2+(-2y)^2+4y^2}=\sqrt{2+8y^2}$, 即 $y^2=\frac{1}{14}$ 且 $y\geq 0$

$\therefore y=\frac{1}{\sqrt{14}}, x=-\frac{2}{\sqrt{14}}, z=\frac{6}{\sqrt{14}}$

$\left(-\frac{2}{\sqrt{14}},\frac{1}{\sqrt{14}},\frac{6}{\sqrt{14}}\right)$ 为所求点.

P143. 4. 法一. $F(x,y,z)=\Phi(x^2+y^2+z^2)-ax-by-cz$，记 $\Phi'=\Phi'(x^2+y^2+z^2)$，

$F_x'=\Phi'\cdot 2x-a=2x\Phi'-a$，$F_y'=\Phi'\cdot 2y-b=2y\Phi'-b$，$F_z'=\Phi'\cdot 2z-c=2z\Phi'-c$

$$\frac{\partial z}{\partial x}=-\frac{F_x'}{F_z'}=-\frac{2x\Phi'-a}{2z\Phi'-c}，\quad \frac{\partial z}{\partial y}=-\frac{F_y'}{F_z'}=-\frac{2y\Phi'-b}{2z\Phi'-c}$$

$$(cy-bz)\frac{\partial z}{\partial x}+(az-cx)\frac{\partial z}{\partial y}=\frac{(cy-bz)\cdot(a-2x\Phi')+(az-cx)\cdot(b-2y\Phi')}{2z\Phi'-c}$$

$$=\frac{acy-2cxy\Phi'-abz+2bxz\Phi'+abz-2ayz\Phi'-bcx+2cxy\Phi'}{2z\Phi'-c}$$

$$=\frac{acy+2bxz\Phi'-2ayz\Phi'-bcx}{2z\Phi'-c}=\frac{(bx-ay)(2z\Phi'-c)}{2z\Phi'-c}=bx-ay.$$

法二. $ax+by+cz=\Phi(x^2+y^2+z^2)$ 两边对 x 求偏导数(此时 z 是关于 x,y 的函数)

得 $a+c\frac{\partial z}{\partial x}=\Phi'(2x+2z\cdot\frac{\partial z}{\partial x})$，$\therefore \frac{\partial z}{\partial x}=\frac{a-2x\Phi'}{2z\Phi'-c}$

$ax+by+cz=\Phi(x^2+y^2+z^2)$ 两边对 y 求偏导数(此时 z 是关于 x,y 的函数)

得 $b+c\frac{\partial z}{\partial y}=\Phi'\cdot(2y+2z\frac{\partial z}{\partial y})$，$\therefore \frac{\partial z}{\partial y}=\frac{b-2y\Phi'}{2z\Phi'-c}$

$\therefore (cy-bz)\frac{\partial z}{\partial x}+(az-cx)\frac{\partial z}{\partial y}=\cdots\cdots=bx-ay$ (同法一)

法三. $ax+by+cz=\Phi(x^2+y^2+z^2)$ 两边求微分得

$$a\,dx+b\,dy+c\,dz=\Phi'\cdot(2x\,dx+2y\,dy+2z\,dz)$$

$$\therefore dz=\frac{a-2x\Phi'}{2z\Phi'-c}dx+\frac{b-2y\Phi'}{2z\Phi'-c}dy$$

$$\therefore \frac{\partial z}{\partial x}=\frac{a-2x\Phi'}{2z\Phi'-c}，\quad \frac{\partial z}{\partial y}=\frac{b-2y\Phi'}{2z\Phi'-c}$$

$(cy-bz)\frac{\partial z}{\partial x}+(az-cx)\frac{\partial z}{\partial y}=\cdots\cdots=bx-ay$. (同法一).

