Impact of Both Nonzero Boresight and Jitter Pointing Error on Outage Capacity of FSO Communication Systems over Strong Turbulence

Kug-Jin Jung, Sung Sik Nam, and Young-Chai Ko
The School of Electrical Engineering
Korea University
Seoul, Korea
Email: {kug0860, ssnam, koyc}@korea.ac.kr

***Abstract*—This paper analyzed and compared the effect of boresight and jitter on outage probability for heterodyne detection technique considering strong atmospheric turbulence in free space optical communication system.**

***Index Terms*—free space optical, pointing error, strong turbulence, heterodyne detection, outage probability**

I. Introduction

Recently, free space optical (FSO) communication has been actively researched as a solution to a limited spectrum of radio frequency (RF) based communication. FSO has superiority in terms of data rate and security compared to RF communication. However, its performance is deteriorated due to pointing error that the detector and the beam trajectory are misaligned each other and atmospheric turbulence where the refractive index is not consistent due to the air flow [1]. Especially, pointing error has two components, fixed displacement between the beam center and the center of detection plane (i.e. boresight) and random offset of the beam center at the detector plane (i.e. jitter) [2]. In the past, the application target of FSO was focused on fixed platforms like buildings. However, since application targets have expanded to communication between fixed and mobile platforms or mobile platforms in recent years, boresight as well as jitter needs to be considered. In this paper, we derive outage probability for a system based on the heterodyne detection technique when boresight and jitter exist in strong turbulence situation and compared the impact of boresight and jitter on outage probability by numerical results.

II. System Model

In this paper, the optical signal is transmitted to the receiver through strong atmospheric turbulence and white Gaussian noise, in the presence of pointing error. The received optical signal is converted into an electrical signal by photodetector and can be expressed as

$$y = \eta h x + n, \tag{1}$$

where x is the transmitted signal, η is the effective photoelectric conversion ratio, and n is AWGN with variance N_0. h is the channel gain and can be modeled as product of h_a and h_p which are the atmospheric fading factor and the pointing error factor, respectively. h_a and h_p are independent random variables and each random variable is modeled as follows.

We consider strong turbulence and h_a can be modeled as a Gamma-Gamma random variable whose PDF can be given as

$$f_{h_a}(h_a) = \frac{2(\alpha\beta)^{\frac{\alpha+\beta}{2}}}{\Gamma(\alpha)\Gamma(\beta)} {h_a}^{\frac{\alpha+\beta}{2}-1} K_{\alpha-\beta}\left(2\sqrt{\alpha\beta h_a}\right). \tag{2}$$

where $\Gamma(\cdot)$ denotes Gamma function [3, eq.(8.310.1)], $K_v(\cdot)$ is the v th-order modified Bessel function of the second kind [3, eq.(8.432.2)], and α, β are the effective number of small-scale and large-scale eddies of scattering environment, respectively.

We assume a Gaussian beam propagating from transmitter to receiver and the fraction of the collected power at the receiver can be approximated as $h_p(r; z) \approx A_0 \exp\left(-\frac{2r^2}{{w_{z_{eq}}}^2}\right)$ [4]. r is the radial displacement between the beam center and the detector center and it can be expressed as $r = \sqrt{x^2+y^2}$, where x and y are the horizontal and the vertical Gaussian displacement, respectively (i.e. $x \sim \mathcal{N}(\mu_x, \sigma^2)$ and $y \sim \mathcal{N}(\mu_y, \sigma^2)$). Then, the PDF of h_p can be given as

$$f_{h_p}(h_p) = \frac{\varepsilon^2 \exp\left(\frac{-s^2}{2\sigma^2}\right)}{{A_0}^{\varepsilon^2}} {h_p}^{\varepsilon^2-1} I_0\left(\frac{s}{\sigma^2}\sqrt{\frac{-{w_{z_{eq}}}^2 \ln\frac{h_p}{A_0}}{2}}\right). \tag{3}$$

where $I_\nu(\cdot)$ denotes the ν th-order modified Bessel function of the first kind [3, eq.(8.431.1)] and $s = \sqrt{{\mu_x}^2 + {\mu_y}^2}$.

