Tri-level Stackelberg Game for Resource Allocation in Radio Access Network Slicing

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***Abstract*—In this paper, we consider a three-level hierarchical structure for resource allocation in the radio access network (RAN) slicing. The infrastructure provider (InP) allocates the RAN slices to the mobile virtual network operators (MVNOs), and the MVNOs then allocate the radio resources to the users. It is challenging for the InP to determine resource allocation strategy efficiently due to the selfish strategic responses of both the MVNOs and the users. To handle this issue, we propose a tri-level Stackelberg game to jointly solve the frequency and power allocation and payment negotiation problem among the three levels. Simulation results verify a general market principle that the more the MVNOs focus on revenue collecting, the lower payoff the InP and the users will obtain.**

I. Introduction

In the incoming 5G, the network slicing is considered as a fundamental capability to support current and future diverse application scenarios. It requires the infrastructure provider (InP) to slice the radio access network (RAN) as well as the core network for the specific demands of the mobile virtual network operators (MVNOs) [1]. Specifically, the RAN slicing allows deployment of multiple logical RANs over a common physical infrastructure in a cost effective way, and enables each MVNO to have its own logically isolated RAN slice with complete control over it [2].

On implementing RAN slicing, one important aspect is the resource allocation problem [3]. In the slicing process, the InP allocates the physical resources, e.g., the frequency resource and the transmit power resource, to the MVNOs, which then allocates them to the users in order to satisfy their quality-of-service (QoS) requirements. Therefore, the resource allocation in RAN slicing follows a *tri-level hierarchical structure*, i.e., the InP-MVNO-user structure [4]. In the tri-level structure, the InP needs to consider the responses in both the MVNO and user levels to determine its resource allocation strategy. However, since the MVNOs and users are generally selfish entities which choose their responses to maximize their own interests, it is challenging for the InP to determine its resource allocation strategy efficiently.

Most existing works on the resource allocation in RAN slicing considered a bi-level structure and neglected the initiative of the users in determining their own resource requirements [5]–[7]. In [5], a bankruptcy game was proposed for dynamic wireless resource allocation among multiple MVNOs to guarantee allocation fairness. In [6], stochastic game based schemes were proposed to cope with the time-varying traffic conditions in wireless networks, where the resource allocation was determined by the VCG mechanism. In [7], two-stage Stackelberg games were applied to optimize the strategies of InP and MVNOs. Few literatures considered the tri-level resource allocation problem, and adopted two levels of auctions to handle it [8], [9]. Authors in [8] proposed to use two levels of combinatorial auctions, and adopted VCG mechanisms in both levels. Authors in [9] proposed to use a proportional auction among the InP and MVNOs, and adopted VCG auction among the MVNOs and the users. Although the auction mechanisms such as VCG have many favorable properties such as social efficiency and truthfulness, whether those properties hold for the coupled bi-level auction is lack of proof.

In this paper, we consider a tri-level RAN slicing where the selfishness of both the MVNO and user levels is taken into account. Unlike the existing works, we propose a tri-level Stackelberg game (TLSG) to jointly solve the frequency and power allocation and payment negotiation problem among three levels, i.e., the InP level, MVNO level and user level. The users, MVNOs, and InP are considered as self-interested individual decision-making entities occupying different levels, i.e., the InP occupying the upper level, the MVNOs occupying the middle level, and the users occupying the lower level. The entities in each level determine their strategies considering the strategies that will be responded by the levels below, in order to maximize their own payoff. Moreover, we solve the equilibrium strategies of the InP, MVNOs, and users in the TLSG by proposing an algorithm which combines the differential evolution (DE) algorithm and local descent method. Unlike the existing works [8], [9], the optimality of the InP's resource allocation strategy is guaranteed. Besides, we consider that the entities in both the InP and MVNO levels aim to achieve a tradeoff between the revenue and users' utility, which is more practical than considering them to maximize either the revenue or users' utility.

The main contributions of this paper are summarized as follows:

1) We analyze the tri-level hierarchical structure in RAN slicing, in which the MVNOs and users are selfish and can determine their own strategies.
2) A TLSG is proposed to solve the tri-level frequency and power allocation problem jointly in RAN slicing.
3) Simulation results verify the market principle that if the MVNOs focus on collecting more revenue, the InP and the users will result in obtaining less payoff.

The rest of this paper is organized as follows. In Section II, the system model is described. In Section III, hierarchical

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resource allocation is formulated as a TLSG and an algorithm combining DE and local descent method is proposed to solve the TLSG. The simulation results are presented in Section IV. Finally, the conclusion is drawn in Section V.

II. System Model

In this section, we first describe the network scenario of the RAN slicing. Following that, the process of the tri-level resource allocation is demonstrated, and the payoff functions of the users, the MVNOs, and the InP are specified, respectively.

