

INFO 7390

Advances in Data Sciences and Architecture

Assignment 1

Professor: Nik Bear Brown

Due: May 22, 2019

Q1 (5 Points)

A certain disease has an incidence rate of 5%. If the false negative rate is 1% and the false positive rate is 3%, compute the probability that a person who tests positive actually has the disease.

Q2 (5 Points)

In a class with $\frac{2}{5}$ women and $\frac{3}{5}$ men, 25% of the women are business majors. Find the probability that a student chosen from the class at random is a female business major.

Q3 (5 Points)

A box contains 5 red balls and 9 green balls. Two balls are drawn in succession **without** replacement. That is, the first ball is selected and its color is noted but it is not replaced, then a second ball is selected. What is the probability that:

- a. the first ball is green and the second ball is green?
- b. the first ball is green and the second ball is red?
- c. the first ball is red and the second ball is green?
- d. the first ball is red and the second ball is red?

Q4 (5 Points)

Stephen Curry hit 77 three-point shots in a row in practice. If his probability of hitting an unguarded three-point shot is 90%, what is the likelihood of Stephen Curry making at least 9 out of 10 three-point shots?

Q5 (5 Points)

A booth at the fair has 200 balloons, 5 of which contain \$10 and 1 of which contains \$20. The rest contain only air. If it costs \$1 to randomly break a balloon, what is the expected return of an individual making such an attempt?

Q6 (5 Points)

How many people must there be before the probability that at least two people have a birthday on October 31 is greater than $1/2$?

Q7 (5 Points)

What is the probability of getting exactly 2 heads after flipping three coins?

Q8 (5 Points)

Consider a six-sided die that gets a 1 with probability $p = 1/6$. How confident are you that you can get a 1 after rolling the die 3 times?

Q9 (5 Points)

Look up the Boolean satisfiability problem (SAT)

https://en.wikipedia.org/wiki/Boolean_satisfiability_problem

Consider a randomized version of SAT called Max-SAT which tries to satisfy as many clauses as possible with a random polynomial-time algorithm. More precisely, we define Max-SAT as follows: Given a set of k clauses $C = \{C_1, C_2 \dots C_k\}$ and n literals $X = \{X_1, X_2 \dots X_n\}$ find a truth assignment satisfying as many clauses as possible. Each clause must have at least one literal in it, and all of the literals within a single clause are distinct.

- A. What is the expected number of satisfied clauses if each clause has just one literal and we randomly assign the truth value by flipping a fair coin?
- B. If each clause has just one literal can we always find a solution that will satisfy all k clauses?

Q10 (5 Points)

Compute the probability of randomly drawing 4 cards from a deck and getting exactly three Queens.

Q11 (5 Points)

Compute the probability of drawing the King of hearts or a Queen from a deck of cards.

Q13 (5 Points)

Compute the likelihood of drawing a Queen from a deck of cards.

Q14 (5 Points)

A group of people decide to do a Xmas gift exchange. In it a bag contains each person's name on a token. Each person selects a token at random. That is the name of the person to give a gift to secretly.

This only works if you get someone else's name, as you would not want to give a gift to yourself.

What is the probability the draw is successful? That is, what is the chance no one selects his or her own name?

Q15 (5 Points)

Suppose $P(A) = 0.30$ and $P(B) = 0.35$. What is $P(A \& B)$?

Q16 (5 Points)

$P(A) = 0.11$, $P(B) = 0.33$, $P(A \cup B) = 0.55$

What is $P(A|B)$?

Q17 (5 Points)

Assume $P(A)$ and $P(B)$ are independent.

Which of the following are true?

- a) $P(A \text{ and } B) = P(A)P(B)$
- b) $P(A|B) = P(A)$
- c) $P(B|A) = P(B)$

Q18 (5 Points)

Suppose we draw two cards from a standard deck. What is the probability that we get a Queen or a King for both?

Q19 (5 Points)

There are 13 audiobooks and 11 flash card sets on a reading list for a college Chinese course. How many different ways can a student select one audiobook and one flash card set to study during the quarter?

Q20 (5 Points)

How many ways can a four-person executive committee (president, vice-president, secretary, treasurer) be selected from a 8-member board of directors of a college club?