```
In [256…
!pip install pandas_datareader
import pandas as pd
import numpy as np
!pip install arch
from arch import arch_model
import statsmodels.api as sm
import statsmodels.formula.api as sm
import statsmodels.formula.api as smf
import pandas_datareader.data as web
from datetime import datetime, timedelta
import matplotlib.pyplot as plt
from statsmodels.graphics.tsaplots import plot_acf, plot_pacf
```

Requirement already satisfied: pandas_datareader in c:\users\bridget\anaconda3\lib\site-
packages (0.10.0)
Requirement already satisfied: lxml in c:\users\bridget\anaconda3\lib\site-packages (fro
m pandas_datareader) (4.6.3)
Requirement already satisfied: pandas>=0.23 in c:\users\bridget\anaconda3\lib\site-packa
ges (from pandas_datareader) (1.2.4)
Requirement already satisfied: requests>=2.19.0 in c:\users\bridget\anaconda3\lib\site-p
ackages (from pandas_datareader) (2.25.1)
Requirement already satisfied: numpy>=1.16.5 in c:\users\bridget\anaconda3\lib\site-pack
ages (from pandas>=0.23->pandas_datareader) (1.20.1)
Requirement already satisfied: pytz>=2017.3 in c:\users\bridget\anaconda3\lib\site-packa
ges (from pandas>=0.23->pandas_datareader) (2021.1)
Requirement already satisfied: python-dateutil>=2.7.3 in c:\users\bridget\anaconda3\lib
\site-packages (from pandas>=0.23->pandas_datareader) (2.8.1)
Requirement already satisfied: six>=1.5 in c:\users\bridget\anaconda3\lib\site-packages
(from python-dateutil>=2.7.3->pandas>=0.23->pandas_datareader) (1.15.0)
Requirement already satisfied: chardet<5,>=3.0.2 in c:\users\bridget\anaconda3\lib\site-
packages (from requests>=2.19.0->pandas_datareader) (4.0.0)
Requirement already satisfied: urllib3<1.27,>=1.21.1 in c:\users\bridget\anaconda3\lib\s
ite-packages (from requests>=2.19.0->pandas_datareader) (1.26.4)
Requirement already satisfied: idna<3,>=2.5 in c:\users\bridget\anaconda3\lib\site-packa
ges (from requests>=2.19.0->pandas_datareader) (2.10)
Requirement already satisfied: certifi>=2017.4.17 in c:\users\bridget\anaconda3\lib\site
-packages (from requests>=2.19.0->pandas_datareader) (2020.12.5)
Requirement already satisfied: arch in c:\users\bridget\anaconda3\lib\site-packages (5.
6.0)
Requirement already satisfied: pandas>=1.0 in c:\users\bridget\anaconda3\lib\site-packag
es (from arch) (1.2.4)
Requirement already satisfied: scipy>=1.3 in c:\users\bridget\anaconda3\lib\site-package
s (from arch) (1.6.2)
Requirement already satisfied: statsmodels>=0.11 in c:\users\bridget\anaconda3\lib\site-
packages (from arch) (0.12.2)
Requirement already satisfied: numpy>=1.17 in c:\users\bridget\anaconda3\lib\site-packag
es (from arch) (1.20.1)
Requirement already satisfied: property-cached>=1.6.4 in c:\users\bridget\anaconda3\lib
\site-packages (from arch) (1.6.4)
Requirement already satisfied: pytz>=2017.3 in c:\users\bridget\anaconda3\lib\site-packa
ges (from pandas>=1.0->arch) (2021.1)
Requirement already satisfied: python-dateutil>=2.7.3 in c:\users\bridget\anaconda3\lib
\site-packages (from pandas>=1.0->arch) (2.8.1)
Requirement already satisfied: six>=1.5 in c:\users\bridget\anaconda3\lib\site-packages
(from python-dateutil>=2.7.3->pandas>=1.0->arch) (1.15.0)
Requirement already satisfied: patsy>=0.5 in c:\users\bridget\anaconda3\lib\site-package
s (from statsmodels>=0.11->arch) (0.5.1)

