

## ASSESSMENT COVER SHEET

|                          |          |  |                              |                         |  |
|--------------------------|----------|--|------------------------------|-------------------------|--|
| <b>Student ID number</b> | 29743338 | Unit Name and Code:  | Parallel Computing - FIT3143 |                         |  |
|                          |          | Campus:  | Clayton                      |                         |  |
|                          |          | Assignment Title:  | ASSIGNMENT 1                 |                         |  |
|                          |          | Name of Lecturer:  | Christopher Watkins          |                         |  |
|                          |          | Name of Tutor:   | Sunny Patel                  |                         |  |
|                          |          | Tutorial Day and Time:   | Thursday 5pm                 |                         |  |
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| <b>Given Name</b>        | James    | Has any part of this assignment been previously submitted as part of another unit/course? <input type="checkbox"/> Yes <input checked="" type="checkbox"/> No  |                              |                         |  |
|                          |          | Due Date: Friday 6th of Sep  |                              | Date Submitted: 6/09/19 |  |
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# Design and Implementation of Parallel Mandelbrot Set Image Generation via OpenMPI

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**Abstract**—The Mandelbrot set is a set of numbers in the complex plane. This set is found via iteration. With some basic quadratic polynomials and a seed, the generation of our set can start. The generation of the set has been referred to as being embarrassingly parallel. This means that it is very easy to split the problem into parallel tasks. Hence the purpose behind this report. This report explores the generation of the Mandelbrot set using a parallel scheme, which the work is split up into rows. The parallelisation of the problem is implemented using the C language and a library called OpenMPI which is apart of the C language. OpenMPI allows us to communicate data across different cores or processors. This means that every process is given a row and will iterate over that row till it reaches the max value, this will be called  $iX_{max}$ . The max number of rows will be called  $iY_{max}$ . This parallel scheme along with discussion of tile based will be analysed via Amdahl's law. It was found that the use of a row-based segmentation with a Dynamic master slave approach for distributing the tasks found the greatest improvement in time taken to complete the Mandelbrot image. After analysing the run time of the serial generation of the Mandelbrot set and the run time of the parallel version, it was found that by doubling the number of cores gave an increase of 43% on average. However, from going from serial to 2 cores gave a 89% increase. The results show a significant change in speed from a single core to multi-cores, showing the parallelisation of the Mandelbrot Set was successful.

**Keywords**— *Mandelbrot set, OpenMPI, MPI, C, parallel computing, distributed computing.*

## I. INTRODUCTION

The purpose of this report was to explore the parallelisability of the Mandelbrot set, understand the intricacies of sending data between process and understand how parallel schemes play a part in the speed of computation.

The Mandelbrot set is a set of complex numbers referred to as  $C$ , in which to calculate  $z$  via the function  $Z = Z^2 + C$  [1]. In this case  $C$  is representing a constant number in which will not change throughout the iterations. The value of  $Z$  itself has no real significance, however the magnitude of the number is the important part of the number. To calculate the magnitude of a complex number is quite hard, so to do this, get the square of the numbers distance to the x-axis and the square of the numbers distance to the y-axis, then add them together and square the result. As the equation is iterated the magnitude of  $Z$  changes. The magnitude of  $Z$  can do two things as its iterated; it will be equal to or less than two or it

will be greater than two forever. If said number is greater than two then it is not a part of the Mandelbrot set. To make the previously said set, you can graph the values on the complex number plane, after plotting 100 to thousands of points a familiar image of the Mandelbrot set starts to generate. To get colour to the said image, the values of the points not in the Mandelbrot set are given colour. [2]

Due to the independent nature of the Mandelbrot Set generation, it was hypothesised that paralleling the serial version of the Mandelbrot Set code via master/slave communication and row-based segmentation of tasks would give a linear increase in speed of the generation.