P_{143}. 7. (1) $F(x,y,z)=x+y+z$，$G(x,y,z)=x^2+y^2+z^2-1$ 均在 R^3 内有连续偏导数

$F(\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}},0)=0$，$G(\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}},0)=0$.

$$J=\frac{\partial(F,G)}{\partial(x,y)}=\begin{vmatrix}F'_x & F'_y\\ G'_x & G'_y\end{vmatrix}=\begin{vmatrix}1 & 1\\ 2x & 2y\end{vmatrix}=2(y-x),\quad J\Big|_{(\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}},0)}=2(y-x)\Big|_{(\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}},0)}=-2\sqrt{2}\neq 0$$

$\therefore \begin{cases}x+y+z=0\\ x^2+y^2+z^2=1\end{cases}$ 在点 $(\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}},0)$ 的邻域内存在隐函数组 $\begin{cases}x=x(z)\\ y=y(z)\end{cases}$

计算 $\frac{dx}{dz}$、$\frac{dy}{dz}$ 的方法：

法一. $$\frac{dx}{dz}=-\frac{\frac{\partial(F,G)}{\partial(z,y)}}{\frac{\partial(F,G)}{\partial(x,y)}}=-\frac{\begin{vmatrix}F'_z & F'_y\\ G'_z & G'_y\end{vmatrix}}{\begin{vmatrix}F'_x & F'_y\\ G'_x & G'_y\end{vmatrix}}=-\frac{\begin{vmatrix}1 & 1\\ 2z & 2y\end{vmatrix}}{\begin{vmatrix}1 & 1\\ 2x & 2y\end{vmatrix}}=-\frac{2y-2z}{2y-2x}=\frac{z-y}{y-x}$$

$$\frac{dy}{dz}=-\frac{\frac{\partial(F,G)}{\partial(x,z)}}{\frac{\partial(F,G)}{\partial(x,y)}}=-\frac{\begin{vmatrix}F'_x & F'_z\\ G'_x & G'_z\end{vmatrix}}{\begin{vmatrix}F'_x & F'_y\\ G'_x & G'_y\end{vmatrix}}=-\frac{\begin{vmatrix}1 & 1\\ 2x & 2z\end{vmatrix}}{\begin{vmatrix}1 & 1\\ 2x & 2y\end{vmatrix}}=-\frac{2z-2x}{2y-2x}=\frac{x-z}{y-x}$$

法二. $\begin{cases}x+y+z=0\\ x^2+y^2+z^2=1\end{cases}$ 两边对 z 求导数（此时 x、y 均是关于 z 的函数）

得 $\begin{cases}\frac{dx}{dz}+\frac{dy}{dz}+1=0,\\ 2x\frac{dx}{dz}+2y\frac{dy}{dz}+2z=0.\end{cases}$ 即 $\begin{cases}\frac{dx}{dz}+\frac{dy}{dz}=-1\\ x\frac{dx}{dz}+y\frac{dy}{dz}=-z\end{cases}$

$$\frac{dx}{dz}=\frac{\begin{vmatrix}-1 & 1\\ -z & y\end{vmatrix}}{\begin{vmatrix}1 & 1\\ x & y\end{vmatrix}}=\frac{-y-(-z)}{y-x}=\frac{z-y}{y-x},\quad \frac{dy}{dz}=\frac{\begin{vmatrix}1 & -1\\ x & -z\end{vmatrix}}{\begin{vmatrix}1 & 1\\ x & y\end{vmatrix}}=\frac{-z-(-x)}{y-x}=\frac{x-z}{y-x}$$

法三. $\begin{cases}x+y+z=0\\ x^2+y^2+z^2=1\end{cases}$ 两边求微分得 $\begin{cases}dx+dy+dz=0,\\ 2xdx+2ydy+2zdz=0.\end{cases}$ 即 $\begin{cases}dx+dy=-dz\\ xdx+ydy=-zdz\end{cases}$

$$dx=\frac{\begin{vmatrix}-dz & 1\\ -zdz & y\end{vmatrix}}{\begin{vmatrix}1 & 1\\ x & y\end{vmatrix}}=\frac{-ydz-(-zdz)}{y-x}=\frac{z-y}{y-x}dz,\quad dy=\frac{\begin{vmatrix}1 & -dz\\ x & -zdz\end{vmatrix}}{\begin{vmatrix}1 & 1\\ x & y\end{vmatrix}}=\frac{-zdz-(-xdz)}{y-x}=\frac{x-z}{y-x}dz$$

$$\therefore \frac{dx}{dz}=\frac{z-y}{y-x},\quad \frac{dy}{dz}=\frac{x-z}{y-x}$$

P_{144}. 8. 设函数 $z=z(x,y)$ 是由方程组 $x=e^{u+v}$, $y=e^{u-v}$, $z=uv$ 所定义的函数，求 dz.

