Math Notation

see conventional math notation

this note describes my custom math notation, meant to solve inconsistencies in conventional math notation. it is not meant to be a fully formal system of mathematics; rather, it is built to be easy to understand and intuitive to use by mere humans.

principles

- all equality operators check for equality and return a boolean, and it is implied that an expression on its own must evaluate to $\top$. this allows for boolean logic operators to be applied on equalities explicitly as opposed informally
- sets are functions that return a boolean (sets are predicates). this way, boolean logic operators and set operators are one and the same. other data structures that work similarly include vectors, matrixes, sequences, multisets, ordered pairs...
- some operators are identical but have different precedence as "more brackets means more explicit, but less brackets means less complex and less confusing"
- $\lfloor a \rfloor$ returns both positive and negative square roots ($\lfloor q2 \rfloor \equiv \ \because q$). the same is true for other reciprocals
- superscripts are modifiers (subscripts with special meanings). this distinction is extremely useful when working with forward propagation and backpropagation in neural networks, for example
- derivatives are not to be written as y', but rather as their complete form $\delta y - \delta x$. this makes calculus notation way more intuitive
- all indices start at 0, as they always should have
- rank polymorphism is supported over all operators

notation

also see trigonometric functions and calculus notation

let:

- M be a matrix
- V be a vector
- P be an ordered pair
- M' be a multiset
- G be a graph
- E be an expression

- x be a variable
- f be a function
- A be a sequence
- B be a series
- a, b be any mathematical objects
- A, B be any mathematical objects with rank greater than 1
- n, i be natural numbers
- b be a boolean
- ω be any number
- $\circ$ be any operator

operator descriptions

notation	description	notes
$a : b$	addition or disjoint union	
$a \cdot b$	subtraction	
$\because$ and $\therefore$	$\pm$ and $\mp$	
$a{\scriptstyle\mid}b$ and $a \mid b$	multiplication	
$a\text{-}b$ and $a - b$	division	
$[a]b$	exponentiation	represents a power by convention
$a[b]$	exponentiation	represents an exponential by convention
$\lfloor a \rfloor b$	b th root of a	$b = 2$ if b is omitted
$\lceil a \rceil b$	base-b logarithm of a	$b = e$ if b is omitted
$x \to E$	function literal	$f = x \to E \equiv f{\leftarrow}x = E$
$f{\leftarrow}E$	function application	uncommon, shorthand preferred
$a = b$	equality	numerical equality by convention
$a < b$ and $a > b$	strict inequality	
$a \leq b$ and $a \geq b$	non-strict inequality	
$a \wedge b$	logical AND or $\min$ function	
$a \vee b$	logical OR or $\max$ function	
$a \mathbin{/} b$	logical difference	$a \wedge b = \bot$
$a \equiv b$	equality	logical equality by convention

notation	description	notes
$a \times b$	nonequality	also serves as logical XOR
$a \vdash b$	implication, subset	a implies b, b for all a
$a \dashv b$	reverse implication, superset	a for all b, b implies a
$a_0 \circ a_1 \circ \ldots a_n$	with $n = 3$, $a_0 \circ a_1 \circ a_2 \circ a_3$	step size is $\because 1$ if $a_1 \circ$ is omitted
$a_0 \ldots a_n$	with $n = 3$, a_0, a_1, a_2, a_3	step size is $\because 1$ if a_1 is omitted
$a \circ \ldots$	the reduce function of $\circ$ on a	
$f \vdots a \circ \ldots b$	$f\ (a \ldots b) \circ \ldots$	
$f \vdots x \to a$	the limit of f as x approaches a	
x_{sub}	the variable x with a subscript sub	
V^n	the n th component of V	
A^i	the i th element of A	
B^i	the i th element of B	
$M^{\langle i,j \rangle}$	the i, j th element of M	uncommon, shorthand preferred
$M^{\intercal}$	the transpose matrix of M	
-M	the multiplicative inverse of M	
P^b	the b th element of P	
$S\ a$	whether a is element of S	
$M'\ a$	the number of elements a in M'	
$G\ a$	whether vertex a is in G	
$G^{\langle a,b \rangle}$	the number of edges from a to b in G	uncommon, shorthand preferred

shorthands

shorthand	definition	notes
$a \nvdash b$, $a \neq b$, $a \nleq b$, $a \nless b$...	$/(a \vdash b)$, $/a = b$, $/a \leq b$, $/a < b$...	
$x\omega$	$[x]\omega$	
ax	$a{\scriptstyle\mid}x$	
$f\ x$	$f{\leftarrow}x$	common, longhand discouraged