A. Network Scenario

As shown in Fig.1, we consider a single-cell time-synchronized OFDMA system consisting of an InP, multiple MVNOs and downlink users. The physical resources in this system are owned by the InP, which can be partitioned into isolated slices. The slices are leased to M MVNOs, which are indexed by i. The MVNOs then use the leased resources to provide services to the users, such as smart electric meters, smart cars, and surveillance cameras, as shown in Fig. 1. Each MVNO i has its own set of subscribed users, denoted as $\mathcal{N}_i$. The number of users in $\mathcal{N}_i$ is denoted as N_i, and the j-th user among them is denoted as user (i, j).

For the physical resources, we consider an available frequency band which is divided into $S_{\max}$ sub-channels (SCs), and the maximum transmit power $P_{\max}$ of the base station. We assume that the RAN slicing is implemented by the InP, which assigns different frequency bands and transmit power to each MVNO i. Therefore, the MVNOs (as well as the users) are isolated with each other and have no interference.

For user (i, j), its data rate is a function of the physical resources allocated to it by the MVNO i, which can be expressed as follows:

$$R_{i,j}(s_{i,j}, p_{i,j}) = s_{i,j} B \cdot \log_2(1 + k_{i,j} \frac{p_{i,j}}{s_{i,j}}), \tag{1}$$

where $s_{i,j}$ and $p_{i,j}$ are the number of SCs and the transmit power in Watt allocated to the user (i, j), respectively, and $k_{i,j}$ indicates the SNR per SC of the MVNO i – user (i, j) link, denoted by $k_{i,j} = H_{i,j}/N_0$. Here $H_{i,j} \sim l_{i,j}^{-\alpha}$ is the channel gain between user (i, j) and the base station with $l_{i,j}$ denoting the distance and α being the decay factor, and N_0 is the power frequency density of the noise at the receivers of the users.

B. Process of Hierarchical Resource Allocation

The resource allocation in the tri-level RAN slicing includes 5 steps, which can be described as follows:

1) The InP determines the unit prices of the frequency resources to be δ_s and power resources to be δ_p, which are together referred to as *resource price* and denoted as $\boldsymbol{\delta} = (\delta_s, \delta_p)$, and announces them to all the MVNOs,
2) Each MVNO i then decides the unit resale prices of the frequency resources to be $\nu_{s,i}$ and that of power resources to be $\nu_{p,i}$ for their users, which are together referred to as *resale price* and denoted as $\boldsymbol{\nu}_i = (\nu_{s,i}, \nu_{p,i})$.
3) Each user (i, j) decides the amount of frequency and power resources that it purchases from the MVNO, i.e., $s_{i,j}$ and $p_{i,j}$, which are referred to as the *purchase amount* of frequency and power resources, respectively, and pays for the resources to its MVNO.
4) Each MVNO i collects the purchase amount from its users, and informs the InP that it will purchase $S_i = \sum_{j=1}^{N_i} s_{i,j}$ frequency resources and $P_i = \sum_{j=1}^{N_i} p_{i,j}$ power resources, and pays the rent to the InP.
5) Finally, the InP creates RAN slices with the required frequency and power resources for the MVNOs, which then allocate the resources to their users.

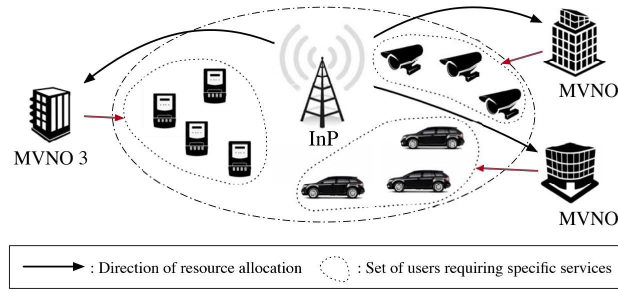

Fig. 1. Illustration on tri-level hierarchical structure in RAN slicing.

C. Payoff Functions

1) Payoff Function of Users: Assume that each user (i, j) has the quasi-linear payoff function in the following form:

$$\mathcal{J}_{i,j}(s_{i,j}, p_{i,j}; \boldsymbol{\nu}_i) = \mathcal{U}_{i,j}(s_{i,j}, p_{i,j}) - \nu_{s,i} s_{i,j} - \nu_{p,i} p_{i,j}, \tag{2}$$

in which $\mathcal{U}_{i,j}(s_{i,j}, p_{i,j})$ is the utility that user (i, j) gets from $s_{i,j}$ SCs and $p_{i,j}$ Watts power. Moreover, we assume that the utility of the users are only relevant to their data rates. Based on [10], the utility function of user (i, j) can be expressed as

$$\mathcal{U}_{i,j}(s_{i,j}, p_{i,j}) = a_{i,j} - b_{i,j} e^{-(R_{i,j}(s_{i,j}, p_{i,j}) - q_{i,j}) \cdot d_{i,j}}, \tag{3}$$

where $a_{i,j}$ is the maximum utility that the user can obtain, $q_{i,j}$ is the required average data rate representing the QoS requirement, $b_{i,j}$ is the loss of utility at the QoS required rate, and $d_{i,j}$ is the parameter to ensure $\mathcal{U}_{i,j}(0) = 0$, i.e. $d_{i,j} = \ln(a_{i,j}/b_{i,j})/q_{i,j}$.

