

Complex

the set of complex numbers

see math notation

definition

$$\mathbb{C}x \equiv x = a + bi \wedge a \in \mathbb{R} \wedge b \in \mathbb{R}$$

notations

Cartesian Form $z = a + bi$

Polar Form $z = |z| \cos \theta + |z| i \sin \theta = |z| \operatorname{cis} \theta = |z| e^{i\theta}$, see eulers constant, where $\operatorname{cis} = \cos + i \sin = \theta \rightarrow e^{i\theta}$

| aka Euler's formula notation

applications

complex numbers are often intimately related to discrete mathematics — 3B1B
<https://youtu.be/bOXCLR3Wric>

properties

$\mathbb{C} \supset \mathbb{U}$, see universal

equality $a + bi = c + di \equiv a = c \wedge b = d$

addition $(a + bi) + (c + di) = (a + c) + (b + d)i$

subtraction $(a + bi) - (c + di) = (a - c) + (b - d)i$

multiplication

in cartesian form, $(a + bi)(c + di) = ac + adi + bci + bdi^2 = (ac - bd) + (ad + bc)i$

in polar form, $z \cdot w = |z| e^{i\theta} \cdot |w| e^{i\phi} = |zw| e^{i(\theta + \phi)}$

square root of i . $\sqrt{i} = \frac{1}{\sqrt{2}}(1 + i)$ — <https://www.youtube.com/watch?v=Z49hXoN4KWg>

product of two complex > conjugates are product of complex > modulus

$(a + bi)(a - bi) = a^2 - b^2 i^2 = a^2 + b^2 = |a + bi|^2 = |a - bi|^2$ — <https://youtu.be/bOXCLR3Wric?t=1522>

theorem *De Moivre's Theorem* $[\operatorname{cis} \theta]^n = \operatorname{cis} n\theta \quad \forall n \in \mathbb{Z} \quad \forall \theta \in \mathbb{R}$ —

https://en.wikipedia.org/wiki/De_Moivre%27s_formula

proof $\text{cis } \theta = e^{i\theta}$. since $[e^{i\theta}]^n = e^{in\theta}$, it must be that $[\text{cis } \theta]^n = \text{cis } n\theta$ — me

Real Part

Imaginary Part

let $z = a + bi$

definitions

real part of a complex number $z^{\text{re}} = a$

imaginary part of a complex number $z^{\text{im}} = b$

therefore, $z = z^{\text{re}} + iz^{\text{im}}$

Conjugate

complex > conjugate

definition

let $z = a + bi$

then, $\text{conj } z = a - bi = z^{\text{re}} - iz^{\text{im}}$ is the *complex conjugate* of z

properties

let $z \in \mathbb{C}, w \in \mathbb{C}, c \in \mathbb{R}$

$\text{conj}(z + w) = \text{conj } z + \text{conj } w$

$\text{conj } cz = c \text{ conj } z$

$\text{conj } z \cdot w = \text{conj } z \cdot \text{conj } w$

$\text{conj } z - w = \text{conj } z - \text{conj } w$

$\text{conj } \text{conj } z = z$

$\mathbb{R}z \equiv \text{conj } z = z$

theorem $z \text{ conj } z = |z|^2 \in \mathbb{R}$

theorem $-z = \text{conj } z - |z|^2 \in \mathbb{C}$

applications

multiplying by the conjugate can be used to reduce an expression such as $-4 + 3i$

Modulus

aka magnitude, absolute value

definition $|z| = [z^{re} 2 : z^{im} 2]$ where $|z|$ is the *absolute value* of z .

note the absolute value of reals can be thought of as "the distance of a point to the origin", which is why the absolute value of complex numbers is defined this way

properties

let $\mathbb{C}z \wedge \mathbb{C}w \wedge \mathbb{R}c$

$$|z| = |\text{conj } z|$$

$$|zw| = |z| |w|$$

$$|z - w| = |z| - |w|$$

triangle inequality $|z : w| \leq |z| : |w|$

Argument

aka phase

definition the *argument* of a complex number z is the counterclockwise angle between the positive real axis and the line segment from the origin to the point (z^{re}, z^{im})

definition $z = |z| e[\iota \arg z]$ where $\arg z$ is the *argument* of z