

Math Notation

see conventional math notation

this note describes my custom math notation, meant to solve inconsistencies in conventional math notation. it is not meant to be a fully formal system of mathematics; rather, it is built to be easy to understand and intuitive to use by mere humans.

principles

- all equality operators check for equality and return a boolean, and it is implied that an expression on its own must evaluate to \top . this allows for boolean logic operators to be applied on equalities explicitly as opposed informally
- sets are functions that return a boolean (sets are predicates). this way, boolean logic operators and set operators are one and the same. other data structures that work similarly include vectors, matrixes, sequences, multisets, ordered pairs...
- some operators are identical but have different precedence as "more brackets means more explicit, but less brackets means less complex and less confusing"
- $[a]$ returns both positive and negative square roots ($[q^2] \equiv \pm q$). the same is true for other reciprocals
- superscripts are modifiers (subscripts with special meanings). this distinction is especially useful when working with forward propagation and backpropagation in neural networks, for example
- derivatives are not to be written as y' , but rather as their complete form $\delta y - \delta x$. this makes calculus notation way more intuitive
- all indices start at 0, as they always should have
- rank polymorphism is supported over all operators

notation

also see trigonometric functions and calculus notation

let:

- M be a matrix
- V be a vector
- P be an ordered pair

- M' be a multiset
- G be a graph
- E be an expression
- x be a variable
- f be a function
- A be a sequence
- B be a series
- a, b be any mathematical objects
- A, B be any mathematical objects with ranks greater than 1
- n, i be natural numbers
- b be a boolean
- ω be any number
- \circ be any operator

operator descriptions

notation	description	notes
$a : b$	addition or disjoint union	
$a \cdot b$	subtraction	
\pm and \mp	\pm and \mp	
$a \backslash b$ and $a \mid b$	multiplication	
$a \div b$ and $a - b$	division	
$[a]b$	exponentiation	represents a power by convention
$a[b]$	exponentiation	represents an exponential by convention
$[a]b$	b th root of a	$b = 2$ if b is omitted
$\lceil a \rceil b$	base- b <u>logarithm</u> of a	$b = e$ if b is omitted
$x \rightarrow E$	<u>function</u> literal	$f = x \rightarrow E \equiv f \leftarrow x = E$
$f \leftarrow E$	<u>function</u> application	uncommon, shorthand preferred
$a = b$	equality	numerical equality by convention
$a < b$ and $a > b$	strict inequality	
$a \leq b$ and $a \geq b$	non-strict inequality	
$a \wedge b$	logical AND or min function	

notation	description	notes
$a \vee b$	logical OR or max function	
a / b	logical difference	$a \wedge b = \perp$
$a \equiv b$	equality	logical equality by convention
$a \times b$	nonequality	also serves as logical XOR
$a \vdash b$	implication, subset	a implies b , b for all a
$a \dashv b$	reverse implication, superset	a for all b , b implies a
$a_0 \circ a_1 \circ \dots a_n$	with $n = 3$, $a_0 \circ a_1 \circ a_2 \circ a_3$	step size is $\therefore 1$ if $a_1 \circ$ is omitted
$a_0 \dots a_n$	with $n = 3$, a_0, a_1, a_2, a_3	step size is $\therefore 1$ if a_1 is omitted
$a \circ \dots$	the <u>reduce function</u> of \circ on a	
x_{sub}	the <u>variable</u> x with a subscript $_{sub}$	
V^n	the n th component of V	
A^i	the i th element of A	
B^i	the i th element of B	
$M^{\langle i,j \rangle}$	the i, j th element of M	uncommon, shorthand preferred
M^\top	the transpose of M	
M^-	the multiplicative inverse of M	
P^b	the b th element of P	
$S a$	whether a is element of S	
$M' a$	the number of elements a in M'	
$G a$	whether vertex a is in G	
$G^{\langle a,b \rangle}$	the number of edges from a to b in G	uncommon, shorthand preferred

shorthands

shorthand	definition	notes
$a \nmid b$, $a \neq b$, $a \not\leq b$, $a \not\leq b \dots$	$/(a \vdash b)$, $/a = b$, $/a \leq b$, $/a < b \dots$	
$x\omega$	$[x]\omega$	
ax	$a \setminus x$	
$f x$	$f \leftarrow x$	common, longhand discouraged

