Radio Resource Allocation for Non-orthogonal Multiple Access (NOMA) Relay Network Using Matching Game

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***Abstract*—In this paper, we study the resource allocation problem for a single-cell non-orthogonal multiple access (NOMA) relay network where an OFDM amplify-and-forward (AF) relay allocates the spectrum and power resources to the source-destination (SD) pairs. We aim to optimize the spectrum and power resource allocation to maximize the total sum-rate. This is a very complicated problem and the optimal approach requires an exhaustive search, leading to a NP hard problem. To solve this problem, we propose an efficient many-to-many two sided SD pair-subchannel matching algorithm in which the SD pairs and sub-channels are considered as two sets of rational and selfish players chasing their own interests. The algorithm converges to a pair-wise stable matching after a limited number of iterations with a low complexity compared with the optimal solution. Simulation results show that the sum-rate of the proposed algorithm approaches the performance of the optimal exhaustive search and significantly outperforms the conventional orthogonal multiple access scheme, in terms of the total sum-rate and number of accessed SD pairs.**

I. Introduction

With rapidly increasing demands for mobile services, this requires ever higher spectral efficiency and massive connectivity in the wireless network [1]. Among several solutions for this challenging requirement, the non-orthogonal multiple access (NOMA) technique has drawn significant attention [2] around this field and became a promising candidate of the fifth generation mobile communication access. Different from the orthogonal frequency division multiple access (OFDMA), NOMA can accommodate multiple users in the same time, frequency, and code domains by differentiating the users through power-domain multiplexing. NOMA has the advantages of a low complexity receiver and high spectrum efficiency due to its multiplexing nature.

On the other side, co-channel interference is unfortunately introduced in the NOMA system since multiple users share the same spectrum resources. To handle this problem, various multi-user detection (MUD) techniques have been proposed, such as the successive interference cancellation (SIC) techniques [3], which can be applied at the end-user receivers to decode the received signals and reduce the inter-user interference effectively. Recently, different aspects of the NOMA scheme with SIC techniques have been discussed in several works. In [4], a novel NOMA transmitter and low complexity receiver was proposed and its performance was compared with the theoretical performance of SIC. In [5], the author studied resource allocation and user scheduling problem for a downlink nonorthogonal multiple access network where the base station allocates spectrum and power resources to a set of users with a joint algorithm.

Although there are several works studying the resource allocation problem in the NOMA scheme, the majority of them focused on the multiple access system and few of them have considered the relay network. In [6], the capacity of a basic decode-and-forward (DF) relay network was studied, and a sub-optimal power allocation scheme for NOMA was given. In [7], the outage probability of an amplify-and-forward (AF) relay network was derived and a lower bound of the outage probability was provided. However, none of these works have considered the radio resource allocation problem in a multi-carrier AF relay network in the NOMA scheme. The well-designed resource allocation can further significantly increase the spectrum efficiency. However, the resource allocation problem in such a network is very complicated and challenging. Because each user and the relay in this network can allocate its own resource to maximize its interest, in which there are sophisticated competition and conflicts among the users.

In this paper, we consider a NOMA wireless network in which an AF relay assigns the sub-channels to a set of source destination (SD) pairs and allocates different levels of power to them. A SD pair consists of a source node and a destination node, and the source node transmits through the relay to its paired destination node. Each SD pair can utilize multiple sub-channels and each sub-channel can be shared by multiple SD pairs. For the SD pairs sharing the same sub-channel, the SIC technique is adopted to remove the inter-user interference. Joint sub-channel and power allocation is then formulated as a non-convex optimization problem to maximize the total sum-rate. However, the complexity of reaching the optimal solution of the problem is extremely high because of the externalities among the users and the dynamic relation between sub-channel allocation and power allocation in this network.

To tackle this problem, we separate the sub-channel and power allocation as two subproblems. In the sub-channel allocation problem, we consider the SD pairs and sub-channels as two sets of selfish and rational players aiming at maximizing their own profits. We then formulate the sub-channel allocation problem as a many-to-many two sided matching game with externalities in which interdependencies exist between the players' preferences due to the co-channel interference. A novel matching algorithm extended from the Gale-Shapley algorithm [8] is proposed for the matching game formulation. After the sub-channel allocation, we use an iterative water filling algorithm to allocate the power and amplifying gains

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of the network. The properties of our matching algorithm are then analyzed in terms of stability, convergence and complexity in the following. Finally, the simulation results show that the proposed matching algorithm in NOMA scheme outperforms OFDMA scheme significantly, and is close to the optimal exhaustive search in terms of the total sum-rate.

