PHILIPPS-UNIVERSITAET MARBURG, THE DEPARTMENT OF PHYSICS

Computational Physics II: Assignment 2

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1 RANDOM WALKS

According to the definition of round function, it has

$$P\left(l_n = b, b+a\right) = \frac{1}{2a}.$$
$$P\left(l_n = b+1, \cdots, b+a-1\right) = \frac{1}{a}.$$

since $l_n = \text{round}\left(a \cdot r_n + b\right)$, where r_n is a uniform random variable in $(0,1)$, while a and b are integers.

Thus the first moment and the second moment can be given out

$$\langle l_n \rangle = \frac{1}{2a} \cdot b + \frac{1}{a}\left(\sum_{i=b+1}^{b+a-1} i\right) + \frac{1}{2a} \cdot (b+a) = b + \frac{a}{2}.$$
$$\left\langle l_n^2 \right\rangle = \frac{1}{2a} \cdot b^2 + \frac{1}{a}\left(\sum_{i=b+1}^{b+a-1} i^2\right) + \frac{1}{2a} \cdot (b+a)^2 = b\left(b+a\right) + \frac{2a^2+1}{6}.$$

Consequently, the diffusion constant D will be

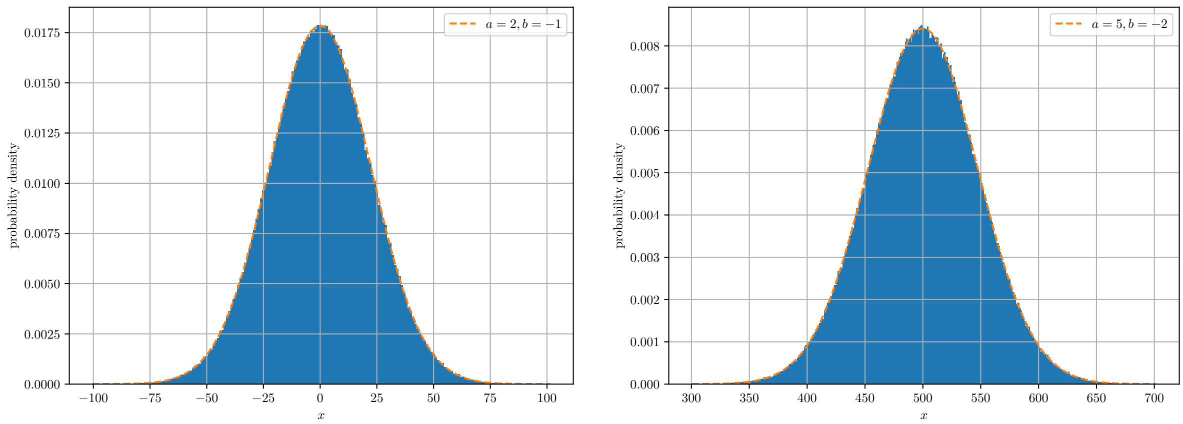
$$D = \left\langle l_n^2 \right\rangle - \langle l_n \rangle^2 = b\left(b+a\right) + \frac{2a^2+1}{6} - \left(b + \frac{a}{2}\right)^2 = \frac{a^2+2}{12}.$$

Then we plot the probability density out, and compare them to the theoretical prediction. It seems the predictions fit the practices very well.

2 MOMENTS OF THE NORMAL DISTRIBUTION

By using Mathematica, the exact results for higher moments

$$\left\langle x^k \right\rangle = \int x^k p\left(x\right) \mathrm{d}x.$$



(a) (b)

Figure 1.1: the position after 1000 steps: (a) $a = 2, b = -1$; (b) $a = 5, b = -2$.

of Gaussian distribution

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{x^2}{2\sigma^2}\right).$$

are determined

$$\left\langle x^k \right\rangle = \begin{cases} 0 & \text{if } k \text{ is odd} \\ (k-1)!! \cdot \sigma^k & \text{if } k \text{ is even} \end{cases}.$$

Hence, the moments are given out

Table 2.1: The high order moments for Gaussian distribution.

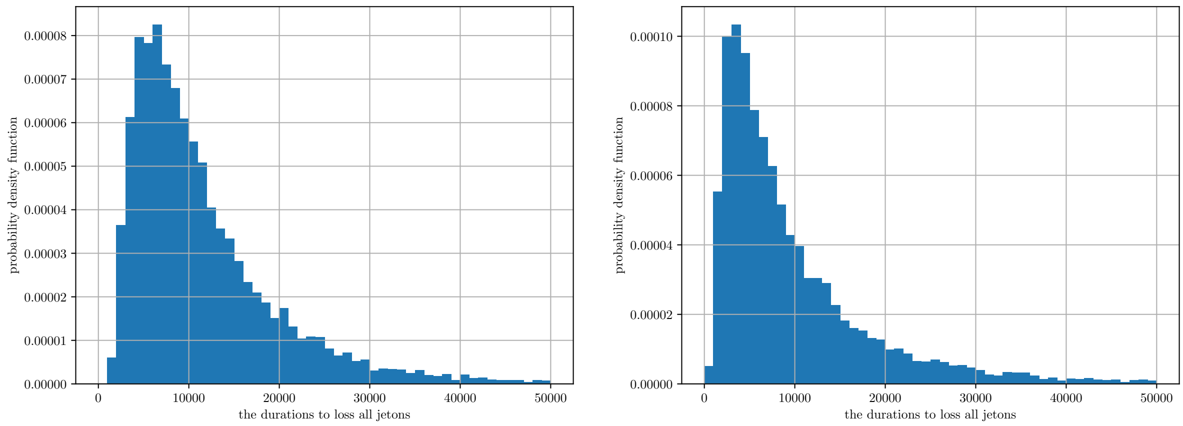
k	2	4	6	8	10
$\left\langle x^k \right\rangle$	σ^2	$3\sigma^4$	$15\sigma^6$	$105\sigma^8$	$945\sigma^{10}$

3 Parrondos paradoxical games

I first tried with strategy A and B separately. The probability density function to loss 100 Jetons after specific time steps are presented in Figure 3.1, who evident that the separately strategy A or B are both losing games.

If I play A and B alternatively, the duration it take to loss 100 Jetons distributes as shown in Figure 3.2.

At last, I tried the sequence AAB. The total amounts of jetons increase from 100 to 50000 very rapidly, which means this kind of playing strategy is a wining games.



(a) (b)

Figure 3.1: the probability to loss 100 Jetons after specific duration: (a) strategy A; (b) strategy B.

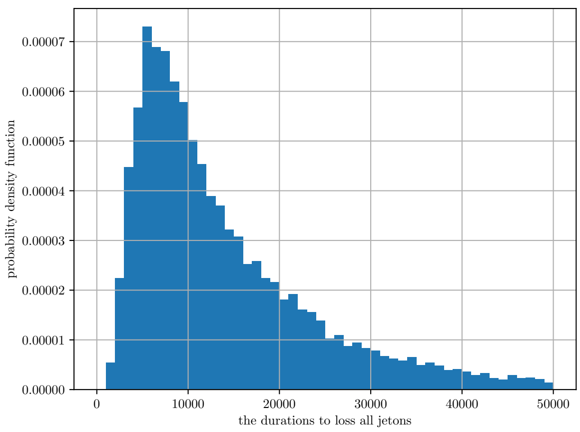


Figure 3.2: the probability to loss 100 Jetons after specific duration for AB sequence.