In [5], finite series approximate composite PDF was derived under the condition $\varepsilon^2 > \max(\alpha, \beta)$ and it can be expressed as

$$f(h) \approx \sum_{j=0}^{J} \left\{ \frac{1}{j!}\left(\frac{\alpha\beta}{A_0}\right)^j \left(v_j(\alpha,\beta)\, h^{\beta-1+j} + v_j(\beta,\alpha)\, h^{\alpha-1+j}\right) \right\}, \tag{4}$$

where $J = \lfloor \varepsilon^2 - \max(\alpha,\beta) \rfloor$, $\lfloor x \rfloor$ denotes the largest integer not greater than x and $v_j(\alpha,\beta)$ is written as (5).

$$v_j(\alpha,\beta) = \frac{\pi\varepsilon^2\left(\frac{\alpha\beta}{A_0}\right)^{\beta}\exp\left(-\frac{s^2}{2\sigma_s{}^2} - \frac{s^2\varepsilon^2/\sigma_s{}^2}{2\beta-2\varepsilon^2+2j}\right)}{\sin\left((\alpha-\beta)\pi\right)\Gamma(\alpha)\Gamma(\beta)\Gamma\left(j-(\alpha-\beta)+1\right)\left(\varepsilon^2-\beta-j\right)} \tag{5}$$

III. Outage Probability Analysis

In the case of heterodyne detection technique, the average electrical SNR can be written as $\mu = \frac{\eta \mathbb{E}_{h_p}[h_p]\mathbb{E}_{h_a}[h_a]}{N_0}$. Utilizing first moment of h_a and h_p in [2], it develops as $\mu = \frac{\eta A_0 \exp\left(-\frac{2s^2}{w_{zeq}{}^2}\right)}{N_0}$. Hence the channel gain h is given as

$$\mu = \frac{\eta A_0 \exp\left(-\frac{2s^2}{w_{zeq}{}^2}\right)}{N_0}. \tag{6}$$

Applying RV transformation of (6) into (4), the PDF of SNR can be written as

$$f(\gamma) = \sum_{j=0}^{J}\left\{\frac{1}{j!}\left(\frac{\alpha\beta}{\mu}\right)^{j}\left(v_j(\alpha,\beta)\left(\frac{A_0c}{\mu}\right)^{\beta}\gamma^{\beta+j-1} + v_j(\beta,\alpha)\left(\frac{A_0c}{\mu}\right)^{\alpha}\gamma^{\alpha+j-1}\right)\right\} \tag{7}$$

where $c = \exp\left(-\frac{2s^2}{w_{zeq}{}^2}\right)$.

Outage probability at SNR threshold γ_0 can be defined as

$$\begin{aligned} P_{out}(\gamma_0) &= \Pr(\gamma < \gamma_0) \\ &= \int_0^{\gamma_0} f_\gamma(\gamma)\, d\gamma. \end{aligned} \tag{8}$$

Substituting (7) into (8) and with some mathematical manipulation, probability of outage can be expressed as

$$P_{out}(R_0) = \sum_{j=0}^{J}\left\{\frac{1}{j!}\left(\frac{\alpha\beta c\gamma_0}{\mu}\right)^{j}\left(v_j(\alpha,\beta)\left(\frac{A_0c\gamma_0}{\mu}\right)^{\beta}\frac{\gamma}{\beta+j} + v_j(\beta,\alpha)\left(\frac{A_0c\gamma_0}{\mu}\right)^{\alpha}\frac{\gamma}{\alpha+j}\right)\right\}. \tag{9}$$

IV. Numerical Results

From Figure 1, the effect of jitter on outage probability can be seen and higher jitter value σ degrades performance of FSO system in case of heterodyne detection technique. We can see the difference between outage probabilities with no boresight, moderate boresight, and strong boresight. It can be observed that the outage probability increases with increasing boresight value when the SNR threshold γ_0 is fixed. Also, since change of boresight value leads to greater gap than those of jitter, borsight has much more impact in terms of outage probability than jitter.

V. Conclusion

Our work was based on FSO communication system utilizing heterodyne detection technique under strong atmospheric turbulence. We also assumed both boresight and jitter and obtained outage probability. In numerical result, we observed

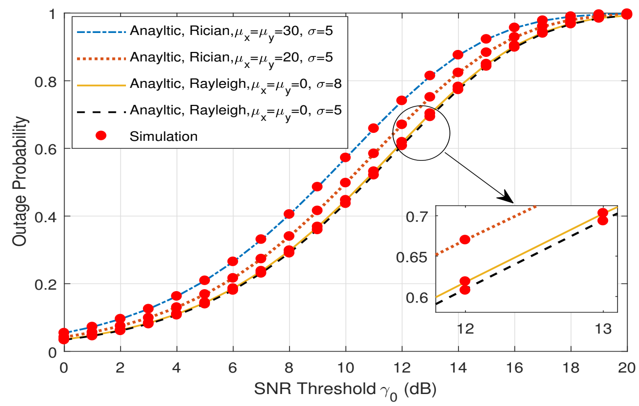

Fig. 1. Outage probability over the Gamma-Gamma fading with Rayleigh and Rician pointing error models (i.e. $\alpha = 4.169, \beta = 1.397, a = 10, w_z = 100$)

performance degradation due to boresight and jitter. First, the outage probability increases with greater boresight value and also the jitter deteriorate the outage probability. In particular, presence of the boresight degraded much more performance than the jitter.

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