A Low Overhead Feedback Scheme of Channel Covariance Matrix for Massive MIMO Systems

Youngrok Jang, Dongheon Lee, and Sooyong Choi
School of Electrical and Electronic Engineering, Yonsei University, Seoul, Korea
Email:{dynamics, alex2923, csyong}@yonsei.ac.kr

***Abstract*—In this paper, we propose a feedback scheme of a channel covariance matrix with low overhead for massive multiple-input multiple-output systems. The proposed scheme decomposes the channel covariance matrix into the phase and amplitude parts. For the phase part, the element-wise uniform scalar quantization is performed. For the amplitude part, the following feedback information is generated: a bitmap which denotes a sign of difference between adjacent elements, the first value, an increment, and a decrement. To calculate the elements of the amplitude part, starting from the first value, when a bitmap is 1, the increment is added, otherwise the decrement is added. Simulation results show that the feedback overhead of the proposed scheme can be significantly reduced from 1.92% to 48.33% while the performance of mean square error can be maintained with that of the conventional scheme.**

I. Introduction

Massive multiple-input multiple-output (MIMO) systems have been of great interest which can improve spectral efficiency [1]. However, a major challenge in realizing the massive MIMO in frequency division duplex system is how to acquire the instantaneous channel state information (ICSI) at the base station (BS) due to a large pilot and feedback overheads [1].

To acquire an ICSI, many recent works utilizing the channel covariance matrix have been investigated in spatially correlated channel [2], [3]. Moreover, instead of acquiring ICSI, transmission schemes based only on the channel covariance matrix have been studied [4], [5]. It is assumed in [2]–[5] that the BS already has a perfect knowledge of the channel covariance matrix to utilize. However, the channel covariance matrix should be fed back since it does not accurate due to a large dimension, and also can be changed over time due to the mobility of users [6]. The conventional feedback scheme of the channel covariance matrix decomposes each element into phase and amplitude parts, and performing an element-wise uniform scalar quantization [7], [8]. However, the feedback overhead significantly increases when the number of BS antennas grows. Hence, an efficient feedback scheme of the channel covariance matrix should be needed.

In this paper, an efficient feedback scheme of the channel covariance matrix with low overhead for massive MIMO systems is proposed. In order to apply the proposed scheme, the channel covariance matrix is decomposed into the phase and amplitude parts. For the phase part, the element-wise uniform scalar quantization is applied. For the amplitude part, in order to reduce the feedback overhead, a bitmap which means a sign of difference between adjacent elements, the first value, an increment, and a decrement can be generated as the feedback information. The elements of the amplitude part can be calculated by starting from the first value and adding the increment when a bitmap is 1 or adding the decrement otherwise. The ratio of the feedback overhead of the proposed scheme to that of the conventional scheme is verified.

The rest of the paper is organized as follows. In Section II, the system model and the conventional scheme are described. In Section III, the proposed feedback scheme for the channel covariance matrix feedback scheme with low overhead is described. Simulation results are shown in Section IV, and Section V concludes the paper.

II. System Model and Conventional Scheme

We consider a spatially correlated massive MIMO system consisting of a base station with M antennas and a single user equipped with a single antenna. By the Karhunen-Loève (KL) transform, the downlink channel vector at time slot t can be written as $\mathbf{h}_t = \mathbf{R}^{\frac{1}{2}}\mathbf{g}_t \sim \mathcal{CN}\left(0, \mathbf{R}\right)$, where $\mathbf{R}^{\frac{1}{2}} \in \mathbb{C}^{M\times r}$ and $\mathbf{g}_t \sim \mathcal{CN}\left(\mathbf{0}, \mathbf{I}_r\right) \in \mathbb{C}^{r\times 1}$ denote a square root of spatial correlation matrix with rank r and an uncorrelated Gaussian random vector, respectively [2]. By using the one-ring model [9], the correlation coefficient between the mth and pth BS antennas can be written as

$$[\mathbf{R}]_{m,p} = \frac{1}{2\Delta}\int_{-\Delta}^{\Delta} e^{j\mathbf{k}^T(\alpha+\theta)(\mathbf{u}_m-\mathbf{u}_p)}d\alpha,$$

where Δ and θ denote the angular spread (AS) and azimuthal angle of arrival (AoA) for the user, respectively. $\mathbf{k}\left(\alpha\right) = -\frac{2\pi}{\lambda}\left(\cos\left(\alpha\right), \sin\left(\alpha\right)\right)^T$ is the wave vector, where λ is a carrier wavelength which can be obtained with $\lambda = \frac{c}{f_c}$, where c and f_c denote the speed of light and carrier frequency, respectively. $\mathbf{u}_m, \mathbf{u}_p \in \mathbb{R}^2$ are the positioning vectors for the mth and pth BS antennas, respectively.

