Question one (30 marks)

Discuss the following accuracy of numbers

(1mark) (Imark)

Qu

- ii. Approximate numbers

 b. A real root of the equation $f(x) = x^3 5x + 1 = 0$ lies in the interval (0,1). Perform g_{xy} iterations of the Newton-Raphson method to obtain this root. Take the initial c. Apply the Runge-Kutta method of order 2 to the differential equation y' = -y + t.
- (5marks) $0 \le t \le 1$, $\gamma(0) = 1$, h = 0.1
- d. Find the relative error in calculation of $\frac{6.213}{0.342}$ where numbers 6.213 & 0.342 are correct to the second state of the second seco 0.342 to three decimal places. Determine the smallest interval in which true results lies (5marks)
- e Perform three iterations of the Newton-Raphson method to solve the system of equations (6marks) $x^2 + xy + y^7 = 7$ $x^{1} + y^{2} = 9$

Take the initial approximation as $x_0 = 1.5$, $y_0 = 0.5$. The exact solution is x = 2, y = 1

f. Given the data in the table below, use the central difference two point formula and (7marks) Richardson extrapolation to find f'(5).

X	1	3	4	5	6	7	9
f(x):	-11	5	37	93	179	301	943

Question two (20marks)

a. Using the Lagrange formula, find the unique polynomial of degree 2 which fits the given

f x	10			(7marks)
f(x)	190	2	4	
D. F. 1	1	8	16	

Perform three iterations of the bisection method to obtain the smallest positive root of the

