

Math Notation

see Conventional Math Notation

this note describes my custom Math Notation, meant to solve inconsistencies in Conventional Math Notation. it is not meant to be a fully formal system of Mathematics; rather, it is built to be easy to understand and intuitive to use by mere humans.

principles

- all equality Operators check for equality and return a Boolean, and it is implied that an Expression on its own must evaluate to \top . this allows for Boolean Logic Operators to be applied on equalities explicitly as opposed informally
- Sets are Functions that return a Boolean (Sets are Predicates). this way, Boolean Logic Operators and Set Operators are one and the same. other Data Structures that work similarly include Vectors, Matrixes, Sequences, Multisets, Ordered Pairs...
- some Operators are identical but have different precedence as "more brackets means more explicit, but less brackets means less complex and less confusing"
- $[a]$ returns both positive and negative square roots ($[q^2] \equiv \pm q$). the same is true for other reciprocals
- superscripts are modifiers (subscripts with special meanings). this distinction is especially useful when working with Forward Propagation and Backpropagation in Neural Networks, for example
- Derivatives are not to be written as y' , but rather as their complete form $\delta y - \delta x$. this makes Calculus Notation way more intuitive
- all indices start at 0, as they always should have

notation

also see Trigonometric Functions and Calculus Notation

operator descriptions

notation	description	notes
$a : b$	addition	
$a \cdot b$	subtraction	

notation	description	notes
\pm and \mp	\pm and \mp	
$a \cdot b$ and $a \mid b$	multiplication	
$a \div b$ and $a \div b$	division	
$[a]b$	exponentiation	represents a power by convention
$a[b]$	exponentiation	represents an exponential by convention
$ a b$	b th root of a	$b = 2$ if b is omitted
$\lceil a \rceil b$	base- b <u>Logarithm</u> of a	$b = e$ if b is omitted
$x \rightarrow E$ where E is an <u>Expression</u>	<u>Function</u> literal	$f = x \rightarrow E \equiv f \leftarrow x = E$
$f \leftarrow E$ where E is an <u>Expression</u>	<u>Function</u> application	uncommon, shorthand is preferred
$a = b$	equality	numerical equality by convention
$a < b$ and $a > b$	strict inequality	
$a \leq b$ and $a \geq b$	non-strict inequality	
$a \wedge b$	logical AND	
$a \vee b$	logical OR	
$a \div b$	logical difference	$a \wedge b = \perp$
$a \equiv b$	equality	logical equality by convention
$a \times b$	nonequality	also serves as logical XOR
$a \vdash b$	implication, subset	a implies b , b for all a
$a \dashv b$	reverse implication, superset	a for all b , b implies a
$x_0 \mid x_1 \mid \dots x_n$ where \mid is any <u>Operator</u>	with $n = 3$, $x_0 \mid x_1 \mid x_2 \mid x_3$	step size is $\therefore 1$ if $x_1 \mid$ is omitted
$x_0 \dots x_n$	with $n = 3$, x_0, x_1, x_2, x_3	step size is $\therefore 1$ if x_1 is omitted
x_{sub}	the variable x with a subscript $_{sub}$	
V^n where V is a <u>Vector</u>	the n th component of V	
a^i where a is a <u>Sequence</u>	the i th element of a	
b^i where b is a <u>Series</u>	the i th element of b	

notation	description	notes
$M^{(i,j)}$ where M is a <u>Matrix</u>	the i, j th element of M	uncommon, shorthand is preferred
M^\top where M is a <u>Matrix</u>	the transpose of A	
M^- where M is a <u>Matrix</u>	the multiplicative inverse of A	
P^b where P is an <u>Ordered Pair</u>	the b th element of P	
$S a$ where S is a <u>Set</u>	whether a is element of S	
$M a$ where M is a <u>Multiset</u>	the number of elements a in M	
$G v$ where G is a <u>Graph</u>	whether vertex v is in G	
$G^{(v,w)}$ where G is a <u>Graph</u>	the number of edges from v to w in G	uncommon, shorthand preferred

