Beamwidth of Base Stations for Maximizing Coverage of Aerial Users

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***Abstract*—In this paper, we investigate cellular-enabled unmanned aerial vehicle (UAV) communications. The UAVs are regarded as aerial users. To operate UAVs safely and efficiently, the proper adjustment of network parameters should be considered. To tackle with this issue, the coverage probability of UAVs in terms of beamwidth of ground base stations (BSs) that have directional antennas is analytically studied. We employ stochastic geometry approach to analyze the performance. Numerical results show that beamwidth of the antennas has a significant effect on the network performance and the optimized beamwidth improves the network performance. This work provides useful insights for establishing networks and a framework for further studying cellular-enabled UAV communications.**

***Index Terms*—Cellular network, coverage probability, beamwidth, stochastic geometry, unmanned aerial vehicle.**

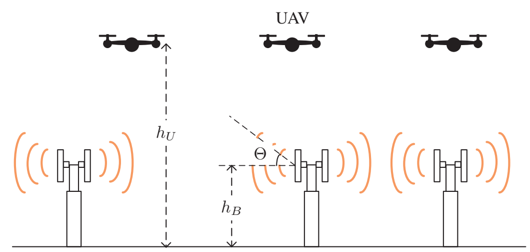

Fig. 1. System model with randomly distributed BSs following homogeneous PPP that have directional antennas of half-power beamwidth Θ. The UAVs fly at a fixed altitude h_U and the BSs have fixed height h_B.

I. Introduction

Over recent years, unmanned aerial vehicles (UAVs) have attracted much attention with their benefits in multiple industries followed from their affordable prices owing to advancements in hardware technology. The UAVs have their benefits in flexibility in movement and they have a lot of chances to employ their benefits. For example, the UAVs could be employed as wireless communication provider [1]–[3]. There have been several projects exploiting aerial platforms in wireless communications such as Google Loon, Facebook Aquila Drone, European Commision project ABSOLUTE [4]–[6]. Furthermore, Amazon PrimeAir project aims to deliver packages with small UAVs [7]. To perform missions well in several industries, safe and reliable operating UAVs is a critical issue. To tackle with this issue, cellular communications with UAVs could be an appealing solution. In such cellular network, the UAVs as aerial users formulate communication links with ground base stations (BSs).

In cellular network, the performance analysis (e.g., coverage probability, outage probability) is worth being studied. However, in order to characterize the performance, most of studies make use of time consuming simulations. Even though UAVs have different channel charactersistics with ground users, there are few literatures studying the performance analysis of UAV cellular communications. In [10], cellular-enabled UAVs are optimized under minimum SNR target constraint and their performance are numerically analyzed. Furthermore, in [8], the downlink coverage analysis for network of UAVs for a finite space was studied, while the coverage probability of downlink from UAV-BS was studied in [9]. In this paper, we study the performance of UAV cellular communications in terms of ground BS antenna beamwidth analytically so that the performance gets characterized much easier and provide insights for BS antenna architecture.

The rest of paper is organized as follows. We introduce our cellular network architecture system model in section II. In section III, coverage probability formulation as well as expressions are presented. In section IV, numerical results provide insights on UAV cellular communications and concluding remarks are followed in section V.

II. System Model

As shown in Fig. 1, we consider UAVs that are cellular-connected. The UAVs are assumed to fly at a fixed altitude h_U, and all ground BSs have the height h_B. The ground BSs are randomly located according to a homogeneous PPP Φ of density λ, and the ground BSs are equipped with directional antennas of which beamwidth is adjustable. Each BS has antenna pattern of horizontally omnidirectional and vertically directional. The half-power beamwidth of antenna pattern is denoted as 2Θ in radians, with $\Theta \in \left(0, \frac{\pi}{2}\right)$. Moreover, the antennas are not tilted to consider both terrestrial and aerial users. Assuming that there is no interference between users (i.e., aerial and terrestrial users) by virtue of a orthogonal multiple access (e.g., frequency division multiple access, time division multiple access). The antenna gain in direction $\theta = \arctan\left(\frac{h_U - h_B}{r}\right)$ is approximately modeled as [3]

$$G(r) = \begin{cases} \frac{G_0}{\Theta^2}, & -\Theta \leq \arctan\left(\frac{h_U - h_B}{r}\right) \leq \Theta \\ 0, & \text{otherwise,} \end{cases} \quad (1)$$

where r denotes the horizontal distance between a UAV and the associated BS and $G_0 = \frac{30000}{2^2} \times \left(\frac{\pi}{180}\right)^2 \approx 2.2846$.

