The Analysis of Rehealing Delay for a Sparse VANET with Unequal Traffic

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Abstract— In this paper, we focus on the analysis of the rehealing delay incurred when restoring a network disconnection in a sparse vehicular ad hoc network (VANET) with unequal traffic on a bidirectional road. In previous works, equal vehicle traffic is assumed to simplify their analyses. To relax this limitation, we make proper revisions to one of the previous works. Numerical results are given to show the effect of unequal traffic on rehealing delay in a steady state.

Keywords—vehicular ad hoc network (VANET), rehealing delay, unequal vehicle traffic.

I. Introduction

Vehicular ad hoc networks (VANETs) are widely regarded as a promising wireless technology to realize intelligent transportation systems (ITS). Many of potential ITS applications, which include, e.g., safety warning [1], real-time route planning [2], etc., would rely heavily on message deliveries between vehicles, or vehicle-to-vehicle (V2V) communications. However, a VANET may suffer from network disconnections, especially during off-peak hours. When a network disconnection occurs, it can typically be restored by opposite-directional vehicles [3], [4], [5]. Due to the restoration, messages may inevitably incur rehealing delay [3], [4]. In either of [3], [4], [5], an analysis is thus performed to calculate the probability distribution of rehealing delay. However, all of them simply assume equal vehicle traffic intensity on a bidirectional road. Though the assumption could be acceptable during off-peak hours, unequal traffic would unarguably be deemed more general. Thus, we revise [4] to achieve this goal.

II. System Model and Imbedded Markov Chain

A. System model

We consider an east-west (E-W) bidirectional highway, with Poisson vehicle intensity μ_E, μ_W (vehicles/m), respectively. Messages sent from an eastbound vehicle are destined for another eastbound vehicle traveling far behind. The delivery is primarily between eastbound vehicles; only when an eastbound network disconnection takes place can westbound vehicles participate in the restoration, which usually entails a store-carry-forward process [3], [4], [5]. The range of a vehicle is R (m) and it is assumed that vehicles travel at a fixed speed, V (m/s).

The occurrence probabilities of an eastbound and a westbound disconnection can respectively be obtained as

$$\tau_E = \int_R^\infty \mu_E e^{-\mu_E x} dx = e^{-\mu_E R}, \quad (1)$$

$$\tau_W = \int_R^\infty \mu_W e^{-\mu_W x} dx = e^{-\mu_W R}. \quad (2)$$

Let α_E, α_W be the lengths of an eastbound and a westbound disconnection, respectively, and their probability density functions (PDFs) can be expressed as

$$f_{\alpha_E}(x) = \mu_E e^{-\mu_E (x-R)}, x > R. \quad (3)$$

$$f_{\alpha_W}(x) = \mu_W e^{-\mu_W (x-R)}, x > R. \quad (4)$$

Same-directional vehicles form a cluster when any pair can communicate with each other via one-/multi-hop transmissions. Denote an eastbound and a westbound cluster length as $\beta_{\mathrm{E}}, \beta_W$, respectively. Their PDFs are approximated as [3], [4]

$$f_{\beta_E}(x) \approx \tau_E \delta(x) + (1-\tau_E)\lambda_E e^{-\lambda_E x}, x \geq 0, \quad (5)$$

$$f_{\beta_W}(x) \approx \tau_W \delta(x) + (1-\tau_W)\lambda_W e^{-\lambda_W x}, x \geq 0, \quad (6)$$

where $\lambda_E = \tau_E / \left(\frac{1}{\mu_E} - \frac{R\tau_E}{1-\tau_E}\right), \lambda_W = \tau_W / \left(\frac{1}{\mu_W} - \frac{R\tau_W}{1-\tau_W}\right)$ [5].

B. Imbedded Markov chain

In order to take the dependence into consideration of the restoration of a network disconnection on that of the previous one, like [4], we develop a mathematical framework utilizing an imbedded Markov chain. In the framework, Π_j is employed to record the system state pertaining to the ramifications of the restoration of the jth network disconnection between S_j and D_j, which is finished at time $t = 0$, as exhibited in Fig. 1. Π_j is a discrete random variable whose range equals the set, $\mathbb{N}$, of all natural numbers. When $\Pi_j = 0$, the jth restoration is similar to Fig. 1(a); on the other hand, when $\Pi_j \in \mathbb{Z}^+$, where $\mathbb{Z}^+$ is the set of all positive integers, it is like Figs. 1(b) or 1(c). Obviously, the rehealing delay $T_j = 0$ in the former, and $T_j > 0$ in the latter.

