Angle-of-Arrival Estimation in Antenna Arrays based on Monopulse Signal

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***Abstract*—Angle-of-Arrival (AoA) estimation supports beamforming that controls array beams in a desired direction while eliminating interference signals. In this paper, we propose a blind AoA estimation method with low complexity by applying a monopulse radar processing method to an antenna array. This paper aims to investigate the statistical characteristics of monopulse signal R for the performance analysis. Monopulse signal ratio R is modeled as the ratio of quadratic random variables in the form of $\mathbf{x}'\mathbf{A}\mathbf{x}/\mathbf{x}'\mathbf{B}\mathbf{x}$, where $\mathbf{A}$ and $\mathbf{B}$ are indefinite and positive semidefinite matrices, respectively. In this paper we obtain the probability density function (PDF) of R. Furthermore, we confirm that the monopulse based AoA estimator (MAoA) is comparable to the single-target estimation performance of subspace-based algorithms with high complexity such as multiple signal classification (MUSIC) and estimation of signal parameters via rotational invariance technique (ESPRIT) in terms of mean square error (MSE).**

***Index Terms*—beamforming, array processing, AoA estimation, monopulse**

I. Introduction

Beamforming technology can be used in LTE and WLAN standards to increase coverage and improve system capacity [1]. Especially, by beamforming in the direction to localize the signal sources, it suppresses noise and interference and improves the signal-to-noise ratio (SNR). To accurately steer the beam into the direction in which the signal is incident, it is required to estimate the AoA and verify the accuracy of the estimation.

In the field of array processing, AoA estimation has been developed over decades. The conventional beamforming such as minimum variance distortionless response (MVDR) [2] has been widely employed due to its simplicity for the implementation. On the other hand, the high resolution subspace-based MUSIC and ESPRIT, which are well known so far, have been developed to attain the near optimal performance in array processing [3], [4]. These algorithms estimate AoA by singular value decomposition (SVD) or eigenvalue decomposition (EVD). Especially, MUSIC is also based on the angle spectrum that exhaustively searches the entire angle. On the other hand, ESPRIT works based on eigenspace invariance to obtain the phase difference between the antennas in the antenna array. However, both algorithms require very high computation complexity for the calculations of SVD and EVD. As the number of antennas increases, the number of computation increases with the cube of the number of antennas, thus, the subspace-based algorithms are prohibited to be utilized in a large-scale antenna array.

To tackle the complexity problem, this paper proposes a low complexity algorithm motivated by the monopulse phase comparison method [5], which can be used in existing radar applications to the antenna array. This proposed MAoA estimates the AoA by using the phase difference between adjacent antennas in the antenna array. Here, the monopulse ratio R is used for AoA estimation which can be expressed in a form of $\mathbf{x}'\mathbf{A}\mathbf{x}/\mathbf{x}'\mathbf{B}\mathbf{x}$, consisting of a random Gaussian vector $\mathbf{x}$, an indefinite matrix $\mathbf{A}$, and a positive semidefinite matrix $\mathbf{B}$. Accordingly, we first obtain the density function of R which is based on non-central Gaussian random variables and an indefinite or a positive semidefinite matrices. Furthermore, we confirm that MAoA is comparable to MUSIC and ESPRIT, which require a high computation intensity.

II. Signal Model

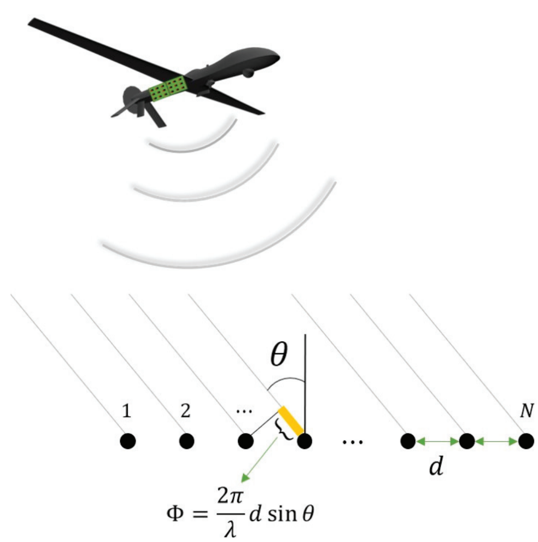

Fig. 1. N uniform linear array signal model

We consider a narrow-band far-field signal waveform impinging on the N uniform linear array (ULA), as depicted in

