Performance Analysis of the Uplink of a Two User NOMA Network under QoS Delay Constraints

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***Abstract*—In fifth generation and beyond (B5G) technologies, delay constrains emerge as a topic of particular interest for ultra reliable low latency communications (e.g., enhanced reality, haptic communications). In this short paper, we study the performance of a two-user uplink NOMA network under quality of service (QoS) delay constraints, captured through each user's effective capacity (EC). We propose novel closed-form expressions for the EC of the NOMA users, validated through Monte Carlo simulations. Interestingly, our study shows that in the high signal to noise ratio (SNR) region, the "strong" NOMA user is more penalized in terms of its EC under the same delay constraint as the "weak" user.**

***Index Terms*—NOMA; QoS; low latency; effective capacity; 5G.**

I. Introduction

Non orthogonal multiple access (NOMA) schemes have attracted a lot of attention recently, allowing multiple users to be served simultaneously with enhanced spectral efficiency; it is known that the boundary of achievable rate pairs (in the case of two users) using NOMA is outside the capacity region achievable with orthogonal multiple access techniques [1]. Superior achievable rates are attainable though the use of superposition coding at the transmitter and of successive interference cancellation (SIC) at the receiver [2], [3]. The SIC receiver decodes multi-user signals with descending received signal power and subtracts the decoded signal(s) from the received multi-user signal, so as to improve the signal-to-interference ratio. The process is repeated until the signal of interest is decoded. In uplink NOMA networks, the strongest user's signal is decoded first (as opposed to downlink NOMA networks in which the inverse order is applied).

Besides, in a number of emerging applications, quality of service (QoS) delay constraints become increasingly important, e.g., ultra reliable low latency (URLLC) systems. Furthermore, in future wireless networks, users are expected to necessitate flexible delay guarantees for achieving different service requirements. Henceforth, in order to satisfy diverse delay requirements, a simple and flexible delay QoS model is imperative to be applied and investigated. In this respect, the effective capacity (EC) theory can be employed [4], [5] [6], with EC denoting the maximum constant arrival rate which can be served by a given service process, while guaranteeing the required statistical delay provisioning.

In this work, we provide a performance evaluation of the uplink of a two-user NOMA network under delay constraints, captured through the users' effective capacities (ECs). We note that the EC is a QoS aware data link layer metric [5], that captures the achievable rate under a delay violation probability threshold. To clarify further, let R_m represent the achievable rate of the m-th user, θ_m its statistical delay QoS exponent, and assume that the service process satisfies Gärtner-Ellis theorem [5]. The notion of the delay exponent θ_m captures how strict the delay constraint is [5]. A slower decay rate can be represented by a smaller θ_m, which indicates that the system is more delay tolerant, while a larger θ_m corresponds to systems with more stringent QoS requirement.

Applying the EC theory in a NOMA setting with multiple users, the m-th user's EC over a block-fading channel, is defined as:

$$E_c^m = -\frac{1}{\theta_m T_f B} \ln\left(\mathbb{E}\left[e^{-\theta_m T_f B R_m}\right]\right) \quad \text{(in b/s/Hz)}, \qquad (1)$$

where T_f is the symbol period, B the block bandwidth and $\mathbb{E}[\cdot]$ denotes expectation over the channel gains.

This short paper is organized as follows: In Section II we investigate the EC of a two user uplink NOMA system under the delay QoS constraints. Simulation results are given in Section III, followed by conclusions in Section IV.

II. Effective Capacity of Two User NOMA Uplink

Assume a two user NOMA uplink with users U_1 and U_2, in a Rayleigh block fading channel with respective gains $|h_1|^2 < |h_2|^2$, transmitting symbols S_1, S_2 respectively, with power $E[|S_1|^2] = P_1$ and $E[|S_2|^2] = P_2$. The superimposed signal received at the base station (BS) can be expressed as:

$$Z = \sqrt{P_1} h_1 S_1 + \sqrt{P_2} h_2 S_2 + w, \qquad (2)$$

where w denotes a zero mean circularly symmetric Gaussian random variable with variance σ^2. The BS will first decode the symbol of the strongest user U_2 treating the transmission of U_1 as interference. After decoding S_2, it suppresses it from Z and will decode S_1. Following the SIC principle, and denoting by $\rho = \frac{1}{\sigma^2}$ the transmit SNR, the achievable rates, in b/s/Hz, for both users U_1 and U_2 are expressed as [7]:

$$R_1 = \log_2[1 + \rho P_1 |h_1|^2], \qquad (3)$$

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$$R_2 = \log_2\left[1 + \frac{\rho P_2 |h_2|^2}{1 + \rho P_1 |h_1|^2}\right]. \tag{4}$$