shortcut	definition	notes
$x \rightarrow y \rightarrow E$	$x \rightarrow y \rightarrow E$	
$\langle \rangle$	$\langle \langle \rangle \rangle$	see empty set
$()$	$(())$	see multiset
V^x, V^y, V^z	V^0, V^1, V^2	
$M^{i,j}$	$M^{\langle i,j \rangle}$	common, longhand discouraged
$M^i,$	the i th row of M	
M^j	the j th column of M	
$S = \langle \langle a \dots b \rangle \rangle$	$Sx \equiv x = a \vee \dots x = b$	see set
$P = \langle f, t \rangle$	$P^\perp = f \wedge P^\top = t$	see ordered pair
$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$	matrix literal	see matrix
$M' = ((1, 2, 2, 2, 3, 3))$	multiset literal	see multiset
$x \rightarrow (a < x < b)$	the interval from a to b	
$A \circ B$	$Ax \circ Bx$ for all x	commonly $\equiv \Vdash$ #think
$A \circ B$	$x \rightarrow Ax \circ Bx$	see rank polymorphism
$\delta y - \delta x$	the derivative of y with respect to x	δ should be used instead of d
$\int y \mid \delta x$	the antiderivative of y with respect to x	δ should be used instead of d

constants

constant	definition	notes
\emptyset	<i>undefined</i>	see improved expression evaluation
\top	logical true	
\perp	logical false	
τ	the ratio of the circumference of a circle to its radius	using π is discouraged
e	Euler's constant	see eulers constant
ι	$[\cdot 1]$	see imaginary , using i is discouraged
Γ	the gamma function	using fact is discouraged

operator properties

in order of high to low precedence

operator	associativity	unary identity	unary description
$() \langle \rangle \llbracket \rrbracket x x_a^i$			
$\llbracket \rrbracket \llbracket \rrbracket \llbracket \rrbracket$			
$\mid -$	left	1	inverse
You can't use 'macro parameter character #' in math mode	right-ish		
$:$ \cdot $\dot{\cdot}$ $\ddot{\cdot}$	left	0	negation
$\mid -$	left	1	inverse
$\int \lim \dots \rightarrow \bmod$	right		
$= \neq > \geq < \leq$	AND	0	is (not) 0
$/$	left	\top	logical NOT
$\wedge \vee$	left		
$\dashv \vdash$	left		
$\equiv \times$	AND	\top	logical NOT
$,$			

- note:** above,
- x represents variables
 - x_a^i represents subscripts and superscripts
 - \leftarrow represents function application
 - \rightarrow represents function literals
 - $\llbracket \rrbracket$ represents matrix literals

note: unary operators have identical precedence to their binary counterparts, but are right associative

definition: let \circ be an operator with *AND* associativity. then,
 $a \circ b \circ c \dots \equiv a \circ b \wedge b \circ c \wedge c \circ \dots$

variable scope

variable scope is currently entirely context-dependent. this is know to cause occasional issues, such as with derivatives: $\delta f x - \delta x$ could represent both the derivative of f with respect to x in the general sense, or the derivative of f with respect to x **at the point** x as $(x \rightarrow \delta f x - \delta x) x \equiv \delta f x - \delta x$.

examples

quadratic formula: $\cdot b : [b^2 \cdot 4ac] - 2a$

definition of the set of complex numbers: $\mathbb{C}x \equiv x = a : b \iota \wedge \mathbb{R}a \wedge \mathbb{R}b$

definition of the implication / subset / superset / “for all” symbol: $a \vdash b \equiv /a \vee b$ and $a \dashv b \equiv a \vee /b$

in set theory, if U is a subset of V and V is a subset of U , then V is U . in this math notation: $(U x \vdash V x) \wedge (U x \dashv V x) \equiv U = V$

the probability density of the normal distribution in conventional math notation: $\frac{1}{\sqrt{2\sigma^2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

compared to in my math notation: $-\lceil \tau \sigma^2 \rceil - e \lceil [x \cdot \mu]^2 - 2\sigma^2 \rceil$

definition of factorials: $\text{fact } n = 1 \mid \dots n$

the negation of an implication in my math notation: $B \vdash C \times B / C$ (B implying C equals not (B without C) or *implication is the negation of set difference* or the negation of “for all B , C ” is “there exists a B such that not C ”)

compared to conventional math notation: $\neg(B \rightarrow C) = B \wedge \neg C$ or $(a \in B \rightarrow a \in C) \iff a \notin B \setminus C$ or $B \subset C \iff \forall a \in C, a \notin B$

the resonant frequency of an LC circuit in conventional math notation: $f = \frac{1}{2\pi\sqrt{LC}}$

compared to in my math notation: $f = -\tau[LC]$

see random math notation formulas for more examples