III. Numerical Analysis

Let Ω represent the westbound cluster helping to restore the jth disconnection, and Γ the westbound stretch affected by it. When $\Pi_j = 0$, let b be the partial length of Ω on the west of $D_j\rceil$ at $t = 0$, where $D_j\rceil$ denotes the east range of D_j. Therefore, the length of Γ, γ, amounts to R plus b, where R accounts for the minimum spacing between two neighboring clusters. On the other hand, when $\Pi_j \in \mathbb{Z}^+$, let ψ be the inter-cluster spacing between Ω_0 and Ω_{-1}, where Ω_0, Ω_{-1} stand respectively for the leading vehicle of Ω and the vehicle immediately ahead of it. It is obvious that $\gamma = \psi$. Since ψ is clearly greater than R, like [4], let $\Pi_j = \lceil \psi - R \rceil$, giving rise to $\Pi_j \in \mathbb{Z}^+$. Note that our discussion hinges on the same speed assumption in Sec. II.A, which ensures network topology ever-unchanged in either direction. From the discussion, we can summarize

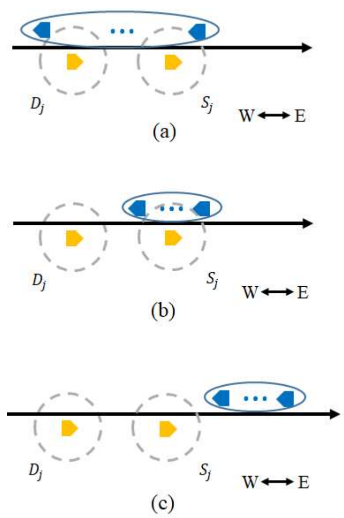

Fig. 1. The restoration of the jth network disconnection between s_j and d_j, where a dashed circle around a vehicle indicates its transmission range and an ellipse surrounding a group of vehicles suggests they belong to the same cluster.

$$\gamma = \begin{cases} b + R & \Pi_j = 0, \\ \Pi_j + R & \Pi_j \in \mathbb{Z}^+. \end{cases} \quad (7)$$

Fig. 2 shows relation between the locations of Γ and the $(j+1)$th disconnection at $t = 0$. To facilitate our analysis, like [4], we introduce two auxiliary random variables, i.e., Φ, Θ. Figs. 2(a), 2(b), and 2(c) correspond to $\Phi = \phi_I, \phi_{II}, \phi_{III}$, respectively. Denote a the eastbound cluster length to which S_{j+1} and D_j both belong, where S_{j+1} denotes the source vehicle of the $(j+1)$th disconnection, as shown in Fig. 2. Clearly, we have

$$\Phi = \begin{cases} \phi_I & \gamma < a + R, \\ \phi_{II} & a + R \le \gamma \le a + 2R, \\ \phi_{III} & \gamma > a + 2R. \end{cases} \quad (8)$$

On the other hand, Θ is used to indicate the relative location to s_{j+1} at $t = 0$ of the westbound vehicle that will relay messages to d_{j+1}. If it is to its west, $\Theta = \theta_W$; otherwise, $\Theta = \theta_E$.

The transition probability from $\Pi_j = m$ to $\Pi_{j+1} = n$, which constitutes the (m,n)th entry of $\boldsymbol{P}$, can thus be expressed as

$$p_{mn} = \sum_{m,\phi,\theta} \begin{pmatrix} P\{\Pi_{j+1} = n | \Pi_j = m, \Phi = \phi, \Theta = \theta\} \times \\ P\{\Theta = \theta | \Pi_j = m, \Phi = \phi\} \times \\ P\{\Phi = \phi | \Pi_j = m\} \end{pmatrix}. \quad (9)$$

The conditional probabilities of (9) are each calculated in the following. After $\boldsymbol{P}$ is found, the probability distribution, $\boldsymbol{\pi}$, of Π_j in a steady state can be obtained by solving simultaneously

$$\boldsymbol{\pi} = \boldsymbol{\pi}\boldsymbol{P}, \quad (10)$$

$$\sum_i \pi_i = 1, \quad (11)$$

where $\boldsymbol{\pi} = (\pi_0, \pi_1, \cdots)$ and $\pi_i = \mathrm{P}\{\Pi_j = i\}$ in a steady state.

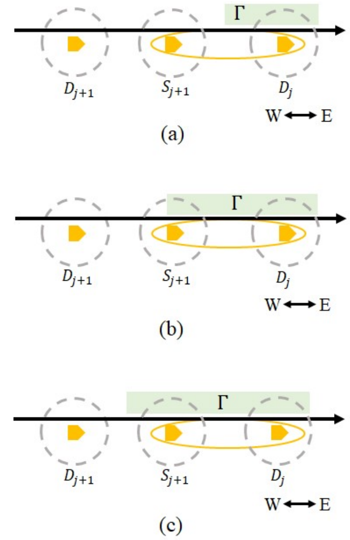

Fig. 2. Relation between γ and the $(j+1)$th disconnection.

