Understanding Decision-focused Learning

Financial Engineering Lab
Department of Industrial Engineering

ACM ICAIF 2025

ULSAN NATIONAL INSTITUTE OF SCIENCE AND TECHNOLOGY

KAIST
Korea Advanced Institute of Science and Technology

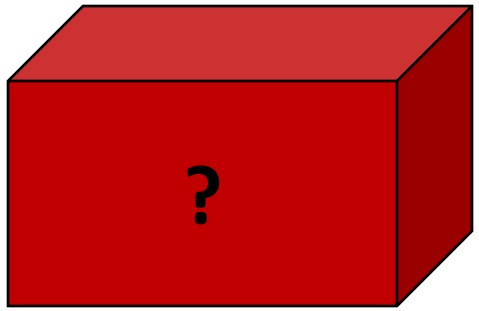
Introduction

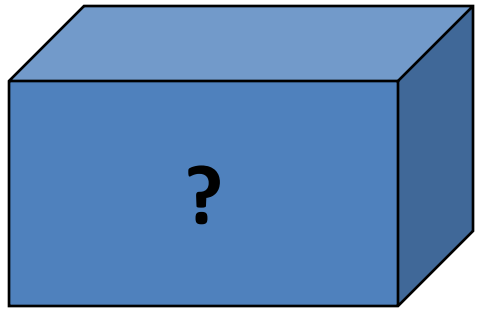
- **Motivating Example**

1. Must choose one box

2. Predict the value of each box

3. Decide the box with a higher value





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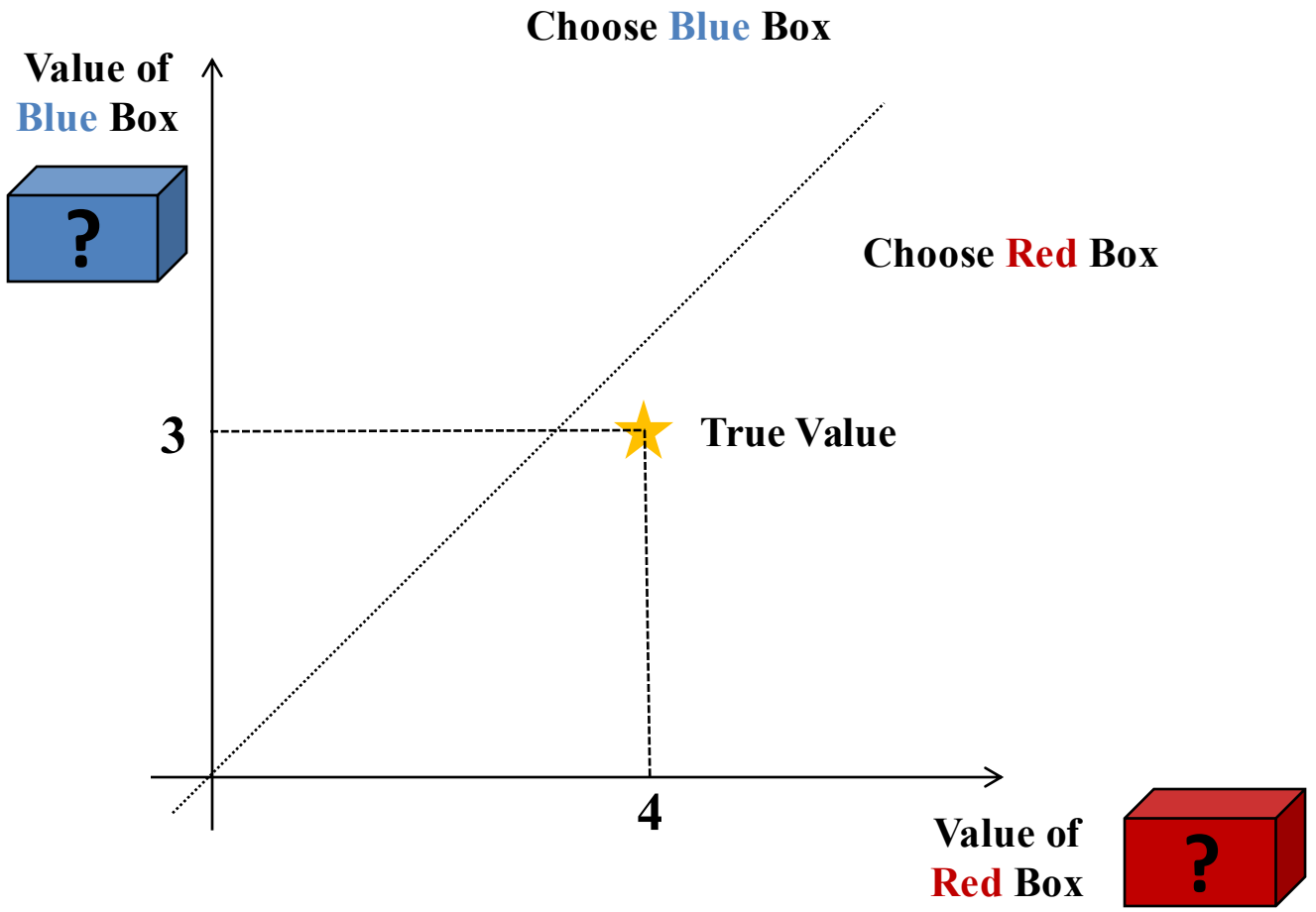