## II. DESIGN AND ANALYSIS OF DESIGN

### A. Parallelisability of Mandelbrot Set

For a calculation to be parallelised, it is needed to see if they meet Bernstein's conditions. The three conditions are:

$$\begin{aligned} I_0 \cap O_1 &= \emptyset \\ I_1 \cap O_0 &= \emptyset \\ O_0 \cap O_1 &= \emptyset \end{aligned}$$

$I_0$  represents the input of the first calculation while  $O_0$  represents the output of the first calculation. Then  $I_1$  and  $O_1$  are the input and output for the second calculation. As the parallel scheme is based on rows our constant value will change dependent on row. The  $z$  iterations is represented by  $j$  and thus for rows  $i$ . This give the equation:

$$\begin{aligned} \text{Row } I &= Z_{ij} = Z_{i,j-1}^2 + i \\ \text{Row } I+1 &= Z_{i+1,j} = Z_{i,j-1}^2 + i+1 \\ \text{Thus } I_0 &= Z_{i,j-1}^2 + i \\ I_1 &= Z_{i,j-1}^2 + i+1, \\ O_0 &= Z_{ij} \\ O_1 &= Z_{i+1,j} \end{aligned}$$

The  $Z$  values are independent to its respected rows. Thus Bernstein's conditions have been met. As,

$$I_0 \cap O_1 = \emptyset$$

$$I_1 \cap O_0 = \emptyset$$

$$O_0 \cap O_1 = \emptyset$$

Thus, the mandelbrot set has met the conditions it can now be parallelised.

### B. Therotical Speed Up of Mandelbrot Set

Amdahl's law is an equation that given the right values give a value of such is the theoretical speed up of a given problem. The serial algorithm has 2 parts. (1) Printing the header to the file which does not make sense to do parallel.

(2) Calculation of the Z values which from Bernstein's conditions are known to be parallelisable. The printing to the file takes like then a thousandth of a second. Compare this to the bulk execution of the algorithm, the calculation gets a 99.998% of the time taken to execute.

$$S_{\text{latency}}(s) = \frac{1}{(1 - p) + \frac{p}{s}} \quad (1)$$

$S_{\text{latency}}$  = Speed up  
 $p$  = Paraellal ratio  
 $s$  = number of processors

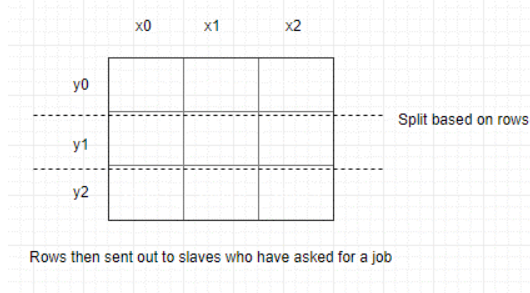
By using Amdahl's law as denoted by (1), the given table 1 is calculated.

|                                   |     |     |     |
|-----------------------------------|-----|-----|-----|
| Number of processors (s)          | 1   | 2   | 4   |
| Speed Up ( $S_{\text{latency}}$ ) | 1.0 | 2.0 | 4.0 |

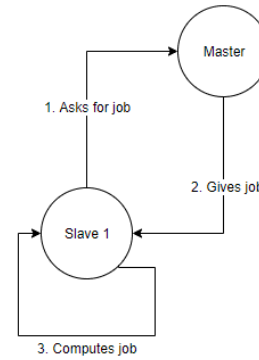
### C. Partionioning Method

The design of the parallel portioning method was developed by using a Dynamic Load Balancing method along with a row-based segmentation approach. DLB is implemented via a Master/Slave communication. The master sends out jobs to the slaves who have asked for said job see (3). This means that the tasks are broken up into small enough tasks that no one process takes to long to complete said task.

Once completed it simply asks for other tasks and the process repeats. By utilizing this method, the rows are not being statically divided to each process. This could potentially give all the hard tasks to a single process, then other process would be idle while a single process finishes its computation. Thus, by sending out jobs when asked for the rows are dynamically allocated to the processors therefore reducing idle time of processors. See (2)

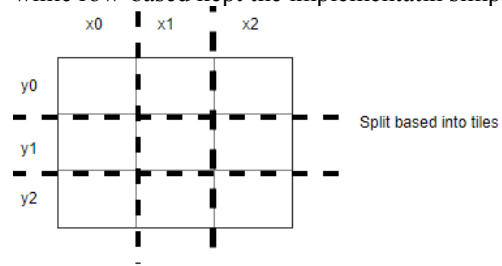


(2)



(3)

