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Department Of Geodetic and Spatial Sciences.

Traverse Adjustment using least squares Method.

BY

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A research project report submitted to the school of Surveying and Geospatial Sciences for the Award of degree of Bachelors of Engineering in Geospatial Engineering.

Declaration

I, Otieno Otieno Brian, Reg No. ESSQ/00530/2018, do hereby declare that this research project is my original work and has not been presented as part of fulfilment of a degree requirement in this or any other university.

Signature…………………………. Date……………………………………

This project has been submitted for examination with my approval as the designated University Supervisor.

Signature……………………………. Date ………………………………….

Name of Supervisor: **Dr. Godfrey Ogonda**

Aknowledgement

I thank the almighty God for granting me good health, safety and continuous providence and guidance throughout my studies and conduct of this project.

I would like to thank my supervisor Dr. Ogonda, Prof. Gordon Wayumba and Mr. Peter Odwe, for the guidance, encouragement and advice they have provided throughout my time as their student.

I would also like to thank the staff of the School of Survey and Spatial Sciences (SSSS) for giving me an opportunity as a student member of the school to make use of the school resources for my research project and all the guidance and support accorded to me.

Dedication

I dedicate this project to my parents, Mr. John Benedict Otieno and Mrs. Rosemary Otieno, and Uncle Erick Ochieng’ for their tremendous support both emotionally and financialy throughout my entire studies.

Abstract

Traversing is a method used in extending horizontal controls in surveying measurements. The observations have to be adjusted to reduce errors. Few of the professionals in the survey field perform their traverse adjustments using least squares method due to its complexity and time-consuming despite proving to be the most rigorous method for traverse adjustments. This method is the most preferred for high accuracy surveying projects compared with Bowditch and transit methods of traverse adjustment. This method aims at optimizing traverse networks by adjusting weights assigned to each link based on real-world network data. The least square utilizes method of creation of observation equations, developed by Ghilani C. D. (2006), from angle, distance and bearing observations and the Jacobian iterative method to solve the equations and minimize the residuals resulting in an improved traverse network. Implementing this method in python allows for easy integration to existing network analysis tools and efficient computation of the adjustments using SciPy, NumPy and pandas libraries of python. The results are then checked using Bowditch method demonstrating an improvement in the traverse network reducing the time and complex nature of the method.

Table of contents

Contents

List of tables

List of Figures

CHAPTER 1

INTRODUCTION

1.1. Background Information

Traversing is a method of control surveying, where the coordinates of series of chosen control points are to be determined, each being intervisible with its adjacent stations (Bannister, Raymond, Baker,1998, p. 161). It is a preferred way of establishing control because of; less reconnaissance and organization needed, the ability to assume any shape thus enabling surveys of any type of terrain and its fewer reading requirement at every station as compared to other methods.

There are two main types of traverses i.e., Open and closed traverse. Open traverses are a type of traverse where the traverse points are computed but the traverse does not terminate at the starting point or at a known control point. Closed traverses are when the traverse either terminates to more than two known control points or at the point where it started.

In surveying, observed quantities have to meet some theoretically proven geometric properties. For example, angles in a loop traverse have to add up to a certain predefined value . However, they very rarely meet these expectations and have to be adjusted mathematically. Least squares adjustment is a statistical originally explored by Laplace (1774) and first published by Legendre (1805). This method is used to statistically reduce the square of the residuals of a set of measurements to a minimum value. It works from a set of equations formed from the observations i.e., observation or conditional equations. The equations are then reduced to the format Ax = B where A is the matrix of coefficients, x is the matrix of unknowns to be determined and B is the matrix of residuals.

X is then solved using any suitable method e.g., Normal equations, Jacobi iterative methods, Cholesky etc.

Least squares adjustment has been adopted in the adjustment of surveying measurements therefore, can be used in traverse observation adjustment as by reducing the residuals, the measurements are more precise.

1.2. Statement of the problem.

In all surveying measurements, there exists an error in the measurements. The error can be random, systematic or gross. These errors are also present in traverse observations, and a method to adjust the errors, or distribute them is required. Currently there are two common ways of adjusting the traverse observations, the transit method and the Bowditch method. The transit method assumes angles are more accurately measured than distances, and therefore distributes the closure error with respect to the ratio of length of the leg's change in northing and the total absolute length of northings in the traverse.

The most commonly used method is the compass or Bowditch method. It assumes that both angles and distances were measured with equal precision. It then distributes the misclosure error in the changes in northings and eastings based on the ratio of the length of the leg in question to the total length of the traverse. This method equally takes into account of both the distance and angular measurements in the correction.

However, both transit and Bowditch methods assume either one measurement is correct or both are correct equally. This is a problem because distance and angular measurements can be made by an instrument with different accuracy, or by two different instruments. In such a scenario then the adjustments methods will just assume both accuracies are same(compass/Bowditch) or discard one(transit).

The least squares method however, takes each individual measurement's accuracies into account and adjusts all the measurements with respect to their unique measurement accuracies.

1.3 Objectives

1.3.1. Main Objective

The main objective of this project is to demonstrate how a link traverse can be adjusted using least squares Adjustment.

1.3.2. Specific objectives

1. To develop an algorithm to solve the formed observation equations.
2. To analyze the results.

3. To compare the results with Bowditch method.

1.4 Significance of the study

Traversing is an important aspect of surveying that is used for densification of control networks. It affects many sectors i.e., construction industry, land surveying sector and also the civil engineering sector. With the implementation of least squares adjustment, more precise coordinates can be established from traversing, and therefore can be used as reference points for high accuracy projects such as tunnel construction, ascertain coordinates of international boundaries and many more. This hence reduces the time and costs of doing tedious traverse surveys for controls establishments.

1.5 Study area

The study area for this project is a secondary data in the Survey of Kenya (pages VI.7 – VI.9) manual conducted with the current surveying machines between confined well established beacons.

1.6 Methodological workflow

Principles of least squares will be applied whereby weights are assigned based on each distance between two points in the network, observation equations based on distance and bearing are then formed, an algorithm for iteration and multiplication of generated matrix is then developed in a python environment for faster and high accurate execution of the adjustment of the traverse. The results are then compared to the Bowditch adjusted coordinates.

1.7 Expected results

Adjusted coordinates of points within the network with very minimized errors within minimum time.

1.8 Conclusion

From this project it is concluded that it is possible to develop an algorithm to adjust traverse data using Least squares method.

1.9 Timeline

The project is estimated to take approximately three months i.e.

First month - Data acquisition and literature compilation.

Second month – Method and algorithm development

Third month – Output analysis

CHAPTER 2

2.1 Literature Review

2.1.1 Traverse Measurements

A traverse is a continuous series of connected lines of known lengths related to one another by known angles. The lengths of the lines are determined by direct measurement of horizontal distances, slope measurement, or by indirect measurement using the methods of stadia or the subtense bar. The line courses run between a series of points are called traverse stations. The angles at the traverse stations, between the lines are measured by tape, transit, theodolite, compass, plane table, or sextant.

These angles can be interior angles, deflection angles, or angles to the right. The lengths and azimuths or bearings of each line of the traverse are estimated through field measurements. The lengths are horizontal distances, and the azimuths or bearings are true, magnetic, assumed, or grid. There are two types or classes of traverses;

An open traverse is called a first class traverse. It starts at a point of known or assumed horizontal position with respect to a horizontal datum, and

terminates at an unknown horizontal position. Thus, open traverses end without closure. Open traverses are used on route surveys, but should be avoided whenever possible since they cannot be properly checked. Measurements in open traverses should be repeated to minimize mistakes.

A closed traverse is called a second class traverse. It starts at a known or assumed horizontal position and terminates at that point (i.e., loop traverse), or it starts at a known horizontal position and terminates at another known horizontal position (i.e., connecting/ray trace traverses). Both the measured angles and lengths in a closed traverse may be checked.

A known horizontal position is defined by its geographic latitude and longitude, its Y- and X-coordinates on a grid system, or by its location on or in relation to a fixed boundary.

Traverses are used to find accurate positions of a small number of marked stations. From these stations, less precise measurements can be made to features to be located without accumulating accidental errors. Thus, traverses usually serve as control surveys. When drawing construction plans, the stations can be used as beginning points from which to lay out work. When new construction of any kind is to be made, a system of traverse stations in the area must be established and surveyed.

Traverse surveys are made for many purposes to include:

- To determine the positions of exiting boundary markers.
- To establish the positions of boundary lines.
- To determine the area encompassed within a boundary.
- To determine the positions of arbitrary points from which data may be obtained for preparing various types of maps (i.e., establish control for map making).
- To establish ground control for photographic mapping.
- To establish control for gathering data regarding earthwork quantities in railroad highway, utility, and other construction work.

- To establish control for locating railroads, highways, and other construction work.

A natural resource professional may need to run a boundary survey for a recreation site, research plot, forest stand, or wildlife habitat area. The shape of the area may be a polygon with three or more sides. The boundary may be located in the field as it is surveyed. Then the polygon is drawn on a map sheet with the area in acres being determined for planning purposes. Open traverses may be run to establish preliminary trail and road locations.

2.1.2 History of Traverse in Kenya

At the beginning of the 20th century the Anglo German Boundary Commission (AGBC) of 1893 that consisted of Belgians, Portuguese, British and Germans was established to carry out delimitation of boundaries of colonies in Central and East Africa.
The delimitation of interterritorial boundaries needed geodetic control points as a basis, but none existed at that time. The colonial powers thus established and observed triangulation networks along the agreed boundaries. The triangulation networks established were not well conditioned insofar as geodetic requirements are concerned and were rather shaped by the boundaries.
The first triangulation network to be observed by the AGBC in East Africa was between Kenya and Tanganyika (current Tanzania) between 1892 and 1893. The second triangulation network was done by the Anglo German Boundary Commission of 1902-1906.

The Adindan Datum
This is a chain of traverses that were measured with the EDM, by the USA government surveyors from Dakar in Senegal to Djibouti to provide for scaling of the Africa continent in the East West Direction. These measurements created the Adindan Datum used for mapping in most of the Sahelian countries including Sudan and Ethiopia. Thus there are two main geodetic datums in Africa, the 30th Meridional Arc running from South Africa to Cairo and the Andidan Arc running from Dakar to Djibouti.

The figure below illustrate the African 30th Arc Meridian.

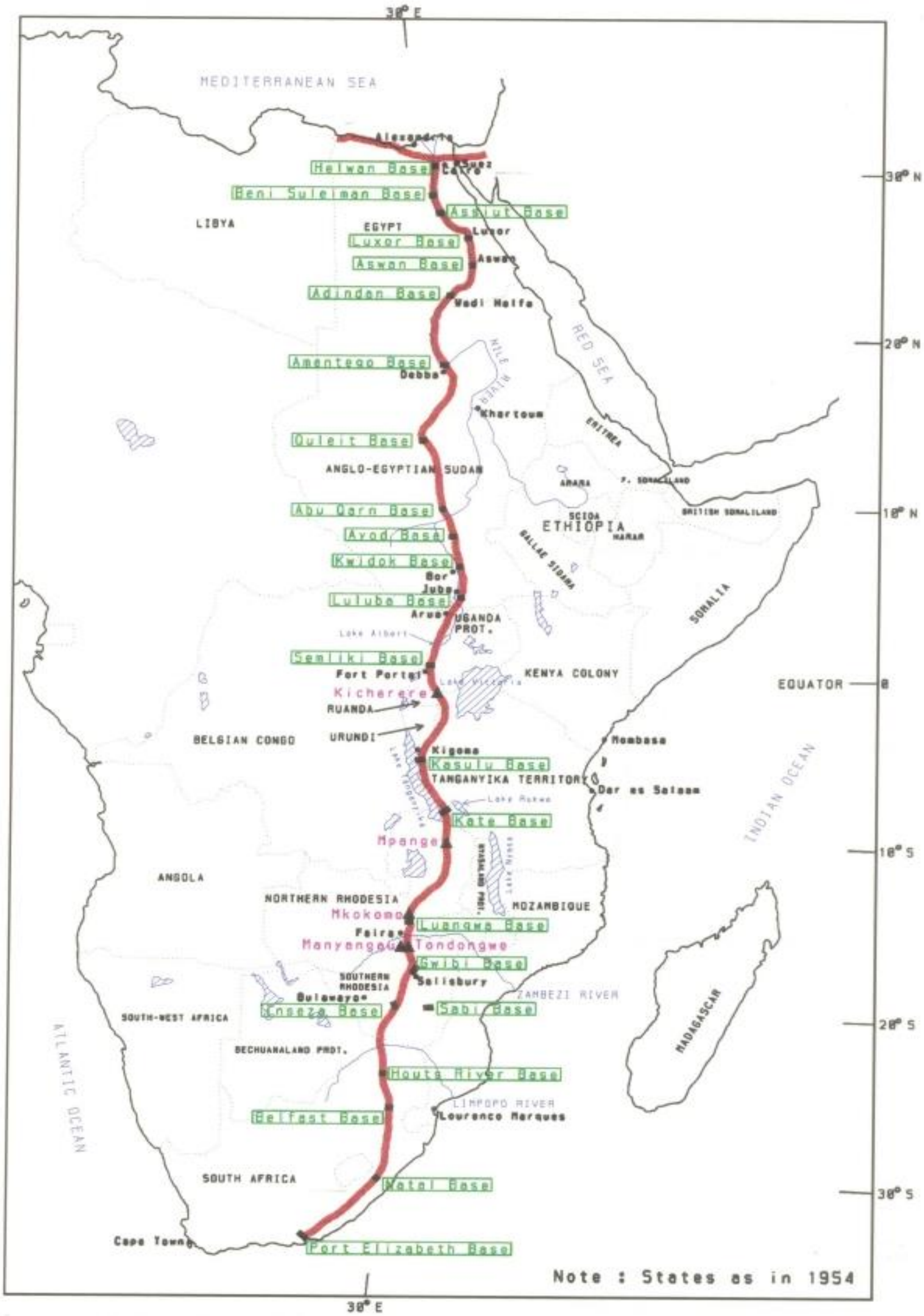


Figure 1a The African 30th Arc Meridian.

Kenya Major Triangulation

By 1906 only two major triangulation chains established by the AGBC existed, these chains were along the Kenya - Tanzania boundary and the edge of Lake Victoria.

This was far away from the areas where surveys for land registration were required. Kenya major triangulation started in 1906 to extend control for title surveys and topographical mapping in the White Highland areas .

Kenya major system was based on Cassini Soldner Projection calculated from origins at the intersections of odd-numbered–degree meridians (as central meridian) with the equator as the reference latitude and extending over successive zones of two degrees of longitude and referenced to Clark 1858 Ellipsoid. Figure 1 shows the extent of this network. Kenya Major derived its datum from AGBC of 1902 at Kisumu where a base existed and its latitude at Athi River. The longitude was derived at Elemoborasha. After this initial work, several other triangulation chains were conducted between 1906 and 1922, see Table 1.

The common points between Kenya Major triangulation network and the AGBC chains enabled re-computation of AGBC 1902-1906 triangulation based on Kenya major sections. The purpose was to generate co-ordinates consistent with Kenya Major triangulation data.

The Kenya Major System was never closed and adjusted as a whole. Consequently, a surveyor working across the triangulation chains would encounter a substantial displacement. To carry out subdivision or re-establishment surveys, it became necessary to know which triangulation chain had controlled the original cadastral survey.

