

Math Notation

see [conventional math notation](#)

this note describes my own [math notation](#), meant to solve inconsistencies in [conventional math notation](#). it is not meant to be a fully formal system of [mathematics](#); rather, it is built to be easy to understand and intuitive to use by mere humans.

this [math notation](#) obviously cannot be used to communicate ideas to people who do not know it, but it has made my own experience of using [mathematics](#) much more enjoyable. being able to use a single relatively well defined notation in various [mathematics fields](#) that conventionally use their own made up notation has been invaluable

principles

- all equality [operators](#) check for equality and return a [boolean](#), and it is implied that an [expression](#) on its own must evaluate to \top . this allows for [boolean logic > operators](#) to be applied on equalities explicitly as opposed informally
- [sets](#) are [functions](#) that return a [boolean](#) ([sets](#) are [predicates](#)). this way, [boolean logic > operators](#) and [set operators](#) are one and the same. other [data structures](#) that work similarly include [vectors](#), [matrixes](#), [sequences](#), [multisets](#), [ordered pairs](#)...
- some [operators](#) are identical but have different precedence as "more brackets means more explicit, but less brackets means less complex and less confusing"
- $[a]$ returns both positive and negative square roots ($[q^2] \equiv \pm q$). the same is true for other reciprocals
- superscripts are modifiers (subscripts with special meanings). this distinction is extremely useful when working with [forward propagation](#) and [backpropagation](#) in [neural networks](#), for example
- [derivatives](#) are not to be written as y' , but rather as their complete form $\delta y - \delta x$. this makes [calculus notation](#) way more intuitive
- all indices start at 0, as they always should have
- [rank polymorphism](#) is supported over all [operators](#)

notation

also see [trigonometric functions](#), [hyperbolic functions](#) and [calculus notation](#)

let:

- M be a [matrix](#)

- V be a vector
- P be an ordered pair
- M' be a multiset
- G be a graph
- E be an expression
- x be a variable
- f be a function
- A be a sequence
- B be a series
- a, b be any mathematical objects
- A, B be any mathematical objects with rank greater than 1
- n, i be natural numbers
- b be a boolean
- ω be any number
- \circ be any operator

operator descriptions

notation	description	notes
$a : b$	addition or disjoint union	
$a \cdot b$	subtraction	
\pm and \mp	\pm and \mp	
$a b$ and $a \mid b$	multiplication	
$a \div b$ and $a \div b$	division	
$[a]b$	exponentiation	represents a power by convention
$a[b]$	exponentiation	represents an exponential by convention
$\lfloor a \rfloor b$	b th root of a	$b = 2$ if b is omitted
$\lceil a \rceil b$	base- b <u>logarithm</u> of a	$b = e$ if b is omitted
$x \rightarrow E$	<u>function</u> literal	$f = x \rightarrow E \equiv f \leftarrow x = E$
$f \leftarrow E$	<u>function</u> application	uncommon, shorthand preferred
$a = b$	equality	numerical equality by convention
$a < b$ and $a > b$	strict inequality	
$a \leq b$ and $a \geq b$	non-strict inequality	

notation	description	notes
$a \wedge b$	logical AND or min function	
$a \vee b$	logical OR or max function	
a / b	logical difference	$a \wedge b = \perp$
$a \equiv b$	equality	logical equality by convention
$a \times b$	nonequality	also serves as logical XOR
$a \vdash b$	implication, <u>set > subset</u>	a implies b , b for all a
$a \dashv b$	reverse implication, <u>set > superset</u>	a for all b , b implies a
$a_0 \circ a_1 \circ \dots a_n$	with $n = 3$, $a_0 \circ a_1 \circ a_2 \circ a_3$	step size is $\therefore 1$ if $a_1 \circ$ is omitted
$a_0 \dots a_n$	with $n = 3$, a_0, a_1, a_2, a_3	step size is $\therefore 1$ if a_1 is omitted
$f \dot{\vdash} a \circ b$	$f a \circ f b$	
$f \dot{\vdash} x \rightarrow a$	the <u>limit</u> of f as x approaches a	
x_{sub}	the <u>variable</u> x with a subscript <i>sub</i>	
V^n	the n th component of V	
A^i	the i th element of A	
B^i	the i th element of B	
$M^{(i,j)}$	the i, j th element of M	uncommon, shorthand preferred
P^b	the b th element of P	
$S a$	whether a is element of S	
$M' a$	the number of elements a in M'	
$G a$	whether vertex a is in G	
$G^{(a,b)}$	the number of edges from a to b in G	uncommon, shorthand preferred

shorthands

shorthand	definition	notes
$a \nvdash b$, $a \neq b$, $a \not\leq b$, $a \not\leq b \dots$	$\neg(a \vdash b)$, $\neg a = b$, $\neg a \leq b$, $\neg a < b \dots$	
$x\omega$	$[x]\omega$	
ax	$a \setminus x$	