Fig. 1. We assume that all antenna elements are omnidirectional and there is no correlation between the received signals of the antenna elements. Then, we can write the received signal vector as

$$\mathbf{y}(t) = \mathbf{a}(\theta)s(t) + \mathbf{n}(t), \tag{1}$$

where $s(t)$ is the incoming pilot signal for AoA estimation and $\mathbf{n}(t)$ is the complex additive white Gaussian noise (AWGN) whose distribution follows $\mathcal{CN}(0, \sigma^2)$. The array response vector, $\mathbf{a}(\theta)$ is given as

$$\mathbf{a}(\theta) = \begin{bmatrix} 1 & e^{j\frac{2\pi}{\lambda}d\sin\theta} & \cdots & e^{j\frac{2\pi}{\lambda}d(N-1)\sin\theta} \end{bmatrix}^T, \tag{2}$$

where λ is the wavelength of the incident signal and the antenna spacing d is half of the wavelength ,that is, $\lambda/2$ [6].

In this system, we estimate AoA based on MAoA, which is often used for monopulse radar system. Note that monopulse radar estimates AoA by obtaining amplitude or phase difference between tilted beams [5]. For the antenna array, AoA can be estimated by calculating the phase shift between adjacent antenna elements. Here, the monopulse ratio R used in the MAoA is calculated as the ratio of the sum to the difference signal between adjacent antennas. We can define the monopulse ratio R as

$$R = \frac{1}{N-1}\sum_{i=1}^{N-1}\frac{d_i}{s_i}, \tag{3}$$

where i denotes the antenna element index and d_i and s_i denote the difference and the sum of the received signals, respectively. The difference and the sum of the received signals are defined as $d_i = \mathbb{E}_t[y_i(t) - y_{i+1}(t)]$ and $s_i = \mathbb{E}_t[y_i(t) + y_{i+1}(t)]$, respectively. Here, $\mathbb{E}_t$ denotes the expectation for time and y_i denotes the received signal of the i-th antenna element, namely, the i-th element of the $\mathbf{y}$. For the sake of simplicity, we here consider two antenna elements case. Then, the monopulse ratio R can be formulated as

$$R = \frac{s(t) - e^{j\Phi}s(t) + n_1(t) - n_2(t)}{s(t) + e^{j\Phi}s(t) + n_1(t) + n_2(t)} \approx -j\tan\frac{\Phi}{2}, \tag{4}$$

where $\Phi = \frac{2\pi}{\lambda}d\sin\theta$ is the phase shift between the adjacent antennas, which can be obtained from the imaginary part of monopulse ratio, i.e., $2\tan^{-1}(-Im\{R\})$. Subsequently, the AoA, θ can be obtained as $\sin^{-1}(\frac{\lambda\Phi}{2\pi d})$. Note that MAoA is a blind estimation method since prior knowledge of signals or channel state information is not required.

III. Statistical Modeling

The monopulse signal is modeled as a ratio of quadratic independent random variables in two adjacent antenna elements. The complex monopulse ratio R is modeled as

$$R = \frac{p + jq}{s + jt} = \frac{ps + qt}{s^2 + t^2} + j\frac{qs - pt}{s^2 + t^2}, \tag{5}$$

where p, q, s, t are non-central Gaussian random variables whose means are $\bar{p}, \bar{q}, \bar{s}, \bar{t}$ and standard deviations are a, a, b, b, respectively. Note that $\mathbf{x}$ represents $\begin{bmatrix} p & q & s & t \end{bmatrix}^T$ whose mean $\mu = \begin{bmatrix} \bar{p} & \bar{q} & \bar{s} & \bar{t} \end{bmatrix}^T$ and covariance matrix Σ are defined as following

$$\mu = \begin{bmatrix} 1-\cos\Phi & -\sin\Phi & 1+\cos\Phi & \sin\Phi \end{bmatrix}^T, \tag{6}$$

$$\Sigma = \begin{bmatrix} a^2 & 0 & 0 & 0 \\ 0 & a^2 & 0 & 0 \\ 0 & 0 & b^2 & 0 \\ 0 & 0 & 0 & b^2 \end{bmatrix}. \tag{7}$$