In [331…
```
df2 = pd.read_excel(r'C:\Users\BRIDGET\Documents\My Spring 2024\My Spring Folder 2024\E
df= df2.dropna()
df.head(5)
```

	observation_date	RAAA	RAAA(t-1)	R3	R3(t-1)	R3(t-2)	IP	IP(t-1)	PW	PW(t-1)	GPW
2	1960-03-01	4.49	4.56	3.31	3.96	4.35	24.5611	24.7836	31.8	31.6	0.006329
3	1960-04-01	4.45	4.49	3.23	3.31	3.96	24.3664	24.5611	31.8	31.8	0.000000
4	1960-05-01	4.46	4.45	3.29	3.23	3.31	24.3386	24.3664	31.7	31.8	-0.003145
5	1960-06-01	4.45	4.46	2.46	3.29	3.23	24.0326	24.3386	31.7	31.7	0.000000
6	1960-07-01	4.41	4.45	2.30	2.46	3.29	23.9492	24.0326	31.7	31.7	0.000000

In [258...

```python
print(df.columns)
```

```
Index(['observation_date', 'RAAA', 'RAAA(t-1)', 'R3', 'R3(t-1)', 'R3(t-2)',
       'IP', 'IP(t-1)', 'PW', 'PW(t-1)', 'GPW'],
      dtype='object')
```

In [389...

```python
#Garch(1,1)
returns = Y.pct_change().dropna()
am = arch_model(returns, vol='GARCH', p=1, q=1)
garch_model = arch_model(residuals, mean='Constant', vol='GARCH', p=1, q=1)
garch_result = garch_model.fit(disp='off')
res = am.fit()
print(res)
```

```
Iteration:      1,    Func. Count:      6,    Neg. LLF: 598460836.4317628
Iteration:      2,    Func. Count:     17,    Neg. LLF: 341771479.05048245
Iteration:      3,    Func. Count:     28,    Neg. LLF: 22363.124385443123
Iteration:      4,    Func. Count:     38,    Neg. LLF: 2416647.98992588
Iteration:      5,    Func. Count:     50,    Neg. LLF: 1459991.4042517906
Iteration:      6,    Func. Count:     60,    Neg. LLF: -1104.9351403419246
Optimization terminated successfully    (Exit mode 0)
            Current function value: -1104.9351389196077
            Iterations: 10
            Function evaluations: 60
            Gradient evaluations: 6
                    Constant Mean - GARCH Model Results
==============================================================================
Dep. Variable:                    RAAA   R-squared:                       0.000
Mean Model:               Constant Mean   Adj. R-squared:                  0.000
Vol Model:                        GARCH   Log-Likelihood:                 1104.94
Distribution:                    Normal   AIC:                           -2201.87
Method:             Maximum Likelihood   BIC:                           -2185.51
                                          No. Observations:                   442
Date:                Sat, Feb 24 2024   Df Residuals:                       441
Time:                        19:24:58   Df Model:                             1
                                Mean Model
==============================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
------------------------------------------------------------------------------
mu           8.5367e-04  7.149e-04      1.194      0.232 [-5.475e-04,2.255e-03]
                             Volatility Model
==============================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
------------------------------------------------------------------------------
omega        1.1383e-05  1.730e-11  6.582e+05      0.000 [1.138e-05,1.138e-05]
alpha[1]         0.2000  3.962e-02      5.048  4.468e-07     [  0.122,  0.278]
beta[1]          0.7800  3.559e-02     21.918 1.758e-106     [  0.710,  0.850]
```

```
========================================================================
```

