BER Analysis of Wavelength Division Multiplexing-Based Multiple Beam Scheduling Scheme based on Gamma Approximation Channel

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***Abstract*—In this paper, we statistically analyze the BER performance of a threshold-based parallel multiple beam selection scheme for a free-space optical (FSO) based system with wavelength division multiplexing (WDM) in cases where a pointing error has occurred under Gamma turbulence conditions, which are a good approximation of Gamma-Gamma distribution. Our derived results based on the Gamma distribution as an approximation of the Gamma-Gamma distribution can be used as approximated performance measure (lower) bounds.**

I. Introduction

Wavelength division multiplexing (WDM) in fiber-optic communications has been shown to increase the capacity and bandwidth [1]. Similarly, in FSO systems, a WDM access network is a realistic proposition and feasible because both conventional fiber-optic communication systems and free-space optical (FSO) systems use similar system components and transmission wavelengths [2]. However, the received signals at the receiver generated based on WDM may not be valid or may not have an acceptable signal-to-noise ratio (SNR) due to atmospheric attenuation, whereas in conventional fiber-optical communication systems, all the received signals are valid.

Based on it, we analyze the BER performance of parallel multiple-beam selection scheme for FSO systems using WDM. More specifically, we consider a parallel multiple-beam selection scheme based on a threshold-based selection scheme [3]. With this scheme, higher spectral efficiency can be obtained while the complexity of implementation caused by applying the selection-based beam selection scheme can be reduced without a considerable performance loss.

II. System and Channel Models

We consider a point-to-point FSO link using heterodyne detection (HD) [4]. We assume a block-turbulence FSO channel in which the turbulence is assumed to be constant but in which the turbulence independently changes in different rounds. We also assume that an FSO link mainly experiences both atmospheric turbulence and pointing errors.

We assume that induced FSO channel turbulence is modeled by the Gamma distribution. The Gamma PDF is found to be a good approximation of the Gamma-Gamma distribution, which are widely accepted conditions in the current literature, by use of the moment-matching method. Approximation using the tractable Gamma distribution can significantly simplify the performance analysis of composite fading channels.

Another factor that affects the reliability of FSO channels is building sway which leads to a misalignment between the transmitter and the receiver, consequently causing pointing errors, which also seriously degrade system performance. For this reason, we also consider the effects of pointing errors.

With the TPMBS scheme, the scheduler selects all the valid optical signals for each time slot with a link condition above a preselected threshold based on the the channel state information back to the transmitter through a reliable feedback path (i.e. RF feedback channel).

III. Average BER Analysis of TPMBS

Based on the mode of operation, the selected beam has a conditional PDF of a truncated (above the threshold, γ_T) random variable (RV). Therefore, the PDF and the CDF can be expressed as for $\gamma \geq \gamma_T$, $f_{\gamma_{TB}}(\gamma) = \frac{f_\gamma(\gamma)}{1-F_\gamma(\gamma_T)}$ and $F_{\gamma_{TB}}(\gamma) = \frac{F_\gamma(\gamma)-F_\gamma(\gamma_T)}{1-F_\gamma(\gamma_T)}$, where $f_\gamma(\cdot)$ and $F_\gamma(\cdot)$ are the PDF and CDF of an instantaneous SNR at the receiver over the various channel models, respectively.

With coherent and non-coherent binary modulation, BER_n can be written as $BER(\gamma) = \frac{\Gamma(p,q\gamma)}{2\Gamma(p)}$, where p and q represent the parameters defining the type of detection mechanism and modulation type, respectively, as $p=\frac{1}{2}$ for coherent detection and $p=1$ for non-coherent or differentially and $q=\frac{1}{2}$ for FSK and $q=1$ for PSK. Then, the average BER can be obtained based on the given turbulence distribution as

$$\overline{BER} = BER(\gamma) F_{\gamma_{TB}}(\gamma)\big|_{\gamma_T}^{\infty} - \int_{\gamma_T}^{\infty} F_{\gamma_{TB}}(\gamma)\, dBER(\gamma). \quad (1)$$

With the help of [5, eq. (6.5.25)], the average BER can be rewritten for mathematical convenience as

$$\overline{BER} = \overline{BER}_I - \overline{BER}_{II}, \quad (2)$$

where

$$\overline{BER_I} = \frac{q^p}{2\Gamma(p)} \cdot \frac{1}{1-F_\gamma(\gamma_T)} \int_{\gamma_T}^{\infty} \gamma^{p-1} \exp(-q\gamma) F_\gamma(\gamma) d\gamma, \quad (3)$$

$$\overline{BER}_{II} = \frac{q^p}{2\Gamma(p)} \cdot \frac{F_\gamma(\gamma_T)}{1-F_\gamma(\gamma_T)} \int_{\gamma_T}^{\infty} \gamma^{p-1} \exp(-q\gamma)\, d\gamma. \quad (4)$$

Here, with the help of [5, eq. (3.381.3)], (4) can be rewritten as

$$\overline{BER_{II}} = \frac{\Gamma(p, q, \gamma_T)}{2\Gamma(p)} \cdot \frac{F_\gamma(\gamma_T)}{1 - F_\gamma(\gamma_T)}. \tag{5}$$

In both (3) and (5), we still need $F_\gamma(\gamma)$.

