MATM 4801 CONCEPTS IN MATHEMATICS EDUCATION

UNIT 1 The History of Mathematics

The development of mathematics from historical perspective to present situation. What changes have been observed? What challenges still affecting learner understanding in some concepts? How do teachers teach to address challenges learners face? What new mathematical concepts have been discovered?

Read The History of mathematics book in the Library for more information.

Unit 2: Proofs and Logic in Mathematics

3.0 Introduction

In this unit you will be introduced to mathematical proofs and logic. This will help you to understand the role of mathematical proofs in the teaching and learning of mathematics. In addition you will also discuss some of the challenges students meet when learning mathematics proofs. Furthermore this will help you suggest better approaches when teaching proofs and logic to your students. Since logic is part of proof arguments it is important for you to gain knowledge of logic in mathematics so that you can teach your students better.

3.1 Learning Outcomes

1. To explain the role of mathematical proofs
2. Discuss challenges associated with proofs and logic
3. Approach to the teaching of proofs and logic

3.2 Prerequisite Knowledge

An assumption is that you have already acquired knowledge of university mathematics and have experienced the teaching of mathematics.

3.3 Time Required 

In order to understand this unit properly and thoroughly you will need 2 hours of independent study.

3.4 Meaning of mathematical proofs and logic

A mathematicalproofis an argument which convinces other people that something is true. A *proof* is an argument from hypotheses (assumptions) to a conclusion. Each step of the argument follows the laws of *logic*. In mathematics, a statement is not accepted as valid or correct unless it is accompanied by a proof. This insistence on proof is one of the things that set mathematics apart from other subjects.It isbelieved that proofs provide the best foundation of logical reasoningabout mathematics structure. The mathematics we are experiencingtoday is as a result of proofs conducted over 2500 years ago by Euclidand Pythagoras (Fauvel J and Maanen V J, 2000). Formal logic originated with the ancient Greek philosopher Aristotle, who believed that operations of valid reasoning were summed up in his list of 14 syllogisms, the most famous being "All men are mortal; all heroes are men; therefore all heroes are mortal."The main application is toward logical reasoning and problem solving in mathematics, but logic has a place in every domain of human thought.

3.5 Understanding Logical Reasoning in Mathematics

In mathematics, we study **statements**, which are declarative sentences that are either true or falsebut not both.

Activity 1

Classify each of these as statement or non statement.

1. Pigs can fly
2. $3 + 4 = 7$

You can also be interested to observe the following examples

a) $2 + 3 = 5$

b) $4 + 2 = 13$ both a) and b) are statements

c) $2x + 3 > 7$ c) is not a statement since x is not specified.

3.6 The Role of Mathematical Proofs and Logic

Mathematics promotes critical thinking and logical reasoning among learners. As such the teaching of mathematics need to foster the

development of skills and knowledge to students so that they understand proofs and logic.

Activity 2 

Explain the role of geometrical theorems and proofs in secondary school curriculum.

3.7 The Challenges Experienced by students when Learning mathematics proofs

All secondary school students take a course in mathematics to prepare them for future use. Some mathematics concepts in the curriculum include proofs for students to understand properly the mathematics behind it. However most students find it hard to grasp the process and logic arguments associated with proofs. Some students have not liked the language (vocabulary) used in proofs which is not familiar to them. Some do not understand the whole process of the mathematical proof since it uses several interrelated concepts. Students are not familiar with some symbols andletters associated with proof concept and failure to actually point out the aim proof is looking for.

The structure of proofs; a proof argument is done after astatement of a theorem which need more understandingbefore working on the proof argument. Furthermore, mostproofs are of higher order reasoning, which does not promote rote memory but understanding, as a result students find it difficult to grasp the content effectively and be able to apply. Unable to recognize its application to real world as such most students would like just to learn mathematics without following this rigorous proofs because they can hardly recognize proofsand its immediate use.

Another challenge is students' attitude towards proofs due to lack of logical reasoning, moststudents decided to ignore anything involving mathematics where proofs are encouraged. Most of them have acarryover effect and look at issues involving mathematicsas something which they cannot manage. Failure to connect previous learnt concepts that could be of more help in the proofs arguments. The problem of connectivity and experiential learning, most students fail to recall and apply in the new mathematics.

Unit 3: MATHEMATICS LITERACY

INTRODUCTION

In this unit you will be introduced to unfamiliar concept which is important in the teaching and learning of mathematics. Most people are aware of literacy as reading and writing which is common in English language however literacy is a cross cutting concept in all areas of real world experiences. You will understand literacy in mathematics, and why it is important in the teaching and learning of mathematics. As such you will recognize the need to promote it as a critical tool in the effective teaching and learning of mathematics.