Covariance estimator: robust

In [390...

```python
#OLS Result
import statsmodels.api as sm
X = df[['RAAA(t-1)', 'R3', 'R3(t-1)', 'R3(t-2)', 'IP', 'IP(t-1)', 'GPW']]
Y = df['RAAA']
x = sm.add_constant(X)
ols_model = sm.OLS(Y, X).fit()
print(ols_model.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                   RAAA   R-squared (uncentered):                   1.000
Model:                            OLS   Adj. R-squared (uncentered):              1.000
Method:                 Least Squares   F-statistic:                          1.377e+05
Date:                Sat, 24 Feb 2024   Prob (F-statistic):                        0.00
Time:                        19:25:12   Log-Likelihood:                          124.30
No. Observations:                 442   AIC:                                      -234.6
Df Residuals:                     435   BIC:                                      -206.0
Df Model:                           7
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
RAAA(t-1)      0.9648      0.008    124.053      0.000       0.949       0.980
R3             0.2805      0.019     15.015      0.000       0.244       0.317
R3(t-1)       -0.2744      0.029     -9.417      0.000      -0.332      -0.217
R3(t-2)        0.0318      0.019      1.676      0.094      -0.005       0.069
IP             0.0628      0.026      2.383      0.018       0.011       0.115
IP(t-1)       -0.0620      0.026     -2.343      0.020      -0.114      -0.010
GPW            4.2845      1.366      3.137      0.002       1.600       6.969
==============================================================================
Omnibus:                       36.058   Durbin-Watson:                   1.486
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              130.748
Skew:                           0.239   Prob(JB):                     4.06e-29
Kurtosis:                       5.621   Cond. No.                      1.15e+04
==============================================================================

Notes:
[1] R² is computed without centering (uncentered) since the model does not contain a constant.
[2] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[3] The condition number is large, 1.15e+04. This might indicate that there are strong multicollinearity or other numerical problems.
```
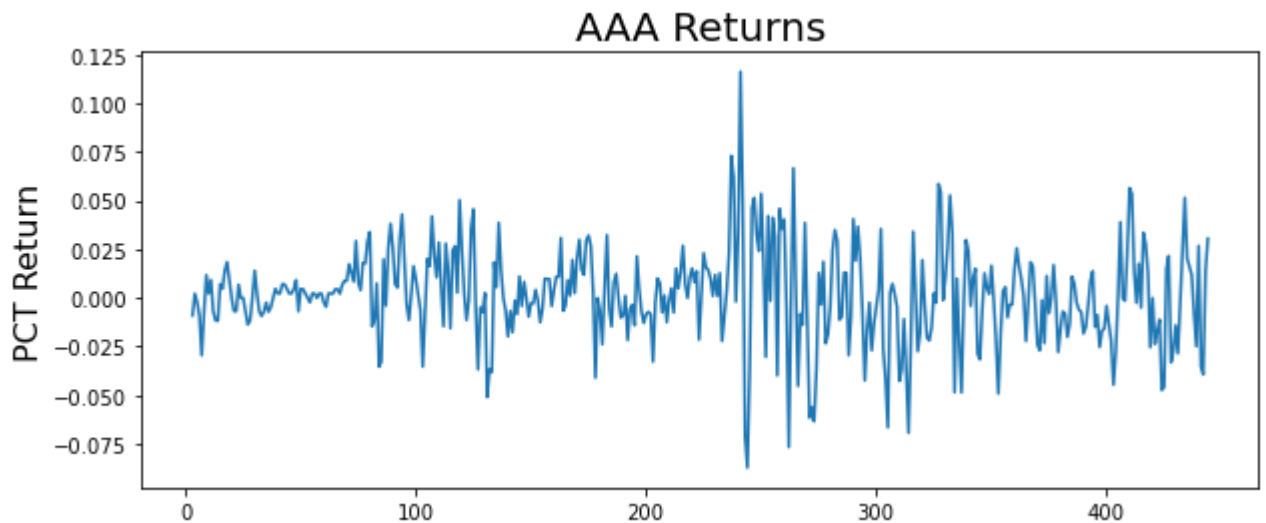
In [352...

```python
#The RAAA regression residual
plt.figure(figsize=(10,4))
plt.plot(returns)
```

```
plt.ylabel('PCT Return', fontsize=16)
plt.title('AAA Returns', fontsize=20)
```

