Low Complexity Polar Code Decoder for HARQ Application

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***Abstract*—In this work, an incremental decoding algorithm is proposed for the punctured polar codes. The incremental decoding is based on the successive cancellation (SC) decoding algorithm. This paper shows that when the punctured polar code is adopted in the HARQ (Hybrid Automatic Repeat Request) protocol, the incremental decoding algorithm can significantly reduce the complexity of the additional decoding attempt for the re-transmission. In addition, the reduction of complexity is not effected by the punctured patterns, and the proposed incremental decoding is suitable for all punctured polar codes.**

I. Introduction

Polar code is proposed by Arikan, it can be decoded by successive cancellation (SC) [1] decoding algorithms. SC decoding algorithm has been proved to provide capacity-achieving error correcting capability when it is applied to an infinite-length code. Polar code [1] has been adopted in upcoming 5G standard [10][11]. In order to cope with the time-varying channel response of the mobile communications, HARQ protocol is required. A punctured polar code can be used in HARQ protocol [2][3][8][9], where the rate is adjusted by the punctured bits.

The HARQ performs multiple re-transmissions and multiple decodings. Only the un-punctured bits are first transmitted. If the first decoding attempt fails, the punctured bits are transmitted and combined with the bits previously transmitted. A long (un-punctured and original) code is combined in the receiver from this two transmissions, and it has a lower code rate to cope with the worse channel condition [4].

The most direct way to process the second decoding attempt is reset decoding. where the decoding results of the first decoding attempt are dropped [5]. However, the decoding results of the first decoding attempt can also be maintained and an incremental decoding can be performed. Thanks for the structured code construction of polar code, the SC decoding can be applied to the incremental decoding of polar code. Following the SC decoding, there are some decoding messages will not be changed during the incremental decoding, and their computations can be saved.

The simulation results shows that the incremental decoding will not effect the error-rate performance, and the reduction in the decoding complexity can be more significant as the code length enlarged.

II. Preliminary

Polar codes are derived from the phenomenon of channel polarization and combination. As shown in Fig. 1, two chan-

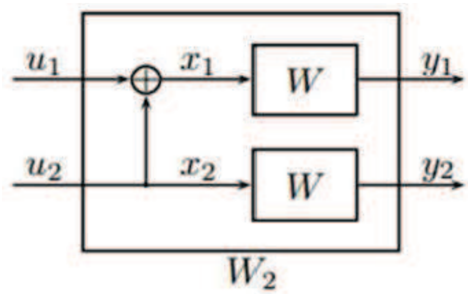

Fig. 1. The function core of polar code encoding.

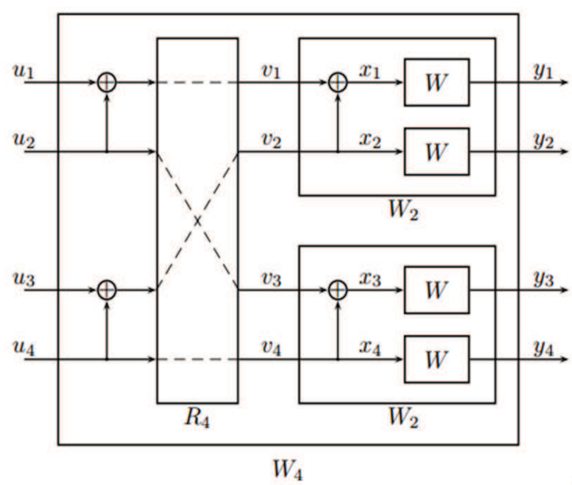

Fig. 2. The polar code encoding for $N = 4$.

nels $W\{y_1|x_1\}$ and $W\{y_2|x_2\}$ are combined to the channel $W\{y_1, y_2|u_1, u_2\}$, then the bit transmitted by u_2 can achieve a lower error rate comparing to that transmitted by u_1. The combination can be extened, and a length $N = 2^n$ polar code can be constructed. The extension of $N = 4$ is shown in Fig. 2.

The effect of polarization can be more significant as the code length extended. Fig. 3 shows an example of a length $N = 16384$ polar code which is designed for a BSC channel with crossover probability $\epsilon = 0.3$. It can be observed that 0.3

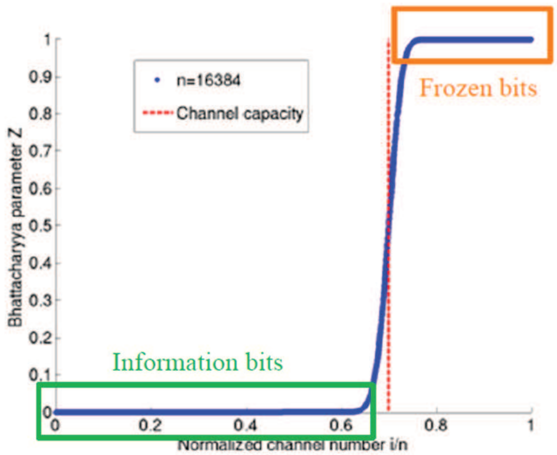

Fig. 3. The polarization for a rate $R = 0.7$ $N = 16384$ polar code.