2) Payoff Function of MVNOs: We consider the payoff of each MVNO i as a tradeoff between the collected revenue and the sum of the users' utility, which can be expressed as

$$\begin{aligned}\mathcal{J}_i(\boldsymbol{\nu}_i; \boldsymbol{s}_i, \boldsymbol{p}_i, \boldsymbol{\delta}) =& \lambda_i \sum_{j=1}^{N_i} \left[(\nu_{s,i} - \delta_s) s_{i,j} + (\nu_{p,i} - \delta_p) p_{i,j}\right] \\ &+ (1 - \lambda_i) \xi_i(\boldsymbol{s}_i, \boldsymbol{p}_i),\end{aligned} \tag{4}$$

where $\boldsymbol{s}_i = (s_{i,1}, s_{i,2}, ... s_{i,N_i})$ and $\boldsymbol{p}_i = (p_{i,1}, p_{i,2}, ... p_{i,N_i})$ are the vectors of users' frequency and power resources purchase amount, $\xi_i(\boldsymbol{s}_i, \boldsymbol{p}_i) = \sum_{j=1}^{N_i} \mathcal{U}_{i,j}(s_{i,j}, p_{i,j})$ denotes the sum of users' utility with regard to users' resource purchase amount vectors, and λ_i is a tradeoff coefficient which indicates how much the MVNO i evaluates between the collected revenue and the utility of the users that it serves.

3) Payoff Function of InP: We assume that the InP also considers a tradeoff between the collected revenue and the utility of users as its payoff, thus the payoff function of InP can be expressed as

$$\mathcal{J}_I(\boldsymbol{\delta}; \boldsymbol{s}_1,...\boldsymbol{s}_M, \boldsymbol{p}_1,...\boldsymbol{p}_M) = \mu \sum_{i=1}^{M} (\delta_s S_i + \delta_p P_i) + (1-\mu) \sum_{i=1}^{M} \xi_i(\boldsymbol{s}_i, \boldsymbol{p}_i), \quad (5)$$

where μ is the tradeoff coefficient indicating how much the InP evaluates between the revenue and the sum of users' utility.

III. Algorithm Design for TLSG

In this section, we propose the TLSG for the tri-level hierarchical resource allocation problem in RAN slicing, and adopt a backward induction approach to find the equilibrium strategy in each level. We transform the TLSG into equivalent optimization problem in each level, for which either an analytical solution or an algorithm is provided.

A. Strategy of Users

For the users of a certain MVNO i, we omit the index i for the sake of convenience in writing, e.g., using s_j to denote $s_{i,j}$. The optimal strategy of the j-th user is the strategy which maximizes its payoff for a given resale price $\boldsymbol{\nu}$, and can be obtained by solving the following optimization problem

$$\max_{s_j, p_j} \quad \mathcal{U}_j(s_j, p_j) - s_j \nu_s - p_j \nu_p, \quad (6)$$

$$s.t. \quad s_j, p_j \geq 0. \quad (6a)$$

It can be proved that the optimization problem (6) is a concave optimization problem, and the solution can be given by the following proposition.

Proposition 1. *The user level optimization problem (6) is a concave optimization problem. For each given resoure resale price $\boldsymbol{\nu}$, there exists an unique optimal resource purchase amount (s_j^*, p_j^*) for user j, which can be calculated as*

$$s_j^*(\boldsymbol{\nu}) = \max\{\frac{\ln(Bd_j a_j k_j) - \ln \nu_p}{Bd_j(W_0(e^{-1}(k_j\kappa - 1)) + 1)} - \frac{1}{Bd_j}, 0\} \quad (7)$$

$$p_j^*(\boldsymbol{\nu}) = (\frac{k_j\kappa - 1}{W_0(e^{-1}(k_j\kappa - 1))} - 1) \cdot \frac{s_j^*(\boldsymbol{\nu})}{k_j} \quad (8)$$

where $\kappa = \nu_s/\nu_p$ is the resale price ratio*, and $W_0(\cdot)$ denotes the zero-th branch of the Lambert W function [11].*

Proof. See Appendix A. ∎

Therefore, for user j, the optimal strategy s_j^* and p_j^* can be seen as functions of the given resale price of its MVNO.

B. Strategy of MVNO

Similar to the former subsection, the subscript i indexing the MVNO is also omitted. Given $\boldsymbol{\delta} = (\delta_s, \delta_p)$, the optimal strategy of the MVNO can be obtained by the following optimization problem:

$$\max_{\boldsymbol{\nu}} \quad \mathcal{J}(\boldsymbol{\nu}; \boldsymbol{s}^*(\boldsymbol{\nu}), \boldsymbol{p}^*(\boldsymbol{\nu}), \boldsymbol{\delta}) \quad (9)$$

$$s.t. \quad \nu_s, \nu_p \geq 0, \quad (9a)$$

where vectors $\boldsymbol{s}^*(\boldsymbol{\nu})$ and $\boldsymbol{p}^*(\boldsymbol{\nu})$ denote the optimal frequency and power resources purchase amount of the users given the resale price $\boldsymbol{\nu}$, respectively. Since the optimization problem (9) is generally not concave, thus, we propose an algorithm combing differential evolution (DE) and local descent method to solve the set of MVNO's optimal responses for a given $\boldsymbol{\delta}$, which is denoted as $\mathcal{R}(\boldsymbol{\delta})$.

