# Math Notation

#### see conventional math notation

this note describes my custom <u>math notation</u>, meant to solve inconsistencies in <u>conventional</u> <u>math notation</u>. it is not meant to be a fully formal system of <u>mathematics</u>; rather, it is built to be easy to understand and intuitive to use by mere humans.

# principles

- all equality <u>operators</u> check for equality and return a <u>boolean</u>, and it is implied that an <u>expression</u> on its own must evaluate to ⊤. this allows for <u>boolean logic operators</u> to be applied on equalities explicitly as opposed informally
- <u>sets</u> are <u>functions</u> that return a <u>boolean</u> (<u>sets</u> are <u>predicates</u>). this way, <u>boolean logic</u> <u>operators</u> and <u>set operators</u> are one and the same. other <u>data structures</u> that work similarly include <u>vectors</u>, <u>matrixes</u>, <u>sequences</u>, <u>multisets</u>, <u>ordered pairs</u>...
- some <u>operators</u> are identical but have different precedence as "more brackets means more explicit, but less brackets means less complex and less confusing"
- $\lfloor a \rfloor$  returns both positive and negative square roots  $(\lfloor q2 \rfloor \equiv \because q)$ . the same is true for other reciprocals
- superscripts are modifiers (subscripts with special meanings). this distinction is especially useful when working with <u>forward propagation</u> and <u>backpropagation</u> in <u>neural networks</u>, for example
- <u>derivatives</u> are not to be written as y', but rather as their complete form  $\delta y \delta x$ . this makes <u>calculus notation</u> way more intuitive
- all indices start at 0, as they always should have

### notation

also see trigonometric functions and calculus notation

#### operator descriptions

notation desc	cription	notes
a:b addi	tion or disjoint union	
$a \cdot b$ subt	subtraction	

notation	description	notes
∵ and ∴	$\pm$ and $\mp$	
$a \mid b \text{ and } a \mid b$	multiplication	
a- $b$ and $a$ – $b$	division	
[a]b	exponentiation	represents a power by convention
a[b]	exponentiation	represents an exponential by convention
$\lfloor a \rfloor b$	b th root of $a$	b=2 if $b$ is omitted
$\lceil a  ceil b$	base- $b \underline{\text{logarithm}}$ of $a$	b = e if $b$ is omitted
$x \to E$ where $E$ is an expression	function literal	$f=x ightarrow E\equiv f{\leftarrow}x=E$
$f \leftarrow E$ where $E$ is an $expression$	function application	uncommon, shorthand preferred
a=b	equality	numerical equality by convention
a < b  and  a > b	strict inequality	
$a \leq b$ and $a \geq b$	non-strict inequality	
$a \wedge b$	logical AND or min function	
a ee b	logical OR or max function	
$a \ / \ b$	logical difference	$a \wedge b = \bot$
$a\equiv b$	equality	logical equality by convention
a  imes b	nonequality	also serves as logical XOR
$a \vdash b$	implication, subset	a implies $b$ , $b$ for all $a$
$a\dashv b$	reverse implication, superset	a for all $b$ , $b$ implies $a$
$x_0 \mid x_1 \mid \dots x_n$ where $\mid$ is any $operator$	with $n=3,\ x_0 \   \ x_1 \   \ x_2 \   \ x_3$	step size is $\because 1$ if $x_1$   is omitted
$x_0 \dots x_n$	with $n=3,x_0,x_1,x_2,x_3$	step size is $:: 1$ if $x_1$ is omitted
$x \lor \dots$ where $\lor$ is any operator	$\vee$ -reducetion of $x$	
$x_{sub}$	the <u>variable</u> $x$ with a subscript $_{sub}$	
$V^n$ where $V$ is a <u>vector</u>	the $n$ th component of $V$	
$a^i$ where $a$ is a <u>sequence</u>	the $i$ th element of $a$	

notation	${\it description}$	notes
$b^i$ where $b$ is a <u>series</u>	the $i$ th element of $b$	
$M^{\langle i,j \rangle}$ where $M$ is a $\underline{\mathrm{matrix}}$	the $i, j$ th element of $M$	uncommon, shorthand preferred
$M^{\intercal}$ where $M$ is a $\underline{\text{matrix}}$	the transpose of $A$	
$M^-$ where $M$ is a $\underline{\text{matrix}}$	the multiplicative inverse of $A$	
$P^b$ where $P$ is an <u>ordered</u> <u>pair</u>	the $b$ th element of $P$	
S a where $S$ is a <u>set</u>	whether $a$ is element of $S$	
M a where $M$ is a <u>multiset</u>	the number of elements $a$ in $M$	
G v where $G$ is a graph	whether vertex $v$ is in $G$	
$G^{\langle v,w angle}$ where $G$ is a graph	the number of edges from $v$ to $w$ in $G$	uncommon, shorthand preferred

