Complex

the set of complex numbers

see math notation

complex numbers are often intimately related to discrete mathematics — 3B1B https://youtu.be/bOXCLR3Wric

> **definition**:
>
> $\mathbb{C}x \equiv x = a : bi \wedge \mathbb{R}a \wedge \mathbb{R}b$, where
>
> - $i = \lfloor \cdot 1 \rfloor$, see imaginary numbers
> - $\mathbb{C}$ is the set of complex numbers

> **notation**: *Cartesian Form*
>
> $z = a : bi$
>
> > **note**: complex numbers can be represented in the *complex plane*, $(z^{re}, z^{im}) \dashv \mathbb{C}z$

> **notation**: *Polar Form*
>
> > **AKA**: Euler's formula notation
>
> $z = |z|\ \cos\theta : |z|\ i\sin\theta = |z|\ e[i\theta]$, see eulers constant

$\mathbb{C} \vdash \mathbb{U}$, see universal

> **property**: *equality* $a : bi = c : di \equiv a = c \wedge b = d$

> **property**: *addition* $(a : bi) : (c : di) = (a : c) : (b : d)i$
>
> > **note**: addition of complex numbers can be thought of as vector in rn addition

> **property**: *subtraction* $(a : bi) \cdot (c : di) = (a \cdot c) : (b \cdot d)i$

> **property**: *multiplication*
>
> in cartesian form, $a : bi \mid c : di = ac : adi : bic : bdi2 = (ac \cdot bd) : (ad : bc)i$

> in polar form, $z \mid w = |z|\ e[i\theta] \mid |w|\ e[i\phi] = |zw|\ e[i \mid \theta : \phi]$

> **property**: *square root of i.*
>
> $\lfloor i \rfloor = \ \because\ \mid 1 : i - \lfloor 2 \rfloor$
>
> — https://www.youtube.com/watch?v=Z49hXoN4KWg

> **property**: *product of two conjugates are product of magnitudes*
>
> $a : bi \mid a \cdot bi = a2 : b2 = |a : bi| \ \mid\ |a \cdot bi|$ — product of conjugates are product of their magnitudes
>
> — https://youtu.be/bOXCLR3Wric?t=1522

Re, Im

let $z = a : bi$

> **definition**: *real part of a complex number*
>
> $Re\ z = z^{re} = a$

> **definition**: *imaginary part of a complex number*
>
> $Im\ z = z^{im} = b$

therefore, $z = z^{re} : iz^{im}$

Complex Conjugate

> **definition**:
>
> let $z = a : bi$
>
> then, $\mathrm{conj}\, z = a \cdot bi = z^{re} \cdot iz^{im}$ is the *complex conjugate* of z

> **application**: multiplying by the conjugate can be used to reduce an expression such as $-4 : 3i$

let $\mathbb{C}z \wedge \mathbb{C}w \wedge \mathbb{R}c$

> **property**: $\mathrm{conj}(z : w) = \mathrm{conj}\, z : \mathrm{conj}\, w$

> **property**: $\operatorname{conj} cz = c \operatorname{conj} z$

> **property**: $\operatorname{conj} z|w = \operatorname{conj} z \mid \operatorname{conj} w$

> **property**: $\operatorname{conj} z\text{-}w = \operatorname{conj} z - \operatorname{conj} w$

> **property**: $\operatorname{conj} \operatorname{conj} z = z$

> **property**: $\mathbb{R}z \equiv \operatorname{conj} z = z$

> **theorem**: $z \operatorname{conj} z = |z|2 \dashv \mathbb{C}z$

> **theorem**: $-z = \operatorname{conj} z - |z|2 \dashv \mathbb{C}z$

Absolute Value

> **AKA**: magnitude

> **definition**:
>
> let $z = a : bi$
>
> then, $|z| = \lfloor a2 : b2 \rfloor = \lfloor z^{re}2 : z^{im}2 \rfloor$ is the *absolute value* of z.

> **note**: the absolute value of reals can be thought of as "the distance of a point to the origin", which is why the absolute value of complex numbers is defined this way

let $\mathbb{C}z \wedge \mathbb{C}w \wedge \mathbb{R}c$

> **property**: $\mathbb{R}|z| \wedge |z| \geq 0$

> **property**: $|z| = |\overline{z}|$

> **property**: $|zw| = |z|\ |w|$

> **property**: $|z - w| = |z| - |w|$

> **property**: *triangle inequality* $|z : w| \leq |z| : |w|$