Energy Efficiency Transceiver Design in Multi-antenna Full-Duplex Cellular Systems

Mong Jai Wang,* Fan-Shuo Tseng,+ and Chao-Yuan Hsu†
*+Institute of Communications Engineering, National Sun Yat-sen University, Kaohsiung, Taiwan
†Chunghwa Telecom Co., Ltd, Taipei, Taiwan.
Email: zxxnnxx@gmail.com; fs.tseng@mail.nsysu.edu.tw; chaoyuanhsu@gmail.com

Abstract—**In this paper, we study the precoder design with an energy efficiency (EE) criterion for a full-duplex (FD) multi-antenna cellular system. Unlike the half-duplex (HD) cellular systems, the extra residual self-interference (RSI) and the co-channel interference (CCI) are involved in the uplink (UL) and downlink (DL) received signals due to the simultaneous transmission and reception in an FD base station (BS). Accordingly, we aim at developing the UL and DL precoders to suppress the influence of the RSI and CCI, where the EE criterion is adopted to reach the goal of the green communication. However, the EE precoder design is a fractional non-convex problem. To find the tractable solution, we propose an alternating optimization (AO) method to conduct the DL and UL precoders iteratively. For given DL precoders, the closed-form solution of UL precoders can be eventually yielded by an iterative water-filling (IWF) procedure. Nevertheless, the similar method cannot directly be applied to optimize the DL precoders. We then resort to the UL-DL duality that transfers the DL precoder design to an equivalent UL precoder design problem. We further apply the primal-dual decomposition to the DL precoder design and solve it by the IWF algorithm. Finally, the performance of the proposed design is verified by computer simulations and the result shows its superiority over the existing design.**

Index Terms **– Full-duplex (FD), residual self-interference (RSI), co-channel interference (CCI), fractional programming, iterative water-filling (IWF), UL-DL duality.**

I. INTRODUCTION

Recently, the full-duplex (FD) transmission is developed to improve the spectrum efficiency. The FD system potentially doubles the spectral efficiency (SE) compared with the conventional half-duplex (HD) system by simultaneous transmission and reception. Nevertheless, it incurs the self-interference (SI) due to the nature of the FD system. Though several passive and active techniques have been developed to cancel/suppress the SI, the residual SI (RSI) still exists and degrades the overall performance [1]. Therefore, it is really crucial to design an approach to further suppress the RSI.

For a multi-user (MU) MIMO FD system, the SE can naturally be enhanced by the benefits of multiple antennas and FD techniques [2]. The RSI can be further mitigated with the precoding schemes [2], [3].

In addition to the SE design, the energy consumption is also an important issue due to the fact that the number of communication terminals is significantly growing. The energy efficiency (EE) design becomes critical and is studied in various kind of wireless communication structures [4]-[7]. Instead of maximizing the SE of a system directly, the EE design aims at maximizing the SE per joule.

In [5], the scheduling and power allocation strategy was proposed to maximize EE. The EE design was also studied in cognitive communications [6]. In [7], the authors devised the downlink (DL) precoder with an EE criterion in a HD MU-MISO system.

The precoder design with SE and EE criteria in a FD MU-MIMO system was first studied in [2]. Comparing with the precoder design in a HD system, the UL signal is subject to the RSI resulted from the DL signal at the FD BS, and the DL users are interfered by the UL users, where the interference is also referred to as the co-channel interference (CCI). To provide a tractable solution, the authors in [2] assumed the DL users are not interfered by the UL users, i.e., the CCI is ignored. Moreover, only the lower bounded objective was adopted in the precoder design with an EE criterion. As a result, the considered scenario is not complete.

To fill up the gap, we consider a more practical FD MU-MIMO system which concurrently suffers from the RSI and CCI. At the BS, we adopt the minimum mean-squared error successive interference cancellation (MMSE-SIC) receiver to recovery the signals of the UL users and use the dirty paper coding (DPC) transmitter for the DL users. To further suppress the RSI and CCI, we propose to jointly optimize the UL and DL precoders with an EE criterion, where the total sum rate per consumed energy is maximized under the restricted power constraints. However, the corresponding optimization problem is a fractional non-convex problem. The optimum solution is not tractable under the limited hardware complexity. Accordingly, we propose an alternating optimization (AO) approach for the precoder design, where the UL precoders are optimized with the given DL precoders and vice versa. When the DL precoders are given, the problem becomes a tractable fractional programing, and the solution can be conducted by Dinkelbach's algorithm [8]. Particularly, the closed-form solution of the UL precoders can be derived by the proposed iterative water-filling (IWF) algorithm. However, owing to the coupled precoders in the objective and the constraints, the IWF cannot be directly applied to conduct the DL precoders.

To overcome this difficulty, we resort to the DL-UL duality to transfer the DL problem to a UL formulation [9]. However, the IWF algorithm cannot be directly used to derive the optimum solution. In this regard, we apply the primal-dual decomposition to decouple the coupled power constraint [11]. Consequently, the optimum DL precoders can be conducted with the IWF algorithm in the primal problem.

Notations: $\mathbf{A}^T$ and $\mathbf{A}^H$ denote the transpose and conjugate transpose of matrix $\mathbf{A}$, respectively. $|\mathbf{A}|$ refers to the determinant of matrix $\mathbf{A}$. $\mathbf{A} \succeq \mathbf{0}$ indicates matrix $\mathbf{A}$ is a positive semidefinite matrix. $[x]^+ = \max(0, x)$. The notation $\mathbf{x} \sim \mathcal{CN}(\mathbf{m}, \mathbf{C})$ indicates that $\mathbf{x}$ is a complex circularly-symmetric Gaussian distributed vector with the mean vector $\mathbf{m}$ and the covariance matrix $\mathbf{C}$.

II. System Model

Consider a FD cellular system shown in Fig. 1. Herein, the BS is equipped with M transmit and receive antennas. The BS simultaneously serves K_D DL users and K_U UL users. The ith DL user is equipped with N_{D_i} antennas and the jth UL user has N_{U_j} antennas.

