Question one (30 marks)

a. Discuss the following accuracy of numbers (1mark) (1mark)
   i. Exact number
   ii. Approximate numbers

b. A real root of the equation $f(x) = x^3 - 5x + 1 = 0$ lies in the interval (0,1). Perform three iterations of the Newton-Raphson method to obtain this root. Take the initial approximation as $x_0 = 0.5$. **(5marks)**

c. Apply the Runge-Kutta method of order 2 to the differential equation $y' = -y + t$, $0 \le t \le 1$, $y(0) = 1$, $h = 0.1$ **(5marks)**

d. Find the relative error in calculation of $\frac{6.213}{0.342}$ where numbers 6.213 & 0.342 are correct to three decimal places. Determine the smallest interval in which true results lies. **(5marks)**

e. Perform three iterations of the Newton-Raphson method to solve the system of equations; **(6marks)**

$$x^2 + xy + y^2 = 7$$
$$x^3 + y^3 = 9$$

Take the initial approximation as $x_0 = 1.5$, $y_0 = 0.5$. The exact solution is $x = 2$, $y = 1$.

f. Given the data in the table below, use the central difference two point formula and Richardson extrapolation to find $f'(5)$. **(7marks)**

x	1	3	4	5	6	7	9
$f(x)$	-11	5	37	93	179	301	943

Question two (20marks)

a. Using the Lagrange formula, find the unique polynomial of degree 2 which fits the given data. **(7marks)**

x	0	2	4
$f(x)$	1	8	16

b. Perform three iterations of the bisection method to obtain the smallest positive root of the equation

$$f(x) = x^3 - 6x + 1 = 0$$

(5marks)

THE TECHNICAL UNIVERSITY OF KENYA

SCHOOL OF SURVEYING AND SPATIAL SCIENCE

DEPARTMENT OF GEOINFORMATION AND EARTH OBSERVATION

END TERM EXAMINATIONS (2022)

EXAMINATION FOR THE DEGREE OF BACHELOR OF ENGINEERING

(GEOSPATIAL ENGINEERING)

EXAMINATIONS FOR THE DEGREE OF B. APPLIED SCIENCE

(GEOICT OPTION)

EEGQ 2203 / EEGR 2202: NUMERICAL METHODS

DATE: JUNE 2022 TIME: 2 HOURS

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination.

- Answer booklet
- Non – Programmable Scientific calculator

Answer question **ONE** and any other **TWO** questions

Maximum marks for each part of a question are as shown

Mobile phones are **prohibited** in examination rooms

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

This paper consists of 4 printed pages

c. By Euler's method;

i. Obtain the difference equation for $\frac{dy}{dx} = -y + 1,\ 0 \le t \le 1,\ \ y(0) = 0$ with $h = 0.2$. **(4marks)**

ii. Use the difference equation in i. above to obtain w_1, w_2 and w_3. **(3marks)**

Question five (20marks)

a. Find the real root of equation $x \log_{10} x = 1.2$ by Bisection method. Apply three approximations only. **(6marks)**

b. Briefly discuss three types of errors in numerical computation. **(3marks)**

c. Use newton Gregory forward difference interpolating polynomial to find $f(0.2)$ given the data in the table below. **(5marks)**

x	-1.0	1.0	3.0	5.0
$f(x)$	4.0	-8.0	4	88

d. The Maclaurin's expansion for e^x is given by;

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots\ldots + \frac{x^{n-1}}{(n-1)!} + \frac{x^n}{n!} e^{\varepsilon},\ 0 < \varepsilon < x$$

Find the number of terms, such that their sum yields the value of e^x correct to 8 decimal places at $x = 1$. **(6marks)**

END

c. Use the Gauss-Legendre two-point formula to evaluate $\int_0^1 \frac{e^x}{1+x^2}dx$ **(4marks)**

d. Determine the error when computing the value of the function $f(x,y,z) = xz - \frac{y}{xz}$ using the following values $x = 2.3,\ y = 1.52,\ z = 5.247$ where each value is assumed to be rounded to the nearest significant digit. **(4marks)**

Question Three (20 marks)

a. Given the data in the table below, use the Newton- Gregory backward difference interpolation polynomial to find $f(5)$ **(7marks)**

x	-4	-2	0	2	4	6
$f(x)$	-207	-35	1	-3	49	253

b. By taking $X = \frac{x_1}{x_2}$ obtain a general formula for error in Division of numbers. **(5marks)**

c. Use Newton- Raphson method to find the root of the equation $e^{2x} - 5x - 7 = 0$ which is close to $x = 1.5$ correct to four decimal places. Apply three approximations. **(5marks)**

d. By taking e_i *and* e_{i+1} as the errors in i^{th} and $(i+1)^{th}$ iterations respectively, show that Bisection method is of first order convergent. **(3marks)**

Question four (20marks)

a. The function $f(x) = Sin\ x$ is defined on the interval $[1,3]$;

i. Obtain the Lagrange linear interpolating polynomial in this interval and find the bound on the truncation error. **(5marks)**

ii. Obtain the approximate values of $f(1.5)$ & $f(2.5)$ **(2marks)**

b. Find the approximate value of $I = \int_0^1 \frac{dx}{1+x}$ and obtain a bound for the errors using;

i. Trapezoidal rule **(3marks)**

ii. Simpson's rule **(3marks)**