Hidden Markov Model based Analysis on the MCS Index of LTE using PDCCH Measurement Data

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***Abstract*—Recent maturity of Long Term Evolution (LTE) system has raised interest in evaluating how efficiently LTE cells are deployed. Noting that the Modulation and Coding Scheme (MCS) index determines the quality of transmission, we analyze the temporal variation of the index of a user equipment. Adopting a widely used Markov assumption on the variation, we exploit a hidden Markov model to estimate transition probability matrices describing the stochastic behavior of the MCS index. In the estimation, we use the real-world measurement data via a recently introduced reliable platform which can decode the control channels of LTE downlink signal. Numerical examples validate our analysis by investigating stationary probabilities.**

***Index Terms*—LTE, MCS index, channel modeling, measurement, hidden Markov model**

I. Introduction

As Long Term Evolution (LTE) has matured enough to serve most terrestrial mobile users, it is getting attention to evaluate how efficiently LTE cells are deployed. Since the wireless channel condition between the base station and a User Equipment (UE) affects the efficiency of transmissions, analyzing the variation of channel quality can be a way of carrying out the evaluation. In LTE, the downlink channel condition of a UE is reflected in the Modulation and Coding Scheme (MCS) index associated with the information bits called Transport Block (TB) transmitted to it. In the open literature, there have been works on Markov modeling for wireless channel variations [1]–[3]. Thus, we focus on analyzing the variation of MCS index of a UE through Markov modeling as in [4].

In this work, we utilize real-world measurement data via a recently introduced platform 'Analyzer of LTE Traffic and Radio Information' (AlteTRi) [5]. AlteTRi is capable of decoding the Physical Downlink Control CHannels (PDCCHs) of downlink signal for every subframe and provides scheduling information for each UE. Since the MCS index is exposed only when a TB is allocated to a UE, we adopt a Hidden Markov Model (HMM). We use the scheduling model given in [4] where an allocation is based on the MCS index and delay with simplified Modified Largest Weighted Delay First (M-LWDF) scheme. Using the Baum-Welch algorithm [6], we estimate the Transition Probability Matrix (TPM) which most likely describes the temporal variation of the MCS index. Some numerical results using the measurement data are also provided.

The rest of the paper is organized as follows. Section II provides detailed analysis based on an HMM. Section III presents numerical examples. Finally, Section IV concludes this work.

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II. Analysis based on a Hidden Markov Model

A. Measurement Data Description

The measurement is performed via the platform AlteTRi [5], which decodes PDCCHs for every subframe and provides information contained in Downlink Control Information (DCI) for each UE. DCI for a UE includes the MCS index and Resource Block (RB) configuration associated with the TB transmitted to the UE. The TB Size (TBS) is also determined by these information using tables given in [7]. Since a UE requiring a service is endowed with a Radio Network Temporary Identifier (RNTI), we can get a sequence of scheduling information for a UE by tracking DCIs for a certain RNTI.

Here, we define some notations for the data we utilize in the analysis. The following notations apply to each RNTI, and let an RNTI be given. The time index of a subframe unit is denoted as t. Consider a data sequence of length $T+1$ from $t=0$ to T, where $t=0$ is an instant when a TB is scheduled for the RNTI. Let S_t be an indicator having a value of 1 if a TB is scheduled for the RNTI at time t, and 0 otherwise. Then we have $S_0 = 1$. Let $\mathcal{T}$ be the set of time instants when the RNTI receives a TB after $t=0$, i.e., $\mathcal{T} = \{t \geq 1 : S_t = 1\}$. Then, the data gives the MCS index M_t and the number of allocated RBs B_t for $t \in \mathcal{T}$. Also, using the tables in [7], we can deduce the TBS R_t from M_t and B_t for $t \in \mathcal{T}$. We let the set of all MCS indices be $\mathcal{M}$.

B. Modeling on Scheduling Discipline

Our modeling is based on the scheduling concept given in [4] where the essentials of the M-LWDF scheme are simply reflected. As in [4], we define the delay W_t at t as the number of subframes elapsed from the last transmission including that subframe. Then, the metric η_t for scheduling at t is defined by $\eta_t = W_t R_t / B_t$, and a TB of size R_t is scheduled if η_t is large enough. At $t \in \mathcal{T}$, since our data sequence provides exact information of scheduling, we have exact values for η_t.

