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Philipps Universität Marburg
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Computational Physics II
SS 2018

Deadline: 9. May 2018
Please turn in a written documentation of your results and submit the programs to Jonathan Prexl at *jonathan.prexl@physik.uni-marburg.de*

3.1. Random walk with memory

Consider a one-dimensional walk on an integer lattice defined by

$$x_n = x_{n-1} + l_n \tag{1}$$

where the steps l_n are ± 1, but with probabilities that depend on the previous steps: there is a probability $1/2 + \varepsilon$ to continue in the same direction, and a probability $1/2 - \varepsilon$ to reverse.
Program the random walk and determine the distribution of points after $n = 1000$ steps for $\varepsilon = 0.1$, 0.2, 0.3, and 0.4.
Compare to a normal distribution

$$p_n(x) = \frac{1}{\sqrt{2\pi D n}} e^{-\frac{(x-n\bar{x})^2}{2Dn}} \tag{2}$$

and determine $D(\varepsilon)$.

3.2. Random walks in 3d

The 3d off-lattice version of the random walks we considered in 1d consists of steps of length ℓ taken in arbitrary directions, uniformly distributed in all directions. The aim of this problem is to compare three models for the determination of uniformly distributed directions:

(i) with φ and θ the angles in spherical coordinates, the direction is given by $\mathbf{n} = (\sin\theta\cos\varphi, \sin\theta\cos\varphi, \cos\theta)$. How do you have to choose φ and θ to pick points uniformly on the sphere?
(ii) pick three random numbers $x_i \in (-1, 1)$, calculate their norm $||x|| = \sqrt{\sum_i x_i^2}$ and keep $\mathbf{n} = x/||x||$ only if $||x|| < 1$.
Question: Why is the requirement $||x|| < 1$ needed?
(iii) pick three normally distributed random numbers x_i, calculate their norm $||x|| = \sqrt{\sum_i x_i^2}$ and take $\mathbf{n} = x/||x||$.
Question: Why is the requirement $||x|| < 1$ not needed in this case?

Program the three choices and verify that your random walk is isotropic in all three directions. Compare the distribution to a Gaussian and determine the diffusion constant $D = \langle \mathbf{x}^2 \rangle(n)/(2n)$.

please turn over

3.3. Brownian motors and ratchets

Consider the stochastic differential equation

$$\dot{x} = -V'(x,t) + \xi \tag{3}$$

with a periodically driven ratchet potential

$$V(x,t) = (1 - \cos 2\omega t)(a \cos x + b \cos 2x) \tag{4}$$

and delta-correlated white noise ξ of strength D.
Such a potential can induce a steady sideways current. Find parameter values for the noise strength D, the amplitudes a and b, and the frequency ω of the force that show such a drift.