1) DE Algorithm: DE algorithm is a parallel direct search method, which utilizes L_p parameter vectors $x_{n,g}$, $n = 1, 2, ..., L_p$ as a population for each generation g [12]. Three operations are performed for each population g to evolve into the next generation, i.e., *mutation*, *crossover* and *selection*.

Mutation: For each vector in the generation g, a *mutant vector* is generated as follows:

$$\boldsymbol{v}_{n,g+1} = \boldsymbol{x}_{r_1,g} + F \cdot (\boldsymbol{x}_{r_2,g} - \boldsymbol{x}_{r_3,g}), \quad (10)$$

where $r_1, r_2, r_3 \in [1, L_p]$ are randomly selected individuals of the population, and $F \in (0, 2]$ is the mutation coefficient. Moreover, a *projection* operation is performed to each mutant vector in order to ensure the prices of the resources are nonnegative, in which the negative elements in $\boldsymbol{v}_{n,g+1}$ are set to zeroes.

Crossover: The crossover is introduced to increase the diversity of the perturbed parameter vectors, in which a *trial vector* $\boldsymbol{u}_{n,g+1}$ is formed for each parameter vector $\boldsymbol{x}_{n,g}$. The l-th element of the n-th trail vector $u_{n,g+1}^{(l)}$ is equal to $v_{n,g+1}^{(l)}$ with probability $C_r \in (0, 1)$, or otherwise equal to $x_{n,g+1}^{(l)}$.

Selection: The trail vector $\boldsymbol{u}_{n,g+1}$ is compared to $\boldsymbol{x}_{n,g}$ in terms of the resulting value of objective function, and the one with larger resulting objective function value will be selected to remain in the next generation.

2) Local Descent Method: Since the DE algorithm is a heuristic algorithm which dose not guarantee the local optimality, a local optimization method is necessary. The optimization problem (9) can be transformed into unconstrained minimization problem using logarithmic barrier function method, in which the objective function can be expressed as

$$\max_{\boldsymbol{\nu}} \quad B_L(\boldsymbol{\nu}; \chi_L, \boldsymbol{\delta}) = -\mathcal{J}(\boldsymbol{\nu}; \boldsymbol{\delta}) - \frac{1}{\chi_L} \ln(\nu_s) - \frac{1}{\chi_L} \ln(\nu_p), \quad (11)$$

where $\chi_L > 0$ is a penalty factor. To solve the unconstrained minimization problem (11), it requires the information on the gradient and hessian of $\mathcal{J}$ with regard to $\boldsymbol{\nu}$, which is derived in the following proposition.

Proposition 2. *The gradient and hessian of the MVNO's payoff function $\mathcal{J}$ with regard to $\boldsymbol{\nu}$ can be calculated by*

$$\nabla \mathcal{J} = \sum_{j \in \mathcal{N}^+} \lambda \begin{bmatrix} s_j^* \\ p_j^* \end{bmatrix} + (\nu_s - \lambda\delta_s) \nabla s_j^* + (\nu_p - \lambda\delta_p) \nabla p_j^*, \quad (12)$$

$$\nabla^2 \mathcal{J} = \sum_{j \in \mathcal{N}^+} (1+\lambda) \frac{\partial(s_j^*, p_j^*)}{\partial(\nu_s, \nu_p)} + (\nu_s - \lambda\delta_s) \nabla^2 s_j^* + (\nu_p - \lambda\delta_p) \nabla^2 p_j^*, \quad (13)$$

where $\mathcal{N}^+$ denotes the set of users who have non-zero optimal resource purchase amount.

Proof. See Appendix B. ∎

Algorithm 1 Algorithm to solve MVNO level optimization.

Input: L_p (population size), G (number of generations), F (mutation coefficient), C_r (crossover rate), $k, g := 1$, $\boldsymbol{\delta}$;

Output: $\mathcal{R}(\boldsymbol{\delta})$;

1: Create a initial population of size L_p of random vectors $\boldsymbol{x}_{n,g},\ n = 1, ...L_p$ with dimension 2, satisfying $\boldsymbol{x}_{n,g} \succ 0$.
2: Evaluate $\mathcal{J}(\boldsymbol{x}_{n,g})$ for $\forall n \in [1, L_p]$;
3: If $g > G$, go to step 8;
4: *Mutation*: For each parameter vector $\boldsymbol{x}_{n,g}$, generate a mutant vector $\boldsymbol{v}_{n,g+1}$.
5: *Crossover*: Perform crossover mechanism between the target vector $\boldsymbol{x}_{n,g}$ and the mutant vector $\boldsymbol{v}_{n,g+1}$ to generate trail vector $\boldsymbol{u}_{n,g+1}$.
6: *Selection*: For each trail vector, evaluate $\mathcal{J}(\boldsymbol{u}_{n,g+1})$. If $\mathcal{J}(\boldsymbol{u}_{n,g+1}) > \mathcal{J}(\boldsymbol{x}_{n,g})$, replace $\boldsymbol{x}_{n,g}$ with $\boldsymbol{u}_{n,g+1}$ in the population.
7: $g := g + 1$, go to step 3.
8: Perform local descent algorithm for L_p times with initial points set as $\boldsymbol{\nu}^{(0)} = \boldsymbol{x}_{n,g},\ n = 1, ...L_p$, and output the set of results as $\mathcal{R}(\boldsymbol{\delta})$.