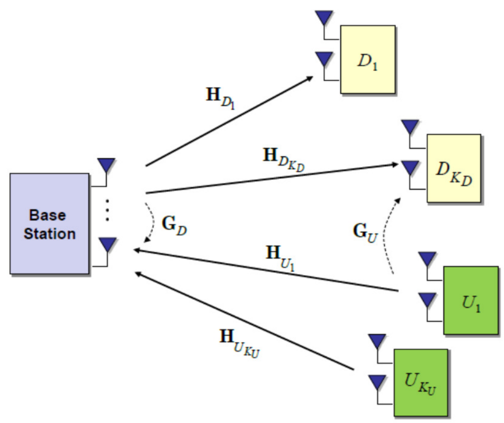

Fig. 1. A FD base station multi-antenna cellular system.

Although several passive and active SI suppression techniques in the analog/digital domain are used, the unavoidable RSI still exists, which degrades the performance of the UL users [12].

For the UL, the received signal vector, denoted as $\mathbf{y}_U$, at the BS is expressed as

$$\mathbf{y}_U = \sum_{j=1}^{K_U} \mathbf{H}_{U_j} \mathbf{x}_{U_j} + \sum_{i=1}^{K_D} \mathbf{G}_D \mathbf{x}_{D_i} + \mathbf{n}_U, \tag{1}$$

where $\mathbf{x}_{U_j} \in \mathbb{C}^{N_{U_j} \times 1}$ and $\mathbf{x}_{D_i} \in \mathbb{C}^{M \times 1}$ are the UL precoded signal of user j and the DL precoded signal of user i, respectively. $\mathbf{H}_{U_j} \in \mathbb{C}^{M \times N_{U_j}}$ is the UL channel matrix associated with the user U_j . $\mathbf{G}_D \in \mathbb{C}^{M \times M}$ is the RSI channel matrix. $\mathbf{n}_U \in \mathbb{C}^{M \times 1} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_M)$ is the AWGN.

In the DL, the received signal of the DL user i is given by

$$\mathbf{y}_{D_i} = \mathbf{H}_{D_i} \mathbf{x}_D + \sum_{j=1}^{K_U} \mathbf{G}_{U_j}^{D_i} \mathbf{x}_{U_j} + \mathbf{n}_{D_i}, \quad i = 1, \cdots, K_D \tag{2}$$

where $\mathbf{H}_{D_i} \in \mathbb{C}^{N_{D_i} \times M}$ is the DL channel matrix regarding to the ith user; $\mathbf{x}_D = \sum_{m=1}^{K_D} \mathbf{x}_{D_m}$ is the overall DL transmit signal at the BS and $\mathbf{x}_{D_m}$ is the DL transmit signal of user m; $\mathbf{G}_{U_j}^{D_i} \in \mathbb{C}^{N_{D_i} \times N_{U_j}}$ is the interference channel from the jth UL user to the ith DL user. $\mathbf{n}_{D_i} \in \mathbb{C}^{N_{D_i} \times 1} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_{N_{D_i}})$ for i = 1,…,K_D .

We assume that the perfect channel state information (CSI) is globally known at all nodes. The BS can adopt an MMSE-SIC detector to sequentially detect the UL data of each user. Herein, we assume the UL users are detected in order. The achievable rate of the jth UL user is [9]

$$R_{U_j} = \log \frac{\left| \mathbf{I} + \mathbf{G}_D \left(\sum_{i=1}^{K_D} \mathbf{Q}_{D_i} \right) \mathbf{G}_D^H + \sum_{l=j}^{K_U} \mathbf{H}_{U_l} \mathbf{Q}_{U_l} \mathbf{H}_{U_l}^H \right|}{\left| \mathbf{I} + \mathbf{G}_D \left(\sum_{i=1}^{K_D} \mathbf{Q}_{D_i} \right) \mathbf{G}_D^H + \sum_{l=j+1}^{K_U} \mathbf{H}_{U_l} \mathbf{Q}_{U_l} \mathbf{H}_{U_l}^H \right|}, \tag{3}$$

where $\mathbf{Q}_{U_l} = E\left[\mathbf{x}_{U_l} \mathbf{x}_{U_l}^H\right] \in \mathbb{C}^{N_{U_l} \times N_{U_l}}$ is the transmit covariance matrix for the lth UL user; $\mathbf{Q}_{D_j} = E\left[\mathbf{x}_{D_j} \mathbf{x}_{D_j}^H\right] \in \mathbb{C}^{M \times M}$ is the transmit covariance matrix for the jth DL user.

To recover the data signal of the DL users, the BS applies the DPC scheme to achieve Shannon capacity region in each cell. Herein, we assume $\mathbf{x}_{D_k}$ is encoded in the order from $\mathbf{x}_{D_{K_D}}$ to $\mathbf{x}_{D_1}$. Accordingly, the achievable rate of the ith DL user is given by [9]

$$R_{D_k} = \log \frac{\left| \mathbf{I} + \sum_{l=1}^{K_U} \mathbf{G}_{U_l}^{D_k} \mathbf{Q}_{U_l} \left(\mathbf{G}_{U_l}^{D_k}\right)^H + \mathbf{H}_{D_k} \left(\sum_{l=k}^{K_D} \mathbf{Q}_{D_l} \right) \mathbf{H}_{D_k}^H \right|}{\left| \mathbf{I} + \sum_{l=1}^{K_U} \mathbf{G}_{U_l}^{D_k} \mathbf{Q}_{U_l} \left(\mathbf{G}_{U_l}^{D_k}\right)^H + \mathbf{H}_{D_k} \left(\sum_{l=k+1}^{K_D} \mathbf{Q}_{D_l} \right) \mathbf{H}_{D_k}^H \right|}. \tag{4}$$

