# Math Notation

see conventional math notation

this note describes my custom math notation, meant to solve inconsistencies in conventional math notation. it is not meant to be a fully formal system of mathematics; rather, it is built to be easy to understand and intuitive to use by mere humans.

## principles

- all equality operators check for equality and return a boolean, and it is implied that an expression on its own must evaluate to $\top$. this allows for boolean logic operators to be applied on equalities explicitly as opposed informally
- sets are functions that return a boolean (sets are predicates). this way, boolean logic operators and set operators are one and the same. other data structures that work similarly include vectors, matrixes, sequences, multisets, ordered pairs...
- some operators are identical but have different precedence as "more brackets means more explicit, but less brackets means less complex and less confusing"
- $\lfloor a \rfloor$ returns both positive and negative square roots ($\lfloor q2 \rfloor \equiv \because q$). the same is true for other reciprocals
- superscripts are modifiers (subscripts with special meanings). this distinction is especially useful when working with forward propagation and backpropagation in neural networks, for example
- derivatives are not to be written as $y'$, but rather as their complete form $\delta y - \delta x$. this makes calculus notation way more intuitive
- all indices start at $0$, as they always should have

## notation

also see trigonometric functions and calculus notation

### operator descriptions

| notation | description | notes |
|---|---|---|
| $a : b$ | addition | |
| $a \cdot b$ | subtraction | |

| **notation** | **description** | **notes** |
|---|---|---|
| $\because$ and $\therefore$ | $\pm$ and $\mp$ | |
| $a{\scriptstyle \vert}b$ and $a \mid b$ | multiplication | |
| $a\text{-}b$ and $a - b$ | division | |
| $[a]b$ | exponentiation | represents a power by convention |
| $a[b]$ | exponentiation | represents an exponential by convention |
| $\lfloor a \rfloor b$ | $b$ th root of $a$ | $b = 2$ if $b$ is omitted |
| $\lceil a \rceil b$ | base-$b$ logarithm of $a$ | $b = e$ if $b$ is omitted |
| $x \to E$ where $E$ is an expression | function literal | $f = x \to E \equiv f \leftarrow x = E$ |
| $f \leftarrow E$ where $E$ is an expression | function application | uncommon, shorthand preferred |
| $a = b$ | equality | numerical equality by convention |
| $a < b$ and $a > b$ | strict inequality | |
| $a \leq b$ and $a \geq b$ | non-strict inequality | |
| $a \wedge b$ | logical AND | |
| $a \vee b$ | logical OR | |
| $a \mathbin{/} b$ | logical difference | $a \wedge b = \bot$ |
| $a \equiv b$ | equality | logical equality by convention |
| $a \times b$ | nonequality | also serves as logical XOR |
| $a \vdash b$ | implication, subset | $a$ implies $b$, $b$ for all $a$ |
| $a \dashv b$ | reverse implication, superset | $a$ for all $b$, $b$ implies $a$ |
| $x_0 \mid x_1 \mid \ldots x_n$ where $\mid$ is any operator | with $n = 3$, $x_0 \mid x_1 \mid x_2 \mid x_3$ | step size is $\because 1$ if $x_1 \mid$ is omitted |
| $x_0 \ldots x_n$ | with $n = 3$, $x_0, x_1, x_2, x_3$ | step size is $\because 1$ if $x_1$ is omitted |
| $x_{sub}$ | the variable $x$ with a subscript $_{sub}$ | |
| $V^n$ where $V$ is a vector | the $n$ th component of $V$ | |
| $a^i$ where $a$ is a sequence | the $i$ th element of $a$ | |
| $b^i$ where $b$ is a series | the $i$ th element of $b$ | |

| notation | description | notes |
|---|---|---|
| $M^{\langle i,j\rangle}$ where $M$ is a matrix | the $i, j$ th element of $M$ | uncommon, shorthand preferred |
| $M^{\intercal}$ where $M$ is a matrix | the transpose of $A$ | |
| $M^{-}$ where $M$ is a matrix | the multiplicative inverse of $A$ | |
| $P^{b}$ where $P$ is an ordered pair | the $b$ th element of $P$ | |
| $S\ a$ where $S$ is a set | whether $a$ is element of $S$ | |
| $M\ a$ where $M$ is a multiset | the number of elements $a$ in $M$ | |
| $G\ v$ where $G$ is a graph | whether vertex $v$ is in $G$ | |
| $G^{\langle v,w\rangle}$ where $G$ is a graph | the number of edges from $v$ to $w$ in $G$ | uncommon, shorthand preferred |

## shorthands

| shorthand | definition | notes |
|---|---|---|
| $a \nvdash b$, $a \neq b$, $a \nleq b$, $a \nless b$... | $/(a \vdash b)$, $/a = b$, $/a \leq b$, $/a < b$ ... | |
| $x\omega$ where $x$ is a variable and $\omega$ is a number | $[x]\omega$ | |
| $ax$ where $x$ is a variable | $a{\shortmid}x$ | |
| $f\ x$ where $f$ is a function | $f{\leftarrow}x$ | common, longhand discouraged |
| $x\ y \to E$ where $E$ is an expression | $x \to y \to E$ | |
| $V^{x}$, $V^{y}$ and $V^{z}$ where $V$ is a vector | the $x$, $y$ and $z$ components of $V$ | |
| $M^{i,j}$ where $M$ is a matrix | $M^{\langle i,j\rangle}$ | common, longhand discouraged |
| $M^{i,}$ where $M$ is a matrix | the $i$ th row of $M$ | |
| $M^{,j}$ where $M$ is a matrix | the $j$ th column of $M$ | |
| $S = \langle\langle a \ldots b\rangle\rangle$ | $S\ x \equiv x = a \vee \ldots x = b$ | see set |
| $P = \langle f, t\rangle$ | $P^{\perp} = f \wedge P^{\top} = t$ | see ordered pair |
| $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ | matrix literal | see matrix |

