Assignment I: Signals and Spectra with MATLAB

MATLAB is a very convenient programming language for signal processing. In this practical you will learn how to use MATLAB to generate and represent signals in time and frequency domain, and how to plot them. For more information on MATLAB commands or functions, you may use the MATLAB documentation or the MATLAB commands ***help*** and ***doc*** - or one of the many MATLAB tutorials on the internet.

Do the following exercises with MATLAB. Exercises 1–6 are preparing you for Exercises 7 and 8.

Problem 1 (*6 points*)

Complex numbers

(a) For the complex number $x = 2 + j$, compute the real part, the imaginary part, the absolute value and the angle;*(4 points)*

(b) For the complex numbers $\{1, j, -1, -j\}$, compute the real part, the imaginary part, the absolute value and the angle, using vector operations. *(2 points)*

Problem 2 (*9 points*)

Function plots

(a) Plot the function $y = x^3$ for $x \in [0, +2]$; *(3 points)*

(b) Plot the function $y = x^{\frac{1}{2}}$ for $x \in [0, +2]$ in red in the same figure; *(3 points)*

(c) To above plot, add labels for the axes and grid lines. Add further title and a legend. *(3 points)*

Problem 3 (*9 points*)

Complex exponential function

(a) Plot the function $y = \sin(\pi t)$ for $t \in [0, 3]$; *(2 points)*

(b) Consider the function $x = e^{j\pi t}$ for $t \in [0, 3]$. For this function, plot the real part, the imaginary part, the absolute value and the angle in four different subplots in the same figure; *(4 points)*

(c) Make a 3D plot of the function $x = e^{j\pi t}$ for $t \in [0, 3]$. Add labels for the three axes. (And rotate the figure with the mouse.) *(3 points)*

Problem 4 (*12 points*)

Plots and sampling period
Consider the signal $x(t) = \sin(2\pi f_0 t)$ for $t \in [0, 2T_0]$, $T_0 = 1/f_0$ and $f_0 = 10^3$ Hz.

(a) Use the sampling period of $T_s = T_0/10$ (*i.e.*, 10 sample points per T_0 seconds) and plot the signal; *(3 points)*

(b) Use the sampling period $T_s = T_0/20$ and plot the signal in the same figure (in a different colour); *(3 points)*

(c) Use the sampling period $T_s = T_0/4$ and plot the signal in the same figure (in a different colour); *(3 points)*

(d) Interpret the results. *(3 points)*

Problem 5 (*9 points*)

The rectangular pulse
Consider the rectangular pulse

$$\texttt{rect}(t) = \begin{cases} 1, & \text{for } |t| \leqslant \frac{1}{2}, \\ 0, & \text{otherwise;} \end{cases}$$

(a) Write a MATLAB function for $\texttt{rect}(t)$; *(3 points)*

(b) Plot $\texttt{rect}(t)$ for $t \in [-2, 2]$; *(3 points)*

(c) Define the x-axis to go from -3 to $+3$, and the y-axis to go from -2 to $+2$. Label the axes. *(3 points)*

Problem 6 (*9 points*)

The sinc-function
Consider the function

$$\text{sinc}(t) = \frac{\sin \pi t}{\pi t},$$

and use the corresponding MATLAB function in the following.

(a) Plot the signal $s(t) = \text{sinc}(f_0 t)$ for $t \in [-10T_0, 10T_0]$, $f_0 = 1/T_0$, $T_0 = 3$ s. Use the sampling period $T_s = T_0/10$; *(4 points)*

(b) Turn on the grid lines in the figure; *(2 points)*

(c) Interpret the zeros of $s(t)$. *(3 points)*

Problem 7 (*15 points*)

Manipulations of signals

(a) Plot the signal $x_1(t) = 2 \cdot \sin(2.5\pi f_1 t)$ with $f_1 = 10$ Hz; *(3 points)*

(b) Plot the signal $x_2(t) = 2 + x_1(t)$; *(3 points)*

(c) Plot the signal $x_3(t) = \sin(2\pi f_3 t)$ with $f_3 = 10 f_1$; *(3 points)*

(d) Plot the signal $x_4(t) = x_1(t) \cdot x_3(t)$; *(3 points)*

(e) Comment your results especially for $x_4(t)$. *(3 points)*

Problem 8 (*31 points*)

Spectra of signals

Use the MATLAB function **`ft.m`** (provided on the LM) to compute the Fourier transform. (This function is only a user-friendly interface to **`fft.m`**.)
Write an m-file to perform steps (a) to (c). This will make it easier for you to do (e), (f), and (g).

(a) Define a sinusoidal signal $x(t)$; *(3 points)*

(b) Compute the Fourier transform $X(f)$ of this signal; *(2 points)*

(c) Use a figure with two subplots, and plot the signal in one subplot and its magnitude spectrum in the other subplot; *(3 points)*

(d) Interpret your results; *(2 points)*

(e) Change the length of the signal $x(t)$ and repeat (b), (c), (d); *(7 points)*

(f) Change the sampling period of the signal $x(t)$ and repeat (b), (c), (d); *(7 points)*

(g) Define a rectangular signal $y(t)$ and repeat (b), (c), (d). *(7 points)*