Table 1: The Kenya Major Triangulation Chains (Source: Caukwell R.A., (1977))

Series No.	Series Name	Year Observed
1	Anglo-German Boundary Chain	1904-1906
2	Lumbwa-sotik	1906
3	Athi-Nyeri	1907
4	Athi-Lumbwa	1907
5	Athi-Mombasa	1908
6	Mombasa-Malindi	1908
7	Malindi-Sabaki River	1909-1910
8	Kitui	1910
9	Laikipia	1910-1911
10	Uasin-Gishu	1910-1911
11	Mombsa-Vanga	1910-1912
12	Trans Nzoia	1911-1912
13	North Rift Valley	1911-1912
14	Machakos	1912-1913
15	Kitui	1912-1913
16	Mumias	1912-1913
17	East Kenya	1912-1913
18	North Kenya	1913-1914
19	Malindi-Kipini	1913-1914

20	Kisii	1913-1914
21	Lamwia-Elemoborasha	1913-1914
22	Kipini-Lamu	1916, 1919 & 1922
23	Voi-Taveta	1910-1920

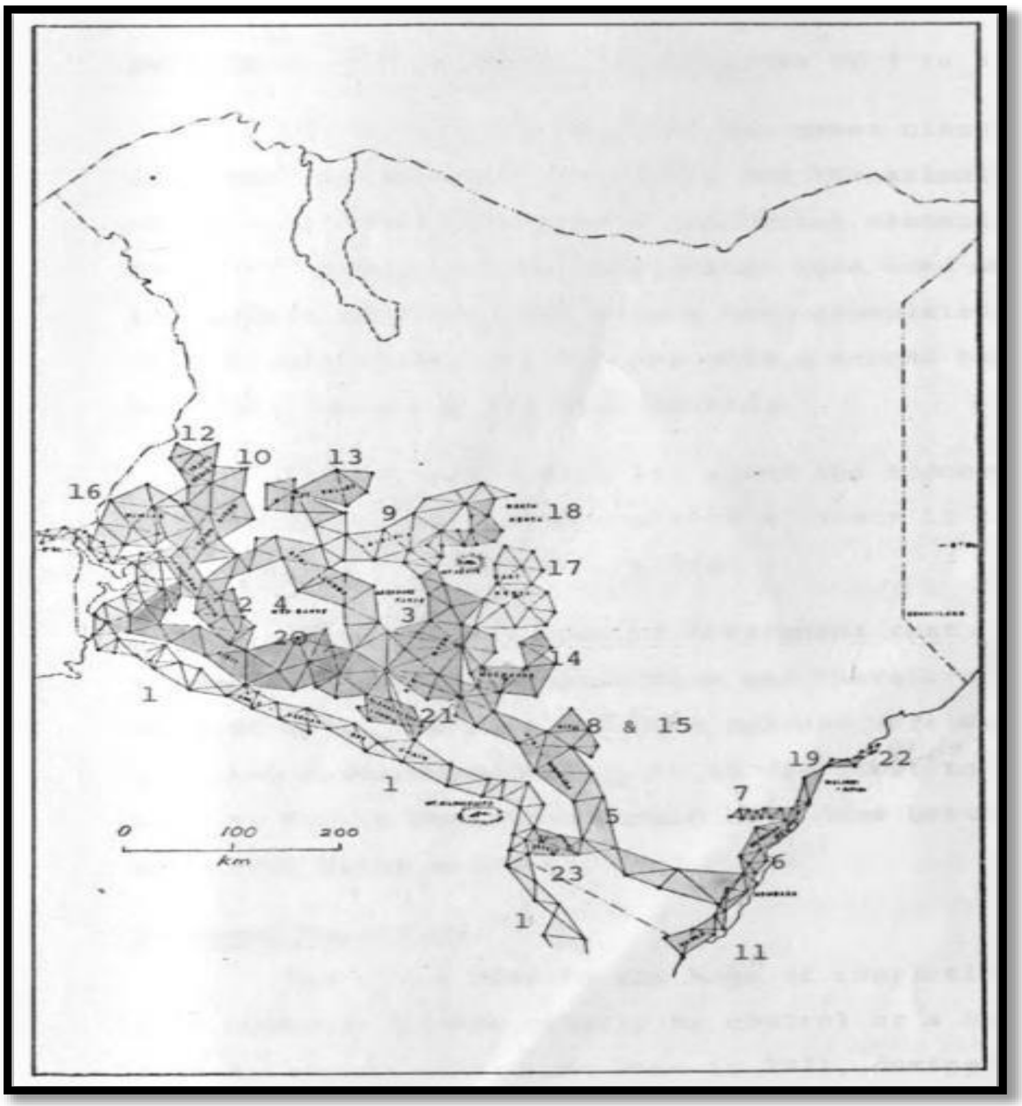


Figure 1b: The Kenya Major Triangulation Chains (Source: Caukwell R.A., (1977))

2.1.3 Extension of The Kenya Major Triangulation

Kenya Major Triangulation was implemented between 1906 and 1914. Due to the world economic recession in 1921, no further extension of control took place as a result of the collapsing of the Trigonometrical and Topographic Sections of the Survey Department and dismissal of all staff below the rank of District Surveyor. The next major extension of control by triangulation resumed between 1939 and 1941 due to the need by the military to use maps. A number of extensions were executed and referred to as the East African War System. The only triangulation conducted from 1921 until 1950 was tertiary breakdown for cadastral purposes in very limited areas.

2.1.4 The Kenya Primary, Secondary and Lower Order Triangulations

The British Government provided survey and mapping services in its colonies through the Directorate of Overseas Surveys (DOS). The DOS executed survey missions in Kenya between 1950 and 1983 during which they established the present Kenya primary, secondary and lower order triangulation networks and also observed traverses. The DOS coordinates were computed using Universal Transverse Mercator (UTM) projection and Clarke 1880 as the reference ellipsoid and these are the controls that are used for all new cadastral surveys in Kenya to date.

A whole new network was observed as an integral part of the East and Central African primary coverage, with three new measured bases in Kenya, and tied to the Arc of the 30th Meridian in western Uganda. This triangulation included the primary traverse from north of *Mount Kenya* down the *Tana River* valley to *Malindi* which was planned in 1908 to close the Kenya Major network, but was not executed until 1958 after the invention of the Tellurometer.

2.1.5 Tellurometer Traverses

For these surveys, traverse distances were measured using the Tellurometer. This exercise was done with the help of the British Survey Squadron and Major Norland in three epochs.

The first epoch was carried out in 1972 headed by Captain Robinson for the Provision of Horizontal and altimeter height control for the 1:100,000 topographical sheets east of *Lake Turkana*. The second was implemented in 1974 east of *Lake Turkana* but at an enlarged scale with the help of Bilby Towers. The third campaign was again carried out east of *Lake Turkana* but with an expanded team between December 1976 and February 1977, the aim was to complete the geodetic loop from each end of the chain. This team was led by *Captain Bruce Burton* and *Professor G.O. Wayumba* was part of this team .

For the three epochs executed by tellurometer the purpose was to connect the old triangulation chain with the Clifford triangulation along the Kenya – Ethiopia border. Another Tellurometer traverse chain running from *Isiolo – Marsabit – Sololo – Moyale – Wajir – Harba Sweni – Kula Mawe* and back to *Isiolo* was carried out between 1976 and 1977.

As earlier mentioned in section 2.2.2 the *Tana River Valley* first order precise traverse running from the base at *Malindi* and closing at the *Isiolo* base, following the road from *Malindi* northwards through *Garsen*, *Garissa* and *Garba Tula* was also observed by Tellurometer in 1957 by the United Kingdom (UK) Directorate of Colonial Surveys in 28 days although the same triangulation traverse had been planned to last two and a half years.

2.1.6 Doppler Station Fixes

The Survey of Kenya, the Defense Mapping Agency of the USA and the Directorate of Military of the UK first carried out Doppler observations in Kenya in 1972 and 1973. The objective of this joint exercise was to; evaluate the accuracy of the primary controls, provide geodetic control in remote and un-surveyed areas of the country, strengthen the triangulation network with precise positions at optimal spacing and contribute to the development of a single well- fitting Datum for the African continent. Doppler positioning using precise ephemeris fixed fifteen (15) stations distributed across the country. See Figure 2 in red.

2.1.7 Clifford Triangulation

This control network was established in 1958, the campaign was led by Major Clifford. The main objective of establishing this control network was to map the Kenya – Ethiopia boundary. This triangulation network was based on the East Africa War System. See Figure 2.

2.1.8 The Kenya – Uganda Border Triangulation

This was carried out in 1975 with the objective of mapping the Kenya – Uganda boundary. This survey was done by Tellurometer traverses. See Figure 2.

2.1.9 Kenya's Vertical Control

Before establishing the Primary Levelling Network in Kenya (1949 - 1958) all the triangulation networks in the country which were observed between 1906 and 1914 for cadastral and topographical surveys had trigonometrical heights derived from:

(a) The original Uganda Railway Datum based on an assumed Mean Low Water Ordinary Spring Tides at Kilindini, Mombasa. This is the height datum that was used for the original railway survey from Mombasa to Kisumu around 1900 and

(b) The New Kenya – Uganda Railway Datum based on more accurate measurements of Mean Low Water Ordinary Spring Tides at Kilindini. This revised datum was found to be 1.65feet below the previously established datum in (a) above.

Except for engineering levels related to rail roads, no levelling network had been established in Kenya before 1949. To establish a uniform system of reduced heights all over the country primary levelling was initiated in 1949. This network was designed to cover routes along railways and main roads connecting towns in Kenya and extending into the neighbouring Uganda and Tanzania. By 1952, approximately 2,000 Kilometers of levelling circuits had been observed and provisional height values determined for 1,290 benchmarks.

Double levelling for the *Nairobi – Kisumu* line with its adjoining circuits was completed by end of 1959 and connections made to three tide gauges along Lake Victoria. Between 1965 – 1966, *Kisumu – Buteba* and *Mombasa – Lungalunga* lines were also levelled making it possible to establish provisional relationship between the three East African Datums.

The primary Levelling Network was referred to the Mean Sea Level (MSL) values deduced from the 1932 – 1933 records of the tide gauge at *Kilindini* in *Mombasa*, this therefore means that Kenya's height Datum is the Mean Sea Level (MSL) referred to a tide gauge installed *at Kilindini* habour in *Mombasa.* Except for the *Webuye – Kisumu – Sirari* line that was levelled between 1970 – 1971, no work has been done to extend the primary levelling network in the country. See figure 2, the levelling loops are shown in colour blue.

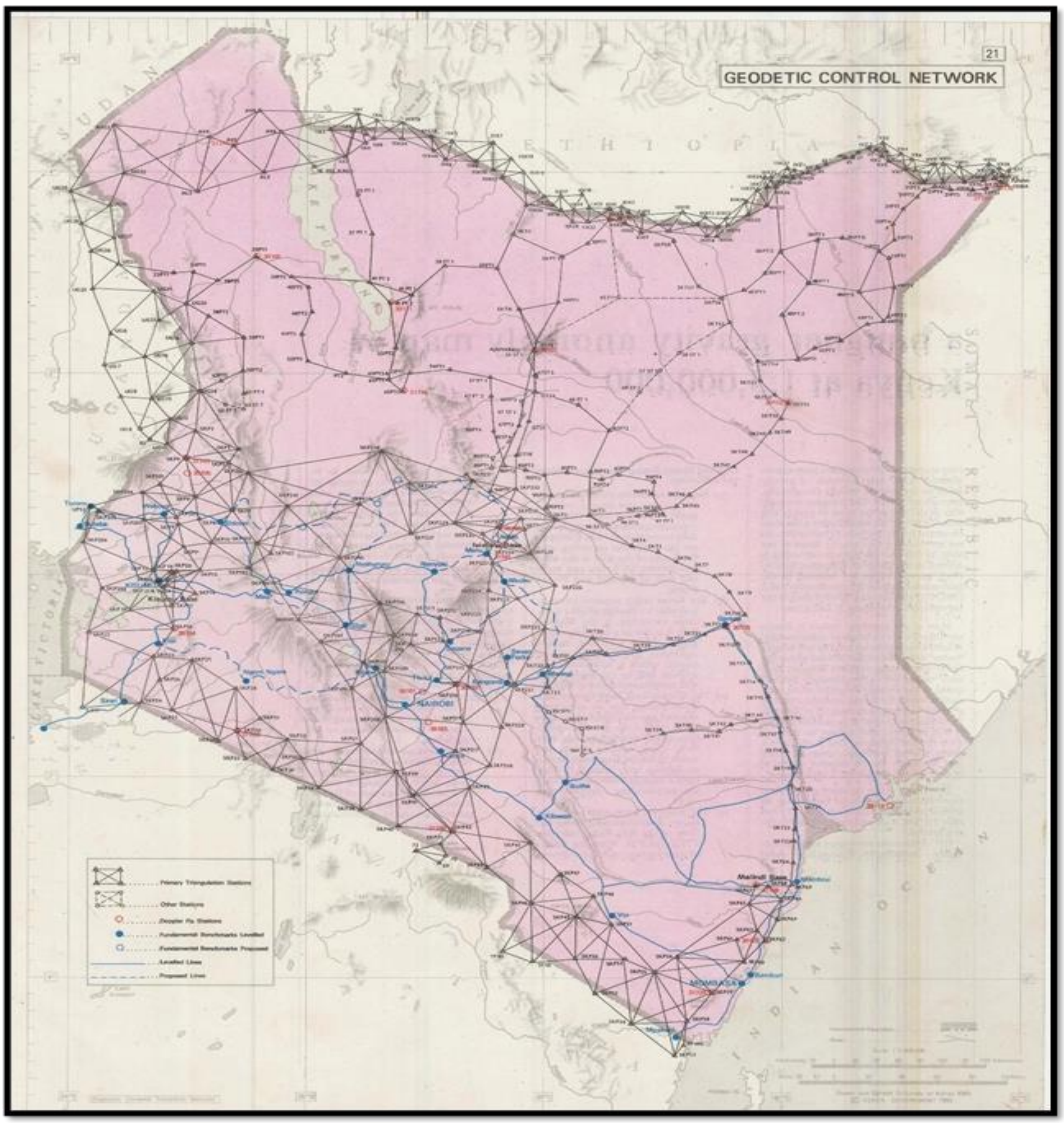


Figure 2: Kenya's Geodetic Network (Kenya National Geodetic Report, (1985))

2.2 GEODETIC DATUMS USED IN KENYA

A datum is any conventional framework onto which observations are related or a reference surface to which the horizontal and vertical coordinates of points are referred. The reference surface may be the geoid which is a natural surface or an ellipsoid

/spheroid which is an artificial entity. We have two categories of ellipsoids:

i. A geocentric ellipsoid e.g., WGS 1984 ellipsoid that is used for worldwide applications or
ii. A regional or local ellipsoid e.g., Clarke 1880 ellipsoid for continental or national level (horizontal) geodetic networks.

Two reference ellipsoids are used for coordinate computation in Kenya: (a.) For Cassini-Soldner projection **Clarke 1858 spheroid** is used while,

(b.) For Universal Transverse Mercator projection (UTM) **Clarke1880 spheroid** is used.

2.2.1 Co-ordinate Systems for Mapping in Kenya

Four co-ordinate systems are used for mapping in Kenya: -

i) The Local Origin Coordinate System

This coordinate system covers a small area or region on the earth surface. It is offset from the geo-center, which in most cases is the origin for global coordinates. In Kenya this system was used in areas that were not covered by the Kenya Major Triangulation network.

Surveys based on a local origin were scattered all over the newly alienated areas in the country and did not have a specific datum. As an example, cadastral surveys for land parcels that were close to the Uganda Railway line were tied to the Centre-line of the railway as a base and referenced by compass measurements to telegraphic posts which were numbered and existed along the Railway corridor all the way from Mombasa to Kisumu. Local origin surveys are captured in the Survey Act and referred to as "isolated" surveys.

The Local Origin Coordinate systems had orthogonal coordinate axes (X, Y) with the Y axis oriented approximately to the North via astronomic Azimuth determination e.g., by sun azimuth method or by campus bearings. The origin of the system was assigned a value of (0, 0) as coordinates or any other arbitrary figure. The scale of the system was obtained by measuring a base-line with a cadastral band (180m) long. Some of the areas where the Local Origin System was used for cadastral surveying included; Londiani, Naivasha, and Molo. In 1903, the Chief Surveyor observed that this situation was not tenable as such isolated surveys would soon result in boundary conflicts. The Chief Surveyor therefore recommended for the establishment of a proper triangulation network to control such surveys, this led to the Kenya Major Triangulation Network that was established starting from 1904.

ii) Cassini-Soldner Coordinate System

The origins of this co-ordinate system are the intersections between the equator and the odd meridians. The odd meridians served as the central meridian for each 2º belt which extends one degree to east and west. For example, the Central meridians can be 35ºE, 37ºE, 39ºE etc., with belt limits extending from 34ºE to 36ºE, 36ºE to 38ºE, 38ºE to 40ºE etc. The reference ellipsoid used is Clarke 1858 and the unit of measurement is the British foot.

Before 1950, nearly all triangulation networks were based on this system. Cadastral surveys in Kenya were also based on this system of co-ordinates and up to today most surveys in urban and former white highlands are based on this system.

iii) The East African War System of Coordinates

The war system was introduced as a military system for East Africa with the extended triangulation to Kenya Major Triangulation north of and around Mt. Kenya being based on this system. The main objective of the East Africa war system was to unify the coordinate system for the British Commonwealth territories in the South, East and Central Africa to avoid discontinuity in topographical mapping and grid references across territorial boundaries. It was also thought that the same system could have been used for cadastral surveys.

Coordinates for this system are based on Transverse Mercator projection of 5° wide belts. Belts were designated C, D, E, etc. and Kenya is covered by Belts H and J as illustrated in Table 2.

Table 2: Zones/Belts of the East Africa War System.