The total power consumption at the BS for the DL can be modeled as the following linear model [13],

$$P_D^{Total} = \eta^{-1} P_D^{TX} + P_D^{\text{cir}}, \tag{5}$$

where $\eta \in [0,1]$ is the power amplifier (PA) efficiency which is related to the PA implementation, $P_D^{TX} = \sum_{i=1}^{K_D} \mathrm{Tr}\left(\mathbf{Q}_{D_i}\right)$ is the transmit power, $P_D^{\text{cir}} = M P_D^{Dyn} + P_D^{\text{sta}}$ is the total circuit power, where P_D^{Dyn} is denoted as the dynamic part, and P_D^{sta} is the static part. Similarly, we can measure the total power consumption at the jth UL user as

$$p_{U_j}^{Total} = \eta^{-1} p_{U_j}^{TX} + p_{U_j}^{\text{cir}}, \tag{6}$$

where $p_{U_j}^{TX} = Tr\left(\mathbf{Q}_{U_j}\right)$ is the transmit power, and $p_{U_j}^{\text{cir}} = N_{U_j} p_{U_j}^{Dyn} + p_{U_j}^{\text{sta}}$ is the circuit power.

III. PROPOSED EE UL/DL PRECODER DESIGN

A. Problem Formulation

The overall EE expression of the considered system is defined as

$$J_E = \frac{\sum_{j=1}^{K_U} R_{U_j} + \sum_{i=1}^{K_D} R_{D_i}}{\frac{\sum_{j=1}^{K_U} \mathrm{Tr}(\mathbf{Q}_{U_j})}{\eta} + \frac{\sum_{i=1}^{K_D} \mathrm{Tr}(\mathbf{Q}_{D_i})}{\eta} + P_{\mathrm{sum}}^{\mathrm{cir}}} \text{ (nats/J)}, \tag{7}$$

where $P_{\mathrm{sum}}^{\mathrm{cir}} = P_D^{\mathrm{cir}} + \sum_{j=1}^{K_U} p_{U_j}^{\mathrm{cir}}$ denotes the overall system circuit power. Therefore, the joint precoder design with an EE criterion is commenced by maximizing J_E with respect to $\left\{\mathbf{Q}_{D_i}\right\}$ and $\left\{\mathbf{Q}_{U_i}\right\}$ provided that the UL/DL power constraints are satisfied and the design is reformulated as

$$\begin{aligned} &\max_{\{\mathbf{Q}_{D_i}\}\{\mathbf{Q}_{U_j}\}} \quad J_E\left(\left\{\mathbf{Q}_{D_i}\right\},\left\{\mathbf{Q}_{U_i}\right\}\right) \\ &\text{s.t.} \quad \mathrm{Tr}(\mathbf{Q}_{U_j}) \le P_{U_j},\ \mathbf{Q}_{U_j} \succeq 0,\ \ j=1,\ldots,K_U \\ &\qquad \sum_{i=1}^{K_D} \mathrm{Tr}(\mathbf{Q}_{D_i}) \le P_D,\ \mathbf{Q}_{D_i} \succeq 0,\ \ i=1,\ldots,K_D. \end{aligned} \tag{8}$$

where P_D is the maximum available transmit power at the BS, P_{U_j} is the power budget for the jth UL user. In light of (8), we can observe that the objective is a fractional form. Furthermore, the nominator part involves the RSI in the UL sum rate and the CCI in the DL sum rate, where the RSI and the CCI make the objective mutually coupled by $\{\mathbf{Q}_{D_i}\}$, $\{\mathbf{Q}_{U_i}\}$. Also, the DL covariance matrix $\mathbf{Q}_{D_i}$, for all i, are mutually coupled. The optimization problem (8) is consequently non-convex and difficult to find the optimum solution.

B. Proposed Design

To find the tractable solution, we resort to Dinkelbach's algorithm to solve the fractional problem [8]. Particularly, Dinkelbach's algorithm can optimally solve a concave-convex fractional problem. Though the considered problem is not concave-convex fractional formulation, we can still apply it to yield a suboptimum solution.

The IWF algorithm can optimally derive the UL transmit covariance matrices when optimizing the UL spectral efficiency in a HD-BS MU MIMO system [10]. Furthermore, with the help of the uplink-downlink (UL-DL) duality, the IWF algorithm can also be applied to conduct the DL transmit covariance matrices. By this concept, we are able to reformulate the DL transmission as the equivalent UL transmission in the FD scenario.

To conduct UL-DL duality, we first rewrite the DL signal for the ith user as

$$\bar{\mathbf{y}}_{D_i} = \bar{\mathbf{H}}_{D_i}\mathbf{x}_{D_i} + \bar{\mathbf{n}}_{D_i}, \tag{9}$$

where

$$\bar{\mathbf{H}}_{D_i} = \left(\mathbf{I}_{N_{D_i}} + \sum_{j=1}^{K_U} \mathbf{G}_{U_j}^{D_i}\mathbf{Q}_{U_j}(\mathbf{G}_{U_j}^{D_i})^H\right)^{-1/2}\mathbf{H}_{D_i}, \tag{10}$$

is the equivalent channel matrix after whitening, and $\bar{\mathbf{n}}_{D_i} \sim \mathcal{CN}\left(\mathbf{0},\mathbf{I}_{N_{D_i}}\right)$. Accordingly, given the UL transmit covariance matrices, the dual uplink received signal of all DL users associated with (9) is written as

$$\mathbf{y}_{DU} = \sum_{i=1}^{K_D} \bar{\mathbf{H}}_{D_i}^H \mathbf{x}_{DU_i} + \mathbf{n}_{DU}, \tag{11}$$

where DU_i denotes the index of the ith DL user, $\mathbf{x}_{DU_i} \in \mathbb{C}^{N_{D_i}\times 1}$ represents the dual UL signal, $\mathbf{n}_{DU} \sim \mathcal{CN}\left(\mathbf{0},\mathbf{I}_M\right)$ is the AWGN. The sum rate of this DU system can be computed as