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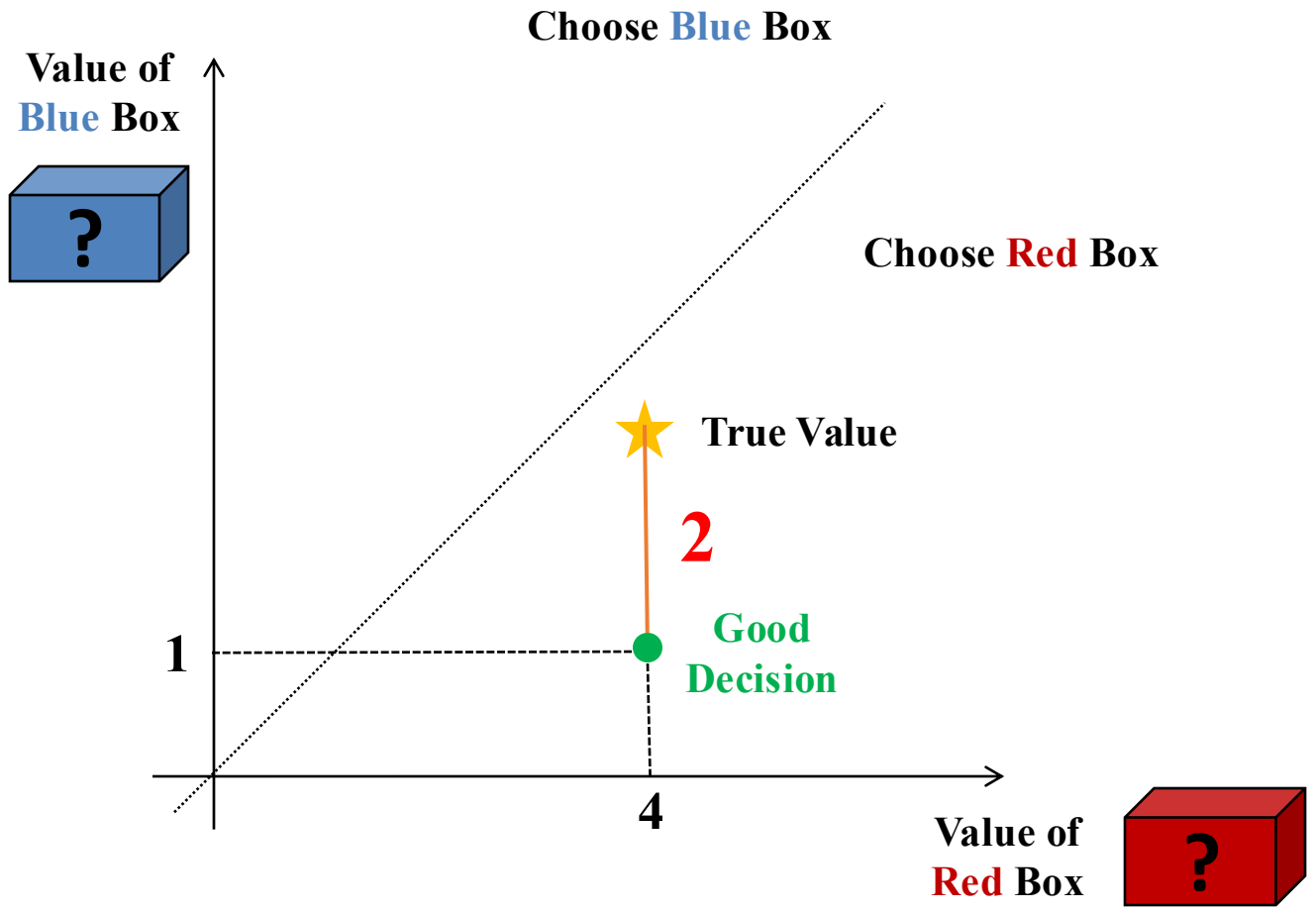

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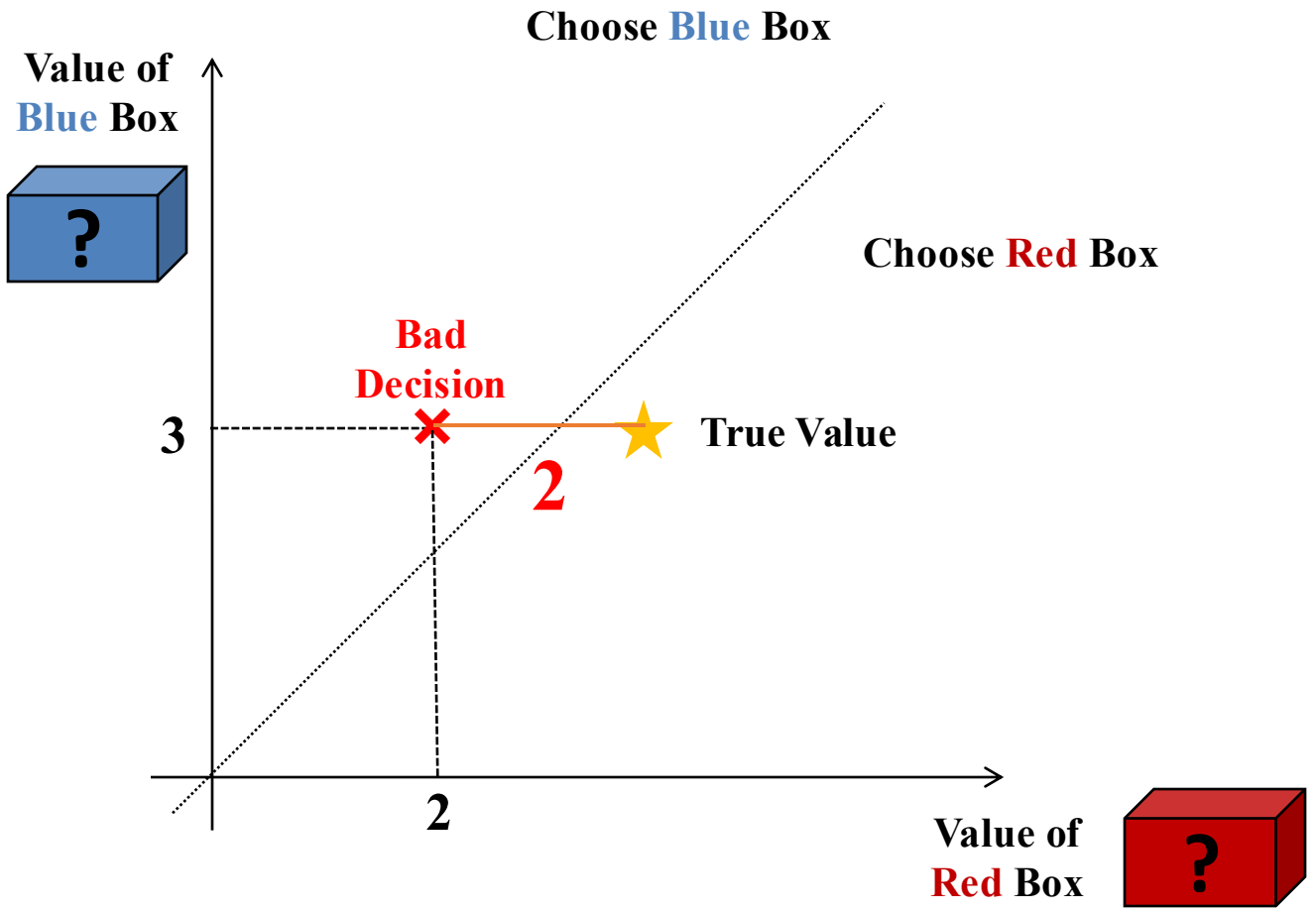