Belt H	Western Limit	35°E
	Central Meridian	37°30’ E
	Eastern Limit	40°E
Belt J	Western Limit	40°E
	Central Meridian	42°30’E
	Eastern Limit	45°E

The origin for this coordinate system was the intersection of the Equator and the Central meridian, with false co-ordinates being; Eastings +400 000 meters and Northings +4,500,000 meters. The scale factor at the central meridian was 1999/2000 approximately equals 0.9995 with the reference spheroid being Clarke 1880. The Co-ordinates in this system have been converted to the Universal Transverse Mercator system.

iv) The Universal Transverse Mercator (UTM) Coordinate System

This coordinate system was introduced in Kenya in 1950 by the Directorate of Overseas Surveys (DOS), when the DOS began implementing control survey work in

Kenya. The system used Clarke 1880 spheroid initially but was later recomputed based on the modified Clarke 1880 spheroid. Kenya is covered by four UTM Zones 36M, 36N, 37M and 37N. The unit of measurement used by this coordinate system is the international metre. The survey department has converted the coordinates of all control points in the country to this co-ordinate reference system.

2.3 Traverse Adjustment methods

As stated earlier, their arise errors in survey measurement which can be categorised as shown below;

a) Gross errors

These are errors that are caused by incorrect measurements, or very wrong measurements. They are also called Blunders and are not easily corrected if not noticed.

b) Systematic errors

They are statistically distributed errors among the readings, they could be due to environment factors, individual bias in setting apparatus or parallax errors, and instrument errors from faulty equipment construction or faulty calibration.

c) Random errors

These occur irregularly and are unpredictable, and occur due to fluctuations in environmental conditions. Could be temperature, voltage or vibrations.

In traverse measurements, random and gross errors are reduced by multiple reading of same measurement, i.e. reverse measurement. However, systematic errors could persist and thus bringing about misclosure errors.

The folowing errors can be minimised using the following adjustment methods classified as either rigorous(least squares method) or approximate(Bowditch/compass rule, Crandall, Transit rule).

2.3.1 Bowditch/compass rule

Also known as the Bowditch method. This method was pioneered by Nathaniel Bowditch, a navigator in 1802. It is a method that assumes that both angles and distances are measured with the same level of accuracy. It therefore distributes the misclosure in the northings and eastings based on the ratio of the length of the traverse leg to the total length of the traverse.

2.3.2 Transit Rule

In this method, angles are assumed to be measured more accurately than distances, and distances receive greater adjustment. The misclosure in the northings and eastings is distributed among the stations based on the ratio of the change in northing of the station, to the total change in nothings, or eastings. This method of error distribution allows for more correction on the distances than the angles.

2.3.3 Least Squares Method

This is a full statistical approach. It uses the least squares adjustment method which was first fronted by Laplace in 1774 and further extended by Legendre in his article “M´ethode des moindres quarr´es” translated to "The Method of least squares” published in 1805. It works by reducing the residuals of the measurements to a minimum. The observations are used to form equations that are then reduced using matrix algebra or iterative methods. It allows for full random error modelling and has the ability to mix different accuracy and precision measurements.

It is very rigorous but is not easily repeatable. It shall be used in the study.

2.3.3.1 Principles of Least Squares

In taking surveying measurements, errors do occur in field observations due to human, environmental or instrumental factors and require computational adjustment towards the most probable value. During the 19th century, the least squares technique was developed, by Legendre, Laplace and Gauss, that minimizes the sum of the squares of differences between the observed measurements and the estimated value. There are variants of the least squares such as Total Least Squares (TLS) and Least Squares Collocation (LSC). The Least Squares technique is conventional for adjusting surveying methods i.e., Traverses.

The fundamental principle of least squares is a solution whose sum of its square of residuals is minimum. i.e

$$\sum_{i=1}^{n}(v_i)^2 = (v_1)^2 + (v_2)^2 + (v_3)^2 \ldots + (v_m)^2 \text{ .... } \mathbf{i}$$

For a weighted system;

$$\sum_{i=1}^{n} p_i(v_i)^2 = p_1(v_1)^2 + p_2(v_2)^2 + p_3(v_3)^2 \ldots + p_m(v_m)^2 \text{ .... } \mathbf{ii}$$

Where v = residuals and p = individual weights.

There are 3 ways of implementing the least squares adjustment method;

a) First principles

b) Observation equations

c) Conditional equations.

In this study the observation method shall be used. In the observation equation method, observation equations are formulated to relate measured values to their residual errors and the unknown parameters. An observation equation is written for every measurement. For unique solutions, the number of equations should be equal to the number of unknowns. However, having more equations than the number of unknowns (from redundant measurements) is also desirable to determine most probable values for unknown parameters using least squares method. An equation for each residual error is obtained for each observation. These residuals (v) are then squared and summed up to give a sum of least squares. Normal equations are then developed, which are equal to the number of unknowns. Solution of the normal equations yields the most probable values of the unknowns.

2.4 Least squares adjustment of a traverse network

For a traverse network there exists two sets of measurements, the distance measurement and the angular measurement. With least squares both can effectively be adjusted at the same time and in a weighted fashion.

2.4.1 Formulation of observation equations.

Traverses contain both distance and bearing measurements. Therefore, to perform least-squares adjustments of horizontal surveys by the method of observation equations, it is necessary to write observation equations for these two types of measurements. The equations are nonlinear, so they are first linearized using Taylor's theorem and then solved iteratively. As defined by Ghilani. C.

The observation equations generally take the form; **A x = B + v**

Where:

A = observation coefficients matrix

x = unknowns' matrix

B = observations

v = residuals

2.4.2 Distance observation equations

These equations relate measured lengths and their random errors to the most probable coordinates of their end points. This is commonly referred to as the method of *variation of coordinates* formed as shown below;

$$\boldsymbol{K_{Lij} + V_{L_{ij}} = \left[\frac{X_{i_0}-X_{j_0}}{IJ_0}\right] dX_i + \left[\frac{Y_{i_0}-Y_{j_0}}{IJ_0}\right] dY_i + \left[\frac{X_{j_0}-X_{i_0}}{IJ_0}\right] dX_j + \left[\frac{Y_{j_0}-Y_{i_0}}{IJ_0}\right] dY_j} \text{....... } \mathbf{iii}$$

Where $K_{Lij} = L_{ij} - (IJ_o)$, and

$$\boldsymbol{(IJ_0) = \sqrt{\left(X_{j_0} - X_{i_0}\right)^2 + \left(Y_{j_0} - Y_{i_0}\right)^2}} \text{ .... } \mathbf{iv}$$

2.4.3 Azimuth observation equations

These equations relate measured bearings and their random errors to the most probable coordinates.

$$\boldsymbol{K_{Lij} + V_{L_{ij}} = \left[\frac{Y_{i_0}-Y_{j_0}}{(IJ_0)^2}\right] dX_i + \left[\frac{X_{j_0}-X_{i_0}}{(IJ_0)^2}\right] dY_i + \left[\frac{Y_{j_0}-Y_{i_0}}{(IJ_0)^2}\right] dX_j + \left[\frac{X_{i_0}-X_{j_0}}{(IJ_0)^2}\right] dY_j} \text{....... } \mathbf{v}$$

Where $K_{Lij} = L_{ij} - (IJ_o)$, and

$$\boldsymbol{(IJ_0) = \sqrt{\left(X_{j_0} - X_{i_0}\right)^2 + \left(Y_{j_0} - Y_{i_0}\right)^2}} \text{ .... } \mathbf{vi}$$

2.4.4 Weights

In surveying, observed data must meet a set of predetermined geometric conditions for the observations to be considered viable. When these conditions are not met, the observations must be adjusted to conform to the conditions and achieve geometric closure. When adjusting observations, those with better precisions and thus smaller variance are assigned with larger weights to influence the adjustment more than those of lower precision.

In the least squares adjustment using least squares, weights are important to balance out the adjustment made on measurements. Given that the distances and angles are adjusted simultaneously, weights are key in distinguishing the different units of measurement. The precision of the machine used in data collection is used to compute the weight which is equivalent to the inverse of the variance.

$w = \frac{1}{\delta^2}$ …………………….. **vii**

To adjust the matrix a diagonal weight matrix is formed for every observation in the adjustment.

2.4.5 Constraints

The least squares technique requires that at least two points in the traverse have fixed coordinates. These provide a basis for corrections to be made to the other points, while not allowing the other points to be indeterminate. It can also be achieved by having one fixed coordinate and holding one bearing fixed. The constraint equations can be expressed as:

$\boldsymbol{Cx - g = 0}$ **…………………….. viii**

Where,

C coefficient matrix of number of constraint equations and unknowns

x vector of unknown corrections applied to the coordinates

g vector of elements derived by difference of constrained values and the approximate traverse coordinates

2.4.6 The Standard Error Ellipse

Error ellipses give a two-dimensional representation of the uncertainties of the adjusted coordinates of points as determined in a least-squares adjustment. They can be plotted at enlarged scales directly on scaled diagrams showing the points in the horizontal survey. When plotted in this manner, their sizes and appearances enable a quick visual analysis to be made of the overall relative precision of all adjusted points. This is useful in planning surveys and in analyzing the results of surveys for acceptance or rejection.

An adjusted point's *error rectangle,* can be defined by the North and East uncertainties.

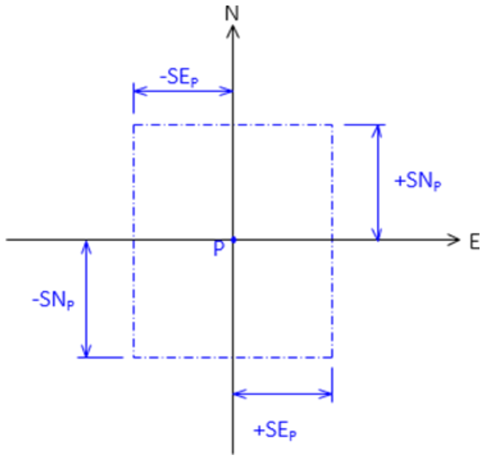
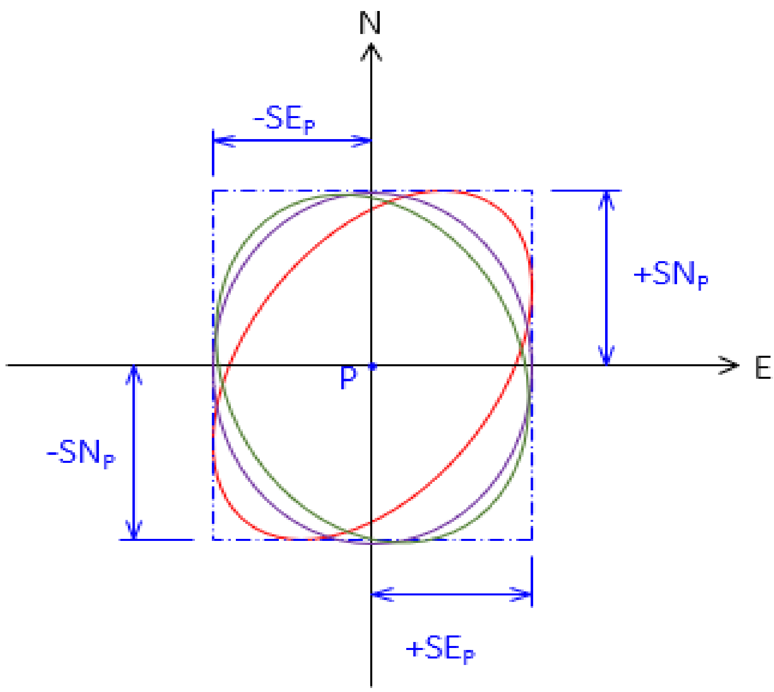


Figure 3: showing Error Rectangle

An error ellipse is tangent to all four sides of the error rectangle. If North and East uncertainties are the same, the error ellipse would be a circle. With differing uncertainties, there are infinite



ellipses tangent to the error rectangle sides.

Figure 4: showing many tangent ellipses

2.4.7 Determining Error Ellipse Parameters

The semi-major axis is the direction in which the adjusted position is weakest, it is strongest in the semi-minor direction. These directions define the auxiliary U-V axis system.

The covariance matrix, [Q], of a horizontal network can be viewed as a series of square *2 x 2* sub-matrices corresponding to each adjusted point,

The structure of the sub-matrices depends on the coefficient arrangement in the covariance matrix. The [Q] matrix is organized by point with North(Y) then East(X) coefficients. If their order is reversed, or they are organized by direction instead of point that is all East coefficients followed by all North coefficients, the sub-matrix structures will be different.

$$[U] = \begin{bmatrix} dN_1 \\ dE_1 \\ dN_2 \\ dE_2 \\ \vdots \\ dN_n \\ dE_n \end{bmatrix} \qquad \begin{array}{cc} & \begin{array}{cccccccc} dN_1 & dE_1 & dN_2 & dE_2 & \cdots & dN_n & dE_n \end{array} \\ [Q] = \begin{array}{c} dN_1 \\ dE_1 \\ dN_2 \\ dE_2 \\ \vdots \\ dN_n \\ dE_n \end{array} & \begin{bmatrix} {}_1q_1 & {}_1q_2 & q_{1,3} & q_{1,4} & \cdots & {}_1q_{2n-1} & {}_1q_{2n} \\ {}_2q_1 & {}_2q_2 & q_{2,3} & q_{2,4} & \cdots & {}_2q_{2n-1} & {}_2q_{2n} \\ {}_3q_1 & {}_3q_2 & q_{3,4} & q_{3,4} & \cdots & {}_3q_{2n-1} & {}_3q_{2n} \\ {}_4q_1 & {}_4q_2 & q_{4,3} & q_{4,4} & \cdots & {}_4q_{2n-1} & {}_4q_{2n} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ {}_{2n-1}q_1 & {}_{2n-1}q_2 & {}_{2n-1}q_3 & {}_{2n-1}q_4 & \cdots & {}_{2n-1}q_{2n-1} & {}_{2n-1}q_{2n} \\ {}_{2n}q_1 & {}_{2n}q_2 & {}_{2n}q_3 & {}_{2n}q_4 & \cdots & {}_{2n}q_{2n-1} & {}_{2n}q_{2n} \end{bmatrix} \end{array}$$

Figure 5: Showing Unknowns and Covariance Matrices.

$${}_i[Q]_i = \begin{bmatrix} {}_Nq_N & {}_Nq_E \\ {}_Eq_N & {}_Eq_E \end{bmatrix}$$

Figure 6: Showing the Sub-matrix for point i.

Diagonal elements of the sub-matrix are used to determine North and East uncertainties as

$$SN = S_o * \sqrt{NqN} \text{ ....ix}$$

$$SE = S_o * \sqrt{EqE} \text{ ....x}$$

The parameters contain the semi-major axes, semi-minor axes, and clockwise rotation angle from the Y-axis to the semi-major axis of the ellipse computed at each station.

2.4.8 How Confident Are We?

Standard deviations in North and East each represent an estimated 68% confidence for the respective direction. But the area of the standard error ellipse only represents about 35% confidence. That will vary somewhat based on the degrees of freedom (DF).

Increasing the confidence interval (CI) increases the size of the standard error ellipse, Figure 7

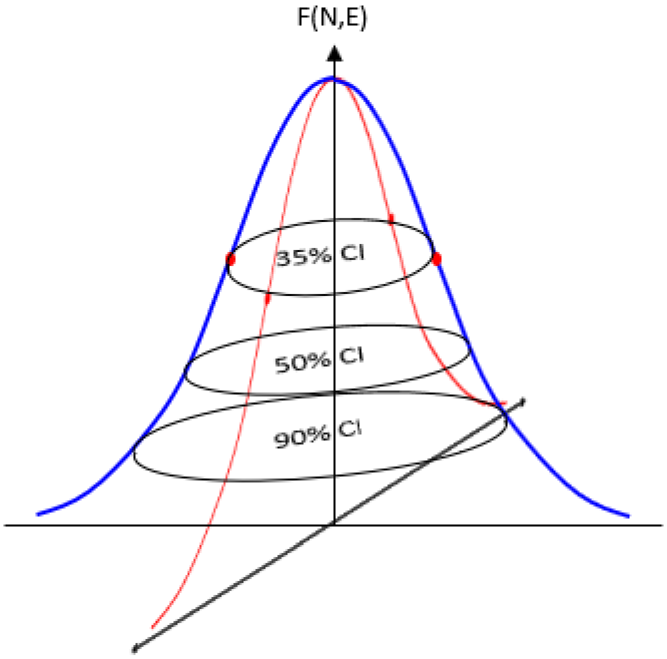


Figure 7
Confidence Intervals

To increase confidence requires use of the *F Statistic* (aka, F Distribution), itself in part dependent on DF. The CI at a specific percentage is computed from Equation xi

$$S_{CI\%} = S \times \sqrt{2 \times F} \quad \text{...xi}$$

F: F Statistic modifier

F-test is described as a type of hypothesis test, that is based on Snedecor f-distribution, under the null hypothesis. The test is performed when it is not known whether the two populations have the same variance.