Text(0.5, 1.0, 'AAA Returns')

```
df.loc[:, 'R3(t-1)_changes'] = df['R3(t-1)'].diff()
R31C = df['R3(t-1)_changes']
print(df.columns)
```

```
Index(['observation_date', 'RAAA', 'RAAA(t-1)', 'R3', 'R3(t-1)', 'R3(t-2)',
       'IP', 'IP(t-1)', 'PW', 'PW(t-1)', 'GPW', 'conditional_volatility**2',
       'conditional_volatility', 'R3(t-1)_changes'],
      dtype='object')
C:\Users\BRIDGET\anaconda3\lib\site-packages\pandas\core\indexing.py:1597: SettingWithCo
pyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable/user_
guide/indexing.html#returning-a-view-versus-a-copy
  self.obj[key] = value
C:\Users\BRIDGET\anaconda3\lib\site-packages\pandas\core\indexing.py:1676: SettingWithCo
pyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable/user_
guide/indexing.html#returning-a-view-versus-a-copy
  self._setitem_single_column(ilocs[0], value, pi)
```

```
#introducing R3(t-1)_changes to (GARCH 1,1) model

df = df.replace([np.inf, -np.inf], np.nan)
df = df.dropna(subset=['R3(t-1)_changes', 'RAAA'])
X = sm.add_constant(df['R3(t-1)_changes'])
model = sm.OLS(df['RAAA'], X).fit()
residuals = model.resid
am = arch_model(residuals, mean='Constant', vol='GARCH', p=1, q=1, rescale=False)
res = am.fit(disp='off')
print(model.summary())
print(res.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                   RAAA   R-squared:                       0.000
Model:                            OLS   Adj. R-squared:                 -0.002
Method:                 Least Squares   F-statistic:                   0.05038
Date:                Sat, 24 Feb 2024   Prob (F-statistic):              0.823
Time:                        15:54:23   Log-Likelihood:                 -1053.6
No. Observations:                 442   AIC:                             2111.
Df Residuals:                     440   BIC:                             2119.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                    coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const              8.1885      0.125     65.454      0.000       7.943       8.434
R3(t-1)_changes   -0.0539      0.240     -0.224      0.823      -0.526       0.418
==============================================================================
Omnibus:                       14.975   Durbin-Watson:                   0.009
Prob(Omnibus):                  0.001   Jarque-Bera (JB):               15.927
Skew:                           0.465   Prob(JB):                     0.000348
Kurtosis:                       2.966   Cond. No.                         1.92
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specifi
ed.
                      Constant Mean - GARCH Model Results
==============================================================================
Dep. Variable:                   None   R-squared:                       0.000
Mean Model:             Constant Mean   Adj. R-squared:                  0.000
Vol Model:                      GARCH   Log-Likelihood:                 -743.765
Distribution:                  Normal   AIC:                             1495.53
Method:            Maximum Likelihood   BIC:                             1511.90
                                        No. Observations:                  442
Date:                Sat, Feb 24 2024   Df Residuals:                      441
Time:                        15:54:23   Df Model:                            1
                                 Mean Model
==============================================================================
                 coef    std err          t      P>|t|     95.0% Conf. Int.
------------------------------------------------------------------------------
mu            -0.1687      0.310     -0.544      0.587  [ -0.776,   0.439]
                              Volatility Model
==============================================================================
                 coef    std err          t      P>|t|       95.0% Conf. Int.
------------------------------------------------------------------------------
omega         0.0263  1.153e-02      2.281  2.253e-02 [3.703e-03,4.889e-02]
alpha[1]      0.9927      0.713      1.393      0.164    [ -0.404,   2.390]
beta[1]       0.0000      0.639      0.000      1.000    [ -1.252,   1.252]
==============================================================================

Covariance estimator: robust
```

```python
#GARRCH (2,2)
X = sm.add_constant(df['R3(t-1)_changes'])
model = sm.OLS(df['returns'], X).fit()
residuals = model.resid
am = arch_model(residuals, mean='Constant', vol='GARCH', p=2, q=2, rescale=False)
res = am.fit(disp='off')
print(model.summary())
print(res.summary())
```

```
                            OLS Regression Results
==============================================================================
```

```
Dep. Variable:                returns   R-squared:                       0.047
Model:                            OLS   Adj. R-squared:                  0.045
Method:                 Least Squares   F-statistic:                     21.80
Date:               Sat, 24 Feb 2024   Prob (F-statistic):           4.03e-06
Time:                      12:03:26    Log-Likelihood:                 1040.0
No. Observations:               443    AIC:                            -2076.
Df Residuals:                   441    BIC:                            -2068.
Df Model:                         1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const           0.0011      0.001      1.012      0.312      -0.001       0.003
R3(t-1)_changes 0.0099      0.002      4.669      0.000       0.006       0.014
==============================================================================
Omnibus:                       27.402   Durbin-Watson:                   1.356
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               88.391
Skew:                          -0.123   Prob(JB):                     6.40e-20
Kurtosis:                       5.175   Cond. No.                         1.92
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specifi
ed.
                  Constant Mean - GARCH Model Results
==============================================================================
Dep. Variable:                   None   R-squared:                       0.000
Mean Model:             Constant Mean   Adj. R-squared:                  0.000
Vol Model:                      GARCH   Log-Likelihood:                1120.69
Distribution:                  Normal   AIC:                          -2229.37
Method:            Maximum Likelihood   BIC:                          -2204.81
                                        No. Observations:                  443
Date:               Sat, Feb 24 2024    Df Residuals:                      442
Time:                      12:03:26     Df Model:                            1
                            Mean Model
==============================================================================
                 coef    std err          t      P>|t|       95.0% Conf. Int.
------------------------------------------------------------------------------
mu         -5.6557e-04  6.508e-04     -0.869      0.385 [-1.841e-03,7.101e-04]
                         Volatility Model
==============================================================================
                 coef    std err          t      P>|t|       95.0% Conf. Int.
------------------------------------------------------------------------------
omega       1.0701e-05  2.097e-10  5.104e+04      0.000 [1.070e-05,1.070e-05]
alpha[1]        0.1000  6.919e-02      1.445      0.148 [-3.561e-02,  0.236]
alpha[2]        0.1000      0.130      0.767      0.443 [ -0.155,  0.355]
beta[1]         0.3900      1.802      0.216      0.829 [ -3.142,  3.922]
beta[2]         0.3900      1.624      0.240      0.810 [ -2.794,  3.574]
==============================================================================

Covariance estimator: robust
```