The rest of this paper is organized as follows. In Section II we describe the system model of the NOMA relay networks. In Section III, we formulate the optimization resource allocation problem as a many-to-many two-sided matching problem, and propose a matching algorithm, followed by the corresponding analysis. Simulation results are presented in Section IV, and finally we conclude the paper in Section V.

II. System Model

We consider a single-cell one-way NOMA network as depicted in Fig.1, with one OFDM AF relay R and N SD pairs. Each SD pair consists of one source node and one destination node, where the source node communicates with the destination node assisted by relay R. Let $\mathcal{S}=\{1,2,\cdots N\}$ denote the set of source nodes and $\mathcal{D}=\{1,2,\cdots N\}$ denote the set of destination nodes. We assume that relay R has full knowledge of the channel side information (CSI). Based on the CSI of each channel, relay R assigns a subset of non-orthogonal sub-channels, denoted as $\mathcal{K}=\{1,2,\cdots K\}$, to the SD pairs and allocates different levels of amplifying gain over the sub-channels. According to the NOMA protocol [9], one sub-channel can be allocated to multiple SD pairs, one SD pair has access to multiple sub-channels in the network, and each SD pair shares the same group of sub-channels. In order to reduce the response time, at most X_{max} SD pairs can have access to each sub-channel. Communication between the source nodes and destination nodes consists of two phases, described specifically as follows.

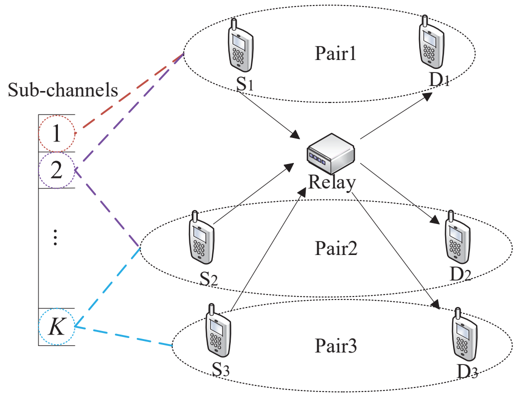

Fig. 1. System model of the single-cell one-way NOMA network.

In the first phase, the source nodes transmit signals to relay R. We denote the mth source node as S_m and the kth sub-channel as SC_k. The transmitting power of S_m over SC_k is denoted as $\mathrm{p}_{k,m}$, satisfying $\sum_{k=1}^{K} p_{k,m} \le P_{SN}$ for each $S_m \in \mathcal{S}$, where P_{SN} is the maximum transmit power of S_m. We consider a block fading channel, for which the channel remains constant within a certain time-slot, but varies independently from one to another. The complex coefficient of SC_k between S_m and relay R is denoted by $h_{k,m}=g_{k,m}/\left(d_m\right)^{\alpha}$, where $g_{k,m}$ denotes the Rayleigh fading channel gain of SC_k from S_m to relay R, d_m denotes the distance between relay R and S_m, and α is the path loss coefficient. Let $x_{k,m}$ be the transmitting information symbol of unit energy from S_m over SC_k. The signal that relay R receives from S_m over SC_k is given by

$$z_{k,m}=h_{k,m}\sqrt{p_{k,m}}x_{k,m}+n_m, \tag{1}$$

where $n_m \sim \mathcal{N}\left(0,\sigma_s^2\right)$ is the additive white Gaussian noise (AWGN), and σ_s^2 is the noise variance.

In the second phase, relay R amplifies the signals received from every source node and broadcasts the superposed signals to the destination nodes consistently [7]. Let G_k denote the amplifying gain of relay R over SC_k and $q_{k,m}$ is the transmitted power that relay R allocates to D_m over SC_k, where D_m is the mth destination node. The relation between $q_{k,m}$ and G_k is given by

$$q_{k,m}=G_k^2\left(p_{k,m}\left|h_{k,m}\right|^2+\sigma_s^2\right). \tag{2}$$

We also assume that relay R has a maximum transmitted power of Q_R, $q_{k,m}$ and Q_R satisfy the following inequation $\sum_{k\in\mathcal{K}}\sum_{m\in\mathcal{D}} q_{k,m} \le Q_R$. Correspondingly, we denote $f_{k,m}$ as the complex coefficient of SC_k between relay R and D_m, and $f_{k,m}=c_{k,m}/\left(b_m\right)^{\alpha}$, where $c_{k,m}$ denotes the Rayleigh fading channel gain of SC_k from relay R to D_m, and b_m is the distance between D_m and relay R. Let U_k be the set of SD pairs that have access to SC_k. The signal that D_m receives from relay R over SC_k is given by

