Localization of Submerged Sensors with a Single Beacon for Non-Parallel Planes State

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***Abstract*— This paper delineates a new method of determining the coordinates of sensors with a single beacon for both the parallel and non-parallel state situations. Cayley-Menger determinant is mostly used when the beacon and deployed sensors are in the state of parallel planes. We have, however, proposed a mathematical model to compute the coordinates of the submerged sensors even when these sensors and beacon are in non-parallel state configuration. As the knowledge of precise coordinates of the sensors is as important as the collected data in underwater wireless sensor networks, the measure of exact distance between the nodes becomes the prime factor for improved accuracy. The proposed mathematical model of coordinate-determination has better immunity from multipath fading and linearization process of non-linear equations, resulting in more precise location of the sensors. Moreover, a single beacon is used to determine the coordinates of the sensor nodes where none of them have *a priori* knowledge about their locations.**

Keywords—localization; submerged sensors; Cayley-Menger determinant; coordinates; non-parallel state; underwater wireless network

I. Introduction

Not long past since researchers have shown fervent interest to explore and fulfil the needs of a multitude of underwater applications, even though terrestrial wireless communication applications have established its prominence. Underwater wireless sensor network (UWSN) is envisioned to enable application for oceanographic data collection and offshore exploration for the profusion of wealth underwater world has. It's not only the wealth; marine life helps determine the very nature of our planet at a very fundamental level and for its sustenance. It, therefore, becomes crucial to obtain accurate environmental data using underwater sensors to provide and maintain sanctuary for the marine life. Besides, advanced robot navigation, autonomous underwater vehicle control and surveillance, finding lost objects, estuary monitoring and pollutant tracking also require accurate localization [1]. In addition to underwater sensors, UWSN may also comprise of surface stations and autonomous underwater vehicles. Regardless of the type of deployment and configuration, the location of sensors needs to be determined for meaningful interpretation of the sensed data [2].

Despite varieties of application of UWSN, idea of submerged wireless communication may still seem far-fetched and got attention of researchers since last decades. As positions of submerged sensors play a vital role for the significance of validity of data, determining the coordinate precisely is very crucial. In UWSNs, acoustic channels are naturally exploited as range measurements using acoustic signals are much more accurate than using radio signals [1, 3]. However, impacts of multipath effects and accurate time synchronization have been the most challenging factors.

In [4], Duff and Muller proposed a method to solve the multilateration equations by means of nonlinear least square optimization when no positions are known. The algorithm is based on a degree-of-freedom analysis – which states enough measurement from different positions will provide enough equations to solve the problem. In [5], a technique is used incorporating extended Kalman filter. However, the degree-of-freedom analysis does not guarantee a unique solution in a system of nonlinear equations, such as trilateration, when the only data available is the distance measured between the nodes [6]. In [7], Guevara et al. introduced a new closed-form solution where no position information of any nodes is required to determine the positions of multiple static beacon nodes, the only information they used is the distance measurement between static beacons and mobile node.

Having critically analyzed the various studies discussed above, in this paper we propose a closed-form solution to determine the coordinates of the submerged sensors having only one beacon node at the surface using both radio and acoustic signals. The precise conditions for obtaining initial subsets of nodes were justified using rigidity theory in [6]. Section II explains the proposed mathematical model to determine the coordinates of sensors for both parallel and non-parallel state situation. Section III covers the simulation and discussions, and finally conclusions are given in Section IV.

II. Coordinates Computation

A. Coordinates with Respect to the Sensor (Parallel plane state)

The objective of localization algorithms is to obtain the exact positions of all the submerged sensors by measuring distances between beacon and nodes. Only measurement available here to compute is the distance and typically it is considered as optimization problem where objective functions to be minimized have residuals of the distance equations. The variables of any localization problem are the 3D coordinates of

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the nodes, where in principle more number of distance equations is needed than number of variables to solve. However, this approach known as degree-of-freedom analysis may not guarantee the unique solution for a nonlinear system.

Trilateration or multilateration techniques are usually applied in nonlinear system to determine the locations or coordinates of the sensors in partial or full. According to Guevara et al. [7] the convergence of optimization algorithms and Bayesian methods depend heavily on initial conditions used and they circumvent the convergence problem by linearizing the trilateration equations.

