Vector in Rn

Vectors in $\mathbb{R}^n$

see [[math-notation]], [[classical-math-notation]], [[vector]] properties

notation

$(1,2)$

$\begin{bmatrix} 1 & 2 \end{bmatrix}$

$[1\,2\,3]$

operations

$(a,b) = (c,d) \equiv a = c \land b = d$ (equality)

$(a,b) \cdot (c,d) \equiv (a \cdot c, b \cdot d)$ (addition)

$0 \equiv (0,0)$ (the zero vector)

$0^m = 0 \land m = 0 \ldots n$, where n is the dimension of the zero vector

$\circ(a,b) \equiv (\circ a, \circ b)$ (negation)

$c(a,b) \equiv (ca, cb) \dashv \mathbb{R}c$ (multiplication by a scalar)

Magnitude

$|V|$, where

V is the vector to find the magnitude of

$|V| = \sqrt{V \cdot V}$ (derived from the Pythagoras theorem)

Unit Vector

where $|V| = 1$

Orientation

$\frac{V}{|V|} = (cos(\theta), sin(\theta))$, where

V is the vector to find the direction of

θ is the angle of the vector

note that $\frac{\vec{V}}{|\vec{V}|}$ is just notation for the direction of the vector V

Angles Between two Vectors

no idea why this actually works

see [[dot-product]], [[cross-product]]

$\cos\theta = \frac{a \cdot b}{|a||b|}$ (use $\cos\theta = \frac{|a \cdot b|}{|a||b|}$ to always get the acute angle solution)

$\sin\theta = \frac{a \times b}{|a||b|}$

Orthogonal Vectors

notation: $u \perp v$

a pair of vectors offset by $90°$

u and v are orthogonal if and only if $u \cdot v = 0$ (see [[dot-product]]), or $u \perp v \equiv u \cdot v = 0$

a set of [[vector]]s is orthogonal if and only if it does not contain the zero [[vector]] and all [[vector]]s in the set are orthogonal to all other [[vector]]s

> **theorem**: an orthogonal set of [[vector]]s is [[linearly-independent]] (think of this visually)

Theorems

any orthogonal set of [[vector]]s in $\mathbb{R}^n$ contains at most n [[vector]]s

any orthogonal set of n [[vector]]s in $\mathbb{R}^n$ is an orthogonal [[basis]] of $\mathbb{R}^n$

orthogonal set $\vdash$ [[linearly-independent]], but not the inverse

> **theorem**: suppose $w_0 \ldots w_m$ is an orthogonal [[basis]] for a subspace W of $\mathbb{R}^n$. then, $w = w_0 \frac{w \cdot w_0}{w_0 \cdot w_0} + \ldots w_m \frac{w \cdot w_m}{w_m \cdot w_m}$ (see [[dot-product]])

Collinear Vectors

a pair of parallel vectors

u and v are colinear if $u = kv$. u is a [[linear-combination]] of the set v

Projections

The scalar projection is equal to the length of the vector projection — Wikipedia

see [[dot-product]]

$|proj_b a| = |a| \cos\theta$, and

$proj_b a = (|a| \cos\theta)\hat{b} = (a \cdot \hat{b})\hat{b} = \frac{a \cdot b}{b \cdot b} b$, where

$proj_b a$ is the *vector projection of a on b*

$|proj_b a|$ is the *scalar projection of a on b*

$\hat{b}$ is the unit vector in the direction of b, $\frac{b}{|b|}$

> **theorem**: suppose $w_0 \ldots w_m$ is an orthogonal [[basis]] for a subspace W of $\mathbb{R}^n$. then for any $\mathbb{R}^n v$, $proj_W v = w_0 \frac{v \cdot w_0}{w_0 \cdot w_0} + \ldots w_m \frac{v \cdot w_m}{w_m \cdot w_m}$ (see [[dot-product]])

properties

see [[math-notation]]

$proj_b a$ is parallel to b

$a \circ proj_b a$ is orthogonal to b

$W(proj_W v)$

$v \circ proj_W v$ is orthogonal to every [[vector]] in W

the [[vector]] $proj_W v$ is the only [[vector]] in $\mathbb{R}^n$ that satisfies the two properties above

$proj_W v$ is the "best approximation" to v by [[vector]]s in W

volume of the [[parallelepiped]] defined by 3 vectors in $\mathbb{R}^3$

does this seem random? well, it is.

see [[dot-product]], [[cross-product]]

$V = |w \cdot (u \times v)|$, where

V is the volume calculated

u, v and w are the three vectors in $\mathbb{R}^3$

todos

replace `\R \N \Z` with `\mathbb{R} \mathbb{N} \mathbb{Z}`

replace `\degree` with `^\circ`

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