Math Notation

see Classical Math Notation

this note describes my custom Math Notation, meant to solve inconsistencies in Classical Math Notation. it is not meant to be a fully formal system of Mathematics; rather, it is built to be easy to understand and intuitive to use by mere humans.

principles

- all equality Operators check for equality and return a Boolean, and it is implied that an Expression on its own must evaluate to $\top$. this allows for Boolean Logic Operators to be applied on equalities explicitly as opposed informally
- Sets are Functions that return a Boolean (Sets are Predicates). this way, Boolean Logic Operators and Set Operators are one and the same. other Data Structures that work similarly include Vectors, Matrixes, Sequences, Multisets, Ordered Pairs...
- some Operators are identical but have different precedence as "more brackets means more explicit, but less brackets means less complex and less confusing"
- $\lfloor a \rfloor$ returns both positive and negative square roots ($\lfloor q2 \rfloor \equiv \ \because q$). the same is true for other reciprocals
- superscripts are modifiers (subscripts with special meanings). this distinction is especially useful when working with Forward Propagation and Backpropagation in Neural Networks, for example
- Derivatives are not to be written as y', but rather as their complete form $\delta y - \delta x$. this makes Calculus Notation way more intuitive
- all indices start at 0, as they always should have

notation

also see Trigonometric Functions and Calculus Notation

operator descriptions

notation	description	notes
$a : b$	addition	
$a \cdot b$	subtraction	

notation	description	notes
$\because$ and $\therefore$	$\pm$ and $\mp$	
$a{\scriptstyle\vert}b$ and $a \mid b$	multiplication	
$a\text{-}b$ and $a - b$	division	
$[a]b$	exponentiation	represents a power by convention
$a[b]$	exponentiation	represents an exponential by convention
$\lfloor a \rfloor b$	b th root of a	$b = 2$ if b is omitted
$\lceil a \rceil b$	base-b Logarithm of a	$b = e$ if b is omitted
$x \to E$ where E is an Expression	Function literal	$f = x \to E \equiv f \leftarrow x = E$
$f \leftarrow E$ where E is an Expression	Function application	uncommon, shorthand is preferred
$a = b$	equality	numerical equality by convention
$a < b$ and $a > b$	strict inequality	
$a \leq b$ and $a \geq b$	non-strict inequality	
$a \wedge b$	logical AND	
$a \vee b$	logical OR	
$a \mathbin{/} b$	logical difference	$a \wedge b = \perp$
$a \equiv b$	equality	logical equality by convention
$a \times b$	nonequality	also serves as logical XOR
$a \vdash b$	implication, subset	a implies b, b for all a
$a \dashv b$	reverse implication, superset	a for all b, b implies a
$x_0 \mid x_1 \mid \ldots x_n$ where $\mid$ is any Operator	with $n = 3$, $x_0 \mid x_1 \mid x_2 \mid x_3$	step size is $\because 1$ if $x_1 \mid$ is omitted
$x_0 \ldots x_n$	with $n = 3$, x_0, x_1, x_2, x_3	step size is $\because 1$ if x_1 is omitted
x_{sub}	the variable x with a subscript $_{sub}$	
V^n where V is a Vector	the n th component of V	
a^i where a is a Sequence	the i th element of a	
b^i where b is a Series	the i th element of b	

notation	description	notes
$M^{\langle i,j \rangle}$ where M is a Matrix	the i, j th element of M	uncommon, shorthand is preferred
$M^{\intercal}$ where M is a Matrix	the transpose of A	
M^{-} where M is a Matrix	the multiplicative inverse of A	
P^{b} where P is an Ordered Pair	the b th element of P	
$S\ a$ where S is a Set	whether a is element of S	
$M\ a$ where M is a Multiset	the number of elements a in M	
$G\ v$ where G is a Graph	whether vertex v is in G	
$G^{\langle v,w \rangle}$ where G is a Graph	the number of edges from v to w in G	uncommon, shorthand preferred

shorthands

shorthand	definition	notes
$a \nvdash b$, $a \neq b$, $a \nleq b$, $a \nless b$...	$/(a \vdash b)$, $/a = b$, $/a \leq b$, $/a < b$ ...	
$x\omega$ where x is a variable and ω is a Number	$[x]\omega$	
ax where x is a variable	$a{\scriptstyle\mid}x$	
$f\ x$ where f is a Function	$f {\leftarrow} x$	common, longhand is discouraged
$x\ y \to E$ where E is an Expression	$x \to y \to E$	
V^{x}, V^{y} and V^{z} where V is a Vector	the x, y and z components of V	
$M^{i,j}$ where M is a Matrix	$M^{\langle i,j \rangle}$	common, longhand is discouraged
$M^{i,}$ where M is a Matrix	the i th row of M	
$M^{,j}$ where M is a Matrix	the j th column of M	
$S = \{a \dots b\}$	$S\ x \equiv x = a \vee \dots x = b$	see Set
$P = \langle f, t \rangle$	$P^{\perp} = f \wedge P^{\top} = t$	see Ordered Pair
$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$	matrix literal	see Matrix