Now we extend these notions to the time instants when nothing is scheduled to the RNTI. For convenience, we use the notation '$t:t'$' for $t \le t'$ to indicate a set $\{t, t+1, \dots, t'\}$. Also, for any variables indexed with t, a set of time indices at the subscript means a sequence for time instants belonging to the set, for e.g., $S_{1:T}$ means $\{S_1, S_2, \dots, S_T\}$. We have the realizations $s_{0:T}$ of $S_{0:T}$ exactly, and since $W_{1:T}$ is uniquely determined by $S_{0:T}$ with the relation

$$W_{t+1} = \begin{cases} W_t + 1 & \text{if } S_t = 0, \\ 1 & \text{if } S_t = 1, \end{cases}$$

we also have the realizations $w_{1:T}$ for $W_{1:T}$. However, for $R_{1:T}$ and $M_{1:T}$, we only have the realizations at $\mathcal{T}$ as $r_{\mathcal{T}}$ and $m_{\mathcal{T}}$, respectively. To deal with the hidden values, we extend the definitions for R_t and M_t given above. First, we let R_t be the required bits at t. Then, for any two consecutive times t_1 and t_2 in $\mathcal{T}$, we can calculate $r_t = \lceil r_{t_2}(t - t_1)/w_{t_2} \rceil$ for $t_1 < t < t_2$ using r_{t_2} and w_{t_2}. For M_t, we define it by the MCS index that a TB of size r_t will be granted if it is scheduled at t as in [4]. Let $b(m, r)$ be the number of RBs required to transmit r bits with the MCS index m based on the tables given in [7]. Then, if an MCS index m_t is given at t, we can calculate the required number of RBs as $b_t = b(m_t, r_t)$ to transmit r_t bits. Here, for $t_1 < t \le t_2$, we assume that the TB of size r_t is not scheduled because the metric $\eta_t = w_t r_t / b_t$ is smaller than the metric η_{t_2} at t_2, which is the next scheduled time after t. Thus, the possible states of the MCS indices at t can be determined as $\{m \in \mathcal{M} : w_t r_t / b(m, r_t) < \eta_{t_2}\}$. Consequently, if the measurement data ($s_{0:T}, w_{1:T}, r_{\mathcal{T}}$, and $m_{\mathcal{T}}$) are given, the possible sequences of MCS indices $\mathcal{M}_s$ can be configured as

$$\begin{aligned} \mathcal{M}_s = \{&\mu_{1:T} \in \mathcal{M}^T : \mu_{\mathcal{T}} = m_{\mathcal{T}}, \text{and, for } t \notin \mathcal{T}, \mu_t \in \mathcal{M} \\ &\text{with } \eta_t < \eta_{t'} \text{ where } t' = \inf\{u > t : s_u = 1\}\} \end{aligned}$$

For later use, we also define $\mathcal{M}_s^{(t)} = \{m \in \mathcal{M} : m \text{ is the } t\text{th element of } m_{1:T} \text{ for some } m_{1:T} \in \mathcal{M}_s\}$.

C. Hidden Markov Model

To apply an HMM to the scheduling procedure, we assume that the MCS index M_t follows a time homogeneous Markov chain whose state space is $\mathcal{M}$, which is a widely used assumption in the open literature [3], [4]. Then the behavior of M_t can be described by the TPM $\mathbf{P} = \{P_{ij}\}_{i,j \in \mathcal{M}}$ where $P_{ij} = P\{M_{t+1} = j | M_t = i\}$ for all $t \ge 0$. When a TPM $\mathbf{P}$ is given, we can calculated the probability of observing a sequence $s_{1:T}$ ($s_0 = 1$ as described above) as the scheduling result. As described above and illustrated in Fig. 1, observed data $s_{1:T}$, $r_{\mathcal{T}}$, and $m_{\mathcal{T}}$ determine the MCS indices $\mathcal{M}_s^{(t)}$ that a TB can be granted for $1 \le t \le T$. For the calculation, we assume that the required bits $r_{1:T}$ are given and the scheduling result at t is determined by m_t and w_t. Then, the probability of observing $s_{1:T}$ can be computed as follows. For brevity, let a lowercase letter indicate a realization for the corresponding uppercase variable.

$$P\{s_{1:T} | \mathbf{P}\} = \sum_{m_{1:T} \in \mathcal{M}_s} \prod_{t=0}^{T-1} P_{m_t m_{t+1}}. \tag{1}$$

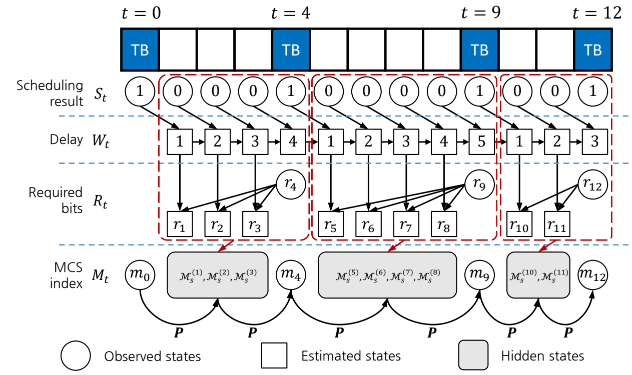


Fig. 1. An example of the scheduling procedure of our model.

