Analyzing LoRa long-range, low-power, wide-area networks using stochastic geometry

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ABSTRACT

In this paper we present a simple, stochastic-geometric model of a wireless access network exploiting the LoRA (Long Range) protocol, which is a non-expensive technology allowing for long-range, single-hop connectivity for the Internet of Things. We assume a space-time Poisson model of packets transmitted by LoRA nodes to a fixed base station. Following previous studies of the impact of interference [8, 10], we assume that a given packet is successfully received when no interfering packet arrives with similar power before the given packet payload phase. This is as a consequence of LoRa using different transmission rates for different link budgets (transmissions with smaller received powers use larger spreading factors) and LoRa intra-technology interference treatment. Using our model, we study the scaling of the packet reception probabilities per link budget as a function of the spatial density of nodes and their rate of transmissions. We consider both the parameter values recommended by the LoRa provider, as well as proposing LoRa tuning to improve the equality of performance for all link budgets. We also consider spatially non-homogeneous distributions of LoRa nodes. We show also how a fair comparison to non-slotted Aloha can be made within the same framework.

CCS CONCEPTS

• **Networks** → **Network performance modeling**; **Network performance analysis**; **Wireless access networks**.

KEYWORDS

Internet of Things; Low-Power, Wide-Area Network; LoRa; stochastic geometry; Poisson process; propagation process; reception probability

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1 INTRODUCTION

Low-power, wide-area networks (LPWANs) will undoubtedly play a crucial role in the development of the Internet of Things. They are wireless cellular networks that, in contrast to LTE (and also WiFi and Bluetooth), do not focus on high data rate communications. The goal of LPWANs is to ensure large coverage areas with reasonable data rates and low energy consumption. A good scaling in terms of the density of nodes is also a key requirement for these systems.

Several systems in the field of LPWANs exist. Essentially, four options are available to customers: Sigfox, operator standards LTE-M or NB-IoT, and several proprietary solutions, LoRa (Long Range) being one of them. Sigfox is offered as an operator service and the technology is proprietary. It operates in the 968/902 MHz licence-free industrial scientific and medical (ISM) radio band. Sigfox acts as both the technology and the service provider and has quickly deployed a large number of base stations in Europe and beyond. Long Term Evolution Machine-to-Machine LTE-M and Narrow Band-IoT are the standards promoted by the traditional telecom operators that expect to propose solutions for machine-to-machine traffic through standardized solutions, in particular in the framework of the forthcoming 5G systems. There is a significant number of proprietary solutions such as Accelus, Cyan's Cynet, Ingenu/On-Ramp, SilverSpring Starfish, etc., but only a limited amount of information about these systems is available.

LoRa is also a proprietary technology developed by Cycleo of Grenoble, France, and acquired by Semtech in 2012. As its name suggests, it mainly aims at long range transmissions with a high robustness, multipath and Doppler resistance, and low power. LoRa networks often operate in the unlicensed ISM band (434 MHz and 868 MHz in Europe and 433 MHz and 915 MHz in the USA), but can also use licensed bands in the spectrum ranging from 137 to 1020 MHz. LoRa uses a kind of Chirp Spread Spectrum (CSS) with Forward Error Correction (FEC), where wideband linear frequency modulated chirp pulses are used to encode information and the spreading factor is directly linked to the power decay between the end device and the base station.

We decided to focus our study on LoRa systems because, even though it is a proprietary technology, a significant amount of information is available on it, see Section 3, making it possible to model and study LoRa networks. Moreover, LoRa industrial momentum seems to be significant and commercial success for LoRa is likely.

LoRa technology has already been studied, both for the evaluation of pairs of simultaneous transmissions [10] and for the scaling evaluation of a large network [11], [8]. However, to the authors' best knowledge, there is no commonly accepted analytical model of LoRa networks. A major difficulty in addressing performances of LoRa consists in its multiparameter nature, with a crucial role played by the spreading factors and, closely related to them, packet transmission time. An additional difficulty is the complexity of the signal-to-interference analysis of packet reception in CSS technology.

In this paper we propose a mathematical model allowing one to calculate packet reception probabilities per link budget, according to the spatial density of nodes and their rate of transmissions, as well as all main LoRa parameters. It is based on a Poisson rain (space-time) model of packet transmissions from the nodes to the base station (gateway). This model has already been used in [5, 6] to study non-slotted Aloha systems.