In order to acquire the channel covariance matrix information at the BS, an approximated channel covariance matrix $\bar{\mathbf{R}}$ can be calculated at the user by using the channel vectors of N adjacent time slots [10], which can be expressed as $\bar{\mathbf{R}} = \frac{1}{N}\sum_{t=1}^{N}\mathbf{h}_t{\mathbf{h}_t}^H$. Since $\bar{\mathbf{R}}$ is a hermitian matrix (i.e., $\bar{\mathbf{R}} = \bar{\mathbf{R}}^H$), $\bar{\mathbf{R}}$ can be decomposed into

$$\bar{\mathbf{R}} = \bar{\mathbf{R}}_{\text{diag}} + \bar{\mathbf{R}}_{\text{off}} + \bar{\mathbf{R}}_{\text{off}}^H, \tag{1}$$

where $\bar{\mathbf{R}}_{\text{diag}} \in \mathbb{R}^{M\times M}$ is a diagonal matrix which contains the diagonal elements of $\bar{\mathbf{R}}$ and $\bar{\mathbf{R}}_{\text{off}} \in \mathbb{C}^{M\times M}$ is an upper or lower triangular matrix which contains the upper or lower

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side of off-diagonal elements of $\bar{\mathbf{R}}$. Hence, for the channel covariance matrix feedback, only non-zero elements of $\bar{\mathbf{R}}_{\text{diag}}$ and $\bar{\mathbf{R}}_{\text{off}}$ can be quantized and fed back to the BS. For the simplicity, the case of the upper triangular matrix is considered for $\bar{\mathbf{R}}_{\text{off}}$ in the sequel.

For the conventional feedback scheme of channel covariance matrix, the element-wise scalar quantization is considered for the phase and amplitude parts of non-zero elements of $\bar{\mathbf{R}}_{\text{diag}}$ and $\bar{\mathbf{R}}_{\text{off}}$ [7], [8]. It is assumed that the elements of the phase and amplitude parts are within the interval $[0, a_{max}]$ and $[0, 2\pi]$, respectively, where a_{max} is the maximum value of the amplitude part. Then, a simple uniform scalar quantizer with given bits is applied to the interval of phase and amplitude part. Then, for given bits to quantize the phase and amplitude parts, the total feedback overhead of the conventional scheme can be expressed as

$$B_{\text{conv}} = \underbrace{B_a M}_{\text{diagonal}} + \underbrace{(B_a + B_p)\frac{M(M-1)}{2}}_{\text{off-diagonal}} \text{ [bits]}, \tag{2}$$

where B_a and B_p are the number of bits to represent the phase and amplitude parts, respectively.

III. Proposed Feedback Scheme of Channel Covariance Matrix with Low Overhead

In this section, we introduce the proposed channel covariance matrix feedback scheme with low overhead. For the proposed scheme, the non-zero elements of $\bar{\mathbf{R}}_{\text{diag}}$ and $\bar{\mathbf{R}}_{\text{off}}$ are also decomposed into the phase and amplitude parts. For the phase part, the element-wise uniform scalar quantization is performed, which is same as the conventional scheme. For the amplitude part, however, the low overhead feedback information can be generated by the proposed scheme.

A. Feedback Information Generation

In this subsection, we present how to generate the feedback information. First, the feedback information of the amplitude part of the diagonal elements can be generated by using $M \times 1$ vector $\mathbf{c}_d \triangleq \bar{\mathbf{R}}_{\text{diag}}\mathbf{1} = \left[c_d^{(1)}, ..., c_d^{(M)}\right]^T$, where $\mathbf{1} \in \mathbb{R}^{M\times 1}$ denotes the all ones vector, i.e., every element is equal to 1. $\mathbf{c}_d$ can be approximated by using the following feedback information : a bitmap $\mathbf{b}_d \in \mathbb{R}^{(M-1)\times 1}$, the first element $c_d^{(1)} \in \mathbb{R}$, an increment $a_{(+)} \in \mathbb{R}$, and a decrement $a_{(\text{-})} \in \mathbb{R}$. $\mathbf{b}_d$ denotes a sign of difference between adjacent elements. The kth element of $\mathbf{b}_d$ can be calculated by

$$b_d^{(k)} = \begin{cases} 1, & c_d^{(k)} \geq c_d^{(k+1)} \\ 0, & c_d^{(k)} < c_d^{(k+1)} \end{cases}, \; 1 \leq k \leq M-1 \tag{3}$$