$$R_{DU} = \log\left|\mathbf{I}_M + \sum_{i=1}^{K_D} \bar{\mathbf{H}}_{D_i}^H \mathbf{Q}_{DU_i}\bar{\mathbf{H}}_{D_i}\right|, \tag{12}$$

where $\mathbf{Q}_{DU_i}$ denotes the transmit covariance matrix of the dual UL user i. By the DL-UL duality, we have the following relation between $\mathbf{Q}_{DU_i}$ and $\mathbf{Q}_{D_i}$

$$\mathbf{Q}_{D_j} = \mathbf{B}_{D_j}^{-1/2}\mathbf{U}_{D_j}\mathbf{V}_{D_j}^H\mathbf{A}_{D_j}^{1/2}\mathbf{Q}_{DU_j}\mathbf{A}_{D_j}^{1/2}\mathbf{V}_{D_j}\mathbf{U}_{D_j}^H\mathbf{B}_{D_j}^{-1/2}, \tag{13}$$

where $\mathbf{A}_{D_j} = \mathbf{I}_{N_{D_j}} + \bar{\mathbf{H}}_{D_j}\left(\sum_{k=1}^{j-1}\mathbf{Q}_{D_k}\right)\bar{\mathbf{H}}_{D_j}^H$; $\mathbf{B}_{D_j} = \mathbf{I}_M + \sum_{k=j+1}^{K_D}\bar{\mathbf{H}}_{D_k}^H\mathbf{Q}_{DU_k}\bar{\mathbf{H}}_{D_k}$; $\mathbf{U}_{D_j}\Lambda_{D_j}\mathbf{V}_{D_j}^H$ is the singular value decomposition (SVD) of the effective channel $\mathbf{B}_{D_j}^{-1/2}\bar{\mathbf{H}}_{D_j}^H\mathbf{A}_{D_j}^{-1/2}$. Considering (13) in (7), we can rewrite the objective as

$$J_E' = \frac{R_U + R_{DU}}{\frac{\sum_{j=1}^{K_U}\mathrm{Tr}(\mathbf{Q}_{U_j})}{\eta} + \frac{\sum_{i=1}^{K_D}\mathrm{Tr}(\mathbf{Q}_{DU_i})}{\eta} + P_{\mathrm{sum}}^{\mathrm{cir}}}. \tag{14}$$

The optimization is then rewritten as

$$\begin{aligned} &\max_{\{\mathbf{Q}_{DU_i}\}\{\mathbf{Q}_{U_j}\}} \quad J_E'\left(\left\{\mathbf{Q}_{DU_i}\right\},\left\{\mathbf{Q}_{U_i}\right\}\right) \\ &\text{s.t.} \quad \mathrm{Tr}(\mathbf{Q}_{U_j}) \le P_{U_j}, \mathbf{Q}_{U_j} \succeq 0,\ j=1,\ldots,K_U \\ &\qquad \sum_{i=1}^{K_D}\mathrm{Tr}(\mathbf{Q}_{DU_i}) \le P_D, \mathbf{Q}_{DU_i} \succeq 0,\ \ i=1,\ldots,K_D. \end{aligned} \tag{15}$$

Now, we can apply Dinkelbach's algorithm to solve the problem of (15). The basic idea of Dinkelbach's algorithm is to transform (15) to an another form by introducing a non-negative scalar λ, then optimizes it and iteratively updates λ. Specifically, by introducing the non-negative scalar λ, we define the following lambda-related problem:

$$F(\lambda) = \max_{\substack{\sum_{i=1}^{K_D}\mathrm{Tr}(\mathbf{Q}_{DU_i})\le P_D \\ \mathrm{Tr}(\mathbf{Q}_{U_j})\le P_{U_j},\ \mathbf{Q}_{U_j}\succeq \mathbf{0},\ \mathbf{Q}_{DU_i}\succeq \mathbf{0}}} R_U\left(\left\{\mathbf{Q}_{U_j}\right\}\right) +$$

$$R_{DU}\left(\left\{\mathbf{Q}_{DU_j}\right\}\right)-\lambda\left(\frac{\sum_{j=1}^{K_U}\mathrm{Tr}(\mathbf{Q}_{U_j})}{\eta}+\frac{\sum_{i=1}^{K_D}\mathrm{Tr}(\mathbf{Q}_{DU_i})}{\eta}+P_{\mathrm{sum}}^{\mathrm{cir}}\right)$$

. (16)

According to Dinkelbach's algorithm [8], we can conduct the solution by the following procedure in Table 1.

TABLE I. DINKELBACH'S ALGORITHM

Dinkelbach's algorithm	
1.	$\varepsilon > 0$; $n = 0$; $\lambda_n = 0$;
	While $F(\lambda_n) > \varepsilon$ do
2.	Find $\left\{\mathbf{Q}_{U_j}^*\right\}$ and $\left\{\mathbf{Q}_{DU_j}^*\right\}$ with (16) and compute $F\left(\lambda_n\right)$.
3.	$\lambda_{n+1}=J_E'\left(\left\{\mathbf{Q}_{DU_i}^*\right\},\left\{\mathbf{Q}_{U_i}^*\right\}\right)$
4.	$n = n + 1$.
	End while

Note that the problem (16) is not convex for $\{\mathbf{Q}_{U_j}^*\}$ and $\{\mathbf{Q}_{DU_j}^*\}$. Hence, we propose the AO approach to locally optimizing (16), where the closed-form expressions of $\{\mathbf{Q}_{U_j}^*\}$ and $\{\mathbf{Q}_{DU_j}^*\}$ can be separately and iteratively obtained with the proposed IWF algorithm which is detailed as follows.