Learning Outcomes

- **Understand literacy as a concept in mathematics**
- **Recognise the competencies required in mathematics literacy**
- **Designing activities for improving mathematics literacy**

4.2 Pre-requisite knowledge : In this unit it is assumed that you have studied college level mathematics and have conducted teaching practicum

4.3 Time Required: In order to complete this unit successfully you require not more than 2hours of study

Activity 4.1

Let students brainstorm the meaning of mathematics literacy

You may also understand that literacy in mathematics is *an individual's capacity to identify and understand the role that mathematics play in the world, to make well-founded judgements and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen.*

Innumeracy, or the inability to handle numbers and data correctly and to evaluate statements regarding problems and situations that invite mental processing and estimating, is a greater problem than our society generally recognizes.Mathematical literacy is not restricted to the ability to apply quantitative aspects of mathematics but involves knowledge of mathematics in the broadest sense

What are the common challenges to developing math literacy?

- Students typically struggle with pattern recognition and the ability to transfer skills.
- Math requires abstract thinking and sometimes that is a difficult transition for students to make.

- Students also find it a challenge to interpret word problems—figuring out exactly what the problem is asking them to do, and the steps that they need to complete to find the answer.

Competencies Needed for Mathematics Literacy

1. **Mathematical thinking and reasoning**.

   Posing questions characteristic of mathematics; knowing the kind of answers thatmathematics offers, distinguishing among different kinds ofstatements; understanding and handling the extent and limitsof mathematical concepts.

2 **Mathematical argumentation.**
   Knowing what proofs are;knowing how proofs differ from other forms of mathematicalreasoning; following and assessing chains of arguments; having a feel for heuristics; creating and expressing mathematicalarguments.
3. **Mathematical communication.**
   Expressing oneself in a varietyof ways in oral, written, and other visual form; understanding someone else's work.
4. **Modeling.**
   Structuring the field to be modeled; translating
   reality into mathematical structures; interpreting mathematical models in terms of context or reality; working with models; validating models; reflecting, analyzing, and offering critiques of models or solutions; reflecting on the modeling
   process.
5. **Problem posing and solving**.
   Posing, formulating, defining,and solving problems in a variety of ways.
6. **Representation**.
   Decoding, encoding, translating, distinguishing between, and interpreting different forms of representations of mathematical objects and situations as well as understanding the relationship among different representations.
7. **Symbols.**
   Using symbolic, formal, and technical language and
   operations.
8. **Tools and technology**.
   Using aids and tools, including technology when appropriate.

Activity 4.2

Discuss in groups how you would promote any three competencies in the teaching of the topic **quadratic expressions** to secondary school students.

To be mathematically literate, individuals need all these competencies to varying degrees, but they also need confidence in their own ability to use mathematics and comfort with quantitativeideas. An appreciation of mathematics from historical, philosophical, and societal points of view is also desirable.

To provide a clearer picture of literacy in mathematics, it seemswise to reflect for a moment on what constitutes mathematics. However we need to address on how mathematics is organized by focusing on the four phenomenological categories for an effective curriculum in order to achieve mathematics literacy properly and effectively. These categories are ***quantity, space and shape, change and relationships, and uncertainty.***

Acticvity 4.3

In groups let students use secondary school mathematics curriculum/syllabus to write down topics under the categories stipulated.

Using these four categories, mathematics content can be organized into a sufficient number of areas to help ensure a spread ofitems across the curriculum, but also a small enough number to
avoid an excessively fine division—which would work against afocus on problems based in real-life situations. Each phenomenological category is an encompassing set of phenomena and concepts that make sense together and may be encountered withinand across a multitude of quite different situations. By their verynature, each idea can be perceived as a general notion dealing witha generalized content dimension.

Quantity:
Important aspects include an understanding of relative size, recognition of numerical patterns, andthe ability to use numbers to represent quantifiable attributes ofreal-world objects (measures).

Space and Shape:
In the study of shapes and constructions, we look for similarities and differences as we analyze
the components of form and recognize shapes in different representations and different dimensions.To achieve this, we must be able to understand the properties of objects and the relative positions of objects; we must beaware of how we see things and why we see them as we do; and wemust learn to navigate through space and through constructions
and shapes.