A. $P\{\Phi = \phi_I | \Pi_j = \pi\}$

Let $\delta = b - a$. Due to the memoryless property for a nonzero cluster length in both (5) and (6), the conditional PDF of δ, given $\Psi_1 = \{b > 0, a > 0\}$, can be expressed as

$$f_{\delta|\Psi_1}(x) = \frac{\lambda_W \lambda_E}{\lambda_W + \lambda_E} \begin{cases} e^{-\lambda_W x} & x \ge 0, \\ e^{\lambda_E x} & x < 0. \end{cases} \quad (12)$$

Because $\gamma = b + R$ when $\Pi_j = 0$ from (7), the conditional probability can be calculated as

$$\begin{aligned} &\mathrm{P}\{\Phi = \phi_I | \Pi_j = 0\} = \mathrm{P}\{\gamma < a + R | b > 0\} \\ &= \mathrm{P}\{b + R < a + R | b > 0\} \\ &= \mathrm{P}\{\delta < 0 | a = 0, b > 0\}\mathrm{P}\{a = 0\} \\ &\quad + \mathrm{P}\{\delta < 0 | a > 0, b > 0\}\mathrm{P}\{a > 0\} = \frac{\lambda_W (1 - \tau_E)}{\lambda_W + \lambda_E}. \end{aligned} \quad (13)$$

On the other hand, $\gamma = \pi + R$ when $\Pi_j = \pi \in \mathbb{Z}^+$ from (7). Clearly, a, Π_j are independent as they are in opposite directions. The conditional probability can thus be computed as

$$\begin{aligned} &\mathrm{P}\{\Phi = \phi_I | \Pi_j = \pi \in \mathbb{Z}^+\} = \mathrm{P}\{a > \pi | \Pi_j = \pi\} \\ &= \int_\pi^\infty f_{\beta_E}(x) dx = (1 - \tau_E) e^{-\lambda_E \pi}. \end{aligned} \quad (14)$$

B. $P\{\Phi = \phi_{II} | \Pi_j = \pi\}$

Let $\sigma = \gamma - (a + R)$. From (7), it can be rewritten as

$$\sigma = \begin{cases} b - a = \delta & \pi = 0, \\ \pi - a & \pi \in \mathbb{Z}^+. \end{cases} \quad (15)$$

Clearly, according to (8), $\{\Phi = \phi_{II}\} \equiv \{0 \le \sigma \le R\}$. Given $\Pi_j = 0$, the conditional PDF can thus be obtained as

$$\begin{aligned} &f_{\sigma|\Pi_j}(x|0) = f_{\delta|b>0}(x) \\ &= \tau_E f_{b|b>0}(x) + (1 - \tau_E) f_{\delta|\Psi_1}(x), \qquad x > 0, \end{aligned} \quad (16)$$

where $f_{b|b>0}(x) = \lambda_W e^{-\lambda_W x}, x > 0$, from (6). On the other hand, given $\Pi_j \in \mathbb{Z}^+$, the conditional PDF of σ can be found as

$$f_{\sigma|\Pi_j}(x|\pi) = f_a(\pi - x) = f_{\beta_E}(\pi - x), \qquad x \le \pi. \quad (17)$$

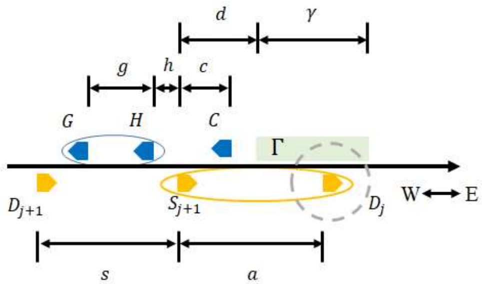

Fig. 3. Relation between random variables when $\Phi = \phi_I$.

The conditional probability can be calculated as

$$\mathrm{P}\{\Phi = \phi_{II} | \Pi_j = \pi \in \mathbb{N}\} = \int_0^R f_{\sigma|\Pi_j}(x|\pi)dx$$
$$= \begin{cases} \frac{\tau_E \lambda_W + \lambda_E}{\lambda_W + \lambda_E}\left(1 - e^{-\lambda_W R}\right) & \pi = 0, \\ 1 - (1-\tau_E)e^{-\lambda_E \pi} & 1 \le \pi \le R, \\ (1-\tau_E)\left(e^{-\lambda_E(\pi - R)} - e^{-\lambda_E \pi}\right) & \pi > R. \end{cases} \quad (18)$$

C. $P\{\Phi = \phi_{III} | \Pi_j = \pi\}$

Similarly, because $\{\Phi = \phi_{III}\} \equiv \{\sigma > R\}$, the conditional probability can be obtained as

$$\mathrm{P}\{\Phi = \phi_{III} | \Pi_j = \pi \in \mathbb{N}\} = \int_R^\infty f_{\sigma|\Pi_j}(x|\pi)dx$$
$$= \begin{cases} \frac{\tau_E \lambda_W + \lambda_E}{\lambda_W + \lambda_E} e^{-\lambda_W R} & \pi = 0, \\ 0 & 1 \le \pi \le R, \\ \tau_E + (1-\tau_E)\left(1 - e^{-\lambda_E(\pi - R)}\right) & \pi > R. \end{cases} \quad (19)$$