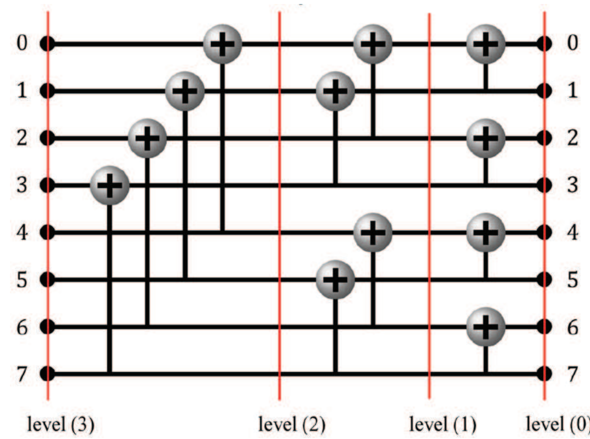

Fig. 5. The decoding for the polar code with $N = 8$.

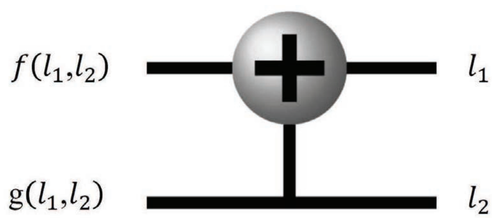

Fig. 4. The function core of polar code decoding.

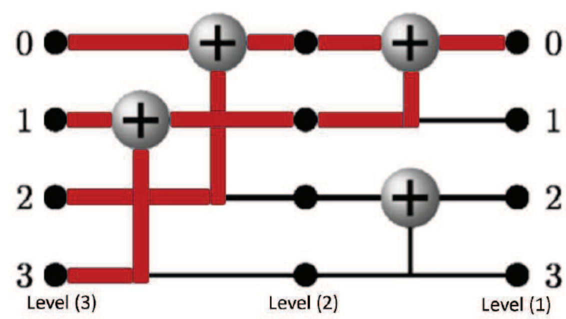

Fig. 6. The incremental decoding for the punctured code.

of total $N = 16384$ bits will have very high error probability, which are set as frozen bits. Frozen bits will not be used to transmit messages, and are usually set to zero. On the other hand, 0.7 of total $N = 16384$ bits can be transmitted in a very low error rate. These bits are used as information bits. Therefore, the rate of the polar code shown in Fig. 3 is $R = 0.7$.

For the SC decoding, its core of function is shown in Fig. 4. l_1 and l_2 are the log-likelihood ratiol (LLR) of the channels, and the decoding results of the corresponding two bits can be described by the functions $f(l_1, l_2)$ and $g(l_1, l_2)$:

$$f(l_1, l_2) = \frac{l_1 l_2 + 1}{l_1 + l_2} \tag{1}$$

$$\begin{cases} g(l_1, l_2) & = l_1 l_2 \text{ if } f(l_1, l_2) \text{ is decided to be } 1 \\ g(l_1, l_2) & = \frac{l_2}{l_1} \quad \text{if } f(l_1, l_2) \text{ is decided to be } 0 \end{cases} \tag{2}$$

The decoding process can be extended to a completed polar code. Fig. 5 shows the decoding process of a polar code with $N = 8$. The decoding results of all functions $f(l_1, l_2)$ in level(0), i.e., directly received messages from channel, can be propagated to the decoding cores of level(1), and the functions $f(l_1, l_2)$ of level(1) generate the informations for the level(2), and so on. The bit-0 will first be decoded. After it is decoded, its binary value can be used to decode the bit-4. The indices of the successive decoding sequence is 0, 4, 2, 6, 1, 5, 3, 7.

III. Incremental decoding

The reduction in the incremental decoding can be explained using the example shown in Fig. 6. Supposed that the 0-th encoding bit (in level(1)) is punctured and is not received for the first decoding attempt. For the punctured bit, its corresponding channel value is not received, and the initial LLR value is set to zero.

If the first decoding attempt fails after a SC decoding is processed, the channel value of the 0-th bit is re-transmitted for the second decoding attempt. According to Fig. 6, the paths denoted in red lines indicate the propagation of the channel value of the 0-th bit. It can be noted that the additional channel values of the 0-th bit will not effect all decoding commutations. The decoding results of the 2-nd and the 3-rd encoding bits of level(1) will keep the same to the first decoding attempt, and the corresponding computation can be saved, consequently reduced the decoding complexity.

It is worth noting that, the reduction in the decoding complexity is not limited to the 0-th bit. For any other punctured bit, the amount of the reduced decoding complexity will be the same.