F-test can also be used to check if the data conforms to a regression model, which is acquired through least square analysis. When there is multiple linear regression analysis, it examines the overall validity of the model or determines whether any of the independent variables is having a linear relationship with the dependent variable. A number of predictions can be made through, the comparison of the two datasets. The expression of the f-test value is in the ratio of variances of the two observations, which is shown as under:

$$F = \frac{\sigma_1^2}{\sigma_2^2}$$ Where, σ^2 = variance

The assumptions on which f-test relies are:

- The population is normally distributed.
- Samples have been drawn randomly.

- Observations are independent.
- H_0 may be one sided or two sided.

1. F-test:

   The F-test is used to compare the variances of two or more groups or populations. It is typically employed in analysis of variance (ANOVA) to test the null hypothesis that the means of multiple groups are equal. The F-test generates an F-statistic, which is the ratio of the variance between groups to the variance within groups. If the F-statistic is significantly different from 1, it indicates that at least one of the group means is different from the others. The F-test is sensitive to normality assumptions and works best when the data follow a normal distribution.

2. Student t-test:

   The Student t-test is used to compare the means of two independent samples or to compare a sample mean with a known population mean when the population standard deviation is unknown. It is most commonly used when dealing with small sample sizes. The t-test calculates a t-statistic, which measures how many standard errors the sample mean is away from the hypothesized population mean. A low p-value from the t-test suggests that the difference between the sample means is statistically significant. There are two main types of t-tests: one-sample t-test and two-sample (independent) t-test.

3. Chi-square test:

   The Chi-square test is used to determine if there is a significant association between two categorical variables. It compares the observed frequencies of different categories to the expected frequencies under the null hypothesis of independence. There are two main types of Chi-square tests: the Chi-square test for independence (for testing association between two categorical variables in a contingency table) and the Chi-square goodness-of-fit test (for testing if an observed frequency distribution fits an expected distribution). The Chi-square test is non-parametric, meaning it does not make assumptions about the underlying distribution of the data.

F-test is more preferable as compared with other statistical analysis method such as student T-test, chi-square test based on its capability to analyze variances, means for large population of data.

A common interval used by surveyors is 95%. Modifiers for 95% CI at different DF are listed in Table 3.

Table 3
F Statistic

DF	F
1	199.5
2	19.0
3	9.55
4	6.94
5	5.79
10	4.10

CHAPTER 3

3.0 METHODOLOGY AND DATA ANALYSIS

3.0.1 Executive summarry

The main objectives for this study is to 1) develop an algorithm to form and solve the least sqaures observation equations 2) analyze the results and 3) compare the least squares results with bowditch method.

3.0.2 Materials

Ms. Excel

Ms. Word

Python 3.11

PyCharm Community Edition 2022.1.2

3.0.3 Methodology overview

This study seeks to adjust a traverse network using the rigorous least squares method. Currently, most professionals use bowditch/compass rule method due to its simplicity which doesn't take into consideration both of the observed parameters leading to unrefined controls. Development of algorithm fastens and simplifies least squares therefore reducing on time and providing accurate and precise data.

3.0.4 Data Acquisition

The data that is used in this project is a secondary data, source being the Survey Manual, new edition, section (VI.7-VI.9).The instrument used for the observations was a theodolite. Points to be adjusted in the network are K6, K7, K8, K9, K10 and K11.The coordinate system used is the Universal Transverse Mercator and coordinates given in meters.

3.0.5 Data Preparation

After the data collection from the field. The curvature corrections were applied to each bearing observations and distance observation to reduce them to the ellipsoid or plane directions since the observations were made at great distant from each instrument station. This correction was done by t-T correction for bearings(Figure 8) and scale factor for distance(Figure 9) since the traverse was covering a londer distance.

VI.8

BEARING SHEET

Station	Observed	t-T	Plane	Orient.		Adj.	Final
At DONGA	FNP. 1						(Cp. 7)
Nyoka	237° 01′ 06″	+1″	237° 01′ 07″	+ 5″			237° 01′ 12″
K 6	251 43 54			+ 5″	43′ 59″	±0″	251 43 59
Twiga	338 58 16	−1″	338 58 15	+ 5″			338 58 20
At K6	FNP1						
Donga	71 43 34			+25			
K7	354 02 54				03 19	+1	354 03 20
At K7	FNP 2						
K6	174 02 52			+27			
K8	43 07 38				08 05	+3	43 08 08
At K8	FNP 2						
K7	223 07 39			+26			
K9	05 20 21				20 47	+4	05 20 51
At K9	FNP3						
K8	185 20 24			+23			
K10	326 19 27				19 50	+6	326 19 56
At K10	FNP3						
K9	146 19 31			+19			
K11	338 06 22				06 41	+7	338 06 48
At K11	FNP4						
K10	158 06 37			+ 4			
Twiga	298 32 26				32 30	+9	298 32 39
At TWIGA	FNP4						
K11	118 32 31			− 1			
Donga	158 58 08	+1	158 58 09		58 08	+12	158 58 20
Nyoka	220 00 45	+2	220 00 47		00 46	+9	220 00 55
					Misclosure +1 0″		(Cp. 7)

Figure 8 t-T correction

Directions observed in the spheroid become curved lines on the projection and it is therefore necessary to reduce the observed geographical directions(T) to the grid direction(t) before any calculations can be done. This correction is called the t-T correction.

The sign of correction is found by inspection. It is observed that the ray always curves away from the central meridian.

When the Y-coordinates are positive, the t-T will be positive for directions from 90 degrees to 270 degrees and when Y-coordinates are negative, the t-T will be negative for directions from 90 degrees to 270 degrees.

The T-t correction formulae is shown below;

$$(t-T)'' = \frac{(N1-N2)(2E1+E2)}{6pvSin1''}$$

Where N_1, E_1 and N_2, E_2 are the co-ordinates of the extremities of the line.

VI.7

TRAVERSE COMPUTATION ON THE
UNIVERSAL TRANSVERSE MERCATOR PROJECTION

Datum Bearings :–
Ref. TC & ST.5043

Twiga	+28162·86	+685828·56	0·384 418	Em	+686100
Donga	+26594·36	+686431·52	1·071 344		4·746X0·16
	– 1568·50	+ 602·96	0·358 821		
			158° 58′ 20″ 1680·40	(t–T)	+0″·8
Twiga	+28162·86	+685828·56	0·839 557		+684000
Nyoka	+23857·59	+682214·04	1·305 700		4·692X0·43
	– 4305·27	– 3614·52	0·642 992		
			220° 00′ 55″ 5621·39		+2″·0
Donga	+26594·36	+686431·52	0·648 911		+684300
Nyoka	+23857·59	+682214·04	1·192 093		4·700X0·27
	– 2736·77	– 4217·48	0·544 346		
			237° 01′ 12″ 5027·63		+1″·3

Conversion of measured distances

1 ft. = 0·30480 metres.

Em +686100
S.F. 1·0000237
+26
1·0000263

Donga – K6	1102·79 ft	F.B. Page 1	336·130	metres.	336·139 metres.
K6 – K7	893·40	"	272·308		272·315
K7 – K8	1573·15	F.B. Page 2	479·495		479·508
K8 – K9	1202·61	"	366·555		366·565
K9 – K10	676·82	F.B. Page 3	206·294		206·299
K10 – K11	1097·32	"	334·463		334·472
K11 – Twiga	1413·76	F.B. Page 4	430·913		430·924
Totals.	7959·85	Check ⟶	2426·158	Check ⟶	2426·222

Figure 9 Scale Factor

To minimize this error, for the U.T.M. projection a scale reduction of -0.04% is introduced on the C.M., increasing to +0.097% at the edges of the belt.

For any line whose extremities have Easting co-ordinates El and E2, the Scale Factor is:-

$$\frac{s}{S} = 0.9996(1 + \frac{E1^2 + E1E2 + E2^2}{6pv}$$

Where

s= projection distance

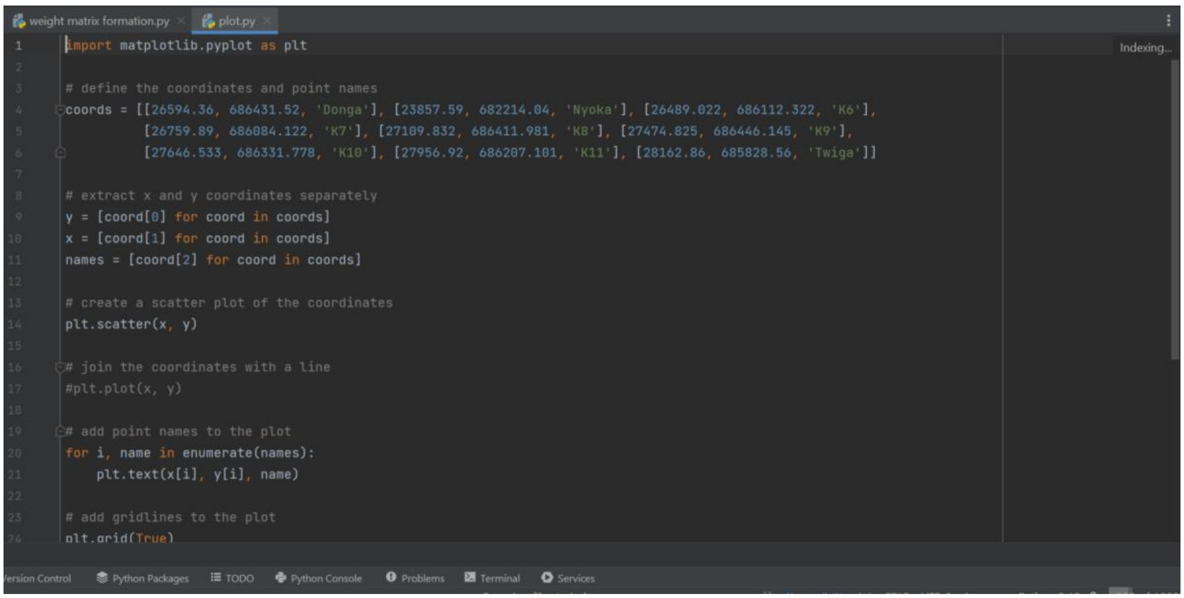
S= true distance

P= the radius of curvature in the meridian section

v = the length of the corresponding normal to the meridian

3.1 Traverse Network

The diagram of the traverse network is shown below plotted in python with the code snippet in Figure 10;



```python
import matplotlib.pyplot as plt

# define the coordinates and point names
coords = [[26594.36, 686431.52, 'Donga'], [23857.59, 682214.04, 'Nyoka'], [26489.022, 686112.322, 'K6'],
          [26759.89, 686084.122, 'K7'], [27109.832, 686411.981, 'K8'], [27474.825, 686446.145, 'K9'],
          [27646.533, 686331.778, 'K10'], [27956.92, 686207.101, 'K11'], [28162.86, 685828.56, 'Twiga']]

# extract x and y coordinates separately
y = [coord[0] for coord in coords]
x = [coord[1] for coord in coords]
names = [coord[2] for coord in coords]

# create a scatter plot of the coordinates
plt.scatter(x, y)

# join the coordinates with a line
#plt.plot(x, y)

# add point names to the plot
for i, name in enumerate(names):
    plt.text(x[i], y[i], name)

# add gridlines to the plot
plt.grid(True)
```


Figure 10

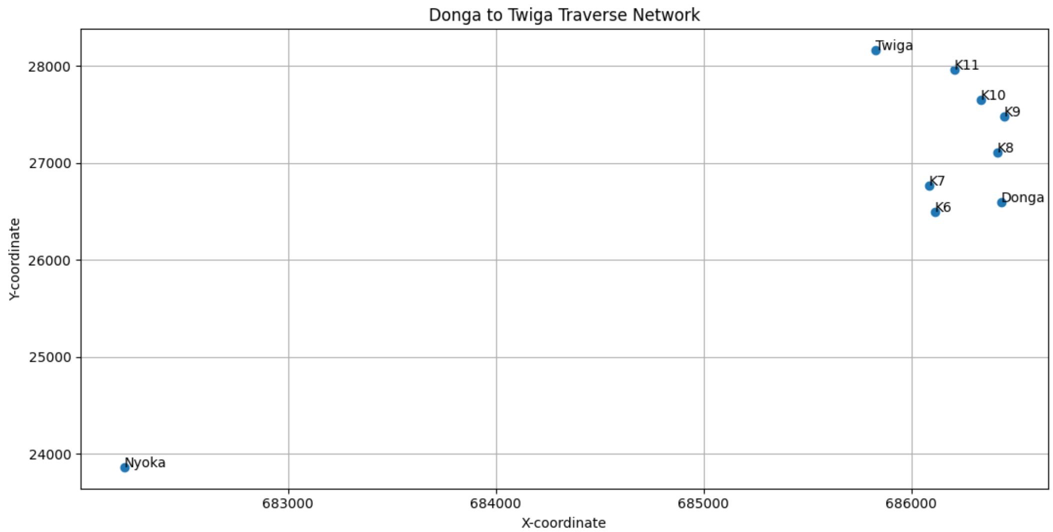
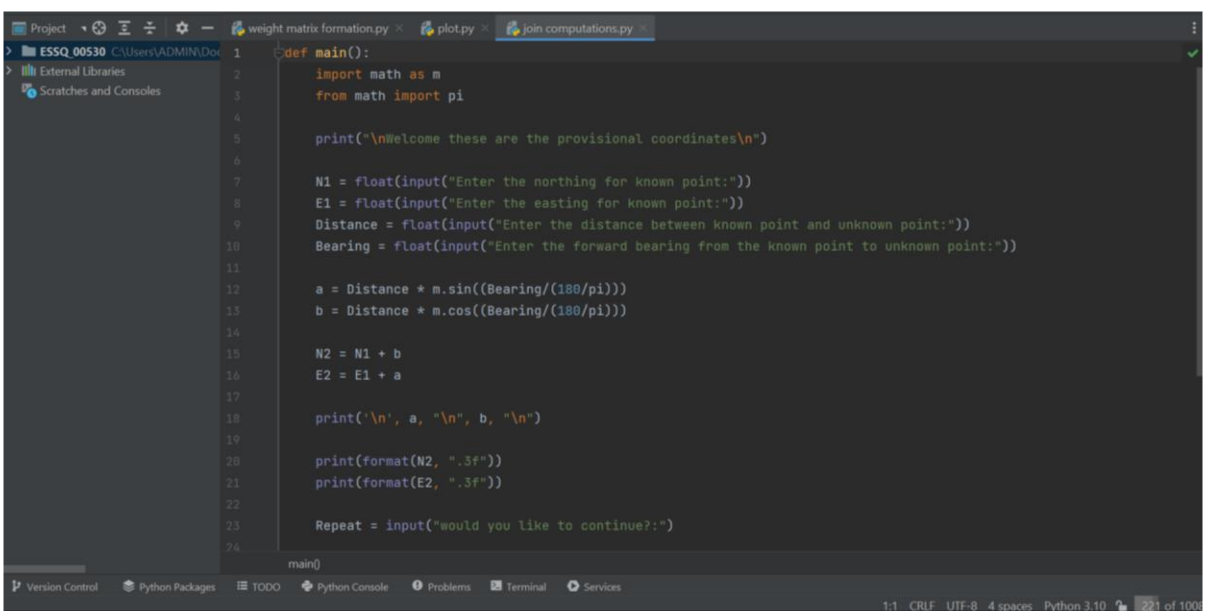


Figure 11 traverse network

Known coordinates for Donga, Twiga and Nyoka from figure 9 above is shown in table 4 below

Point Name	Northings	Eastings
Donga	26594.36	686431.52
Nyoka	23857.59	682214.04
Twiga	28162.86	685828.56