D. $P\{\Theta = \theta | \Pi_j = \pi, \Phi = \phi_I\}$

Fig. 3 shows relation between random variables when $\Phi = \phi_I$. As depicted, since Γ is all to the west of S_{j+1}, it is independent of whether $\Theta = \theta_W$ or $\Theta = \theta_E$. Thus, $\forall \pi \in \mathbb{N}$, the conditional probabilities can be written as

$$\mathrm{P}\{\Theta = \theta_W | \Pi_j = \pi, \Phi = \phi_I\} = 1 - e^{-\mu_W R}, \quad (20)$$
$$\mathrm{P}\{\Theta = \theta_E | \Pi_j = \pi, \Phi = \phi_I\} = e^{-\mu_W R}. \quad (21)$$

E. $P\{\Theta = \theta | \Pi_j = \pi, \Phi = \phi_{II}\}$

Fig. 4 shows relation between random variables when $\Phi = \phi_{II}$. As shown, the west edge of Γ extends well beyond S_{j+1}, though it is still inside the west range of S_{j+1}, denoted as $\lceil S_{j+1}$.

When $\pi = 0$, there exist no westbound vehicles between the west edge of Γ and S_{j+1}. Let K be the westbound vehicle to the west of and nearest the west edge of Γ at $t = 0$, and k be the distance between them. It is clear that if $\xi = k + \sigma < R$, $\Theta = \theta_W$; otherwise, $\Theta = \theta_E$.

To calculate the conditional probability that $\xi < R$, we need to obtain the conditional PDF of σ, which can be computed as

$$f_{\sigma|\Pi_j,\Phi}(x|0, \phi_{II}) = \frac{f_{\sigma|\Pi_j}(x|0)}{\mathrm{P}\{\Phi = \phi_{II} | \Pi_j = 0\}}, \quad (22)$$

where $\mathrm{P}\{\Phi = \phi_{II} | \Pi_j = 0\}$ is given in (18), $f_{\sigma|\Pi_j}(x|0)$ in (16).

The conditional PDF of ξ can thus be expressed as

$$f_{\xi|\Pi_j,\Phi}(x|0, \phi_{II}) = \left(f_{\sigma|\Pi_j,\Phi} * f_k\right)(x|0, \phi_{II}), \quad (23)$$

where $x > 0$ and $(f_1 * f_2)$ denotes the convolution of f_1 and f_2.

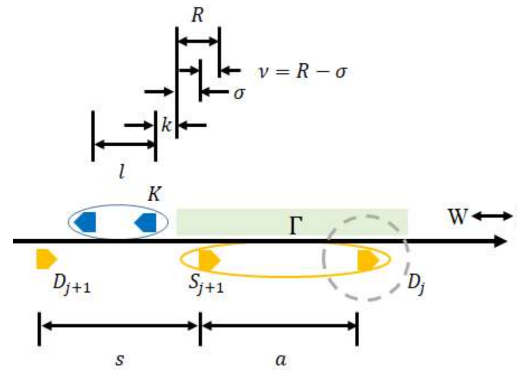

Fig. 4. Relation between random variables when $\Phi = \phi_{II}$.

The conditional probability can then be found as

$$\mathrm{P}\{\Theta = \theta_W | \Pi_j = 0, \Phi = \phi_{II}\} = \mathrm{P}\{\xi < R | \Pi_j = 0, \Phi = \phi_{II}\}$$
$$= \int_0^R f_{\xi|\Pi_j,\Phi}(x|0, \phi_{II})\,dx, \quad (24)$$
$$\mathrm{P}\{\Theta = \theta_E | \Pi_j = 0, \Phi = \phi_{II}\}$$
$$= 1 - \mathrm{P}\{\Theta = \theta_W | \Pi_j = 0, \Phi = \phi_{II}\}. \quad (25)$$

On the other hand, when $\pi \in \mathbb{Z}^+$, there is a westbound vehicle right at the west edge of Γ. Since the westbound vehicle is between S_{j+1} and $\lceil S_{j+1}$, it is clear that

$$\mathrm{P}\{\Theta = \theta_W | \Pi_j = \pi, \Phi = \phi_{II}\} = 1, \quad (26)$$
$$\mathrm{P}\{\Theta = \theta_E | \Pi_j = \pi, \Phi = \phi_{II}\} = 0. \quad (27)$$

F. $P\{\Theta = \theta | \Pi_j = \pi, \Phi = \phi_{III}\}$

Fig. 5 shows relation between random variables when $\Phi = \phi_{III}$, where Γ is beyond S_{j+1}, and its west edge is outside $\lceil S_{j+1}$.