Based on the acquired gradient and hessian of $\mathcal{J}$, Newton methods can be adopted to solve the unconstrained optimization problem (11), and the original constrained optimization problem can be solved using the logarithmic barrier method.

In summary, based on the DE algorithm and the local descent algorithm, Alg. 1 is proposed to solve the optimization problem (9). Since the DE algorithm is a heuristic approach, the results may not be global optimal. However, as the local descent method is combined with the DE algorithm, the results are at least local optimal.

C. Strategy of InP

In TLSG, the InP decides its strategy considering the strategies responded by the MVNOs and users, in order to maximize its payoff $\mathcal{J}_I$ defined in (5). The InP's strategy in the equilibrium of TLSG can be obtained by solving the following bi-level optimization problem, with the constraints that the MVNOs and users choose their optimal strategies found in former parts of this section.

$$\max_{\boldsymbol{\delta}\succeq 0}\ \mathcal{J}_I(\boldsymbol{\delta}; \boldsymbol{s}_1^*(\boldsymbol{\nu}_1^*), ...\boldsymbol{s}_M^*(\boldsymbol{\nu}_M^*), \boldsymbol{p}_1^*(\boldsymbol{\nu}_1^*), ...\boldsymbol{p}_M^*(\boldsymbol{\nu}_M^*)) \tag{14}$$

$$s.t.\ \sum_{i=1}^{M} S_i^*(\boldsymbol{\nu}_i^*) \le S_{max},\ \sum_{i}^{M} P_i^*(\boldsymbol{\nu}_i^*) \le P_{max}, \tag{14a}$$

$$\boldsymbol{\nu}_i^* = \underset{\boldsymbol{\nu}_i\in\mathcal{R}_i(\boldsymbol{\delta})}{\operatorname{argmin}}\ \mu(\delta_s S_i^*(\boldsymbol{\nu}_i) + \delta_p P_i^*(\boldsymbol{\nu}_i)) + (1-\mu)\xi_i(\boldsymbol{s}_i^*(\boldsymbol{\nu}_i), \boldsymbol{p}_i^*(\boldsymbol{\nu}_i)), \tag{14b}$$

$$\mathcal{R}_i(\boldsymbol{\delta}) = \underset{\boldsymbol{\nu}_i\succeq 0}{\operatorname{argmax}}\ \mathcal{J}_i(\boldsymbol{\nu}_i; \boldsymbol{s}_i^*(\boldsymbol{\nu}_i), \boldsymbol{p}_i^*(\boldsymbol{\nu}_i), \boldsymbol{\delta}), \tag{14c}$$

where $\forall i \in [1, M]$. The constraint (14b) indicates that the InP takes a pessimistic position [13], i.e., it considers that the MVNOs choose the optimal strategies which lead to the worst objective function values for it.

TABLE I

SIMULATION PARAMETERS

Parameter	$S_{\max}$	$P_{\max}$	a
Value	1200 SCs	49.5 dBm	200
Parameter	b/a	q	k
Value	10%	50 Kbps	30 dB/SC
Parameter	N	M	L_m
Value	1000	2	50
Parameter	G	F	C_r
Value	50	0.5	0.1

For a complex bi-level problem as (14), classical methods often fail due to the non-convexity and disconnectedness, and thus, a nested DE algorithms is adopted to solve (14) as in [14], which is further combined with local descent method to ensure the local optimality. Since $\boldsymbol{\nu}_i^*,\ \forall i \in [1, M]$ can be seen as a function of $\boldsymbol{\delta}$, thus the payoff function of the InP can be denoted as $\mathcal{J}_I(\boldsymbol{\delta})$. Therefore, using penalty function, the optimization problem (14) can be transformed into an unconstrained optimization problem as

$$\max_{\boldsymbol{\delta}\succeq 0}\quad P_I(\boldsymbol{\delta}; \chi_P) = \mathcal{J}_I(\boldsymbol{\delta}) - \frac{1}{2}\chi_P C(\boldsymbol{\delta}), \tag{15}$$

where $C(\boldsymbol{\delta})$ denotes the exterior penalty function for the constraints (14a), and $\chi_P > 0$ is the penalty coefficient. Specifically, the quadratic penalty function is adopted for the constraints (14a), i.e., $C(\boldsymbol{\delta}) = \min\{0, S_{\max} - \sum_i^M S_i^*(\boldsymbol{\nu}_i^*(\boldsymbol{\delta}))\}^2 + \min\{0, P_{\max} - \sum_i^M P_i^*(\boldsymbol{\nu}_i^*(\boldsymbol{\delta}))\}^2$.

