CS5000 HW03

Bridger Jones A02314787

September 24, 2021

Problem 1

If L_1 and L_2 are regular, then we can construct a DFA that represents the intersection of them. The cartesian product of M_1 and M_2 will result in DFA M_3 that accepts $L_1 \cap L_2$. The formal proof is from Lecture 5 slide. This can also be proven clearly with construction and drawing out the two DFAs and then the cartesian product of them.

Proof:

Proof : If L_1 and L_2 are regular, there are two DFAs $M_1 = (Q, \Sigma, \delta_1, q_0, F_1)$ such that $L_1 = L(M_1)$ and $M_2 = (R, \Sigma, \delta_2, r_0, F_2)$ such that $L_2 = L(M_2)$. We construct a new DFA $M_3 = (Q \times R, \Sigma, \delta_3, (q_0, r_0), F_1 \times F_2)$, where $\delta_3((q, r), a) = (\delta_1(q, a), \delta_2(r, a))$, where $q \in Q, r \in R, a \in \Sigma$. For any $x \in \Sigma^*$, $x \in L_1 \cap L_2 \Leftrightarrow$ $\delta_1^*(q_0, x) \in F_1$ and $\delta_2^*(r_0, x) \in F_2 \Rightarrow$ $(\delta_1^*(q_0, x), \delta_2^*(r_0, x)) \in F_1 \times F_2 \Leftrightarrow$ $\delta_3^*((q_0, r_0), x) \in F_1 \times F_2 \Leftrightarrow x \in L_1 \cap L_2$.

Problem 2

If L_1 and L_2 are regular, and we know that they are closed under both intersection and union, and we know that they are closed with the complement, we can say that $L_2 - L_1 = L_2 \cap \bar{L}_1$. This shows that they are regular.

Problem 3