# Math Notation

see conventional math notation

this note describes my own math notation, meant to solve inconsistencies in conventional math notation. it is not meant to be a fully formal system of mathematics; rather, it is built to be easy to understand and intuitive to use by mere humans.

this math notation obviously cannot be used to communicate ideas to people who do not know it, but it has made my own experience of using mathematics much more enjoyable. being able to use a single relatively well defined notation in various mathematics fields that conventially use their own made up notation has been invaluable

## principles

- all equality operators check for equality and return a boolean, and it is implied that an expression on its own must evaluate to $\top$. this allows for boolean logic operators to be applied on equalities explicitly as opposed informally
- sets are functions that return a boolean (sets are predicates). this way, boolean logic operators and set operators are one and the same. other data structures that work similarly include vectors, matrixes, sequences, multisets, ordered pairs...
- some operators are identical but have different precedence as "more brackets means more explicit, but less brackets means less complex and less confusing"
- $\lfloor a \rfloor$ returns both positive and negative square roots ($\lfloor q2 \rfloor \equiv \because q$). the same is true for other reciprocals
- superscripts are modifiers (subscripts with special meanings). this distinction is extremely useful when working with forward propagation and backpropagation in neural networks, for example
- derivatives are not to be written as $y'$, but rather as their complete form $\delta y - \delta x$. this makes calculus notation way more intuitive
- all indices start at $0$, as they always should have
- rank polymorphism is supported over all operators

## notation

also see trigonometric functions and calculus notation

let:

- $M$ be a matrix

- $V$ be a vector
- $P$ be an ordered pair
- $M'$ be a multiset
- $G$ be a graph
- $E$ be an expression
- $x$ be a variable
- $f$ be a function
- $A$ be a sequence
- $B$ be a series
- $a, b$ be any mathematical objects
- $A, B$ be any mathematical objects with rank greater than $1$
- $n, i$ be natural numbers
- $b$ be a boolean
- $\omega$ be any number
- $\circ$ be any operator

## operator descriptions

| notation | description | notes |
|---|---|---|
| $a : b$ | addition or disjoint union | |
| $a \cdot b$ | subtraction | |
| $\because$ and $\therefore$ | $\pm$ and $\mp$ | |
| $a{\scriptstyle\mid}b$ and $a \mid b$ | multiplication | |
| $a\text{-}b$ and $a - b$ | division | |
| $[a]b$ | exponentiation | represents a power by convention |
| $a[b]$ | exponentiation | represents an exponential by convention |
| $\lfloor a \rfloor b$ | $b$ th root of $a$ | $b = 2$ if $b$ is omitted |
| $\lceil a \rceil b$ | base-$b$ logarithm of $a$ | $b = e$ if $b$ is omitted |
| $x \to E$ | function literal | $f = x \to E \equiv f{\leftarrow}x = E$ |
| $f{\leftarrow}E$ | function application | uncommon, shorthand preferred |
| $a = b$ | equality | numerical equality by convention |
| $a < b$ and $a > b$ | strict inequality | |
| $a \leq b$ and $a \geq b$ | non-strict inequality | |

| notation | description | notes |
|---|---|---|
| $a \wedge b$ | logical AND or $\min$ function | |
| $a \vee b$ | logical OR or $\max$ function | |
| $a \;/\; b$ | logical difference | $a \wedge b = \bot$ |
| $a \equiv b$ | equality | logical equality by convention |
| $a \times b$ | nonequality | also serves as logical XOR |
| $a \vdash b$ | implication, subset | $a$ implies $b$, $b$ for all $a$ |
| $a \dashv b$ | reverse implication, superset | $a$ for all $b$, $b$ implies $a$ |
| $a_0 \circ a_1 \circ \ldots a_n$ | with $n = 3$, $a_0 \circ a_1 \circ a_2 \circ a_3$ | step size is $\because 1$ if $a_1 \circ$ is omitted |
| $a_0 \ldots a_n$ | with $n = 3$, $a_0, a_1, a_2, a_3$ | step size is $\because 1$ if $a_1$ is omitted |
| $a \circ \ldots$ | the reduce function of $\circ$ on $a$ | |
| $f \vdots\ a \circ \ldots b$ | $f\ (a \ldots b) \circ \ldots$ | |
| $f \vdots\ x \to a$ | the limit of $f$ as $x$ approaches $a$ | |
| $x_{sub}$ | the variable $x$ with a subscript $_{sub}$ | |
| $V^n$ | the $n$ th component of $V$ | |
| $A^i$ | the $i$ th element of $A$ | |
| $B^i$ | the $i$ th element of $B$ | |
| $M^{\langle i,j \rangle}$ | the $i, j$ th element of $M$ | uncommon, shorthand preferred |
| $M^\intercal$ | the transpose matrix of $M$ | |
| -$M$ | the multiplicative inverse of $M$ | |
| $P^b$ | the $b$ th element of $P$ | |
| $S\ a$ | whether $a$ is element of $S$ | |
| $M'\ a$ | the number of elements $a$ in $M'$ | |
| $G\ a$ | whether vertex $a$ is in $G$ | |
| $G^{\langle a,b \rangle}$ | the number of edges from $a$ to $b$ in $G$ | uncommon, shorthand preferred |