The remainder of this paper is organized as follows. In Section 2 we present some related studies, which are crucial to our approach. In Section 3 we present LoRa system assumptions and in Section 4 we present our model with its analysis. Section 5 presents numerical results. In Section 6 we briefly sketch how LoRa can be compared to non-slotted Aloha using a Poisson rain model of packet transmissions. Finally, Section 7 concludes the article.

2 RELATED STUDIES

A few papers, such as [12], present measurements of existing LoRa systems and study the actual performance of the end devices with respect to their relative distance to the gateways. They aim at optimising the parametrization of LoRa networks. However, due to the limited number of end devices considered in these studies, it is difficult to gauge the scaling properties of LoRa networks, for instance in terms of the maximum number of end devices supported with a given goodput rate. Moreover, these studies do not permit a fine-tuned analysis of collisions in LoRa networks.

In contrast, [10] scrutinizes interference in LoRa in order to establish packet collision rules, which are reproduced next in a simulation model allowing the authors to study the scalability issues of LoRa networks. A real platform to test capture in LoRa networks is used in [8]. The conclusions drawn from the tests lead to a collision model close to that derived in [10] and to similar scalability evaluation results. We recall the collision rules established in [10] and [8] in Section 3.2 since we integrate them in our stochastic-geometric LoRa network model.

3 LORA SYSTEM DESCRIPTION AND ASSUMPTIONS

LoRa systems are highly parameterizable. The carrier frequency can be chosen from 137 to 1020 MHz by steps of 61 kHz and the bandwidth (the width of frequencies in the radio band) can be set to 125 MHz, 250 MHz or 500 MHz. LoRa can use transmission powers from -4 dBm to 20 dBm.

A key parameter of LoRa is the spreading factor, which is the ratio between the symbol rate and the chip rate. Denoted in what follows by SF, and expressed at the logarithmic scale at base 2 (thus an increase by 1 of the SF multiplies the chip per symbol rate by 2), the SF of LoRa varies from 6 to 12. With SF = 6 the symbol rate is the highest but the transmission is the least robust. The value of SF is fixed for individual transmissions depending on the received power, as explained in Section 3.1.

The physical layer of LoRa also incorporates Forward Error Correction (FEC) that permits the recovery of bits of information in the case of corruption by interference. This requires a small overhead of additional encoding of the data in the transmitted packet thus reducing the Coding Rate (CR). The available values of CR are $4/5, 5/6, 4/7, 4/8$ (with smaller values providing more reliability).

3.1 Spreading factor and received power

The spreading factor parameter SF, which is the logarithm at base 2 of the ratio between the symbol and the chip rate, is a key parameter of LoRa. Even if in principle it can be set arbitrarily, it is obvious that its value must be appropriately chosen in order to optimize the performance of the network. A reasonable choice is to make it inversely related to the received power: higher received power allows one to use a smaller spreading factor and thus reduce the packet transmission time, as specified in Section 3.3. As an example, Table 1 gives the values of SF proposed in [2] for different thresholds of the received power, called *sensitivity*, for transmissions with 125 kHz bandwidth.

Table 1: Sensitivity (minimum received power thresholds) and corresponding spreading factors

SF	Sensitivity in dBm (for 125 kHz bandwidth system)
6	-121
7	-124
8	-127
9	-130
10	-133
11	-135
12	-137

For example, when the signal is received with at least -121 dBm of power then the spreading factor is set to its minimal value SF = 6. When the received power is in the interval $[-124, -121)$ then SF = 7, etc. The smallest acceptable received power -137 dBm triggers SF = 12.

3.2 Collision model

A precise (signal to interference) analysis of the reception of packets in LoRa chirp spread spectrum systems is complicated. We will use a simplified *collision model*, to this end, inspired by the results presented in [8] and [10]. The first assumption we make is that *two transmissions using different values of SF do not interfere with each other*; this assumption is retained in the collision model described in [8]. In other words, the codes with different spreading factors are assumed orthogonal.

The second assumption concerns collisions caused by *simultaneous transmissions using the same value of SF*. Different cases of this, identified in [10], depend on the time overlap of the (two or more) simultaneous transmissions and their received powers, called also Received Signal Strength Indications (RSSIs). More specifically, [10] distinguishes five cases of possible overlap of the transmission period of a given packet with an interfering transmission, presented in Figure 1, combined with two cases regarding the difference between their RSSI values. It shows that the given packet is always successfully received if its RSSI is significantly greater than that of the interfering packet or, in the case of similar RSSI values, when the interfering transmission starts relatively late, precisely during the payload part of the given packet (case 5 in Figure 1). It is not successful when an interfering packet with a similar RSSI starts before the payload part of the given packet (depicted as cases 1 to 4 in Figure 1).