Fig. 1 shows the initial subset composed of the beacon node $S_j, j = 4,5....9$ and three sensor nodes $S_i, i = 1,2,3$. Without loss of generality, a coordinate system can be defined using one of the sensor nodes S_i as the origin $(0,0,0)$ of the coordinate system. The following method of coordinates determination of submerged sensors with a single beacon have been devised in [8].

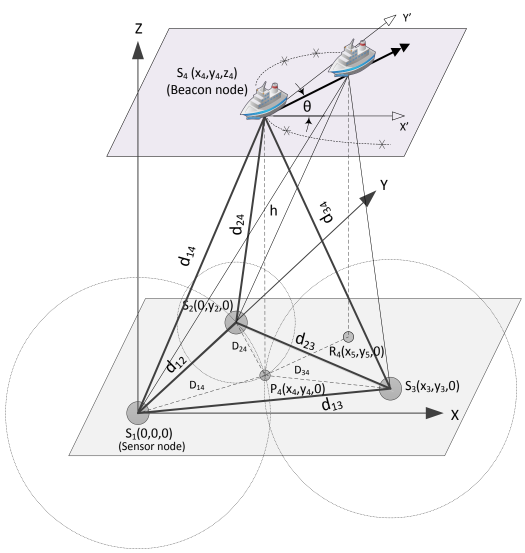


Fig. 1. Coordinate determination

The distances between beacon and sensors $d_{14}, d_{24}, d_{34}....$ which are known (measured) data, and inter-sensor distances d_{12}, d_{13}, d_{23} and the volume of tetrahedron V_t (formed by the beacon and sensors) are unknown. Based on the local positioning system configuration of Fig. 1, we need to write equations that will include all known and unknown distances. For that matter, we express the volume of tetrahedron V_t using Cayley-Menger determinant as following [8]:

$$288 V_t^2 = \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & d_{12}^2 & d_{13}^2 & d_{14}^2 \\ 1 & d_{12}^2 & 0 & d_{23}^2 & d_{24}^2 \\ 1 & d_{13}^2 & d_{23}^2 & 0 & d_{34}^2 \\ 1 & d_{14}^2 & d_{24}^2 & d_{34}^2 & 0 \end{vmatrix} \tag{1}$$

By expanding (1), we obtain:

$$d_{34}^2 d_{23}^2 - d_{34}^2 d_{12}^2 + d_{34}^2 d_{13}^2 - \frac{d_{14}^2 d_{23}^4}{d_{12}^2} + d_{23}^2 d_{14}^2 + \frac{d_{13}^2 d_{14}^2 d_{23}^2}{d_{12}^2} - \frac{d_{24}^2 d_{13}^4}{d_{12}^2} + \frac{d_{13}^2 d_{23}^2 d_{24}^2}{d_{12}^2} +$$
$$d_{13}^2 d_{24}^2 - d_{13}^2 d_{23}^2 - 144\frac{V_t^2}{d_{12}^2} + \frac{d_{14}^2 d_{23}^2 d_{24}^2}{d_{12}^2} + \frac{d_{14}^2 d_{23}^2 d_{34}^2}{d_{12}^2} - \frac{d_{23}^2 d_{24}^2 d_{34}^2}{d_{12}^2} - \frac{d_{14}^4 d_{23}^2}{d_{12}^2} + \frac{d_{13}^2 d_{24}^2 d_{34}^2}{d_{12}^2}$$
$$- \frac{d_{13}^2 d_{14}^2 d_{34}^2}{d_{12}^2} + \frac{d_{13}^2 d_{14}^2 d_{24}^2}{d_{12}^2} - \frac{d_{13}^2 d_{24}^4}{d_{12}^2} - d_{34}^4 + d_{24}^2 d_{34}^2 + d_{14}^2 d_{34}^2 - d_{14}^2 d_{24}^2 = 0$$

Grouping known and unknown variables, we get

$\left(\frac{d_{23}^4}{d_{12}^2} - d_{23}^2 - \frac{d_{13}^2 d_{23}^2}{d_{12}^2}\right)$, $\frac{d_{23}^2}{d_{12}^2}$, $\frac{d_{13}^2}{d_{12}^2}$, $\left(\frac{d_{13}^4}{d_{12}^2} - \frac{d_{13}^2 d_{23}^2}{d_{12}^2} - d_{13}^2\right)$,

$\left(d_{12}^2 - d_{23}^2 - d_{13}^2\right)$, and $\left(144\frac{V_t^2}{d_{12}^2} + d_{13}^2 d_{23}^2\right)$ as unknown terms.

