MZUZU UNIVERSITY

FACULTY OF SCIENCE, TECHNOLOGY & NNOVATION

DEPARTMENT OF ICT

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COURSE CODE	:	ARTIFICIAL INTELLIGENCE
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TASK	:	ARTIFICIAL INTELLIGENCE TAKE HOME TEST
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1. Propositional Logic, PL, constitutes a vital concept in Artificial Intelligence.

(a) Convert the following PL sentences to canonical normal form, CNF:

(i) $A \Leftrightarrow (B \vee E)$

a) Eliminate $\Leftrightarrow$ and replace $\alpha \Leftrightarrow \beta$ with $(\alpha \Longrightarrow \beta) \wedge (\beta \Longrightarrow \alpha)$

$$(A \Longrightarrow (B \vee E)) \wedge ((B \vee E) \Longrightarrow A)$$

b) Eliminate $\Longrightarrow$ $and\ replace$ $\alpha \Leftrightarrow \beta$ with $\neg\alpha \vee \beta$

$$(\neg A \vee B \vee E) \wedge (\neg\,(B \vee E) \vee A)$$

c) Move $\neg$ inwards using de Morgan's rules and double-negation:

$\neg(\neg\alpha) \equiv \alpha$ $(double\ negation\ elimination)$

$\neg(\alpha \wedge \beta) \equiv (\neg\alpha \vee \neg\beta)$

$\neg(\alpha \vee \beta) \equiv (\neg\alpha \wedge \neg\beta)$

d) Now $\wedge$ and $\vee$ operator applied in literal, have been nested. apply the distributivity law $\vee$ over $\wedge$ wherever possible.

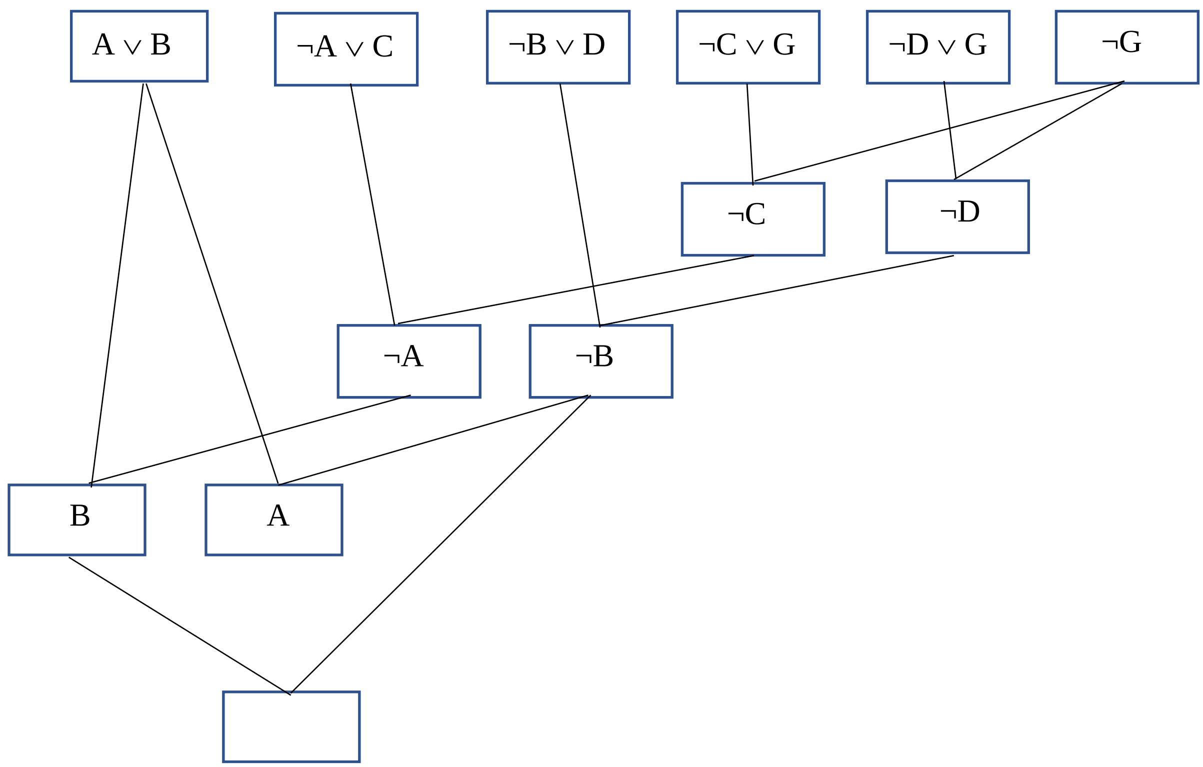
The canonical normal form CNF is: $(\neg A \vee B \vee E) \wedge (\neg B \vee A) \wedge (\neg E \vee A)$