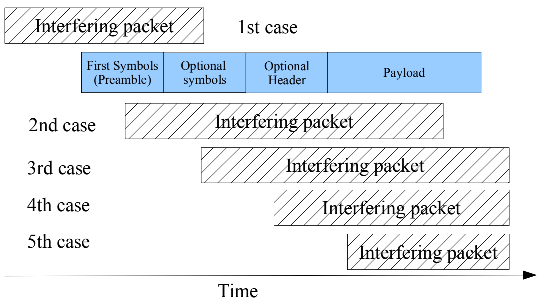

Figure 1: Collision model: different cases of a given packet transmission (in blue) overlapping with an interfering transmission. The reception of the given packet is successful if its received power (RSSI) is significantly greater than that of the interfering packet or, in the case of similar RSSI values, when the interfering transmission starts during the payload (case 5). Cases 1–4 with similar RSSI values give collisions.

An additional advantage of relating the spreading factors to the received powers, as described in Section 3.1, is that any two transmissions with significantly different received powers will use different spreading factors and thus, according to the assumption made above, will not interfere. Consequently, we can assume that a *given packet is successfully received when no interfering packet arrives with similar power before its payload phase.*

3.3 Packet transmission times

The packet transmission (on air) time depends on its composition (cf Figure 1) and the symbol transmission time T_{symbol}, which is related to the spreading factor SF and the bandwidth BW via a fundamental relation

$$T_{\text{symbol}} = \frac{2^{\text{SF}}}{\text{BW}}\,. \tag{1}$$

The following specific expressions for transmission times (in seconds) of the two main parts of the packet are given in [1].

Preamble time (with with the optional symbols):

$$(4.25 + np)T_{\text{symbol}}.$$

Payload time (with the optional header):

$$\left(8 + \max\left(\left\lceil \frac{8\text{PL} - 4\text{SF} + 28 + 16 - 20\text{H}}{4(\text{SF} - 2\text{DE})} \right\rceil (\text{CR} + 4), 0\right) T_{\text{symbol}},$$

with notation explained in Table 2.

Table 2: LoRa main parameters

BW	bandwidth in Hz
SF	Spreading Factor
n_p	number of additional preamble symbols
PL	payload in Bytes
H	H=0 if header, 1 if no header present
DE	DE=1 if low data rate optimization, 0 if not
CR	Coding Rate

4 NETWORK MODEL AND ANALYSIS

In this section we propose a stochastic geometric model of LoRa and present some results of its analysis.

4.1 Poisson LoRa model with collisions

Our LoRA network model consists of one base station located (without loss of generality) at the origin of the plane $\mathbb{R}^2$ and a space-time Poisson process of packets sent to it. This model, called *Poisson rain* in [5, 6], assumes packet transmissions initiated from points $\{X_i\}$ on the plane at time instants $\{T_i\}$, with $\Phi = \{(X_i, T_i)\}$ being a Poisson point process on $\mathbb{R}^2 \times \mathbb{R}$. Our default assumption is that Φ is homogeneous in space and time, with the expected number of transmissions initiated per unit of time and space denoted by λ. However, in Section 4.2.2 we shall also consider some space-non-homogeneous, radially symmetric Poisson model of packet transmissions.

We assume that the packets are transmitted with power P_{tr} and suffer from path-loss modeled by the usual power-law function $l(r) = (\kappa r)^\beta$, of the transmission distance r with some constants $\kappa > 0$ and $\beta > 2$, as well as independent propagation effects (such as those from multi-path fading, shadowing or other seemingly random phenomena perturbing the base-station-to-device signal) modeled by a generic random variable F. Thus, a packet transmitted from a location X_i (regardless of time) is received at the base station with power equal to

$$P_{\text{rec}}(X_i) := \frac{P_{\text{tr}} F_i}{l(|X_i|)} = \frac{P_{\text{tr}} F_i}{(\kappa |X_i|)^\beta}$$

where, given $\{X_i\}$, F_i are i.i.d. with common distribution of F and $|\cdot|$ is the Euclidean distance on the plane.