$$y_{k,m}=G_k f_{k,m}\sum_{i\in U_k} z_{k,i}+w_m, \tag{3}$$

where $w_m \sim \mathcal{N}\left(0,\sigma_d^2\right)$ is AWGN. Assuming that $\sigma_d^2=\sigma_s^2$, when we substitute (1) into (3), it can be reformed as

$$y_{k,m}= \quad G_k f_{k,m}\sum\nolimits_{i\in U_k}\left(h_{k,i}\sqrt{p_{k,i}}x_{k,i}+n_i\right)+w_m. \tag{4}$$

After receiving the signals, the destination nodes perform SIC to reduce the interference from the source nodes of other SD pairs with a smaller channel gain over SC_k [10]. For simplicity, we only consider the channel gain of the second phase, i.e., $f_{k,m}, k\in\mathcal{K}, m\in\mathcal{D}$. For example, for $S_i, S_j \in U_k$, if $\left|f_{k,i}\right|^2 > \left|f_{k,j}\right|^2$, D_i can cancellate the signal-to-signal interference of S_j from S_i while decoding. The order for decoding is based on the increasing channel gains described above, which guarantees that the upper bound on the capacity region can be reached [11]. The interference that D_m receives over SC_k is shown as below,

$$I_{k,m}=\sum\nolimits_{\left|f_{k,i}\right|^2>\left|f_{k,m}\right|^2} G_k^2\left|f_{k,i}\right|^2 p_{k,m}\left|h_{k,i}\right|^2. \tag{5}$$

Note that the noise and interference for D_m over SC_k consists of three parts: the noise at D_m, the amplified noise forwarded by relay R, and interference from other source nodes. Therefore, the data rate of D_m over SC_k is given by

$$R_{k,m}=\log_2\left(1+\frac{G_k^2\left|f_{k,m}\right|^2 p_{k,m}\left|h_{k,m}\right|^2}{\sigma_d^2+G_k^2\left|f_{k,m}\right|^2\sum\sigma_s^2+I_{k,m}}\right). \tag{6}$$

To better describe the resource allocation, we define a binary $N\times K$ SD pair-subchannel pairing matrix $\mathbf{\Phi}$, in which $\phi_{m,k}=$

1 denotes that S_m and D_m are paired with SC_k. The objective is to maximize the sum-rate of the network by jointly allocating $\{\phi_{m,k}, p_{m,k}, q_{m,k}\}$ which can be formulated as:

$$\max_{\boldsymbol{\Phi}, \boldsymbol{p}, \boldsymbol{q}} \sum_{k=1}^{K} \sum_{m=1}^{N} R_{k,m}(p,q)\,\phi_{k,m}, \tag{7a}$$

$$\textbf{\textit{s.t.}} \sum_{m=1}^{N} \phi_{m,k} \le X_{max}, \forall k \in \mathcal{K}, \tag{7b}$$

$$\sum_{k=1}^{K} p_{k,m} \le P_{SN}, \forall m \in \mathcal{S}, \tag{7c}$$

$$\sum_{k \in \mathcal{K}} \sum_{m \in \mathcal{D}} q_{k,m}\phi_{k,m} \le Q_R, \tag{7d}$$

$$p_{k,m} \ge 0, \forall k \in \mathcal{K}, \forall m \in \mathcal{S}, \tag{7e}$$

$$q_{k,m} \ge 0, \forall k \in \mathcal{K}, \forall m \in \mathcal{S}, \tag{7f}$$

$$\phi_{k,m} \in \{0,1\}, \tag{7g}$$

where $(7b)$ shows that each sub-channel can be allocated to at most X_{max} SD pairs, $(7c)$ and $(7d)$ are the power restrictions of the source nodes and relay R respectively. Constraints $(7e)$ and $(7f)$ show that the transmitting power is no less than 0.