When $\pi = 0$, though there exists an empty stretch of length R inside and at the west end of Γ, it is clear that the leading vehicle of Ω is to the west of S_{j+1} at $t = 0$. Therefore, we have

$$\mathrm{P}\{\Theta = \theta_W | \Pi_j = 0, \Phi = \phi_{III}\} = 1, \quad (28)$$
$$\mathrm{P}\{\Theta = \theta_E | \Pi_j = 0, \Phi = \phi_{III}\} = 0. \quad (29)$$

On the other hand, when $\pi > R$, there exist no westbound vehicles between the leading vehicle of Ω and the westbound vehicle at the west edge of Γ, where the former should be right at $D_j\rceil$ at $t = 0$ and the latter should be outside the west range of S_{j+1}. Therefore,

$$\mathrm{P}\{\Theta = \theta_W | \Pi_j = \pi, \Phi = \phi_{III}\} = 0, \quad (30)$$
$$\mathrm{P}\{\Theta = \theta_E | \Pi_j = \pi, \Phi = \phi_{III}\} = 1. \quad (31)$$

G. $P\{\Pi_{j+1} = \pi' | \Pi_j = \pi, \Phi = \phi_I, \Theta = \theta_W\}$

As displayed in Fig. 3, when $\Theta = \theta_W$, the $(j+1)$ th restoration is clearly independent of Γ. Let H be the westbound vehicle to the west of and nearest to S_{j+1}, h be the distance separating them, g the cluster length to which H belongs, and G the leading vehicle of the cluster. It is clear that if $X = s - h - g - R > 0$, X is the distance G needs to travel in order to restore the $(j+1)$ th disconnection whose length is s; otherwise, $T_{j+1} = 0$, and $\Pi_{j+1} = 0$.

In order to calculate the PDF of X, we first calculate the PDF of $s' = (s - R)$, which can readily be obtained from (3) as

$$f_{s'}(x) = \mu_E e^{-\mu_E x}, x > 0, \quad (32)$$

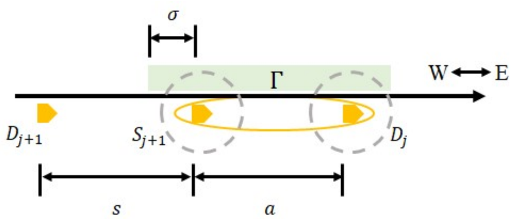

Fig. 5. Relation between random variables when $\Phi = \phi_{III}$.

and the PDFs of $g' = -g, h' = -h$ can be written as

$$f_{g'}(x) = f_{\beta_W}(-x), x \le 0, \tag{33}$$

$$f_{h'|\Theta}(x|\theta_W) = \frac{\mu_W e^{\mu_W x}}{1-e^{-\mu_W R}}, -R < x < 0. \tag{34}$$

The conditional PDF of X, $\forall \pi \in \mathbb{N}$, can then be found as

$$f_{X|\Pi_j,\Phi,\Theta}(x|\pi,\phi_I,\theta_W) = \left(f_{s'} * f_{g'} * f_{h'|\Theta}\right)(x|\pi,\phi_I,\theta_W), -\infty < x < \infty. \tag{35}$$

Finally, the transition probability can thus be calculated as

$$\mathrm{P}\{\Pi_{j+1} = \pi' | \Pi_j = \pi, \Phi = \phi_I, \Theta = \theta_W\} = \begin{cases} 1-\eta_1 & \pi' = 0, \\ \eta_1 \int_{\pi'-1}^{\pi'} \mu_W e^{-\mu_W x} dx & \pi' \in \mathbb{Z}^+, \end{cases} \tag{36}$$

where $\eta_1 = \mathrm{P}\{X > 0 | \Pi_j \in \mathbb{N}, \Phi = \phi_I, \Theta = \theta_W\}$.

H. $P\{\Pi_{j+1} = \pi' | \Pi_j = \pi, \Phi = \phi_I, \Theta = \theta_E\}$

As shown in Fig. 3, denote C the westbound vehicle to the east of and nearest S_{j+1} at $t = 0$, and c the distance between them. If $c < d$, where d is the distance from the west boundary of Γ to S_{j+1} at $t = 0$, $X = c + s'$; otherwise, the $(j+1)$th restoration must resort to the leading vehicle of Γ; as a result, $X = s' + d + R$ for $\Pi_j = 0$ and $X = s' + d$ for $\Pi_j \in \mathbb{Z}^+$.