So, the above expansion can be rewritten as follows:

$$d_{14}^2 X_1 + d_{24}^2 X_2 + d_{34}^2 X_3 - \left(d_{14}^2 - d_{34}^2\right)\left(d_{24}^2 - d_{14}^2\right) X_4$$
$$- \left(d_{24}^2 - d_{14}^2\right)\left(d_{34}^2 - d_{24}^2\right) X_5 + X_6 = \left(d_{24}^2 - d_{34}^2\right)\left(d_{34}^2 - d_{14}^2\right) \tag{2}$$

Equation (2) in fact resembles the linear form of $a_1 x_1 + a_2 x_2 + \cdots + a_n x_n = b_1$, where we have six unknowns. So we need at least six measurements to solve this system of linear equations, which could be done following the same procedure steering the beacon node $S_j, j = 4,5....9$ to six different locations and measuring the distances in the vicinity of S_4 as in Figure 1.

Then, the system of equations can be written as: AX=b, where

$$A = \begin{bmatrix} d_{14}^2 & d_{24}^2 & d_{34}^2 & -\left(d_{14}^2 - d_{34}^2\right)\left(d_{24}^2 - d_{14}^2\right) & -\left(d_{24}^2 - d_{14}^2\right)\left(d_{34}^2 - d_{24}^2\right) & 1 \\ d_{15}^2 & d_{25}^2 & d_{35}^2 & -\left(d_{15}^2 - d_{35}^2\right)\left(d_{25}^2 - d_{15}^2\right) & -\left(d_{25}^2 - d_{15}^2\right)\left(d_{35}^2 - d_{25}^2\right) & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ d_{19}^2 & d_{29}^2 & d_{39}^2 & -\left(d_{19}^2 - d_{39}^2\right)\left(d_{29}^2 - d_{19}^2\right) & -\left(d_{29}^2 - d_{19}^2\right)\left(d_{39}^2 - d_{29}^2\right) & 1 \end{bmatrix}$$

$$X = \begin{bmatrix} \left(\frac{d_{23}^4}{d_{12}^2} - d_{23}^2 - \frac{d_{13}^2 d_{23}^2}{d_{12}^2}\right) \\ \left(\frac{d_{13}^4}{d_{12}^2} - \frac{d_{13}^2 d_{23}^2}{d_{12}^2} - d_{13}^2\right) \\ \left(d_{12}^2 - d_{23}^2 - d_{13}^2\right) \\ \frac{d_{23}^2}{d_{12}^2} \\ \frac{d_{13}^2}{d_{12}^2} \\ \left(144\frac{V_t^2}{d_{12}^2} + d_{13}^2 d_{23}^2\right) \end{bmatrix} \qquad b = \begin{bmatrix} \left(d_{24}^2 - d_{34}^2\right)\left(d_{34}^2 - d_{14}^2\right) \\ \left(d_{25}^2 - d_{35}^2\right)\left(d_{35}^2 - d_{15}^2\right) \\ \vdots \\ \left(d_{29}^2 - d_{39}^2\right)\left(d_{29}^2 - d_{19}^2\right) \end{bmatrix}$$

From the above representation, after finding X_1, X_2, X_3, X_4, X_5 and X_6 we calculate d_{12}, d_{13} and d_{23} as follows:

$$d_{12}^2 = \frac{X_3}{\left(1 - X_4 - X_5\right)}, \quad d_{13}^2 = \frac{X_3 X_5}{\left(1 - X_4 - X_5\right)}, \quad d_{23}^2 = \frac{X_3 X_4}{\left(1 - X_4 - X_5\right)}$$