Provisional coordinates for K6, K7, K8, K9, K10 and K11 were then calculated using the join computation code illustrated in Figure12 snippet giving the following results in Table 5 below;



```
def main():
    import math as m
    from math import pi

    print("\nWelcome these are the provisional coordinates\n")

    N1 = float(input("Enter the northing for known point:"))
    E1 = float(input("Enter the easting for known point:"))
    Distance = float(input("Enter the distance between known point and unknown point:"))
    Bearing = float(input("Enter the forward bearing from the known point to unknown point:"))

    a = Distance * m.sin((Bearing/(180/pi)))
    b = Distance * m.cos((Bearing/(180/pi)))

    N2 = N1 + b
    E2 = E1 + a

    print('\n', a, "\n", b, "\n")

    print(format(N2, ".3f"))
    print(format(E2, ".3f"))

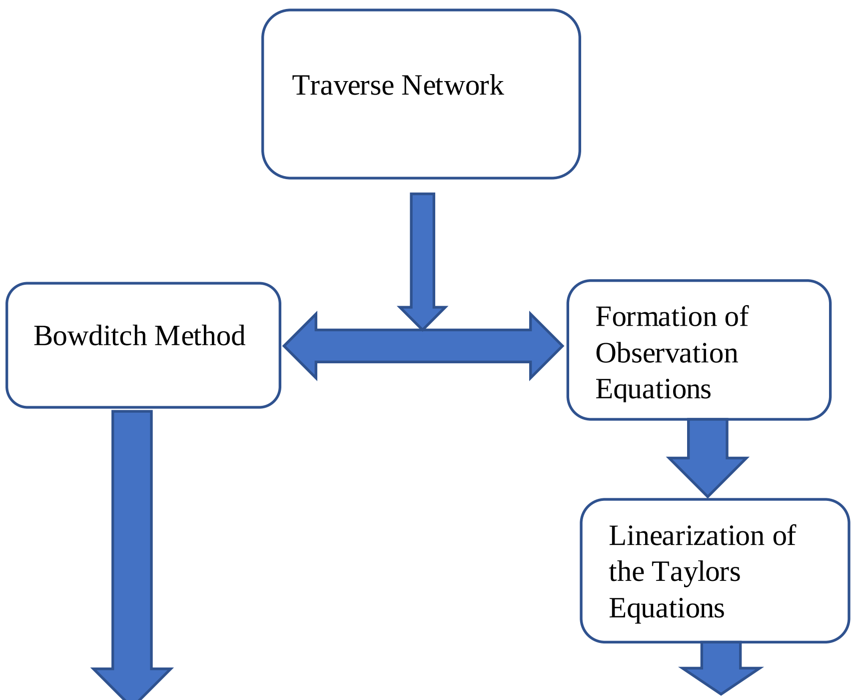
    Repeat = input("would you like to continue?:")
```

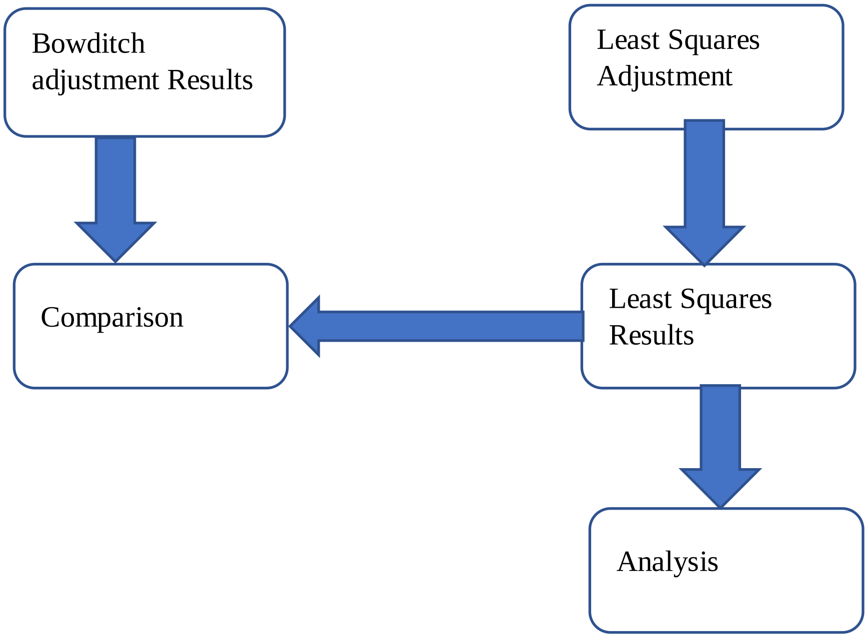
Figure 12

Provisional coordinates		
Name	Eastings	Nothings
K6	686112.32	26489.00
K7	686084.12	26759.85
K8	686411.97	27109.77
K9	686446.07	27474.74
K10	686331.70	27646.44
K11	686207.02	27956.80

Table 5

3.2 Work flow framework

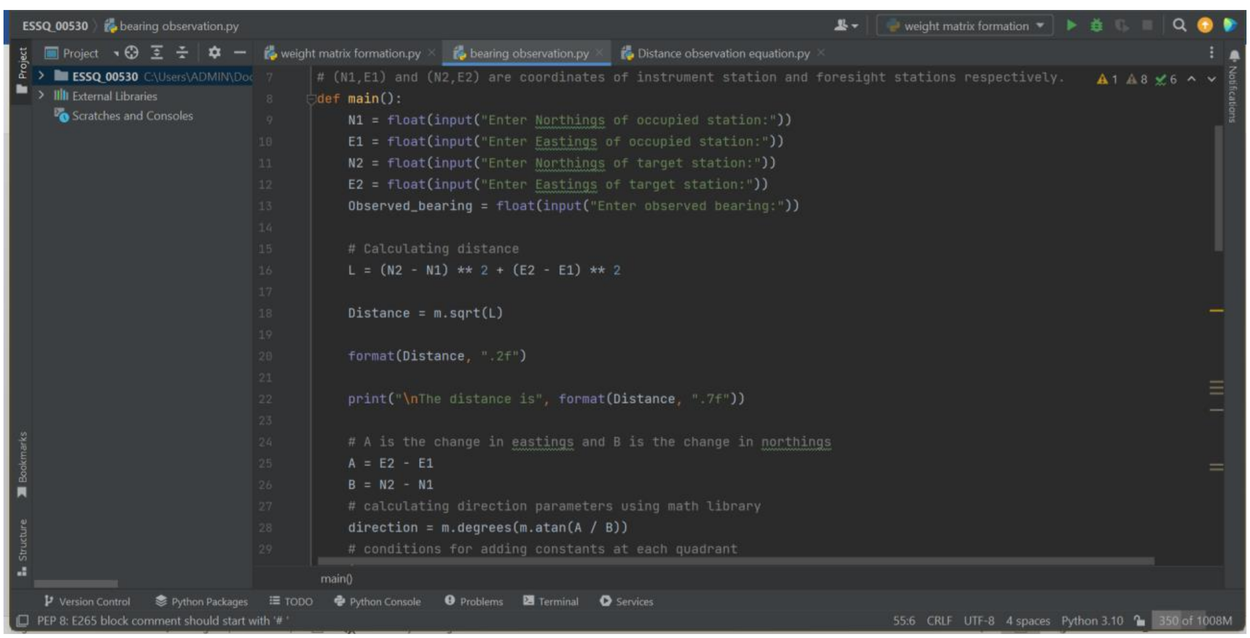


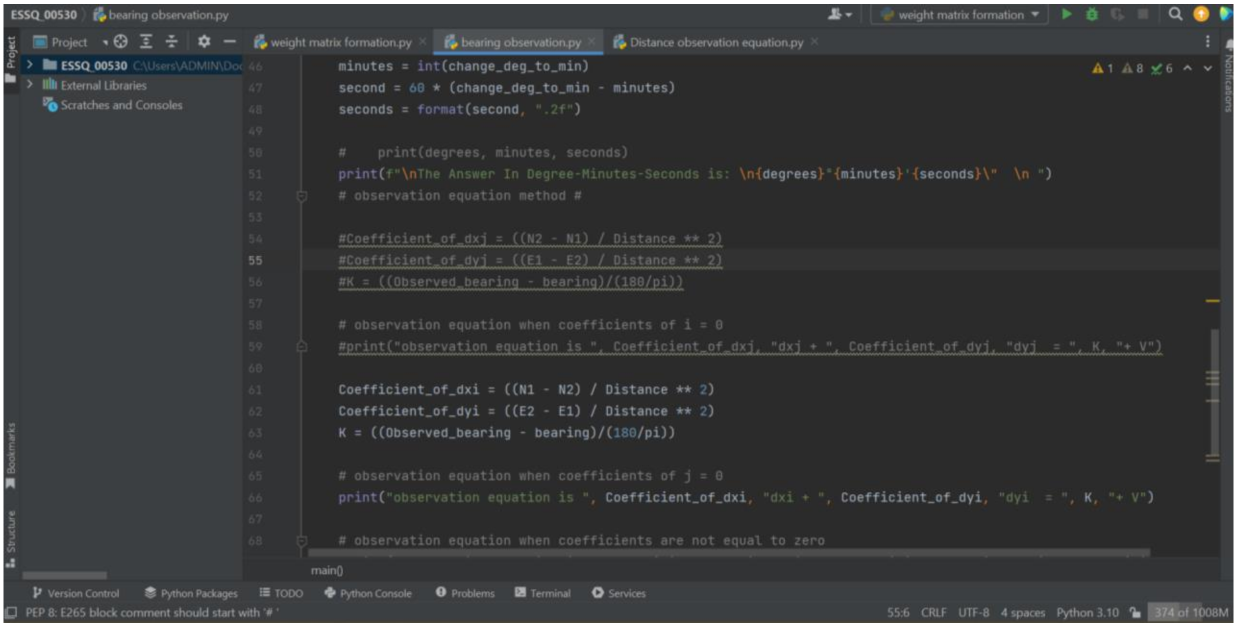


3.3 Observation equations

3.3.1 Formation of Observation equations

As explained earlier, using the general equations iii and v in Chapter 2, an algorithm is developed with i_o as the occupied station and j_o as observed station giving out linearized equations as illustrated in the code snippet below;





A sample observation equations for Donga to K6 with the code is given below;

Bearing observation equations

Donga to K6

Enter Northings of occupied station:26594.36

Enter Eastings of occupied station:686431.52

Enter Northings of target station:26489.02759957

Enter Eastings of target station:686112.31079995

Enter observed bearing:251.7330556

The distance is 336.1390010

251.73818413835082

The Answer In Degree-Minutes-Seconds is:

251°44'17.46"

observation equation is **-0.0009322323540883762 dxj + 0.0028251244896580407 dyj = -8.950988003656155e-05 + V**

Distance observation equations with weights

Donga to K6

Enter Northings of occupied station:26594.36

Enter Eastings of occupied station:686431.52

Enter Northings of target station:26489.02759957

Enter Eastings of target station:686112.31079995

Enter observed distance:336.139

The distance is 336.13900097563265

observation equation is **-0.949634523585448 dxj + -0.31335965218042905 dyj = -9.756326448950858e-07 + V**

3.3.2 Jacobian matrix formation

From the linearized equations, a J matrix is then formed with the coefficients following the structure illustrated below where i= occupied station and j= observed station

$$\begin{array}{cccccccccccc}
\multicolumn{2}{c}{K6} & \multicolumn{2}{c}{K7} & \multicolumn{2}{c}{K8} & \multicolumn{2}{c}{K9} & \multicolumn{2}{c}{K10} & \multicolumn{2}{c}{K11}
\end{array}$$

$$\left.\begin{matrix}
Xj & Yj & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
Xi & Yi & Xj & Yj & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & Xi & Yi & Xj & Yj & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & Xi & Yi & Xj & Yj & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & Xi & Yi & Xj & Yj & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Xi & Yi & Xj & Yj \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Xi & Yi
\end{matrix}\right\}\text{Distance}$$

$$\left.\begin{matrix}
Xj & Yj & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
Xi & Yi & Xj & Yj & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & Xi & Yi & Xj & Yj & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & Xi & Yi & Xj & Yj & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & Xi & Yi & Xj & Yj & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Xi & Yi & Xj & Yj \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Xi & Yi
\end{matrix}\right\}\text{Bearing}$$

giving a 14(no of observations)*12(no of unknowns) Jackobian matrix below;

```
J = [-0.9496073991182035, -0.31344184076150555, 0, 0, 0, 0, 0, 0, 0, 0, 0,0],
    [0.10355688980872348, -0.9946235320829404, -0.10355688980872348,
0.9946235320829404, 0, 0, 0, 0, 0, 0, 0,
                0],
               [0, 0, -0.6837191102832774, -0.7297452831183245,
0.6837191102832774, 0.7297452831183245, 0, 0, 0, 0, 0,
                0],
               [0, 0, 0, 0, -0.093027174127288, -0.9956635701249148,
0.093027174127288, 0.9956635701249148, 0, 0, 0, 0],
               [0, 0, 0, 0, 0, 0, 0.554375763652679, -0.8322664913803205, -
0.554375763652679, 0.8322664913803205, 0, 0],
               [0, 0, 0, 0, 0, 0, 0, 0, 0.3727717720310866, -
0.9279230603755915,
                -0.3727717720310866, 0.9279230603755915],
               [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0.8782589316816722, -
0.47818537087762075],
               [-0.0009324771026951448, 0.0028250445252501304, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0],
               [-0.0036524865075619325, -0.00038028473144942576,
0.0036524865075619325, 0.00038028473144942576, 0, 0, 0,
                0, 0, 0, 0, 0],
               [0, 0, -0.001521856933680386, 0.0014258710439731266,
0.001521856933680386, -0.0014258710439731266, 0, 0,
                0, 0, 0, 0],
               [0, 0, 0, 0, -0.002716239539890643, 0.0002537846078039494,
0.002716239539890643, -0.0002537846078039494,
                0, 0, 0, 0],
               [0, 0, 0, 0, 0, 0, -0.004034173049938968, -
0.002687177470713505, 0.004034173049938968,
                0.002687177470713505, 0, 0],
               [0, 0, 0, 0, 0, 0, 0, 0, -0.002774330474213178, -
0.0011145235324291711, 0.002774330474213178,
                0.0011145235324291711],
               [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -0.0011096828541267896, -
0.0020380984808928506]]
```

3.3.3 Formation of K matrix

A 14*1 K matrix was then formed by obtaining differences between calculated values from observed values

```
K = [0.00008585877071709547],
    [0.0009120427358766392],
    [-0.0017797751220882674],
    [0.005440173770296042],
    [-0.005112658952896254],
      [0.00461444501928554],
      [0.0032471927438564308],
      [-0.0000029610522722522066],
      [-0.000007247024909343016],
      [0.000010472840011369102],
      [-0.000004927587572501664],
      [-0.000000839661529836532],
      [-0.00000009018681120618016],
      [-0.0003976898549065242]
```

K = [0.000086]
[0.000912]
[-0.00178]
[0.00544]
[-0.005113]
[0.004614]
[0.003247]
[-0.000003]
[-0.000007]
[0.00001]
[-0.000005]
[-0.000001]
[-0.000000]
[-0.000398]

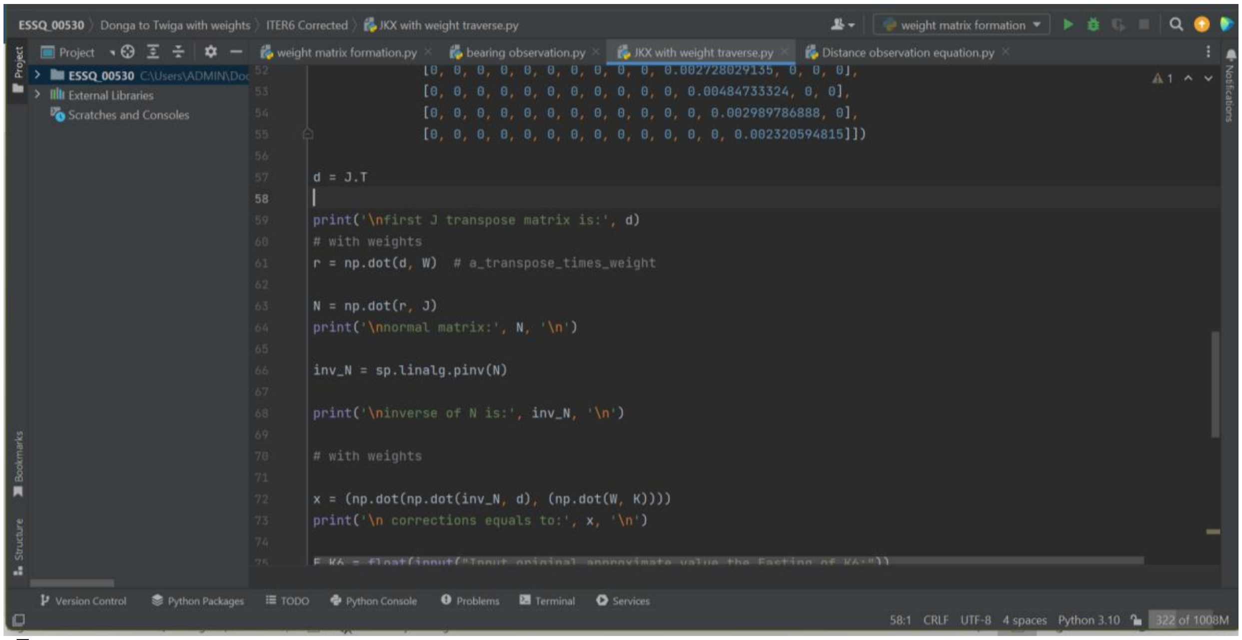
3.3.4 Weight matrix formation

A 14*14 weighted matrix was then formed based on the inverse distance for each of the observations

```
W =             [0.002975039419, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0],
                [0, 0.003672217836, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0],
                [0, 0, 0.002085470941, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0],
                [0, 0, 0, 0.002728029135, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0],
                [0, 0, 0, 0, 0.00484733324, 0, 0, 0, 0, 0, 0, 0, 0, 0],
                [0, 0, 0, 0, 0, 0.002989786888, 0, 0, 0, 0, 0, 0, 0, 0],
                [0, 0, 0, 0, 0, 0, 0.002320594815, 0, 0, 0, 0, 0, 0, 0],
                [0, 0, 0, 0, 0, 0, 0, 0.002975039419, 0, 0, 0, 0, 0, 0],
                [0, 0, 0, 0, 0, 0, 0, 0, 0.003672217836, 0, 0, 0, 0, 0],
                [0, 0, 0, 0, 0, 0, 0, 0, 0, 0.002085470941, 0, 0, 0, 0],
                [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0.002728029135, 0, 0, 0],
                [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0.00484733324, 0, 0],
                [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0.002989786888, 0],
                [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0.002320594815]
```