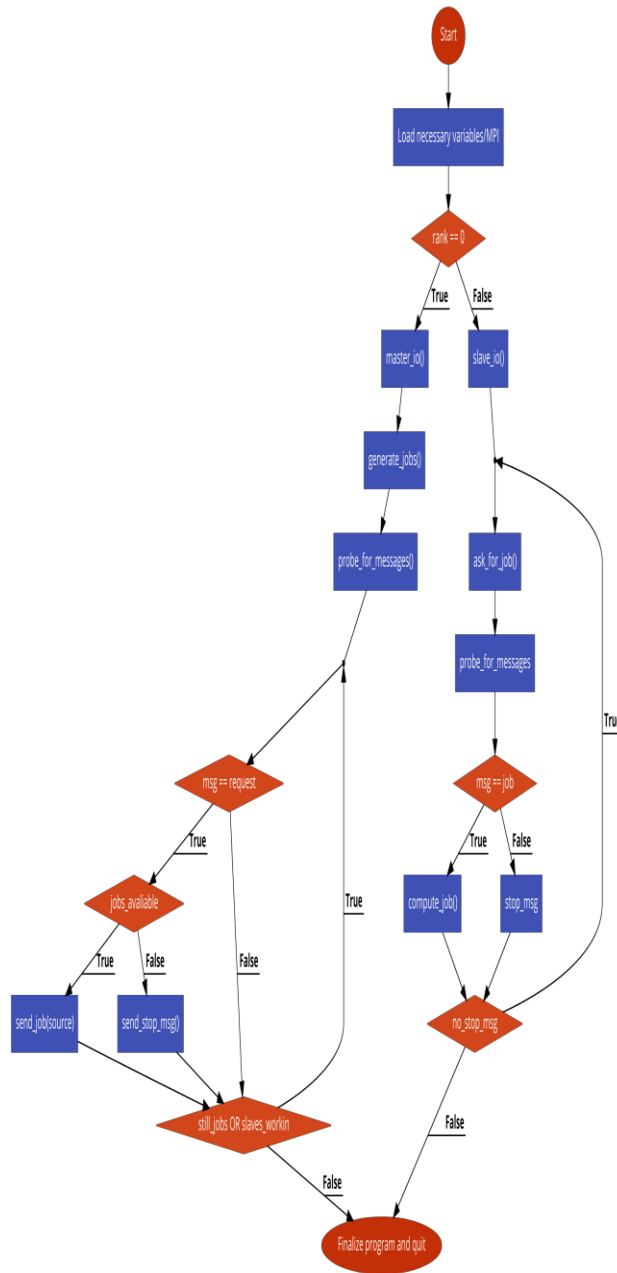
Another approach at parralising the task is to segment it into tiles see (4) However, this method makes the jobs to small such that the master has trouble keeping up with the requests of jobs. When testing both implementations, with a 8 core system, I found that the time increased from 6 seconds to 8 seconds. This time could be better increased via making the tiles larger, however this method also increased the complexity of the code and the algorithm, while row-based kept the implementatin simpiler and faster.



Tiles then sent out to slaves who have asked for a job (4)

(4)

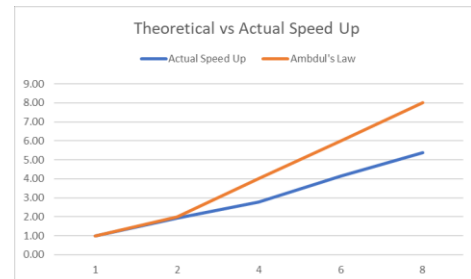
#### D. Pseudocode of Master/Slave Mandelbrot