To perform local descent method to the unconstrained optimization problem (15), the gradient of $\mathcal{J}_I(\boldsymbol{\delta})$ with regard to $\boldsymbol{\delta}$ is necessary, which is derived in Appendix C. Based on the gradient information, the quasi-newton method can be adopted to solve the problem (15), and the combined nested DE and local descent algorithm can be adopted to solve the bi-level optimization problem (14). The main procedures of the algorithm are similar with those in Alg. 1, while the difference is that for each InP level parameter vector, another DE algorithm is performed to solve the MVNO level problem (14c), by generating and evolving the MVNO level parameter vectors. Moreover, for the last generation of population, the local descent method is performed. Therefore, the results obtained by the proposed algorithm is at least local optimal.

IV. SIMULATION RESULTS

In this section the simulation results are presented to verify the proposed algorithm for the TLSG, and to reveal the relationship between the InP's revenue and users' utility with regard to the tradeoff coefficients of the MVNOs and InP. The simulation parameters are presented in Table I.

A. Verification of Proposed Algorithm

Fig. 2 shows the payoff of the InP versus the resource price δ_s and δ_p, as well as the results generated by the proposed algorithm. It shows that our combined algorithm succeeds in finding the strategies of the InP to maximize its payoff. Besides, it can be seen that after the local descent method is perfomed, the results do not converge to one point. Instead,

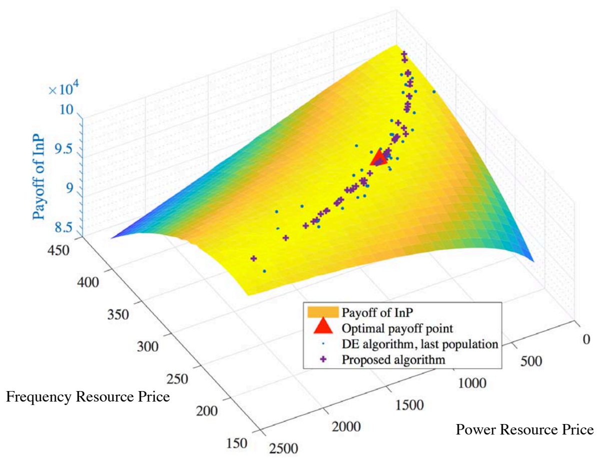

Fig. 2. InP's payoff versus the frequency resource price and the power resource price, $\lambda_1 = \lambda_2 = 0.5,\ \mu = 0.3$.

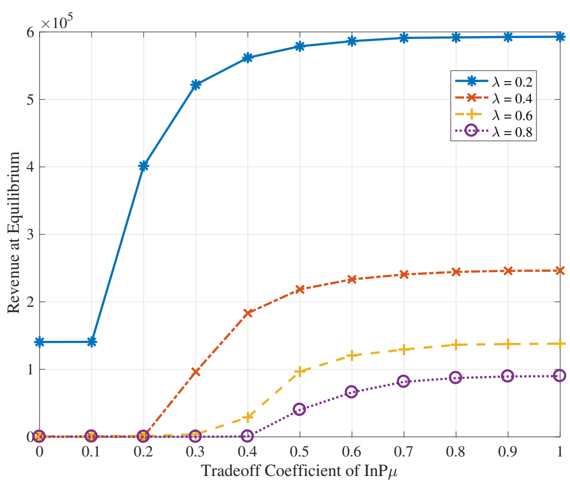

Fig. 3. Revenue of the InP versus InP's tradeoff coefficient μ, in different cases of MVNOs' tradeoff coefficient λ values.

they compose a curve which is generally in the shape of $\delta_p \sim \delta_s^{-1}$. This is because the descent algorithm stops when the change of the objective function value is lower than a certain threshold, which means the difference between the resulting payoff of these resource prices is relatively small. Nevertheless, in the simulation, the optimal point is selected to be the price setting with the maximum payoff of InP among all the local optimal points obtained by the proposed algorithm.

B. Influence of Tradeoff Coefficients μ and λ on InP's Revenue and Users' Utility

In Fig. 3 and Fig. 4, the influence of the tradeoff coefficient values of the InP and MVNOs are shown. In the simulated scenarios, there are two MVNOs, who have the same number of users with same utility functions. Besides, the tradeoff coefficients $\lambda_i,\ i \in [1, 2]$ of the two MVNOs are the same.

