Complex

the set of complex numbers

see math notation

definition

$\mathbb{C}x \equiv x = a : b\iota \wedge \mathbb{R}a \wedge \mathbb{R}b$

notations

Cartesian Form

$z = a : b\iota$

> **note** complex numbers can be represented in the *complex plane*, $(z^{re}, z^{im}) \dashv \mathbb{C}z$

Polar Form

> **AKA** Euler's formula notation

$z = |z|\ \cos\theta : |z|\ \iota \sin\theta = |z|\ e[\iota\theta]$, see eulers constant

applications

complex numbers are often intimately related to discrete mathematics — 3B1B https://youtu.be/bOXCLR3Wric

properties

$\mathbb{C} \vdash \mathbb{U}$, see universal

equality $a : b\iota = c : d\iota \equiv a = c \wedge b = d$

addition $(a : b\iota) : (c : d\iota) = (a : c) : (b : d)\iota$

> **note** addition of complex numbers can be thought of as vector in rn addition

subtraction $(a : b\iota) \cdot (c : d\iota) = (a \cdot c) : (b \cdot d)\iota$

multiplication

in cartesian form, $a : b\iota \mid c : d\iota = ac : ad\iota : b\iota c : bd\iota 2 = (ac \cdot bd) : (ad : bc)\iota$

in polar form, $z \mid w = |z|\ e[\iota\theta] \mid |w|\ e[\iota\phi] = |zw|\ e[\iota \mid \theta : \phi]$

square root of ι. $\lfloor \iota \rfloor = \ \because \ \mid 1 : \iota - \lfloor 2 \rfloor$ — https://www.youtube.com/watch?v=Z49hXoN4KWg

product of two conjugates are product of magnitudes
$a : b\iota \mid a \cdot b\iota = a2 : b2 = |a : b\iota| \ \mid \ |a \cdot b\iota|$ — https://youtu.be/bOXCLR3Wric?t=1522

Re and Im Parts

let $z = a : b\iota$

definitions

real part of a complex number $z^{re} = a$

imaginary part of a complex number $z^{im} = b$

therefore, $z = z^{re} : \iota z^{im}$

Complex Conjugate

definition

let $z = a : b\iota$

then, $\operatorname{conj} z = a \cdot b\iota = z^{re} \cdot \iota z^{im}$ is the *complex conjugate* of z

properties

let $\mathbb{C}z \wedge \mathbb{C}w \wedge \mathbb{R}c$

$\operatorname{conj}(z : w) = \operatorname{conj} z : \operatorname{conj} w$

$\operatorname{conj} cz = c \operatorname{conj} z$

$\operatorname{conj} z{\mid}w = \operatorname{conj} z \mid \operatorname{conj} w$

$\operatorname{conj} z\text{-}w = \operatorname{conj} z - \operatorname{conj} w$

$\operatorname{conj} \operatorname{conj} z = z$

$\mathbb{R}z \equiv \operatorname{conj} z = z$

theorems

theorem $z \operatorname{conj} z = |z|2 \dashv \mathbb{C}z$

theorem $-z = \operatorname{conj} z - |z|2 \dashv \mathbb{C}z$

applications

multiplying by the conjugate can be used to reduce an expression such as $-4 : 3\iota$

Absolute Value

> **AKA** magnitude

definition

let $z = a : b\iota$

then, $|z| = \lfloor a2 : b2 \rfloor = \lfloor z^{re}2 : z^{im}2 \rfloor$ is the *absolute value* of z.

> **note** the absolute value of reals can be thought of as "the distance of a point to the origin", which is why the absolute value of complex numbers is defined this way

properties

let $\mathbb{C}z \wedge \mathbb{C}w \wedge \mathbb{R}c$

let $\mathbb{R}|z| \wedge |z| \geq 0$

$|z| = |\operatorname{conj} z|$

$|zw| = |z|\ |w|$

$|z - w| = |z| - |w|$

triangle inequality $|z : w| \leq |z| : |w|$