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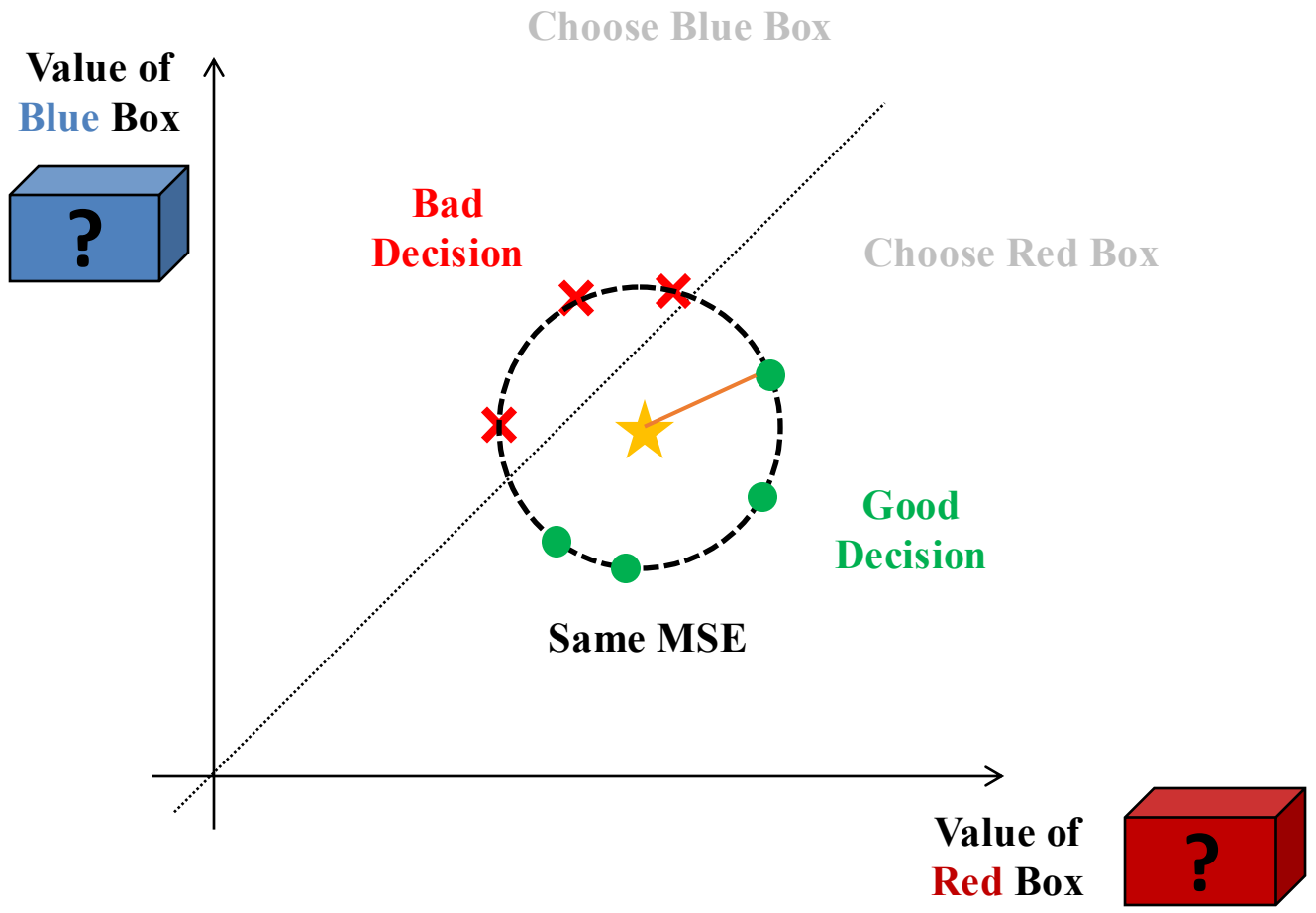
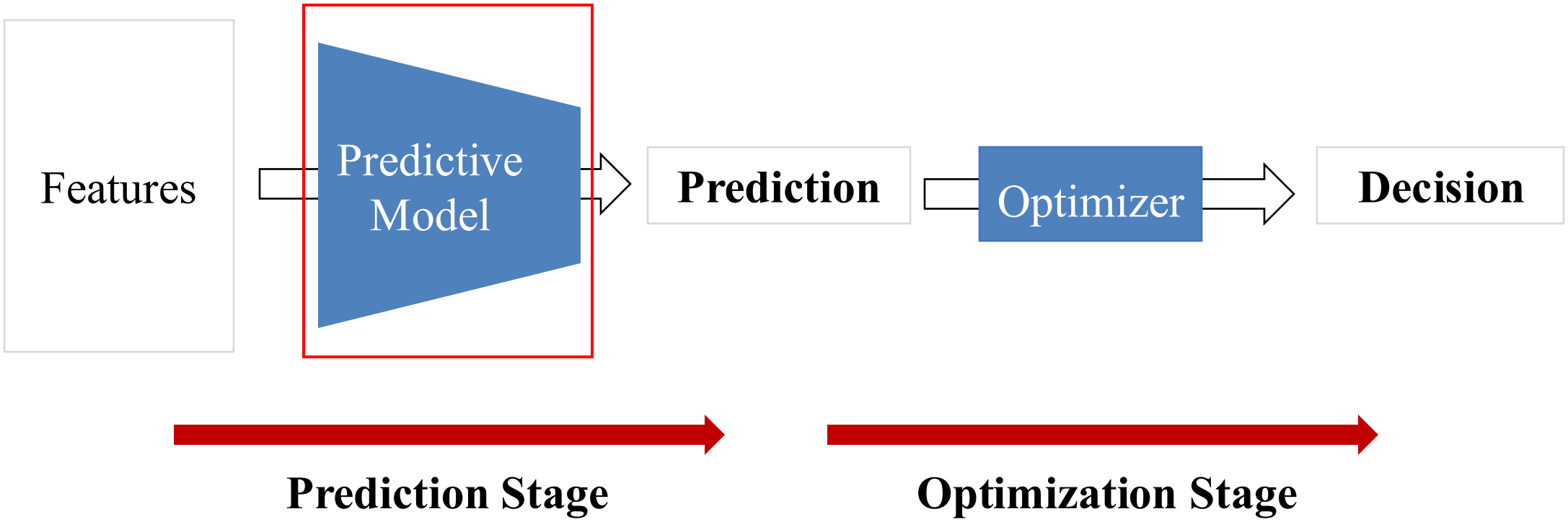