3.3.5 Formation of X, J^TWJ matrices

From the above matrices, X which is unknown and J^TWJ matrices are formed using the least squares algorithm in the snippet below;



J^TWJ = [2.72218688e-03 5.07269027e-04 -3.94299619e-05 3.78233775e-04
  0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
  0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00]
 [5.07269027e-04 3.92514623e-03 3.78233775e-04 -3.63283739e-03
  0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
  0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00]
 [-3.94299619e-05 3.78233775e-04 1.01433369e-03 6.62288230e-04
 -9.74903730e-04 -1.04052201e-03 0.00000000e+00 0.00000000e+00
  0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00]
 [3.78233775e-04 -3.63283739e-03 6.62288230e-04 4.74341368e-03
 -1.04052201e-03 -1.11057628e-03 0.00000000e+00 0.00000000e+00
  0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00]
 [0.00000000e+00 0.00000000e+00 -9.74903730e-04 -1.04052201e-03
  9.98532372e-04 1.29320046e-03 -2.36286418e-05 -2.52678458e-04
  0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00]
 [0.00000000e+00 0.00000000e+00 -1.04052201e-03 -1.11057628e-03
  1.29320046e-03 3.81499708e-03 -2.52678458e-04 -2.70442080e-03
  0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00]
 [0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
 -2.36286418e-05 -2.52678458e-04 1.51345051e-03 -1.98377219e-03
 -1.48982187e-03 2.23645064e-03 0.00000000e+00 0.00000000e+00]
 [0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
 -2.52678458e-04 -2.70442080e-03 -1.98377219e-03 6.06204606e-03
  2.23645064e-03 -3.35762526e-03 0.00000000e+00 0.00000000e+00]
 [0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
  0.00000000e+00 0.00000000e+00 -1.48982187e-03 2.23645064e-03
  1.90530206e-03 -3.27061922e-03 -4.15480192e-04 1.03416857e-03]
 [0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
  0.00000000e+00 0.00000000e+00 2.23645064e-03 -3.35762526e-03

```
 -3.27061922e-03  5.93195868e-03  1.03416857e-03 -2.57433342e-03]
[ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
 -4.15480192e-04  1.03416857e-03  2.20544776e-03 -2.00874486e-03]
[ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  1.03416857e-03 -2.57433342e-03 -2.00874486e-03  3.10497317e-03]
```

```
X = [-0.00920005]
    [ 0.02759957]
    [ 0.00180841]
    [ 0.02966297]
    [-0.01939766]
    [ 0.04709338]
    [ 0.00145245]
    [ 0.05060956]
    [ 0.01159043]
    [ 0.05121959]
    [ 0.04064833]
    [ 0.06786607]
```

3.3.6 Iterations

The corrections (X) were then added to the provisional coordinates to generate new J and K matrices until their existed no change in the adjusted coordinates leading to iteration of the process. The process terminated in the 8th iteration where bearing and distance corrections remained constant despite further iterations.

CHAPTER 4

4.1 Results and Analysis

From the method above, the data acquired is then used to perform analysis on the least squares method compared with the Bowditch method used in adjusting traverse measurements. This will be achieved by using the statistical methods.

Adjustment was done by both the Bowditch method and the developed least squares algorithm.

4.2 J Matrix before iteration

```
J = [[-0.9496073991182035, -0.31344184076150555, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0],
              [0.10355688980872348, -0.9946235320829404, -0.10355688980872348,
0.9946235320829404, 0, 0, 0, 0, 0, 0, 0,
               0],
              [0, 0, -0.6837191102832774, -0.7297452831183245,
0.6837191102832774, 0.7297452831183245, 0, 0, 0, 0, 0,
               0],
              [0, 0, 0, 0, -0.093027174127288, -0.9956635701249148,
0.093027174127288, 0.9956635701249148, 0, 0, 0, 0],
              [0, 0, 0, 0, 0, 0, 0.554375763652679, -0.8322664913803205, -
0.554375763652679, 0.8322664913803205, 0, 0],
              [0, 0, 0, 0, 0, 0, 0, 0, 0.3727717720310866, -0.9279230603755915,
               -0.3727717720310866, 0.9279230603755915],
              [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0.8782589316816722, -
0.47818537087762075],
              [-0.0009324771026951448, 0.0028250445252501304, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0],
              [-0.0036524865075619325, -0.00038028473144942576,
0.0036524865075619325, 0.00038028473144942576, 0, 0, 0,
               0, 0, 0, 0, 0],
              [0, 0, -0.001521856933680386, 0.0014258710439731266,
0.001521856933680386, -0.0014258710439731266, 0, 0,
               0, 0, 0, 0],
              [0, 0, 0, 0, -0.002716239539890643, 0.0002537846078039494,
0.002716239539890643, -0.0002537846078039494,
               0, 0, 0, 0],
              [0, 0, 0, 0, 0, 0, -0.004034173049938968, -0.002687177470713505,
0.004034173049938968,
               0.002687177470713505, 0, 0],
              [0, 0, 0, 0, 0, 0, 0, 0, -0.002774330474213178, -
0.0011145235324291711, 0.002774330474213178,
               0.0011145235324291711],
              [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -0.0011096828541267896, -
0.0020380984808928506]]
```

14*12

4.3 K Matrix before iteration

```
K =          [[0.00008585877071709547],
              [0.0009120427358766392],
              [-0.0017797751220882674],
              [0.005440173770296042],
              [-0.005112658952896254],
              [0.00461444501928554],
              [0.0032471927438564308],
```

```
                [-0.0000029610522722522066],
                [-0.0000007247024909343016],
                [0.000010472840011369102],
                [-0.000004927587572501664],
                [-0.000000839661529836532],
                [-0.00000009018681120618016],
                [-0.0003976898549065242]]
```

14*1

4.4 Solution for unknowns

The solution for unknowns was determined using the formula

$$x = (J^T wJ)^{-1} * (J^T wK) \text{ ....} \mathbf{xii}$$

The solution was then used to adjust the previous coordinates and compute values for the next iteration.

After the end of the iterations, the J matrix and k matrix are different and can be seen below;

```
J = [[-0.9496345283824282, -0.31335963764320873, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0],
               [0.1035161218163475, -0.9946277758659784, -0.1035161218163475,
0.9946277758659784, 0, 0, 0, 0, 0, 0, 0,
                0],
               [0, 0, -0.6836774222853305, -0.7297843395533272,
0.6836774222853305, 0.7297843395533272, 0, 0, 0, 0, 0,
                0],
               [0, 0, 0, 0, -0.09308266182988835, -0.9956583842195388,
0.09308266182988835, 0.9956583842195388, 0, 0, 0, 0],
               [0, 0, 0, 0, 0, 0, 0.5543403620732157, -0.8322900714153306, -
0.5543403620732157, 0.8322900714153306, 0, 0],
               [0, 0, 0, 0, 0, 0, 0, 0, 0.3726797561512006, -
0.9279600203430542,
                -0.3726797561512006, 0.9279600203430542],
               [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0.8783466240383936, -
0.4780242755764156],
               [-0.0009322323143231423, 0.0028251245144822914, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0],
               [-0.003652489855103975, -0.00038013374847161184,
0.003652489855103975, 0.00038013374847161184, 0, 0, 0,
                0, 0, 0, 0, 0],
               [0, 0, -0.0015219440316764541, 0.001425789395639762,
0.0015219440316764541, -0.001425789395639762, 0, 0,
                0, 0, 0, 0],
               [0, 0, 0, 0, -0.0027161850781158257, 0.00025393221319763056,
0.0027161850781158257, -0.00025393221319763056,
                0, 0, 0, 0],
               [0, 0, 0, 0, 0, 0, -0.004034387325894431, -
```

```
0.0026870724616200395, 0.004034387325894431,
                 0.0026870724616200395, 0, 0],
                [0, 0, 0, 0, 0, 0, 0, 0, -0.002774402699382652, -
0.0011142330475497394, 0.002774402699382652,
                 0.0011142330475497394],
                [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -0.001109300655273114, -
0.0020382866213809274]]
```

```
K =            [[0.00000028003535135212587],
                [-0.0000002664865519363957],
                [-0.0000005336688673196477],
                [-0.00000039025729847708135],
                [-0.00000011665599686239148],
                [-0.00000024334605086551164],
                [-0.0000000031590161597705446],
                [-0.00008952518826300032],
                [-0.00004823530246578505],
                [0.00006759809089774087],
                [-0.00006065710179284415],
                [-0.00004337540978869235],
                [-0.00009925147640591013],
                [-0.00021427326462130608]]
```

J^TWJ = [[2.72230916e-03 5.07209375e-04 -3.93989613e-05 3.78086487e-04

0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00

0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00]

[5.07209375e-04 3.92502394e-03 3.78086487e-04 -3.63286840e-03

0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00

0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00]

[-3.93989613e-05 3.78086487e-04 1.01418381e-03 6.62427762e-04

-9.74784850e-04 -1.04051425e-03 0.00000000e+00 0.00000000e+00

0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00]

[3.78086487e-04 -3.63286840e-03 6.62427762e-04 4.74356356e-03

-1.04051425e-03 -1.11069516e-03 0.00000000e+00 0.00000000e+00

0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00]

[0.00000000e+00 0.00000000e+00 -9.74784850e-04 -1.04051425e-03

9.98441663e-04 1.29334210e-03 -2.36568128e-05 -2.52827856e-04

0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00]

[0.00000000e+00 0.00000000e+00 -1.04051425e-03 -1.11069516e-03

1.29334210e-03 3.81508779e-03 -2.52827856e-04 -2.70439262e-03

0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00]

```
[ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
 -2.36568128e-05 -2.52827856e-04  1.51328843e-03 -1.98354333e-03
 -1.48963162e-03  2.23637118e-03  0.00000000e+00  0.00000000e+00]
[ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
 -2.52827856e-04 -2.70439262e-03 -1.98354333e-03  6.06220814e-03
  2.23637118e-03 -3.35781552e-03  0.00000000e+00  0.00000000e+00]
[ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00 -1.48963162e-03  2.23637118e-03
  1.90490673e-03 -3.27032566e-03 -4.15275114e-04  1.03395448e-03]
[ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00  2.23637118e-03 -3.35781552e-03
 -3.27032566e-03  5.93235402e-03  1.03395448e-03 -2.57453850e-03]
[ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
 -4.15275114e-04  1.03395448e-03  2.20560014e-03 -2.00829972e-03]
[ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  1.03395448e-03 -2.57453850e-03 -2.00829972e-03  3.10482078e-03]]
```

Adjusted coordinates

$$\text{Corrections X} = \begin{array}{lcl} \mathbf{[[\ 7.73071385e\text{-}10]\ +E_{K6}} & = & \mathbf{686112.31079953} \\ \mathbf{[\ 3.80506868e\text{-}09]\ +N_{K6}} & = & \mathbf{26489.02760485} \end{array} \Bigg\} \ \mathbf{K6}$$

$$\begin{array}{lcl} \mathbf{[\ 3.47620267e\text{-}09]\ +E_{K7}} & = & \mathbf{686084.12180679} \\ \mathbf{[-2.89576789e\text{-}09]\ +N_{K7}} & = & \mathbf{26759.8796679} \end{array} \Bigg\} \ \mathbf{K7}$$

$$\begin{array}{lcl} \mathbf{[-1.42104580e\text{-}09]\ +E_{K8}} & = & \mathbf{686411.95060056} \\ \mathbf{[\ 3.57759334e\text{-}09]\ +N_{K8}} & = & \mathbf{27109.81709738} \end{array} \Bigg\} \ \mathbf{K8}$$

$$\begin{array}{lcl} \mathbf{[\ 2.24958791e\text{-}09]\ +E_{K9}} & = & \mathbf{686446.07144653} \\ \mathbf{[\ 2.24184958e\text{-}09]\ +N_{K9}} & = & \mathbf{27474.79061338} \end{array} \Bigg\} \ \mathbf{K9}$$

$$\begin{array}{lcl} \mathbf{[\ 4.80273843e\text{-}09]\ +E_{K10}} & = & \mathbf{686331.71158411} \\ \mathbf{[-2.10288915e\text{-}09]\ +N_{K10}} & = & \mathbf{27646.49122292} \end{array} \Bigg\} \ \mathbf{K10}$$

$$\begin{array}{lcl} \mathbf{[-2.48575704e\text{-}09]\ +E_{K11}} & = & \mathbf{686207.06064062} \\ \mathbf{[-1.32870336e\text{-}09]]\ +N_{K11}} & = & \mathbf{27956.86786707} \end{array} \Bigg\} \ \mathbf{K11}$$

4.5 Bowditch method results

The results are also adjusted using bowditch method in ms. Excel as illustrated below in table 6 below;

station	distance	bearing	departure(sin)	dep misc	adj dep	latitude(cos)	lat misc	adj lat
Donga			686431.52		686431.52	26594.36		26594.36
Donga to K6	336.139	251.7333333	-319.2002802	-0.002172367	-319.2024526	-105.3594249	-0.00666168	-105.3660866
K6 to K7	272.315	354.0555556	-28.20205722	-0.000191933	-28.20224915	270.8507028	0.01712539	270.8678282
K7 to K8	479.508	43.13611111	327.8558406	0.002231274	327.8580719	349.9120887	0.0221243	349.934213
K8 to K9	366.565	5.347222222	34.16063795	0.000232486	34.16087043	364.9697933	0.02307637	364.9928697
K9 to K10	206.299	326.3319444	-114.3681422	-0.00077835	-114.3689206	171.6950944	0.01085596	171.7059504
K10 to K11	334.472	338.1138889	-124.6787384	-0.00084852	-124.6795869	310.3654797	0.01962384	310.3851035
K11 to Twiga	430.924	298.5444444	-378.5437725	-0.002576238	-378.5463487	205.9128605	0.01301949	205.9258799
			685828.5435		685828.5394	28162.70659		28162.80576
Twiga			685828.56			28162.86		
misclosure			0.01651203			0.153405536		
	2426.222							
	linear misc	0.154291626						
	relative precision	6.35934E-05						

Giving a linear misclosure of 0.15.

4.6 Statistical analysis of least squares results

4.6.1 Variance of residuals

The variance of residuals is a measure of how far from the solution the adjusted measurements are from the line of best fit.

It is calculated by having the square root of the sum of the square of the residuals divided by the degrees of freedom.

$$s = \sqrt{\frac{\sum_{i=1}^{n} v_i}{dof}} \qquad \textbf{................xiii}$$

The variance residual was computed from the individual residuals for the distance and angle measurements and the results found in table 7 below.