| shorthand | definition | notes |
|---|---|---|
| $x \to (a < x < b)$ | the closed interval from $a$ to $b$ | same can be used for open intervals |
| $A \vdash B$ where $\vdash$ is any #think operator | $A\,x \vdash B\,x$ for all $x$ | commonly $\equiv\dashv\Vdash$ #think |
| $A \cdot B$ where $\cdot$ is any #think operator | $x \to A\,x \cdot B\,x$ | commonly $: \cdot \mid -$ #think |
| $\delta y - \delta x$ | the derivative of $y$ with respect to $x$ | $\delta$ should be used instead of $d$ |
| $\int y \mid \delta x$ | the antiderivative of $y$ with respect to $x$ | $\delta$ should be used instead of $d$ |

## constants

| constant | definition | notes |
|---|---|---|
| $\varnothing$ | *undefined* | see improved expression evaluation |
| $\top$ | logical true | |
| $\bot$ | logical false | |
| $\tau$ | the ratio of the circumference of a circle to its radius | using $\pi$ is discouraged |
| $e$ | Euler's constant | see eulers constant |
| $i$ | $\lfloor \cdot 1 \rfloor$ | see imaginary |

## operator properties

*in order of high to low precedence*

| operator | associativity | unary identity | unary description |
|---|---|---|---|
| $() \ \langle\rangle \ \Big[\Big] \ x \ x_a^i$ | | | |
| $[] \ \lfloor\rfloor \ \lceil\rceil$ | | | |
| $\scriptstyle \mid -$ | left | 1 | inverse |
| $\delta \ \sin \ \leftarrow$ | right-ish | | |
| $: \ \cdot \ \because \ \therefore$ | left | 0 | negation |
| $\mid -$ | left | 1 | inverse |
| $\int \lim \ \ldots \ \to$ | right | | |

| operator | associativity | unary identity | unary description |
|---|---|---|---|
| $= \neq > \geq < \leq$ | AND | 0 | is (not) 0 |
| $/$ | left | $\top$ | logical NOT |
| $\wedge\ \vee$ | left | | |
| $\dashv\ \vdash$ | left | | |
| $\equiv\ \times$ | AND | $\top$ | logical NOT |
| , | | | |

> **note**: above,
>
> - $x$ represents variables
> - $x_a^i$ represents subscripts and superscripts
> - $\leftarrow$ represents function application
> - $\rightarrow$ represents function literals
> - $\begin{bmatrix} \end{bmatrix}$ represents matrix literals

> **note**: unary operators have identical precedence to their binary counterparts, but are right associative

> **definition**: let $=$ be an operator with *AND* associativity. then, $a = b = c = \ldots \quad \equiv \quad a = b \wedge b = c \wedge c = \ldots$

# variable scope

variable scope is currently entirely context-dependent. this is know to cause occasional issues, such as with derivatives: $\delta\ f\ x - \delta x$ could represent both the derivative of $f$ with respect to $x$ in the general sense, or the derivative of $f$ with respect to $x$ **at the point** $x$ as $(x \rightarrow \delta\ f\ x - \delta x)\ x \equiv \delta\ f\ x - \delta x$.

# examples

quadratic formula: $\cdot b : \lfloor b2 \cdot 4ac \rfloor - 2a$

definition of the set of complex numbers: $\mathbb{C}x \equiv x = a : b\lfloor \cdot 1 \rfloor \wedge \mathbb{R}a \wedge \mathbb{R}b$

definition of the implication / subset / superset / "for all" symbol: $a \vdash b \equiv /a \vee b$ and $a \dashv b \equiv a \vee /b$

in set theory, if $U$ is a subset of $V$ and $V$ is a subset of $U$, then $V$ is $U$. in this math notation: $(U\ x \vdash V\ x) \wedge (U\ x \dashv V\ x) \equiv U = V$

the probability density of the normal distribution in conventional math notation: $\frac{1}{\sqrt{2\sigma^2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

compared to in my math notation: $-\lfloor \tau\sigma 2 \rfloor - e[\ [x \cdot \mu]2 - 2\sigma 2\ ]$

definition of factorials: $\mathsf{fact}\, n = 1 \mid \ldots n$

the negation of an implication in my math notation: $B \vdash C \times B \mathbin{/} C$ (*$B$ implying $C$ equals not ($B$ without $C$)* or *implication is the negation of set difference* or *the negation of "for all B, C" is "there exists a B such that not C"*)

compared to conventional math notation: $\neg(B \to C) = B \wedge \neg C$ or $(a \in B \to a \in C) \iff a \notin B \backslash C$ or $B \subset C \iff \forall a \in C, a \notin B$

the resonant frequency of an LC circuit in conventional math notation: $f = \frac{1}{2\pi\sqrt{LC}}$

compared to in my math notation: $f = -\tau\lfloor LC \rfloor$

see random math notation formulas for more examples