If we let the coordinates of the submerged sensors S_1, S_2 and S_3 to be $(0,0,0)$, $(0, y_2, 0)$ and $(x_3, y_3, 0)$ respectively, then the inter-sensor distances could be written with respect to coordinates of the sensors as follows:

$$d_{12}^2 = y_2^2,\ d_{13}^2 = x_3^2 + y_3^2,\ d_{23}^2 = x_3^2 + (y_3 - y_2)^2$$

From the above values, the unknown variables can be computed as:

$$y_2 = d_{12},\ y_3 = \frac{d_{12}^2 + d_{13}^2 - d_{23}^2}{2d_{12}},\ x_3 = \sqrt{\left(d_{13}^2 - \left(\frac{d_{12}^2 + d_{13}^2 - d_{23}^2}{2d_{12}}\right)^2\right)}$$

where d_{12}, d_{13} and d_{23} are known computed distances. Table I summarizes the coordinates of the sensors for the proposed problem domain.

TABLE I. COORDINATES OF THE SENSORS

Sensors	Coordinates
S_1	$(0,0,0)$
S_2	$(0, d_{12}, 0)$
S_3	$\left(\sqrt{\left(d_{13}^2 - \left(\frac{d_{12}^2 + d_{13}^2 - d_{23}^2}{2d_{12}}\right)^2\right)}, \frac{d_{12}^2 + d_{13}^2 - d_{23}^2}{2d_{12}}, 0\right)$

B. Coordinates with Respect to the Beacon (Parallel plane state)

Up to now we have been able to determine the coordinates of the sensor nodes with respect to S_1. In order to find the coordinate with respect to the beacon node we follow the following steps.

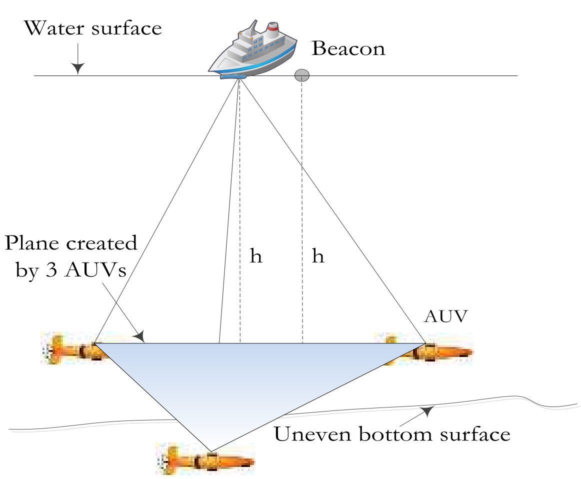


Fig. 2. Parallel plane state scenario

We assume that with the use of appropriate sensors, the depth h in Fig. 2 can be measured as depicted in [9]. After measuring the vertical distance h in between the beacon node $S_4(x_4, y_4, z_4)$ and the XY plane, we can assume the projected coordinate of the beacon node $S_4(x_4, y_4, z_4)$ on the plane XY is $P_4(x_4, y_4, 0)$. To find x_4 and y_4, we can apply trilateration in the following manner assuming the distances between S_1, S_2, S_3 and P_4 are D_{14}, D_{24} and D_{34} respectively and device following relations.

$$D_{14}^2 = x_4^2 + y_4^2 \quad (3)$$

$$D_{24}^2 = x_4^2 + (y_4 - y_2)^2 \quad (4)$$

$$D_{34}^2 = (x_4 - x_3)^2 + (y_4 - y_3)^2 \quad (5)$$

From (3), (4) and (5) we obtain the projected beacon's coordinates $P_4(x_4, y_4, 0)$, where

$$x_4 = \frac{1}{2d_{12}}\sqrt{\left(4d_{12}^2 D_{14}^2 - (D_{14}^2 - D_{24}^2 + d_{12}^2)^2\right)},$$

$$y_4 = \frac{1}{2d_{12}}\left(D_{14}^2 - D_{24}^2 + d_{12}^2\right)$$

As d_{14}, d_{24} and d_{34} are the hypotenuse of the $\Delta S_1 P_4 S_4$, $\Delta S_2 P_4 S_4$ and $\Delta S_3 P_4 S_4$ respectively, so it is possible to obtain D_{14}, D_{24} and D_{34} using Pythagorean theorem. So the coordinate of the beacon node $S_4(x_4, y_4, z_4)$ would be (x_4, y_4, h) where all the elements are known.