1) Optimize $\{\mathbf{Q}_{U_j}\}$ *with fixed* $\{\mathbf{Q}_{DU_j}\}$

The proposed IWF approach is commenced by optimizing $\mathbf{Q}_{U_i}$ with given fixed $\{\mathbf{Q}_{U_j}\}_{j\neq i}$ and $\{\mathbf{Q}_{DU_j}\}$. Accordingly, $\mathbf{Q}_{U_i}^{(n)}$ at the nth iteration can be solved by the following subproblem

$$\begin{aligned}Y_{U_j}^{(n)}(\lambda)=&\max_{\substack{\mathbf{Q}_{U_j}\succeq\mathbf{0}\\ \mathrm{Tr}(\mathbf{Q}_{U_j})\le P_{U_j}}}\log\left|\mathbf{I}+\tilde{\mathbf{H}}_{U_j}^{(n)}\mathbf{Q}_{U_j}\tilde{\mathbf{H}}_{U_j}^{(n)H}\right|\\&+a_{U_j}^{(n)}-\lambda\left(\frac{\mathrm{Tr}(\mathbf{Q}_{U_j})}{\eta}+b_{U_j}^{(n)}\right)\end{aligned}, \quad (17)$$

where

$$\begin{aligned}\tilde{\mathbf{H}}_{U_j}^{(n)}=&\left(\sigma^2\mathbf{I}_M+\sum_{i=1}^{K_D}\mathbf{G}_{D_i}\mathbf{Q}_{D_i}^{(n-1)}\mathbf{G}_{D_i}^H+\sum_{k=1}^{j-1}\mathbf{H}_{U_k}\mathbf{Q}_{U_k}^{(n)}\mathbf{H}_{U_k}^H\right.\\&\left.+\sum_{k=j+1}^{K_U}\mathbf{H}_{U_k}\mathbf{Q}_{U_k}^{(n-1)}\mathbf{H}_{U_k}^H\right)^{-1/2}\mathbf{H}_{U_j}\end{aligned} \quad (18)$$

$$\begin{aligned}a_{U_j}^{(n)}=&\log\left|\mathbf{I}+\frac{1}{\sigma^2}\sum_{i=1}^{K_D}\mathbf{G}_{D_i}\mathbf{Q}_{D_i}^{(n-1)}\mathbf{G}_{D_i}^H+\frac{1}{\sigma^2}\sum_{k=1}^{j-1}\mathbf{H}_{U_k}\mathbf{Q}_{U_k}^{(n)}\mathbf{H}_{U_k}^H\right.\\&\left.\frac{1}{\sigma^2}\sum_{k=j+1}^{K_U}\mathbf{H}_{U_k}\mathbf{Q}_{U_k}^{(n-1)}\mathbf{H}_{U_k}^H\right|-\log\left|\mathbf{I}+\frac{1}{\sigma^2}\sum_{i=1}^{K_D}\mathbf{G}_D\mathbf{Q}_{D_i}^{(n-1)}\mathbf{G}_D^H\right|\\&+\log\left|\mathbf{I}+\sum_{i=1}^{K_D}\bar{\mathbf{H}}_{D_i}^{(n-1)H}\mathbf{Q}_{D_i}^{(n-1)}\bar{\mathbf{H}}_{D_i}^{(n-1)}\right|\end{aligned} \quad (19)$$

$$b_{U_j}^{(n)}=\frac{\sum_{k=1}^{j-1}\mathrm{Tr}(\mathbf{Q}_{U_j}^{(n)})+\sum_{k=j+1}^{K_U}\mathrm{Tr}(\mathbf{Q}_{U_j}^{(n-1)})}{\eta}+\frac{\sum_{i=1}^{K_D}\mathrm{Tr}(\mathbf{Q}_{DU_i}^{(n-1)})}{\eta}+P_{\mathrm{sum}}^{\mathrm{cir}}. \quad (20)$$

Herein, $\mathbf{Q}_{D_i}^{(n-1)}$ can be computed from $\mathbf{Q}_{DU_i}^{(n-1)}$ by the DL-UL duality of (13) and $\bar{\mathbf{H}}_{D_i}^{(n)}$ is defined as

$$\bar{\mathbf{H}}_{D_i}^{(n)}=\left(\mathbf{I}+\sum_{j=1}^{K_U}\mathbf{G}_{U_j}^{D_i}\mathbf{Q}_{U_j}^{(n)}(\mathbf{G}_{U_j}^{D_i})^H\right)^{-1/2}\mathbf{H}_{D_i}. \quad (21)$$

For each $\mathbf{Q}_{U_j}$, the subproblem (17) can be optimized by the following proposition.

Proposition 1: The subproblem (17) is convex in $\mathbf{Q}_{U_j}$. The corresponding optimum solution is expressed as

$$\mathbf{Q}_{U_j}^{(n)}=\mathbf{U}_{U_j}^{(n)}\mathbf{S}_{U_j}^{(n)}\mathbf{U}_{U_j}^{(n)H},\ \text{for each } j, \quad (22)$$

where $\mathbf{U}_{U_j}^{(n)}\in\mathbb{C}^{M\times M}$ is composed of the eigenvectors of $\tilde{\mathbf{H}}_{U_j}^{(n)H}\tilde{\mathbf{H}}_{U_j}^{(n)}=\mathbf{U}_{U_j}^{(n)}\mathbf{D}_{U_j}^{(n)}\mathbf{U}_{U_j}^{(n)H}$, and $\mathbf{D}_{U_j}^{(n)}$ is a diagonal matrix with entries being eigenvalues $\{d_{U_j,i}^{(n)}\}$. $\mathbf{S}_{U_j}^{(n)}$ is a diagonal matrix with diagonal elements $\{s_{U_j,i}^{(n)}\}$, and

$$s_{U_j,k}^{(n)}=\left[\frac{1}{\frac{\lambda}{\eta}+\mu^*}-\frac{1}{d_{U_j,k}^{(n)}}\right]^+,\quad k=1,...,L, \quad (23)$$

where $\mu^*>0$ and $\left(\lambda\eta^{-1}+\mu^*\right)^{-1}$ is the water-level satisfing $\sum_{i=1}^{L}s_{U_j,i}^{(n)}=P_{U_j}$. Otherwise, $\mu^*=0$ and

$$s_{U_j,k}^{(n)}=\left[\frac{\eta}{\lambda}-\frac{1}{d_{U_j,k}^{(n)}}\right]^+,\quad k=1,...,L. \quad (24)$$

Proof: We skip the derivation due to the limited space.