Then, $\mathbf{b}_{d,(+)}$ and $\mathbf{b}_{d,(\text{-})}$, which denote the cumulative count of 1 and 0, respectively, can be calculated by using $\mathbf{b}_d$. The kth element of $\mathbf{b}_{d,(+)}$ and $\mathbf{b}_{d,(\text{-})}$ can be expressed as

$$b_{d,(+)}^{(k)} = \sum_{n=1}^{k} b_d^{(n)}, \; b_{d,(\text{-})}^{(k)} = k - \sum_{n=1}^{k} b_d^{(n)}, \tag{4}$$

respectively. Finally, $\tilde{\mathbf{c}}_d$, which is an approximated version of $\mathbf{c}_d$, can be calculated by using $\mathbf{b}_{d,(+)}$, $\mathbf{b}_{d,(\text{-})}$, $c_d^{(1)}$, $a_{(+)}$, and $a_{(\text{-})}$. The kth element of $\tilde{\mathbf{c}}_d$ can be expressed as

$$\tilde{c}_d^{(k)} = c_d^{(1)} + a_{(+)} b_{d,(+)}^{(k-1)} + a_{(\text{-})} b_{d,(\text{-})}^{(k-1)}. \tag{5}$$

Using $\mathbf{c}_d$ and $\tilde{\mathbf{c}}_d$, the optimization problem is formulated as

$$\min_{a_{(+)}>0, a_{(\text{-})}<0} \left\|\mathbf{c}_d - \tilde{\mathbf{c}}_d\right\|^2. \tag{6}$$

Since the objective function of (6) is a bivariate quadratic function of $a_{(+)}$ and $a_{(\text{-})}$ [11], it can be arranged in descending order of $a_{(+)}$ and $a_{(\text{-})}$ as

$$\begin{aligned} \left\|\mathbf{c}_d - \tilde{\mathbf{c}}_d\right\|^2 &= \sum_{k=2}^{M} \left\{ c_d^{(k)} - \left(c_d^{(1)} + a_{(+)} b_{d,(+)}^{(k-1)} + a_{(\text{-})} b_{d,(\text{-})}^{(k-1)} \right) \right\}^2 \\ &= A a_{(+)}^2 + B a_{(\text{-})}^2 + C a_{(+)} + D a_{(\text{-})} + E a_{(+)} a_{(\text{-})} + F, \end{aligned} \tag{7}$$

where $A = \sum_{k=2}^{M} \left(b_{d,(+)}^{(k-1)}\right)^2$, $B = \sum_{k=2}^{M} \left(b_{d,(\text{-})}^{(k-1)}\right)^2$, $C = -2\sum_{k=2}^{M} b_{d,(+)}^{(k-1)} (c_d^{(k)} - c_d^{(1)})$, $D = -2\sum_{k=2}^{M} b_{d,(\text{-})}^{(k-1)} (c_d^{(k)} - c_d^{(1)})$, $E = 2\sum_{k=2}^{M} \left(b_{d,(+)}^{(k-1)} b_{d,(\text{-})}^{(k-1)}\right)$, and $F = \sum_{k=2}^{M} \left(c_d^{(k)} - c_d^{(1)}\right)^2$. Since $4AB - E^2 > 0$ is satisfied by using the arithmetic-geometric mean inequality and $A > 0$, the bivariate quadratic function, which is the objective function of (6) in this case, always has the minimum value [11]. Therefore, the optimal solutions $a_{(+)}^*$ and $a_{(\text{-})}^*$ for (6) can be expressed as

$$a_{(+)}^* = -\frac{2BC - DE}{4AB - E^2}, \; a_{(\text{-})}^* = -\frac{2AD - CE}{4AB - E^2}. \tag{8}$$

