Low Complexity Beam Searching Algorithm Using Asymptotic Property of Massive MIMO Systems

Heeyoung Kim*, Jaemin Jung*, Seongbae Han*, Seokki Kim†, Seungkwon Baek†, and Sooyong Choi*
*School of Electrical and Electronic Engineering, Yonsei University, Seoul, Korea.
Email: {khy93, jjmm1000, hsb1531, csyong}@yonsei.ac.kr
†Electronics and Telecommunications Research Institute (ETRI), Daejeon, Korea
Email: {kimsk0729, skback}@etri.re.kr

***Abstract*—This paper considers a low complexity codebook-based beam searching algorithm for massive MIMO (Multiple Input Multiple Output) systems in limited scattering environments. Based on the asymptotic property of the massive MIMO systems that the channel array response vector converges to the optimal precoding vector derived from channel singular value decomposition (SVD), we propose a independent selection method for the transmitter (TX) and receiver (RX) beam vectors. Due to the independent selection of the Tx and Rx beam vectors from the asymptotic property, the proposed algorithm has a much lower complexity compared to the full search beam selection method. Furthermore, the proposed algorithm shows the similar data rates to the full search beam selection method at low signal to noise ratio regions in a 6-bit codebook system with 64×64 antennas and 2 RF chains at each TX and RX.**

I. Introduction

In massive multiple input multiple output (MIMO) systems with limited scattering environment, channel array response vector can simply converges to optimal precoding based on channel singular vector as the number of antenna increases [1]. Therefore, the optimal precoding can be realized only with equal gain and analog phase shifters in massive MIMO systems [2], [3]. With such a constrained precoder and combiner architecture, we propose a low complexity codebook-based beam searching algorithm using the asymptotic property of massive MIMO systems.

Although several low complexity beam searching schemes have been proposed, they require several iterations to converge [4], [5]. By exploiting the asymptotic property of precoder and combiner in the massive MIMO systems, we can find half of the beam combination in the analog codebook using the closed formula. That is, half of the beam can be obtained from the singular vector of the channel. Then, we are able to select the remaining beam combination with full search beam selection method. Therefore, we just investigate much smaller size of beam combination sets for half of the beam. The proposed scheme assumes environment such as Xhaul network where obtaining channel state information is possible [6].

Notation : We use the boldface uppercase and lowercase letters for matrices and vectors, respectively. The operators $(.)^T$ and $(.)^H$ denote transpose and conjugate transpose, respectively. $\|\mathbf{A}-\mathbf{B}\|$ is the Euclidean distance between $\mathbf{A}$ and $\mathbf{B}$. $|\mathbf{A}|$ is the determinant. $\mathbf{I}_N$ is the $N\times N$ identity matrices. $\mathbb{E}$ denotes the expectation operation. $\mathcal{CN}$ is a complex Gaussian.

II. System Model

A. System Model

We consider a single user massive MIMO system as shown in Fig. 1, where a base station is equipped with N_t TX antennas, N_r RX antennas and 2 RF chains [7]. Fig. 1 shows the simplified hardware architecture including RF signal processing at TX and RX. We do not include the baseband processing part since a phase shifter by itself can produce optimal precoder at massive MIMO environment with limited scattering environment [1]. Since we do not consider the baseband processing, we assume that the number of data streams, N_s, is equal to the number of RF chains as 2. Therefore, the transmitted signal is $\mathbf{F}_{RF}\mathbf{s}$ where $\mathbf{F}_{RF}$ is the $N_t\times N_s$ RF precoder matrix with all elements having equal norm and $\mathbf{s}$ is the $N_s\times 1$ signal vector satisfying $\mathbb{E}[\mathbf{ss}^*]=\frac{1}{N_s}\mathbf{I}_{N_s}$. The received signal $\mathbf{y}$ is given by $\mathbf{y}=\sqrt{\rho}\mathrm{H}\mathbf{F}_{RF}\mathbf{s}+\mathbf{n}$, where $\mathbf{y}$ is the $N_r\times 1$ vector, H is the $N_r\times N_t$ MIMO channel matrix and $\mathbf{n}$ is $N_r\times 1$ additive white Gaussian noise vector with $\mathcal{CN}(0,N_0\mathbf{I}_{N_r})$ where the N_0 is noise variance. The processed signal $\widetilde{\mathbf{y}}$ at RX base station can be written as