2) Optimize $\{\mathbf{Q}_{DU_j}\}$ with fixed $\{\mathbf{Q}_{U_j}\}$

When $\{\mathbf{Q}_{U_j}^{(n)}\}$ is treated as a constant, we can utilize the IWF algorithm to conduct dual uplink covariance matrices $\{\mathbf{Q}_{DU_j}\}$. In this regard, we optimize the following subproblem

$$\begin{aligned}\max_{\left\{\mathbf{Q}_{DU_i}\right\},\left\{P_{DU_i}\right\}}&\log\left|\mathbf{I}+\sum_{i=1}^{K_D}\bar{\mathbf{H}}_{D_i}^H\mathbf{Q}_{DU_i}\bar{\mathbf{H}}_{D_i}\right|-\\&\lambda\left(\frac{\sum_{j=1}^{K_U}\mathrm{Tr}(\mathbf{Q}_{U_j}^{(n)})}{\eta}+\frac{\sum_{i=1}^{K_D}\mathrm{Tr}(\mathbf{Q}_{DU_i})}{\eta}+P_{\mathrm{sum}}^{\mathrm{cir}}\right)\end{aligned} \quad (25)$$

$$s.t.\quad Tr\left(\mathbf{Q}_{DU_i}\right)\le P_{DU_i},\quad \sum_{i=1}^{K_D}P_{DU_i}\le P_D,\quad \mathbf{Q}_{DU_i}\succeq\mathbf{0}.$$

Since the introduced P_{DU_i} are mutually coupled, the proposed method in (15) cannot directly be applied to optimize the subproblem (24). Herein, we propose the dual decomposition to relax the coupled constraint [11], and the partial Lagrangian regarding to the subprolbem (24) can be expressed as

$$\mathcal{L}\left(\left\{\mathbf{Q}_{DU_i}\right\},\left\{P_{DU_i}\right\},v\right) = -\log\left|\mathbf{I}+\sum_{i=1}^{K_D}\overline{\mathbf{H}}_{D_i}^{H}\mathbf{Q}_{DU_i}\overline{\mathbf{H}}_{D_i}\right| - \lambda\left(\frac{\sum_{j=1}^{K_U}\mathrm{Tr}(\mathbf{Q}_{U_j}^{(n)})}{\eta}+\frac{\sum_{i=1}^{K_D}\mathrm{Tr}(\mathbf{Q}_{DU_i})}{\eta}+P_{\mathrm{sum}}^{\mathrm{cir}}\right)+v\left(\sum_{i=1}^{K_D}P_{DU_i}-P_D\right). \tag{26}$$

For a fixed *v*, the subproblem now becomes

$$\begin{aligned}\max_{\left\{\mathbf{Q}_{DU_i}\right\},\left\{P_{DU_i}\right\}} \quad & \log\left|\mathbf{I}+\sum_{i=1}^{K_D}\overline{\mathbf{H}}_{D_i}^{H}\mathbf{Q}_{DU_i}\overline{\mathbf{H}}_{D_i}\right| - \lambda\left(\frac{\sum_{j=1}^{K_U}\mathrm{Tr}(\mathbf{Q}_{U_j}^{(n)})}{\eta}+\frac{\sum_{i=1}^{K_D}\mathrm{Tr}(\mathbf{Q}_{DU_i})}{\eta}+P_{\mathrm{sum}}^{\mathrm{cir}}\right)+v\left(\sum_{i=1}^{K_D}P_{DU_i}-P_D\right). \\ s.t. \quad & Tr\left(\mathbf{Q}_{DU_i}\right)\le P_{DU_i}, \quad \mathbf{Q}_{DU_i} \succeq \mathbf{0}\end{aligned} \tag{27}$$

Now, the IWF algorithm can directly be applied to individually optimize $\mathbf{Q}_{DU_i}$ and P_{DU_i} for each user *i*, and the result is given by

$$\begin{aligned}\max_{\mathbf{Q}_{DU_i},P_{DU_i}} \quad & \log\left|\mathbf{I}_M+\widehat{\mathbf{H}}_{DU_i}^{(n)H}\mathbf{Q}_{DU_i}\widehat{\mathbf{H}}_{DU_i}^{(n)}\right| \\ & -\lambda^{(n)}\left(\frac{\mathrm{Tr}(\mathbf{Q}_{DU_i})}{\eta}+b_{DU_i}^{(n)}\right)+v^{(n)}P_{DU_i}+a_{DU_i}^{(n)}, \\ s.t. \quad & Tr\left(\mathbf{Q}_{DU_i}\right)\le P_{DU_i}, \quad \mathbf{Q}_{DU_i} \succeq \mathbf{0}\end{aligned} \tag{28}$$

where

$$\widehat{\mathbf{H}}_{DU_i}^{(n)} = \left(\mathbf{I}_M+\sum_{k=1,k\ne i}^{K_D}\overline{\mathbf{H}}_{D_k}^{(n)H}\mathbf{Q}_{DU_k}^{(n-1)}\overline{\mathbf{H}}_{D_k}^{(n)}\right)^{-1/2}\overline{\mathbf{H}}_{D_i}^{(n)H}, \tag{29}$$