Following the system description presented in Section 3.1, we assume that the spreading factors SF used for transmissions of different packets are related to their received powers. More specifically, we consider a set of threshold powers (sensitivities) $P_1 < P_2 < \cdots < P_N$, for some $N \geq 1$, with the corresponding spreading factors SF_n, $n = 1, \ldots, n$, and we assume that the value SF_n is used for the transmission from X_i when $P_{\text{rec}}(X_i) \in [P_n, P_{n+1})$, with $P_{N+1} := \infty$. No successful reception (packet loss) is assumed when $P_{\text{rec}}(X_i) < P_1$.

We have seen in Section 3.3 that packet transmission times depend on many parameters, in particular the spreading factor SF. Consequently, we denote by B_n the transmission time of a packet using spreading factor value SF_n and by Δ_n the transmission time of the initial part, required for the base station to lock on the transmission. During this latter part, an interfering packet of similar power can lead to a collision, as explained in Section 3.2. More precisely, we assume that a given packet transmitted from X_i and reaching the base station with power $P_{\mathrm{rec}}(X_i) \in [P_n, P_{n+1})$ is successfully received if no any other packet is on air with received power in $[P_n, P_{n+1})$ during the initial time Δ_n of the given packet's transmission.

4.2 Performance analysis

We begin by recalling a useful result regarding the process of received powers in an homogeneous Poisson network model with power-low path-loss model; see e.g [3, Lemma 4.2.2]. In the context of our Poisson rain network model it can be formulated as follows.

Lemma 4.1. *The values of powers received at the origin from all packet transmissions initiated in the Poisson rain model Φ during any time interval of length T form a non-homogeneous Poisson point process on $(0, \infty)$ with intensity measure*

$$\frac{2aT}{\beta} t^{-2/\beta-1} dt,$$

where

$$a = \frac{\pi\lambda P_{\mathrm{tr}}^{2/\beta} E[F^{2/\beta}]}{\kappa^2},$$

F is the generic random propagation in the wireless channel, and λ is the expected number of transmissions initiated per unit of time and space.

4.2.1 Successful reception probabilities. We now consider any given (arbitrary) packet transmission arriving at the base station with power within the interval, $[P_n, P_{n+1})$ for some $n = 1, \dots, N$. We denote by Π_n the probability that this transmission is successfully received by the base station. As previously stated, the transmission of this packet takes time B_n, with the initial time Δ_n required for the base station to lock on this transmission. Using Lemma 4.1 we obtain our first result.

Proposition 4.2.

$$\Pi_n = \exp\Big(- a_n (P_n^{-2/\beta} - P_{n+1}^{-2/\beta})\Big)$$

for $n = 1, \dots, N-1$, and

$$\Pi_N = \exp\Big(- a_N P_N^{-2/\beta}\Big),$$

where

$$a_n = a(B_n + \Delta_n).$$

Proof. According to the adopted collision model, Π_n is equal to the probability that there is no transmission in the Poisson rain model during the interval of time of duration $B_n + \Delta_n$, reaching the base station with the power in the same interval $[P_n, P_{n+1})$, assuming the convention $P_{N+1} = \infty$. By Lemma 4.1 and the well known formula for Poisson void probabilities (cf e.g., [3, Section 3.4.1])

$$\Pi_n = \exp\Big(\frac{-2a_n}{\beta} \int_{P_n}^{P_{n+1}} t^{-2/\beta-1} dt\Big).$$

Direct evaluation of the above integral completes the proof. □

Note that the transmission times B_n and Δ_n depend on the spreading factor SF_n used for the transmissions with received powers in $[P_n, P_{n+1})$, as well as on other parameters, as explained in Section 3.3.

4.2.2 Equalizing the reception probabilities. We assume now that all LoRa model parameters are given except the power thresholds $P_1, \dots, P_N$ (sensitivities), which we want to fix so that the packets arriving with all powers not smaller than P_1 have the same reception probability. We have the following result.

Proposition 4.3. *For a given $\Pi \in (0, 1)$ assume*

$$P_n = \Big(- \sum_{i=n}^{N} \frac{1}{a_i} \log(\Pi)\Big)^{-\beta/2}$$

for $n = 1, \dots, N$. Then the reception probabilities are constant $\Pi_n = \Pi$.

Proof. The result follows directly from Proposition 4.2 by substituting Π to Π_n in the expressions for Π_n, for $n = 1, \dots, N$. □

Obviously, the larger target value Π, the larger all power thresholds P_n, in particular P_1, meaning that more transmissions will remain unsuccessful because having a received power smaller than the minimal acceptable value P_1.