## shorthands

| shorthand | definition | notes |
|---|---|---|

| shorthand | definition | notes |
|---|---|---|
| $a \nvdash b$, $a \neq b$, $a \nleq b$, $a \nless b$... | $/(a \vdash b)$, $/a = b$, $/a \leq b$, $/a < b$... | |
| $x\omega$ | $[x]\omega$ | |
| $ax$ | $a{\scriptstyle\vert}x$ | |
| $f\ x$ | $f{\leftarrow}x$ | common, longhand discouraged |
| $x\ y \to E$ | $x \to y \to E$ | |
| $\langle\ \rangle$ | $\langle\langle\ \rangle\rangle$ | see empty set |
| $(\ )$ | $((\ ))$ | see multiset |
| $V^x, V^y, V^z$ | $V^0, V^1, V^2$ | |
| $M^{i,j}$ | $M^{\langle i,j\rangle}$ | common, longhand discouraged |
| $M^{i,}$ | the $i$ th row of $M$ | |
| $M^{,j}$ | the $j$ th column of $M$ | |
| $P = \langle f, t\rangle$ | $P^\perp = f \wedge P^\top = t$ | see ordered pair |
| $S = \langle\langle a \ldots b\rangle\rangle$ | $S\ x \equiv x = a \vee \ldots x = b$ | see set |
| $V = (a \ldots b)$ | $V^0 = a \wedge \ldots V^n = b$ | see vector in rn |
| $M' = ((1, 2, 2, 2, 3, 3))$ | multiset literal | see multiset |
| $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ | matrix literal | see matrix |
| $x \to (a < x < b)$ | the interval from $a$ to $b$ | |
| $A \circ B$ | $x \to A\ x \circ B\ x$ | see rank polymorphism |
| $A \circ B$ | $A\ x \circ B\ x$ for all $x$ | $(\top \ldots)$ is treated as $\top$ |
| $\delta y - \delta x$ | the derivative of $y$ with respect to $x$ | $\delta$ should be used instead of $d$ |
| $\int y \mid \delta x$ | the antiderivative of $y$ with respect to $x$ | $\delta$ should be used instead of $d$ |

## constants

| constant | definition | notes |
|---|---|---|
| $\varnothing$ | *undefined* | see improved expression evaluation |
| $\top$ | logical true | |

| constant | definition | notes |
|---|---|---|
| $\bot$ | logical false | |
| $\tau$ | the ratio of the circumference of a circle to its radius | using $\pi$ is discouraged |
| $e$ | Euler's constant | see eulers constant |
| $\iota$ | $\lfloor \cdot 1 \rfloor$ | see imaginary, using $i$ is discouraged |
| $\Pi$ | the pi function | using $\mathrm{fact}$ is discouraged |

## operator properties

*in order of high to low precedence*

#todo fix asymmetry between $= \neq > \geq < \leq$ and $\wedge \vee \dashv \vdash \equiv \times$

| operator | associativity | unary identity | unary description |
|---|---|---|---|
| $()\ \langle\rangle\ \Big[\Big]\ x\ x_a^i$ | | | |
| $[]\ \lfloor\rfloor\ \lceil\rceil$ | | | |
| ı - | left | $1$ | inverse |
| $\delta\ \sin\ \#\ \leftarrow$ | right-ish | | |
| $:\ \cdot\ \because\ \therefore$ | left | $0$ | negation |
| $\mid\ -$ | left | $1$ | inverse |
| $\int\ \vdots\ \ldots\ \rightarrow\ \mathrm{mod}$ | right-ish | | |
| $= \neq > \geq < \leq$ | AND | $0$ | is (not) $0$ |
| $/$ | left | $\top$ | boolean logic NOT |
| $\wedge\ \vee$ | left | | |
| $\dashv\ \vdash$ | left | | |
| $\equiv\ \times$ | AND | $\top$ | boolean logic NOT |
| $,$ | | | |

> **note**: above,
>
> - $x$ represents variables
> - $x_a^i$ represents subscripts and superscripts
> - $\leftarrow$ represents function application
> - $\rightarrow$ represents function literals

> - $[]$ represents matrix literals

> **note**: unary operators have identical precedence to their binary counterparts, but are right associative

**definition** let $\circ$ be an operator with *AND* associativity. then,
$a \circ b \circ c \circ \ldots \quad \equiv \quad a \circ b \wedge b \circ c \wedge c \circ \ldots$

# variable scope

variable scope is currently entirely context-dependent. this is know to cause occasional issues, such as with derivatives: $\delta\ f\ x - \delta x$ could represent both the derivative of $f$ with respect to $x$ in the general sense, or the derivative of $f$ with respect to $x$ **at the point** $x$

# examples

quadratic formula: $\cdot b : \lfloor b2 \cdot 4ac \rfloor - 2a$

definition of the set of complex numbers: $\mathbb{C}x \equiv x = a : b\iota \wedge \mathbb{R}a \wedge \mathbb{R}b$

definition of the implication / subset / superset / “for all” symbol: $a \vdash b \equiv /a \vee b$ and $a \dashv b \equiv a \vee /b$

in set theory, if $U$ is a subset of $V$ and $V$ is a subset of $U$, then $V$ is $U$. in this math notation: $(U\ x \vdash V\ x) \wedge (U\ x \dashv V\ x) \equiv U = V$

the probability density of the normal distribution in conventional math notation:
$\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}$

compared to in my math notation: $-\lfloor \tau\sigma 2 \rfloor - e[\ [x \cdot \mu - \sigma]2 - 2\ ]$

definition of factorials: $\mathrm{fact}\, n = 1 \mid \ldots n$

the negation of an implication in my math notation: $B \vdash C \times B\ /\ C$ (*B implying C equals not (B without C)* or *implication is the negation of set difference* or *the negation of "for all B, C" is "there exists a B such that not C"*)

compared to conventional math notation: $\neg(B \to C) = B \wedge \neg C$ or $(a \in B \to a \in C) \iff a \notin B \backslash C$ or $B \subset C \iff \forall a \in C, a \notin B$

the resonant frequency of an LC circuit in conventional math notation: $f = \frac{1}{2\pi\sqrt{LC}}$

compared to in my math notation: $f = -\tau\lfloor LC \rfloor$

see random math notation formulas for more examples