

# Math Notation

see [Classical Math Notation](#)

this note describes my custom [Math Notation](#), meant to solve inconsistencies in [Classical Math Notation](#). it is not meant to be a fully formal system of [Mathematics](#); rather, it is built to be easy to understand and intuitive to use by mere humans.

## principles

- all equality [Operators](#) check for equality and return a [Boolean](#), and it is implied that an [Expression](#) on its own must evaluate to  $\top$ . this allows for [Boolean Logic Operators](#) to be applied on equalities explicitly as opposed informally
- [Sets](#) are [Functions](#) that return a [Boolean](#) ([Sets](#) are [Predicates](#)). this way, [Boolean Logic Operators](#) and [Set Operators](#) are one and the same. other [Data Structures](#) that work similarly include [Vectors](#), [Matrixes](#), [Sequences](#), [Multisets](#), [Ordered Pairs](#)...
- some [Operators](#) are identical but have different precedence as "more brackets means more explicit, but less brackets means less complex and less confusing"
- $[a]$  returns both positive and negative square roots ( $[q^2] \equiv \pm q$ ). the same is true for other reciprocals
- superscripts are modifiers (subscripts with special meanings). this distinction is especially useful when working with [Forward Propagation](#) and [Backpropagation](#) in [Neural Networks](#), for example
- [Derivatives](#) are not to be written as  $y'$ , but rather as their complete form  $\delta y - \delta x$ . this makes [Calculus Notation](#) way more intuitive
- all indices start at 0, as they always should have

## notation

also see [Trigonometric Functions](#) and [Calculus Notation](#)

## operator descriptions

notation	description	notes
$a : b$	addition	
$a \cdot b$	subtraction	

notation	description	notes
$\pm$ and $\mp$	$\pm$ and $\mp$	
$a \cdot b$ and $a \mid b$	multiplication	
$a \div b$ and $a \div b$	division	
$[a]b$	exponentiation	represents a power by convention
$a[b]$	exponentiation	represents an exponential by convention
$ a b$	$b$ th root of $a$	$b = 2$ if $b$ is omitted
$\lceil a \rceil b$	base- $b$ <u>Logarithm</u> of $a$	$b = e$ if $b$ is omitted
$x \rightarrow E$ where $E$ is an <u>Expression</u>	<u>Function</u> literal	$f = x \rightarrow E \equiv f \leftarrow x = E$
$f \leftarrow E$ where $E$ is an <u>Expression</u>	<u>Function</u> application	uncommon, shorthand is preferred
$a = b$	equality	numerical equality by convention
$a < b$ and $a > b$	strict inequality	
$a \leq b$ and $a \geq b$	non-strict inequality	
$a \wedge b$	logical AND	
$a \vee b$	logical OR	
$a \div b$	logical difference	$a \wedge b = \perp$
$a \equiv b$	equality	logical equality by convention
$a \times b$	nonequality	also serves as logical XOR
$a \vdash b$	implication, subset	$a$ implies $b$ , $b$ for all $a$
$a \dashv b$	reverse implication, superset	$a$ for all $b$ , $b$ implies $a$
$x_0 \mid x_1 \mid \dots x_n$ where $\mid$ is any <u>Operator</u>	with $n = 3$ , $x_0 \mid x_1 \mid x_2 \mid x_3$	step size is $\therefore 1$ if $x_1 \mid$ is omitted
$x_0 \dots x_n$	with $n = 3$ , $x_0, x_1, x_2, x_3$	step size is $\therefore 1$ if $x_1$ is omitted
$x_{sub}$	the variable $x$ with a subscript $_{sub}$	
$V^n$ where $V$ is a <u>Vector</u>	the $n$ th component of $V$	
$a^i$ where $a$ is a <u>Sequence</u>	the $i$ th element of $a$	
$b^i$ where $b$ is a <u>Series</u>	the $i$ th element of $b$	

notation	description	notes
$M^{(i,j)}$ where $M$ is a <u>Matrix</u>	the $i, j$ th element of $M$	uncommon, shorthand is preferred
$M^\top$ where $M$ is a <u>Matrix</u>	the transpose of $A$	
$M^-$ where $M$ is a <u>Matrix</u>	the multiplicative inverse of $A$	
$P^b$ where $P$ is an <u>Ordered Pair</u>	the $b$ th element of $P$	
$S a$ where $S$ is a <u>Set</u>	whether $a$ is element of $S$	
$M a$ where $M$ is a <u>Multiset</u>	the number of elements $a$ in $M$	
$G v$ where $G$ is a <u>Graph</u>	whether vertex $v$ is in $G$	
$G^{(v,w)}$ where $G$ is a <u>Graph</u>	the number of edges from $v$ to $w$ in $G$	uncommon, shorthand preferred