Since angular information is included only in the imaginary part of R, we therefore focus on the imaginary part and denote $Im\{R\}$ as R_{im} for simplicity. The imaginary part in (5) can be re-formulated as using the random vector $\mathbf{x}$,

$$R_{im} = \frac{\mathbf{x}'\mathbf{A}\mathbf{x}}{\mathbf{x}'\mathbf{B}\mathbf{x}}. \tag{8}$$

Note that $\mathbf{A}$ and $\mathbf{B}$ can be written respectively as,

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 & -c \\ 0 & 0 & c & 0 \\ 0 & c & 0 & 0 \\ -c & 0 & 0 & 0 \end{bmatrix}, \ \mathbf{B} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 2c & 0 \\ 0 & 0 & 0 & 2c \end{bmatrix}, \tag{9}$$

where c is an arbitrary constant, $\mathbf{A}$ is an indefinite matrix and $\mathbf{B}$ is a positive semidefinite matrix. To investigate the statistical properties of random variable R_{im}, let us find the probability density function of R_{im}. Furthermore, we calculate the density function by differentiating the cumulative density function (CDF) of R_{im} as

$$\begin{aligned} f_{R_{im}}(r) = \frac{2b}{a}e^{-C}\sum_{n=0}^{\infty}\sum_{m=0}^{n}\frac{\Gamma(n+3)}{(n-m)!m!\Gamma(n-m+2)\Gamma(m+2)} \\ \times C_1^m(r)C_2^{n-m}(r)\left(\frac{1}{2\sqrt{(\frac{b}{a}r)^2+1}}\right)^{n+3}(1-A+n), \end{aligned} \tag{10}$$

where $C_1(r)$ and $C_2(r)$ are given as

$$\begin{aligned} C_1(r) &= \frac{C + 2(A\frac{b}{a}r + B)\left(\frac{b}{a}r + \sqrt{(\frac{b}{a}r)^2+1}\right)}{2\sqrt{(\frac{b}{a}r)^2+1}} \\ C_2(r) &= \frac{C + 2(A\frac{b}{a}r + B)\left(\frac{b}{a}r - \sqrt{(\frac{b}{a}r)^2+1}\right)}{2\sqrt{(\frac{b}{a}r)^2+1}}. \end{aligned} \tag{11}$$

The details of the derivation of (10) can be found in [7]. Note that the parameters A, B, and C in (11) are found to be as

$$\begin{aligned} A &= \frac{1}{2b^2}\left(\left(\frac{b}{a}\cdot\frac{\bar{q}}{\bar{s}}\right)^2\bar{s}^2 + \left(-\frac{b}{a}\cdot\frac{\bar{p}}{\bar{t}}\right)^2\bar{t}^2\right), \\ B &= \frac{1}{2b^2}\left(\left(\frac{b}{a}\cdot\frac{\bar{q}}{\bar{s}}\right)\bar{s}^2 + \left(-\frac{b}{a}\cdot\frac{\bar{p}}{\bar{t}}\right)\bar{t}^2\right), \\ C &= \frac{1}{2b^2}\left(\bar{s}^2 + \bar{t}^2 + \left(\frac{b}{a}\cdot\frac{\bar{q}}{\bar{s}}\right)^2\bar{s}^2 + \left(\frac{b}{a}\cdot-\frac{\bar{p}}{\bar{t}}\right)^2\bar{t}^2\right). \end{aligned} \tag{12}$$

IV. SIMULATIONS

In this section, we present simulation results for performance analysis of proposed MAoA. In order to identify the monopulse ratio distribution for the dual antennas case, the random variables, i.e., p, q, s, and t are generated as a histogram. We compare the theoretical value of the closed-form expression with the simulation value of the histogram. In Fig. 2, we consider that the AoA is 40 degrees and SNR is 10 dB as an example. As the two distributions are almost in agreement, we can identify that the closed-form PDF in (10) is an accurate theoretical distribution.