# shorthands

shorthand	definition	notes
$a \nvdash b, \ a \neq b, \ a \nleq b, \ a \nless b$	$/(a dash b), \ /a = b, \ /a \leq b, \ /a < b$	
$x\omega$ where $x$ is a <u>variable</u> and $\omega$ is a <u>number</u>	$[x]\omega$	
ax where $x$ is a <u>variable</u>	aı $x$	
f x where $f$ is a function	$f \leftarrow x$	common, longhand discouraged
$x \ y \to E$ where $E$ is an $expression$	x o y o E	
$V^x$ , $V^y$ and $V^z$ where $V$ is a <u>vector</u>	the $x$ , $y$ and $z$ components of $V$	
$M^{i,j}$ where $M$ is a matrix	$M^{\langle i,j angle}$	common, longhand discouraged
$M^{i,}$ where $M$ is a $\underline{\text{matrix}}$	the $i$ th row of $M$	
$M^{,j}$ where $M$ is a $\underline{\text{matrix}}$	the $j$ th column of $M$	
$S = \langle \langle a \dots b  angle  angle$	$Sx\equiv x=a\vee\ldots x=b$	see <u>set</u>
$P=\langle f,t angle$	$P^\perp = f \wedge P^\top = t$	see <u>ordered pair</u>

shorthand	definition	notes
$M = egin{bmatrix} a & b \ c & d \end{bmatrix}$	matrix literal	see <u>matrix</u>
x  o (a < x < b)	the closed interval from $a$ to $b$	same can be used for open intervals
$A \vdash B$ where $\vdash$ is any $\underline{\#think}$ operator	$A x \vdash B x \text{ for all } x$	commonly $\equiv \dashv \vdash \# \underline{\text{think}}$
$A \cdot B$ where $\cdot$ is any $\frac{\text{\#think}}{\text{operator}}$	$x  o A \ x \cdot B \ x$	commonly : $\cdot \mid - \frac{\# \text{think}}{}$
$\delta y - \delta x$	the derivative of $y$ with respect to $x$	$\delta$ should be used instead of $d$
$\int y \mid \delta x$	the antiderivative of $y$ with respect to $x$	$\delta$ should be used instead of $d$

# constants

constant	definition	notes
Ø	undefined	see <u>improved expression</u> <u>evaluation</u>
Т	logical true	
$\perp$	logical false	
au	the ratio of the circumference of a $\underline{\text{circle}}$ to its radius	using $\pi$ is discouraged
e	Euler's constant	see <u>eulers constant</u>
i	$\lfloor \cdot 1 \rfloor$	see <u>imaginary</u>

# operator properties

in order of high to low precedence

operator	associativity	unary identity	unary description
$()\ \left\langle\right\rangle \left[\right]\ x\ x_a^i$			
1 -	left	1	inverse
$\delta  \sin  \leftarrow $	right-ish		
: • :: ::	left	0	negation

operator	associativity	unary identity	unary description
-	left	1	inverse
$\int \lim \; \dots \;  o$	right		
=#>><<	AND	0	is (not) 0
/	left	Т	logical NOT
$\wedge$ $\vee$	left		
<b>⊣</b> ⊢	left		
$\equiv$ ×	AND	Т	logical NOT
,			

note: above,

- x represents <u>variables</u>
- ullet  $x_a^i$  represents subscripts and superscripts
- $\leftarrow$  represents <u>function</u> application
- $\rightarrow$  represents <u>function</u> literals
- | represents matrix literals

**note**: unary <u>operators</u> have identical precedence to their binary counterparts, but are right associative

**definition**: let = be an <u>operator</u> with *AND* associativity. then,  $a=b=c=\dots$   $\equiv$   $a=b \land b=c \land c=\dots$ 

# variable scope

<u>variable scope</u> is currently entirely context-dependent. this is know to cause occasional issues, such as with <u>derivatives</u>:  $\delta f x - \delta x$  could represent both the <u>derivative</u> of f with respect to x in the general sense, or the <u>derivative</u> of f with respect to x at the point x as  $(x \to \delta f x - \delta x) x \equiv \delta f x - \delta x$ .

# examples

 $\underline{\text{quadratic formula}} \colon \cdot b : \lfloor b2 \cdot 4ac \rfloor - 2a$ 

definition of the set of complex numbers:  $\mathbb{C}x \equiv x = a: b\lfloor \cdot 1 \rfloor \wedge \mathbb{R}a \wedge \mathbb{R}b$ 

definition of the implication / sub<u>set</u> / super<u>set</u> / "for all" symbol:  $a \vdash b \equiv /a \lor b$  and  $a \dashv b \equiv a \lor /b$ 

in <u>set theory</u>, if U is a sub<u>set</u> of V and V is a sub<u>set</u> of U, then V is U. in this math notation:  $(U x \vdash V x) \land (U x \dashv V x) \equiv U = V$ 

the probability density of the normal distribution in <u>conventional math notation</u>:  $\frac{1}{\sqrt{2\sigma^2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ 

compared to in my math notation:  $-|\tau\sigma 2| - e[|x\cdot\mu|^2 - 2\sigma^2]$ 

definition of factorials: fact  $n = 1 \mid \dots n$ 

the negation of an implication in my math notation:  $B \vdash C \times B / C$  (B implying C equals not (B without C) or implication is the negation of set difference or the negation of "for all B, C" is "there exists a B such that not C")

compared to <u>conventional math notation</u>:  $\neg(B \to C) = B \land \neg C$  or  $(a \in B \to a \in C) \iff a \notin B \backslash C$  or  $B \subset C \iff \forall a \in C, a \notin B$ 

the resonant frequency of an LC circuit in <u>conventional math notation</u>:  $f = \frac{1}{2\pi\sqrt{LC}}$ 

compared to in my math notation:  $f = -\tau |LC|$ 

see <u>random math notation formulas</u> for more examples