In [302…
```python
X = sm.add_constant(df[['R3(t-1)_changes', 'R3(t-1)', 'R3(t-2)']])
model = sm.OLS(df['returns'], X).fit()
residuals = model.resid
am = arch_model(residuals, mean='Constant', vol='GARCH', p=2, q=2, rescale=False)
res = am.fit(disp='off')
print(model.summary())
print(res.summary())
```

```
                      OLS Regression Results
==============================================================================
Dep. Variable:                returns   R-squared:                       0.047
```

```
Model:                            OLS   Adj. R-squared:                 0.041
Method:                 Least Squares   F-statistic:                    7.252
Date:                Sat, 24 Feb 2024   Prob (F-statistic):          9.30e-05
Time:                        12:11:36   Log-Likelihood:                 1040.0
No. Observations:                 443   AIC:                           -2072.
Df Residuals:                     439   BIC:                           -2056.
Df Model:                           3
Covariance Type:            nonrobust
==============================================================================
                    coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const             0.0005      0.003      0.195      0.846      -0.005       0.006
R3(t-1)_changes   0.0102      0.058      0.174      0.862      -0.104       0.125
R3(t-1)          -0.0002      0.058     -0.004      0.997      -0.115       0.114
R3(t-2)           0.0003      0.058      0.006      0.996      -0.114       0.115
==============================================================================
Omnibus:                       28.220   Durbin-Watson:                  1.358
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              90.546
Skew:                          -0.150   Prob(JB):                    2.18e-20
Kurtosis:                       5.194   Cond. No.                        865.
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specifi
ed.
                   Constant Mean - GARCH Model Results
==============================================================================
Dep. Variable:                   None   R-squared:                      0.000
Mean Model:             Constant Mean   Adj. R-squared:                 0.000
Vol Model:                      GARCH   Log-Likelihood:                1120.84
Distribution:                  Normal   AIC:                          -2229.67
Method:            Maximum Likelihood   BIC:                          -2205.11
                                        No. Observations:                 443
Date:                Sat, Feb 24 2024   Df Residuals:                     442
Time:                        12:11:36   Df Model:                           1
                              Mean Model
==============================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
------------------------------------------------------------------------------
mu        -4.0568e-04  6.521e-04     -0.622      0.534 [-1.684e-03,8.725e-04]
                            Volatility Model
==============================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
------------------------------------------------------------------------------
omega      1.0702e-05  1.729e-10  6.189e+04      0.000 [1.070e-05,1.070e-05]
alpha[1]       0.1000  6.055e-02      1.652  9.863e-02 [-1.868e-02,  0.219]
alpha[2]       0.1000      0.125      0.800      0.424 [ -0.145,  0.345]
beta[1]        0.3900      1.608      0.242      0.808 [ -2.762,  3.542]
beta[2]        0.3900      1.449      0.269      0.788 [ -2.449,  3.229]
==============================================================================

Covariance estimator: robust
```

