# Modeling and Analysis of Distance-based Registration in Metropolitan Area

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Abstract— In this paper, we develop a 4-directional mobility model applicable in the metropolitan area to calculate the number of registrations per mobile station (MS) for the zone-based (ZBR) and distance-based (DBR) registration schemes. The analytical results are presented with the assumptions of the MS's uniform distribution in the location area (LA). The simulation studies are also performed to verify the analytical results. The numerical results show that, if the MS's moving distance between turning points is smaller than the distance between incoming calls or LA's radius, the DBR always outperforms the ZBR.

Keywords—Mobility management; location registration; zonebased registration; distance-based registration; RAPTOR

# I. INTRODUCTION

In a mobile communication network, the location of the mobile station (MS) must be known prior to the successful delivery of an incoming call to the MS. The mobile network keeps track of the location of the MS through the location registration (LR). When an incoming call arrives the MS, the paging operations are performed to find the MS. Since the trade-off relation between LR and paging schemes exists, many studies have been performed [1,2,4]. In our study, we consider the zone-based (ZBR) and distance-based (DBR) registration schemes.

In the ZBR, the mobile network consists of fixed zones (location area, LA), and the registration occurs only at the border cells of the LA. Therefore, since the zigzag movement of the MS near the border of LA, the LR's are frequently occurred in the borders. On the other hand, in the DBR, the MS requires the registrations whenever the distance between the current base station and the base station where it was last registered exceeds a threshold. The previous results [3,5] show that the DBR can avoid the unnecessary registrations along the border of LA. In this paper, new versions of analysis for the ZBR and DBR are presented with 4-directional mobility model proper in the metropolitan area.

### II. MOBILITY MODELING

To calculate the number of registrations, we introduce the mobility model assuming MS's moving in the environments of metropolitan area as shown in the followings.

- An MS moves straight, until it reaches the turning point.
- · When the MS reaches the turning point, it has four

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directions,  $n \times 90^{\circ}$  (n=0, 1, 2, 3) with the same probabilities.

• The distance, X, between two successive turning points is exponentially distributed with mean  $\delta$ .

The following notations are used throughout our study.

2d: the length of side in square-shaped LA in ZBR

R: the radius of a circle LA in DBR, for comparison with ZBR,  $(2d)^2 = \pi R^2$ , that is,  $R = 2d/\sqrt{\pi}$ 

L: the moving distance between two incoming calls

 $K_{\rm ct}$ : the number of registrations between a call occurrence point and next turning point

 $K_{\rm tt}$ : the number of registrations between two successive turning points

S: the distance from a turning point to the border of LA

N: the number of registrations per MS between two incoming calls

# 1. Zone-based registration (ZBR)

As shown in Fig. 1, we assume that the shape of an LA is a square in ZBR, and the MS moves to the one direction among four types of directions  $(S_1, S_2, S_3, \text{ and } S_4)$  at each turning point. The probability density function (pdf) of the S is

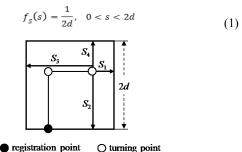


Figure 1. Zone-based registration

The expected number of registrations between two adjacent turning points can be written by

$$E[K_{tt}] = \sum_{i=1}^{\infty} \{i \times Pr[S + 2(i-1)d \le X < S + 2id]\} = \frac{\delta}{2d}$$
 (2)

Thus, the mean number of registrations between two incoming calls for ZBR is [4]

$$E[N] = \frac{E[L]}{E[X]} \times E[K_{tt}] = \frac{E[L]}{2d}$$
 (3)

# 2. Distance-based registration (DBR)

Under the DBR, the MS registers when the distance between the current base station and the last registered base station exceeds a threshold (R) [7]. Therefore, as shown in Fig. 2, we assume that the shape of LA is a circle in DBR, where the radius of the circle is the *R*.