shorthand	definition	notes
$f E$	$f \leftarrow E$	common, longhand discouraged
$x y \rightarrow E$	$x \rightarrow y \rightarrow E$	
$\langle \rangle$	$\langle \langle \rangle \rangle$	see empty set
$()$	$(())$	see multiset
V^x, V^y, V^z	V^0, V^1, V^2	
$M^{i,j}$	$M^{\langle i,j \rangle}$	common, longhand discouraged
$M^i,$	the i th row of M	
M^j	the j th column of M	
$P = \langle f, t \rangle$	$P^\perp = f \wedge P^\top = t$	see ordered pair
$S = \langle \langle a \cdots b \rangle \rangle$	$S x \equiv x = a \vee \cdots x = b$	see set
$V = (a \cdots b)$	$V^0 = a \wedge \cdots V^n = b$	see vector in rn
$M' = ((1, 2, 2, 2, 3, 3))$	multiset literal	see multiset
$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$	matrix literal	see matrix
$x \rightarrow (a < x < b)$	the interval from a to b	
$A \circ B$	$x \rightarrow A x \circ B x$	see rank polymorphism
$A \circ B$	$A x \circ B x$ for all x	why not $\wedge A \circ B$ #think
$\circ A$	the reduce function of A with \circ	
$\delta y - \delta x$	the derivative of y with respect to x	δ should be used instead of d
$\int y \mid \delta x$	the antiderivative of y with respect to x	δ should be used instead of d

constants

constant	definition	notes
\emptyset	<i>undefined</i>	see improved expression evaluation
\top	logical true	
\perp	logical false	
τ	the ratio of the circumference of a circle to its radius	using π is discouraged

constant	definition	notes
e	Euler's constant	see eulers constant
ι	$[\cdot 1]$	see imaginary , using i is discouraged
Π	the pi function	using fact is discouraged
$\#$	the size of the range of a function	

operator properties

in order of high to low precedence

#todo fix asymmetry between $=\neq>\geq<\leq$ and $\wedge \vee \dashv \vdash \equiv \times$

operator	associativity	unary identity	unary description
$() \langle \rangle \boxed{}$			
$\boxed{} \sqcup \sqcap$			
$\mid -$	left	1	inverse
$\delta \sin \# \leftarrow$	right-ish		
$: \cdot \therefore \therefore$	left	0	negation
$\mid -$	left	1	inverse
$\int \dot{} \cdots \rightarrow \bmod$	right-ish		
$=\neq>\geq<\leq$	AND	0	is (not) 0
$/$	left	\top	boolean logic NOT
$\wedge \vee$	left		
$\dashv \vdash$	left		
$\equiv \times$	AND	\top	boolean logic NOT
$,$			

note: above,

- x represents [variables](#)
- x_a^i represents subscripts and superscripts
- \leftarrow represents [function](#) application
- \rightarrow represents [function](#) literals
- $\boxed{}$ represents [matrix](#) literals

note: unary operators have identical precedence to their binary counterparts, but are right associative

definition let \circ be an operator with *AND associativity*. then,
 $a \circ b \circ c \circ \dots \equiv a \circ b \wedge b \circ c \wedge c \circ \dots$

variable scope

variable scope is currently entirely context dependent. this is know to cause occasional issues, such as with derivatives: $\delta f x - \delta x$ could represent both the derivative of f with respect to x in the general sense, or the derivative of f with respect to x **at the point** x

examples

description	in this <u>math notation</u>	in <u>conventional math notation</u>
the <u>quadratic formula</u>	$x = \cdot b : [b^2 \cdot 4ac] - 2a$	$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
definition of <u>complex numbers</u>	$\mathbb{C}x \equiv x = a : b\iota \wedge \mathbb{R}a \wedge \mathbb{R}b$	$\mathbb{C} = \{a + bi : a, b \in \mathbb{R}\}$
the <u>gaussian function</u>	$G^\sigma p = -[\tau\sigma^2] - e[: p^2 - 2\sigma^2]$	$G(x, y, \dots) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2+y^2+\dots}{2\sigma^2}}$
<u>limit</u> form of a <u>derivative</u>	$f x \cdot f a - x \cdot a : x \rightarrow a$	$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$
definition of factorials	$\text{fact } n = 1 \mid \dots n$	$n! = 1 \cdot 2 \cdot \dots n$ (informally)
the resonant <u>frequency</u> of an LC circuit	$f = -\tau[LC]$	$f = \frac{1}{2\pi\sqrt{LC}}$
definition of the <u>dot product</u>	$: xy$	$x \cdot y = \sum_{i=1}^n x_i y_i$
definition of <u>vector in rn > magnitude</u>	$[: v^2]$	$ v = \sqrt{x^2 + y^2 + \dots}$ with $v = (x, y, \dots)$
definition of <u>set difference</u>	$A/B \equiv A \wedge \times B$	$A \setminus B = \{x \in A : x \notin B\}$

definition of the boolean logic > implication / set > subset / set > superset / quantifier > universal quantifier in this math notation: $a \vdash b \equiv /a \vee b$ and $a \dashv b \equiv a \vee /b$

in set theory, if U is a set > subset of V and V is a set > subset of U , then V is U . in this [[math notation]]: $(U x \vdash V x) \wedge (U x \dashv V x) \equiv U = V$

the negation of a boolean logic > implication in this math notation: $B \vdash C \times B / C$

in conventional math notation: $\neg(B \rightarrow C) = B \wedge \neg C$ or $(a \in B \rightarrow a \in C) \iff a \notin B \setminus C$ or $B \subset C \iff \forall a \in C, a \notin B$