The table 7 below outlines the residuals and their corresponding variance for the dataset used.

	bearing bowditch	bearing least squares	distance bowditch	distance least squares	bearing resids	distance resids
Donga to k6	251.7330556	251.738185	336.139	336.1389997	-0.00512939999999	0.00000030000001
k6 to k7	354.0555556	354.0583193	272.315	272.3150003	-0.00276370000000	-0.00000030000001
k7 to k8	43.1355556	43.1316825	479.508	479.5080005	0.00387310000001	-0.00000050000000
k8 to k9	5.3375	5.341	366.565	366.5650004	-0.00350000000000	-0.00000039999998
k9 to k10	326.3322222	326.3347074	206.299	206.2990001	-0.00248520000002	-0.00000009999999
k10 to k11	338.1133333	338.11902	334.472	334.4720002	-0.00568669999996	-0.00000020000004
k11 to Twiga	298.5441667	298.5564437	430.924	430.924	-0.01227699999998	0.00000000000000
					0.00025043919059	0.00000000000064
					7.00000000000000	
				variance	0.00646064	0.0000003266

The variance is very minimal attributed by the fact that weighted least square adjusted was carried out.

4.7 Computation of variance and covariance from covariant matrix

A covariant matrix is used to compute the variances and covariances for the coordinate values.

The variance in the individual coordinates is derived from the covariant matrix

A sample covariant matrix

$$\begin{bmatrix} a & b \\ b & c \end{bmatrix}$$

Where;

a is the variance in the Northings

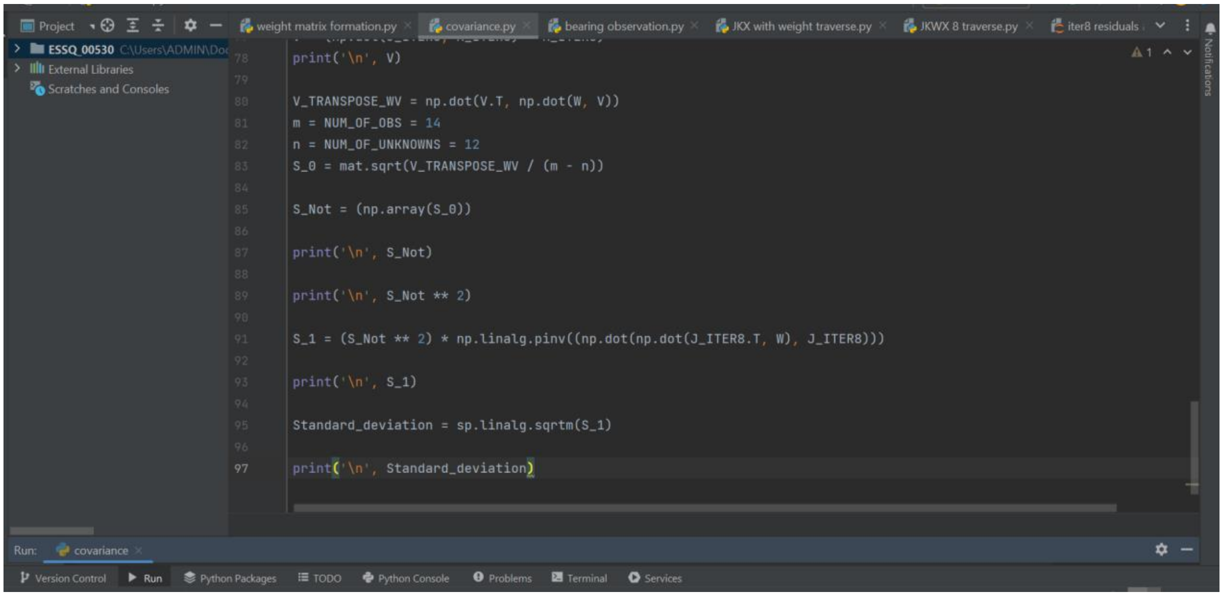
c is the variant in the Eastings

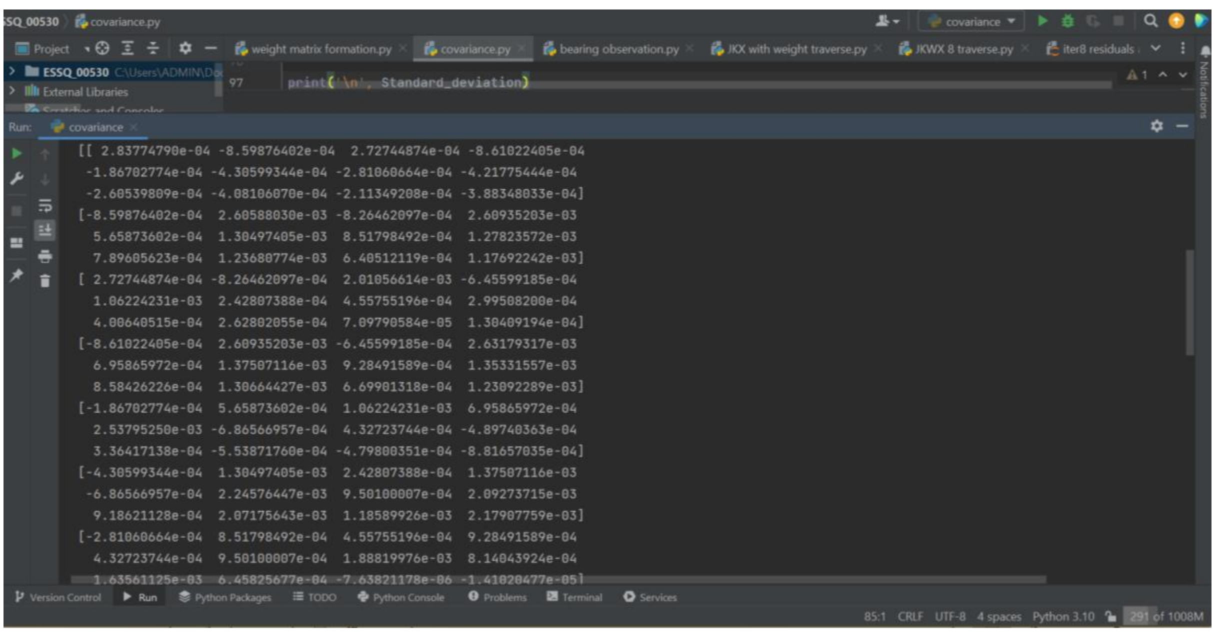
b is the covariance of the two coordinates to each other

The link between the coordinates can be expressed as the Pearson Correlation Coefficient. This is a measure of how the coordinates are related and to what degree and direction. This coefficient is then used to compute the semi major and the semi minor axes of the error ellipse. Its formula can be derived from equation 4.4:

$$\rho = \frac{\sigma_{ne}}{\sqrt{\sigma_n \, x \, \sigma_e}} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \mathbf{xiv}$$

The code snippet below shows the calculation for the covariant matrix





Giving the following variance and covariance matrix as given in the table 8 below

[2.83774790e-04 -8.59876402e-04 2.72744874e-04 -8.61022405e-04, -1.86702774e-04 -4.30599344e-04 -2.81060664e-04 -4.21775444e-04, -2.60539809e-04 -4.08106070e-04 -2.11349208e-04 -3.88348033e-04]

[-8.59876402e-04 2.60588030e-03 -8.26462097e-04 2.60935203e-03, 5.65873602e-04 1.30497405e-03 8.51798492e-04 1.27823572e-03, 7.89605623e-04 1.23680774e-03 6.40512119e-04 1.17692242e-03]

[2.72744874e-04 -8.26462097e-04 2.01056614e-03 -6.45599185e-04 1.06224231e-03 2.42807388e-04 4.55755196e-04 2.99508200e-04, 4.00640515e-04 2.62802055e-04 7.09790584e-05 1.30409194e-04]

[-8.61022405e-04 2.60935203e-03 -6.45599185e-04 2.63179317e-03 6.95865972e-04 1.37507116e-03 9.28491589e-04 1.35331557e-03, 8.58426226e-04 1.30664427e-03 6.69901318e-04 1.23092289e-03]

[-1.86702774e-04 5.65873602e-04 1.06224231e-03 6.95865972e-04, 2.53795250e-03 -6.86566957e-04 4.32723744e-04 -4.89740363e-04, 3.36417138e-04 -5.53871760e-04 -4.79800351e-04 -8.81657035e-04]

[-4.30599344e-04 1.30497405e-03 2.42807388e-04 1.37507116e-03, -6.86566957e-04 2.24576447e-03 9.50100007e-04 2.09273715e-03, 9.18621128e-04 2.07175643e-03 1.18589926e-03 2.17907759e-03]

[-2.81060664e-04 8.51798492e-04 4.55755196e-04 9.28491589e-04, 4.32723744e-04 9.50100007e-04 1.88819976e-03 8.14043924e-04, 1.63561125e-03 6.45825677e-04 -7.63821178e-06 -1.41020477e-05]

[-4.21775444e-04 1.27823572e-03 2.99508200e-04 1.35331557e-03, -4.89740363e-04 2.09273715e-03 8.14043924e-04 1.97086676e-03, 7.97174408e-04 1.95961616e-03 1.14176618e-03 2.09798709e-03]

[-2.60539809e-04 7.89605623e-04 4.00640515e-04 8.58426226e-04, 3.36417138e-04 9.18621128e-04 1.63561125e-03 7.97174408e-04, 1.94916845e-03 1.00601945e-03 1.55518147e-04 2.85688440e-04]

[-4.08106070e-04 1.23680774e-03 2.62802055e-04 1.30664427e-03, -5.53871760e-04 2.07175643e-03 6.45825677e-04 1.95961616e-03 1.00601945e-03 2.19952589e-03 1.25043826e-03 2.29766581e-03]

[-2.11349208e-04 6.40512119e-04 7.09790584e-05 6.69901318e-04, -4.79800351e-04 1.18589926e-03 -7.63821181e-06 1.14176618e-03, 1.55518147e-04 1.25043826e-03 8.27435283e-04 1.52029412e-03

[-3.88348033e-04 1.17692242e-03 1.30409194e-04 1.23092289e-03, -8.81657035e-04 2.17907759e-03 -1.41020477e-05 2.09798709e-03, 2.85688440e-04 2.29766581e-03 1.52029412e-03 2.79350908e-03]]

Covariance matrix

Station	Variance in N	Variance in E	Covariance
K6	0.000283775	0.00260588	-0.000859876
K7	0.002010566	0.002631793	-0.000645599
K8	0.002537953	0.002245764	-0.000686567
K9	0.0018882	0.001970867	0.000814044
K10	0.001949168	0.002199526	0.001006019
K11	0.000827435	0.002793509	0.001520294
Variance	0.000000716167	0.000000099745	

Table 9

From the table 9 above, the coordinates obtained from Least squares method have a variance of 0.0000000716167m in the northings and 0.000000099745m in the eastings.

The variance levels show that the coordinates are adjusted to the minimum variations from the line of best fit.

4.8 Error Ellipses

Error ellipses are ways of showing the precision of an adjusted coordinate at a traverse station. They can be used to analyze and visually assess the strong and weak position fixes in a network.

The interpretation is visually as elongated ellipses indicate a weak fix, and the fix gets stronger as the ellipse gets more circular.

The ellipse parameters as derived by Mikhail, E.M., (1976, pp 30-31) as in eq xv and xvi are used to compute the parameters for the ellipses for the analysis

$$a^2 = \frac{1}{2}\left(\delta_N^2 + \delta_E^2 + \sqrt{(\delta_N^2 - \delta_E^2)^2 + (2\delta_{NE})^2}\right)\text{.................}\mathbf{xv}$$

$$b^2 = \frac{1}{2}\left(\delta_N^2 + \delta_E^2 - \sqrt{(\delta_N^2 - \delta_E^2)^2 + (2\delta_{NE})^2}\right)\text{.................}\mathbf{xvi}$$

Deakin, R.E., (1991) described the orientation angle of the error ellipse from the north is given by ;

$$tan\ 2\theta = \frac{2\delta_{NE}}{\delta_N^2 - \delta_E^2}\text{.................}\mathbf{xvii}$$

The signs of the numerator and denominator are used to determine the quadrant of 2θ

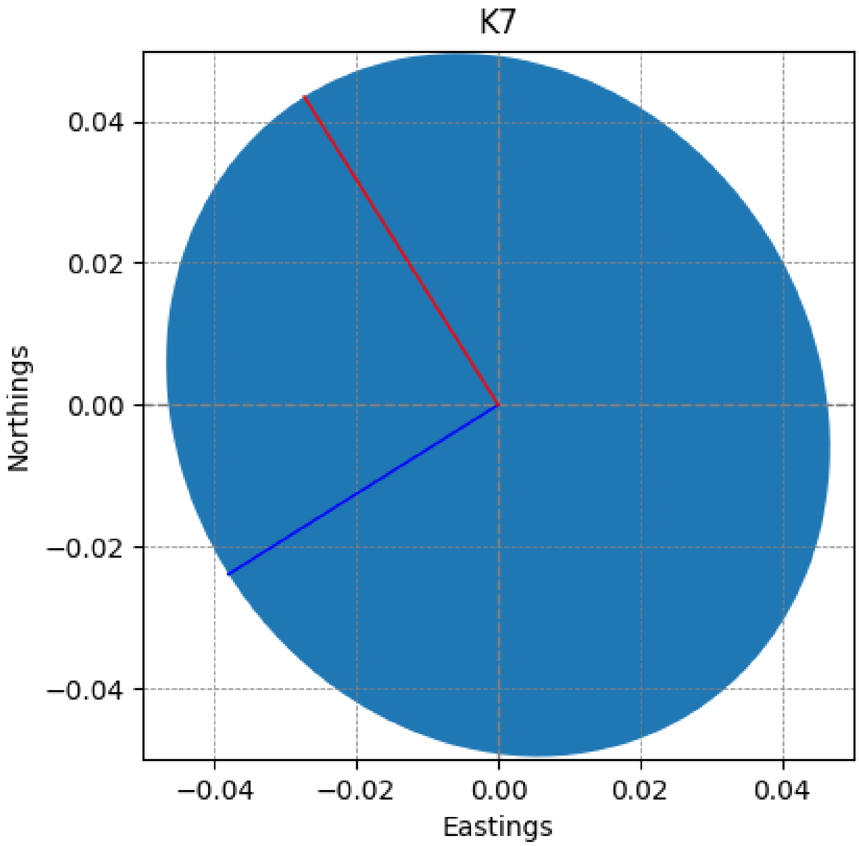
${}_Nq_E$	${}_Nq_N - {}_Eq_E$	*Quad*	*T*	*Az_U*
+	+	NE	0°	t
−	+	SE	180°	t+180°
−	−	SW	180°	t+180°
+	−	NW	360°	t+360°

Figure 1: A conversion matrix that provides reference for signs accorded to the angle of the error ellipse from the north.

The error ellipse parameters for every station are summarized in the table 10 below;

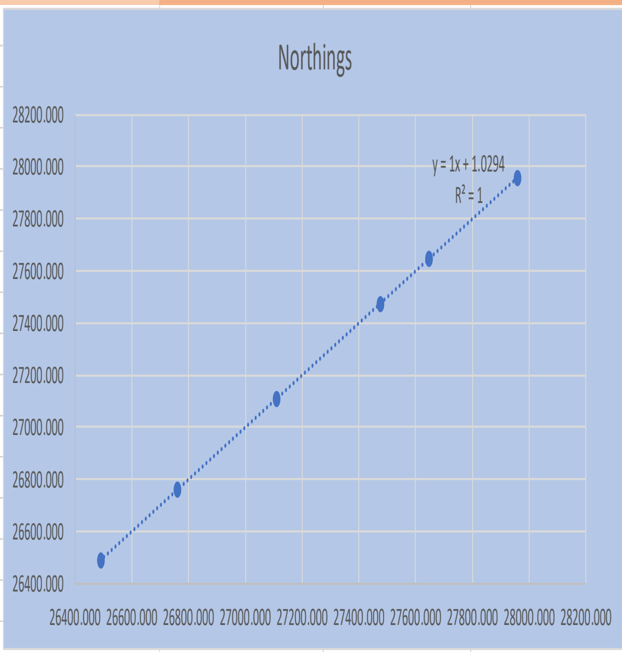
Station	Semi major axis	semi minor axis	Quadrant	t	T	Azu
K6	0.051047824	0.016845616	SE	44.88822302	180	224.888223
K7	0.051301006	0.044839337	SE	44.93601355	180	224.9360136
K8	0.047389497	0.050378095	SW	-44.97083866	180	135.0291613
K9	0.044394445	0.043453421	NE	-44.99438656	360	315.0056134
K10	0.046899103	0.044149388	NE	-44.98521136	360	315.0147886
K11	0.052853657	0.028765175	NE	-44.93292574	360	315.0670743

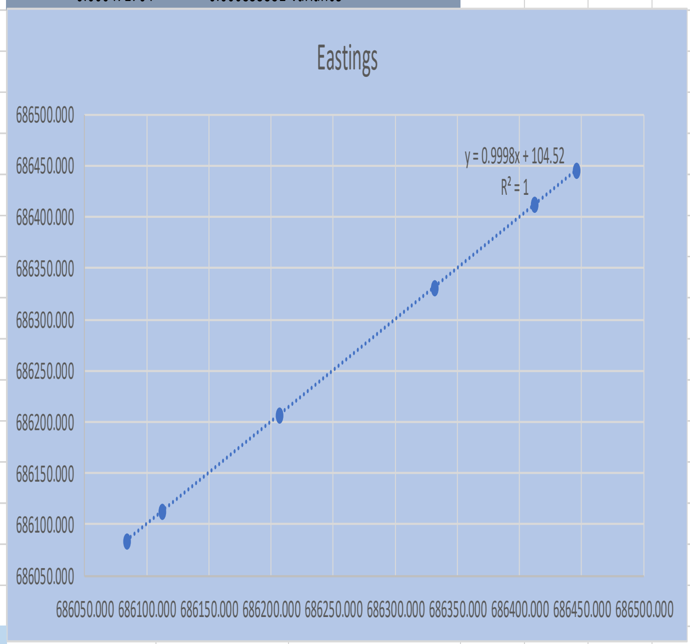
A sample error ellipse for point K7 is shown figure 12 below;



4.9 Comparison of Bowditch and Least square results

Name	Northings(bowditch)	Northings(least squares)	Eastings(bowditch)	Eastings(least squares)	Northing residuals	Eastings residuals			
K6	26489.022	26489.028	686112.322	686112.311	-0.006	0.011			
K7	26759.890	26759.880	686084.122	686084.122	0.010	0.000			
K8	27109.832	27109.817	686411.981	686411.951	0.015	0.030			
K9	27474.825	27474.791	686446.145	686446.071	0.034	0.074			
K10	27646.533	27646.491	686331.778	686331.712	0.042	0.066			
K11	27956.920	27956.868	686207.101	686207.061	0.052	0.040		0.000472	0.000855
					0.006	0.012			
					0.032	0.046	RMSE		
	Correlation	1		0.999999994	0.019827673	0.026697867	Stdev		
					0.000471764	0.000855331	variance		





Thus the above indicates a perfect linear relationship between the coordinates.