In [381…
```python
##Introducing conditional Volatility (Standard deviation) into the Garch model
garch_model = arch_model(residuals, mean='Constant', vol='GARCH', p=1, q=1, rescale=Fal
garch_result = garch_model.fit(disp='off')
conditional_volatility = garch_result.conditional_volatility
df['conditional_volatility'] = conditional_volatility
df_clean = df.dropna()
X = sm.add_constant(df_clean[['conditional_volatility']])
y = df_clean['RAAA']
ols_model = sm.OLS(returns, X).fit()
```

```python
print(ols_model.summary())
print(garch_result.summary())
```

```
                            OLS Regression Results
=================================================================================
==
Dep. Variable:                      RAAA   R-squared:                       0.001
Model:                               OLS   Adj. R-squared:                 -0.001
Method:                    Least Squares   F-statistic:                    0.5045
Date:                   Sat, 24 Feb 2024   Prob (F-statistic):              0.478
Time:                           17:30:23   Log-Likelihood:                 1026.7
No. Observations:                    442   AIC:                            -2049.
Df Residuals:                        440   BIC:                            -2041.
Df Model:                              1
Covariance Type:               nonrobust
=================================================================================
==
                            coef    std err          t      P>|t|      [0.025
 0.97
5]
---------------------------------------------------------------------------------
--
const                     0.0024      0.002      1.351      0.178      -0.001
 0.0
06
conditional_volatility   -0.0005      0.001     -0.710      0.478      -0.002
 0.0
01
=================================================================================
==
Omnibus:                          23.395   Durbin-Watson:                   1.178
Prob(Omnibus):                     0.000   Jarque-Bera (JB):               70.216
Skew:                              0.037   Prob(JB):                     5.66e-16
Kurtosis:                          4.951   Cond. No.                         4.46
=================================================================================
==

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specifi
ed.
                   Constant Mean - GARCH Model Results
=================================================================================
Dep. Variable:                      None   R-squared:                       0.000
Mean Model:                Constant Mean   Adj. R-squared:                  0.000
Vol Model:                         GARCH   Log-Likelihood:               -743.765
Distribution:                     Normal   AIC:                           1495.53
Method:               Maximum Likelihood   BIC:                           1511.90
                                           No. Observations:                  442
Date:                   Sat, Feb 24 2024   Df Residuals:                      441
Time:                           17:30:23   Df Model:                            1
                               Mean Model
=================================================================================
                 coef    std err          t      P>|t|  95.0% Conf. Int.
---------------------------------------------------------------------------------
mu            -0.1687      0.310     -0.544      0.587 [ -0.776,  0.439]
                             Volatility Model
=================================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
---------------------------------------------------------------------------------
omega         0.0263  1.153e-02      2.281  2.253e-02 [3.703e-03,4.889e-02]
alpha[1]      0.9927      0.713      1.393      0.164  [ -0.404,  2.390]
beta[1]       0.0000      0.639      0.000      1.000  [ -1.252,  1.252]
=================================================================================

Covariance estimator: robust
```

```python
#GARCH in Mean specification model.
returns = df['returns']
garch_m_model = arch_model(returns, mean='Constant', vol='GARCH', p=1, q=1, dist='norma
```

```
garch_m_result = garch_m_model.fit(update_freq=5)
print(garch_m_result.summary())
```

```
Iteration:      5,    Func. Count:      48,    Neg. LLF: 74086.03951722595
Optimization terminated successfully    (Exit mode 0)
            Current function value: -1108.6834886985837
            Iterations: 9
            Function evaluations: 78
            Gradient evaluations: 7
                    Constant Mean - GARCH Model Results
==============================================================================
Dep. Variable:                returns   R-squared:                       0.000
Mean Model:             Constant Mean   Adj. R-squared:                  0.000
Vol Model:                      GARCH   Log-Likelihood:                 1108.68
Distribution:                  Normal   AIC:                           -2209.37
Method:            Maximum Likelihood   BIC:                           -2192.99
                                        No. Observations:                   443
Date:              Sat, Feb 24 2024     Df Residuals:                       442
Time:                      13:52:47     Df Model:                             1
                               Mean Model
==============================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
------------------------------------------------------------------------------
mu          7.6706e-04  7.133e-04      1.075      0.282 [-6.310e-04,2.165e-03]
                            Volatility Model
==============================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
------------------------------------------------------------------------------
omega       1.1227e-05  3.322e-11  3.379e+05      0.000 [1.123e-05,1.123e-05]
alpha[1]        0.2000  3.925e-02      5.095  3.491e-07 [  0.123,  0.277]
beta[1]         0.7800  3.572e-02     21.840 9.727e-106 [  0.710,  0.850]
==============================================================================

Covariance estimator: robust
```