$$\therefore S_4(x_4, y_4, h) = S_4\left(\frac{1}{2d_{12}}\sqrt{\left(4d_{12}^2 D_{14}^2 - (D_{14}^2 - D_{24}^2 + d_{12}^2)^2\right)}, \frac{1}{2d_{12}}\left(D_{14}^2 - D_{24}^2 + d_{12}^2\right), h\right)$$

The origin of the Cartesian system is transferred on to the coordinate of the beacon node and sensors coordinates are found with respect to the beacon node S_4.

TABLE II. COORDINATES WITH RESPECT TO THE BEACON (PARALLEL PLANES STATE)

Sensors	Coordinates
S_4	$(0,0,0)$
S_1	$\left(\frac{\sqrt{4d_{12}^2 D_{14}^2 - \left(D_{14}^2 - D_{24}^2 + d_{12}^2\right)^2}}{2d_{12}},\ -\frac{1}{2d_{12}}\left(D_{14}^2 - D_{24}^2 + d_{12}^2\right),\ -h\right)$
S_2	$\left(\frac{\sqrt{4d_{12}^2 D_{14}^2 - \left(D_{14}^2 - D_{24}^2 + d_{12}^2\right)^2}}{2d_{12}},\ \frac{1}{2d_{12}}\left(d_{12}^2 - D_{14}^2 + D_{24}^2\right),\ -h\right)$
S_3	$\left(\left(\sqrt{\left(d_{13}^2 - \left(\frac{d_{12}^2 + d_{13}^2 - d_{23}^2}{2d_{12}}\right)^2\right)} - \frac{\sqrt{4d_{12}^2 D_{14}^2 - \left(D_{14}^2 - D_{24}^2 + d_{12}^2\right)^2}}{2d_{12}}\right), \frac{1}{2d_{12}}\left(d_{13}^2 - d_{23}^2 - D_{14}^2 + D_{24}^2\right),\ -h\right)$

C. Mathematical Model for Non-Parallel Planes State

Previous section assumes the plane that is created by the three underwater sensors and the plane (water surface) on which the beacon surfs while taking distance measurements were assumed to be in parallel state. In case of non-parallel plane situation, aforesaid mathematical model will be followed with additional adjustment as depicted in this section. Fig. 3 shows the scenario when the two planes are in non-parallel state. Where the planes are defined by;

$$\prod_{S_1, S_2, S_3} : (a_1 x + b_1 y + c_1 z + \delta_1 = 0)$$

which is created by the underwater deployed sensors S_1, S_2 and S_3; and the plane

$$\prod_{Beacon}: (a_2 x + b_2 y + c_2 z + \delta_2 = 0)$$

which is where the beacon surfs around i.e., water surface.

The situation as in Fig. 3 would be more common in the real world scenario, unless if AUV/UUVs are used to maintain a specific depth as in Fig. 2 or if an artificial water container is used where the bottom surface is always parallel with the water surface. Regardless of the situation, if the depth of any of the sensors is different than the others we could conclude that both the planes are in non-parallel state.

The system of equations mentioned in previous sections is only valid if the planes are in parallel state; because the volume of multiple tetrahedrons created by three sensors and the multiple apexes (different beacon's positions) are equal. The variation in vertical distance will depend on the dihedral angle α as in (6) and changes can be minimized if the beacon's positions are in close proximity while measuring distances. As the proposed method requires six distance measurements from beacon to underwater sensors, the total process should be done in quicker fashion so that the mobility of the sensors will not contribute more errors in coordinates that are being determined.