First, $d = (a + R) - \gamma > 0$, as can be seen in Fig. 3. The conditional PDF of d can then be obtained as

$$f_{d|\Pi_j,\Phi}(x|\pi \in \mathbb{N}, \phi_I) = \lambda_E e^{-\lambda_E x}, x > 0. \tag{37}$$

The conditional marginal PDF of c for $c < d$ can be found as

$$f_{c,c<d|\Pi_j,\Phi,\Theta}(x|\pi,\phi_I,\theta_E) = \mu_W e^{-\mu_W x} \int_x^\infty f_{d|\Pi_j,\Phi}(y|\pi,\phi_I) dy, x > 0. \tag{38}$$

On the other hand, the conditional marginal PDF of d for $c > d$ can similarly be calculated as

$$f_{d,c>d|\Pi_j,\Phi,\Theta}(x|\pi,\phi_I,\theta_E) = f_{d|\Pi_j,\Phi}(x|\pi,\phi_I) \int_x^\infty \mu_W e^{-\mu_W y} dy, \quad x > 0. \tag{39}$$

Next, the conditional PDF of X can then be expressed as

$$f_{X|\Pi_j,\Phi,\Theta}(x|\pi,\phi_I,\theta_E) = \left(f_{s'} * f_{c,c<a|\Pi_j,\Phi,\Theta}\right)(x|\pi,\phi_I,\theta_E) + \left(f_{a,c>a|\Pi_j,\Phi,\Theta} * g_{s'}\right)(x|\pi,\phi_I,\theta_E), \quad x > 0, \tag{40}$$

where

$$g_{s'}(x) = \begin{cases} f_{s'}(x-R) & \pi = 0, \\ f_{s'}(x) & \pi \in \mathbb{Z}^+. \end{cases} \tag{41}$$

Finally, the transition probability can be calculated as

$$\mathrm{P}\{\Pi_{j+1} = \pi' | \Pi_j = \pi, \Phi = \phi_I, \Theta = \theta_E\} = \begin{cases} 0 & \pi' = 0, \\ \int_{\pi'-1}^{\pi'} f_{X|\Pi_j,\Phi,\Theta}(x|\pi,\phi_I,\theta_E) dx & \pi' \in \mathbb{Z}^+. \end{cases} \tag{42}$$

I. $P\{\Pi_{j+1} = \pi' | \Pi_j = \pi, \Phi = \phi_{II}, \Theta = \theta_W\}$

As shown in Fig. 4, when $\pi = 0$, $X = s' - \xi - l$, where l is the cluster length to which K belongs. The conditional PDF of ξ can be obtained as

$$f_{\xi|\Pi_j,\Phi,\Theta}(x|0,\phi_{II},\theta_W) = \frac{f_{\xi|\Pi_j,\Phi}(x|0,\phi_{II})}{\mathrm{P}\{\Theta = \theta_W | \Pi_j = 0, \Phi = \phi_{II}\}}, \tag{43}$$

where $0 < x < R$. Let $\xi' = -\xi, l' = -l$. The conditional PDF of ξ' and the PDF of l' can respectively be expressed as

$$f_{\xi'|\Pi_j,\Phi,\Theta}(x|0,\phi_{II},\theta_W) = f_{\xi|\Pi_j,\Phi,\Theta}(-x|0,\phi_{II},\theta_W), \quad -R < x < 0, \tag{44}$$

$$f_{l'}(x) = f_{\beta_W}(-x), x \le 0. \tag{45}$$

Then, the conditional PDF of X can be written as

$$f_{X|\Pi_j,\Phi,\Theta}(x|0,\phi_{II},\theta_W) = \left(f_{s'} * f_{\xi'|\Pi_j,\Phi,\Theta} * f_{l'}\right)(x|0,\phi_{II},\theta_W). \tag{46}$$

On the other hand, when $\pi \in \mathbb{Z}^+$, there is a westbound vehicle right at the west edge of Γ, and it is the last vehicle of a westbound cluster of length m. Clearly, $X = s' - \sigma - m$, where $0 < \sigma < R$. The conditional PDF of σ can be written as

$$f_{\sigma|\Pi_j,\Phi}(x|\pi,\phi_{II}) = \frac{f_{\sigma|\Pi_j}(x|\pi)}{\mathrm{P}\{\Phi = \phi_{II} | \Pi_j = \pi\}}, \tag{47}$$

where $0 < x < min(\pi, R)$ and $f_{\sigma|\Pi_j}(x|\pi)$ is given in (17).

Since $\mathrm{P}\{\Theta = \theta_W | \Pi_j \in \mathbb{Z}^+, \Phi = \phi_{II}\} = 1$ in (26), we have

$$f_{\sigma|\Pi_j,\Phi,\Theta}(x|\pi,\phi_{II},\theta_W) = f_{\sigma|\Pi_j,\Phi}(x|\pi,\phi_{II}). \tag{48}$$

Let $\sigma' = -\sigma, m' = -m$. Their conditional PDFs can be obtained as

$$f_{\sigma'|\Pi_j,\Phi,\Theta}(x|\pi,\phi_{II},\theta_W) = f_{\sigma|\Pi_j,\Phi,\Theta}(-x|\pi,\phi_{II},\theta_W), \quad -R < x < 0, \tag{49}$$

$$f_{m'}(x) = f_{\beta_W}(-x), \quad x \le 0. \tag{50}$$

Then, the conditional PDF of X can be computed as

$$f_{X|\Pi_j,\Phi,\Theta}(x|\pi \in \mathbb{Z}^+,\phi_{II},\theta_W) = \left(f_{s'} * f_{\sigma'|\Pi_j,\Phi,\Theta} * f_{m'}\right)(x|\pi,\phi_{II},\theta_W), \tag{51}$$

where $-\infty < x < \infty$.

