

习题讲解

陈建文

习题

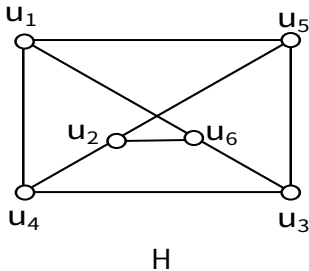
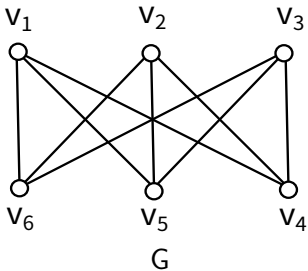
画出具有4个顶点的所有无向图（同构的只算一个）。

习题

画出具有4个顶点的所有无向图（同构的只算一个）。

定义1

设 $G = (V, E)$, $H = (U, F)$ 为两个图，如果存在一个一一对应 $\phi: V \rightarrow U$ ，使得 $\{u, v\} \in E$ 当且仅当 $\{\phi(u), \phi(v)\} \in F$ ，则称 G 与 H 同构。



习题

画出具有4个顶点的所有无向图（同构的只算一个）。



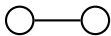
A

习题

画出具有4个顶点的所有无向图（同构的只算一个）。



A



B

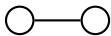


习题

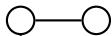
画出具有4个顶点的所有无向图（同构的只算一个）。



A



B



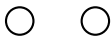
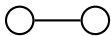
C

习题

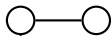
画出具有4个顶点的所有无向图（同构的只算一个）。



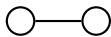
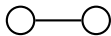
A



B



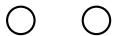
C



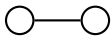
D

习题

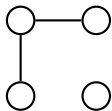
画出具有4个顶点的所有无向图（同构的只算一个）。



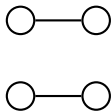
A



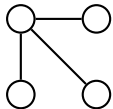
B



C



D



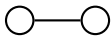
E

习题

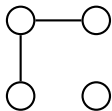
画出具有4个顶点的所有无向图（同构的只算一个）。



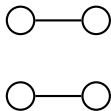
A



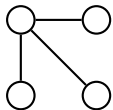
B



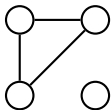
C



D



E



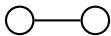
F

习题

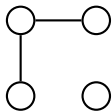
画出具有4个顶点的所有无向图（同构的只算一个）。



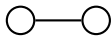
A



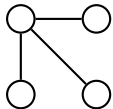
B



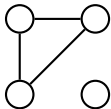
C



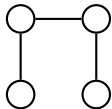
D



E



F



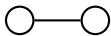
G

习题

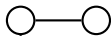
画出具有4个顶点的所有无向图（同构的只算一个）。



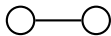
A



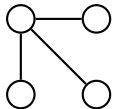
B



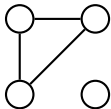
C



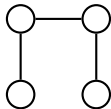
D



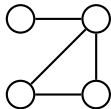
E



F



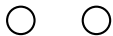
G



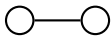
H

习题

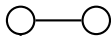
画出具有4个顶点的所有无向图（同构的只算一个）。



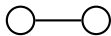
A



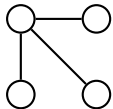
B



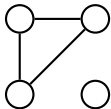
C



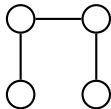
D



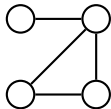
E



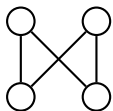
F



G



H



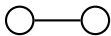
I

习题

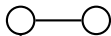
画出具有4个顶点的所有无向图（同构的只算一个）。



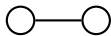
A



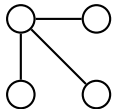
B



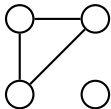
C



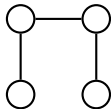
D



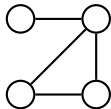
E



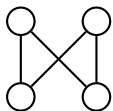
F



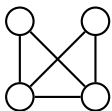
G



H



I



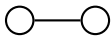
J

习题

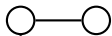
画出具有4个顶点的所有无向图（同构的只算一个）。



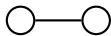
A



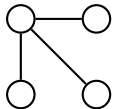
B



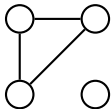
C



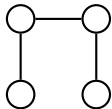
D



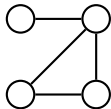
E



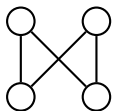
F



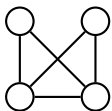
G



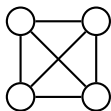
H



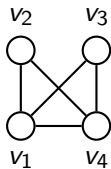
I



J



K



J

$$J = (V, E)$$

$$V = \{v_1, v_2, v_3, v_4\}$$

$$E = \{\{v_1, v_2\}, \{v_1, v_3\}, \{v_1, v_4\}, \{v_2, v_4\}, \{v_3, v_4\}\}$$

定义2

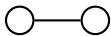
设 $G = (V, E)$ 为一个图，图 $G^c = (V, \mathcal{P}_2(V) \setminus E)$ 称为 G 的补图。
如果 G 与 G^c 同构，则称 G 为自补图。