解：由 $\begin{cases} x=e^{u+v} \\ y=e^{u-v} \end{cases}$ 知 $\begin{cases} dx=e^{u+v}(du+dv) \\ dy=e^{u-v}(du-dv) \end{cases}$ 即 $\begin{cases} du+dv=e^{-u-v}dx \\ du-dv=e^{v-u}dy \end{cases}$

$\therefore du=\frac{1}{2}e^{-u-v}dx+\frac{1}{2}e^{v-u}dy$, $dv=\frac{1}{2}e^{-u-v}dx-\frac{1}{2}e^{v-u}dy$

$$dz=udv+vdu=u\cdot(\tfrac{1}{2}e^{-u-v}dx-\tfrac{1}{2}e^{v-u}dy)+v\cdot(\tfrac{1}{2}e^{-u-v}dx+\tfrac{1}{2}e^{v-u}dy)$$

$$=\frac{u+v}{2e^{u+v}}dx+\frac{v-u}{2e^{u-v}}dy=\frac{\ln x}{2x}dx-\frac{\ln y}{2y}dy.$$

注：若问题是书上的：求 $u=0$, $v=0$ 时的 dz，可以这么做：

$$dz=d(uv)=udv+vdu=0dv+0du=0.$$

P152. 5. 设内接于椭球 $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ 的长方体在第一卦限中的顶点为 (x,y,z)，

则此长方体体积为 $V=2x\cdot 2y\cdot 2z=8xyz$，其中 $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$，$x>0$，$y>0$，$z>0$.

令 $L(x,y,z,\lambda)=8xyz+\lambda\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}-1\right)$

由 $\begin{cases}\frac{\partial L}{\partial x}=8yz+\frac{2x}{a^2}\lambda=0\\ \frac{\partial L}{\partial y}=8xz+\frac{2y}{b^2}\lambda=0\\ \frac{\partial L}{\partial z}=8xy+\frac{2z}{c^2}\lambda=0\\ \frac{\partial L}{\partial \lambda}=\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}-1=0\end{cases}$ 得 $x=\frac{a}{\sqrt{3}}$，$y=\frac{b}{\sqrt{3}}$，$z=\frac{c}{\sqrt{3}}$，$\lambda=-\frac{4}{\sqrt{3}}abc$.

$V_{max}=8\cdot\frac{a}{\sqrt{3}}\cdot\frac{b}{\sqrt{3}}\cdot\frac{c}{\sqrt{3}}=\frac{8\sqrt{3}}{9}abc$.

P151. 1. (2) $L(x,y,z,\lambda_1,\lambda_2)=xyz+\lambda_1(x^2+y^2+z^2-1)+\lambda_2(x+y+z)$

由 $\begin{cases}\frac{\partial L}{\partial x}=yz+2\lambda_1x+\lambda_2=0\\ \frac{\partial L}{\partial y}=xz+2\lambda_1y+\lambda_2=0\\ \frac{\partial L}{\partial z}=xy+2\lambda_1z+\lambda_2=0\\ \frac{\partial L}{\partial \lambda_1}=x^2+y^2+z^2-1=0\\ \frac{\partial L}{\partial \lambda_2}=x+y+z=0\end{cases}$

得驻点为 $\left(\pm\frac{\sqrt{6}}{6},\pm\frac{\sqrt{6}}{6},\mp\frac{\sqrt{6}}{3},\pm\frac{1}{2\sqrt{6}},\frac{1}{6}\right)$，$\left(\pm\frac{\sqrt{6}}{6},\mp\frac{\sqrt{6}}{3},\pm\frac{\sqrt{6}}{6},\pm\frac{1}{2\sqrt{6}},\frac{1}{6}\right)$

$\left(\mp\frac{\sqrt{6}}{3},\pm\frac{\sqrt{6}}{6},\pm\frac{\sqrt{6}}{6},\pm\frac{1}{2\sqrt{6}},\frac{1}{6}\right)$

所求极值为 $\pm\frac{\sqrt{6}}{6}\cdot\frac{\sqrt{6}}{6}\cdot\frac{\sqrt{6}}{3}=\pm\frac{\sqrt{6}}{18}$.