III. Many to Many Matching for NOMA

As shown in $(7a)$, the sum-rate of the network is determined by both power allocation and sub-channel allocation. Note that sub-channel allocation and power control of the transmitter users are performed by relay R and source nodes, respectively. Therefore, we decouple the resource allocation problem into two subproblems, sub-channel allocation and power allocation, to reach a sub-optimal solution. We first assume that each source node allocates its power equally over the sub-channels, and the amplifying gains of relay R over each sub-channel are the same. Thus, the sub-channel allocation can be formulated as a many-to-many two-sided matching problem, which can be solved by utilizing the matching theory. Afterwards, each source node allocates its own transmitting power over each sub-channel with water filling algorithm [12], and relay R determines its amplifying gain over each sub-channel, as will be discussed in Section III.C.

A. Two Sided Many-to-Many Matching Problem Formulation

Sub-channel allocation can be considered by relay R as a matching process in which the set of sub-channels and the set of SD pairs aim at matching with each other. To better describe the matching process between the SD pairs and the sub-channels [1], we consider the set of SD pairs and the set of sub-channels as two disjoint sets of selfish and rational players aiming to maximize their own interests. Since the source and destination nodes are already paired, we can transform the matching problem between the set of SD pairs and the set of sub-channels into another one between source nodes $\mathcal{S}$ and sub-channels $\mathcal{K}$ for convenience. We denote (S_m, SC_k) as a matching pair if SC_k is assigned to S_m. To describe the competition behavior and decision process of each player, we assume that each player has preferences over the subsets of the opposite set. The preference of the source node is based on its achievable data rates, while the preference of the sub-channel is determined by the total sum-rate of the subset of SD pairs over it. Given V and V' as two subsets of source node $\mathcal{S}$, SC_k's preference over different subsets of source nodes can be written as

$$\begin{gathered} V \succ_{SC_k} V', V \subseteq S, V' \subseteq S \Leftrightarrow \\ \sum_{l \in V} R_{k,l} > \sum_{l' \in V'} R_{k,l'}, \end{gathered} \tag{8}$$

which implies that SC_k prefers V to V' because the former subset of source nodes provides higher data rate than the latter one. Given U and U' as two subsets of sub-channel $\mathcal{K}$, the preference of S_m over these subsets of sub-channels can be represented as

$$\begin{gathered} U \succ_{S_m} U', U \subseteq K, U' \subseteq K \Leftrightarrow \\ \sum_{t \in U} \frac{G_0^2 p_0 \left|f_{t,m}\right|^2 \left|h_{t,m}\right|^2}{\sigma_d^2 + G_0^2 \left|f_{t,m}\right|^2 \sigma_s^2 + I_{t,m}} > \sum_{t' \in U'} \frac{G_0^2 p_0 \left|f_{t',m}\right|^2 \left|h_{t',m}\right|^2}{\sigma_d^2 + G_0^2 \left|f_{t',m}\right|^2 \sigma_s^2 + I_{t',m}}, \end{gathered} \tag{9}$$

which implies that S_m prefers V to V' because of the higher channel gain. G_0 and p_0 are fixed amplifying gain and source node power in the sub-channel matching process.

We then denote the set of preference lists of the source nodes and sub-channels as $\mathbf{P} = \{P(S_1), \cdots, P(S_N), P(SC_1), \cdots, P(SC_K)\}$, where $P(S_m)$ and $P(SC_k)$ are the preference lists of S_m and SC_k.

We assume that the preferences of the source nodes and sub-channels are *transitive* and *substitutable* . *Transitive* means that for all $L \succ_{S_M} L'$ and $L' \succ_{S_M} L''$, we have $L \succ_{S_M} L''$. Given a player i in $\mathcal{K} \cup \mathcal{S}$, let T be the set of i's potential partners and $L \subseteq T$. Player i's preferences are called *substitutable* if for any players $l, l' \in L$, then $l \in L \backslash \{l'\}$. With the notion of the preference list, we can then formulate the optimization problem as a many-to-many two-sided matching game [13].