Table 11

Table 11 above is a summary of the comparison between the Bowditch and the Least squares method. The table comprises of the coordinates of both the methods and deviations.

The two variables were derived from the formula

Northing residuals = $\text{Northing}_{\text{Bowditch}}$ – $\text{Northing}_{\text{LSA}}$ **……….......xviii**

Easting residuals = $\text{Easting}_{\text{Bowditch}}$ – $\text{Easting}_{\text{LSA}}$ **---------------xix**

The results show that there is a very small variation between the two methods, with K11 having the highest difference in Northings and K9 highest difference in eastings making it a remarkable difference in the adjusted coordinates in the eastings.

CHAPTER 5
CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

The researcher concluded that it is possible to develop an algorithm to adjust survey traverse data using Least squares method. The developed algorithm was able to adjust the traverse observations presented to it with excellent results. It also proved that it can be run with very many datasets and perform as expected regardless of the data given to the program.

The researcher was able to derive and produce the observation equations of a link traverse, which an algorithm was built upon to automatically generate the observation equations, for both the distance and bearing observations, and from which Jacobian matrix representing the observation equations was generated from.

An algorithm was successfully developed that solved the observation equations by iterative methods to produce solutions to adjust the calculated provisional coordinates.

From the output data analysis performed in chapter 4, it was noticed that the Least squares method is a good method of adjusting measurements and observations. It provides corrected coordinates, as well as corrected observations and their respective residuals.

5.2 Recommendations

The researcher recommends deployment of traverse adjustment to a a web application or a stand alone offline app in further studies to allow for easier interoperability allowing surveyors and engineers to correct field data when they are in the field.

The researcher recommends a unified reference frame being fought for by Kefref and afref to be given much emphasis for a standard reference frame be used by professionals in the whole country.

The researcher recommends further research to facilitate the development of an algorithm to enable the adjustment of linked traverses with multiple opening and closing stations.

The algorithm has been developed and tested on two separate datasets, the performance was the same, however it is recommended that caution be taken when using it, as it has not been tested against numerous datasets to identify tailbacks in the system.

Further testing be done to the traverse system by numerous traverse data to identify any unseen bugs in the algorithm for a better and more robust algorithm.

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Appendix
Code
Area of interest plot

```
import matplotlib.pyplot as plt

# define the coordinates and point names
coords = [[26594.36, 686431.52, 'Donga'], [23857.59, 682214.04, 'Nyoka'], [26489.022, 686112.322,
'K6'],
        [26759.89, 686084.122, 'K7'], [27109.832, 686411.981, 'K8'], [27474.825, 686446.145, 'K9'],
        [27646.533, 686331.778, 'K10'], [27956.92, 686207.101, 'K11'], [28162.86, 685828.56, 'Twiga']]

# extract x and y coordinates separately
y = [coord[0] for coord in coords]
x = [coord[1] for coord in coords]
names = [coord[2] for coord in coords]

# create a scatter plot of the coordinates
plt.scatter(x, y)

# join the coordinates with a line
#plt.plot(x, y)

# add point names to the plot
for i, name in enumerate(names):
    plt.text(x[i], y[i], name)

# add gridlines to the plot
plt.grid(True)

# set the title and labels
plt.title('Donga to Twiga Traverse Network')
plt.xlabel('X-coordinate')
plt.ylabel('Y-coordinate')

# display the plot
plt.show()
```

Join computations code for provisional coordinates

```
def main():
    import math as m
    from math import pi
```

```
    print("\nWelcome these are the provisional coordinates\n")

    N1 = float(input("Enter the northing for known point:"))
    E1 = float(input("Enter the easting for known point:"))
    Distance = float(input("Enter the distance between known point and unknown point:"))
    Bearing = float(input("Enter the forward bearing from the known point to unknown point:"))

    a = Distance * m.sin((Bearing/(180/pi)))
    b = Distance * m.cos((Bearing/(180/pi)))

    N2 = N1 + b
    E2 = E1 + a

    print('\n', a, "\n", b, "\n")

    print(format(N2, ".3f"))
    print(format(E2, ".3f"))

    Repeat = input("would you like to continue?:")

    if Repeat == "Yes":
        main()
    elif Repeat == "No":
        exit()


main()
```

Weight matrix formation

```
import numpy as np

d1 = float(input("Please input d1:"))
d2 = float(input("Please input d2:"))
d3 = float(input("Please input d3:"))
d4 = float(input("Please input d4:"))
d5 = float(input("Please input d5:"))
d6 = float(input("Please input d6:"))
d7 = float(input("Please input d7:"))

f = np.array([(1 / d1), (1 / d2), (1 / d3), (1 / d4), (1 / d5), (1 / d6),
              (1 / d7), (1 / d1), (1 / d2), (1 / d3), (1 / d4), (1 / d5),
              (1 / d6), (1 / d7)])
```

```
w = np.diag(f)

print(w)
```

Observation equations matrix

```
# Distance observation equation
import math as m  # Import math library used in join computation
import numpy as np
import scipy as sp

print("Hello and welcome!!!. \n Experience least squares with ease.\n")


# (N1,E1) and (N2,E2) are coordinates of instrument station and foresight stations respectively.
def main():
  N1 = float(input("Enter Northings of occupied station:"))
  E1 = float(input("Enter Eastings of occupied station:"))
  N2 = float(input("Enter Northings of target station:"))
  E2 = float(input("Enter Eastings of target station:"))
  Observed_distance = float(input("Enter observed distance:"))

  # Calculating distance
  L = (N2 - N1) ** 2 + (E2 - E1) ** 2

  Distance = m.sqrt(L)

  format(Distance, ".3f")

  print("\nThe distance is", Distance)

  # observation equation method #

  #Coefficient_of_dxj = ((E2 - E1) / Distance)
  #Coefficient_of_dyj = ((N2 - N1) / Distance)
  #K = Observed_distance - Distance

  # observation equation when coefficients of i = 0
  #print("observation equation is ", Coefficient_of_dxj, "dxj + ", Coefficient_of_dyj, "dyj  = ", K, "+ V")

  Coefficient_of_dxi = ((E1 - E2) / Distance)
  Coefficient_of_dyi = ((N1 - N2) / Distance)
  K = Observed_distance - Distance

  # observation equation when coefficients of j = 0
```

```
    print("observation equation is ", Coefficient_of_dxi, "dxi + ", Coefficient_of_dyi, "dyi  = ", K, "+ V")

    # observation equation when coefficients are not equal to zero
    #print("observation equation is ", Coefficient_of_dxi, "dxi + ", Coefficient_of_dyi, " dyi +",
Coefficient_of_dxj,
          #"dxj + ", Coefficient_of_dyj, "dyj  = ", K, "+ V")

    Repeat = input("would you like to continue?:")

    if Repeat == "Yes":
        main()
    elif Repeat == "No":
        exit()


main()

import math as m  # Import math library used in join computation
from math import pi

print("Hello and welcome!!!. \n Experience a great tasking.\n")


# (N1,E1) and (N2,E2) are coordinates of instrument station and foresight stations respectively.
def main():
    N1 = float(input("Enter Northings of occupied station:"))
    E1 = float(input("Enter Eastings of occupied station:"))
    N2 = float(input("Enter Northings of target station:"))
    E2 = float(input("Enter Eastings of target station:"))
    Observed_bearing = float(input("Enter observed bearing:"))

    # Calculating distance
    L = (N2 - N1) ** 2 + (E2 - E1) ** 2

    Distance = m.sqrt(L)

    format(Distance, ".2f")

    print("\nThe distance is", format(Distance, ".7f"))

    # A is the change in eastings and B is the change in northings
    A = E2 - E1
    B = N2 - N1
    # calculating direction parameters using math library
    direction = m.degrees(m.atan(A / B))
```

```
    # conditions for adding constants at each quadrant
    if A > 0 and B > 0:
        bearing = abs(direction)
    elif A < 0 and B < 0:
        bearing = abs(direction) + 180
    elif A < 0 and B > 0:
        bearing = 360 - abs(direction)
    elif A > 0 and B < 0:
        bearing = 180 - abs(direction)
    else:
        print("done")
    # bearing in decimal degrees
    print(bearing)
    # print bearing in degrees,minutes and seconds
    decimal_degrees = bearing
    degrees = int(decimal_degrees)
    change_deg_to_min = 60 * (decimal_degrees - degrees)
    minutes = int(change_deg_to_min)
    second = 60 * (change_deg_to_min - minutes)
    seconds = format(second, ".2f")

    #    print(degrees, minutes, seconds)
    print(f"\nThe Answer In Degree-Minutes-Seconds is: \n{degrees}°{minutes}'{seconds}\"  \n ")
    # observation equation method #

    #Coefficient_of_dxj = ((N2 - N1) / Distance ** 2)
    #Coefficient_of_dyj = ((E1 - E2) / Distance ** 2)
    #K = ((Observed_bearing - bearing)/(180/pi))

    # observation equation when coefficients of i = 0
    #print("observation equation is ", Coefficient_of_dxj, "dxj + ", Coefficient_of_dyj, "dyj  = ", K, "+ V")

    Coefficient_of_dxi = ((N1 - N2) / Distance ** 2)
    Coefficient_of_dyi = ((E2 - E1) / Distance ** 2)
    K = ((Observed_bearing - bearing)/(180/pi))

    # observation equation when coefficients of j = 0
    print("observation equation is ", Coefficient_of_dxi, "dxi + ", Coefficient_of_dyi, "dyi  = ", K, "+ V")

    # observation equation when coefficients are not equal to zero
    #print("observation equation is ", Coefficient_of_dxi, "dxi + ", Coefficient_of_dyi, " dyi +",
Coefficient_of_dxj,
          #"dxj + ", Coefficient_of_dyj, "dyj  = ", K, "+ V")
```

```
    Repeat = input("would you like to continue?:")

    if Repeat == "Yes":
        main()
    elif Repeat == "No":
        exit()


main()
```

Least squares

```
import scipy as sp
import numpy as np

J = np.array([J Matrix])

K = np.array([K Matrix])

W = np.array([Weight Matrix])

d = J.T

print('\nfirst J transpose matrix is:', d)
# with weights
r = np.dot(d, W)  # a_transpose_times_weight

N = np.dot(r, J)
print('\nnormal matrix:', N, '\n')

inv_N = sp.linalg.pinv(N)

print('\ninverse of N is:', inv_N, '\n')

# with weights

x = (np.dot(np.dot(inv_N, d), (np.dot(W, K))))
print('\n corrections equals to:', x, '\n')

E_K6 = float(input("Input original approximate value the Easting of K6:"))
N_K6 = float(input("Input original approximate value the Northing of K6:"))
E_K7 = float(input("Input original approximate value the Easting of K7:"))
N_K7 = float(input("Input original approximate value the Northing of K7:"))
E_K8 = float(input("Input original approximate value the Easting of K8:"))
N_K8 = float(input("Input original approximate value the Northing of K8:"))
```

```
E_K9 = float(input("Input original approximate value the Easting of K9:"))
N_K9 = float(input("Input original approximate value the Northing of K9:"))
E_K10 = float(input("Input original approximate value the Easting of K10:"))
N_K10 = float(input("Input original approximate value the Northing of K10:"))
E_K11 = float(input("Input original approximate value the Easting of K11:"))
N_K11 = float(input("Input original approximate value the Northing of K11:"))

list1 = np.array([[E_K6], [N_K6], [E_K7], [N_K7], [E_K8], [N_K8], [E_K9], [N_K9], [E_K10], [N_K10],
[E_K11], [N_K11]])
list2 = list1 + x

# print(x)

print(np.array(list2))
```

Residuals and covariance

```
import numpy as np
import math as mat
import scipy as sp

W = np.array([Weight Matrix])

J_ITER8 = np.array([Last Iteration J Matrix])
K_ITER8 = np.array([Last Iteration K Matrix])

X_ITER8 = np.array([Last Iteration X Matrix])

V = (np.dot(J_ITER8, X_ITER8) - K_ITER8)
print('\n', V)

V_TRANSPOSE_WV = np.dot(V.T, np.dot(W, V))
m = NUM_OF_OBS = 14
n = NUM_OF_UNKNOWNS = 12
S_0 = mat.sqrt(V_TRANSPOSE_WV / (m - n))

S_Not = (np.array(S_0))

print('\n', S_Not)

print('\n', S_Not ** 2)

S_1 = (S_Not ** 2) * np.linalg.pinv((np.dot(np.dot(J_ITER8.T, W), J_ITER8)))

print('\n', S_1)
```

```
Standard_deviation = sp.linalg.sqrtm(S_1)

print('\n', Standard_deviation)
```

Error ellipse parameters

```
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.patches import Ellipse

cov = np.array([[‘station var in N’, ‘station covariance’], [‘station covariance’, ‘station variance in E’]])
mean = np.array([0, 0])
scale = 2 # represents the scale of the error ellipse, e.g., 2 standard deviations

# Calculate standard deviations
std_dev = np.sqrt(np.diag(cov))

# Get the rotation angle of the ellipse
angle = np.degrees(np.arctan2(*np.linalg.eig(cov)[1][:,0][::-1]))

# Plot the error ellipse
fig, ax = plt.subplots(subplot_kw={'aspect': 'equal'})
ell = Ellipse(xy=mean, width=scale*std_dev[0], height=scale*std_dev[1], angle=angle)
width=scale*std_dev[0]
height=scale*std_dev[1]
angle=angle
print('semi minor axis:', width/2)
print('semi major axis:', height/2)
print('angle equals:', (180-angle))
ax.add_artist(ell)
ax.set_xlim(mean[0]-0.05, mean[0]+0.05)
ax.set_ylim(mean[1]-0.05, mean[1]+0.05)
#plot semi-major and semi-minor axis
ax.axhline(y=mean[1], color='gray', linestyle='--', linewidth=1)
ax.axvline(x=mean[0], color='gray', linestyle='--', linewidth=1)
ax.plot([mean[0], mean[0]+(width/2)*np.cos(np.radians(angle))], [mean[1],
mean[1]+(width/2)*np.sin(np.radians(angle))], color='blue', linestyle='-', linewidth=1)
ax.plot([mean[0], mean[0]+(height/2)*np.sin(np.radians(angle))], [mean[1], mean[1]-
(height/2)*np.cos(np.radians(angle))], color='red', linestyle='-', linewidth=1)
#add grids
ax.grid(color='gray', linestyle='--', linewidth=0.5)

plt.xlabel('Eastings')
plt.ylabel('Northings')
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plt.title('station name')
plt.show()
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