PHILIPPS-UNIVERSITAET MARBURG, THE DEPARTMENT OF PHYSICS

XXXXXX XXXXXX: Assignment X

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1 RANDOM HAMILTONIANS

As Central Limit Theorem, given a sufficiently large sample size, the samples will follow an approximate normal distribution with the original mean, with all variances being approximately equal to the variance of the population divided by ea wich sample's size.

—— RandomHamiltoniansType1 ————-

What will be happen with vector samples rather than scalar? With random matrix, we will see the eigen value distribution. A matrix H is randomly determined with following consition for elements follow normal distribution.

$$\mathbb{E}H_{ij} = 0, \quad \mathbb{E}|H_{ij}|^2 = \frac{1}{n} \tag{1.1}$$

In oder to deal with real eigenvalues, we adopt symmetry condition in the matrix. To generate such a matrix H, first we determine a n by n matrix A with random elements from normal distribution.

$$A'_{ij} = \frac{A_{ij}}{\sqrt{n}} \times \sqrt{2} \rightarrow H = \frac{A' + A'^T}{2} \tag{1.2}$$

Then density of eigenvalue $\rho(\lambda) = \frac{1}{2\pi}\sqrt{4-\lambda^2}; \int \rho(\lambda) d\lambda = n$

—— RandomHamiltoniansType2 ————-

From the given question,

$$H = \frac{A + A^T}{2 * \sqrt{2}} \quad \rightarrow \quad \mathbb{E}H_{ij} = 0, \quad \mathbb{E}|H_{ij}|^2 = \frac{1}{4} \tag{1.3}$$

$$\text{Eigenvalue} \quad h_i, \quad \int \rho(h)dh = n, \quad dh = \sqrt{n/4}d\lambda, \quad \rho(h) = 1/\sqrt{n}\pi\sqrt{4-(4/n)h^2}$$

$$\text{Modified eigenvalue} \quad x_i = h_i/\sqrt{n}, \quad \rho(h)dh \to \rho(x)dx = \frac{2}{\pi}\sqrt{1-x^2}dx$$

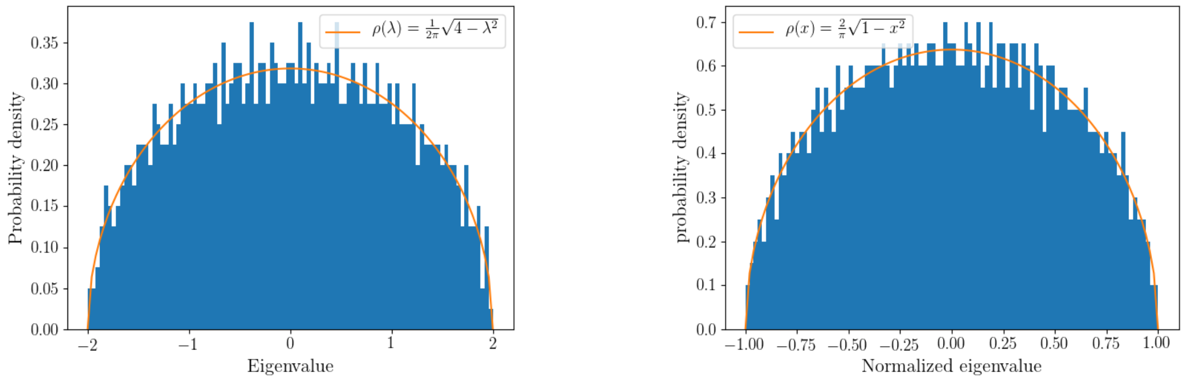

Figure 1.1: 7.1 eigenvalue density with type1(left) and type2 (right)

Integrated density

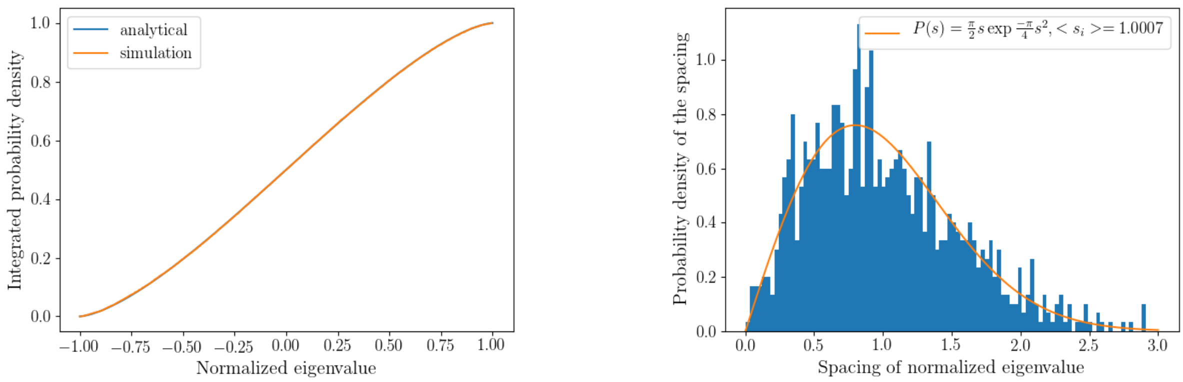

Figure 1.2: 7.1 Integrated density(left) and spacing density(right) of normalized eigenvalues

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