$$\widetilde{\mathbf{y}}=\sqrt{\rho}\mathbf{P}_{RF}^*\mathrm{H}\mathbf{F}_{RF}\mathbf{s}+\mathbf{P}_{RF}^*\mathbf{n}, \tag{1}$$

where $\mathbf{P}_{RF}$ is the $N_r\times N_s$ RF combiner with unit norm elements. In this paper, we represent the specific notation of codebook-based precoder $\mathbf{F}_{RF}$ as $\mathbf{W}$ which is the $N_t\times N_s$ quantized precoder at TX. Also codebook-based combiner $\mathbf{P}_{RF}$ is assigned to notation of $\mathbf{C}$ which is the $N_r\times N_s$ quantized combiner at RX. Since we consider 2 RF chains, let us represent $\mathbf{W}=[\mathbf{w}_1,\mathbf{w}_2]$ and $\mathbf{C}=[\mathbf{c}_1,\mathbf{c}_2]$ where $\mathbf{w}_1$ and $\mathbf{w}_2$ are the $N_t\times 1$ beam vectors corresponding to each 2 RF chains at TX, respectively. In a similar way, $\mathbf{c}_1$ and $\mathbf{c}_2$ are the $N_r\times 1$ beam vectors associated with each RF chain at RX.

B. channel model and analog codebook setup

In this paper, the Saleh Valenzuela model is considered for a clustered scattering environment. The Saleh Valenzuela model is often used in mmWave channel modeling [8], [9]. Therefore, the channel H can be represented as

$$\mathrm{H}=\sqrt{\frac{N_rN_t}{2}}\sum_{i}^{L}\alpha_i\mathbf{f}_t(\phi_i^t)\mathbf{f}_r(\phi_i^r)^*,$$

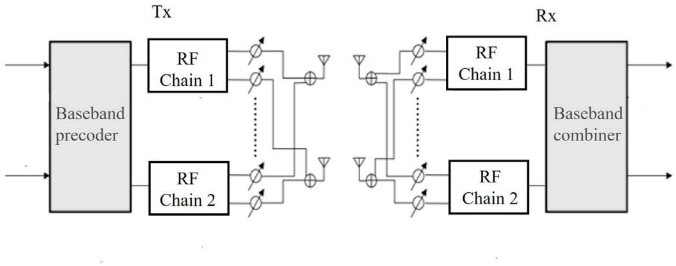

Fig. 1. Hardware architecture of the massive MIMO systems with RF chains

where L is the number of propagation paths, $\alpha_i \in \mathcal{CN}$ is the channel gain of the ith path and ϕ_i^t and ϕ_i^r are the azimuths of angle of departure (AoD) and angle of arrival (AoA) of the ith path, respectively. As we consider uniform linear arrays (ULAs) in both TX and RX antennas, $\mathbf{f}(\phi_i^t)$ is the TX channel array response vector and $\mathbf{f}(\phi_i^r)$ is the RX channel array response vector. $\mathbf{f}(\phi_i^t)$ and $\mathbf{f}(\phi_i^r)$ are represented as

$$\mathbf{f}(\phi_i^t) = \frac{1}{\sqrt{N_t}}[1, e^{jkdsin(\phi_i^t)}, \dots, e^{j(N_t-1)kdsin(\phi_i^t)}]^T, \quad (2)$$

$$\mathbf{f}(\phi_i^r) = \frac{1}{\sqrt{N_r}}[1, e^{jkdsin(\phi_i^r)}), \dots, e^{j(N_r-1)kdsin(\phi_i^r)}]^T, \quad (3)$$