Next, the feedback information of the amplitude part of the off-diagonal elements can be generated by using $\{\mathbf{c}_{o,k}\}_{k=1}^{M-1}$ vectors that kth vector contains the kth diagonal element of $\bar{\mathbf{R}}_{\text{diag}}$ as the first value and non-zero elements of the kth row of $\bar{\mathbf{R}}_{\text{off}}$, which can be expressed as

$$\mathbf{c}_{o,k} = \left[c_d^{(k)}, \bar{\mathbf{R}}_{\text{off}}^{(k,k+1)}, ..., \bar{\mathbf{R}}_{\text{off}}^{(k,M)}\right]^T \in \mathbb{R}^{(M-k+1)\times 1}, \tag{9}$$

where $\bar{\mathbf{R}}_{\text{off}}^{(a,b)}$ denotes the element in ath row and bth column of $\bar{\mathbf{R}}_{\text{off}}$. By using the same method for the amplitude part of the diagonal elements, $\mathbf{c}_{o,k}$ can be approximated by using bitmap $\mathbf{b}_{o,k} \in \mathbb{R}^{(M-k)\times 1}$, increment $a_{o,(+),k} \in \mathbb{R}$, and decrement $a_{o,(\text{-}),k} \in \mathbb{R}$ for $1 \leq k \leq M-1$. The already generated diagonal element can be used as the first value. Based on (5)–(8), the optimal $a_{o,(+),k}^*$ and $a_{o,(\text{-}),k}^*$ of kth row can be obtained.

Then, an uniform scalar quantizer can be applied to the first value, increments, and decrements for diagonal and off-diagonal elements. Finally, the quantized scalars $\hat{c}_d^{(1)}$, $\hat{a}_{(+)}^*$, $\hat{a}_{(\text{-})}^*$, $\hat{a}_{o,(+),1}^*, ..., \hat{a}_{o,(+),M-1}^*$, $\hat{a}_{o,(\text{-}),1}^*, ..., \hat{a}_{o,(\text{-}),M-1}^*$, and bitmaps $\mathbf{b}_d$ and $\mathbf{b}_{o,1}, ..., \mathbf{b}_{o,M-1}$ are fed back from the user to the BS. The total feedback overhead of the proposed scheme can be expressed as

$$B_{\text{prop}} = \underbrace{3B_a + M - 1}_{\text{diagonal}} + \underbrace{2B_a (M-1) + \frac{M(M-1)}{2}}_{\text{off-diagonal, amplitude}} + \underbrace{B_p \frac{M(M-1)}{2}}_{\text{off-diagonal, phase}} \text{ [bits]}, \tag{10}$$

where $3B_a$ and $M-1$ are feedback bits for 3 scalars (first value, increment, and decrement) and a bitmap of feedback information of the diagonal elements, respectively. For the feedback information of the off-diagonal elements, since there are $M-1$ rows, and 2 scalars (increment and decrement) and a bitmap of length k are generated for kth row, $2B_a(M-1)$ and $\frac{M(M-1)}{2}$ bits are needed for scalars and bitmaps for all rows, respectively. Based on (2) and (10), as M goes to infinity, the ratio of feedback overhead of the proposed scheme to that of the conventional scheme can be approximated as

$$\lim_{M\to\infty} \frac{B_{\text{prop}}}{B_{\text{conv}}} = \frac{B_p + 1}{B_p + B_a}. \tag{11}$$

Since the proposed scheme approximates the amplitude part of non-zero elements of $\tilde{\mathbf{R}}$, it is important how many B_a can be used. However, although more B_a are used, it can be shown from (11) that the ratio of feedback overheads can be reduced when M becomes larger.

B. Reconstruction

Based on the feedback information of the diagonal and off-diagonal elements, the approximated channel covariance matrix $\tilde{\mathbf{R}}$ can be reconstructed at the BS. First, the diagonal matrix $\tilde{\mathbf{R}}_{\text{diag}}$ can be reconstructed by using a bitmap $\mathbf{b}_d$, first value $\hat{c}_d^{(1)}$, an increment $\hat{a}^*_{(+)}$, and a decrement $\hat{a}^*_{(-)}$. The kth diagonal element of $\tilde{\mathbf{R}}_{\text{diag}}$ can be expressed as

$$\tilde{\mathbf{R}}_{\text{diag}}^{(k,k)} = \hat{c}_d^{(1)} + \hat{a}^*_{(+)} b_{d,(+)}^{(k-1)} + \hat{a}^*_{(-)} b_{d,(-)}^{(k-1)}, 1 \le k \le M. \tag{12}$$