Here, we try to find a TPM that maximizes the probability in (1). To this end, we apply the Baum-Welch algorithm given in [6] to our model. The following probabilities for $1 \le t \le T$ facilitate the application of the algorithm.

- *Forward probability*: $\alpha_t(m) = P\{s_{1:t}, M_t = m | \mathbf{P}\}$
- *Backward probability*: $\beta_t(m) = P\{s_{1:t} | \mathbf{P}, M_t = m, w_t\}$

As given in [6], these probabilities can be computed by induction. For forward probabilities, with $\alpha_0(m_0) = 1$, $\alpha_t(m)$'s are computed by $\alpha_t(m) = \sum_{j \in \mathcal{M}_s^{(t-1)}} \alpha_{t-1}(j) P_{jm}$ for $1 \le t \le T$ and $m \in \mathcal{M}_s^{(t)}$. Once the forward probabilities are obtained, the observation probability in (1) can be calculated as $P\{s_{1:T} | \mathbf{P}\} = \sum_{m \in \mathcal{M}_s^{(T)}} \alpha_T(m)$. For backward probabilities, with $\beta_T(m_T) = 1$ for $m_T \in \mathcal{M}_s^{(T)}$, $\beta_t(m)$'s are computed by $\beta_t(m) = \sum_{j \in \mathcal{M}_s^{(t+1)}} P_{mj} \beta_{t+1}(j)$ for $1 \le t \le T-1$ and $m \in \mathcal{M}_s^{(t)}$. Then essential probabilities for the Baum-Welch algorithm are defined and computed as follows:

$$\begin{aligned} \gamma_t(i) &= P\{M_t = i | \mathbf{P}, s_{1:T}\} \\ &= \frac{\alpha_t(i)\beta_t(i)}{\sum_{u \in \mathcal{M}_s^{(t)}} \alpha_t(u)\beta_t(u)}, \text{ for } i \in \mathcal{M}_s^{(t)}, \end{aligned} \tag{2}$$

$$\begin{aligned} \xi_t(i,j) &= P\{M_t = i, M_{t+1} = j | \mathbf{P}, s_{1:T}\} \\ &= \frac{\alpha_t(i) P_{ij} \beta_{t+1}(j)}{\sum_{u \in \mathcal{M}_s^{(t)}, v \in \mathcal{M}_s^{(t+1)}} \alpha_t(u) P_{uv} \beta_{t+1}(v)}, \\ &\quad \text{for } i \in \mathcal{M}_s^{(t)}, j \in \mathcal{M}_s^{(t+1)}. \end{aligned} \tag{3}$$

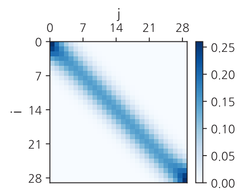
Let a set of D observations $\mathcal{S}_D = \left\{ s_{1:T}^{(1)}, \dots, s_{1:T}^{(D)} \right\}$ of length T be given. The observations are assumed to be independently and identically distributed with the HMM model. Then the Baum-Welch algorithm can be applied as shown in Table I.

III. Numerical Results

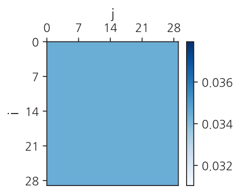
The measurement is performed on a cell operating at sub-GHz frequency with 10MHz bandwidth. As given in [7], we let the MCS index take values on $\mathcal{M} = \{0, 1, \dots, 28\}$. We use the scheduling data of four different RNTIs, which appeared for a sufficiently long time as tens of thousands of subframes. We extract multiple sequences of length 501 ($T = 500$) from the data so that each one begins with a scheduled subframe ($s_0 = 1$). For initializing a TPM, we consider two cases. Binomial

TABLE I
THE BAUM-WELCH ALGORITHM.

