

习题. 设 A, B, C, D 为任意四个集合, 证明

$$(A \cap B) \times (C \cap D) = (A \times C) \cap (B \times D)$$

$$\begin{aligned} & \forall x \forall y (x, y) \in (A \cap B) \times (C \cap D) \\ & \Leftrightarrow x \in A \cap B \wedge y \in C \cap D \\ & \Leftrightarrow x \in A \wedge x \in B \wedge y \in C \wedge y \in D \\ & \Leftrightarrow x \in A \wedge y \in C \wedge x \in B \wedge y \in D \\ & \Leftrightarrow (x, y) \in A \times C \wedge (x, y) \in B \times D \\ & \Leftrightarrow (x, y) \in (A \times C) \cap (B \times D) \end{aligned}$$

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□