Change and Relationships:
Some change processes can be modeled by straightforward mathematical functions:linear, exponential, periodic or logistic, discrete or continuous.But many relationships fall into different categories, and dataanalysis is often essential to determine the kind of relationshippresent. mathematical relationships often take the shape of equations or inequalities, but relations of a more general nature (e.g.equivalence, divisibility) may appear as well. Functional thinking—that is, thinking in terms of and about relationships—is oneof the fundamental disciplinary aims of the teaching of mathematics. Relationships can take a variety of different representations,
including symbolic, algebraic, graphic, tabular, and geometric. Asa result, translation between representations is often of key importance in dealing with mathematical situations.

Uncertainty:
Uncertainty is intended tosuggest two related topics: data and chance, phenomena that are the subject of mathematical study in statistics and probability,respectively.

An important part of mathematical literacy is using, doing, andrecognizing mathematics in a variety of situations. In dealing withissues that lend themselves to a mathematical treatment, the
choice of mathematical methods and representations often depends on the situations in which the problems are presented.Teachers of mathematics often complain that students have difficulty applying the mathematics they have learned in differentcontexts. It is observed that nonscience students often dislike contexts involving physics applications in mathematics because they do not understand the physics.Building from this, I think we need to examine the wisdom ofconfronting nonscience students with mathematics applicationsthat need specific science literacy at a nonbasic level. As has beenpointed out before, to effectively transfer their knowledge fromone area of application to another, students need experience solving problems in many different situations and contexts (de Lange1987). Making competencies a central emphasis facilitates thisprocess: competencies are independent of the area of application.Students should be offered real-world situations relevant to them,either real-world situations that will help them to function asinformed and intelligent citizens or real-world situations that arerelevant to their areas of interest, either professionally or educationally.

Activity 4.4
Let students design activities that will promote mathematics literacy to their learners in the teaching and learning of mathematics.

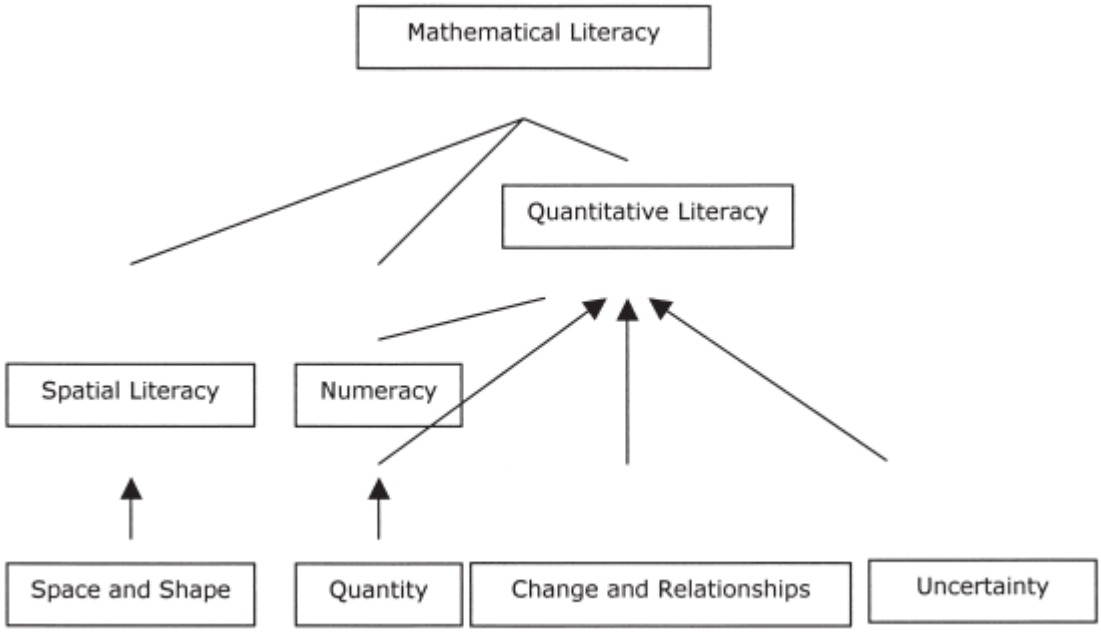
Improving math literacy skills for learners

Teachers need to **b**uild math skills or review the basic concepts before embarking on something more complex. The better the basic conceptsare understood, the easier that math becomes.