4.2.3 A non-homogeneous spatial density of transmission. We consider now a non-homogeneous pattern of transmissions. More specifically, let the process of transmissions form a space-time Poisson point process with the local density of transmission initiated at distance r from the origin being equal to λr^α, for some parameter α. A natural assumption is $\alpha < 0$, meaning that the density decays within the distance, but $\alpha > 0$ is also possible. Obviously $\alpha = 0$ is the previously considered homogeneous case.

In order to study this case we need the following extension of Lemma 4.1.

Lemma 4.4. *The values of powers received during any time interval of length T at the origin in the Poisson rain model, with intensity of transmissions $\lambda_s r^\alpha$ at distance r from the origin, form a non-homogeneous Poisson point process on $(0, \infty)$ with intensity measure*

$$\frac{2a'T}{\beta} t^{-(\alpha+2)/\beta-1} dt,$$

where

$$a' = \frac{\pi\lambda P_{\mathrm{tr}}^{(\alpha+2)/\beta} \mathbf{E}[F^{(\alpha+2)/\beta}]}{\kappa^{\alpha+2}}.$$

Remark 1. *Note that the intensity of received powers in our non-homogeneous model, given in Lemma 4.4, coincides with that of the homogeneous model presented in Lemma 4.1, with the path-loss exponent β and the intensity of transmissions λ replaced respectively by*

$$\beta' = \frac{2\beta}{\alpha+2}$$

$$\lambda' = \frac{2\lambda}{(\alpha+2)\kappa^\alpha}.$$

Consequently, we observe that the performance of the considered non-homogeneous LoRa network (regarding metrics based on the process of received powers, as e.g. the reception probabilities Π_n) is equivalent to the performance the homogeneous LoRa model with the above parameter modifications. This is yet another instance of the network equivalence property formulated in [4].

Proof of Lemma 4.4. By the displacement theorem for Poisson processes (see e.g. [3, Theorem 4.4.2]) the process of received powers is a Poisson process. To compute its mean measure denoted by $\Lambda(t)$, we calculate the mean number of transmissions started within a unit interval of time and received with power greater than $t > 0$

$$\Lambda(t) = \mathbf{E}[\#\{(X_i, T_i) \in \Phi : (\kappa|X_i|)^{-\beta} P_{\text{tr}} F_i > t, T_i \in [0,1]\}].$$

Writing the condition on the received power in the form

$$|X_i| < \Big(\frac{P_{\text{tr}} F_i}{t}\Big)^{1/\beta} 1/\kappa$$

and using the spatial density of transmissions, we obtain

$$\Lambda(t) = 2\pi\lambda \mathbf{E}\Big[\int_0^{(P_{\text{tr}} F_i/t)^{1/\beta} 1/\kappa} r^{\alpha} r dr\Big]$$
$$= \frac{2\pi\lambda P_{tr}^{(\alpha+2)/\beta}}{(\alpha+2)\kappa^{\alpha+2}} \mathbf{E}\Big[F^{(\alpha+2)/\beta}\Big] t^{-(\alpha+2)/\beta}.$$

Differentiating $\Lambda(t)$ with respect to t and multiplying by -1 one concludes the proof. □

5 NUMERICAL RESULTS

In this section we present numerical results obtained using the model and its analysis developed in Section 4, and assuming parameter values typical for a rural LoRa network.

We first consider a homogeneous deployment of nodes parametrized by the mean number N_{nodes} of nodes within a radius of $R =$ 8000m around the base station, which corresponds to the spatial density $\lambda_s = \frac{N_{\text{nodes}}}{\pi R^2}$ of nodes per per m^2. Suppose that each node transmits a packet to the base station every 16.666 minutes on average, which is equivalent to the rate $\lambda_t = 0.001$ transmissions per second. In order to represent this network we use the Poisson rain model with space-time intensity $\lambda := \lambda_s \lambda_t$ of transmissions per second per m^2.