P163. 1. (1) $t=1$ 时, $x=a, y=b, z=c$. $\left.\frac{dx}{dt}\right|_{t=1}=a$, $\left.\frac{dy}{dt}\right|_{t=1}=\left.2bt\right|_{t=1}=2b$. $\left.\frac{dz}{dt}\right|_{t=1}=\left.3ct^2\right|_{t=1}=3c$.

切线方程: $\frac{x-a}{a}=\frac{y-b}{2b}=\frac{z-c}{3c}$

法平面方程: $a(x-a)+2b(y-b)+3c(z-c)=0$, 即 $ax+2by+3cz=a^2+2b^2+3c^2$.

(2) $t=\frac{\pi}{2}$ 时, $x=1, y=1, z=0$, $\left.\frac{dx}{dt}\right|_{t=\frac{\pi}{2}}=\left.(-\sin t+2\sin t\cos t)\right|_{t=\frac{\pi}{2}}=-1$,

$\left.\frac{dy}{dt}\right|_{t=\frac{\pi}{2}}=\left.[\cos t(1-\cos t)+\sin t\cdot\sin t]\right|_{t=\frac{\pi}{2}}=1$, $\left.\frac{dz}{dt}\right|_{t=\frac{\pi}{2}}=\left.[-\sin t]\right|_{t=\frac{\pi}{2}}=-1$.

切线方程: $\frac{x-1}{-1}=\frac{y-1}{1}=\frac{z-0}{-1}$

法平面方程: $-1(x-1)+1\cdot(y-1)+(-1)\cdot(z-0)=0$, 即 $x-y+z=0$

(3) $\begin{cases} x=x \\ y=\sqrt{x} \\ z=x^2 \end{cases}$ 在 $(1,1,1)$ 处 $\frac{dy}{dx}=\frac{1}{2\sqrt{x}}=\frac{1}{2}$, $\frac{dz}{dx}=2x=2$.

切线方程: $\frac{x-1}{1}=\frac{y-1}{\frac{1}{2}}=\frac{z-1}{2}$

法平面方程: $1\cdot(x-1)+\frac{1}{2}\cdot(y-1)+2\cdot(z-1)=0$, 即 $2x+y+4z-7=0$.

(4) $F(x,y,z)=2x^2+y^2+z^2-45$, $G(x,y,z)=x^2+2y^2-z$

$$\left.\frac{\partial(F,G)}{\partial(y,z)}\right|_{(-2,1,6)}=\left.\begin{vmatrix} F'_y & F'_z \\ G'_y & G'_z \end{vmatrix}\right|_{(-2,1,6)}=\left.\begin{vmatrix} 2y & 2z \\ 4y & -1 \end{vmatrix}\right|_{(-2,1,6)}=\left.[-2y-8yz]\right|_{(-2,1,6)}=-50$$

$$\left.\frac{\partial(F,G)}{\partial(z,x)}\right|_{(-2,1,6)}=\left.\begin{vmatrix} F'_z & F'_x \\ G'_z & G'_x \end{vmatrix}\right|_{(-2,1,6)}=\left.\begin{vmatrix} 2z & 4x \\ -1 & 2x \end{vmatrix}\right|_{(-2,1,6)}=\left.[4xz+4x]\right|_{(-2,1,6)}=-56$$

$$\left.\frac{\partial(F,G)}{\partial(x,y)}\right|_{(-2,1,6)}=\left.\begin{vmatrix} F'_x & F'_y \\ G'_x & G'_y \end{vmatrix}\right|_{(-2,1,6)}=\left.\begin{vmatrix} 4x & 2y \\ 2x & 4y \end{vmatrix}\right|_{(-2,1,6)}=\left.[16xy-4xy]\right|_{(-2,1,6)}=\left.12xy\right|_{(-2,1,6)}=-24$$

切~~法~~向量为 $(-50,-56,-24)$ 或 $(25,28,12)$

切线方程: $\frac{x+2}{25}=\frac{y-1}{28}=\frac{z-6}{12}$

法平面方程: $25(x+2)+28(y-1)+12(z-6)=0$, 即 $25x+28y+12z=50$.