It is assumed that all BSs transmit at the same power, and the receiver suffers from small scale fading. Then, the received power of an aerial user at horizontal distance r can be modeled as follows [2]:

$$\mathrm{P}_{\mathrm{Rx}}(r) = \mathrm{P}_{\mathrm{Tx}} G(r) d^{-\alpha} \eta^{-1} \Omega, \quad (2)$$

where P_{Tx} denotes fixed transmit power, $d = \sqrt{r^2 + (h_U - h_B)^2}$ denotes the distance between the UAV and the associated BS, α denotes the path loss exponent, η denotes excessive path loss, and Ω denotes the small scale fading gain. A Nakagami-m fading model is assumed that the channel gain Ω follows a gamma distribution as follows:

$$f_\Omega(\omega) = \frac{m^m \omega^{m-1}}{\Gamma(m)} \exp(-m\omega), \quad (3)$$

where m represents an integer-valued fading parameter and $\Gamma(\cdot)$ denotes gamma function. For Nakagami-m fading model, if the parameter $m = 1$, the fading model is equivalent to Rayleigh fading model and, $m \to \infty$ denotes that there is no fading. Then, the signal-to-noise (SNR) at the receiver of horizontal distance r from the associated BS is represented as follows:

$$\mathrm{SNR} = \frac{\mathrm{P}_{\mathrm{Rx}}(r)}{\sigma^2}, \quad (4)$$

where σ^2 denotes the noise power. Assuming the fixed height of both BSs and UAVs, the SNR is a function of horizontal distance r.

III. Coverage Probability

In this section, the coverage probability is obtained. For a given SNR threshold T, the coverage probability P_{cov} of UAV is represented as follows:

$$\begin{aligned} \mathrm{P}_{cov} &= \mathbb{P}(\mathrm{SNR} > T) \\ &= \int_0^\infty \mathbb{P}\left[\mathrm{SNR} > T | r\right] f_R(r) dr, \end{aligned} \quad (5)$$

where $f_R(r)$ denotes the probability density function (pdf) of the associated BS's horizontal distance.

Theorem 1. *The coverage probability of the link between a UAV and the associated BS is obtained as*

$$\mathrm{P}_{cov} = \int_{r_m}^\infty \sum_{k=0}^{m-1} \frac{s^k}{k!} \exp\left(-s\right) f_R(r) dr, \quad (6)$$

where s is given by

$$s = \frac{T\sigma^2 m \Theta^2}{\mathrm{P}_{\mathrm{Tx}} G_0 d^{-\alpha} \eta^{-1}}, \quad (7)$$

$f_R(r)$ denotes the pdf of the distance of the nearest BS that is farther than r_m, and $r_m = \frac{h}{\tan(\Theta)}$.

Proof. The coverage probability is rewritten as follows:

$$\begin{aligned} &\mathbb{P}\left[\mathrm{SNR} > T | r\right] \\ &= \mathbb{P}\left[\frac{\mathrm{P}_{\mathrm{Rx}}(r)}{\sigma^2} > T\right] \\ &= \mathbb{P}\left[\frac{\mathrm{P}_{\mathrm{Tx}} G(r) d^{-\alpha} \eta^{-1} \Omega}{\sigma^2} > T\right] \\ &= \mathbb{P}\left[\Omega > \frac{T\sigma^2}{\mathrm{P}_{\mathrm{Tx}} G(r) d^{-\alpha} \eta^{-1}}\right] \\ &\overset{(a)}{=} \sum_{k=0}^{m-1} \frac{s^k}{k!} \exp\left(-s\right), \end{aligned} \quad (8)$$

where $s = \frac{T\sigma^2 m}{\mathrm{P}_{\mathrm{Tx}} G(r) d^{-\alpha} \eta^{-1}}$. The (a) follows from gamma random variable with an integer parameter m. Meanwhile, the antenna gain $G(r)$ is zero if $\arctan\left(\frac{h_U - h_B}{r}\right) > \Theta$. As a result, the coverage probability is zero if r is smaller than r_m. Then, assuming that UAVs associate with the closest BS with non-zero power gain, the coverage probability can be rewritten as (6). □

Lemma 1. *The pdf of the distance of the nearest BS farther than r_m, $f_R(r)$ has the distribution of*

$$f_R(r) = 2\lambda\pi r \exp\left(-\lambda\pi\left(r^2 - r_m^2\right)\right), r \geq r_m. \quad (9)$$

Proof. For homogeneous PPP in $\mathbb{R}^2$, the number of points in Borel set A is represented as $\left(\lambda L(A)\right)^n / n! \cdot \exp\left(-\lambda L(A)\right)$ where $L(A)$ denotes the area of A (see [11] for more details). Then, the contact distribution function in the area between outer circle of radius r and inner circle of radius r_m is noted as

$$1 - \exp\left(-\lambda\pi\left(r^2 - r_m^2\right)\right), \quad (10)$$

which also represents the cumulative distribution function (cdf) of the distance of the nearest point that is farther than r_m. Therefore, the pdf is derived as (9). □