where $k = \frac{2\pi}{\lambda}$, λ denotes the wavelength of the signal and d is the antenna spacing. Now let $\mathbf{A}$ denotes the analog codebook matrix for both TX and RX. If we use a B_t-bit analog codebook with N antenna array, $\mathbf{A}$ is represented as

$$\mathbf{A} = [\mathbf{a}(\phi_1), \dots, \mathbf{a}(\phi_{2^{B_t}})],$$

where each beam vector is represented as

$$\mathbf{a}(\phi_i) = \frac{1}{\sqrt{N}}[1, e^{jkdsin(\phi_i)}, \dots, e^{jk(N-1)dsin(\phi_i)})]^T,$$

where $\phi_i = \frac{2\pi i}{2^{B_t}}$ for $i \in 1, \dots, 2^{B_t}$.

III. Proposed low complexity beam searching algorithm

The proposed beam searching algorithm combines a full search method with closed formula for the selection of beam vectors to reduce the computational complexity. The asymptotic optimal precoder (combiner) in massive MIMO system, i.e. right (left) channel singular vector, can be implemented only with phase shifters in massive MIMO systems [1]. Therefore, it can be assumed that the optimal analog beamformer with large antenna arrays are the channel singular vector. There is a constraint on quantization of codebook. In codebook-based communication, however, beam vectors can't be exactly the same as the channel singular vector. Thus we independently select half of the codebook-based beam vectors at TX and RX which has the minimum Euclidean distance to channel singular vector based on the closed formula given in [1]. Then, the remaining beam vectors can be searched to maximize the data rate of (1) with full search beam selection method.

In the searching process, we need the full channel state information since we right singular vector for closed formula. For convenience, we use the notation of a column of right singular vector as $\mathbf{v}^1$ and a column of left singular vector as $\mathbf{u}^1$ which are the largest singular values.

TABLE I
Proposed beam search algorithm

▷ Pre-step : Do singular vector decomposition of the channel H

Pre-step solution : Right channel singular vector = $\mathbf{v}$
Left channel singular vector = $\mathbf{u}$

▷ Step1-1 : $\mathbf{w}_1^* = \arg\min_{\mathbf{w}_1} \|\mathbf{w}_1 - \mathbf{v}^1\|$

▷ Step1-2 : $\mathbf{c}_1^* = \arg\min_{\mathbf{c}_1} \|\mathbf{c}_1 - \mathbf{u}^1\|$

Step 1 solution : $\mathbf{W} = [\mathbf{w}_1^*, \mathbf{w}_2]$, $\quad \mathbf{C} = [\mathbf{c}_1^*, \mathbf{c}_2]$

▷ Step 2 : $(\mathbf{w}_2^*, \mathbf{c}_2^*) = \arg\max_{\mathbf{w}_2, \mathbf{c}_2} \log_2(|\mathbf{I}_{N_s} + \mathbf{R}_\mathbf{N}^{-1}\mathbf{C}^H\mathbf{H}\mathbf{W}\mathbf{W}^H\mathbf{H}^H\mathbf{C}|)$

Final solution : $\mathbf{W}^* = [\mathbf{w}_1^*, \mathbf{w}_2^*]$, $\quad \mathbf{C}^* = [\mathbf{c}_1^*, \mathbf{c}_2^*]$

TABLE I represents the proposed algorithm with system parameters such as $\mathbf{W} = [\mathbf{w}_1, \mathbf{w}_2]$ and $\mathbf{C} = [\mathbf{c}_1, \mathbf{c}_2]$. As shown in TABLE I, we select the beam vector $\mathbf{w}_1$ and $\mathbf{c}_1$ in step 1 that satisfy the minimum distance with the channel singular vectors. Since we can search $\mathbf{w}_1$ and $\mathbf{c}_1$ independently in step 1-1 and step 1-2, we can obtain the computational complexity gains compared to the joint search. Although we exploit the full search beam selection method in step 2, we can reduce the number of beam combinations to search by selecting first $\mathbf{w}_1^*$ and $\mathbf{c}_1^*$ in step 1.