P163. 2. 曲线在点(t,t^2,t^3)的切向量为$(1,2t,3t^2)$，此切向量平行于平面$x+2y+z=4$，

所以$(1,2t,3t^2)\cdot(1,2,1)=0$，即$1+4t+3t^2=0$，解得$t=-1$或$-\frac{1}{3}$

所求点为$(-1,1,-1)$或$(-\frac{1}{3},\frac{1}{9},-\frac{1}{27})$

4. (2) $S=\int_0^2\sqrt{2^2+(2t)^2+(-2t)^2}\,dt=2\int_0^2\sqrt{1+2t^2}\,dt=\sqrt{2}\int_0^2\sqrt{1+(\sqrt{2}t)^2}\,d(\sqrt{2}t)$

$=\sqrt{2}\left[\frac{1}{2}\sqrt{2}t\sqrt{1+(\sqrt{2}t)^2}+\frac{1}{2}\ln(\sqrt{2}t+\sqrt{1+(\sqrt{2}t)^2})\right]\Big|_0^2$

$=\sqrt{2}\left[\frac{1}{2}\sqrt{2}\cdot 2\cdot\sqrt{1+2\cdot 2^2}+\frac{1}{2}\ln(2\sqrt{2}+\sqrt{1+2\cdot 2^2})\right]=\sqrt{2}\left[3\sqrt{2}+\frac{1}{2}\ln(2\sqrt{2}+3)\right]$

$=6+\frac{\sqrt{2}}{2}\ln(2\sqrt{2}+3)$

(3) $\begin{cases} x=x \\ y=\frac{x^2}{3} \\ z=\frac{2xy}{9}=\frac{2}{27}x^3 \end{cases}$

$S=\int_0^3\sqrt{1+(\frac{2}{3}x)^2+(\frac{2}{27}\cdot 3x^2)^2}\,dx=\int_0^3\sqrt{1+\frac{4}{9}x^2+(\frac{2}{9}x^2)^2}\,dx=\int_0^3(1+\frac{2}{9}x^2)dx$

$=\left[x+\frac{2}{27}x^3\right]\Big|_0^3=3+\frac{2}{27}\cdot 3^3=5.$

P170. 2. 令$F(x,y,z)=\sqrt{x^2+y^2}-z$，则

$F_x'=\frac{x}{\sqrt{x^2+y^2}}$，$F_y'=\frac{y}{\sqrt{x^2+y^2}}$，$F_z'=-1$

锥面$z=\sqrt{x^2+y^2}$在$(3,4,5)$处的法向量为$(F_x',F_y',F_z')\big|_{(3,4,5)}=\left(\frac{x}{\sqrt{x^2+y^2}},\frac{y}{\sqrt{x^2+y^2}},-1\right)\Big|_{(3,4,5)}$

$=(\frac{3}{5},\frac{4}{5},-1)$

$\|n\|=\sqrt{(\frac{3}{5})^2+(\frac{4}{5})^2+(-1)^2}=\sqrt{2}$

$\cos\alpha=\frac{\frac{3}{5}}{\sqrt{2}}=\frac{3}{5\sqrt{2}}$，$\cos\beta=\frac{\frac{4}{5}}{\sqrt{2}}=\frac{4}{5\sqrt{2}}$，$\cos\gamma=\frac{-1}{\sqrt{2}}$。$\frac{\partial u}{\partial x}=yz$，$\frac{\partial u}{\partial y}=xz$，$\frac{\partial u}{\partial z}=xy$

所求方向导数为$\frac{\partial u}{\partial x}\big|_{(3,4,5)}\cos\alpha+\frac{\partial u}{\partial y}\big|_{(3,4,5)}\cos\beta+\frac{\partial u}{\partial z}\big|_{(3,4,5)}\cos\gamma=20\cdot\frac{3}{5\sqrt{2}}+15\cdot\frac{4}{5\sqrt{2}}+12\cdot\frac{-1}{\sqrt{2}}=6\sqrt{2}.$

当法向量$\vec{n}=(-\frac{3}{5},-\frac{4}{5},1)$时，所求方向导数为$-6\sqrt{2}$.