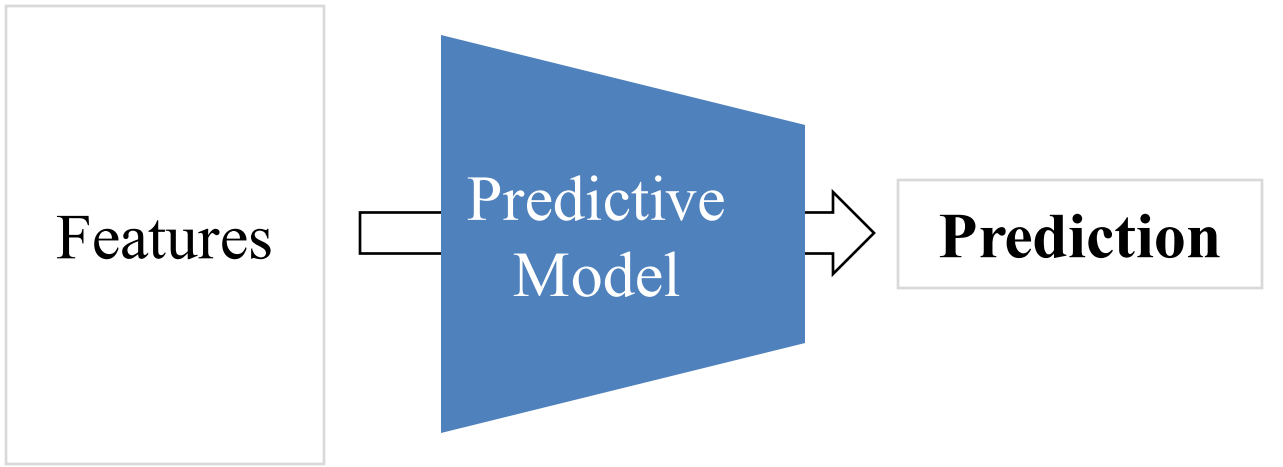
Prediction & Optimization Framework

- **Overview**

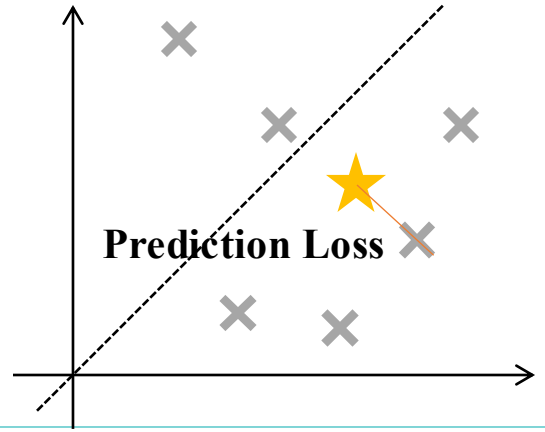


Prediction & Optimization Framework

- **Prediction-focused Learning (PFL)**

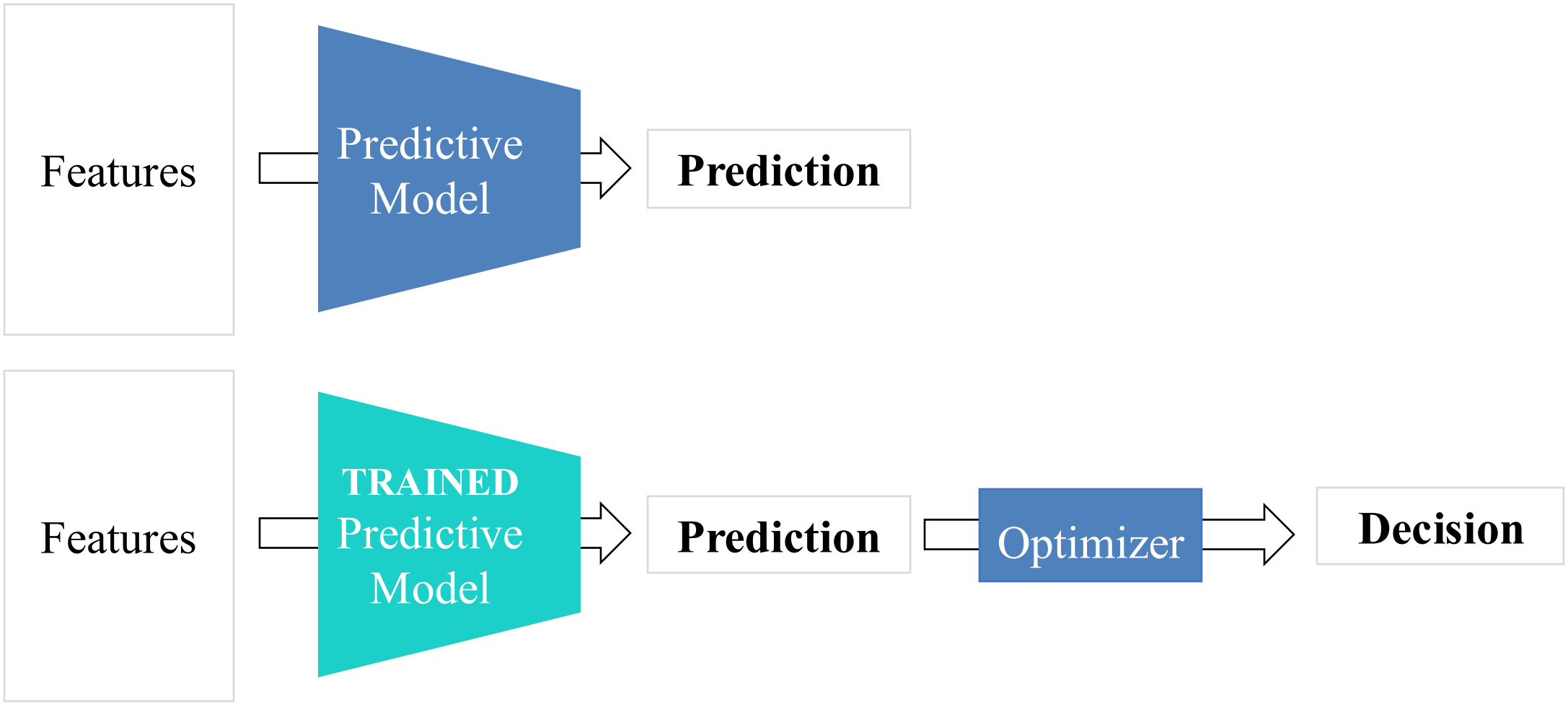


$$\frac{\partial \mathcal{L}_{\text{pred}}}{\partial \theta} = \frac{\partial \mathcal{L}_{\text{pred}}}{\partial (\text{prediction})} \cdot \frac{\partial (\text{prediction})}{\partial \theta}$$