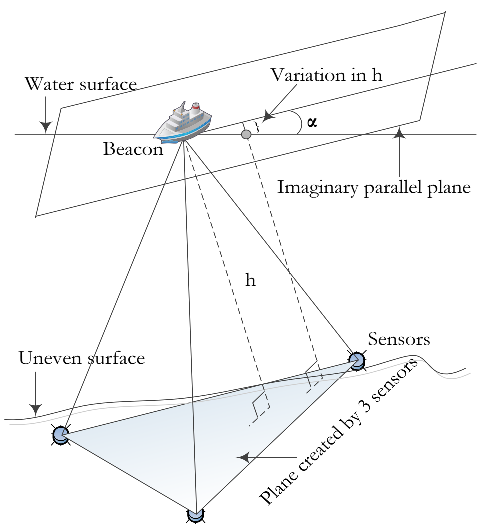


Fig. 3. Parallel and non-parallel plane orientations

The system of equations mentioned in previous sections is only valid if the planes are in parallel state; because the volume of multiple tetrahedrons created by three sensors and the multiple apexes (multiple beacons' positions) are equal.

$$\cos\alpha = \frac{\hat{n}_1 . \hat{n}_2}{|\hat{n}_1||\hat{n}_2|} = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}} \tag{6}$$

where $\hat{n}_1 = (a_1, b_1, c_1)$ and $\hat{n}_2 = (a_2, b_2, c_2)$ are normal vector to the planes.

As mentioned earlier, all the deployed sensors or nodes will be equipped with a pressure sensor, which will read pressure. Once the pressure reading is sent to beacon, it will be converted in to depth measurement. Once all the depth of the sensors is determined the following method could be applied to determine the coordinates as depicted in Fig. 4.

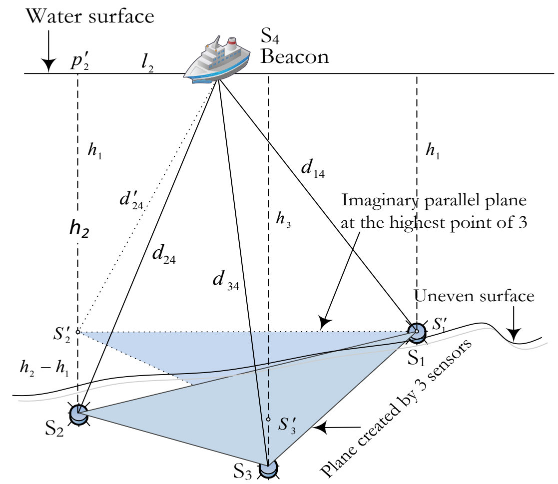


Fig. 4. Determination of coordinates for non-parallel planes state

Let the depths of the deployed sensors are h_1, h_2 and h_3 for the sensors S_1, S_2 and S_3 respectively, where assuming $h_3 > h_2 > h_1$, then S_1 is the highest point among all the three sensors with depth h_1. So the plane Π_{s_1,s_2,s_3} created by three points S_1, S_2 and S_3 would be in non-parallel state with the plane Π_{beacon}(water surface).

Let that highest point be S_1', where $S_1' = S_1$, we also imagine two more points S_2' and S_3' with depth same as h_1 right above S_2 and S_3 respectively. Since S_2' and S_3' are right above S_2 and S_3, x and y component of its coordinates will be same as S_2 and S_3 respectively, only z component of the coordinates will be different. So the plane $\Pi_{s_1' s_2' s_3'}$ and plane Π_{beacon} would be in parallel state. For $\Delta S_4 S_2 p_2'$, where p_2' is the projection of S_2, and d_{24} is the measured distance which would be taken according to the process discussed in [10]; h_2 is measured depth from built-in pressure sensor associated with deployed nodes. So l_2 i.e. the distance between p_2' and S_4, can be calculated as:

$$l_2 = \sqrt{d_{24}^2 - h_2^2} \tag{7}$$

Once l_2 is known, the imaginary distance d'_{24} can be achieved from the $\Delta S_4 S_2' p_2'$.

$$d'_{24} = \sqrt{l_2^2 + h_1^2} \tag{8}$$

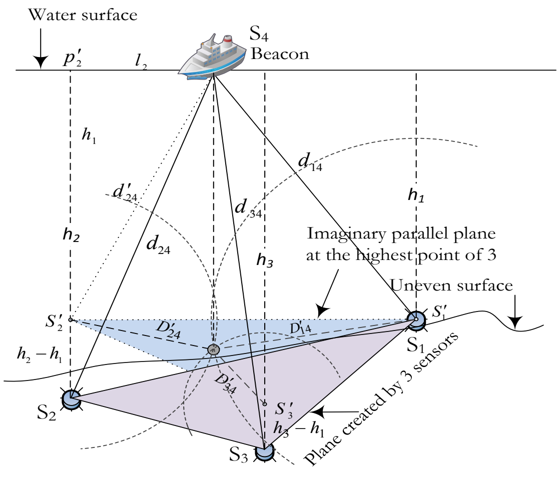


Fig. 5. Coordinates computation for non-parallel planes state

TABLE III. COORDINATES WITH RESPECT TO THE BEACON (NON- PARALLEL PLANES STATE)