(ii) $(A \vee B) \wedge (A \Longrightarrow C) \wedge (B \Longrightarrow D) \wedge (C \Longrightarrow G) \wedge (D \Longrightarrow G)$

Following steps above, the PL sentence is already at stage 2, as it does not have any biconditional

$$(\boldsymbol{A} \vee \boldsymbol{B}) \wedge (\neg\boldsymbol{A} \vee \boldsymbol{C}) \wedge (\neg\boldsymbol{B} \vee \boldsymbol{D}) \wedge (\neg\boldsymbol{C} \vee \boldsymbol{G}) \wedge (\neg\boldsymbol{D} \vee \boldsymbol{G})$$

(b) Prove, using resolution, that the sentence in 1(a)(ii) entails G



Therefore, having empty clause means KFBG

(c) A sentence is in disjunctive normal form, DNF, if it is the disjunction of conjunctions of literals. For example, $(A \wedge D) \vee (\neg B \wedge C)$ is DNF

(i) Any PL sentence is logically equivalent to the assertion that some possible world in which it would be true is in fact the case. From this observation, prove that any sentence can be written in DNF.

It is evident that every possible world can be written as a conjunction of literal for instance; $(A \wedge B \wedge \neg C)$.

Asserting that a possible world is not the case can be written by negating, e.g. $\neg (A \wedge B \wedge \neg C)$, which can be rewritten as $(\neg A \vee \neg B \vee C)$.

This is the form of a clause; a conjunction of these clauses is a CNF sentence, and can list the negations of all the possible worlds that would make the sentence false.

(ii) From your understanding of the CNF algorithm, construct a non-trivial algorithm that converts any PL sentence into DNF.

a) Eliminating ⇔, replacing $\alpha \Leftrightarrow \beta$ with $(\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)$.

b) Eliminating ⇒, replacing $\alpha \Rightarrow \beta$ with $\neg \alpha \vee \beta$.

c) Move ¬ inwards using de Morgan's rules and double-negation.

d) Apply distributive law (∨ over ∧) and flatten.

(iii) Convert the knowledge base, $KB = (A \Rightarrow B) \wedge (B \Rightarrow C) \wedge (C \Rightarrow \neg A)$ into DNF applying the algorithm constructed in (c)(ii)

$(\neg A \vee B) \wedge (\neg B \vee C) \wedge (\neg C \vee \neg A)$

$\neg A \wedge (\neg B \vee C) \vee B \wedge (\neg B \vee C)$

$\{(\neg A \wedge \neg B) \vee (\neg A \wedge C) \vee (B \wedge \neg B) \vee (B \wedge C)\} \wedge (\neg C \vee \neg A)$

$\neg C((\neg A \wedge \neg B) \vee (\neg A \wedge C) \vee (B \wedge \neg B) \vee (B \vee C)) \vee \neg A((\neg A \wedge \neg B) \vee (\neg A \wedge C) \vee (B \wedge \neg B) \vee (B \wedge C))$

$(\neg C \wedge \neg A \wedge \neg B) \vee (\neg C \wedge \neg A \wedge C) \vee (\neg C \wedge \neg B \wedge B) \vee (\neg C \wedge \neg B \wedge C) \vee (\neg A \wedge \neg A \wedge \neg B) \vee (\neg A \wedge \neg A \wedge \neg C) \vee (\neg A \wedge B \wedge \neg B) \vee (\neg A \wedge B \wedge C)$

2. From your understanding of First Order Logic, FOL, and the vocabulary given in Table 1, write the following paragraph in FOL:

(i) *There exists a lecturer all of whose clients are students.*

$\exists y\ D(y, \mathrm{L})) \wedge \forall x\ C(x, y) \Rightarrow D(x, s)$

(ii) *Every student is a client of a lecturer.*

$\forall x\ D(x, s) \Rightarrow \exists y\ D(y, \mathrm{L})) \wedge C(x, y)$ s

(iii) *Thondoya has a father who is a lecturer.*

$\exists x F(x, T)\ ) \wedge D(x, \mathrm{L})$

(iv) *Thondoya is neither a student nor a lecturer*.

D (T, $\neg S) \wedge D(T, \neg\mathrm{L})$