Remark 2. *When we use the Poisson rain model we are making an approximation of the real network. In fact, "real" nodes are fixed in space and send (assume independent Poisson) streams of packets in time. Nodes in Poisson rain model are not fixed in space: we may see them rather as "born" at some time, transmitting a packet and "dying" immediately after. While it is possible to come up with a model representing fixed nodes in space, see e.g. the Poisson-renewal model developed in [5, 6] for non-slotted Aloha, its analysis is more complicated, with less closed-form results. Moreover, most importantly, the two models provide very close results. This can be theoretically explained by the convergence of the space-time pattern of transmissions in the real situation (with fixed nodes) to the Poisson rain model when* $\lambda_s \to \infty$ *and* $\lambda_t \to 0$ *with* $\lambda = \lambda_s \lambda_t =$ constant. *Numerical results provided in [5, 6] confirm this statement.*

We use the Hata model [9] to determine a suitable value for the path-loss exponent β. In this model the path-loss in dB at distance r is $(44.9 - 6.55\ \log_{10}(h_B)) \log_{10}(r)$ where h_B is the height of the base station antenna in meters. We assume that the height of the base station antenna is 30 m and obtain the path-loss function $35.22\ \log_{10}(r)$ in dB. Thus we choose $\beta = 3.5$. We assume the path loss constant to be $\kappa = 2$. Our default assumption regarding propagation effects is Rayleigh fading of mean 1. With this choice we have $E(F^{2/\beta}) = \frac{2\Gamma(2/\beta)}{\beta}$. However, in Section 5.3 we shall also consider the no-fading case and log-normal shadowing.

We assume that all nodes transmit with power $P_{\text{tr}} = 10$ dBm. Other LoRa parameters are specified in Table 3.

Table 3: LoRa parameters used in our model

BW	125 kHz
SF	$\{6, \ldots, 12\}$
n_p	6
PL	20 Bytes
H	H=0 (with header)
DE	DE=0 (no low data rate optimization)
CR	4/5

Note that we have $N = 7$ different values of the spreading factor SF (running from 6 to 12). The packet and preamble transmission times B_n and Δ_n, $n = 1, \ldots, 7$, corresponding to these values are calculated using the expressions presented in Section 3.3.

5.1 Reception probabilities

Let us assume received power thresholds (sensitivities) for different spreading factors as in Table 1. That is, $P_1, \ldots, P_7$ expressed in dBm are as in the right column of this table from bottom to top ($P_1 = 10^{-13.7}$mW, $P_2 = 10^{-13.5}$mW, etc, $P_7 = 10^{-12.1}$mW and the corresponding values of the spreading factor are $\text{SF}_1 = 12, \text{SF}_2 = 11$, etc. , $S_7 = 6$). Using the expressions given in Proposition 4.2 we calculate the probability of successful reception Π_n of packets receiving with powers in successive intervals $[P_n, P_{n+1})$, $n = 1, \ldots, 7$. These probabilities, calculated assuming Rayleigh fading, are shown in Figure 2. Note that increasing SF values correspond to nodes received with smaller powers, thus located statistically in larger distances to the base station. (When some fading is assumed then this relation is not a deterministic function of the distance.) We observe that, up to the density of $N_{\text{nodes}} = 2000$ nodes in the disk of radius 8 km, all transmissions have a very high reception probability (close to 1), except for the three weakest power categories (SF = 10, 11, 12).

5.2 Equalizing reception probabilities for all nodes

We assume the density of nodes corresponding to $N_{\text{nodes}} = 2500$ and we use the same values as in Section 5 and a Rayleigh fading of mean 1. Using the results of Section 4.2.2, we calculate the modified values of the sensitivity which allow all nodes to have the same reception probability equal to $\Pi \approx 0.99$. These modified sensitivities are presented in Table 4, where we also recall the values recommended by the provider of the LoRa technology: Semtech. We observe that the values we obtain are greater than those recommended by Semtech. We also have very small power intervals for large values of the SFs, which leads to an impractical system for large values of the SFs and also a waste in the performance of the system.

However if we use the values recommended by Semtech, we can have for $N_{\text{nodes}} = 700$ and for all the values of the SFs proposed

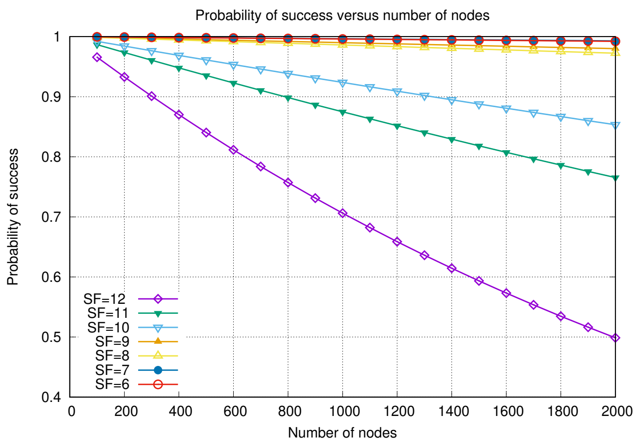

Figure 2: Probability of successful reception versus the node density for all spreading factors SF in the Rayleigh fading case.