The off-diagonal matrix $\tilde{\mathbf{R}}_{\text{off}}$ can be reconstructed by row-wise. Using $\tilde{\mathbf{R}}_{\text{diag}}^{(k,k)}$ as the first value of kth row, the element in kth row and tth column of $\tilde{\mathbf{R}}_{\text{off}}$ can be expressed as

$$\tilde{\mathbf{R}}_{\text{off}}^{(k,t)} = \tilde{\mathbf{R}}_{\text{diag}}^{(k,k)} + \hat{a}^*_{o,(+),k} b_{d,(+),k}^{(t-1)} + \hat{a}^*_{o,(-),k} b_{d,(-),k}^{(t-1)}, \quad 1 \le k \le M-1, \ k+1 \le t \le M. \tag{13}$$

Finally, the reconstructed $\tilde{\mathbf{R}}$ can be expressed as

$$\tilde{\mathbf{R}} = \tilde{\mathbf{R}}_{\text{diag}} + \tilde{\mathbf{R}}_{\text{off}} + \tilde{\mathbf{R}}_{\text{off}}^H. \tag{14}$$

Fig. 1 shows an example comparing original and reconstructed amplitude parts of diagonal elements. In Fig. 1, the unquantized first value $c_d^{(1)}$, an increment $a^*_{(+)}$, and a decrement $a^*_{(-)}$ can be used to reconstruct ampltidues of diagonal elements. Based on (3) and (5)–(8), $\mathbf{b}_d = [0,0,1,0,0,1,1]^T$, $c_d^{(1)} = 1$, $a^*_{(+)} = 0.2185$, and $a^*_{(-)} = -0.2723$ can be obtained. As shown in Fig. 1, every increment and decrement of the reconstructed amplitude parts of diagonal elements have an equal value as $a^*_{(+)}$ and $a^*_{(-)}$, respectively.

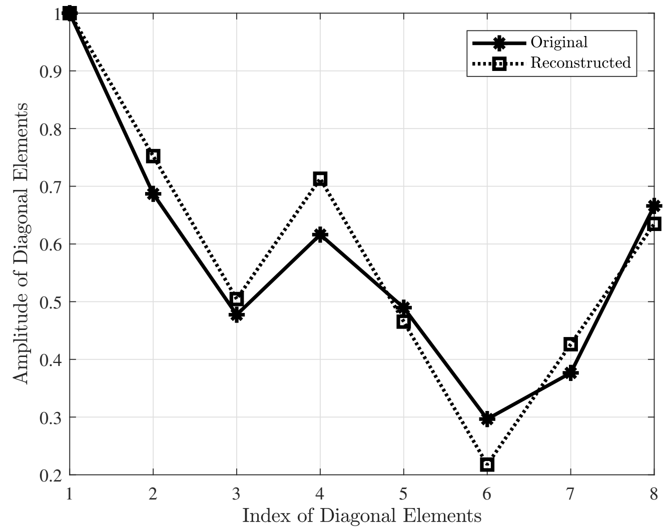

Fig. 1. Example of comparing original and reconstructed amplitude parts of diagonal elements by applying the proposed scheme where $M = 8$.

TABLE I
SIMULATION PARAMETERS RELATED TO $\mathbf{R}$

Parameter	Value
Carrier Frequency (f_c)	3 GHz
Antenna Configuration	Uniform Linear Array
Antenna Spacing	0.5λ
AS (Δ)	$15°$
AoA (θ)	Randomly generated in $[-\pi, \pi]$

IV. SIMULATION RESULTS

In this section, the simulation results for evaluating the performance of the proposed scheme are presented. For generating $\mathbf{R}$, the simulation parameters in Table I are used. N is set to 50 to generate $\bar{\mathbf{R}}$. $a_{max} = 1.2$ and $B_p = 3$ bits are used for both the conventional and proposed schemes. For the performance metric, we consider the mean square error (MSE) which can be expressed as

$$\text{MSE} = \frac{1}{M^2} \left\| \bar{\mathbf{R}} - \tilde{\mathbf{R}} \right\|_F^2, \tag{15}$$

where $\|\mathbf{A}\|_F$ denotes the Frobenius norm of a matrix $\mathbf{A}$.

Fig. 2 shows the MSE of the proposed scheme with quantized and unquantized feedback information with respect to the number of BS antennas M for different $B_a = 2, 3, 4$, and 5. When $B_a = 2$, the MSE of the proposed scheme with quantized information significantly degrades. However, when B_a increases, the MSE of the proposed scheme with quantized information can be converged to that of the proposed scheme with unquantized one.