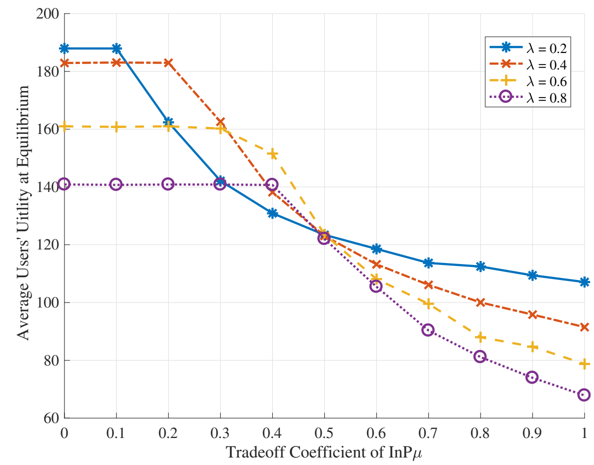

Fig. 4. Average utility of users versus different tradeoff coefficients of the InP and MVNOs.

From the curves with the same λ value in the two figures, it can be seen that when the InP focuses more on collecting revenue, its revenue grows while the average users' utility becomes lower. Besides, from the two figures, we also observe that when the MVNOs focus more on revenue collecting, the revenue of InP and the average users' utility both become lower. This indicates that the more selfish the MVNOs are, the lower payoff the InP and the users will obtain.

V. Conclusion

In this paper, we considered the tri-level hierarchical structure in RAN slicing, in which the MVNOs and users are selfish and can determine their own strategies. To jointly solve the frequency and power allocation, we proposed a TLSG, which achieved an equilibrium such that each level selected the optimal strategy considering the strategic responses of the levels below. To solve the equilibrium, an algorithm combing the DE algorithm and local descent method was proposed. Simulation results verified the proposed algorithm. Besides, it also verified the market principle that the more selfish the MVNOs are, the lower payoff the InP and users will be.

Appendix A
Proof of Proposition 1

The quadratic form of the hessian $\nabla^2 R_j(s_j, p_j)$ can be calculated as follows:

$$\boldsymbol{x}^T \nabla^2 R_j(s_j, p_j)\boldsymbol{x} = -\frac{Bk_j^2}{s_j^2(s_j + k_j p_j)^2}(x_1 p_j - x_2 s_j)^2 \preceq 0,$$

which indicates that $R_j(s_j, p_j)$ is concave with regard to s_j and p_j given $B > 0$. Based on the vector composition rule in convex function, since $R_j(s_j, p_j)$ is a concave function, $\mathcal{U}_j(R_j) = a_j - b_j e^{-(R_j - q_j)d_j}$ is concave and non-decreasing in R_j, thus $\mathcal{U}_j(s_j, p_j)$ is a concave function with regard to s_j and p_j.

Suppose that the user j purchases none-zero amount of frequency and power resources, then one KKT condition of the optimization problem (6) can be given by

$$\frac{\partial \mathcal{U}_j(s_j^*,p_j^*)}{\partial s_j} = \nu_s, \quad \frac{\partial \mathcal{U}_j(s_j^*,p_j^*)}{\partial p_j} = \nu_p. \tag{16}$$

The partial derivatives can be calculated as follows

$$\frac{Ba_jd_j(\ln(1+k_jp_j^*/s_j^*) - \frac{k_jp_j^*/s_j^*}{1+k_jp_j^*/s_j^*})}{(1+k_jp_j^*/s_j^*)^{Bd_js_j^*}} = \nu_s, \tag{17a}$$

$$\frac{Ba_jd_jk_j}{(1+k_jp_j^*/s_j^*)(1+k_jp_j^*/s_j^*)^{Bd_js_j^*}} = \nu_p. \tag{17b}$$

Divide the first equation by the second equation and denote $\kappa = \nu_s/\nu_p$, it can be derived that

$$\ln(1+k_jp_j^*/s_j^*)(1+k_jp_j^*/s_j^*) - k_jp_j^*/s_j^* = k_j\kappa.$$

Based on the Lambert W function, it can be derived that

$$k_jp_j^*/s_j^* = \frac{k_j\kappa - 1}{W_0(e^{-1}(k_j\kappa-1))} - 1, \tag{18}$$

where $W_0(\cdot)$ denotes the zero-th branch of the Lambert W function. Substituting (18) into (17a) and (17b), the Proposition 1 can be derived, where the lower bound 0 is due to that the resource purchase amount should be nonnegative. ■

APPENDIX B
GRADIENT AND HESSIAN OF MVNO'S PAYOFF FUNCTION

For $\nabla\mathcal{J}$, substitute the KKT condition (16) into the definition of $\nabla\mathcal{J}$, then (12) can be derived directly. For $\nabla^2\mathcal{J}$, based on (12), it can be derived that