P71. 5. 令 $F(x,y,z)=f(ax-bz,\ ay-cz)$，则

$F'_x=f'_1\cdot a$ ，$F'_y=f'_2\cdot a$，$F'_z=f'_1\cdot(-b)+f'_2\cdot(-c)$

曲面 $f(ax-bz,ay-cz)=0$ 上任一点 (x,y,z) 处的法向量是 $(af'_1,af'_2,-bf'_1-cf'_2)$.

因为 $af'_1\cdot b+af'_2\cdot c+(-bf'_1-cf'_2)\cdot a=0$，从而该法向量垂直于向量 (b,c,a)，

曲面 $f(ax-bz,ay-cz)=0$ 上任一点的切平面都与方向向量为 (b,c,a) 的直线平行.

6. 曲线 $\begin{cases}x=y^2\\y=y\\z=3(y-1)\end{cases}$ 在 $y=1$ 处的切向量为 $(2y,1,3)|_{y=1}=(2,1,3)$. $y=1$ 时，$x=1,z=0$.

切线方程为 $\frac{x-1}{2}=\frac{y-1}{1}=\frac{z-0}{3}$ 或 $\begin{cases}\cancel{x=2y}\ \frac{x-1}{2}=\frac{y-1}{1}\\ \frac{y-1}{1}=\frac{z-0}{3}\end{cases}$ 即 $\begin{cases}x-2y+1=0\\3y-z-3=0\end{cases}$

过此切线的平面方程为 $\alpha(x-2y+1)+\beta(3y-z-3)=0$，

即 $\alpha x+(3\beta-2\alpha)y-\beta z+\alpha-3\beta=0$.

设该平面与曲面 $x^2+y^2=4z$ 相切的切点为 (x,y,z)，则

$$\begin{cases}\alpha x+(3\beta-2\alpha)y-\beta z+\alpha-3\beta=0\\x^2+y^2=4z\\ \frac{\alpha}{2x}=\frac{3\beta-2\alpha}{2y}=\frac{-\beta}{-4}\end{cases}$$

解得 $\beta=\alpha$ 或 $\beta=\frac{5}{6}\alpha$.

$\beta=\alpha$ 时，所求平面方程为 $\alpha x+\alpha y-\alpha z-2\alpha=0$，即 $x+y-z-2=0$

$\beta=\frac{5}{6}\alpha$ 时，所求平面方程为 $\alpha x+(3\cdot\frac{5}{6}\alpha-2\alpha)y-\frac{5}{6}\alpha+\alpha-3\cdot\frac{5}{6}\alpha=0$，即 $6x+3y-5z-9=0$

8. 曲线 $\begin{cases}3x^2+2y^2=12\\z=0\end{cases}$ 绕 y 轴旋转一周所得旋转曲面方程为 $3(x^2+z^2)+2y^2=12$，即 $3x^2+2y^2+3z^2=12$，

设 $F(x,y,z)=3x^2+2y^2+3z^2-12$，则 $(F'_x,F'_y,F'_z)|_{(0,\sqrt3,\sqrt2)}=(6x,4y,6z)|_{(0,\sqrt3,\sqrt2)}=(0,4\sqrt3,6\sqrt2)$

旋转曲面在 $(0,\sqrt3,\sqrt2)$ 处由内部指向外部的法向量 $\vec{n}=(0,2\sqrt3,3\sqrt2)$，$\|\vec{n}\|=\sqrt{0^2+(2\sqrt3)^2+(3\sqrt2)^2}=\sqrt{30}$

所求单位法向量为 $(0,\frac{2\sqrt3}{\sqrt{30}},\frac{3\sqrt2}{\sqrt{30}})$.