$$\begin{aligned}a_{DU_i}^{(n)} = & \log\left|\mathbf{I}_M+\sum_{k=1,k\ne i}^{K_D}\overline{\mathbf{H}}_{D_k}^{(n)H}\mathbf{Q}_{DU_k}^{(n-1)}\overline{\mathbf{H}}_{D_k}^{(n)}\right| \\ & +\log\left|\mathbf{I}_M+\sigma^{-2}\sum_{i=1}^{K_D}\mathbf{G}_D\mathbf{Q}_{D_i}^{(n)}\mathbf{G}_D^H+\sigma^{-2}\sum_{j=1}^{K_U}\mathbf{H}_{U_j}\mathbf{Q}_{U_j}\mathbf{H}_{U_j}^H\right| \\ & -\log\left|\mathbf{I}_M+\sigma^{-2}\sum_{i=1}^{K_D}\mathbf{G}_D\mathbf{Q}_{D_i}^{(n)}\mathbf{G}_D^H\right|\end{aligned}, \tag{30}$$

$$b_{DU_i}^{(n)} = \frac{\sum_{k=1}^{i-1}\mathrm{Tr}\left(\mathbf{Q}_{DU_k}^{(n)}\right)+\sum_{k=i+1}^{K_D}\mathrm{Tr}\left(\mathbf{Q}_{DU_k}^{(n-1)}\right)}{\eta} + \frac{\sum_{j=1}^{K_U}\mathrm{Tr}(\mathbf{Q}_{U_j}^{(n)})}{\eta}+P_{\mathrm{sum}}^{\mathrm{cir}}+v^{(n)}\left(\sum_{k=1}^{i-1}P_{DU_k}^{(n)}+\sum_{k=i+1}^{K_D}P_{DU_k}^{(n-1)}-P_D\right), \tag{31}$$

Unlike the problem (15), the additional variable $v^{(n)}P_{DU_i}$ is involved in the objective. Following the same derivation in Proposition 1, we have the optimum solution of $\mathbf{Q}_{DU_i}$ and P_{DU_i} by KKT conditions, expressed as

$$\mathbf{Q}_{DU_j}^{(n)} = \mathbf{U}_{DU_j}^{(n)}\mathbf{S}_{DU_j}^{(n)}\mathbf{U}_{DU_j}^{(n)H}, \quad \text{for each } j, \tag{32}$$

where $\mathbf{S}_{DU_j}^{(n)}$ is a diagonal matrix whose diagonal element is

$$s_{DU_j,i}^{(n)} = \left[\left(\frac{\lambda}{\eta}+v^{(n)}\right)^{-1}-\frac{1}{d_{DU_j,i}^{(n)}}\right]^{+}. \tag{33}$$

Herein, $\widehat{\mathbf{H}}_{DU_j}^{(n)H}\widehat{\mathbf{H}}_{DU_j}^{(n)} = \mathbf{U}_{DU_j}^{(n)}\mathbf{D}_{DU_j}^{(n)}\mathbf{U}_{DU_j}^{(n)H}$ is the EVD of $\widehat{\mathbf{H}}_{DU_j}^{(n)H}\widehat{\mathbf{H}}_{DU_j}^{(n)}$, and $\mathbf{D}_{DU_j}^{(n)}$ is a diagonal matrix with the eigenvalues $\{d_{DU_j,i}^{(n)}\}$ on its diagonal. $\left(\lambda\eta^{-1}+\mu\right)^{-1}$ is treated as the water-level.

Now, we have to optimize the dual problem

$$\min_{v} g\left(v\right) \qquad s.t. \quad v \ge 0, \tag{34}$$

where $g\left(v\right)$ is the dual function regarding to (28). By the subgradient method, the dual variable is updated by

$$v(n+1) = \left[v(n)-\alpha\left(P_D-\sum_{k=1}^{K_D}P_{DU_k}^{(n)}\right)\right]^{+}. \tag{35}$$

The proposed method is summarized in Table II.

TABLE II. PROPOSED IWF ALGORITHM

1.	Initialize $\left\{\mathbf{Q}_{U_i}^{(0)}\right\}$ and $\left\{\mathbf{Q}_{DU_i}^{(0)}\right\}$, λ, $v(0)$.
2.	Transfer $\left\{\mathbf{Q}_{DU_i}^{(0)}\right\}$ to $\left\{\mathbf{Q}_{D_i}^{(0)}\right\}$.
	Repeat
3.	For *i* = 1:*Ku*
4.	Use (22) and (23) to determine $\left\{\mathbf{Q}_{U_i}^{(n)}\right\}$
	End
	For *i* = 1: K_D.
5.	Use (32) and (33) to determine $\left\{\mathbf{Q}_{DU_i}^{(n)}\right\}$
6.	Transfer $\left\{\mathbf{Q}_{DU_i}^{(n)}\right\}$ to $\left\{\mathbf{Q}_{D_i}^{(n)}\right\}$
	End
7.	$\lambda_{n+1} = J_E'\left(\left\{\mathbf{Q}_{DU_i}^{*}\right\},\left\{\mathbf{Q}_{U_i}^{*}\right\}\right)$
8.	Update dual master problem (35).
9.	*n* = *n* + 1.
10.	***Until*** λ_{n+1} is converged.

IV. NUMERICAL RESULTS

The performance of the proposed energy efficiency design is compared with that of [2] in a FD MU-MIMO system. Unless otherwise mentioned, the each element of uplink channel $\mathbf{H}_{U_i}$ is assumed to be independent and identically distributed (i.i.d.) circularly complex Gaussian random variable with zero mean and unit variance. The same definition is applied for $\mathbf{H}_{D_i}$. For all i's and j's, each entry of $\mathbf{G}_S$ and $\mathbf{G}_{U_j}^{D_i}$ follows $\mathcal{CN}\left(0,\sigma_{SI}^2\right)$ and $\mathcal{CN}\left(0,\sigma_{CCI}^2\right)$, respectively. For simplicity, we consider two users for each link direction (UL and DL) and each user is equipped with two antennas, i.e., $K_D = K_U = 2$, and $N_{D_1} = N_{D_1} = N_{U_1} = N_{U_2} = 2$. The BS has 8 antennas (M = 8). Moreover, the initial values of $\left\{\mathbf{Q}_{D_i}^{(0)}\right\}$ are selected as identity matrices. The noise power at the BS and the DL channel is set to be unity. We also let P_{U_i} =0.7 P_D. The dynamic circuit power is set to P_D^{Dyn} = 38 dBm and P_D^{Sta} = 27 dBm. The error tolerance for conducting the iteration process is set to 10^{-6}.

