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Nov 27, 2017
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Computational Physics I

Deadline: Dec 04, 2017

6.1. Many body equilibria in 2d

Consider a system of N particles in the plane, trapped by a harmonic potential $V_h = \mathbf{x}^2/2$ and repelling each other by vortices forces, $V_{12}(\mathbf{x}_1, \mathbf{x}_2) = -\log|\mathbf{x}_1 - \mathbf{x}_2|$.

In order to reduce the rotational symmetry, fix one particle to move along the x-axis, and allow all others to move freely in the plane. Find equilibrium positions for $N = 2, \ldots, 4$ particles.

6.2 LU decomposition for tridiagonal matrices

Matrices in which only the diagonal and the elements above and below the diagonal are non-zero are called tri-diagonal. They arise, for instance, when second order derivatives are computed with finite differences.

Consider the matrix

$$A = \begin{pmatrix} 2 & -1 & 0 & 0 & \cdots \\ -1 & 2 & -1 & 0 & \cdots \\ 0 & -1 & 2 & -1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \tag{1}$$

and determine the LU decomposition $A = LU$. Since the matrix is tridiagonal, only the elements along the diagonal and one line above or below the diagonal are non-zero. Therefore, not the full matrix but only the corresponding diagonals need to be stored.

Verify your LU decomposition by solving the equation $A\mathbf{x} = \mathbf{b}$ with $\mathbf{b} = (1, 0, \ldots, 0)$ for a 10×10 matrix and compare to the solution obtained with the corresponding Python routines.