In [391...

```
##Introducing lagged-1 Variance into the Garch model

garch_model = arch_model(returns, mean='Constant', vol='GARCH', p=1, q=1, dist='normal'
garch_result = garch_model.fit(disp='off')
conditional_variances = garch_result.conditional_volatility**2
conditional_variances.index = returns.index
lagged_variance = conditional_variances.shift(1)
df_for_regression = pd.DataFrame({
    'returns': returns,
    'lagged_variance': lagged_variance
}).dropna()
X = sm.add_constant(df_for_regression[['lagged_variance']])
model = sm.OLS(df_for_regression['returns'], X).fit()
print(model.summary())
print(garch_result.summary())
```

```
                        OLS Regression Results
==============================================================================
Dep. Variable:                returns   R-squared:                       0.000
Model:                            OLS   Adj. R-squared:                 -0.002
Method:                 Least Squares   F-statistic:                   0.002777
Date:              Sat, 24 Feb 2024     Prob (F-statistic):              0.958
Time:                      19:44:19     Log-Likelihood:                 1023.7
No. Observations:                 441   AIC:                            -2043.
Df Residuals:                     439   BIC:                            -2035.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
```

```
                          coef    std err           t      P>|t|      [0.025      0.975]
-------------------------------------------------------------------------------------
const                   0.0014     0.002       0.889      0.374      -0.002       0.004
lagged_variance         0.1012     1.919       0.053      0.958      -3.671       3.873
=====================================================================================
Omnibus:                         23.807  Durbin-Watson:                    1.180
Prob(Omnibus):                    0.000  Jarque-Bera (JB):                73.015
Skew:                            -0.009  Prob(JB):                       1.40e-16
Kurtosis:                         4.993  Cond. No.                       1.69e+03
=====================================================================================
```

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 1.69e+03. This might indicate that there are
strong multicollinearity or other numerical problems.

```
                 Constant Mean - GARCH Model Results
==============================================================================
Dep. Variable:                  RAAA   R-squared:                       0.000
Mean Model:             Constant Mean   Adj. R-squared:                  0.000
Vol Model:                      GARCH   Log-Likelihood:                 1104.94
Distribution:                  Normal   AIC:                           -2201.87
Method:            Maximum Likelihood   BIC:                           -2185.51
                                        No. Observations:                  442
Date:                Sat, Feb 24 2024   Df Residuals:                      441
Time:                        19:44:19   Df Model:                            1
                             Mean Model
==============================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
------------------------------------------------------------------------------
mu           8.5367e-04  7.149e-04      1.194      0.232 [-5.475e-04,2.255e-03]
                          Volatility Model
==============================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
------------------------------------------------------------------------------
omega        1.1383e-05  1.730e-11  6.582e+05      0.000 [1.138e-05,1.138e-05]
alpha[1]         0.2000  3.962e-02      5.048  4.468e-07     [  0.122,  0.278]
beta[1]          0.7800  3.559e-02     21.918 1.758e-106     [  0.710,  0.850]
==============================================================================
```