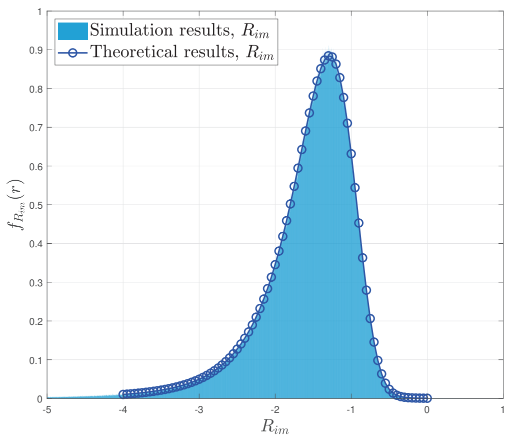

Fig. 2. The probability density function of monopulse ratio R_{im}

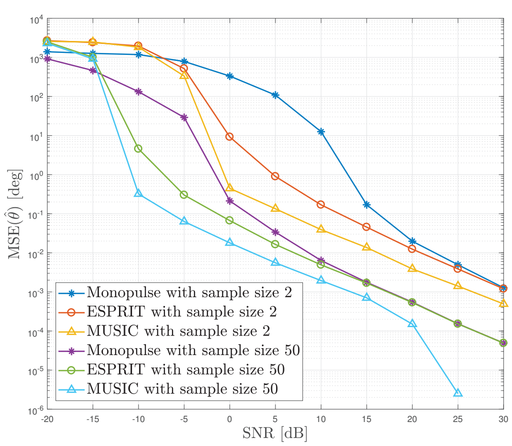

Fig. 3. The mean square error of three AoA estimators versus SNR in [dB] over AWGN

For the performance comparison, we plot the MSE of MAoA, ESPRIT, and MUSIC versus SNR in [dB] as shown in Fig. 3. The simulation parameters are as follows : the number of antenna elements is $N = 8$, sample size are $K = 2$ and $K = 50$, respectively, the AoA, θ is generated from the uniform distribution $[-\frac{\pi}{3}, \frac{\pi}{3}]$, and the angular resolution of the MUSIC spectrum is $\delta = 0.05$. Here, we discuss the computational complexity among MAoA, ESPRIT [4], and MUSIC [3]. For subspace-based algorithms, the main computational complexity comes from EVD such as $O(N^3)$. Accordingly, the total complexity of ESPRIT and MUSIC are $O(N^3 + KN^2 + N)$ and $O(N^3 + KN^2 + N^2(\frac{180}{\delta}))$, respectively. On the other hand, the complexity of the MAoA is $O(n^2)$ for n-digit numbers, which is negligible because only scalar calculations are performed. From Fig. 3, increasing the sample size, the SNR is reduced by 15 dB for the desired MSE $= 10^{-3}$ for the MAoA. Also, as sample size increases to 50, the performance of the MAoA comes to that of ESPRIT in high SNR region. This is because the MAoA is more sensitive to sample size than ESPRIT in that monopulse signal is based on the sample mean of the sum and the difference of the received signals, respectively, as explained in (3). By further increasing the sample size, the performance of MAoA will be more comparable to that of ESPSRIT. Thus, we can confirm that the performance of MAoA with much lower complexity is nearly the same as that of ESPRIT in high SNR region and is comparable to that of subspace-based algorithms in low SNR region. In addition, note that for MUSIC algorithm, an accurate comparison is difficult because simulation performance depends on angular resolution δ of the spectrum.

V. CONCLUSIONS

In this paper, we showed the statistical analysis of monopulse signal, which is represented as the ratio of independent Gaussian quadratic random variables. We confirmed that the derived PDF, of which derivation is not shown due to the limited page, is matched to the simulation results. We also compared the MSE performance among MAoA, ESPRIT, and MUSIC and found that the MSE of MAoA approaches MSE of ESPRIT in high SNR region, although the computation complexity of MAoA is much lower.

VI. ACKNOWLEDGMENT

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