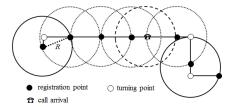


Figure 2. Distance-based registration

Also, note that, in DBR, when a call arrives to an MS, LA of the MS is reset so that current cell becomes the center of a new LA. So, in this case, the distance to the border of LA is clearly R. To find the pdf of S in DBR, we assume the mobiles are distributed uniformly over the LA. The polar coordinates of the MS are denoted by  $(r, \theta)$ , where r and  $\theta$  represent the radial and angular distance of an MS from the center of its LA. When the mobiles are uniformly distributed over the LA, the cumulative distribution function of  $(r, \theta)$  is

$$G(r, \theta) = \frac{r^2 \theta}{2\pi R^2}, \quad 0 < r < R, \quad 0 < \theta < 2\pi$$
 (4)

Thus, the pdf is

$$g(r,\theta) = \frac{r}{\pi R^2}, \quad 0 < r < R, \ 0 < \theta < 2\pi$$
 (5)

Making a change of variable [8],  $x = r\cos\theta$ ,  $y = r\sin\theta$ , we have

$$f(x,y) = \frac{1}{\pi R^2}, \qquad 0 < x^2 + y^2 < R^2, \quad -R < x, y < R$$
 (6)

As shown in Fig. 3, the S is divided into four types of distance  $(S_1, S_2, S_3, \text{ and } S_4)$  according to MS's moving direction. However, with the rotational displacement, the pdf of each  $S_i$ 's is the same. Then, the pdf of  $S_1$  ( $S_1 = \sqrt{R^2 - y^2} - x$ ) is to be found to obtain the pdf of S.

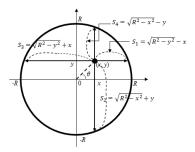


Figure 3. The distance to the LA boundary

Let, 
$$s = \sqrt{R^2 - y^2} - x$$
, and  $t = x$ , we have

x=t,  $y=\pm\sqrt{R^2-(s+t)^2}$ ,  $x^2+y^2=R^2-s^2-2st$  and determinant (Jacobian) of order 2,

$$J = \begin{vmatrix} \frac{\partial x}{\partial s} & \frac{\partial x}{\partial t} \\ \frac{\partial y}{\partial s} & \frac{\partial y}{\partial t} \end{vmatrix} = \left| \mp \frac{0}{\sqrt{R^2 - (s+t)^2}} + \mp \frac{1}{\sqrt{R^2 - (s+t)^2}} \right| = \left| \pm \frac{s+t}{\sqrt{R^2 - (s+t)^2}} \right|$$
(7)

From the Jacobian and the conditions of  $0 < x^2 + y^2 = R^2 - s^2 - 2st < R^2, R^2 - (s+t)^2 > 0$ , the joint

pdf of 
$$(s, t)$$
 is given by
$$f(s, t) = \frac{2(s+t)}{\pi R^2 \sqrt{R^2 - (s+t)^2}}, \quad -\frac{s}{2} < t < R - s, 0 < s < 2R$$
(8)

The marginal pdf of S is then

$$f_{s}(s) = \int_{-\frac{s}{2}}^{R-s} f(s,t) dt = \frac{2}{\pi R^{2}} \int_{-\frac{s}{2}}^{R-s} \frac{s+t}{\sqrt{R^{2} - (s+t)^{2}}} dt$$

$$= \frac{2}{\pi R^{2}} \sqrt{R^{2} - \left(\frac{s}{2}\right)^{2}}, 0 < s < 2R$$
(9)

Thus, the number of registrations between an incoming call and turning point,  $E[K_{ct}]$  becomes

$$E[K_{ct}] = \sum_{i=1}^{\infty} \{i \times Pr[iR \le X < (i+1)R]\} = \frac{e^{-R/\delta}}{1 - e^{-R/\delta}}$$
 (10)  
On the other hand, the expected number of registrations

between two successive turning points is given by

$$E[K_{tt}] = \sum_{i=1}^{\infty} \{i \times Pr[S + (i-1)R \le X < S + iR]\}$$
  
=  $\frac{2}{\pi R^2 (1 - e^{-R/\delta})} \int_0^{2R} e^{-s/\delta} \sqrt{R^2 - (s/2)^2} ds$  (11)

Then, we have the expected number of registrations per MS for DBR between two incoming calls assuming the MS's are uniformly distributed after the turning moves.

$$E[N] = \begin{cases} \frac{E[L]}{E[X]} E[K_{ct}], & \text{if } E[L] \le E[X] \\ E[K_{ct}] + \left\{\frac{E[L]}{E[X]} - 1\right\} E[K_{tt}], & \text{otherwise} \end{cases}$$
(12)

However, note that the MS's are not uniformly distributed after the turning. So, the simulation studies are required to have the exact results for the DBR.