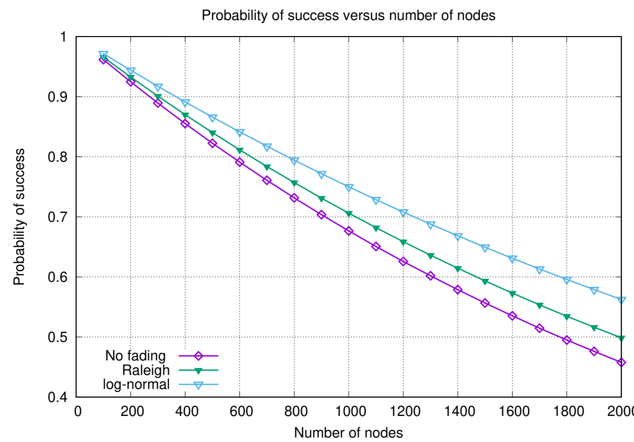

Figure 3: Impact of the propagation effects on the probability of successful transmission of the weakest transmissions (SF = 12). Log-normal variance is taken $\sigma = 2$dB.

in the technology $\Pi > 0.90$; which could be considered as a weak fairness property.

Table 4: Sensitivities equalizing reception probabilities to $\Pi \approx 0.99$ for all spreading factors in the network of density $N_{\text{nodes}} = 2500$ nodes in a radius of 8 km, compared to the sensitivities recommended by the LoRa technology provider. Note that for all the values of the SFs, and especially for large values, the required powers are much larger than those recommended by Semtech and thus packets with some small powers are lost with respect to the original configuration.

	Sensitivity in dBm	
SF	equalizing reception	LoRa recommended
6	-121.13	-121
7	-124.38	-126
8	-125.71	-129
9	-126.34	-131
10	-126.63	-133
11	-126.79	-135
12	-126.87	-137

5.3 Effect of the fading law

Now, we study the influence of the propagation effects modeled by the distribution of F. We consider three cases: no fading i.e. $F \equiv 1$, Rayleigh fading of mean 1 (exponential F) and log-normal fading of mean 1 ($F = \exp(-\sigma^2/2 + \sigma Z)$ with Z standard normal). In our model this distribution impacts the results only via its moment $\mathbf{E}(F^{2/\beta})$. Recall that

$$\mathbf{E}(F^{2/\beta}) = \begin{cases} 1 & \text{when no fading,} \\ 2\Gamma(\frac{2}{\beta})/\beta & \text{for Rayleigh fading,} \\ \exp(\sigma^2(2-\beta)/\beta^2 & \text{for log-normal shadowing.} \end{cases}$$

In Figure 3 we show how these three different cases of propagation effects impact the probability of successful reception of the weakest transmissions, that is using the spreading factor SF = 12. We observe that the performance of this category of transmissions is slightly improved by random propagation effects. This can be generalized to all transmissions using Jensen's inequality $\mathbf{E}(F^{2/\beta}) \leq (\mathbf{E}(F))^{2/\beta} = 1$ (recall $\beta > 2$, $\mathbf{E}(F) = 1$) and observing that a network with some random propagation effects is equivalent to a sparser network without propagation effects, cf [4] and hence generates smaller interference.

5.4 Effect of decaying density of nodes

Finally we consider a non-homogeneous distribution of nodes, which we assume decreasing with the distance r to the base station as the function $\lambda_s r^{-1/5}$. Here λ_s and all LoRa parameters are assumed as in Section 5.1. We use the results of Section 4.2.3 to compute the probability of successful transmission for various spreading factors. The results are shown in Figure 4. We observe a very significant improvement and equalization of the probabilities of success for all categories of transmissions, with all values presented being bigger than 0.9.

6 COMPARISON TO ALOHA

Performance of LoRa can be compared to that of the classical, non-slotted Aloha. This can be done assuming the same Poisson rain model of packet transmissions. Let us briefly recall Aloha model assumptions and results obtained in [7] (the context being that of sensors sending packets to a sink-node).

