PHILIPPS-UNIVERSITAET MARBURG, THE DEPARTMENT OF PHYSICS

Computational Physics II: Assignment 1

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April 24, 2018

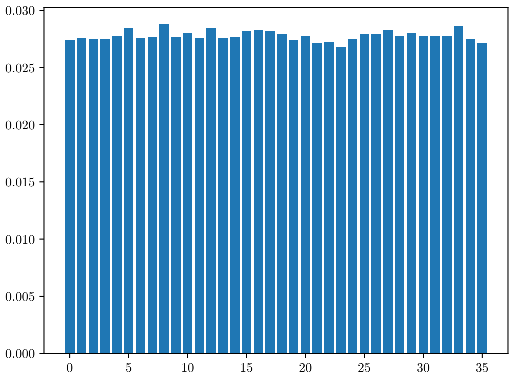
1 DICE

The key point of this task is to find a proper way plotting the histograms. Since the duplets and the triplets are multi-dimensional random vectors, plot the probabilities of each sample versus a multi-dimensional phase space may not be such evident. My approach is to use a senary scale to transfer the digits to natural integers, for example, I allocate duplets and triplets as

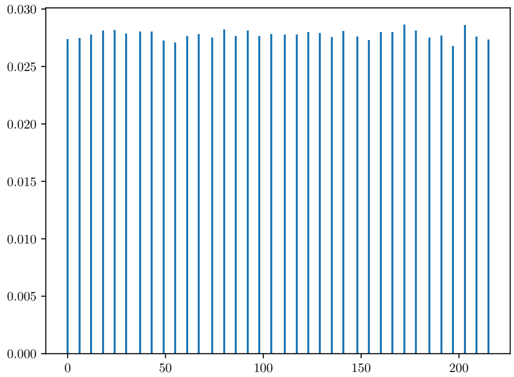
$$
\begin{aligned}
(11)_6 &\sim (0)_{10} & \qquad (111)_6 &\sim (0)_{10} \\
(12)_6 &\sim (1)_{10} & (112)_6 &\sim (1)_{10} \\
&\vdots & &\vdots \\
(ij)_6 &\sim ((i-1)\times 6 + (j-1))_{10} & (ijk)_6 &\sim \big((i-1)\times 6^2 + (j-1)\times 6 + (k-1)\big)_{10} \\
&\vdots & &\vdots \\
(66)_6 &\sim (35)_{10} & (666)_6 &\sim (215)_{10}
\end{aligned}
$$

Thus, each sample can be represent by a given integer. Then I use this integer as the x-axis while the probability as the y-axis to do plotting, the histograms of dulplets and triplets are presented in Figure 1.1.

The distribution of duplets seem to be a uniform distribution, which evident the pair correlation of the random number generator is nearly ignorable. However, the distribution of the triplets provides some strange information. There should be 216 bars in the histogram instead of 36 bars, since our sample space has a size of 216 rather than 36. I think the some of the samples are not covered by the random triplets due to the collapse of the zero correlation, resulting in such a phenomenon. In the end, a conclusion, the random number generator can work without the pair correlation but still may crash if being involved with triplets correlation, can be extracted.



(a)



(b)

Figure 1.1: The histograms for the duplets and triplets of dice rolling arrays with a size of 1×10^6: (1.1a) the distribution of the duplets; (1.1b) the distribution of the triplets.

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The comparing relation is going as:

$$\text{rank}(31) < \text{rank}(32) < \text{rank}(41) < \text{rank}(42) < \text{rank}(43) < \text{rank}(51) < \text{rank}(52) < \text{rank}(53) < \text{rank}(54) < \text{rank}(61) < \text{rank}(62) < \text{rank}(63) < \text{rank}(64) < \text{rank}(65) < \text{rank}(11) < \text{rank}(22) < \text{rank}(33) < \text{rank}(44) < \text{rank}(55) < \text{rank}(66) < \text{rank}(21).$$

By using python, I generated about 1×10^6 such duplet groups. Then use the comparing relation to check the duration of non-decreasing series. The distribution of the non-decreasing series is presented in Figure 2.1.

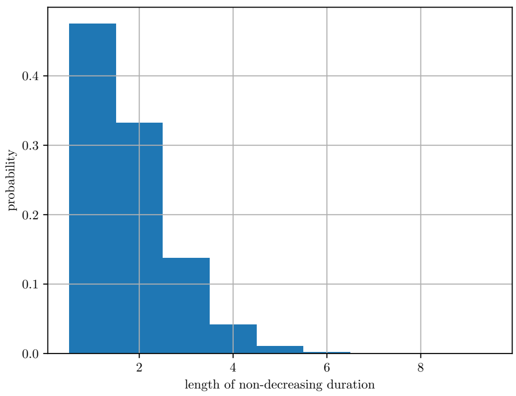

Figure 2.1: The distribution of the non-decreasing duration.