$$\begin{aligned}\frac{\partial^2\mathcal{J}(\boldsymbol{\nu})}{\partial\nu_s^2} &= \sum_{j\in\mathcal{N}^+}\lambda(2\frac{\partial s_j^*}{\partial\nu_s} + \nu_s\frac{\partial^2 s_j^*}{\partial\nu_s^2} + \nu_p\frac{\partial p_j^*}{\partial\nu_s}) \\ &+ (1-\lambda)\Big(\frac{\partial^2\mathcal{U}_j(s_j^*,p_j^*)}{\partial s_j^2}(\frac{\partial s_j^*}{\partial\nu_s})^2 + 2\frac{\partial^2\mathcal{U}_j(s_j^*,p_j^*)}{\partial s_j\partial p_j}(\frac{\partial s_j^*}{\partial\nu_s}\cdot\frac{\partial p_j^*}{\partial\nu_s}) \\ &+ \frac{\partial^2\mathcal{U}_j(s_j^*,p_j^*)}{\partial p_j^2}(\frac{\partial p_j^*}{\partial\nu_s})^2 + \frac{\partial\mathcal{U}_j(s_j^*,p_j^*)}{\partial s_j}\frac{\partial^2 s_j^*}{\partial\nu_s^2} + \frac{\partial\mathcal{U}_j(s_j^*,p_j^*)}{\partial p_j}\frac{\partial^2 p_j^*}{\partial\nu_s^2}\Big),\end{aligned} \tag{19}$$

where $\mathcal{N}^+$ denotes the set of users with non-zero resource purchase amount. Taking the partial derivatives on the both sides of the equations in (16) with regard to ν_s, it can be derived that

$$\begin{bmatrix}\frac{\partial^2\mathcal{U}_j(s_j^*,p_j^*)}{\partial s_j^2} & \frac{\partial^2\mathcal{U}_j(s_j^*,p_j^*)}{\partial s_j\partial p_j} \\ \frac{\partial^2\mathcal{U}_j(s_j^*,p_j^*)}{\partial s_j\partial p_j} & \frac{\partial^2\mathcal{U}_j(s_j^*,p_j^*)}{\partial p_j^2}\end{bmatrix}\cdot\begin{bmatrix}\frac{\partial s_j^*}{\partial\nu_s} \\ \frac{\partial p_j^*}{\partial\nu_s}\end{bmatrix} = \begin{bmatrix}1\\0\end{bmatrix}. \tag{20}$$

Substituting (16) and (20) into (19), it can be derived that

$$\frac{\partial^2\mathcal{J}(\boldsymbol{\nu})}{\partial\nu_s^2} = \sum_{j\in\mathcal{N}^+}(1+\lambda)\frac{\partial s_j^*}{\partial\nu_s} + \nu_s\frac{\partial s_j^{*2}}{\partial\nu_s^2} + \nu_p\frac{\partial p_j^{*2}}{\partial\nu_s^2}.$$

By the similar token, $\partial^2\mathcal{J}(\boldsymbol{\nu})/\partial\nu_s\partial\nu_p$ and $\partial^2\mathcal{J}(\boldsymbol{\nu})/\partial\nu_p^2$ can also be derived. Therefore, based on the definition of $\nabla^2\mathcal{J}(\boldsymbol{\nu})$, (13) can be proved.

APPENDIX C
GRADIENT OF INP'S PAYOFF FUNCTION $\nabla\mathcal{J}_I(\boldsymbol{\delta})$ WITH REGARD TO $\boldsymbol{\delta}$

Taking partial derivatives on the both side of the KKT condition $\nabla\mathcal{J}_i(\boldsymbol{\nu}_i^*) = 0$ with regard to δ_s and δ_p, it can be derived that

$$\nabla^2\mathcal{J}_i\begin{bmatrix}\frac{\partial\nu_{s,i}^*}{\partial\delta_s} \\ \frac{\partial\nu_{p,i}^*}{\partial\delta_s}\end{bmatrix} = \lambda_i\nabla S_i^*(\boldsymbol{\nu}_i^*), \quad \nabla^2\mathcal{J}_i\begin{bmatrix}\frac{\partial\nu_{s,i}^*}{\partial\delta_p} \\ \frac{\partial\nu_{p,i}^*}{\partial\delta_p}\end{bmatrix} = \lambda_i\nabla P_i^*(\boldsymbol{\nu}_i^*).$$

Suppose that the hessian $\nabla^2\mathcal{J}_i$ is non-singular, thus, it can be derived that

$$\begin{aligned}\frac{\partial\mathcal{J}_I(\boldsymbol{\delta})}{\partial\delta_s} &= -(1-\mu)\sum_i^M\lambda_i(\nabla\xi_i^*)^T(\nabla^2\mathcal{J}_i)^{-1}\nabla S_i^* \\ &-\mu(\sum_i^M S_i^* + \lambda_i(\nabla S_i^*)^T(\nabla^2\mathcal{J}_i)^{-1}\nabla S_i^* + \lambda_i(\nabla P_i^*)^T(\nabla^2\mathcal{J}_i)^{-1}\nabla S_i^*),\end{aligned}$$

$$\begin{aligned}\frac{\partial\mathcal{J}_I(\boldsymbol{\delta})}{\partial\delta_p} &= -(1-\mu)\sum_i^M\lambda_i(\nabla\xi_i^*)^T(\nabla^2\mathcal{J}_i)^{-1}\nabla P_i^* \\ &-\mu(\sum_i^M S_i^* + \lambda_i(\nabla S_i^*)^T(\nabla^2\mathcal{J}_i)^{-1}\nabla P_i^* + \lambda_i(\nabla P_i^*)^T(\nabla^2\mathcal{J}_i)^{-1}\nabla P_i^*).\end{aligned}$$

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