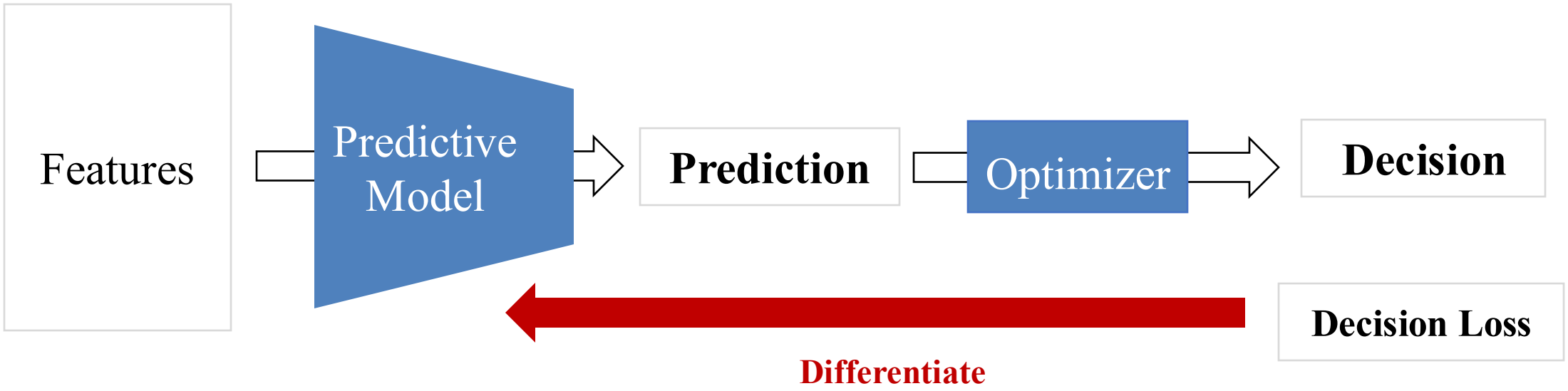
Prediction & Optimization Framework

- **Prediction-focused Learning (PFL)**



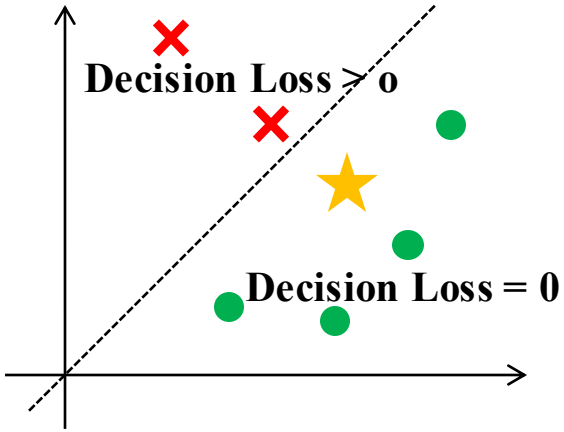
Prediction & Optimization Framework

- **Decision-focused Learning (DFL)**



$$\frac{\partial \mathcal{L}_{\text{dec}}}{\partial \theta} = \frac{\partial \mathcal{L}_{\text{dec}}}{\partial(\text{decision})} \cdot \boxed{\frac{\partial(\text{decision})}{\partial(\text{prediction})}} \cdot \frac{\partial(\text{prediction})}{\partial \theta}$$

How?



Prediction & Optimization Framework

- **Decision-focused Learning (DFL)**

$$\frac{\partial \mathcal{L}_{\text{dec}}}{\partial \theta} = \frac{\partial \mathcal{L}_{\text{dec}}}{\partial (\text{decision})} \cdot \boxed{\frac{\partial (\text{decision})}{\partial (\text{prediction})}} \cdot \frac{\partial (\text{prediction})}{\partial \theta}$$

Exact Differentiation

- Derive exact gradient $\frac{\partial (\text{decision})}{\partial (\text{prediction})}$ for certain optimization formulations

Smoothing, then Differentiation

- Smoothen the prediction-to-decision mapping, then derive $\frac{\partial (\text{decision})}{\partial (\text{prediction})}$

Surrogate Loss

- Define $\mathcal{L}_{\text{surrogate}}$ that gives an easy $\frac{\partial \mathcal{L}_{\text{surrogate}}}{\partial (\text{prediction})}$

- then derive $\frac{\partial \mathcal{L}_{\text{dec}}}{\partial \theta} \approx \frac{\partial \mathcal{L}_{\text{surrogate}}}{\partial \theta} = \frac{\partial \mathcal{L}_{\text{surrogate}}}{\partial (\text{prediction})} \cdot \frac{\partial (\text{prediction})}{\partial \theta}$