Replacing $R_m, m = 1, 2$ of both users given in (3)-(4) into (1), we obtain the following expressions for the ECs of the two NOMA users:

$$E_c^1 = \frac{1}{\beta_1} \log_2 \mathbb{E}\Big[(1 + \rho P_1 |h_1|^2)^{\beta_1}\Big], \tag{5}$$

$$E_c^2 = \frac{1}{\beta_2} \log_2 \mathbb{E}\left[\left(1 + \frac{\rho P_2 |h_2|^2}{1 + \rho P_1 |h_1|^2}\right)^{\beta_2}\right], \tag{6}$$

where $\beta_i = -\frac{\theta_i T_f B}{\ln 2}$ is the normalized (negative) QoS exponent for $i = \{1, 2\}$.

The account for the ordering of the channel gains we make use of the theory of order statistics in the following analysis [8]. Assume a network of two users' with $|h_1|^2 < |h_2|^2$, and $\gamma_i = \rho |h_i|^2$, $i = \{1, 2\}$ to denote independent and identically distributed (i.i.d.) random variables with corresponding probability density function (pdf) $f(\gamma_i)$. Ranking these random variables using order statistics we obtain:

$$f_{\gamma_{1:2}}(\gamma_1) = \psi_1 f(\gamma_1) F(\gamma_1), \tag{7}$$

$$f_{\gamma_{2:2}}(\gamma_2) = \psi_2 f(\gamma_2)(1 - F(\gamma_2)), \tag{8}$$

where $f_{\gamma_{i:M}}$ denotes the pdf of the i-th ordered random variable in a population of M (in the two-user case $M = 2$), $\psi_i = \frac{1}{B(i, M-i+1)}$, and, $B(a, b)$ is the beta function $B(a, b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$. Finally, the cumulative density function (cdf) of the the random variable γ_i is denoted by $F(\gamma_i)$, for $i = \{1, 2\}$.

Assuming a Rayleigh wireless environment, the NOMA gains of the channel, denoted by γ_i, are exponentially distributed with respective pdf and cdf

$$f(\gamma_i) = \frac{1}{\rho} e^{\frac{-\gamma_i}{\rho}}, \;\; F(\gamma_i) = 1 - e^{\frac{-\gamma_i}{\rho}}, \tag{9}$$

for $i = 1, 2$ [9]. As a result, the ordered channel gains have respective pdfs:

$$f_{\gamma_{1:2}}(\gamma_1) = \frac{1}{B(1,2)} f(\gamma_1)(1 - F(\gamma_1)) = \frac{2}{\rho} e^{\frac{-2\gamma_1}{\rho}}, \tag{10}$$

$$f_{\gamma_{2:2}}(\gamma_2) = \frac{1}{B(2,1)} f(\gamma_2) F(\gamma_2) = \frac{2}{\rho} e^{\frac{-\gamma_2}{\rho}} (1 - e^{\frac{-\gamma_2}{\rho}}). \tag{11}$$

In this context, the EC of U$_i$, denoted by E_c^i is expressed as:

$$E_c^1 = \frac{1}{\beta_1} \log_2 \left(\int_0^\infty (1 + P_1 \gamma_1)^{\beta_1} f_{\gamma_{1:2}}(\gamma_1) d\gamma_1 \right), \tag{12}$$

$$E_c^2 = \frac{1}{\beta_2} \log_2 \left(\int_0^\infty \int_{\gamma_1}^\infty \left(1 + \frac{P_2 \gamma_2}{1 + P_1 \gamma_1}\right)^{\beta_2} f_{\gamma_{2:2}}(\gamma_2) \, f_{\gamma_{1:2}}(\gamma_1) \, d\gamma_2 d\gamma_1 \right). \tag{13}$$

