

# Numerical Solution for the Line Curvature

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## 1 Formular

The expression of curvature:

$$\kappa = \frac{\ddot{\vec{r}} \times \dot{\vec{r}}}{|\dot{\vec{r}}|^3} \quad (1)$$

In 2D:

$$\kappa = \frac{x''y' - x'y''}{((x')^2 + (y')^2)^{3/2}} \quad (2)$$

With three points  $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ , to estimate the curvature, we firstly fit the three point to a 2D expression of parametric equation:

$$\begin{cases} x = a_1 + a_2t + a_3t^2 \\ y = b_1 + b_2t + b_3t^2 \end{cases} \quad (3)$$

With upper and lower limit of  $t_a$  and  $t_b$ , we can apply the three points

$$\begin{aligned} (x, y)|_{t=-t_a} &= (x_1, y_1) \\ (x, y)|_{t=0} &= (x_2, y_2) \\ (x, y)|_{t=t_b} &= (x_3, y_3) \end{aligned}$$

to the parametric equation:

$$\begin{cases} x_1 &= a_1 - a_2t_a + a_3t_a^2 \\ x_2 &= a_1 \\ x_3 &= a_1 + a_2t_b + a_3t_b^2 \end{cases}$$

## 2 Usage