Finally, like (36), the transition probability can be found as

$$\mathrm{P}\{\Pi_{j+1} = \pi' | \Pi_j \in \mathbb{N}, \Phi = \phi_{II}, \Theta = \theta_W\} = \begin{cases} 1-\eta_2 & \pi' = 0, \\ \eta_2 \int_{\pi'-1}^{\pi'} \mu_W e^{-\mu_W x} dx & \pi' \in \mathbb{Z}^+, \end{cases} \tag{52}$$

where $\eta_2 = \mathrm{P}\{X > 0 | \Pi_j \in \mathbb{N}, \Phi = \phi_{II}, \Theta = \theta_W\}$.

J. $P\{\Pi_{j+1} = \pi' | \Pi_j = \pi, \Phi = \phi_{II}, \Theta = \theta_E\}$

Since $\mathrm{P}\{\Theta = \theta_E | \Pi_j \in \mathbb{Z}^+, \Phi = \phi_{II}\} = 0$, we only calculate $\mathrm{P}\{\Pi_{j+1} = \pi' | \Pi_j = 0, \Phi = \phi_{II}, \Theta = \theta_E\}$. As shown in Fig. 4, the westbound vehicle nearest and to the east of S_{j+1} at $t = 0$ should be R behind the west edge of Γ. Given $\xi = k + \sigma > R$, the conditional marginal PDF of σ can thus be expressed as

$$f_{\sigma|\Pi_j,\Phi,\Theta}(x|0,\phi_{II},\theta_E) = f_{\sigma|\Pi_j,\Phi}(x|0,\phi_{II}) \int_{R-x}^\infty f_k(y) dy, \quad 0 \le x \le R. \tag{53}$$

Let $\nu = R - \sigma$. The conditional PDF of ν can be obtained as

$$f_{\nu|\Pi_j,\Phi,\Theta}(x|0,\phi_{II},\theta_E) = \frac{f_{\sigma|\Pi_j,\Phi,\Theta}(R-x|0,\phi_{II},\theta_E)}{\mathrm{P}\{\Theta = \theta_E | \Pi_j = 0, \Phi = \phi_{II}\}}, \quad 0 \le x \le R. \tag{54}$$

As $X = s' + \nu$ in Fig. 4, its conditional PDF can be found as

$$f_{X|\Pi_j,\Phi,\Theta}(x|0,\phi_{II},\theta_E) = \left(f_{\nu|\Pi_j,\Phi,\Theta} * f_{s'}\right)(x|0,\phi_{II},\theta_E), \quad x > 0. \tag{55}$$

Finally, the transition probability can be calculated as

$$\mathrm{P}\{\Pi_{j+1} = \pi' | \Pi_j = 0, \Phi = \phi_{II}, \Theta = \theta_E\} = \begin{cases} 0 & \pi' = 0, \\ \int_{\pi'-1}^{\pi'} f_{X|\Pi_j,\Phi,\Theta}(x|0,\phi_{II},\theta_E)dx & \pi' \in \mathbb{Z}^+. \end{cases} \tag{56}$$

K. $P\{\Pi_{j+1} = \pi' | \Pi_j = \pi, \Phi = \phi_{III}, \Theta = \theta_W\}$

Due to that $\mathrm{P}\{\Theta = \theta_W | \Pi_j = \pi, \Phi = \phi_{III}\} = 0$ for $\pi > R$ and $\mathrm{P}\{\Phi = \phi_{III} | \Pi_j = \pi\} = 0$ for $1 \leq \pi \leq R$, we only calculate $\mathrm{P}\{\Pi_{j+1} = \pi' | \Pi_j = 0, \Phi = \phi_{III}, \Theta = \theta_W\}$. As shown in Fig. 5, $X = s' - \sigma + R$. The conditional PDF of σ can be written as

$$f_{\sigma|\Pi_j,\Phi}(x|0,\phi_{III}) = \frac{f_{\sigma|\Pi_j}(x|0)}{\mathrm{P}\{\Phi = \phi_{III} | \Pi_j = 0\}}, \quad x > R, \tag{57}$$

$$f_{\sigma|\Pi_j,\Phi,\Theta}(x|0,\phi_{III},\theta_W) = f_{\sigma|\Pi_j,\Phi}(x|0,\phi_{III}), \quad x > R. \tag{58}$$

The conditional PDF of $\sigma' = -\sigma + R$ can be obtained as

$$f_{\sigma'|\Pi_j,\Phi,\Theta}(x|0,\phi_{III},\theta_W) = f_{\sigma|\Pi_j,\Phi,\Theta}(R - x|0,\phi_{III},\theta_W), \quad x < 0. \tag{59}$$