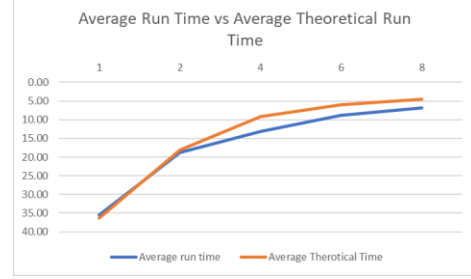
### III. RESULTS & DISCUSSION

#### A. Table of Findings

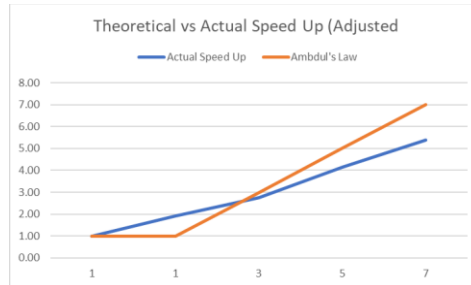
|  |                |   |       |       |      |      |  |
|--|----------------|---|-------|-------|------|------|--|
| Computer Specifications                  |                | a) Intel Core i7-7700k<br>b) 8 Logical Processors<br>c) 16Gb of RAM<br>d) 82 Mbps |       |       |      |      |  |
| Value of iXmax                           |                | 8000  |       |       |      |      |  |
| Value of iYmax                           |                | 8000  |       |       |      |      |  |
| Value of IterationMax                    |                | 1000  |       |       |      |      |  |
|  | Serial Program | Parallel Program  |       |       |      |      |  |
|  |                | MPI   |       |       |      |      |  |
|  |                | 1   | 2     | 4     | 6    | 8    |  |
| Run #1                                   | 37.14          | 35.27   | 18.64 | 13.09 | 8.61 | 6.68 |  |
| Run #2                                   | 36.69          | 35.41   | 18.89 | 13.14 | 8.69 | 6.77 |  |
| Run #3                                   | 35.75          | 35.47   | 19.03 | 12.98 | 8.91 | 6.72 |  |
| Run #4                                   | 36.07          | 35.42   | 18.84 | 13.31 | 8.59 | 6.80 |  |
| Run #5                                   | 35.73          | 36.23   | 18.67 | 13.19 | 9.08 | 6.76 |  |
| Average run time                         | 36.28          | 35.56   | 18.81 | 13.14 | 8.78 | 6.75 |  |
| Actual Speed up                          | 3.04           | 0.98  | 1.93  | 2.76  | 4.13 | 5.38 |  |
| Ambdul's Law                             | 4.20           | 1.00  | 2.00  | 4.00  | 6.00 | 8.00 |  |
| Average Theoretical Time                 | 36.28          | 36.28   | 18.14 | 9.07  | 6.05 | 4.53 |  |
| Difference between Actual vs Theoretical |                | 0.28  |       |       |      |      |  |



(1)



(2)



(3)

#### B. Discussion

From the results, you can see a clear increase in speed when implementing the parallel algorithm. However, it does not meet our theoretical analysis of the problem. This however is because of the way the algorithm is designed vs the way the formula works out the theoretical result. As this approach is a master slave algorithm, the algorithm is using 1

core as a master no matter how many cores are added. This means that when the program has access to 6 cores, it is only utilizing 5 cores, which would account for the difference in values from theoretical vs actual speed times. As you can see via the graph (3) when plotted against the adjusted values our speed is much closer to the theoretical amount. The rest of the differences can be purely based on implementation problems due to an inexperienced programmer, CPU interrupts, feedback and a range of other interferences that could alter times.

In terms of scalability, from the graphs and data, it shows a close to linear growth with regards to increase of processors, this gives us strong scalability. If the data didn't suggest that the run time decreased to by a fairly strong amount, then the scalability of the algorithm would be considered weak. These results could be proved further however at the time of testing the MonARCH system was not available, this in turn does not allow the testing of any further calls

#### IV. CONCLUSION

The report shows the findings of implementing a row-based segmentation using a Master Slave communication on the Mandelbrot Set. This was implemented with the programming language C as well as a library OpenMPI which allowed the Master Slave Implementation. The results from the algorithm showed that the Mandelbrot set benefits greatly from the use of parallel schemes. It also showed the scheme with the master/slave communication approach scaled greatly when extra processors are used. The small differences between theoretical and actual run time were identified after completing said project.

#### REFERENCES

- [1] "What is the Mandelbrot set?," 26-Jul-2018. [Online]. Available: <https://plus.maths.org/content/what-mandelbrot-set>.
- [2] D. Dewey, *Introduction to the Mandelbrot Set*, Sep-2002. [Online]. Available: <http://www.ddewey.net/mandelbrot/>.