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Computational Physics II
SS 2018

Deadline: 2. May 2018
Please turn in a written documentation of your results and submit the programs to Jonathan Prexl at *jonathan.prexl@physik.uni-marburg.de*

2.1. Random walks

Consider a one-dimensional walk on an integer lattice defined by

$$x_n = x_{n-1} + l_n \tag{1}$$

where the steps l_n are generated by the rule

$$l_n = \text{round}\,(a \cdot r_n + b) \tag{2}$$

where r_n is a random variable uniformly distributed in $(0, 1)$ and a and b are parameters. Which steps are possible and with which probability do they occur?
What are the drift $\langle x \rangle = \bar{x} = \int lp(l)dl$ and the diffusion constant $D = \langle l^2 \rangle - \langle l \rangle^2 = \int l^2 p(l)dl - \bar{x}^2$ for this process? If the general expressions become too complicated, consider the specific values $(a = 2,\, b = -1)$ and $(a = 5,\, b = -2)$.
Program the random walk and compare the distribution of points after n steps to the normal distribution

$$p_n(x) = \frac{1}{\sqrt{2\pi D n}} e^{-\frac{(x-n\bar{x})^2}{2Dn}} \tag{3}$$

2.2. Moments of the normal distribution

For a normally distributed random variable x,

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}} \tag{4}$$

determine the higher moments $\langle x^k \rangle = \int dx x^k p(x)$ for $k = 2,\, 4,\, 6,\, 8,$ and 10. Calculate the moments analytically and compare the exact values to the ones you find numerically with $N = 10^\ell$ values, where $\ell = 3, \ldots$.
How big are the relative errors (numerical - exact)/exact ?

please turn over

2.3. Parrondos paradoxical games
We start out with the following three uncorrelated games:

- in game 1 you win 1 jeton with probability $p_1 = 1/10 - \epsilon$ and loose one with probability $1 - p_1$.
- in game 2 you win 1 jeton with probability $p_2 = 3/4 - \epsilon$ and loose one with probability $1 - p_2$.
- in game 3 you win 1 jeton with probability $p_3 = 1/2 - \epsilon$ and loose one with probability $1 - p_3$.

Now games 1 and 2 are lumped together through the value M of the number of jetons: if M is divisible by 3, play game 1, if M is not divisible, play game 2. This lumped game is called A, and the third, almost fair game, will be B.

Verify, for instance for $\epsilon = 0.005$, that both the combined game A and game B are typically loosing games.

Now demonstrate that various combinations of A and B are winning strategies: study, for instance, the sequences AB, AAB, AAB, etc., or random combinations.