Complex

the [set](#) of [complex](#) numbers

see [math notation](#)

definition

$$\mathbb{C}x \equiv x = a : b\iota \land \mathbb{R}a \land \mathbb{R}b$$

notations

Cartesian Form $z = a : b\iota$

Polar Form $z = |z|\,\cos\theta : |z|\,\iota\sin\theta = |z|\,\text{cis}\,\theta = |z|\,e[\iota\theta]$, see [eulers constant](#), where $\text{cis} = \cos : \iota\sin = \theta \to e[\iota\theta]$

> **aka** Euler's formula notation

applications

[complex](#) numbers are often intimately related to [discrete mathematics](#) — 3B1B
https://youtu.be/bOXCLR3Wric

properties

$\mathbb{C} \vdash \mathbb{U}$, see [universal](#)

equality $a : b\iota = c : d\iota \equiv a = c \land b = d$

addition $(a : b\iota) : (c : d\iota) = (a : c) : (b : d)\iota$

subtraction $(a : b\iota) \cdot (c : d\iota) = (a \cdot c) : (b \cdot d)\iota$

multiplication

in cartesian form, $a : b\iota \mid c : d\iota = ac : ad\iota : b\iota c : bd\iota 2 = (ac \cdot bd) : (ad : bc)\iota$

in polar form, $z \mid w = |z|\,e[\iota\theta] \mid |w|\,e[\iota\phi] = |zw|\,e[\iota \mid \theta : \phi]$

square root of ι. $\lfloor\iota\rfloor = \ \because \ \mid 1 : \iota - \lfloor 2 \rfloor$ — https://www.youtube.com/watch?v=Z49hXoN4KWg

product of two [complex > conjugate](#)s are product of [complex > modulus](#)es
$a : b\iota \mid a \cdot b\iota = a2 : b2 = |a : b\iota| \ \mid \ |a \cdot b\iota|$ — https://youtu.be/bOXCLR3Wric?t=1522

theorem *De Moivre's Theorem* $[\text{cis}\,\theta]n = \text{cis}\,n\theta \dashv \mathbb{Z}n \dashv \mathbb{R}\theta$ —
https://en.wikipedia.org/wiki/De_Moivre%27s_formula

proof $\operatorname{cis}\theta = e[\iota\theta]$. since $[e[\iota\theta]]n = e[n\iota\theta]$, it must be that $[\operatorname{cis}\theta]n = \operatorname{cis}n\theta$ — me

Real Part

Imaginary Part

let $z = a : b\iota$

definitions

real part of a complex number $z^{re} = a$

imaginary part of a complex number $z^{im} = b$

therefore, $z = z^{re} : \iota z^{im}$

Conjugate

complex > conjugate

definition

let $z = a : b\iota$

then, $\operatorname{conj}z = a \cdot b\iota = z^{re} \cdot \iota z^{im}$ is the *complex conjugate* of z

properties

let $\mathbb{C}z \wedge \mathbb{C}w \wedge \mathbb{R}c$

$\operatorname{conj}(z : w) = \operatorname{conj}z : \operatorname{conj}w$

$\operatorname{conj}cz = c\operatorname{conj}z$

$\operatorname{conj}z_{|}w = \operatorname{conj}z \mid \operatorname{conj}w$

$\operatorname{conj}z\text{-}w = \operatorname{conj}z - \operatorname{conj}w$

$\operatorname{conj}\operatorname{conj}z = z$

$\mathbb{R}z \equiv \operatorname{conj}z = z$

theorem $z\operatorname{conj}z = |z|2 \dashv \mathbb{C}z$

theorem $-z = \operatorname{conj}z - |z|2 \dashv \mathbb{C}z$

applications

multiplying by the conjugate can be used to reduce an expression such as $-4 : 3\iota$

Modulus

> **aka** magnitude, absolute value

definition $|z| = \lfloor z^{re}2 : z^{im}2 \rfloor$ where $|z|$ is the *absolute value* of z.

> **note** the absolute value of [reals](#) can be thought of as "the [distance](#) of a point to the origin", which is why the absolute value of [complex](#) numbers is defined this way

properties

let $\mathbb{C}z \wedge \mathbb{C}w \wedge \mathbb{R}c$

$|z| = |\operatorname{conj} z|$

$|zw| = |z|\,|w|$

$|z - w| = |z| - |w|$

triangle inequality $|z : w| \leq |z| : |w|$

Argument

> **aka** phase

definition the *argument* of a [complex](#) number z is the counterclockwise [angle](#) between the positive [real](#) axis and the [line](#) segment from the origin to the point (z^{re}, z^{im})

definition $z = |z|\, e[\iota \arg z]$ where $\arg z$ is the *argument* of z