

# Math Notation

see [conventional math notation](#)

this note describes my own [math notation](#), meant to solve inconsistencies in [conventional math notation](#). it is not meant to be a fully formal system of [mathematics](#); rather, it is built to be easy to understand and intuitive to use by mere humans.

this [math notation](#) obviously cannot be used to communicate ideas to people who do not know it, but it has made my own experience of using [mathematics](#) much more enjoyable. being able to use a single relatively well defined notation in various [mathematics fields](#) that conventionally use their own made up notation has been invaluable

## principles

- all equality [operators](#) check for equality and return a [boolean](#), and it is implied that an [expression](#) on its own must evaluate to  $\top$ . this allows for [boolean logic > operators](#) to be applied on equalities explicitly as opposed informally
- [sets](#) are [functions](#) that return a [boolean](#) ([sets](#) are [predicates](#)). this way, [boolean logic > operators](#) and [set operators](#) are one and the same. other [data structures](#) that work similarly include [vectors](#), [matrixes](#), [sequences](#), [multisets](#), [ordered pairs](#)...
- some [operators](#) are identical but have different precedence as "more brackets means more explicit, but less brackets means less complex and less confusing"
- $[a]$  returns both positive and negative square roots ( $[q^2] \equiv \pm q$ ). the same is true for other reciprocals
- superscripts are modifiers (subscripts with special meanings). this distinction is extremely useful when working with [forward propagation](#) and [backpropagation](#) in [neural networks](#), for example
- [derivatives](#) are not to be written as  $y'$ , but rather as their complete form  $\delta y - \delta x$ . this makes [calculus notation](#) way more intuitive
- all indices start at 0, as they always should have
- [rank polymorphism](#) is supported over all [operators](#)

## notation

also see [trigonometric functions](#), [hyperbolic functions](#) and [calculus notation](#)

let:

- $M$  be a [matrix](#)

- $V$  be a vector
- $P$  be an ordered pair
- $M'$  be a multiset
- $G$  be a graph
- $E$  be an expression
- $x$  be a variable
- $f$  be a function
- $A$  be a sequence
- $B$  be a series
- $a, b$  be any mathematical objects
- $A, B$  be any mathematical objects with rank greater than 1
- $n, i$  be natural numbers
- $b$  be a boolean
- $\omega$  be any number
- $\circ$  be any operator

## operator descriptions

notation	description	notes
$a : b$	addition or disjoint union	
$a \cdot b$	subtraction	
$\pm$ and $\mp$	$\pm$ and $\mp$	
$a b$ and $a \mid b$	multiplication	
$a \div b$ and $a \div b$	division	
$[a]b$	exponentiation	represents a power by convention
$a[b]$	exponentiation	represents an exponential by convention
$\lfloor a \rfloor b$	$b$ th root of $a$	$b = 2$ if $b$ is omitted
$\lceil a \rceil b$	base- $b$ <u>logarithm</u> of $a$	$b = e$ if $b$ is omitted
$x \rightarrow E$	<u>function</u> literal	$f = x \rightarrow E \equiv f \leftarrow x = E$
$f \leftarrow E$	<u>function</u> application	uncommon, shorthand preferred
$a = b$	equality	numerical equality by convention
$a < b$ and $a > b$	strict inequality	
$a \leq b$ and $a \geq b$	non-strict inequality	