3 Non-uniform random variables

The transformation from a uniform distribution to a power distribution both at interval $(0, 1)$ goes as below:

$$
\begin{aligned}
Ax^{\alpha}\mathrm{d}x &= \mathrm{d}\xi \\
A\left(\alpha+1\right)^{-1}\mathrm{d}x^{\alpha+1} &= \mathrm{d}\xi \\
x &= \left[A^{-1}\left(\alpha+1\right)\xi\right]^{\frac{1}{\alpha+1}} \\
x &= \xi^{\frac{1}{\alpha+1}}
\end{aligned}
$$

where A is a normalization factor. The restriction for α is that the integral of the power distribution shall exist for $(0, 1)$. Only when $\alpha > -1$, can this integral exist.

Figure 3.1 gives the histograms of power distribution with different α.

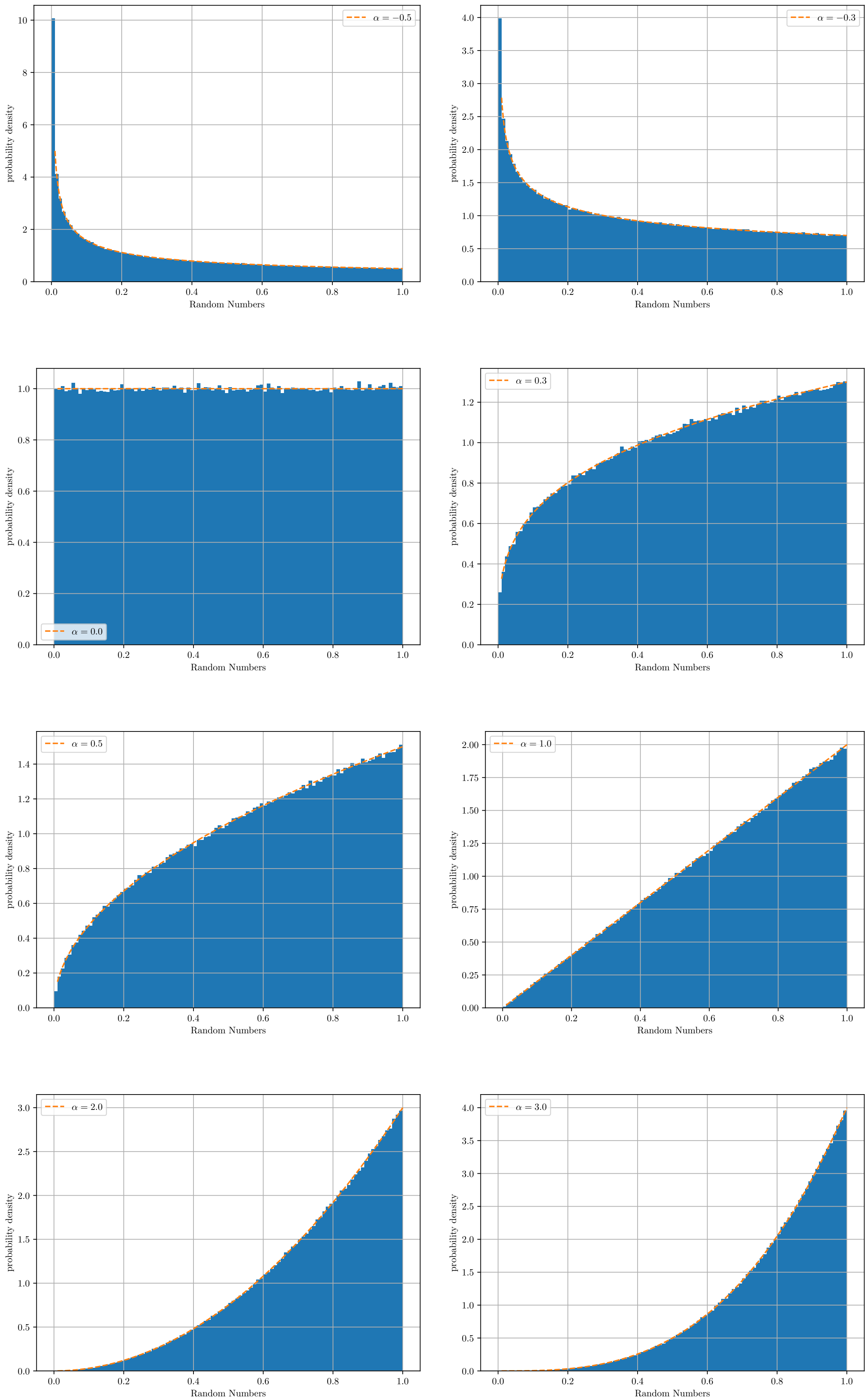

Figure 3.1: the power random distribution transformed from uniform distribution with different α.