Complex

the set of complex numbers

see math notation

definition

$\mathbb{C}x \equiv x = a : b\iota \wedge \mathbb{R}a \wedge \mathbb{R}b$

notations

Cartesian Form $z = a : b\iota$

Polar Form $z = |z|\ \cos\theta : |z|\ \iota\sin\theta = |z|\,\mathrm{cis}\,\theta = |z|\ e[\iota\theta]$, see eulers constant, where $\mathrm{cis} = \cos : \iota\sin = \theta \to e[\iota\theta]$

> **AKA** Euler's formula notation

applications

complex numbers are often intimately related to discrete mathematics — 3B1B https://youtu.be/bOXCLR3Wric

properties

$\mathbb{C} \vdash \mathbb{U}$, see universal

equality $a : b\iota = c : d\iota \equiv a = c \wedge b = d$

addition $(a : b\iota) : (c : d\iota) = (a : c) : (b : d)\iota$

subtraction $(a : b\iota) \cdot (c : d\iota) = (a \cdot c) : (b \cdot d)\iota$

multiplication

in cartesian form, $a : b\iota \mid c : d\iota = ac : ad\iota : b\iota c : bd\iota 2 = (ac \cdot bd) : (ad : bc)\iota$

in polar form, $z \mid w = |z|\ e[\iota\theta] \mid |w|\ e[\iota\phi] = |zw|\ e[\iota \mid \theta : \phi]$

square root of ι. $\lfloor\iota\rfloor = \ \because\ \mid 1 : \iota - \lfloor 2\rfloor$ — https://www.youtube.com/watch?v=Z49hXoN4KWg

product of two conjugates are product of magnitudes $a : b\iota \mid a \cdot b\iota = a2 : b2 = |a : b\iota|\ \mid\ |a \cdot b\iota|$ — https://youtu.be/bOXCLR3Wric?t=1522

theorem *De Moivre's Theorem* $[\mathrm{cis}\,\theta]n = \mathrm{cis}\,n\theta \dashv \mathbb{Z}n \dashv \mathbb{R}\theta$ — https://en.wikipedia.org/wiki/De_Moivre%27s_formula

> **proof** $\operatorname{cis}\theta = e[\iota\theta]$. since $[e[\iota\theta]]n = e[n\iota\theta]$, it must be that $[\operatorname{cis}\theta]n = \operatorname{cis} n\theta$ — me

Re and Im Parts

let $z = a : b\iota$

definitions

real part of a complex number $z^{re} = a$

imaginary part of a complex number $z^{im} = b$

therefore, $z = z^{re} : \iota z^{im}$

Complex Conjugate

definition

let $z = a : b\iota$

then, $\operatorname{conj} z = a \cdot b\iota = z^{re} \cdot \iota z^{im}$ is the *complex conjugate* of z

properties

let $\mathbb{C}z \wedge \mathbb{C}w \wedge \mathbb{R}c$

$\operatorname{conj}(z : w) = \operatorname{conj} z : \operatorname{conj} w$

$\operatorname{conj} cz = c \operatorname{conj} z$

$\operatorname{conj} z|w = \operatorname{conj} z \mid \operatorname{conj} w$

$\operatorname{conj} z\text{-}w = \operatorname{conj} z - \operatorname{conj} w$

$\operatorname{conj}\operatorname{conj} z = z$

$\mathbb{R}z \equiv \operatorname{conj} z = z$

theorem $z \operatorname{conj} z = |z|2 \dashv \mathbb{C}z$

theorem $-z = \operatorname{conj} z - |z|2 \dashv \mathbb{C}z$

applications

multiplying by the conjugate can be used to reduce an expression such as $-4 : 3\iota$

Modulus

> **AKA** magnitude, absolute value

definition $|z| = \lfloor z^{re}2 : z^{im}2 \rfloor$ where $|z|$ is the *absolute value* of z.

> **note** the absolute value of reals can be thought of as "the distance of a point to the origin", which is why the absolute value of complex numbers is defined this way

properties

let $\mathbb{C}z \wedge \mathbb{C}w \wedge \mathbb{R}c$

let $\mathbb{R}|z| \wedge |z| \geq 0$

$|z| = |\operatorname{conj} z|$

$|zw| = |z| \ |w|$

$|z - w| = |z| - |w|$

triangle inequality $|z : w| \leq |z| : |w|$

Argument

> **AKA** phase

definition the *argument* of a complex number z is the counterclockwise angle between the positive real axis and the line in rn segment from the origin to the point (z^{re}, z^{im})

definition $z = |z| \ e[\iota \arg z]$ where $\arg z$ is the *argument* of z