Definition 1: Given two disjoint sets, $\mathcal{S} = \{1, 2, \cdots, \mathrm{S}\}$ of the source nodes, and $\mathcal{K} = \{1, 2, \cdots, \mathrm{K}\}$ of the sub-channels, a many-to-many matching $\boldsymbol{\Psi}$ is a mapping from the set $\mathcal{S} \cup \mathcal{K}$ into the set of all subsets of $\mathcal{S} \cup \mathcal{K}$ such that for every $S_m \in \mathcal{S}$, and $\mathrm{SC}_k \in \mathcal{K}$:

1) $\boldsymbol{\Psi}(S_m) \subseteq \mathcal{K}$,
2) $\boldsymbol{\Psi}(SC_k) \subseteq \mathcal{S}$,
3) $\boldsymbol{\Psi}\left|(SC_k)\right| \le X_{max}$,
4) $SC_k \in \boldsymbol{\Psi}(S_m) \Leftrightarrow S_m \in \boldsymbol{\Psi}(SC_k)$

Conditions 1) and 2) state that each source node is matched with a subset of sub-channels, and each sub-channel can be allocated to a subset of source nodes. Condition 3) implies that each sub-channel can be allocated to no more than X_{max} SD pairs.

The matching model above is more complicated than the conventional two-sided matching models. In our model any member of either set can be matched with any subsets of the opposite set rather than a single member, because of the interdependencies between the source nodes sharing the

[1]The outcome of the matching is the solution for relay R to allocate the sub-channels as the interests of the sub-channels and relay R are identical.

same sub-channel. Therefore, the number of potential matching combinations can be extremely huge with the increment of the members in each set. This makes the problem quite intractable even when the power allocation is not considered. Therefore, to solve this matching problem, we develop an extended version of the Gale-Shapley algorithm [8] and propose a new matching algorithm in Section III.B.

B. SD pairs-SCs Matching Algorithm

In this subsection, we propose a low-complexity SD pair-subchannel matching algorithm (SD-SCMA). To decrease the complexity of the matching process, we simplify the preference list of the source nodes. Since different sub-channels have a linear superposition impact over the interest of each source node, the subsets of $\mathcal{K}$ can be replaced with the sub-channels in $\mathcal{K}$ in the preference lists of the source nodes. Given k and k' as two different sub-channels, (9) can be simplified as

$$k \succ_{S_m} k^{'}, k \subseteq K, k^{'} \subseteq K \Leftrightarrow$$
$$\frac{G_0^2 p_0 \left|f_{k,m}\right|^2 \left|h_{k,m}\right|^2}{\sigma_d^2 + G_0^2 \left|f_{k,m}\right|^2 \sigma_s^2 + I_{k,m}} > \frac{G_0^2 p_0 \left|f_{k',m}\right|^2 \left|h_{k',m}\right|^2}{\sigma_d^2 + G_0^2 \left|f_{k',m}\right|^2 \sigma_s^2 + I_{k',m}}, \quad (10)$$

However, the preference lists of the sub-channel are unreducible because there exists interdependencies between the source nodes who share the same sub-channel.

The key idea of SD-SCMA is that each source node figures out the best sub-channel which has not refused it yet, and proposes itself to this sub-channel. After all the source nodes have proposed themselves to their preferred sub-channels, each sub-channel has right to accept or reject these offers. When all the source nodes have made an decision once and all the proposed sub-channels have responded to the source nodes, we say one round of proposals is performed. Before describing the algorithm in detail, we first introduce how the sub-channels choose from the offers of the proposing source nodes by introducing the concept of *blocking pair*.

Definition 2: Given a matching $\mathbf{\Psi}$ and a pair (S_m, SC$_k$) with $S_m \notin \mathbf{\Psi}(SC_k)$ and SC$_k \notin \mathbf{\Psi}(S_m)$. ($S_m$, SC$_k$) is a *blocking pair* if (1) $L \succ_{SC_k} \mathbf{\Psi}(SC_k)$, $L \subseteq \{S_m\} \cup \mathbf{\Psi}(SC_k)$, and $S_m \in L$, (2) SC$_k \succ_{S_m} SC_l$, and $SC_l \in \mathbf{\Psi}(S_m)$.

With the definition above, we can describe the decision of each sub-channel as below. Since the aim of each sub-channel is to maximize its own interest, S_i and SC_j form a *blocking pair* if a subset of $\{S_i\} \cup \mathbf{\Psi}(SC_j)$ can provide a higher sum-rate than $\mathbf{\Psi}(SC_j)$ over SC_j. Under this condition, SC_j will accept the proposal from S_i and regroup the set of its matched source nodes $\mathbf{\Psi}(SC_j)$. The process of the proposed SD-SCMA is to find and eliminate the potential *blocking pairs* round by round.