Covariance estimator: robust
```

In [371... `df.head()`

Out[371...

| | observation_date | RAAA | RAAA(t-1) | R3 | R3(t-1) | R3(t-2) | IP | IP(t-1) | PW | PW(t-1) | GPW | condit |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 3 | 1960-04-01 | 4.45 | 4.49 | 3.23 | 3.31 | 3.96 | 24.3664 | 24.5611 | 31.8 | 31.8 | 0.000000 | |
| 4 | 1960-05-01 | 4.46 | 4.45 | 3.29 | 3.23 | 3.31 | 24.3386 | 24.3664 | 31.7 | 31.8 | -0.003145 | |
| 5 | 1960-06-01 | 4.45 | 4.46 | 2.46 | 3.29 | 3.23 | 24.0326 | 24.3386 | 31.7 | 31.7 | 0.000000 | |
| 6 | 1960-07-01 | 4.41 | 4.45 | 2.30 | 2.46 | 3.29 | 23.9492 | 24.0326 | 31.7 | 31.7 | 0.000000 | |
| 7 | 1960-08-01 | 4.28 | 4.41 | 2.30 | 2.30 | 2.46 | 23.9213 | 23.9492 | 31.6 | 31.7 | -0.003155 | |

In [388...

```python
#Introducing Variance into the Garch model
garch_model = arch_model(returns, mean='Constant', vol='GARCH', p=1, q=1, dist='normal'
garch_result = garch_model.fit(disp='off')
```

```python
conditional_variances = garch_result.conditional_volatility**2
conditional_variances.index = returns.index
df_for_regression = pd.DataFrame({
    'returns': returns,
    'variance': conditional_variances
}).dropna()

X = sm.add_constant(df_for_regression[['variance']])
model = sm.OLS(df_for_regression['returns'], X).fit()
print(model.summary())
print(garch_result.summary())
```

```
                           OLS Regression Results
==============================================================================
Dep. Variable:                returns   R-squared:                       0.000
Model:                            OLS   Adj. R-squared:                 -0.002
Method:                 Least Squares   F-statistic:                  0.0005036
Date:                Sat, 24 Feb 2024   Prob (F-statistic):              0.982
Time:                        17:59:39   Log-Likelihood:                 1026.4
No. Observations:                 442   AIC:                            -2049.
Df Residuals:                     440   BIC:                            -2041.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0014      0.002      0.896      0.371      -0.002       0.004
variance       0.0430      1.917      0.022      0.982      -3.725       3.811
==============================================================================
Omnibus:                       23.905   Durbin-Watson:                   1.180
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               73.493
Skew:                          -0.003   Prob(JB):                     1.10e-16
Kurtosis:                       4.998   Cond. No.                     1.70e+03
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specifi
ed.
[2] The condition number is large, 1.7e+03. This might indicate that there are
strong multicollinearity or other numerical problems.
                  Constant Mean - GARCH Model Results
==============================================================================
Dep. Variable:                   RAAA   R-squared:                       0.000
Mean Model:              Constant Mean   Adj. R-squared:                  0.000
Vol Model:                       GARCH   Log-Likelihood:                 1104.94
Distribution:                   Normal   AIC:                            -2201.87
Method:            Maximum Likelihood   BIC:                            -2185.51
                                         No. Observations:                  442
Date:                Sat, Feb 24 2024   Df Residuals:                       441
Time:                        17:59:39   Df Model:                             1
                                 Mean Model
==============================================================================
                 coef    std err          t      P>|t|     95.0% Conf. Int.
------------------------------------------------------------------------------
mu          8.5367e-04  7.149e-04      1.194      0.232 [-5.475e-04,2.255e-03]
                              Volatility Model
==============================================================================
                 coef    std err          t      P>|t|     95.0% Conf. Int.
------------------------------------------------------------------------------
omega       1.1383e-05  1.730e-11  6.582e+05      0.000 [1.138e-05,1.138e-05]
alpha[1]        0.2000  3.962e-02      5.048  4.468e-07   [  0.122,   0.278]
beta[1]         0.7800  3.559e-02     21.918 1.758e-106   [  0.710,   0.850]
==============================================================================
```

Covariance estimator: robust