Then, from (6) and (9), the coverage probability is obtained. The fading parameter m, path loss exponent α, and the excessive path loss η represents the various channel model. For the particular case of $m = 1$, which represents the Rayleigh fading channel, the coverage probability is obtained as follows:

$$\mathrm{P}_{cov} = \int_{r_m}^\infty \exp\left(-s\right) 2\lambda\pi r \exp\left(-\lambda\pi\left(r^2 - r_m^2\right)\right) dr, r \geq r_m. \quad (11)$$

IV. Numerical Results

In this section, we provide insights through numerical results. The simulation parameters are set as Table I. The air-to-ground channel is usually characterized as dominant line-of-sight links. Therefore, the excessive path loss η is set as small value and Nakagami-m fading parameter m is set as 3 that reflects less fading than Rayleigh fading.

The coverage probability in terms of beamwidth with varying BS density is shown in Fig. 2, when the UAV flies at

TABLE I
PARAMETERS FOR NUMERICAL SIMULATIONS

Parameter	Value
$\mathrm{P_{Tx}}$ (dB)	20
h_B (m)	30
T (dB)	30
Bandwidth (MHz)	10
Noise spectral density (dBm/Hz)	-101
α	2.09
η (dB)	2.3
m	3

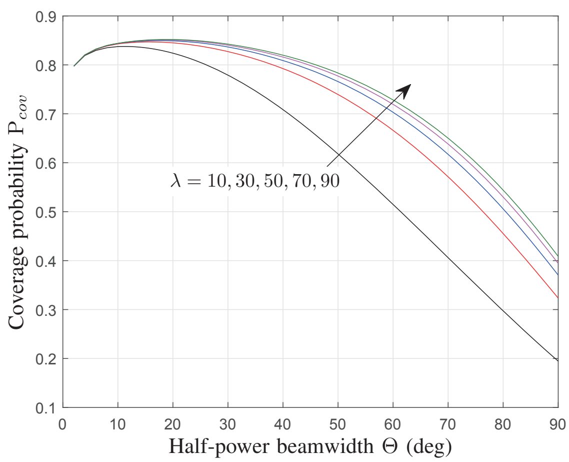

Fig. 2. Coverage probability in terms of half-power beamwidth Θ for ground BSs of density $\lambda = 10, 30, 50, 70, 90$ (BSs/km^2) when $h_U = 230$ m.

the altitude $h_U = 230$ m. The coverage probability has optimal point that maximizes coverage probability. The coverage probability gets large as density of BSs gets large. However, the coverage probability converges as more BSs are deployed since transmit power is limited. It is worth being noted that the optimal beamwidth is not full covering $\pi/2$ that the *phisically closest BS may not associate with the aerial users*.

Fig. 3 shows the coverage probability in terms of beamwidth with different UAV altitudes when BS density $\lambda = 50$ (BSs/km^2). As the altitude UAV flies gets higher, the coverage probability decreases. However, the overall tendency of coverage probability in terms of beamwidth is similar. Remarkably, the optimal beamwidth value is about 18° for varying altitudes that does not vary a lot. We can find the rationale that the transmit power degradation owing to increasing beamwidth would be more dominant than others.

V. CONCLUSION

In this work, we investigated the performance of cellular-enabled UAVs (i.e., aerial users) in terms of BS antenna beamwidth. We modeled the cellular network architecture with BSs following homogenous PPP that have directional antennas. Then, the coverage probability was presented. The

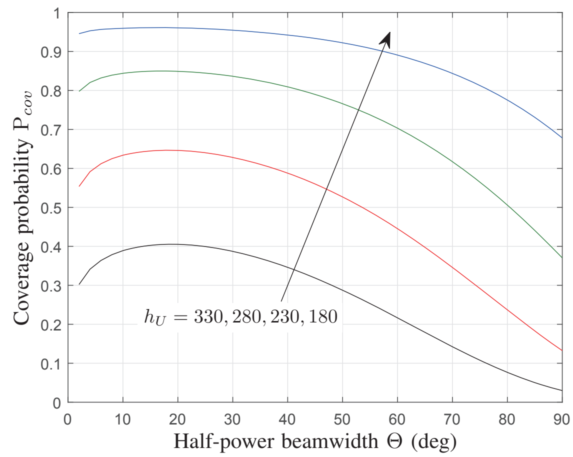

Fig. 3. Coverage probability in terms of half-power beamwidth Θ for ground BSs of density $h_U = 330, 280, 230, 180$ m (black, red, green, blue in order) when $\lambda = 50$ (BSs/km^2).

numerical results provide nontrivial insights. The results in this paper would be a framework for further studying cellular-enabled UAV communications. The network performance analysis considering both terrestrial and aerial users, multi-cell interference analysis remains as a further work.

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