- The earlier that students repair their math skill gaps, the greater the chance that they can become math literate.
- Students need to stop thinking of math as something that they will only use in the classroom.
- Math is everywhere and it **IS** used in everyday life from cooking, to fitness, home décor, landscaping, nursing, driving, even art. What is computer science? Math. What is creativity? Math.
- Often, a lack of confidence and/or motivation gets in the way of students achieving math literacy.
- It is very important for students to have positive experiences with math from an early age with many opportunities to achieve success. Feeling capable and competent goes a long way toward achieving math literacy.
- Use real life examples, this can help to motivate students to make the effort needed to become math literate.

Summary:

You have learned mathematics literacy and its importance for learners in mathematics learning . The figure below has given as a summary of important ideas what mathematics literacy connects with the syllabus.



UNIT 4: Mathematics Professional Development

4.0 Introduction

In this unit you will learn that professional development is one of the most important resources in helping to advance effective teaching skills and techniques for the mathematics teacher. This shows that theultimate goal of professional development is improving students' learning, through the mechanism of improving instruction. The role of professional development for mathematics teachers is key to the learning of students. This will lead you to discuss the importance of an INSET and how it should be organized for professional development.

4.1 Learning outcomes

- Describing professional development
- Discussing opportunities for professional development
- Identifying ways for supporting both newly and experienced teachers
- Organising an INSET for mathematics teachers

4.2 Pre-Requisite Knowledge

Classroom teaching of mathematics and knowledge of mathematics a

4.3 Time Required

In order for you to understand the unit effectively you required not more than 2 hours of studying.

Activity 1

Discuss the meaning of *professional development* for mathematics teachers

Research evidence to date suggests that mathematics professional development should promote the growth of mathematics teachers in four major areas.

Build teachers' mathematical knowledge and their capacity to use it in practice.

Teachers' mathematical knowledge matters and significantly predicts gains in students' achievement .As such teachers need mathematical knowledge that extends beyond an understanding of mathematical procedures and concepts Teachers must be able to choose appropriate mathematical tasks, judge the advantages of particular representations of a mathematical concept, help students make connections among mathematical ideas, and grasp and respond to students' mathematical arguments and solutions. Professional development that attends to dimensions of teachers' mathematical knowledge is more effective than professional development that focuses only on pedagogy or generic teaching skills.

It is believed that teachers can develop their mathematical content knowledge in a number of differentways, *including solving and discussing mathematics problems, studying students' mathematical thinking, collaborating with other teachers to plan or discuss instruction, analyzing instances of classroom practice, and using new curricular materials*

2. **Build teachers' capacity to notice, analyze, and respond to students' thinking.**

A number of studies provide evidence that professional development can help teachers learn to notice, value, and analyze students' mathematical thinking. Professional development that helps teachers attend to students' thinking can shift teachers' focus from simply evaluating students' work as correct or incorrect to analyzing the particulars of students' thinking

They also learned to make principled decisions about choosing mathematics problems that would engage and extend each student's current level of reasoning . As teachers learn to notice and analyze students' thinking, they

gain a more accurate picture of the strengths and weaknesses in students' mathematical understandings

3. **Build teachers' productive habits of mind.**

Learning to improve one's teaching practice is challenging, effortful work. An important goal of professional development is to help teachers develop the beliefs, habits, and
dispositions needed to improve practice on an ongoing basis. For example, teachers' beliefs about mathematics , curriculum , and students' capacity for learning all influence what teachers learn from professional development opportunities. Hence, an important criterion for selecting a professional development program is whether it nurtures beliefs and dispositions, that result in continued learning in daily practice.
Professional learning experiences that involve learning mathematics related to teaching can build teachers' desire to learn more mathematics, perhaps by building the sense of efficacy, identity as a mathematics learner, or collegial support for learning

4. **Build collegial relationships and structures that support continued learning.**

One way that professional development can support teachers' ongoing learning is by catalyzing changes in collegial relationships and structures for collegial work. Recent research has pointed to the value of collaboration for the learning of teachers. Collaboration with colleagues can spark the need for teachers to explain their practices and to articulate rationales for instructional decisions, helping teachers make tacit ideas visible and subject to shared scrutiny and develop deeper, more widely shared understandings of students' learning
Teachers value the kinds of professional relationships that can be built through shared inquiry into practice; such interactions with colleagues can support teachers' sense of competence as they engage in the work of changing
In addition, three features of professional development design appear to be important for supporting progress toward these goals:

- *time;*
- *systemic support for teachers' learning; and*
- *opportunities for teachers' active learning.*

Unit 5: Extracurricular activities and mathematics learning

Being developed…….