We now describe the whole process of SD-SCMA to solve the sub-channel allocation problem. The specific details of the proposed SD-SCMA are described in Table I, consisting of an initialization phase and a matching phase. In the initialization phase, each source node and sub-channel constructs its own preference list according to the CSI of the network. In the matching phase, each source node figures out whether there is any sub-channel in its preference list that can provide a higher data rate, and if any, propose itself to the sub-channel. Then each sub-channel SC$_k$ decides whether to accept or not based on its preference $\succ_{SC_k}$. The matching process stops when no source node is willing to make new offers.

TABLE I
SD PAIR-SUBCHANNEL MATCHING ALGORITHM (SD-SCMA)

1) **Initialization Phase**
   a) Each source node constructs its preference list $P(S_i), i \in \mathcal{S}$.
   b) Each sub-channel constructs its preference list $P(SC_j), j \in \mathcal{K}$.
2) **Matching Phase**

```
while In the first round or any source node proposes itself in previous round
  for i=1:N
    if P(S_i) is not empty
      Propose itself to the most-preferred sub-channel in P(S_i);
      Remove the sub-channel from P(S_i);
  for i=1:K
    if SC_k has received any proposal
      if It's a blocking pair
        Accept the proposing source node;
        Regroup Ψ(SC_k) to maximize SC_k's utility;
      else Refuse the proposing source node;
```

3) **Matching Finished**

C. Water Filling Power Allocation

Power allocation can be implemented after the SD pair-subchannel matching. We divide the power allocation into two phases. In the first phase, the transmitting power of source nodes is allocated through the water filling algorithm, which can be presented as

$$p_{k,m} = \left[\lambda_k - \frac{1}{\left|h_{k,m}\right|^2/\sigma_s^2}\right]^+, \quad (11)$$

where

$$\lambda_k = \frac{1}{|W_m|}\left(P_{SN} + \sum_{i \in W_m} \frac{1}{\left|h_{k,i}\right|^2/\sigma_s^2}\right), \quad (12)$$

is the water filling level of S_m over SC_k, and W_m is the set of sub-channels allocated to S_m.

In the second phase, relay R allocates its amplifying gain over different sub-channels. We assume that the maximum power that relay R allocates to each sub-channel is identical, i.e., $Q_K = Q_R/K$. To maximize the sum data rate, relay R provides the maximum power level over every sub-channel, so that the amplifying gain G_k can be given by

$$G_k = \sqrt{\frac{Q_K}{\sum_{m=1}^{N} p_{k,m}\phi_{k,m}}}. \quad (13)$$

D. Stability, Convergence and Complexity

With the definition of *blocking pair* and the substitutable preference lists explained above, we then introduce the conception of pairwise-stability as below and prove that the proposed SD-SCMA converges to a *pairwise stable* matching.

Definition 3: A matching $\mathbf{\Psi}$ is defined as *pairwise stable* if it is not blocked by any pair which does not exist in $\mathbf{\Psi}$.

Lemma 1: If the proposed SD-SCMA converges to a matching $\mathbf{\Psi}^*$, then $\mathbf{\Psi}^*$ is a *pairwise stable* matching.

Proof: If $\mathbf{\Psi}^*$ is not a *pairwise stable* matching, it means that there exists a pair (S_m, SC_k), such that $L \succ_{SC_k} \mathbf{\Psi}(SC_k)$,

$L \subseteq \{S_m\} \cup \mathbf{\Psi}(SC_k)$, $S_m \in L$, and SC$_k \succ_{S_m} SC_l$, $SC_l \in \mathbf{\Psi}(S_m)$. According to the the proposed SD-SCMA, S_m must have proposed itself to SC$_k$ before since it can provide a higher utility than SC$_l$. We assume that SC$_k$ eliminate S_m in the tth round, that is, $\mathbf{\Psi}^t(SC_k) \succ_{SC_k} L, L \subseteq \{S_m\} \cup \mathbf{\Psi}^t(SC_k), S_m \in L$. While SC$_k$ only accepts the proposals that provide a larger benefit, we have $\mathbf{\Psi}(SC_k) \succ_{SC_k} \mathbf{\Psi}^t(SC_k)$. Finally, we have $L \succ_{SC_k} \mathbf{\Psi}(SC_k)$, $\mathbf{\Psi}^t(SC_k) \succ_{SC_k} L$, and $\mathbf{\Psi}(SC_k) \succ_{SC_k} \mathbf{\Psi}^t(SC_k)$, which is contradictory to the *transitive* property of the preference list. Hence, theorem 1 is proved. ■

Theorem 1: The the proposed SD-SCMA converges to a *pairwise stable* matching $\mathbf{\Psi}^*$ after limited iterations.