Sensors	Coordinates
S_4	(0,0,0)
S_1	$\left(\frac{\sqrt{4d_{12}'^2 D_{14}'^2 - \left(D_{14}'^2 - D_{24}'^2 + d_{12}'^2\right)^2}}{2d_{12}'},\ -\frac{1}{2d_{12}'}\left(D_{14}'^2 - D_{24}'^2 + d_{12}'^2\right),\ -h_1\right)$
S_2	$\left(\frac{\sqrt{4d_{12}'^2 D_{14}'^2 - \left(D_{14}'^2 - D_{24}'^2 + d_{12}'^2\right)^2}}{2d_{12}'},\ \frac{1}{2d_{12}'}\left(d_{12}'^2 - D_{14}'^2 + D_{24}'^2\right),\ -h_2\right)$
S_3	$\left(\left(\sqrt{\left(d_{13}'^2 - \left(\frac{d_{12}'^2 + d_{13}'^2 - d_{23}'^2}{2d_{12}'}\right)^2\right)} - \frac{\sqrt{4d_{12}'^2 D_{14}'^2 - \left(D_{14}'^2 - D_{24}'^2 + d_{12}'^2\right)^2}}{2d_{12}'}\right),\ \frac{1}{2d_{12}'}\left(d_{13}'^2 - d_{23}'^2 - D_{14}'^2 + D_{24}'^2\right),\ -h_3\right)$

Same technique and procedure will be followed to find the distance between S_4 and imaginary point S_3'. Once all distances from beacon S_4 to imaginary points S_2' and S_3' are calculated, section II will be followed to determine coordinates of S_1', S_2' and S_3'. Table II shows the determined coordinates of the sensors for parallel state with respect to the beacon S_4. Table III shows the computed coordinates of sensors S_1, S_2 and S_3 for non-parallel state.

III. SIMULATION RESULTS AND DISCUSSIONS

In order to validate the mathematical model, the proposed method has been simulated using Matlab in [11]. A group of three sensors are placed at (0,0,0), (0,75,0), and (80,40,0) and the mobile beacon moved randomly in a plane, which is parallel to the bottom plane where the sensors are deployed. Simulation for coordinate determination is done for 100m water column with 30°C surface and 15°C bottom temperature having salinity variation of 0.5ppt. We have incorporated Gaussian noise ($\mu=0$, $\sigma=1$) to 15°C bottom temperature and to the flight time $T_{ij(travel)}$ as both of them have more uncertainties.

To get distance measurement from six different positions of the beacon, it has been moved around in six different coordinates randomly within close proximity. However, mobility of the sensors is not considered in the proposed mathematical model. Errors in coordinates for S_2 and S_3 are shown in Fig. 6 for 100 iterations. It should be noted that sensor S_1 is placed at the reference coordinate (0,0,0); hence producing no error in coordinates determination for S_1.