## shorthands

shorthand	definition	notes
$a \nmid b, a \neq b, a \not\leq b, a \not< b...$	$/(a \vdash b), /a = b, /a \leq b, /a < b$ ...	
$x\omega$ where $x$ is a variable and $\omega$ is a <u>Number</u>	$[x]\omega$	
$ax$ where $x$ is a variable	$a \setminus x$	
$f x$ where $f$ is a <u>Function</u>	$f \leftarrow x$	common, longhand is discouraged
$x y \rightarrow E$ where $E$ is an <u>Expression</u>	$x \rightarrow y \rightarrow E$	
$V^x, V^y$ and $V^z$ where $V$ is a <u>Vector</u>	the $x, y$ and $z$ components of $V$	
$M^{i,j}$ where $M$ is a <u>Matrix</u>	$M^{(i,j)}$	common, longhand is discouraged
$M^i$ , where $M$ is a <u>Matrix</u>	the $i$ th row of $M$	
$M^j$ where $M$ is a <u>Matrix</u>	the $j$ th column of $M$	
$S = \{a \dots b\}$	$S x \equiv x = a \vee \dots x = b$	see <u>Set</u>
$P = \langle f, t \rangle$	$P^\perp = f \wedge P^\top = t$	see <u>Ordered Pair</u>
$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$	matrix literal	see <u>Matrix</u>

shorthand	definition	notes
$x \rightarrow (a < x < b)$	the closed interval from $a$ to $b$	same can be used for open intervals
$A \vdash B$ where $\vdash$ is any <a href="#">#think Operator</a>	$A x \vdash B x$ for all $x$	commonly $\equiv \Vdash$ <a href="#">#think</a>
$A \cdot B$ where $\cdot$ is any <a href="#">#think Operator</a>	$x \rightarrow A x \cdot B x$	commonly $:\cdot  -$ <a href="#">#think</a>
$\delta y - \delta x$	the <u>Derivative</u> of $y$ with respect to $x$	$\delta$ should be used instead of $d$
$\int y   \delta x$	the <u>Antiderivative</u> of $y$ with respect to $x$	$\delta$ should be used instead of $d$

### constants

constant	definition	notes
$\emptyset$	<i>undefined</i>	see <a href="#">Improved Expression Evaluation</a>
$\top$	logical true	
$\perp$	logical false	
$\tau$	the ratio of the circumference of a <a href="#">Circle</a> to its radius	using $\pi$ is discouraged
$e$	Euler's constant	see <a href="#">E</a>
$i$	[.1]	see <a href="#">Imaginary</a>

### operator properties

*in order of high to low precedence*

operator	associativity	unary identity	unary description
$() \{ \} \langle \rangle \square$		$x x_a^i$	
$\square \sqcup \sqcap$			
$\mid -$	left	1	inverse
$\delta \sin \leftarrow$	right-ish		
$:$ $\cdot$ $\therefore$ $\therefore$	left	0	negation
$  -$	left	1	inverse
$\int \lim \dots \rightarrow$	right		

operator	associativity	unary identity	unary description
$=\neq>\geq<\leq$	AND	0	is (not) 0
/	left	$\top$	logical NOT
$\wedge \vee$	left		
$\dashv \vdash$	left		
$\equiv \times$	AND	$\top$	logical NOT
,			

**note:** above,

- $x$  represents variables
- $x^i_a$  represents subscripts and superscripts
- $\leftarrow$  represents Function application
- $\rightarrow$  represents Function literals
- $\boxed{\phantom{x}}$  represents Matrix literals

**note:** unary Operators have identical precedence to their binary counterparts, but are right associative

**definition:** let  $\equiv$  be an Operator with *AND* associativity. then,  
 $a = b = c = \dots \equiv a = b \wedge b = c \wedge c = \dots$

## variable scope

variable scope is currently entirely context-dependent. this is know to cause occasional issues, such as with Derivatives:  $\delta f x - \delta x$  could represent both the Derivative of  $f$  with respect to  $x$  in the general sense, or the Derivative of  $f$  with respect to  $x$  **at the point**  $x$  as  $(x \rightarrow \delta f x - \delta x) x \equiv \delta f x - \delta x$ .

## examples

Quadratic Formula:  $\cdot b : \lfloor b^2 \cdot 4ac \rfloor - 2a$

definition of the Set of Complex numbers:  $\mathbb{C}x \equiv x = a : b \lfloor \cdot 1 \rfloor \wedge \mathbb{R}a \wedge \mathbb{R}b$

definition of the implication / subset / superset / “for all” symbol:  $a \vdash b \equiv /a \vee b$  and  $a \dashv b \equiv a \vee /b$

in Set Theory, if  $U$  is a subset of  $V$  and  $V$  is a subset of  $U$ , then  $V$  is  $U$ . in this math notation:

$$(U \times V \cap V \times U) \cap (U \times V \cup V \times U) \equiv U = V$$

the probability density of the normal distribution in Classical Math Notation:  $\frac{1}{\sqrt{2\sigma^2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

compared to in my Math Notation:  $-\lceil \tau \sigma^2 \rceil - e[\lceil x \cdot \mu \rceil^2 - 2\sigma^2]$

definition of factorials:  $\text{fact } n = 1 \mid \dots n$

the negation of an implication in my Math Notation:  $B \vdash C \times B / C$  ( $B$  implying  $C$  equals not ( $B$  without  $C$ ) or implication is the negation of set difference or the negation of "for all  $B$ ,  $C$ " is "there exists a  $B$  such that not  $C$ ")

compared to Classical Math Notation:  $\neg(B \rightarrow C) = B \wedge \neg C$  or  $(a \in B \rightarrow a \in C) \iff a \notin B \setminus C$  or  $B \subset C \iff \forall a \in C, a \notin B$

see Random Math Notation Formulas for more examples