# III. RAPTOR VISUAL SIMULATION

The mean number of registrations, E[N], developed theoretically under ZBR and DBR was examined through simulation studies. The Fig. 4 shows the simulation model. When an incoming call arrives, the initial coordinates (X, Y) =(0,0) are determined. Then, the distance between two successive turning points, X, is randomly determined from exponentially distributed distance with the mean ( $\delta$ ). If the X is greater than L, the MS moves along the L. Otherwise, the MS moves along X until the sum of  $X_i$ 's (the distance to the  $i^{th}$ turning point) reaches the L. In this model, the following two different cases are compared for the new location of MS after turning in DBR: (1) MS is uniformly located in LA (DBR uniform), and (2) The location of MS is determined based on the moving directions (DBR) as shown in Fig. 3. Note that, in the ZBR's simulation, the locations of MS after turning are determined based on the MS's moving in the square-shaped LA as shown in Fig. 1.

We performed the computer simulation by using RAPTOR [6], which is free flowcharting software package to provide the interfaces for users to create executable flowcharts. In Fig. 4, we repeated the processes for 1,000 MS's, and the simulations are performed 10 times, then final results for the number of registrations for 10,000 MS's are obtained.

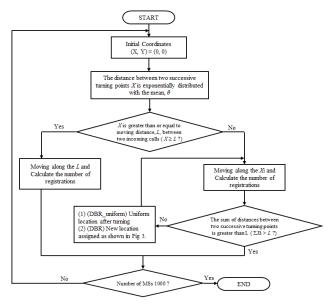


Figure 4. Simulation model

# IV. NUMERICAL RESULTS

The performance for ZBR and DBR are compared. To make the same paging load for two schemes, the area of LA for ZBR and DBR is assumed to be the same. Then, R = 5.6 for d = 5 km. Figs. 5 and 6 show the number of registrations per MS versus  $\delta$ , when L=6.7 and L=10, respectively. From those results, we have the following observations.

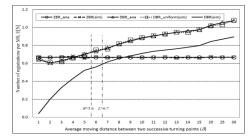


Figure 5. Number of registrations versus  $\delta$  (L=6.7)

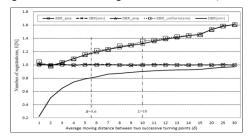


Figure 6. Number of registrations versus  $\delta$  (L=10)

1) The analytical and simulation results for ZBR and DBR are very close, where the 'ana' and 'sim' represent the analytical and simulation results, respectively. However, under the non-uniform locations of MS after turning in DBR, the

simulation results (DBR(sim)) are not identical to DBR\_ana (or DBR\_uniform(sim)). It indicates that the MS's are not randomly located in the LA after turning moves in DBR. However, noting that the results of ZBR\_ana and ZBR(sim) are almost the same, it is observed that the MS's are uniformly located in the square-shaped LA in ZBR.

- 2) The number of registrations in DBR increases as E[X] or E[L] increases. On the other hand, in ZBR, E[N] has the same value as E[X] increases(Eq'(3)). The simulation results in DBR(sim) show that, in the case of  $E[X] \le E[L]$ , the number of registration is smaller than ZBR. In specific, when E[X] is smaller than R, the DBR outperforms the ZBR. It indicates that if the MS dwells in near center after turning in DBR, the number of registrations in DBR is always smaller than the ZBR.
- 3) On the other hand, when E[X] > E[L], the E[N] in DBR converges to L/R, then the E[N] in DBR can be much more than the ZBR.

#### V. CONCLUSIONS

Using the mobility model suitable in the metropolitan area, we have analyzed the number of registrations of the zone-based (ZBR) and distance-based (DBR) schemes. We obtained the analytical results with the assumptions of uniformly distributed MS's in the location area (LA), and compared the simulation studies with (or without) the assumptions. The numerical results showed that the MS's are uniformly distributed in the LA after turning moves in ZBR, but they are not uniformly distributed in LA for DBR. It was also observed that, when the MS resides in the LA near center after turning in DBR, that is, if the moving distance between turning points is smaller than the distance between incoming calls or LA's radius, the DBR always outperforms the ZBR. Further study will be performed to obtain accurate analytical performance in DBR.

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