Definitions

Decision Problem

- A decision problem is one where, given an input, the output is either true or false (https://en.wikipedia.org/wiki/Decision_problem).

- A function *f(x)* which, for some input *x*, yields a Boolean result is called a *predicate*.

Search Problem

- A search problem applies to a space $\boldsymbol{S}$ of potential (candidate) solutions where for each s_i there is a corresponding decision problem $f(s_i)$.
  - For a more formal definition, see for example Wikipedia.
  - Most (all?) of the algorithms which we will study are designed to solve search problems.
  - Example: you have an array of N elements x_i and you wish to sort the array. This is a “search problem” where each candidate solution is one of the $N!$ possible permutations P_j. The corresponding decision problem (predicate) is $f(P)$ where f is true if for each element x_i in P where i runs from 1 to N-1:

$$x_{i-1} <= x_i.$$

Entropy

- Entropy is the degree of disorder. This is true for both:
  - Thermodynamic (Boltzmann) entropy; and
  - Information (Shannon) entropy.
- I will also use Entropy to describe the number of candidate solutions remaining in a search problem. As with information entropy, we measure this by taking the logarithm of the number of solutions.
  - For example, when sorting an array of N elements x_i, if we start with a random array, there will be $\log(N!)$ entropy.
  - When we are finished with sorting, there will be one (and only one) solution and so there will be zero entropy.

Logarithms

- Notation for logarithms is not consistent throughout the world. I know this will cause confusion.
- The following is the notation I will be using:
  - $ln(x)$ is the <u>natural</u> logarithm of x, *i.e.* $log_e(x)$.
  - $lg(x)$ is the <u>binary</u> logarithm of x, *i.e.* $log_2(x)$.
  - $log(x)$ is simply used when we don't really care to distinguish—remember that the only difference between logarithms with different bases is a constant factor. We typically ignore constant factors when we study complexity.
  - We will never ever use logarithms to the base 10.