Numerical Solution for the Line Curvature

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1 Formular

The expression of curvature:

$$\kappa = \frac{\ddot{\vec{r}} \times \dot{\vec{r}}}{\left|\dot{\vec{r}}\right|^3} \tag{1}$$

In 2D:

$$\kappa = \frac{x''y' - x'y''}{\left((x')^2 + (y')^2\right)^{3/2}} \tag{2}$$

With three points $(x_1, y_1), (x_2, y_2), (x_3, y_3)$, to estimate the curvature, we firstly fit the three point to a 2D expression of parametric equation:

$$\begin{cases} x = a_1 + a_2 t + a_3 t^2 \\ y = b_1 + b_2 t + b_3 t^2 \end{cases} \tag{3}$$

With upper and lower limit of t_a and t_b, we can apply the three points

$$(x, y)|_{t=-t_a} = (x_1, y_1) \tag{4}$$
$$(x, y)|_{t=0} = (x_2, y_2) \tag{5}$$
$$(x, y)|_{t=t_b} = (x_3, y_3) \tag{6}$$

to the parametric equation:

$$\begin{cases} x_1 &= a_1 &-a_2 t_a + a_3 t_a^2 \\ x_2 &= a_1 \\ x_3 &= a_1 &+a_2 t_b + a_3 t_b^2 \end{cases} \tag{7}$$

and

$$\begin{cases} y_1 &= b_1 &-b_2 t_a + b_3 t_a^2 \\ y_2 &= b_1 \\ y_3 &= b_1 &+b_2 t_b + b_3 t_b^2 \end{cases} \tag{8}$$

2 Usage