Input	Observation sequences: $\mathcal{S}_D = \left\{ s_{1:T}^{(1)}, \ldots, s_{1:T}^{(D)} \right\}$
	A stopping threshold: κ
Output	A TPM on the MCS indices: $\hat{\mathbf{P}} = \left\{ \hat{P}_{ij} \right\}_{i,j \in \mathcal{M}}$
Step 1	Initialize a TPM $\mathbf{P} = \{P_{ij}\}_{i,j \in \mathcal{M}}$.
Step 2	Calculate $\gamma_t^{(d)}(i)$ and $\xi_t^{(d)}(i,j)$ in (2) and (3) for each $1 \le d \le D$ and $1 \le t \le T$.
Step 3	Calculate $\tilde{P}_{ij} = \frac{\sum_{d=1}^{D} \sum_{t=1}^{T-1} \xi_t^{(d)}(i,j)}{\sum_{d=1}^{D} \sum_{t=1}^{T-1} \gamma_t^{(d)}(i)}$.
Step 4	With $\tilde{\mathbf{P}} = \left\{ \tilde{P}_{ij} \right\}_{i,j \in \mathcal{M}}$,
	if $\sum_{d=1}^{D} (\log P\{s_{1:T}^{(d)} \vert \tilde{\mathbf{P}}\} - \log P\{s_{1:T}^{(d)} \vert \mathbf{P}\}) > \kappa$,
	$\mathbf{P} \leftarrow \tilde{\mathbf{P}}$ and return to **Step 2**.
	else, $\hat{\mathbf{P}} \leftarrow \tilde{\mathbf{P}}$.



(a) BTPM



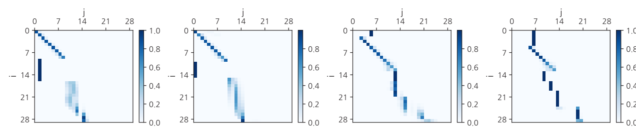
(b) UTPM

Fig. 2. Considered initial TPMs. The color of each grid cell indicates the value of P_{ij}.

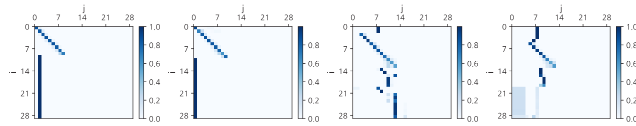
TPM (BTPM) is based on a binomial distribution $\mathcal{B}(28, 0.5)$, where the probability mass function of it is shifted and rescaled so that the diagonal of the TPM has the highest probabilities. Uniform TPM (UTPM) is based on a uniform distribution on $\mathcal{M}$ for each row. These TPMs are described in Fig. 2.

In the following figures, four considered RNTIs are arranged in an increasing order of the efficiency of scheduled RBs, which is calculated by dividing the total amount of received bits by the number of total scheduled RBs. The efficiency of scheduled RBs reflects overall channel quality that the RNTI experiences. Fig. 3 shows the obtained TPMs $\hat{\mathbf{P}}$ as the results of applying the Baum-Welch algorithm. As can be seen in the figure, the algorithm results in a form of TPM in which the transition behavior of the MCS index is concentrated within a similar range, even though the initial TPM is quite different. The transition probabilities on these concentrated ranges are also consistent with the Markov models in [1], [2] which allow transitions to adjacent states only.

Using the TPM $\hat{\mathbf{P}}$, we can obtain steady state probabilities $\boldsymbol{\pi}$ as a solution of $\boldsymbol{\pi}\hat{\mathbf{P}} = \boldsymbol{\pi}$. Fig. 4 shows the steady state probabilities. The efficiency of scheduled RBs for each RNTI is indicated in parentheses in the legend. The figure shows that the steady state probabilities for two initial TPMs are the same. Also we can check a reasonable tendency that the larger the efficiency of scheduled RBs, the more likely the steady state probabilities are concentrated on higher MCS indices.

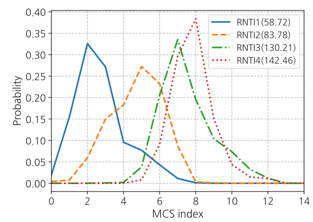


(a) Initialized with BTPM

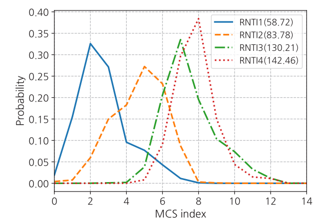


(b) Initialized with UTPM

Fig. 3. Results of $\hat{\mathbf{P}}$ from the Baum-Welch algorithm for 4 RNTIs with the initial TPMs of BPTM and UTPM.



(a) Initialized with BTPM



(b) Initialized with UTPM

Fig. 4. Steady state probabilities for 4 RNTIs with the initial TPMs of BPTM and UTPM.

IV. CONCLUSION

In this work, we provided an HMM based analysis on the temporal variation of the MCS index of a UE in LTE system. The analysis was based on real-world measurement data via a reliable platform capable of decoding the control channels of LTE downlink signal. Some TPMs that describe the stochastic behavior of MCS index were estimated, and the validity of the estimation was checked through investigating associated stationary probabilities. The future work will cover the channel conditions of UEs utilizing multiple antennas.

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