证明:

$$
\begin{aligned}
& A \cap B = A \\
\iff & \forall x((x \in A \wedge x \in B) \leftrightarrow x \in A) && \text{(外延原则、集合交定义)} \\
\iff & \forall x(((x \in A \wedge x \in B) \to x \in A) \wedge (x \in A \to (x \in A \wedge x \in B))) && \text{(等价联结词定义)} \\
\iff & \forall x((\neg(x \in A \wedge x \in B) \vee x \in A) \wedge (\neg x \in A \vee (x \in A \wedge x \in B))) && \text{(蕴涵等值式)} \\
\iff & \forall x((\neg x \in A \vee \neg x \in B \vee x \in A) \wedge (\neg x \in A \vee (x \in A \wedge x \in B))) && \text{(命题逻辑德·摩根律)} \\
\iff & \forall x((\neg x \in A \vee x \in A \vee \neg x \in B) \wedge (\neg x \in A \vee (x \in A \wedge x \in B))) && \text{(命题逻辑交换律)} \\
\iff & \forall x((\neg x \in A \vee x \in A \vee \neg x \in B) \wedge \\
& \qquad ((\neg x \in A \vee x \in A) \wedge (\neg x \in A \vee x \in B))) && \text{(命题逻辑分配律)} \\
\iff & \forall x((1 \vee \neg x \in B) \wedge (1 \wedge (\neg x \in A \vee x \in B))) && \text{(命题逻辑排中律)} \\
\iff & \forall x(1 \wedge (1 \wedge (\neg x \in A \vee x \in B))) && \text{(命题逻辑零律)} \\
\iff & \forall x(\neg x \in A \vee x \in B) && \text{(命题逻辑同一律)} \\
\iff & \forall x(x \in A \to x \in B) && \text{(蕴涵等值式)} \\
\iff & A \subseteq B && \text{(子集关系定义)}
\end{aligned}
$$

□

(2) 答：$A \cup B = A$ 当且仅当 $B \subseteq A$。

证明:

$$
\begin{aligned}
& A \cup B = A \\
\iff & \forall x((x \in A \vee x \in B) \leftrightarrow x \in A) && \text{(外延原则、集合并定义)} \\
\iff & \forall x(((x \in A \vee x \in B) \to x \in A) \wedge (x \in A \to (x \in A \vee x \in B))) && \text{(等价联结词定义)} \\
\iff & \forall x((\neg(x \in A \vee x \in B) \vee x \in A) \wedge (\neg x \in A \vee x \in A \vee x \in B)) && \text{(蕴涵等值式)} \\
\iff & \forall x(((\neg x \in A \wedge \neg x \in B) \vee x \in A) \wedge (\neg x \in A \vee x \in A \vee x \in B)) && \text{(命题逻辑德·摩根律)} \\
\iff & \forall x(((\neg x \in A \vee x \in A) \wedge (\neg x \in B \vee x \in A)) \wedge \\
& \qquad (\neg x \in A \vee x \in A \vee x \in B)) && \text{(命题逻辑分配律)} \\
\iff & \forall x((1 \wedge (\neg x \in B \vee x \in A)) \wedge (1 \vee x \in B)) && \text{(命题逻辑排中律)} \\
\iff & \forall x((1 \wedge (\neg x \in B \vee x \in A)) \wedge 1) && \text{(命题逻辑零律)} \\
\iff & \forall x(\neg x \in B \vee x \in A) && \text{(命题逻辑同一律)} \\
\iff & \forall x(x \in B \to x \in A) && \text{(蕴涵等值式)} \\
\iff & B \subseteq A && \text{(子集关系定义)}
\end{aligned}
$$

□

(3) 答：$A \oplus B = A$ 当且仅当 $B = \varnothing$。

证明: 充分性。若 $B = \varnothing$，则：

$$
\begin{aligned}
A \oplus B &= A \oplus \varnothing && (B = \varnothing) \\
&= A && \text{(教材例 1.7(4))}
\end{aligned}
$$

必要性。若 $A \oplus B = A$，则：

$$
\begin{aligned}
B &= \varnothing \oplus B && \text{(教材例 1.7(4))} \\
&= (A \oplus A) \oplus B && \text{(教材例 1.7(5))}
\end{aligned}
$$