6.1 Aloha SINR collision model

In Aloha systems the base station needs to be synchronized before receiving a packet. This can be done when the base station is not locked on a different packet at the given packet arrival time. If this condition is satisfied, the base station starts and continues

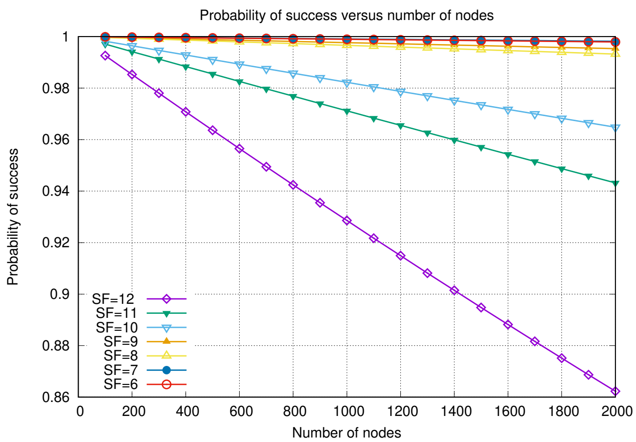

Figure 4: Probabilities of successful transmission in an inhomogeneous network with spatial density of nodes decaying with the distance to the base station as $r^{-1/5}$.

receiving the packet until the end of the transmission. However, the transmission may be lost because of the interference created by other packet emissions started during the reception, in which case the error will be detected only at the end of the reception. (These interfering packets will be lost as well since the base station was not idle to lock on them at their arrival epochs).

6.2 Packet admission policy

In order to improve Aloha efficiency, a packet admission policy is introduced. Once the base station detects a packet transmission it may decide to receive or ignore the detected packet according to some (possibly randomized) packet admission policy, based, for example, on the received power. The idea is to allow the base station to ignore some very weak packets, whose successful reception is hardly likely, as well as to drop some fraction of very strong packets (which will be transmitted at another time) in order to let the base station be more often available for packets with a moderate received power. In the model proposed and studied in [7] this admission policy took a form of the probability $d(x)$ of packet drop (even if the base station is available) applied for packets sent from the location x on the plane. For a given path-loss model it can be related to the mean received power. It was shown how to optimize it so as to maximize the coverage or total throughput in the network.

6.3 Aloha model analysis

Assuming the same Poisson rain model of packet transmission as used in the present paper, with Rayleigh fading, the above Aloha SINR collision model, and some arbitrary probabilistic packet admission policy, it is possible to express the probability of successful reception of packets versus the transmission location x on the plane. The analysis is however more complicated than that of LoRa due to a more complicated (detailed) SINR collision model.

More specifically, denoting by λ the space-time density of transmissions, B packet transmission time (the same for all transmissions), the probability of successful reception of a packet transmitted from location x, by the base station located at the origin, can be factorized as follows

$$\Pi_x = \frac{d(x)}{1+\lambda B}\,\Pi'_x,$$

where $\frac{1}{1+\lambda B}$ is the (well known Erlang's formula for the) probability that the packet finds the base station idle (ready to lock on the transmission) when it arrives, $d(x)$ is the probability that it is not dropped by the admission policy, and Π'_x is the conditional probability that it is correctly received (SINR condition satisfied), given the base station starts receiving it. Explicit evaluation of Π'_x requires a fine analysis of the interference (averaged over the reception period) from different types of packets: non-admissible (too weak power to lock on) and admissible ones arriving before and during the current reception. Assuming Rayleigh fading, Π'_x can be expressed using the Laplace transforms of these components of the interference and of the noise, as shown in [7, Fact 3.2 with the Laplace transforms given by (9.1), (9.2), (9.3)]. These formulas and their approximations developed in [7] allow one to solve various optimization problems regarding Π_x with respect to the packet admission policy $d(x)$.

A detailed comparison of the performance of LoRa and Aloha using the Poisson rain framework is left for future work.

7 CONCLUSION

We have presented a simple model to study the performance of the node-to-base-station link in a realistic, spatial deployment of a LoRa network, which is a type of low-power, wide-area technology destined for the Internet of Things. Crucial components of the model are the space-time Poisson process of packet transmissions (called in the literature the Poisson rain model) and a packet collision model accounting for LoRa intra-technology interference treatment, which has already been validated with respect to measurements in laboratory conditions. Our model allows for an explicit optimization of the packet reception probabilities with respect to numerous LoRa parameters, taking into account possibly non-homogeneous node deployment on the plane and various types of wireless channel propagation effects. In particular, it sheds light on how the, crucial to LoRa technology, link budget thresholds (called sensitivities) impact the performance of different categories of transmissions. We have studied only one base-station scenario; extensions to multi-base station networks are left for future work. A fair comparison of LoRa to non-slotted Aloha within the proposed framework is also possible and desirable.

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