In Fig. 2, we investigate the EE and SE designs in [2] and the proposed design for different values of P_D^{Tx}. In this simulation, we let P_{U_j} = 32 dBm, σ_{CCI}^2 = 0, and σ_{SI}^2 = -20 dBm. As shown in this figure, the performance of the SE design in [2] is close to that of the proposed EE design, while the gap becomes significant as P_D increases. The reason follows that the higher SI power is, the lower SINR can be achieved for the UL users. In this case, the UL users prefer to save power to reach a higher EE performance. The result corresponds to the deviation of Eq. (24). The proposed method outperforms the EE design in [2] due to the fact that [2] essentially optimizes a lower bound of EE, while the proposed method uses the exact EE value for optimization, achieving a better result.

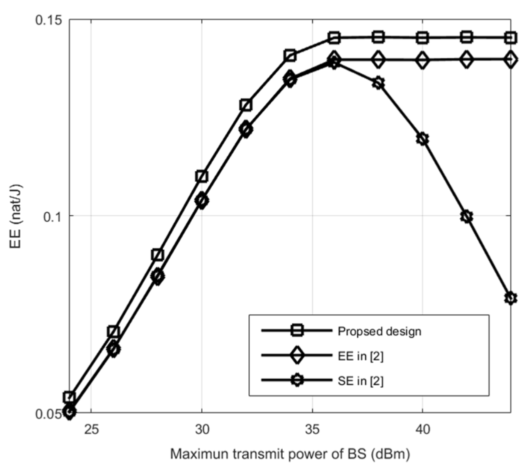

Fig. 2 EE versus P_D^{Tx}.

V. CONCLUSIONS

This paper studies the UL and DL precoder design to suppress the influence of the RSI and CCI for a FD multi-antenna cellular system. We first propose the AO approach which solves the UL precoders with given DL precoders, and vice versa. Then we apply Dinkelbach's algorithm to sequentially optimize the UL precoders, and the closed-from solutions are obtained by the IWF algorithm. For the DL precoders, we apply the UL-DL duality and the primal and dual decomposition so that the coupled objective and constraints are decoupled. The closed-form solution of the DL precoders is finally yielded. Simulation results show that the proposed design outperforms the exiting methods.

REFERENCES

[1] Z. Zhang, X. Chai, K. Long, A. Vasilakos, L. Hanzo, "Full duplex technologies for 5G networks: self-interference cancellation, protocal design, and relay selection," *IEEE Commun., Mag.*, vol. 53, no. 5, pp. 128-137, May 2015.

[2] D. Nguyen, L.-N. Tran, P. Pirinen, and M. Latva-aho, "Precoding for full duplex multiuser MIMO systems: Spectral and energy efficiency maximization," *IEEE Trans. Signal Process.*, vol. 61, no. 16, pp. 4038–4050, Aug. 2013.

[3] R. Sultan, L. Song, K. G. Seddik, and Zhu Han, "Full-duplex meets multiuser MIMO: Comarisons and analysis," *IEEE Trans. Vehicular Tech.*, vol. 66, no. 1, pp. 455-467, Jan. 2017.

[4] S. Buzzi, C.-L. I, T. E. Klein, H. V. Poor, C. Yang, and A. Zappone, "A survey of energy-efficient techniques for 5G networks and challenges ahead," *IEEE Journal on Selected Areas in Communications*, vol. 34, no. 4, pp. 697-709, April 2016.

[5] L. Venturino, A. Zappone, C. Risi, and S. Buzzi, "Energy-efficient scheduling and power allocation in downlink OFDMA networks with base station coordination," *IEEE Trans. Wireless Commun.*, vol. 14, no. 1, pp. 1–14, Jan. 2015.

[6] F. Gabry, A. Zappone, R. Thobaben, E. A. Jorswieck, and M. Skoglund, "Energy efficiency analysis of cooperative jamming in cognitive radio networks with secrecy constraints," *IEEE Wireless Commun. Lett.*, vol. 4, no. 4, pp. 437–440, Aug. 2015.

[7] X. Gui, K.-J. Lee, Z. Zhu, J. Lung, and I. Lee, "Energy efficiency optimization with nonlinear precoding in multi-cell MISO broadcast channels," in *Proc. IEEE ICC 2015*.

[8] W. Dinkelbach, "On nonlinear fractional programming," *Management Science*, vol. 13, no. 7, pp. 492-498, March 1967.

[9] L. Zhang, R. Zhang, Y.-C. Liang, Y. Xin, and H.-V. Poor, "On Gaussian MIMO BC-MAC duality with multiple transmit covariance constraints," *IEEE Trans. Inf. Theory,* vol. 58, no. 4, Apr. 2012.

[10] G. Scutari, D. P. Palomar, and S. Barbarssa, "The MIMO iterative waterfilling algorithm," *IEEE Trans. Signal Process.*, vol. 57, no. 5, pp. 1917-1953, May 2009.

[11] Daniel P. Palomar, "Convex Primal Decomposition for Multicarrier Linear MIMO Transceivers," *IEEE Trans. Signal Processing*, vol. 53, no. 12, pp. 4661-4674, Dec. 2005.

[12] H. Li, J. V. Kerrebrouck, O. Caytan, H. Rogier, J. Bauwelinck, P. Demeester, and G. Torfs, "Self-interference cancellation enabling high-throughput short-reach wireless full-duplex communication," *IEEE Trans. Wireless Commun.*, 2018, In press.

[13] O. Arnold, F. Richter, G. Fettweis, and O. Blume, "Power consumption modeling of different base station types in heterogeneous cellular networks," in *Proc. 19th Future Netw. MobileSummit (ICT Summit'10)*, Florence, Italy, Jun. 2010, pp. 1–8.