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Computational Physics I

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2.1 Computational costs

The hyperbolic tangent can be evaluated in several ways, for instance

$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{e^{2x}-1}{e^{2x}+1} = \left.\frac{y-1}{y+1}\right|_{y=e^{2x}}. \tag{1}$$

Alternatively, it can be approximated by the first elements of a Taylor series for small arguments x,

$$\tanh(x) \approx x - \frac{x^3}{3} + \frac{2x^5}{15} - \frac{17x^7}{315}. \tag{2}$$

Compare the runtimes of the different approaches.
Hint: To reach measurable times, evaluate the functions for sufficiently many numbers from the interval $[0, 1]$.

2.2 Computing a series

Consider the sum

$$\sum_{n=1}^{N} \frac{2n+1}{n^2(n+1)^2}. \tag{3}$$

Compare the results for large N and summations in the direction of increasing and decreasing n. What happens? What is the (more) correct result? How many terms do you have to add to reach an accuracy of 10^{-5}?

2.3 Quadratic iterations

The iteration

$$x_{n+1} = 4x_n(1 - x_n) \tag{4}$$

with an initial value $x_0 \in [0, 1]$ is an example of a chaotic map (the 'fully developed parabola'). It is analytically conjugate to the map

$$\phi_{n+1} = 1 - 2|\phi_n - 1/2| \tag{5}$$

with the identification $x_n = \sin^2(\pi\phi_n/2)$. (Verify!)

Pick a $\phi_0 \in [0, 1]$ and compare the results from the two iterations.