The conditional PDF of X can be expressed as

$$f_{X|\Pi_j,\Phi,\Theta}(x|0,\phi_{III},\theta_W) = \left(f_{s'} * f_{\sigma'|\Pi_j,\Phi,\Theta}\right)(x|0,\phi_{III},\theta_W). \tag{60}$$

Like (37) and (53), the transition probability can be found as

$$\mathrm{P}\{\Pi_{j+1} = \pi' | \Pi_j = 0, \Phi = \phi_{III}, \Theta = \theta_W\} = \begin{cases} 1 - \eta_3 & \pi' = 0, \\ \eta_3 \int_{\pi'-1}^{\pi'} \mu_W e^{-\mu_W x} dx & \pi' \in \mathbb{Z}^+, \end{cases} \tag{61}$$

where $\eta_3 = \mathrm{P}\{X > 0 | \Pi_j = 0, \Phi = \phi_{III}, \Theta = \theta_W\}$.

L. $P\{\Pi_{j+1} = \pi' | \Pi_j = \pi, \Phi = \phi_{III}, \Theta = \theta_E\}$

As a result of $\mathrm{P}\{\Phi = \phi_{III} | \Pi_j = \pi\} = 0$ for $1 \leq \pi \leq R$ and $\mathrm{P}\{\Theta = \theta_E | \Pi_j = 0, \Phi = \phi_{III}\} = 0$, we calculate the conditional transition probabilities for $\pi > R$ only, which can be found as

$$\mathrm{P}\{\Pi_{j+1} = \pi' | \Pi_j > R, \Phi = \phi_{III}, \Theta = \theta_E\} = \begin{cases} 1 & \pi' = \pi, \\ 0 & \pi' \neq \pi. \end{cases} \tag{62}$$

M. The PDF of Rehealing Delay

The PDF of T, which is the rehealing delay in a steady state, can then be obtained as

$$f_T(x) = \sum_{i,\phi,\theta} \left\{ \begin{array}{c} f_{X|\Pi_j,\Phi,\Theta}(2Vx|i,\phi,\theta) \\ \times P\{\Theta = \theta | \Pi_j = i, \Phi = \phi\} \\ \times P\{\Phi = \phi | \Pi_j = i\} \\ \times \pi_i \end{array} \right\}. \tag{63}$$

IV. Numerical Results and Discussions

In this section, analytic and simulation results of the CDF of rehealing delay are compared. Analytic results are calculated using MATLAB Symbolic Toolbox, whereas their simulation counterparts are each obtained from 1,000,000 independent samples. Three different scenarios are considered, including $\mu_E/\mu_W = 0.02/0.01, 0.01/0.01$, and $0.01/0.02$. Relevant system parameters include $R = 250$ m, $V = 30$ (m/s).

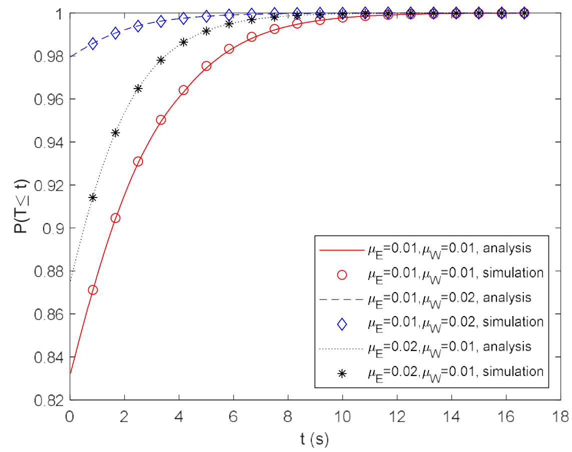

Fig. 6. The CDF of rehealing delay T in a steady state.

Fig. 6 shows the cumulative distribution function (CDF) of rehealing delay T in a steady state. As displayed, when $\mu_E/\mu_W = 0.01/0.01$, $\mathrm{P}\{T = 0\}$ in the steady state is the lowest among the three different scenarios considered. However, when $\mu_E/\mu_W = 0.02/0.01$ or $0.01/0.02$, $\mathrm{P}\{T = 0\}$ in the steady state increases. This suggests that the increase of intensity in either direction can help to increase the chance of a network disconnection being restored without incurring any rehealing delay, like the one in Fig. 1(a). Comparing further between $\mu_E/\mu_W = 0.02/0.01$ and $0.01/0.02$, raising the intensity in the opposite direction have a more significant effect on lowering T.

V. Conclusions

In this paper, we analyze the rehealing delay incurred when restoring a network disconnection in a sparse vehicular ad hoc network (VANET) with unequal traffic on a bidirectional road. In our analysis, in order to taking the dependence into account of the restoration of a disconnection on that of the previous one, an imbedded Markov chain is utilized to keep track of the system state. Numerical results demonstrate the excellent accuracy of our analysis; in addition, increasing the intensity of the opposite directional traffic is more effective on reducing rehealing delay than increasing that of the same directional one.

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