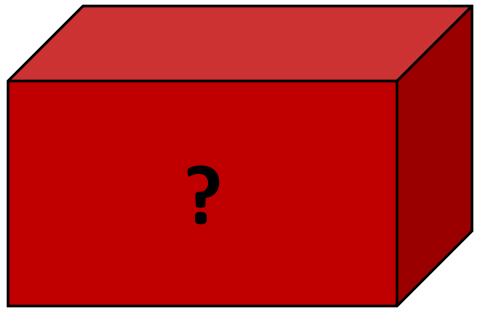
Hands-on Exercise

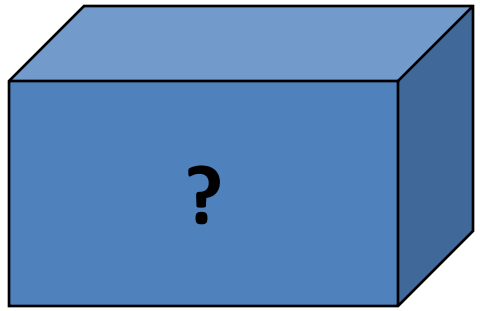
- **Recall: Motivating Example**

1. Must choose one box

2. Predict the value of each box

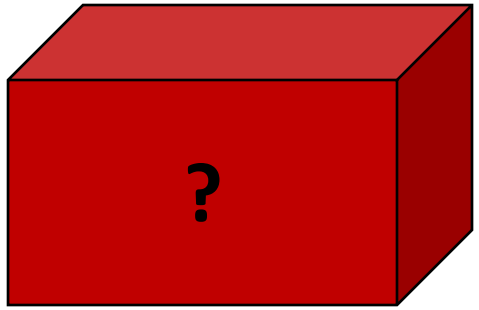
3. Decide the box with a higher value

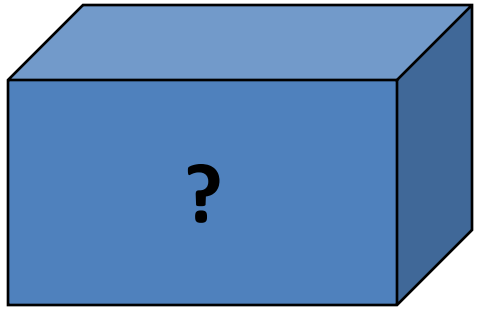




Hands-on Exercise

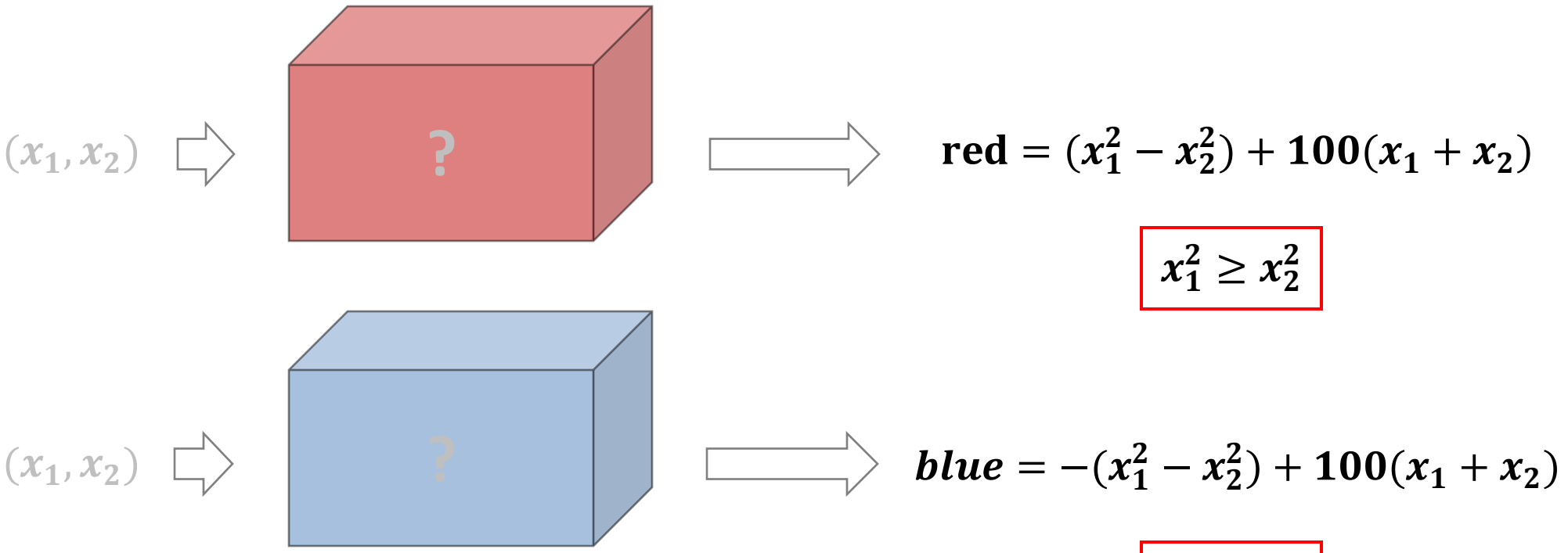
- **True underlying reward of each box**