IV. Simulation results

We evaluate the proposed algorithm in terms of the achievable rates and complexity compared to the full search beam selection method. The AoA and AoD in (2) and (3) are generated randomly with uniform distribution within $[0, \pi]$. The channel gain α_i follows $\mathcal{CN}(0, 1)$ and the number of propagation path L is set to be 2.The antenna arrays at both TX and RX are ULAs with antenna spacing $d = \frac{1}{\lambda}$. The number of RF chain at both TX and RX is fixed as 2. The achievable data rate is measured as SNR increases from -20 dB to 0 dB. We use the 5-bit and 6-bit analog codebooks.

Fig. 2 shows the achievable rates according to the number of codebook bit such as $B_t = 5$ and $B_t = 6$ for 64×64 massive MIMO systems. As the number of antenna increases, we can obtain the higher data rates of the proposed algorithm with the 6-bit codebook system compared to the full search beam selection method with the 5-bit codebook system. We can observe that the proposed scheme follows the achievable rates of the full search beam selection method regardless of the number of antenna and codebook bit in Fig. 2.

To compare complexities of the full search beam selection method and the proposed algorithm, the number of possible beam combination that has to be considered in the 6-bit and 5-bit codebook-based beam searching with 2 RF chains is presented in TABLE II. We ignore the complexity of SVD of channel matrix because especially for 6-bit codebook systems, it accounts for only about 0.21% of beam searching time using MATLAB. In the 6-bit codebook-based beam searching, the

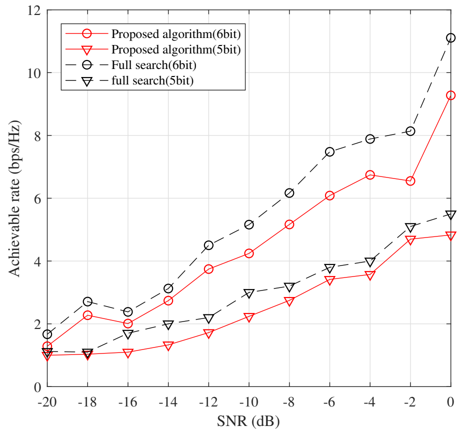

Fig. 2. Achievable rate of the proposed algorithm according to the number of codebook bit for 64×64 massive MIMO systems

TABLE II
COMPLEXITY OF THE PROPOSED ALGORITHM AND THE FULL SEARCH BEAM SELECTION METHOD

The number of possible beam combination		
Codebook bit	5-bit	6-bit
Full search beam selection method	1.0×10^6	1.6×10^7
Proposed algorithm	1.0×10^3	4.2×10^3

proposed algorithm shows the 4.2×10^3 complexity, which can reduce the complexity of full search beam selection as much as 0.026%. In terms of the achievable rates, 6-bit codebook-based proposed scheme for 64×64 massive MIMO systems shows the higher data rates than the 5-bit codebook-based full search beam selection method with much lower complexity.

V. CONCLUSION

In this paper, we proposed a low complexity codebook-based beam searching algorithm for massive MIMO systems with limited scattering channels. The proposed algorithm can reduce the beam searching complexity by combining the closed formula with full search beam selection method. In the simulation result, the 6-bit codebook-based proposed algorithm shows the higher achievable rates than the 5-bit codebook-based full search beam selection algorithm with much lower complexity for 64×64 massive MIMO systems. If the antenna array size increases, the achievable complexity gains increase. Therefore, in massive MIMO systems, the proposed algorithm can lead to a better performance in the achievable rates with much lower complexity compared to the full search beam selection method. Our future work will be focused on extending to more general architectures.

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