Fig. 3 shows the MSE and feedback overheads of the conventional and proposed schemes with respect to B_a for different $M = 8, 16, 32, 64$, and 128. Fig. 3 also shows the ratio of feedback overheads for each M and B_a presented over the bar chart of each subplot at the right side. According to Fig. 3, it is observed that the MSE performance of the proposed scheme can be maintained with that of the conventional

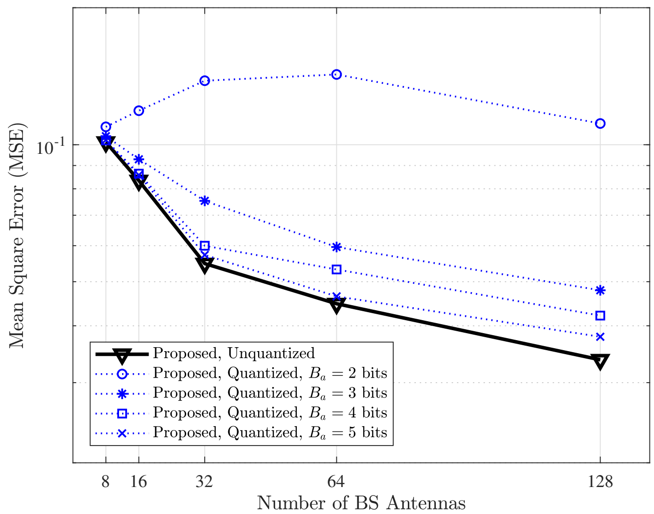

Fig. 2. MSE of the proposed scheme with quantized and unquantized feedback information with respect to the number of BS antennas for different $B_a = 2, 3, 4$, and 5.

scheme achieving the reduction of the feedback overhead from 1.92% to 48.33%. Also, as M becomes larger, the ratio of feedback overheads converges to the derived ratio of (11).

V. Conclusion

A feedback scheme of the channel covariance matrix feedback scheme with low overhead is proposed for massive MIMO systems. The proposed scheme decomposes the channel covariance matrix into the phase and amplitude parts. For the phase part, the element-wise uniform scalar quantization is performed. For the amplitude part, a bitmap, the first value, an increment, and a decrement are generated as the feedback information. Starting from the first value, when a bitmap is 1, the increment is added, otherwise the decrement is added to calculate the amplitude part. Simulation results show that the proposed scheme achieve a similar MSE performance of the conventional scheme with reduced feedback overhead.

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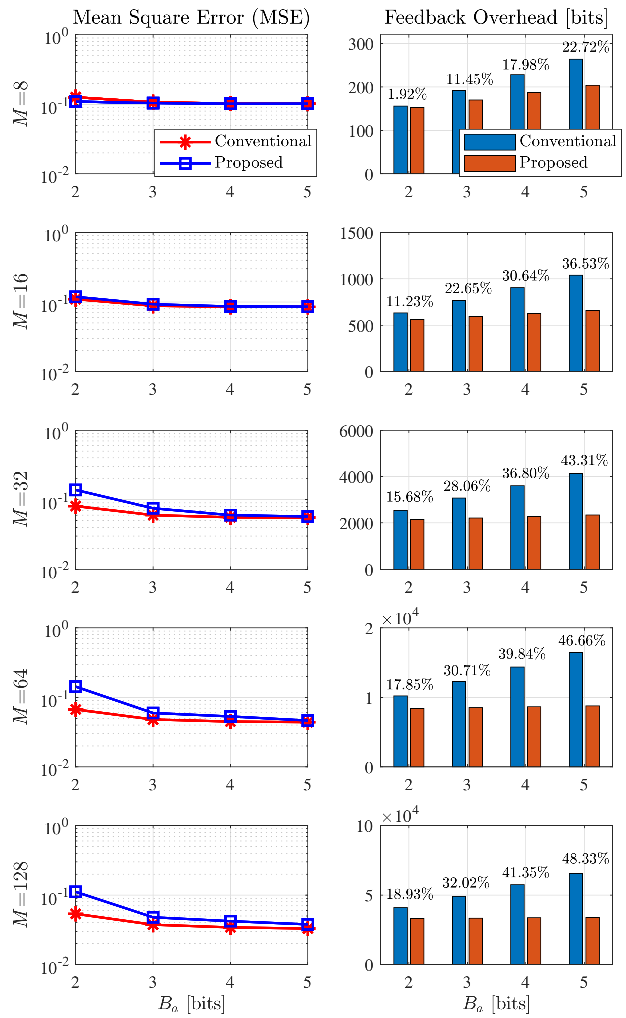

Fig. 3. MSE and feedback overheads of the conventional and proposed schemes with respect to B_a for different $M = 8, 16, 32, 64$, and 128.

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