(x_1, x_2) ⇨  ⇨ $\mathbf{red} = (x_1^2 - x_2^2) + 100(x_1 + x_2)$

(x_1, x_2) ⇨  ⇨ $\boldsymbol{blue} = -(x_1^2 - x_2^2) + 100(x_1 + x_2)$

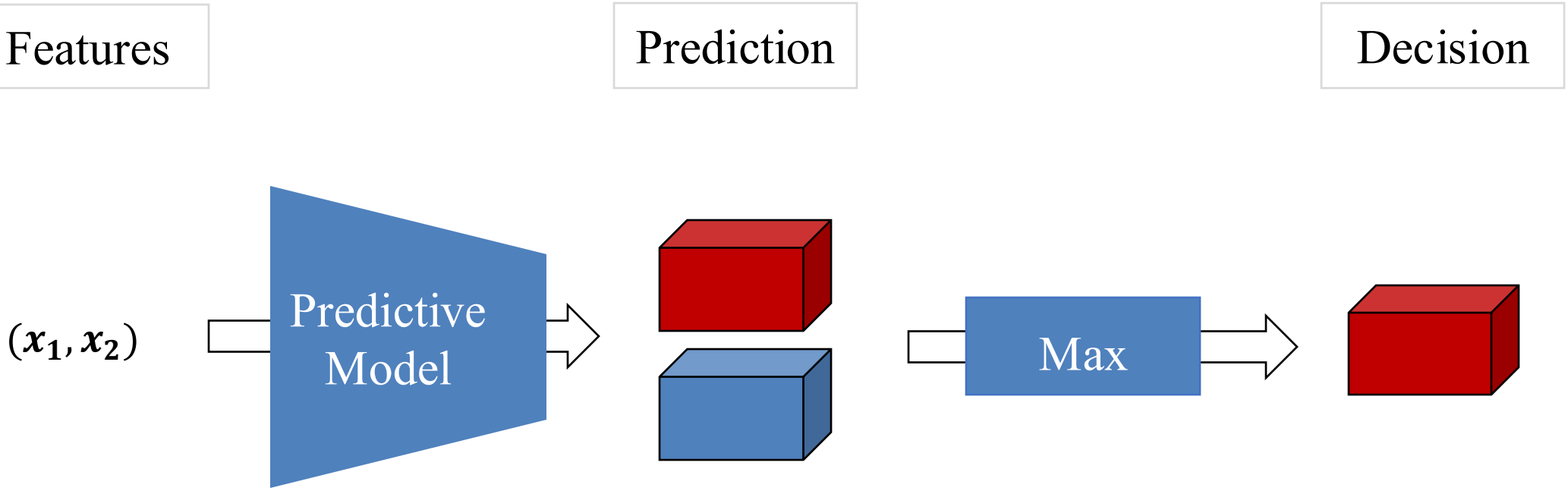
Hands-on Exercise

- **True optimal decision**



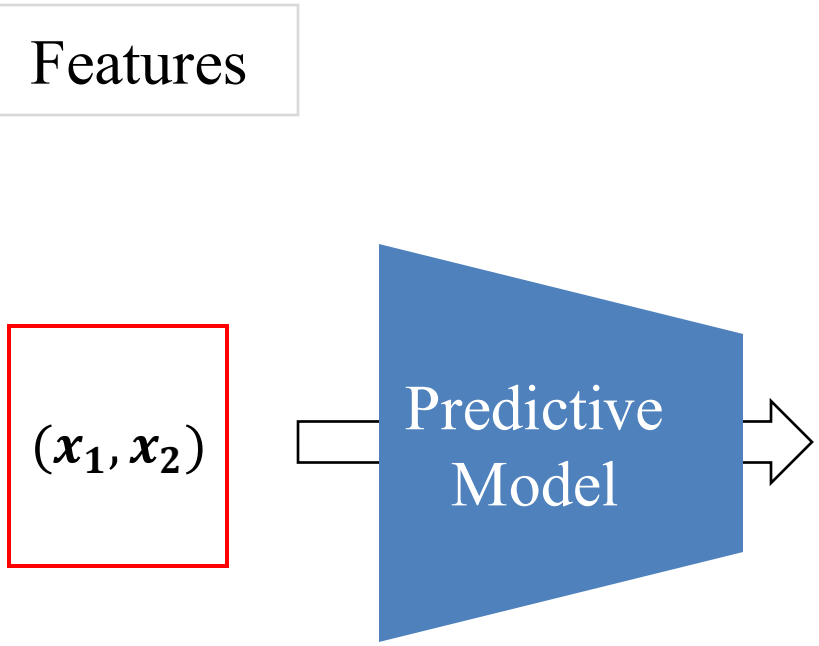
Hands-on Exercise

- **Overview of Problem Framework**



Hands-on Exercise

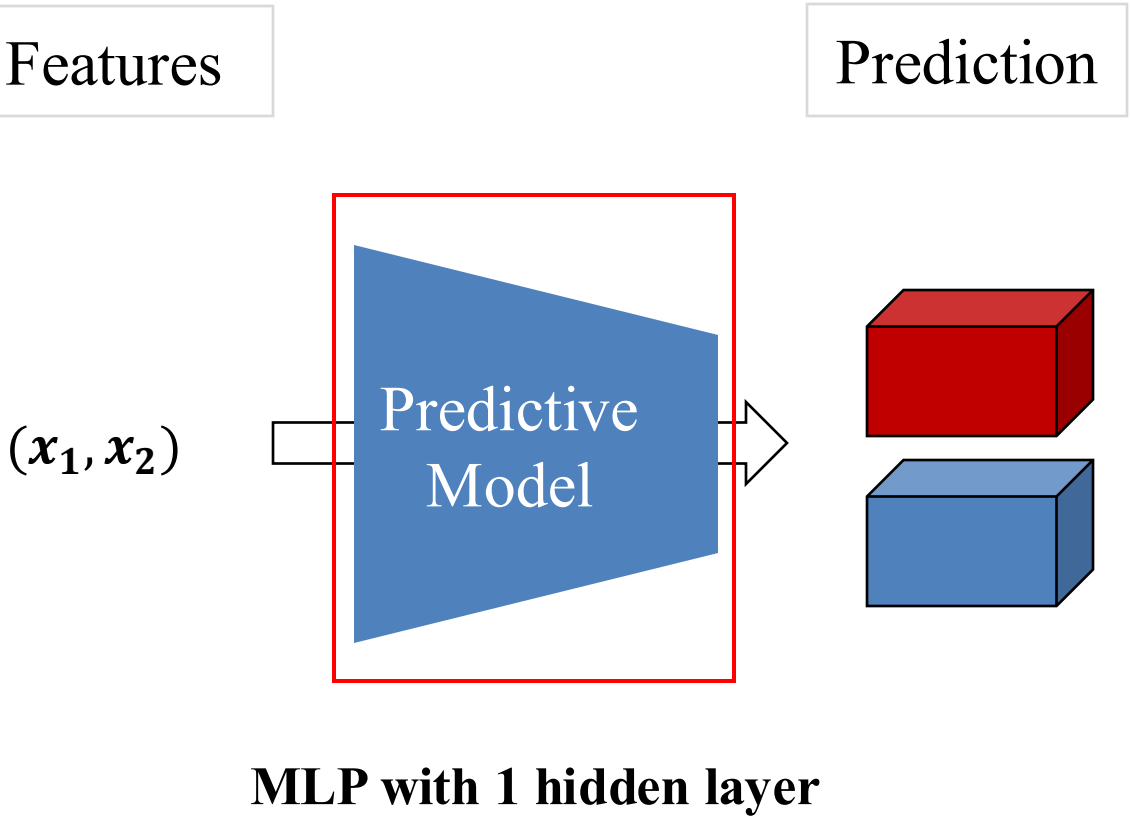
- **Step 1: Data Generation**



$$x_i \sim U(-1, 1)$$