By replacing (10) into (12) we obtain:

$$\begin{aligned} E_c^1 &= \frac{1}{\beta_1} \log_2 \left(\frac{2}{\rho} \int_0^\infty (1 + P_1 \gamma_1)^{\beta_1} e^{\frac{-2\gamma_1}{\rho}}) d\gamma_1 \right) \\ &= \frac{1}{\beta_1} \log_2 \left(\left(\frac{P_1 \rho}{2}\right)^{\beta_1} e^{\frac{2}{P_1 \rho}} \Gamma\Big(1 + \beta_1, \frac{2}{P_1 \rho}\Big) \right), \end{aligned} \tag{14}$$

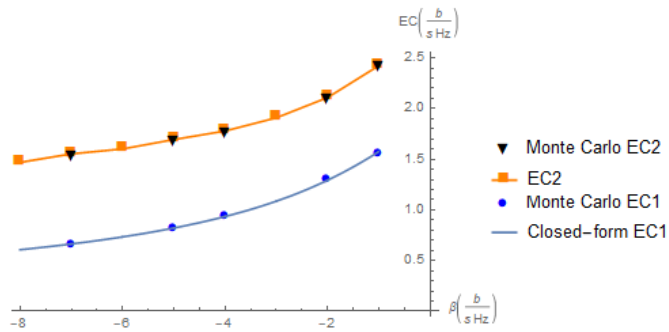

Fig. 1. Effective capacities E_c^1 and E_c^2 in a two-user NOMA uplink network as functions of the normalized QoS delay exponents β_1 and β_2, respectively.

given that $\rho > 0$.

Furthermore, replacing (11) into (13) we have that,

$$\begin{aligned} E_c^2 &= \frac{1}{\beta_2} \log_2 \Bigg(\frac{4}{\rho^2} \int_0^\infty \int_{\gamma_1}^\infty \left(1 + \frac{P_2 \gamma_2}{1 + P_1 \gamma_1}\right)^{\beta_2} \\ &\quad e^{\frac{-\gamma_2}{\rho}} (1 - e^{\frac{-\gamma_2}{\rho}}) e^{\frac{-2\gamma_1}{\rho}} d\gamma_2 d\gamma_1 \Bigg) \\ &= \frac{1}{\beta_2} \log_2 \Bigg(\frac{4}{\rho^{-\beta_2 + 1}} \Big(\int_0^\infty e^{\frac{1 + P_1\gamma_1 - 2P_2\gamma_1}{P_2 \rho}} \left(\frac{P_2}{1 + P_1 \gamma_1}\right)^{\beta_2} \\ &\quad \Gamma\Big(\beta_2 + 1, \frac{1 + (P_1 + P_2)\gamma_1}{P_2 \rho}\Big) d\gamma_1 + 2^{\beta_2} \int_0^\infty e^{\frac{2 + 2P_1\gamma_1 - 2P_2\gamma_1}{P_2\rho}} \\ &\quad \left(\frac{P_2}{1 + P_1 \gamma_1}\right)^{\beta_2} \Gamma\Big(\beta_2 + 1, \frac{2 + 2(P_1 + P_2)\gamma_1}{P_2 \rho}\Big) d\gamma_1 \Big) \Bigg), \end{aligned} \tag{15}$$

where $\Gamma(s, x)$ is the incomplete Gamma function i.e., $\Gamma(s, x) = \int_x^\infty t^{s-1} e^{-t} dt$.

III. Numerical Results

In the following, we validate using Monte Carlo simulations [10] the accuracy of the mathematical expressions for the effective capacity of both users E_c^1 and E_c^2. We plot the ECs over the following parameters (β_i,ρ,P_i), $i = \{1, 2\}$.

A. Effective Capacities vs the Normalized Delay Exponents

Fig. 1 shows the curves of the E_c^1 and E_c^2 in a scenario where the power of the user with stronger channel condition U$_2$ is P_2 = 1 Watt and that of the weaker user U$_1$ is P_1 = 0.8 Watt. The plots confirm the validity of the proposed expressions in (14) and (15). Note than in the case of E_c^2 only integer values of β are examined in order to reduce the computation time. The numerical results in Fig.1 show that as the normalized delay exponent β_i, $i = \{1, 2\}$ decreases (i.e., the application has stricter delay-constraints), the value of the EC for both users decreases, i.e., the achievable data link layer rates while guaranteeing a certain level of delay QoS constraint decrease. This result is in agreement with the respective trend in OMA networks [11].