习题

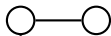
画出具有4个顶点的所有无向图（同构的只算一个）。



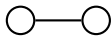
A



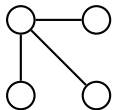
B



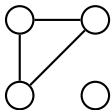
C



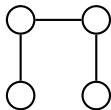
D



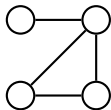
E



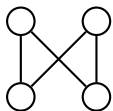
F



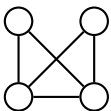
G



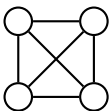
H



I



J



K

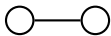
哪些图为自补图？

习题

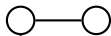
画出具有4个顶点的所有无向图（同构的只算一个）。



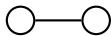
A



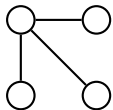
B



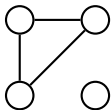
C



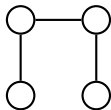
D



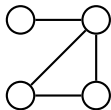
E



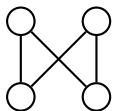
F



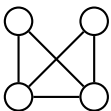
G



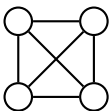
H



I



J



K

哪些图为自补图？ G

定义3

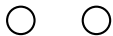
设 $G = (V, E)$ 为一个图, 如果 G 的顶点集 V 有一个二划分 $\{V_1, V_2\}$, 使得 G 的任一条边的两个端点一个在 V_1 中, 另一个在 V_2 中, 则称 G 为偶图。如果 $\forall u \in V_1, v \in V_2$ 均有 $uv \in E$, 则称 G 为完全偶图, 记为 $K_{m,n}$, 其中 $|V_1| = m, |V_2| = n$ 。