shorthand	definition	notes
$x\ y \to E$	$x \to y \to E$	
$\langle\ \rangle$	$\langle\langle\ \rangle\rangle$	see empty set
$(\ )$	$((\ ))$	see multiset
V^x, V^y, V^z	V^0, V^1, V^2	
$M^{i,j}$	$M^{\langle i,j\rangle}$	common, longhand discouraged
$M^{i,}$	the i th row of M	
$M^{,j}$	the j th column of M	
$S = \langle\langle a \ldots b\rangle\rangle$	$S\ x \equiv x = a \vee \ldots x = b$	see set
$P = \langle f, t\rangle$	$P^{\perp} = f \wedge P^{\top} = t$	see ordered pair
$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$	matrix literal	see matrix
$M' = ((1,2,2,2,3,3))$	multiset literal	see multiset
$x \to (a < x < b)$	the interval from a to b	
$A \circ B$	$x \to A\ x \circ B\ x$	see rank polymorphism
$A \circ B$	$A\ x \circ B\ x$ for all x	$(\top \ldots)$ is treated as $\top$
$\delta y - \delta x$	the derivative of y with respect to x	δ should be used instead of d
$\int y \mid \delta x$	the antiderivative of y with respect to x	δ should be used instead of d

constants

constant	definition	notes
$\varnothing$	*undefined*	see improved expression evaluation
$\top$	logical true	
$\perp$	logical false	
τ	the ratio of the circumference of a circle to its radius	using π is discouraged
e	Euler's constant	see eulers constant
ι	$\lfloor \cdot 1 \rfloor$	see imaginary, using i is discouraged
Π	the pi function	using fact is discouraged

operator properties

in order of high to low precedence

#todo fix asymmetry between $=\neq>\geq<\leq$ and $\wedge \vee \dashv \ \vdash \ \equiv \times$

operator	associativity	unary identity	unary description
$() \ \langle\rangle \ \begin{bmatrix} \\ \end{bmatrix} \ x \ x_a^i$			
$[] \ \lfloor\rfloor \ \lceil\rceil$			
ı -	left	1	inverse
$\delta \ \sin \ \# \leftarrow$	right-ish		
$: \ \cdot \ \because \ \therefore$	left	0	negation
$\mid -$	left	1	inverse
$\int \ \vdots \ \ldots \ \rightarrow \ \mod$	right-ish		
$=\neq>\geq<\leq$	AND	0	is (not) 0
$/$	left	$\top$	boolean logic NOT
$\wedge \vee$	left		
$\dashv \ \vdash$	left		
$\equiv \times$	AND	$\top$	boolean logic NOT
$,$			

> **note**: above,
>
> - x represents variables
> - x_a^i represents subscripts and superscripts
> - $\leftarrow$ represents function application
> - $\rightarrow$ represents function literals
> - $\begin{bmatrix} \\ \end{bmatrix}$ represents matrix literals

> **note**: unary operators have identical precedence to their binary counterparts, but are right associative

definition let $\circ$ be an operator with *AND* associativity. then,
$a \circ b \circ c \circ \ldots \quad \equiv \quad a \circ b \wedge b \circ c \wedge c \circ \ldots$

variable scope

variable scope is currently entirely context-dependent. this is know to cause occasional issues, such as with derivatives: $\delta \ f \ x - \delta x$ could represent both the derivative of f with respect to x in the general sense, or the derivative of f with respect to x **at the point** x

examples

quadratic formula $\cdot b : \lfloor b2 \cdot 4ac \rfloor - 2a$

definition of the set of complex numbers: $\mathbb{C}x \equiv x = a : b\iota \wedge \mathbb{R}a \wedge \mathbb{R}b$

definition of the implication / subset / superset / “for all” symbol: $a \vdash b \equiv /a \vee b$ and $a \dashv b \equiv a \vee /b$

in set theory, if U is a subset of V and V is a subset of U, then V is U. in this math notation: $(U\ x \vdash V\ x) \wedge (U\ x \dashv V\ x) \equiv U = V$

the probability density of the normal distribution in conventional math notation: $\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}$

compared to in my math notation: $-\lfloor \tau\sigma 2 \rfloor - e[\ [x \cdot \mu - \sigma]2 - 2\ ]$

definition of factorials: $\mathrm{fact}\, n = 1 \mid \ldots n$

the negation of an implication in my math notation: $B \vdash C \times B\ /\ C$ (*B implying C equals not (B without C)* or *implication is the negation of set difference* or *the negation of "for all B, C" is "there exists a B such that not C"*)

compared to conventional math notation: $\neg(B \to C) = B \wedge \neg C$ or $(a \in B \to a \in C) \iff a \notin B \backslash C$ or $B \subset C \iff \forall a \in C, a \notin B$

the resonant frequency of an LC circuit in conventional math notation: $f = \frac{1}{2\pi\sqrt{LC}}$

compared to in my math notation: $f = -\tau\lfloor LC \rfloor$

see random math notation formulas for more examples