shorthands

shorthand	definition	notes
$a \nmid b, a \neq b, a \not\leq b, a \not< b...$	$/(a \vdash b), /a = b, /a \leq b, /a < b$...	
$x\omega$ where x is a variable and ω is a <u>Number</u>	$[x]\omega$	
ax where x is a variable	$a \setminus x$	
$f x$ where f is a <u>Function</u>	$f \leftarrow x$	common, longhand is discouraged
$x y \rightarrow E$ where E is an <u>Expression</u>	$x \rightarrow y \rightarrow E$	
V^x, V^y and V^z where V is a <u>Vector</u>	the x, y and z components of V	
$M^{i,j}$ where M is a <u>Matrix</u>	$M^{(i,j)}$	common, longhand is discouraged
M^i , where M is a <u>Matrix</u>	the i th row of M	
M^j where M is a <u>Matrix</u>	the j th column of M	
$S = \{a \dots b\}$	$S x \equiv x = a \vee \dots x = b$	see <u>Set</u>
$P = \langle f, t \rangle$	$P^\perp = f \wedge P^\top = t$	see <u>Ordered Pair</u>
$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$	matrix literal	see <u>Matrix</u>

shorthand	definition	notes
$x \rightarrow (a < x < b)$	the closed interval from a to b	same can be used for open intervals
$A \vdash B$ where \vdash is any #think Operator	$A x \vdash B x$ for all x	commonly $\equiv \Vdash$ #think
$A \cdot B$ where \cdot is any #think Operator	$x \rightarrow A x \cdot B x$	commonly $:\cdot -$ #think
$\delta y - \delta x$	the <u>Derivative</u> of y with respect to x	δ should be used instead of d
$\int y \delta x$	the <u>Antiderivative</u> of y with respect to x	δ should be used instead of d

constants

constant	definition	notes
\emptyset	<i>undefined</i>	see Improved Expression Evaluation
\top	logical true	
\perp	logical false	
τ	the ratio of the circumference of a Circle to its radius	using π is discouraged
e	Euler's constant	see E
i	[.1]	see Imaginary

operator properties

in order of high to low precedence

operator	associativity	unary identity	unary description
$() \{ \} \langle \rangle \square$		$x x_a^i$	
$\square \sqcup \sqcap$			
$\mid -$	left	1	inverse
$\delta \sin \leftarrow$	right-ish		
$:$ \cdot \therefore \therefore	left	0	negation
$ -$	left	1	inverse
$\int \lim \dots \rightarrow$	right		

operator	associativity	unary identity	unary description
$=\neq>\geq<\leq$	AND	0	is (not) 0
/	left	\top	logical NOT
$\wedge \vee$	left		
$\dashv \vdash$	left		
$\equiv \times$	AND	\top	logical NOT
,			

note: above,

- x represents variables
- x_a^i represents subscripts and superscripts
- \leftarrow represents Function application
- \rightarrow represents Function literals
- $\boxed{}$ represents Matrix literals

note: unary Operators have identical precedence to their binary counterparts, but are right associative

definition: let $=$ be an Operator with *AND* associativity. then,
 $a = b = c = \dots \equiv a = b \wedge b = c \wedge c = \dots$

variable scope

variable scope is currently entirely context-dependent. this is know to cause occasional issues, such as with Derivatives: $\delta f\, x - \delta x$ could represent both the Derivative of f with respect to x in the general sense, or the Derivative of f with respect to x **at the point** x as $(x \rightarrow \delta f\, x - \delta x)\, x \equiv \delta f\, x - \delta x$.

examples

Quadratic Formula: $\cdot b : \lfloor b^2 \cdot 4ac \rfloor - 2a$

definition of the Set of Complex numbers: $\mathbb{C}x \equiv x = a : b \lfloor \cdot 1 \rfloor \wedge \mathbb{R}a \wedge \mathbb{R}b$

definition of the implication / subset / superset / “for all” symbol: $a \vdash b \equiv /a \vee b$ and $a \dashv b \equiv a \vee /b$

in Set Theory, if U is a subset of V and V is a subset of U , then V is U . in this math notation:

$$(U \ x \vdash V \ x) \wedge (U \ x \dashv V \ x) \equiv U = V$$

the probability density of the normal distribution in Conventional Math Notation:

$$\frac{1}{\sqrt{2\sigma^2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

compared to in my Math Notation: $-\lceil\tau\sigma^2\rceil - e[[x \cdot \mu]^2 - 2\sigma^2]$

definition of factorials: $\text{fact } n = 1 \mid \dots n$

the negation of an implication in my Math Notation: $B \vdash C \times B / C$ (B implying C equals not (B without C) or *implication is the negation of set difference* or the negation of "for all B , C " is "there exists a B such that not C ")

compared to Conventional Math Notation: $\neg(B \rightarrow C) = B \wedge \neg C$ or $(a \in B \rightarrow a \in C) \iff a \notin B \setminus C$ or $B \subset C \iff \forall a \in C, a \notin B$

see Random Math Notation Formulas for more examples