Proof: As shown in Table I, in each iteration, every source node will propose itself to the most-preferred sub-channel that hasn't reject it. No matter whether the source node is accepted or not, it will not propose itself to this sub-channel again, which means that as the matching goes on, potential choices for each source node keeps decreasing. So the number of iterations is no more than K, where K is the number of sub-channels, and the proposed SD-SCMA will converge within K iterations. According to Lemma 1, the proposed SD-SCMA converges to a *pairwise stable* matching. ■

Theorem 2: The complexity of the optimal exhaustive search is $O(2^{KN})$, while the complexity of the proposed SD-SCMA is $O(NK^2)$.

Proof: For the optimal exhaustive search, relay R exhaustively searches the best subset of SD pairs over every sub-channel. Since every source node and sub-channel can be paired with each other, there exists $K \times N$ possible combinations and the complexity of the optimal exhaustive search is $O(2^{KN})$. For the proposed SD-SCMA, in initialization phase, every source node converges its own preference list. The initialization of each preference list is considered as a sorting problem with the complexity of $O(K^2)$, and the total complexity of the initialization phase is $O(NK^2)$. In the matching phase, the number of iterations is no more than K, and in each iteration, at most N source nodes make proposals, so the complexity is $O(NK)$. The total complexity of the proposed SD-SCMA is $O(NK^2) + O(NK) = O(NK^2)$. ■

The whole resource allocation for the NOMA relay network is shown in Table II.

TABLE II
WHOLE PROCESS OF THE RESOURCE ALLOCATION PROBLEM IN THE NOMA RELAY NETWORK

1) **Sub-channel Allocation**
   a) Allocate the power of source nodes over each sub-channel equally.
   b) SD-SCMA (In Table I)
2) **Power Allocation**
   a) Relay R assigns the sub-channels to the SD pairs according to the outcome of SD-SCMA.
   b) Each source node allocates its transmitting power over each sub-channel using water filling algorithm.
   c) Relay R allocates the amplifying gain of each sub-channel.
3) **Resource Allocation Finished**

IV. Simulation Results

In this section, we evaluate the performance of the proposed SD-SCMA in NOMA scheme, and compare it with the OFDMA scheme and the optimal exhaustive search with MATLAB. In the OFDMA scheme, each sub-channel can only be allocated to one SD pair at one time, while one SD pair may have access to multiple sub-channels. For the exhaustive search, we assume that $X_{max} = \infty$, and power allocation is performed over each sub-channel. However, exhaustive search is unrealistic since its complexity is extremely high. In the simulation, we set the peak power of each source node P_{SN} as 46 dBm, and the maximum power of relay R, Q_R as 86 dBm so that the maximum power over each sub-channel Q_K is approximately around 46 dBm. The noise variance σ^2 is -90 dBm , the path loss coefficient α is set as 2, and all user nodes are uniformly distributed in an square area with the size of length 200 m, with relay R in the center of the square. We obtain the simulation results as shown below, and all curves are generated based on averaging over 10000 instances of the algorithms.

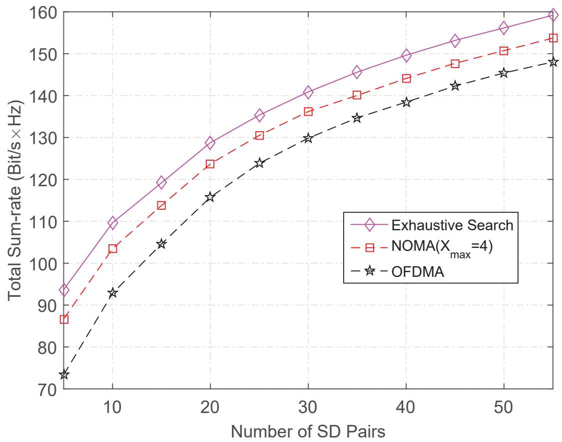

Fig. 2. Total Sum-rate vs. Number of SD Pairs in different schemes (K=10).

Fig.2 illustrates the relation between total sum-rate and the number of SD pairs with the number of sub-channel K=10. The performance of the proposed SD-SCMA in NOMA scheme is much better than the OFDMA scheme, and is close to the exhaustive search. The proposed SD-SCMA in NOMA scheme outperforms OFDMA scheme because it takes a more sufficient utilization over the spectrum resource. The total sum-rate increases as the number of SD pair grows, and the growth becomes slower as N turns larger because of the saturation of the channel capacity. But when the number of SD pairs is much larger than the number of sub-channels, the total sum-rate continues to increase at a low speed due to the multiuser diversity gain.

