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Computational Physics II

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9.1. Pattern recognition in Hopfield spin glasses

In an influential article in 1982, JJ Hopfield suggested analogies between the operation of neurons in the brain and the dynamics of spin glasses. The two spin states ± 1 are related to a neuron being active or inactive, and the coupling between neurons in a node is represented by an interaction potential J_{ij}. Every spin is updated asynchronously to minimize its local energy (a Metropolis type dynamics at $T = 0$). To demonstrate the power of such a system, he suggested an application to associative memory, the ability to recognize a pattern in noisy or otherwise defective input signals. The example given here follows W. Kinzel, Z. Phys. B. **60**, 205 (1985).

Let $(s_i = \pm 1)$ be the spins and $H = -\sum_{ij} J_{ij} s_i s_j$ the Hamiltonian. The coupling matrix is defined as $J_{ij} = \sum_p s_i^{(p)} s_j^{(p)}$, where each $(s_i^{(p)})$ defines a pattern. Take a 20×20 grid to define patterns of your choice (letters, drawings, random patterns), compute the corresponding J_{ij} and try to retrieve your pattern from perturbed initial conditions. For the perturbation, switch a fraction of $q\%$ of all spins. As a measure of the recovery of the initial pattern, use the overlap $r = (1/N) \sum_i s_i^{(p)} s_i$ (which equals 1 if the recovery is perfect). Study $r(q)$ as a function of the perturbation of the pattern. How many realizations do you have to run so that r is reliably estimated?

9.2. Fourier transforms

(a) The convolution $f * g$ of two functions is defined as

$$(f * g)(x) = \int_{-\infty}^{\infty} f(y) g(x-y) dy \tag{1}$$

Show that the Fourier transform of the convolution $f * g$ equals the product of the Fourier transforms, $(f * g)\hat{} = \hat{f} \cdot \hat{g}$.

(b) The correlation function between two signals is defined as

$$C(x) = \int_{-\infty}^{\infty} f(y) f(y+x) dy \,. \tag{2}$$

What does the convolution theorem say about the Fourier transform of the correlation function?

(c) Use the convolution theorem to relate the Fourier transforms of a signal $f(t)$ (assumed to be defined for all t) and that of a segment where $f(t)$ is available on the interval $[-T, T]$ only.

(d) Compute the Fourier transform of $f(t) = \exp(-t^2/T^2)$.