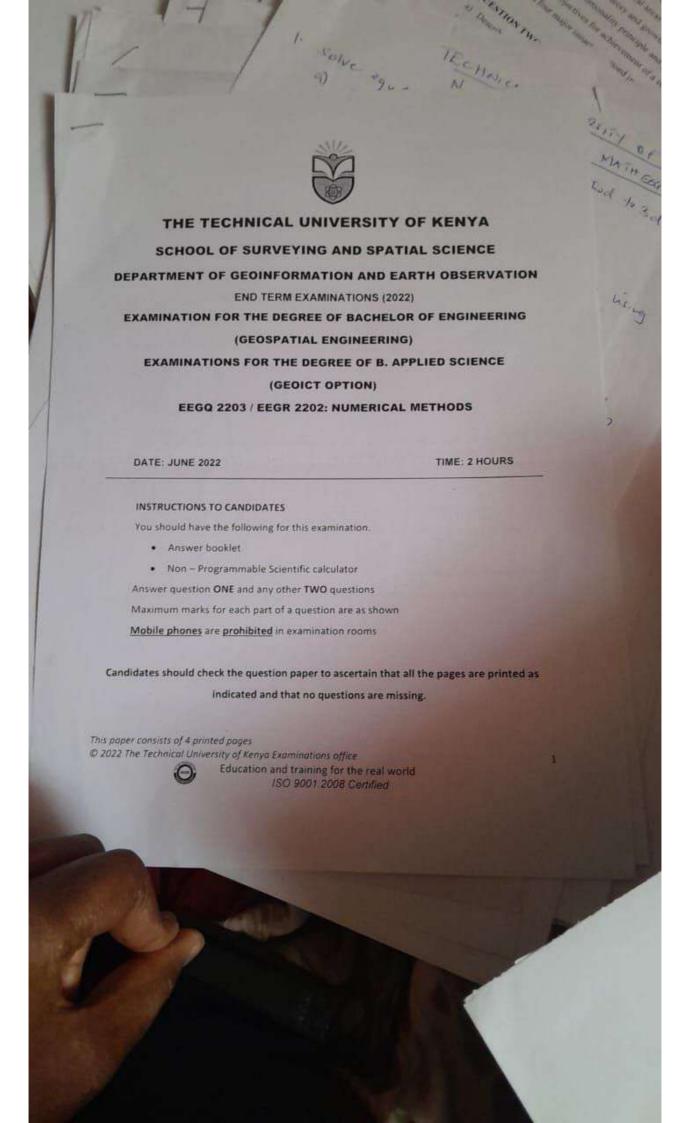
$$f(x) = x^3 - 6x + 1 = 0$$

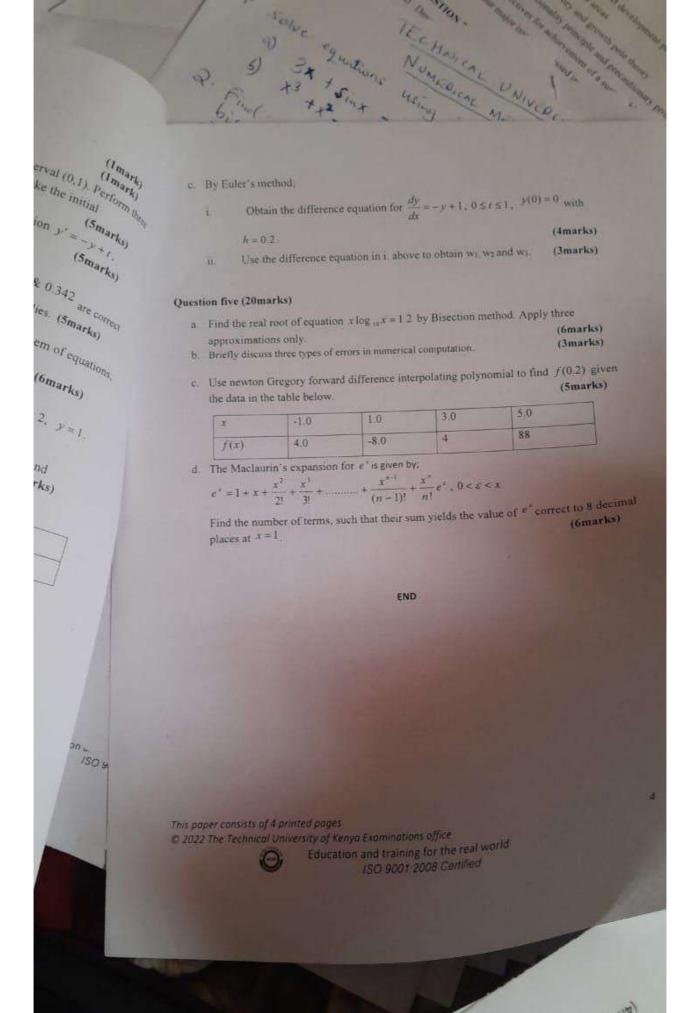
(5marks)

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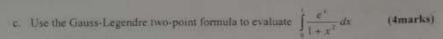


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d. Determine the error when computing the value of the function $f(x, y, z) = xz - \frac{y}{xz}$ using the following values x - 2.3, y = 1.52, z = 5.247 where each value is assumed to be rounded to the nearest significant digit. (4marks)

Question Three (20 marks)

a. Given the data in the table below, use the Newton-Gregory backward difference interpolation polynomial to find f(5) (7mar

x	-4	-2	0	2	4	6
f(x)	-207	35	1	-3	49	253

b. By taking $X = \frac{x_1}{x_2}$ obtain a general formula for error in Division of numbers.

(5marks)

c. Use Newton-Raphson method to find the root of the equation $e^{2x} - 5x - 7 = 0$ which is close to x = 1.5 correct to four decimal places. Apply three approximations.

(5marks)

d. By taking e_i and e_{i+1} as the errors in i^{th} and $(i+1)^{th}$ iterations respectively, show that Bisection method is of first order convergent. (3marks)

Question four (20marks)

- a. The function f(x) = Sin x is defined on the interval [1,3];
 - Obtain the Lagrange linear interpolating polynomial in this interval and find the bound on the truncation error. (5marks)
 - Obtain the approximate values of f(1.5) & f(2.5) (2marks)
- b. Find the approximate value of $I = \int_{0}^{1} \frac{dx}{1+x}$ and obtain a bound for the errors using;
 - i. Trapezoidal rule

(3marks)

ii. Simpson's rule

(3marks)

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