Hands-on Exercise

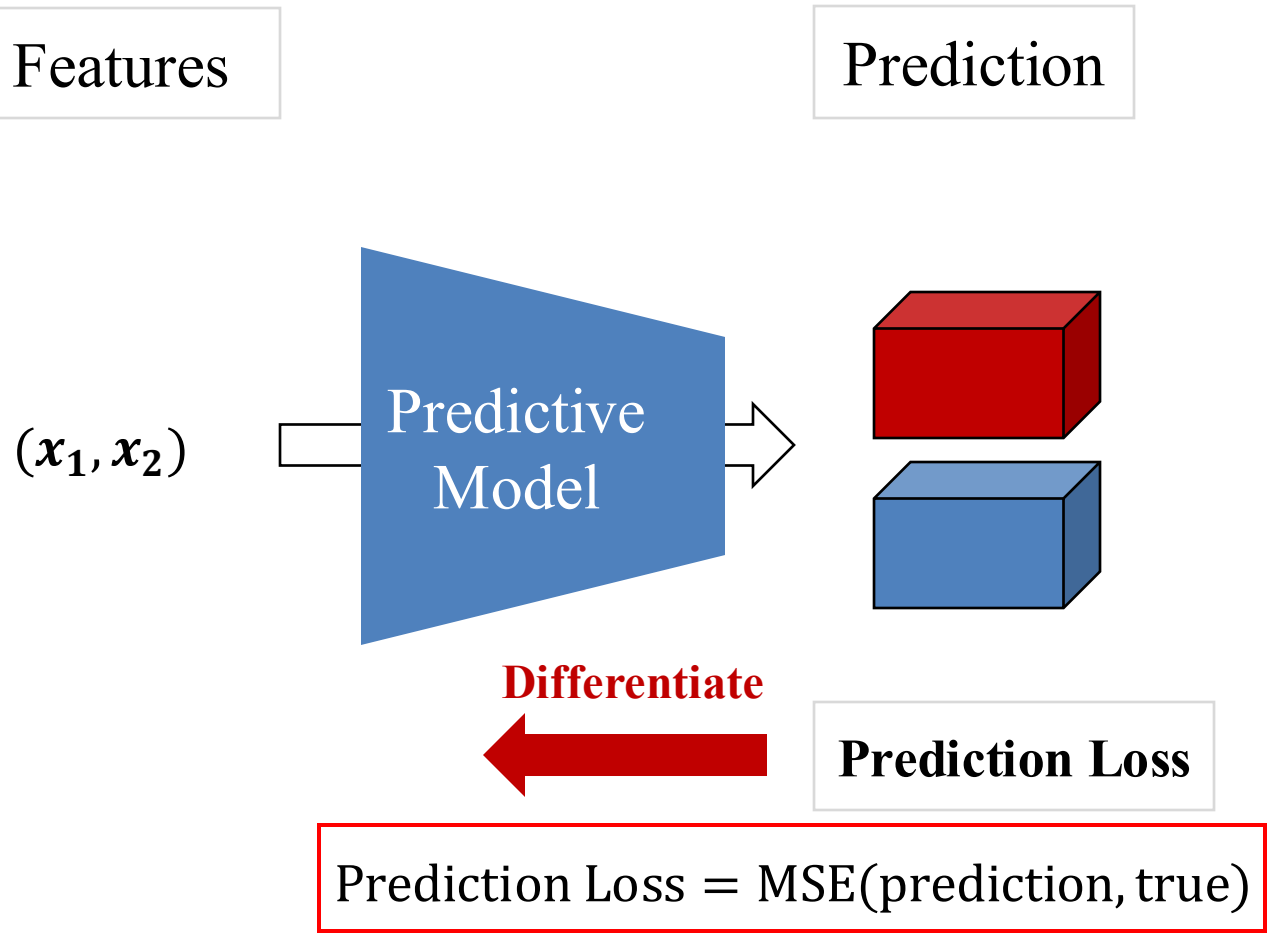
- **Step 2: Building Predictive Model**



MLP with 1 hidden layer

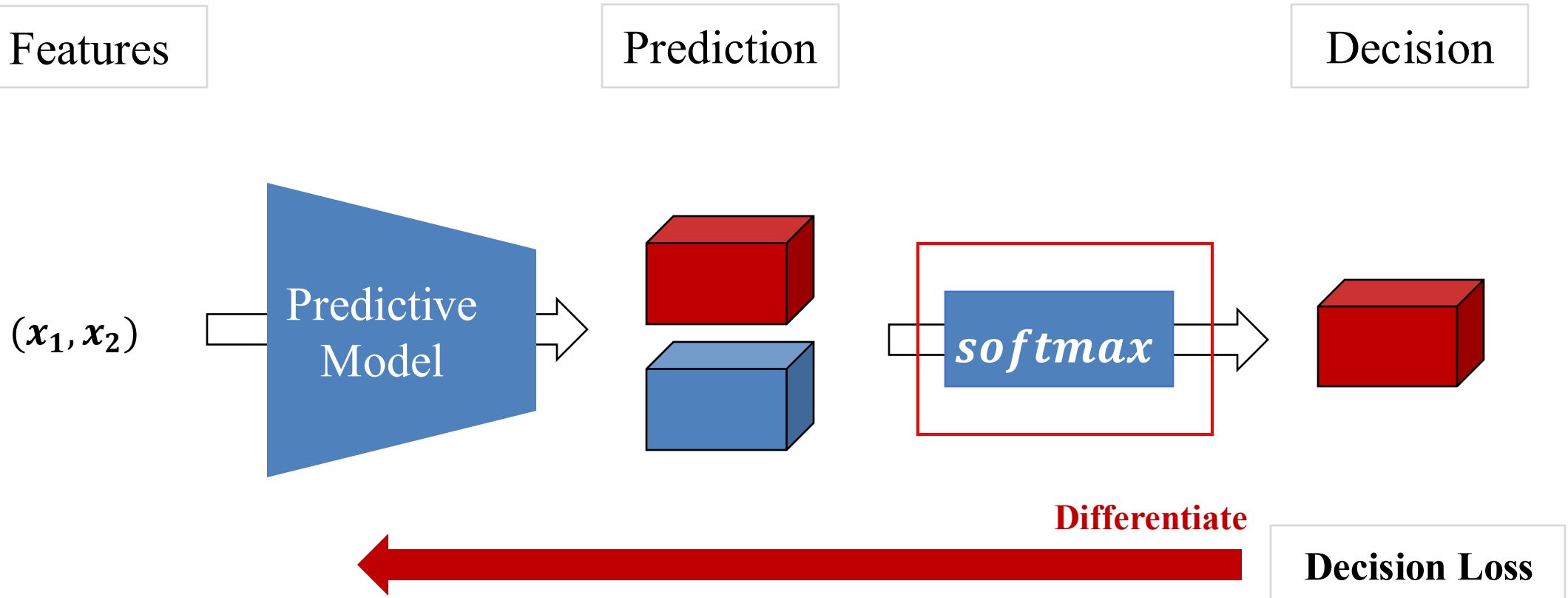
Hands-on Exercise

- **Step 3A: Train with Prediction-focused Learning (PFL)**



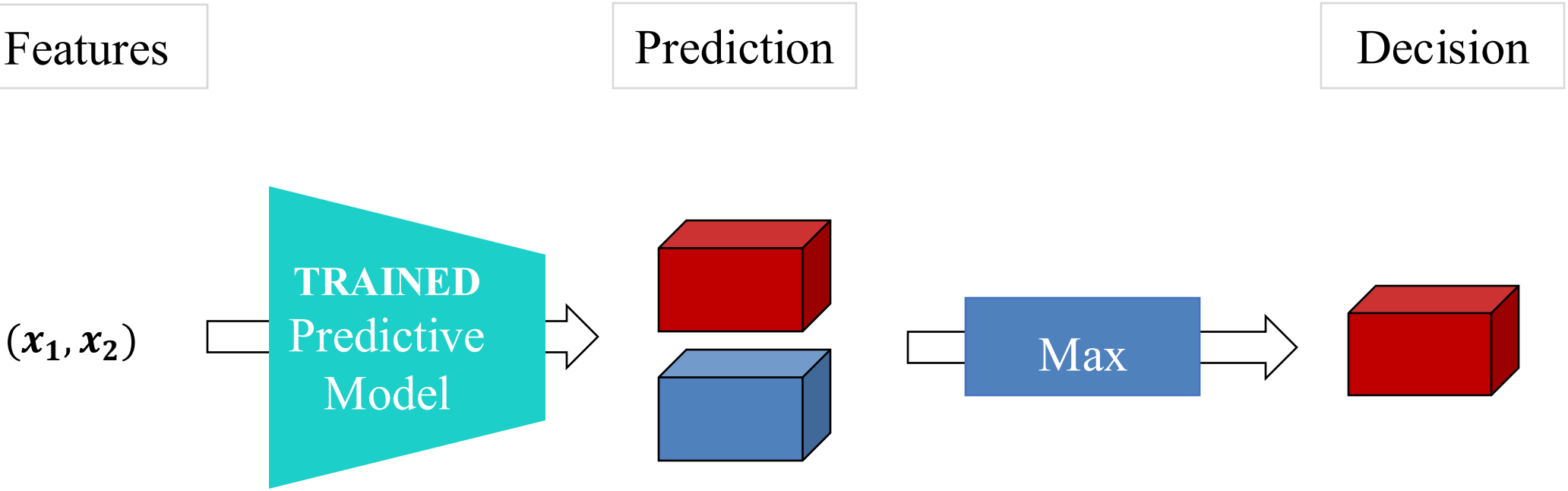
Hands-on Exercise

- **Step 3B: Train with Decision-focused Learning (DFL)**



Hands-on Exercise

- **Step 4: Test and Plot Results**



Hands-on Exercise

Github Link