Fig.3 depicts the number of SD pairs vs. the total sum-rate in the NOMA scheme with different X_{max}, and the number of sub-channel K=10. By comparing the curves, we find that the total sum-rate is higher with a larger X_{max}, due to a more efficient utilization of the sub-channels. When X_{max} increases, the marginal increment of the total sum-rate becomes smaller. Because as the inter-signal interference is increasing, it is more

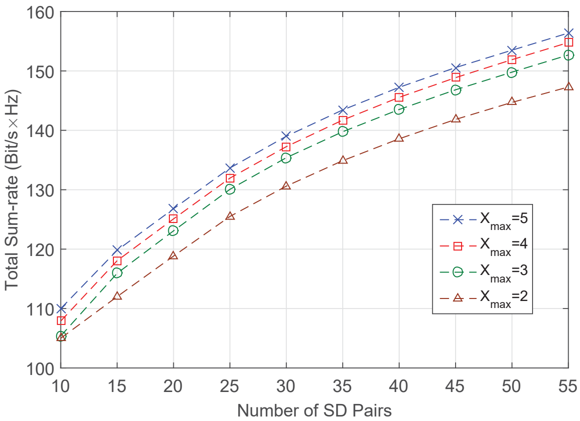

Fig. 3. Total Sum-rate vs. Number of SD Pairs with different X_{max} (K=10).

difficult for a new coming SD pair to enhance the total sum-rate over the sub-channel, and when $X_{max} \to \infty$, the total sum-rate in NOMA scheme can not be larger than the exhaustive search. When $X_{max} = 1$, it'll be identical to the OFDMA scheme in Fig.2.

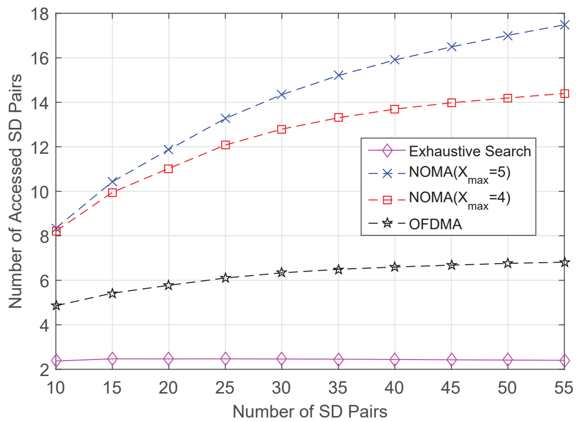

Fig. 4. Number of Accessed SD Pairs vs. Number of SD Pairs (K=10).

Fig.4 shows the number of accessed SD pairs vs. the number of SD pairs, with the number of sub-channels K=10. The number of accessed SD pairs in exhaustive search is around 2 to 3 regardless of the total number of SD pairs, because several SD pairs with the best channel conditions always occupy all the sub-channels, which is unfair to other SD pairs. The OFDMA scheme performs better than the exhaustive search because of the limitation on the assignment. However, the number of the accessed SD pairs still turns saturated since each sub-channel can be allocated to no more than one SD pair. The NOMA scheme outperforms OFDMA scheme and exhaustive search significantly because of its non-orthogonal nature. When X_{max} increase, the number of accessed SD pairs will increase since each sub-channel can be allocated to more SD pairs.

V. Conclusion

In this paper, we studied the resource allocation problem in a NOMA wireless network with a one-way OFDM AF relay by optimizing the sub-channel assignment and the power allocation. By formulating the problem as a many-to-many two-sided matching problem, we proposed a near optimal SD pair-subchannel matching algorithm in which the SD pairs and sub-channels can be matched and converge to a stable matching. NOMA scheme greatly outperforms OFDMA scheme in terms of both the total sum-rate and the number of accessed SD pair number. The proposed SD-SCMA can reach a total sum-rate close to the exhaustive search with a much lower complexity and more accessed SD pairs. The receiver complexity influences the total sum-rate and the number of accessed SD pairs on different directions, and the trade-off between the total sum-rate and number of accessed SD pair number can be adjusted by maximum number of SD pairs that each sub-channel can be allocated to (X_{max}).

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