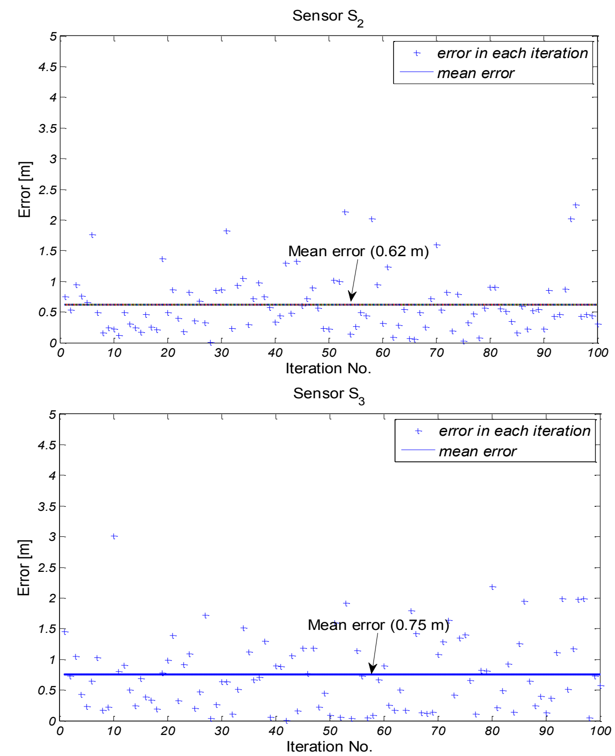
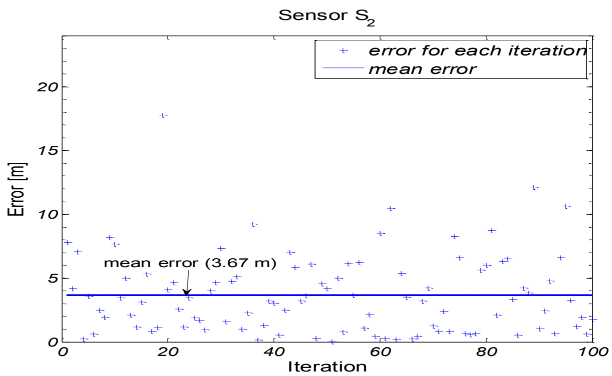


Fig. 6. Distance errors of sensor S_2 and S_3 from original position

Table IV compares the positional error of sensor S_2 and S_3 when distances between the beacon and sensors are with and without Gaussian noise. Positional error with true Euclidean distance is almost negligible which validates the proposed mathematical model. This negligible error appeared even after considering true Euclidean distances are due to linearization process. It also shows that the mean distance errors for both the sensors are 0.62m and 0.75m with standard deviation of 0.478 and 0.603 respectively where Gaussian noise has been added to distance measurement only. In addition to that Fig. 7 shows positional error of sensors after incorporating Gaussian noise in distance measurements and bearings; it is conspicuous that positional errors are within acceptable range considering physical sizes of the sensors for a 100m deep water column. This result will surely improve if the outliers are not considered in simulations.



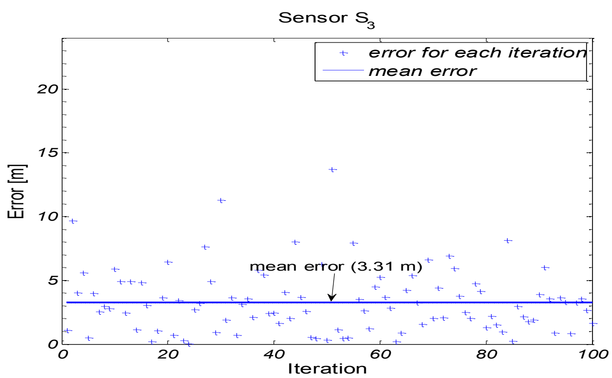


Fig. 7. Distance errors of sensor S_2 and S_3 from original position after incorporating Gaussian noise in distance measurements and bearings

TABLE IV. DISTANCE ERRORS IN COORDINATE DETERMINATION

Sensors	**Distance Error (m) (due to linearization, without noise)**	**Distance Error (with Gaussian noise)**	
		(m)	**σ**
S_2	1.06×10^{-10}	0.62	0.478
S_3	2.75×10^{-10}	0.75	0.603

IV. CONCLUSIONS

In this paper we have presented a mathematical model to determine the coordinates of submerged sensors using a single beacon for both parallel state and non-parallel state scenarios. The method computes the coordinates with respect to one sensor node that alleviates a number of problems in the domain of localization. Results from simulation validate the proposed mathematical model that generates negligible error in coordinates determination of the sensors when distances between beacon and sensors are true Euclidean. The proposed method can also determine the coordinates of the deployed submerged sensors using a single surfaced beacon for a real life orientation. It further shows that coordinates are within acceptable error range when Gaussian noise is applied to distance determinations. In future study we plan to consider involuntary mobility of the sensors due to currents in the proposed model.

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