分析：可微 $\Leftrightarrow \dfrac{\Delta z - f_x'(x_0,y_0)\Delta x - f_y'(x_0,y_0)\Delta y}{\rho} \to 0 \quad (\rho\to 0)$

证：$\Delta z = f(x_0+\Delta x, y_0+\Delta y) - f(x_0, y_0)$

$= [f(x_0+\Delta x, y_0+\Delta y) - f(x_0, y_0+\Delta y)] + [f(x_0, y_0+\Delta y) - f(x_0, y_0)]$

$f(x_0+\Delta x, y_0+\Delta y) - f(x_0, y_0+\Delta y)$

$= f_x'(x_0+\theta_1\Delta x, y_0+\Delta y)\cdot\Delta x \qquad 0<\theta_1<1$

$\because f_x'(x,y)$ 在 (x_0,y_0) 连续.

$\therefore f_x'(x_0+\theta_1\Delta x, y_0+\Delta y) = f_x'(x_0,y_0) + \alpha$ 其中 $\lim\limits_{\substack{\Delta x\to 0\\ \Delta y\to 0}} \alpha = 0$

同理：$f(x_0, y_0+\Delta y) - f(x_0, y_0) = f_y'(x_0, y_0+\theta_2\Delta y)\cdot\Delta y$ ，$0<\theta_2<1$

而 $f_y'(x_0, y_0+\theta_2\Delta y) = f_y'(x_0, y_0) + \beta$，其中 $\lim\limits_{\substack{\Delta x\to 0\\ \Delta y\to 0}} \beta = 0$

故 $\Delta z = [f_x'(x_0,y_0)+\alpha]\Delta x + [f_y'(x_0,y_0)+\beta]\cdot\Delta y$

$= f_x'(x_0,y_0)\Delta x + f_y'(x_0,y_0)\Delta y + \alpha\Delta x + \beta\cdot\Delta y$

$\therefore \lim\limits_{\rho\to 0} \dfrac{\Delta z - f_x'(x_0,y_0)\Delta x - f_y'(x_0,y_0)\Delta y}{\rho}$

$= \lim\limits_{\rho\to 0} \dfrac{\alpha\Delta x + \beta\cdot\Delta y}{\rho} = 0$

事实上 $0 \le \left|\dfrac{\alpha\Delta x+\beta\Delta y}{\rho}\right| = \left|\dfrac{\alpha\Delta x+\beta\Delta y}{\sqrt{(\Delta x)^2+(\Delta y)^2}}\right| \le |\alpha|\dfrac{|\Delta x|}{\sqrt{(\Delta x)^2+(\Delta y)^2}} + |\beta|\dfrac{|\Delta y|}{\sqrt{(\Delta x)^2+(\Delta y)^2}}$

$\le |\alpha| + |\beta| \to 0 \qquad (\rho\to 0)$

例7. 求 $z = x^2+3xy+y^2$ 全微分

解：$\dfrac{\partial z}{\partial x} = 2x+3y$， $\dfrac{\partial z}{\partial y} = 3x+2y$ 连续.

$\therefore dz = (2x+3y)dx + (3x+2y)dy$

例8. 讨论 $f(x,y) = |xy|$ 在点 $(0,0)$ 处可微性及偏导数连续性.

解：$f_x'(0,0) = \lim\limits_{\Delta x\to 0} \dfrac{f(\Delta x,0)-f(0,0)}{\Delta x} = \lim\limits_{\Delta x\to 0}\dfrac{0-0}{\Delta x} = 0$