notation	description	notes
$a \wedge b$	logical AND or min function	
$a \vee b$	logical OR or max function	
$a / b$	logical difference	$a \wedge b = \perp$
$a \equiv b$	equality	logical equality by convention
$a \times b$	nonequality	also serves as logical XOR
$a \vdash b$	implication, <u>set &gt; subset</u>	$a$ implies $b$ , $b$ for all $a$
$a \dashv b$	reverse implication, <u>set &gt; superset</u>	$a$ for all $b$ , $b$ implies $a$
$a_0 \circ a_1 \circ \dots a_n$	with $n = 3$ , $a_0 \circ a_1 \circ a_2 \circ a_3$	step size is $\therefore 1$ if $a_1 \circ$ is omitted
$a_0 \dots a_n$	with $n = 3$ , $a_0, a_1, a_2, a_3$	step size is $\therefore 1$ if $a_1$ is omitted
$f \dot{\vdash} a \circ b$	$f a \circ f b$	
$f \dot{\vdash} x \rightarrow a$	the <u>limit</u> of $f$ as $x$ approaches $a$	
$x_{sub}$	the <u>variable</u> $x$ with a subscript <i>sub</i>	
$V^n$	the $n$ th component of $V$	
$A^i$	the $i$ th element of $A$	
$B^i$	the $i$ th element of $B$	
$M^{(i,j)}$	the $i, j$ th element of $M$	uncommon, shorthand preferred
$P^b$	the $b$ th element of $P$	
$S a$	whether $a$ is element of $S$	
$M' a$	the number of elements $a$ in $M'$	
$G a$	whether vertex $a$ is in $G$	
$G^{(a,b)}$	the number of edges from $a$ to $b$ in $G$	uncommon, shorthand preferred

## shorthands

shorthand	definition	notes
$a \nvdash b, a \neq b, a \not\leq b, a \not\prec b...$	$/(a \vdash b), /a = b, /a \leq b, /a < b...$	
$x\omega$	$[x]\omega$	
$ax$	$a \setminus x$	

shorthand	definition	notes
$f E$	$f \leftarrow E$	common, longhand discouraged
$x y \rightarrow E$	$x \rightarrow y \rightarrow E$	
$\langle \rangle$	$\langle \langle \rangle \rangle$	see <a href="#">empty set</a>
$()$	$(( ))$	see <a href="#">multiset</a>
$V^x, V^y, V^z$	$V^0, V^1, V^2$	
$M^{i,j}$	$M^{\langle i,j \rangle}$	common, longhand discouraged
$M^i,$	the $i$ th row of $M$	
$M^j$	the $j$ th column of $M$	
$P = \langle f, t \rangle$	$P^\perp = f \wedge P^\top = t$	see <a href="#">ordered pair</a>
$S = \langle \langle a \cdots b \rangle \rangle$	$S x \equiv x = a \vee \cdots x = b$	see <a href="#">set</a>
$V = (a \cdots b)$	$V^0 = a \wedge \cdots V^n = b$	see <a href="#">vector in rn</a>
$M' = ((1, 2, 2, 2, 3, 3))$	<a href="#">multiset</a> literal	see <a href="#">multiset</a>
$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$	<a href="#">matrix</a> literal	see <a href="#">matrix</a>
$x \rightarrow (a < x < b)$	the interval from $a$ to $b$	
$A \circ B$	$x \rightarrow A x \circ B x$	see <a href="#">rank polymorphism</a>
$A \circ B$	$A x \circ B x$ for all $x$	why not $\wedge A \circ B$ <span>#think</span>
$\circ A$	the <a href="#">reduce function</a> of $A$ with $\circ$	
$A \dot{\circ} B$	the <a href="#">outer product</a> of $A$ and $B$ with $\circ$	
$\delta y - \delta x$	the <a href="#">derivative</a> of $y$ with respect to $x$	$\delta$ should be used instead of $d$
$\int y \mid \delta x$	the <a href="#">antiderivative</a> of $y$ with respect to $x$	$\delta$ should be used instead of $d$

## constants

constant	definition	notes
$\emptyset$	<i>undefined</i>	see <a href="#">improved expression evaluation</a>
$\top$	logical true	
$\perp$	logical false	

constant	definition	notes
$\tau$	the ratio of the circumference of a <a href="#">circle</a> to its radius	using $\pi$ is discouraged
$e$	Euler's constant	see <a href="#">eulers constant</a>
$\iota$	$[\cdot 1]$	see <a href="#">imaginary</a> , using $i$ is discouraged
$\Pi$	the <a href="#">pi function</a>	using $\text{fact}$ is discouraged
$\#$	the size of the range of a <a href="#">function</a>	
$\psi$	the <a href="#">multiset</a> of prime factors of a <a href="#">natural</a>	see <a href="#">psi function in mat2348</a>