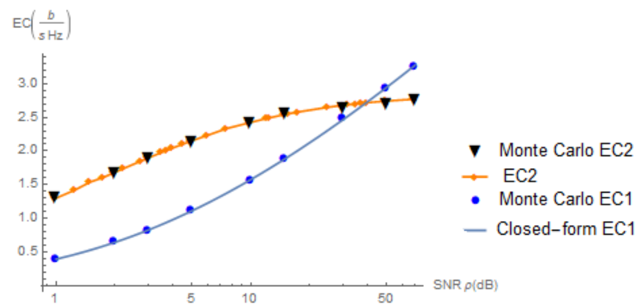

Fig. 2. Effective capacities E_c^1 and E_c^2 in a two-user NOMA uplink network as functions of the transmit SNR ρ.

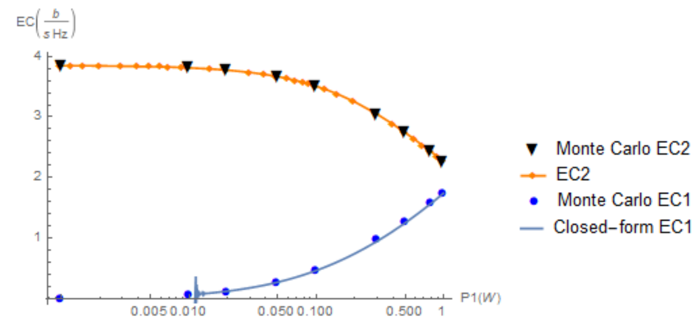

Fig. 3. Effective capacities E_c^1 and E_c^2 in a two-user NOMA uplink network as functions of the transmit power levels P_1.

B. Effective Capacities vs the Transmit SNR

Here we consider the scenario in which both users have the same QoS delay requirements (they are both users of the same "class"), i.e., the normalized delay exponents are the same, e.g., $\beta_1 = \beta_2 = -1$. Besides, the two users U_1 and U_2 transmit their signals with power P_1 = 0.8 Watt and P_2 = 1 Watt, respectively, as in the previous scenario.

As shown in Fig. 2, the Monte Carlo numerical results agree with the evaluation of E_c^1 and E_c^2 obtained from the analysis presented in Section II, further validating the proposed expressions. Quite interestingly, at very high transmit SNR values ($\rho \geq 45$ dB), we see that E_c^1 surpasses E_c^2.

Unlike the results presented in Subsection III.A, this effect is counter-intuitive. Our analysis demonstrates that for users of the same "class" (i.e., with the same delay exponents), the strong NOMA user can be more penalized in terms of its EC than the weak user. The reason for this is that the signal emitted from the weak user acts as interference for the strong NOMA user in an uplink network. As a result, the weak user's EC is not constrained by interference, while the strong user's is, and this effect is more evident in the high SNR region.

C. Effective Capacities vs Power Levels

Finally, in this third set of results, we evaluated E_c^1 and E_c^2 for various values of $P_1 \in [10^{-2}, 1]$ Watts while we fixed $P_2 =$ 1 Watt and $\beta_1 = \beta_2 = -1$; The curves in Fig. 3 showcase the agreement between the Monte Carlo simulation of E_c^1 and E_c^2 and the expressions in (14) and (15). Furthermore, when the transmitted powers are equal ($P_1 = P_2 =$1W), the EC of the strong user is only slightly higher than the EC of the weak one. This trend is however not universal and depends on the delay exponents as well, which here were taken equal (both users belong to the same QoS class).

IV. Conclusions and Future Work

The concept of the effective capacity enables us to study the achievable data link layer rates when QoS delay guarantees are in place in the form of delay exponents. In this paper, we have investigated the effective capacity of the uplink of a two-user NOMA network, assuming a block Rayleigh fading channel. We have derived novel closed-form expressions for the effective capacity of the weak user and proposed novel expressions for the strong user. We validated the proposed mathematical formulas with Monte Carlo simulations and were able to study the impact of the delay exponent, the transmit SNR and the transmit power in the ECs of the two users.

Intuitively, the ECs of both users decrease as the delay constraints become stringer. On the other hand, in the high transmit SNR regime, the EC of the weak user can surpass the EC of the strong user, as the latter is interference limited. Finally, the EC of the weak user increases with increasing its transmit power, while the EC of the strong user decreases when the weak user increases its transmit power. In the future we will generalize this analysis to NOMA uplink networks with multiple superimposed users.

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