定理1

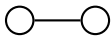
图 G 为偶图的充分必要条件为它的所有圈都是偶数长。

习题

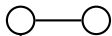
画出具有4个顶点的所有无向图（同构的只算一个）。



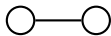
A



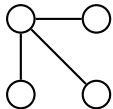
B



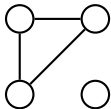
C



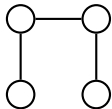
D



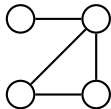
E



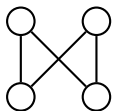
F



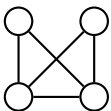
G



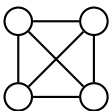
H



I



J



K

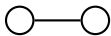
哪些图为偶图？

习题

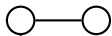
画出具有4个顶点的所有无向图（同构的只算一个）。



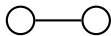
A



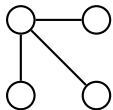
B



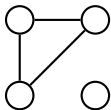
C



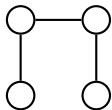
D



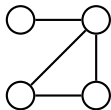
E



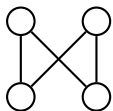
F



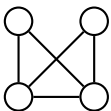
G



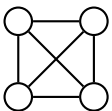
H



I



J



K

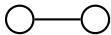
哪些图为偶图？ABCDEFGHI

习题

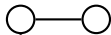
画出具有4个顶点的所有无向图（同构的只算一个）。



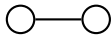
A



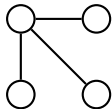
B



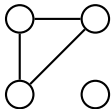
C



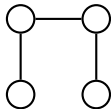
D



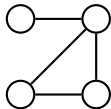
E



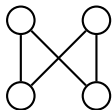
F



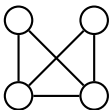
G



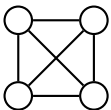
H



I



J



K

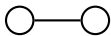
哪些图为偶图？ABCDEFGHI
哪些图为完全偶图？

习题

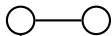
画出具有4个顶点的所有无向图（同构的只算一个）。



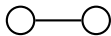
A



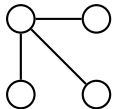
B



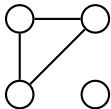
C



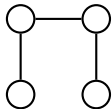
D



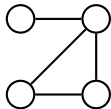
E



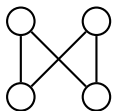
F



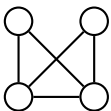
G



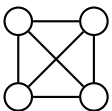
H



I



J



K

哪些图为偶图？ABCDEFGHI
哪些图为完全偶图？EI

定义4

包含图的所有顶点和所有边的闭迹称为欧拉闭迹。存在一条欧拉闭迹的图称为欧拉图。

定理2

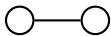
图 G 为欧拉图当且仅当 G 为连通的且每个顶点的度为偶数。

习题

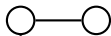
画出具有4个顶点的所有无向图（同构的只算一个）。



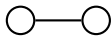
A



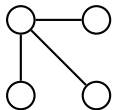
B



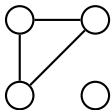
C



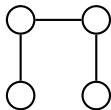
D



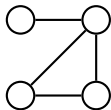
E



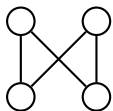
F



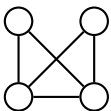
G



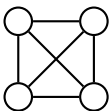
H



I



J



K

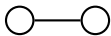
哪些图为欧拉图？

习题

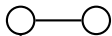
画出具有4个顶点的所有无向图（同构的只算一个）。



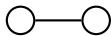
A



B



C



D



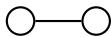
E



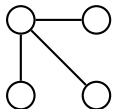
F



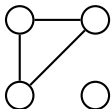
G



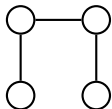
H



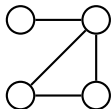
I



J



K



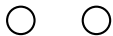
哪些图为欧拉图？

定义5

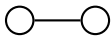
图 G 的一条包含所有顶点的路称为 G 的一条哈密顿路;图 G 的一个包含所有顶点的圈称为 G 的一个哈密顿圈。具有哈密顿圈的图称为哈密顿图。

习题

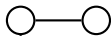
画出具有4个顶点的所有无向图（同构的只算一个）。



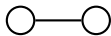
A



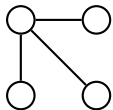
B



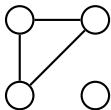
C



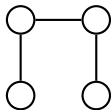
D



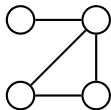
E



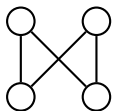
F



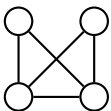
G



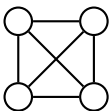
H



I



J



K

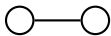
哪些图有哈密顿路？

习题

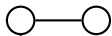
画出具有4个顶点的所有无向图（同构的只算一个）。



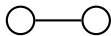
A



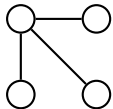
B



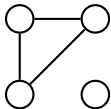
C



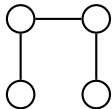
D



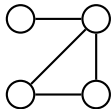
E



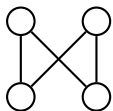
F



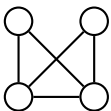
G



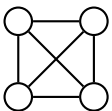
H



I



J

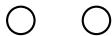


K

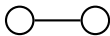
哪些图有哈密顿路？ GHIJK

习题

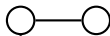
画出具有4个顶点的所有无向图（同构的只算一个）。



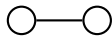
A



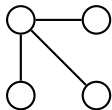
B



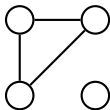
C



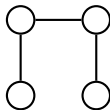
D



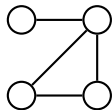
E



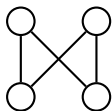
F



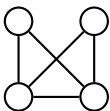
G



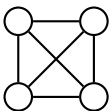
H



I



J



K

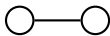
哪些图有哈密顿路？ GHIJK
哪些图为哈密顿图？

习题

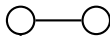
画出具有4个顶点的所有无向图（同构的只算一个）。



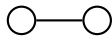
A



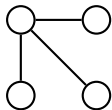
B



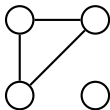
C



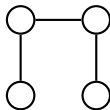
D



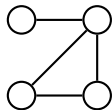
E



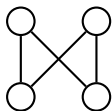
F



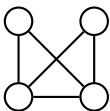
G



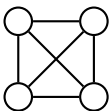
H



I



J



K

哪些图有哈密顿路？ GHIJK
哪些图为哈密顿图？ IJK

$$\frac{100!}{2^{40} * 3600 * 24 * 365} > \frac{2^{100}}{2^{40} * 2^{20} * 2^{10} * 2^{10}} = 2^{20}$$

$$S = \{1, 2, 7, 14, 49, 98, 343, 686, 2409, 2793, 16808, 17206, 117705, 117993\}$$

$$t = 138457$$

$$S' = \{1, 2, 7, 98, 343, 686, 2409, 17206, 117705\}$$

$$a^3 + b^3 = c^3$$