# operator properties

in order of high to low precedence

#todo fix asymmetry between  $\neq > \geq < \leq$  and  $\wedge \vee \dashv \vdash \equiv \times$

operator	associativity	unary identity	unary description
$() \langle \rangle \boxed{\phantom{x}}$			
$\boxed{\phantom{x}} \sqcup \sqcap$			
$\mid -$	left	1	inverse
$\delta \sin \# \leftarrow$	right-ish		
$: \cdot \therefore \ddots$	left	0	negation
$\mid -$	left	1	inverse
$\int \dot{\phantom{x}} \dots \rightarrow \bmod$	right-ish		
$\neq > \geq < \leq$	AND	0	is (not) 0
$/$	left	$\top$	<a href="#">boolean logic</a> NOT
$\wedge \vee$	left		
$\dashv \vdash$	left		
$\equiv \times$	AND	$\top$	<a href="#">boolean logic</a> NOT
$,$			

**note:** above,

- $x$  represents [variables](#)
- $x_a^i$  represents subscripts and superscripts
- $\leftarrow$  represents [function](#) application

- $\rightarrow$  represents function literals
- $\boxed{\phantom{x}}$  represents matrix literals

**note:** unary operators have identical precedence to their binary counterparts, but are right associative

**definition** let  $\circ$  be an operator with *AND associativity*. then,

$$a \circ b \circ c \circ \dots \equiv a \circ b \wedge b \circ c \wedge c \circ \dots$$

## variable scope

variable scope is currently entirely context dependent. this is know to cause occasional issues, such as with derivatives:  $\delta f x - \delta x$  could represent both the derivative of  $f$  with respect to  $x$  in the general sense, or the derivative of  $f$  with respect to  $x$  **at the point**  $x$

## examples

description	in this <u>math notation</u>	in <u>conventional math notation</u>
the <u>quadratic formula</u>	$x = .b : [b^2 \cdot 4ac] - 2a$	$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
definition of <u>complex</u> numbers	$\mathbb{C}x \equiv x = a : b \wedge \mathbb{R}a \wedge \mathbb{R}b$	$\mathbb{C} = \{a + bi : a, b \in \mathbb{R}\}$
the <u>gaussian function</u>	$G^\sigma p = -[\tau\sigma^2] - e[:p^2 - 2\sigma^2]$	$G(x, y, \dots) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2+y^2+\dots}{2\sigma^2}}$
<u>limit</u> form of a <u>derivative</u>	$f x \cdot f a - x \cdot a : x \rightarrow a$	$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$
definition of factorials	$\text{fact } n = 1 \mid \dots n$	$n! = \prod_{i=1}^n i$
the resonant <u>frequency</u> of an LC circuit	$f = -\tau[LC]$	$f = \frac{1}{2\pi\sqrt{LC}}$
definition of the <u>dot product</u>	$:xy$	$x \cdot y = \sum_{i=1}^n x_i y_i$
definition of the <u>outer product</u>	$x \mid y$	$(x \otimes y)_{i,j} = x_i \times y_j$
definition of the <u>cartesian product</u>	$x ; y$	$x \times y = \{(x, y) \mid x \in X \text{ and } y \in Y\}$
definition of <u>vector</u> in <u>rn &gt; magnitude</u>	$[:v^2]$	$ v  = \sqrt{x^2 + y^2 + \dots}$ with $v = (x, y, \dots)$

description	in this <u>math notation</u>	in <u>conventional math notation</u>
definition of <u>set difference</u>	$A/B \equiv A \wedge \times B$	$A \setminus B = \{x \in A : x \notin B\}$

definition of the boolean logic > implication / set > subset / set > superset / quantifier > universal quantifier in this math notation:  $a \vdash b \equiv /a \vee b$  and  $a \dashv b \equiv a \vee /b$

in set theory, if  $U$  is a set > subset of  $V$  and  $V$  is a set > subset of  $U$ , then  $V$  is  $U$ . in this math notation:  $(U \ x \vdash V \ x) \wedge (U \ x \dashv V \ x) \equiv U = V$

the negation of a boolean logic > implication in this math notation:  $B \vdash C \times B / C$

in conventional math notation:  $\neg(B \rightarrow C) = B \wedge \neg C$  or  $(a \in B \rightarrow a \in C) \iff a \notin B \setminus C$  or  $B \subset C \iff \forall a \in C, a \notin B$