If only a single bits is punctured, the complexity of the incremental decoding is shown in Table. I. The first column is

TABLE I
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N	Rst. SC	Inc. SC	Reduced Rate
8	24	14	0.4167
16	64	30	0.5313
32	160	62	0.6125
64	384	126	0.6719
128	896	254	0.7165
256	2048	510	0.7510
512	4608	1022	0.7782
1024	10240	2046	0.8002

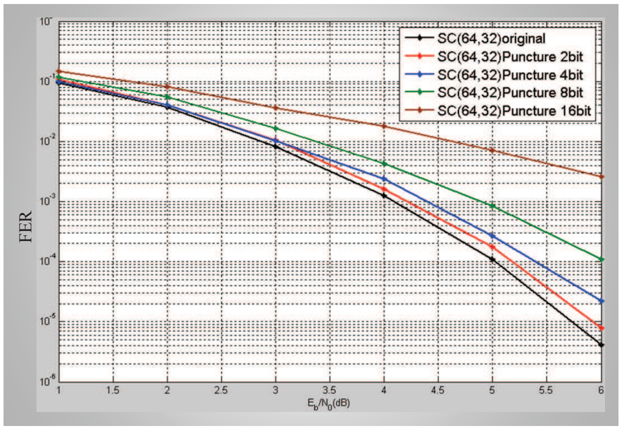

Fig. 7. FER for the punctured polar code.

shows the considered code length, the range covers $N = 8$ to $N = 1024$. The second column shows the decoding complexity of the reset decoding, the values are the total required numbers of the sum of $f(l_1, l_2)$ and $g(l_1, l_2)$ operations. The third column shows the decoding complexity of the incremental decoding, where only the $f(l_1, l_2)$ and $g(l_1, l_2)$ operations on the propagation path of the LLR value of the punctured bit are required. The rate of the reduction complexities are shown in the forth column. It can be observed that, for a longer code, the reduction is more significant.

Fig 7 shows the frame error rate (FER) of the (64,32) polar code with different number of punctured bits. Since the reduction in the complexity introduced by the proposed incremental decoding provide exactly the same decoding messages, it can be expected that the decoding performance will also be exactly the same comparing to that of the reset decoding.

IV. CONCLUSION

In this paper, the complexity of the incremental decoding for punctured polar codes is analyzed. It shows that based on the SC decoding, the proposed incremental decoding can provide exactly the same performance using much lower decoding complexity comparing to the reset decoding. The advantage of low complexity can be more significant long codes.

REFERENCES

[1] E. Arkan, "Channel polarization: A method for constructing capacity-achieving codes for symmetric binary-input memoryless channels," *IEEE Trans. Inf. Theory,* vol. 55, no. 7, pp. 3051-3073, Jul. 2009.S.

[2] Kai Niu, Jincheng Dai, Kai Chen, Jiaru Lin, Q. T. Zhang, and Athanasios V. Vasilakos, "Rate-Compatible Punctured Polar Codes: Optimal Construction Based on Polar Spectra," *IEEE Trans. Inf. Theory,* 5 Dec. 2016.

[3] ZHU Hongbin, DAI Shengchen , KANG Kai , and QIAN Hua "An Improved HARQ Scheme with Polar Codes," *Journal of Electronics and Information Technology,* Vol.39, No.5, May 2017.

[4] K. Chen, K. Niu, Z. He, and J. Lin, "Polar coded HARQ scheme with Chase combining," *in Proc. IEEE Wireless Commun. Netw. Conf. (WCNC),* Apr. 2014, pp. 474479.

[5] Frenger P, Parkvall S, and Dahlman E. "Performance comparison of HARQ with chase combining and incremental redundancy for HSDPA," *in Proc. IEEE Vehicular Technology Conference,* Atlantic City, USA, 2001, pp. 1829-1833.

[6] Niu K, Chen K, Lin J R. "Beyond turbo codes: Rate-compatible punctured polar codes," *in Proc. IEEE International Conference on Communications,* 2013, pp. 3423-3427.

[7] Yi Ming , Ji Xin-sheng , Huang Kai-zhi , Jin Liang , and Wang Jing "A Method Based on Puncturing Polar Codes for Physical Layer Security," *Journal of Electronics and Information Technology,* Vol.36, No.12, Dec. 2014.

[8] Sha Wang, Jian Jiao, Bowen Feng, Shaohua Wu, Shushi Gu, Qinyu Zhang, "Efficient HARQ scheme based on rate-compatible punctured polar codes," *in Proc. IEEE/CIC International Conference on Communications in China (ICCC)*, 2017, pp.1 - 6;

[9] Hao Liang, Aijun Liu, Yingxian Zhang, Qingshuang Zhang, "Analysis and Adaptive Design of Polar Coded HARQ Transmission Under SC-List Decoding," *IEEE Wireless Communications Letters,* Vol. 6, Iss. 6, 2017, pp. 798 - 801.

[10] N. Ahmed, H, Rahman, and Md.I. Hussain, "A comparison of 802.11ah and 802.15.4 for IoT," *ICT Express,* Vol.2, No. 3, Sep. 2016, pp. 100-102

[11] Bin Liu, Zhisheng Yan, and Chang Wen Chen, "Medium Access Control for Wireless Body Area Networks with QoS Provisioning and Energy Efficient Design," *IEEE Transactions on Mobile Computing,* Vol. 16, No. 2, Feb. 2017, pp. 422-434.