shorthand	definition	notes
$x \to (a < x < b)$	the closed interval from a to b	same can be used for open intervals
$A \vdash B$ where $\vdash$ is any #think Operator	$A\,x \vdash B\,x$ for all x	commonly $\equiv \dashv \vdash$ #think
$A \cdot B$ where $\cdot$ is any #think Operator	$x \to A\,x \cdot B\,x$	commonly $: \cdot \vert -$ #think
$\delta y - \delta x$	the Derivative of y with respect to x	δ should be used instead of d
$\int y \vert \delta x$	the Antiderivative of y with respect to x	δ should be used instead of d

constants

constant	definition	notes
$\varnothing$	*undefined*	see Improved Expression Evaluation
$\top$	logical true	
$\bot$	logical false	
τ	the ratio of the circumference of a Circle to its radius	using π is discouraged
e	Euler's constant	see E
i	$\lfloor \cdot 1 \rfloor$	see Imaginary

operator properties

in order of high to low precedence

operator	associativity	unary identity	unary description
$() \; \{\} \; \langle\rangle \; \Big[\Big] \; x \; x_a^i$			
$[] \; \lfloor\rfloor \; \lceil\rceil$			
ı -	left	1	inverse
δ sin $\leftarrow$	right-ish		
$: \; \cdot \; \because \; \therefore$	left	0	negation
$\vert \; -$	left	1	inverse
$\int$ lim $\ldots \; \to$	right		

operator	**associativity**	**unary identity**	**unary description**
$=\neq>\geq<\leq$	AND	0	is (not) 0
$/$	left	$\top$	logical NOT
$\wedge\ \vee$	left		
$\dashv\ \vdash$	left		
$\equiv\ \times$	AND	$\top$	logical NOT
$,$			

> **note**: above,
>
> - x represents variables
> - x_a^i represents subscripts and superscripts
> - $\leftarrow$ represents Function application
> - $\rightarrow$ represents Function literals
> - $[\,]$ represents Matrix literals

> **note**: unary Operators have identical precedence to their binary counterparts, but are right associative

> **definition**: let $=$ be an Operator with *AND* associativity. then, $a = b = c = \ldots \quad \equiv \quad a = b \wedge b = c \wedge c = \ldots$

variable scope

variable scope is currently entirely context-dependent. this is know to cause occasional issues, such as with Derivatives: $\delta\ f\ x - \delta x$ could represent both the Derivative of f with respect to x in the general sense, or the Derivative of f with respect to x **at the point** x as $(x \rightarrow \delta\ f\ x - \delta x)\ x \equiv \delta\ f\ x - \delta x$.

examples

Quadratic Formula: $\cdot b : \lfloor b2 \cdot 4ac \rfloor - 2a$

definition of the Set of Complex numbers: $\mathbb{C}x \equiv x = a : b\lfloor \cdot 1 \rfloor \wedge \mathbb{R}a \wedge \mathbb{R}b$

definition of the implication / subset / superset / "for all" symbol: $a \vdash b \equiv /a \vee b$ and $a \dashv b \equiv a \vee /b$

in Set Theory, if U is a subset of V and V is a subset of U, then V is U. in this math notation: $(U\ x \vdash V\ x) \wedge (U\ x \dashv V\ x) \equiv U = V$

the probability density of the normal distribution in Classical Math Notation: $\frac{1}{\sqrt{2\sigma^2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

compared to in my Math Notation: $-\lfloor\tau\sigma 2\rfloor - e[[x \cdot \mu]2 - 2\sigma 2]$

definition of factorials: $\mathsf{fact}\, n = 1 \mid \ldots n$

the negation of an implication in my Math Notation: $B \vdash C \times B \ /\ C$ (*B implying C equals not (B without C)* or *implication is the negation of set difference* or *the negation of "for all B, C" is "there exists a B such that not C"*)

compared to Classical Math Notation: $\neg(B \to C) = B \wedge \neg C$ or $(a \in B \to a \in C) \iff a \notin B \setminus C$ or $B \subset C \iff \forall a \in C, a \notin B$

see Random Math Notation Formulas for more examples