With the help of [6] and then utilizing [7, Eq.(06.06.21.0002.01)], $F_\gamma(\gamma)$ can be given as

$$\begin{aligned} F_\gamma(\gamma) =& \frac{1}{\Gamma(k)}\left[\left\{\frac{\xi^2\gamma}{\theta(1+\xi^2)\mu_{HD}}\right\}^{\xi^2}\Gamma\left(k-\xi^2, \frac{\xi^2\gamma}{\theta(1+\xi^2)\mu_{HD}}\right)\right. \\ &\left. +\Gamma(k) - \Gamma\left(k, \frac{\xi^2\gamma}{\theta(1+\xi^2)\mu_{HD}}\right)\right], \end{aligned} \tag{6}$$

where μ_{HD} is the electrical average SNR, ξ is the ratio between the equivalent beam radius, and the pointing error displacement standard deviation (jitter) at the receiver. A_0 is the pointing loss. $k = \frac{\alpha\beta}{1+\alpha+\beta}$, $\theta = \frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\alpha\beta}$, and α and β are the atmospheric turbulence conditions. Then, for the closed-form expression of $\overline{BER_{II}}$, we can directly obtain a closed-form result by substituting the closed-form results given in (6) into (5).

For $\overline{BER_I}$, with (6), we can be rewrite (3) as

$$\begin{aligned} \overline{BER_I} =& \frac{q^p}{2\Gamma(p)} \cdot \frac{1}{1 - F_\gamma(\gamma_T)} \cdot \frac{1}{\Gamma(k)} \\ &\times \left[A^{\xi^2}\int_{\gamma_T}^{\infty}\gamma^{\xi^2+p-1}\exp(-q\gamma)\,\Gamma\left(k-\xi^2, A\gamma\right)d\gamma\right. \\ &\left. +\Gamma(k)\int_{\gamma_T}^{\infty}\gamma^{p-1}\exp(-q\gamma)d\gamma - \int_{\gamma_T}^{\infty}\gamma^{p-1}\exp(-q\gamma)\Gamma(k, A\gamma)d\gamma\right], \end{aligned} \tag{7}$$

where $A = \frac{\xi^2}{(1+\xi^2)\theta\mu_{HD}}$. Here, for the second integral term, with the help of [5, eq. (3.381.3)], the closed-form expression can be obtained as

$$\Gamma(k)\int_{\gamma_T}^{\infty}\gamma^{p-1}\exp(-q\gamma)\,d\gamma = \Gamma(k)\,q^{-p}\,\Gamma(p, q\gamma_T). \tag{8}$$

For the first and third integral terms, by utilizing [7, Eq.(06.06.21.0002.01)], the closed-form expressions of each integral terms can be obtained as

$$\begin{aligned} &A^{\xi^2}\int_{\gamma_T}^{\infty}\gamma^{\xi^2+p-1}\exp(-q\gamma)\,\Gamma\left(k-\xi^2, A\gamma\right)d\gamma \\ &= \sum_{n=0}^{\infty}\frac{(-q)^n\left\{\Gamma(n+p+k, A\gamma_T) - (A\gamma_T)^{\xi^2+n+p}\Gamma\left(k-\xi^2, A\gamma_T\right)\right\}}{n!A^{n+p}(\xi^2+n+p)}, \end{aligned} \tag{9}$$

and

$$\begin{aligned} &\int_{\gamma_T}^{\infty}\gamma^{p-1}\exp(-q\gamma)\,\Gamma(k, A\gamma)\,d\gamma \\ &= \sum_{n=0}^{\infty}\frac{(-q)^n\left\{\Gamma(n+p+k, A\gamma_T) - (A\gamma_T)^{n+p}\Gamma(k, A\gamma_T)\right\}}{n!A^{n+p}(n+p)}. \end{aligned} \tag{10}$$

IV. Numerical Results and Discussions

In this section, we present some selected result, especially the average BER of the selected beams. We verified these results via computer-based Monte-Carlo simulations, we evaluated the analytical results numerically and show them under both varying turbulent FSO channel conditions and pointing errors. In the figures, a marker and a line are given for the analytical and simulation results, respectively.

The FSO link is modeled as a strong turbulence channel based on the FSO channels in [8]. Specifically, we vary the fading/scintillation parameters (i.e., α and β, respectively) and the pointing error parameters (i.e., ξ). In Fig. 1, the average BER performance comparisons based on the Gamma distribution and the Gamma-Gamma distribution under HD technique, especially considering worst case, strong turbulence (i.e., $\alpha = 2.064$ and $\beta = 1.342$) and strong pointing error (i.e., $\xi = 1$). The result shows that under the worst conditions, the approximate Gamma turbulence model provides almost the same performance. With this sufficiently accurate approximation using the Gamma distribution, we can simplify the performance analysis as a function of tractable function (i.e., Gamma function). Note that our derived results based on

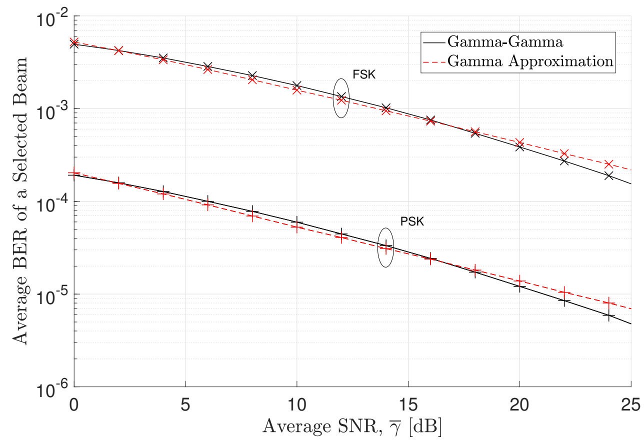

Fig. 1. Average BER comparison between Gamma-Gamma and Gamma approximation over strong turbulence with strong pointing error under HD with $L = 5$ and $\gamma_T = 7.1$ dB.

the Gamma distribution as an approximation of the Gamma-Gamma distribution can still be used as lower bounds on the approximated considered performance measures.

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