



$$H_{\omega, U_a}(s) = \frac{\omega(s)}{U_a(s)} = \frac{1/k_e}{1 + sT_m + s^2 T_m T_a}$$

$$H_{i_a, U_a}(s) = \frac{i_a(s)}{U_a(s)} = \frac{sT_m / R_a}{1 + sT_m + s^2 T_m T_a}$$

$$H_{\omega, m_s}(s) = \frac{\omega(s)}{m_s(s)} = -\frac{R_a}{k_m k_e} \cdot \frac{1 + sT_a}{1 + sT_m + s^2 T_m T_a}$$

$$H_{i_a, m_s}(s) = \frac{i_a(s)}{m_s(s)} = -\frac{1/k_m}{1 + sT_m + s^2 T_m T_a}$$

$$T_a = \frac{L_a}{R_a}$$

$$T_m = \frac{J R_a}{k_m k_e}$$

Ecuațiile aferente PC :

$$\begin{cases} \frac{L_a}{R_a} \frac{d i_a(t)}{dt} + i_a(t) = \frac{1}{R_a} (u_a(t) - e_u(t)) \\ J \frac{d\omega(t)}{dt} = m_a(t) - m_f(t) - m_s(t) \end{cases}$$

$$m_a(t) = k_m i_a(t)$$

Notăm

$$m_f(t) = k_f \omega(t)$$

$$x_1(t) = i_a(t)$$

$$e_u(t) = k_e \omega(t)$$

$$x_2(t) = \omega(t)$$

$$T_a = \frac{L_a}{R_a}$$

$$\begin{cases} \frac{L_a}{R_a} \dot{x}_1(t) + x_1(t) = \frac{1}{R_a} [u_a(t) - e_u(t)] \\ J \dot{x}_2(t) = m_a(t) - m_f(t) - m_s(t) \end{cases}$$

$$k_f \approx 0$$

$$\begin{cases} L_a \dot{x}_1(t) + R_a x_1(t) = u_a(t) - k_e x_2(t) \\ J \dot{x}_2(t) = k_m x_1(t) - m_s(t) \end{cases}$$

$$\begin{cases} L_a \dot{x}_1(t) = -R_a x_1(t) - k_e x_2(t) + u_q(t) \\ J \dot{x}_2(t) = k_m x_1(t) - m_J(t) \end{cases}$$

$$\rightarrow \begin{cases} \dot{x}_1(t) = -\frac{1}{L_a} x_1(t) - \frac{k_e}{L_a} x_2(t) + \frac{1}{L_a} u_q(t) \\ \dot{x}_2(t) = \frac{k_m}{J} x_1(t) - \frac{1}{J} m_J(t) \end{cases}$$

↓ matrice

$$\dot{x}(t) = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \underbrace{\begin{bmatrix} -\frac{1}{L_a} & -\frac{k_e}{L_a} \\ \frac{k_m}{J} & 0 \end{bmatrix}}_A \underbrace{\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}}_{x(t)} + \underbrace{\begin{bmatrix} \frac{1}{L_a} & 0 \\ 0 & -\frac{1}{J} \end{bmatrix}}_B \underbrace{\begin{bmatrix} u_q(t) \\ m_J(t) \end{bmatrix}}_{u(t)}$$

$$y(t) = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_C \underbrace{\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}}_{x(t)}$$

$$\dot{x}(t) = A x(t) + B u(t)$$

$$y(t) = C x(t) + \underbrace{D u(t)}_{=0}$$

$$A = \begin{bmatrix} -\frac{1}{T_a} & -\frac{k_e}{L_a} \\ \frac{k_m}{J} & 0 \end{bmatrix} \quad B = \begin{bmatrix} \frac{1}{L_a} & 0 \\ 0 & -\frac{1}{J} \end{bmatrix} \quad C = \frac{T}{2}$$

$$H(s) = \frac{Y(s)}{U(s)} = C (sI - A)^{-1} B$$

$$sI = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} \quad sI - A = \begin{bmatrix} s + \frac{1}{T_a} & \frac{k_e}{L_a} \\ -\frac{k_m}{J} & s \end{bmatrix}$$

$$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \rightarrow M^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\det(sI - A) = \left(s^2 + \frac{1}{T_a}\right) + \frac{k_e k_m}{L_a J}$$

$$T_a = \frac{L_a}{R_a}$$

$$T_m = \frac{J R_a}{k_e k_m}$$

$$\frac{k_e k_m}{L_a J} = \frac{1}{T_m T_a}$$

$$\det = s^2 + \frac{1}{T_a} + \frac{1}{T_m T_a} \Rightarrow \frac{s^2 T_m T_a + s T_m + 1}{T_m T_a}$$

$$(sI - A)^{-1} = \frac{1}{\det} \cdot \begin{bmatrix} 1 & -\frac{k_c}{L_a} \\ \frac{k_m}{J} & s + \frac{1}{T_a} \end{bmatrix}$$

$$C \cdot (sI - A)^{-1} = I_2 \cdot (sI - A)^{-1} = (sI - A)^{-1}$$

$$(sI - A)^{-1} \cdot B = \frac{1}{\det} \begin{bmatrix} 1 & -\frac{k_c}{L_a} \\ \frac{k_m}{J} & s + \frac{1}{T_a} \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{L_a} & 0 \\ 0 & -\frac{1}{J} \end{bmatrix}$$

$$= \frac{1}{\det} \begin{bmatrix} \frac{s}{L_a} \overset{x_{1,u_a}}{k_c} & \frac{k_c}{L_a J} \overset{x_{1,m_s}}{1} \\ \frac{k_m}{L_a J} \overset{x_{2,u_a}}{1} & -\frac{s T_a + 1}{T_a J} \overset{x_{2,m_s}}{1} \end{bmatrix}$$

$$H_{ia} u_a = \frac{s}{L_a} \cdot \frac{1}{\det} = \frac{s}{L_a} \cdot \frac{1}{\left( s^2 + \frac{1}{T_a} \right) + \frac{k_c k_m}{L_a J}}$$

$$= \frac{s}{L_a} \cdot \frac{T_m T_a}{1 + T_m s + T_m T_a s^2} \cdot \frac{s \cdot T_m / T_a}{1 + T_m s + T_m T_a s^2}$$

$$H_{ia, m_s} = \frac{1}{\det L_a J} = \frac{k_e}{L_a J} \cdot \frac{T_m T_a}{1 + T_m s + T_m T_a s^2}$$

$$\frac{T_a}{L_a} = \frac{1}{R_a} \quad \frac{k_e k_m}{J R_a} = \frac{1}{T_m} \quad \rightarrow \frac{k_e T_m}{J R_a} = \frac{1}{k_m}$$

$$\frac{k_e T_m T_a}{L_a J} = \frac{\cancel{k_e} \cdot \cancel{J R_a} \cdot \frac{L_a}{R_a}}{\cancel{L_a J} \cdot \cancel{k_m k_e}} = \frac{1}{k_m}$$

$$H_{ia, m_s} = \frac{1/k_m}{1 + T_m s + T_m T_a s^2}$$

$$H_{u, u_a} = \frac{1}{\det L_a J} = \frac{k_m}{L_a J} \cdot \frac{T_m T_a}{1 + T_m s + T_m T_a s^2}$$

$$\frac{k_m}{\cancel{L_a J}} \cdot \frac{\cancel{J R_a}}{\cancel{k_m k_e}} \cdot \frac{L_a}{\cancel{R_a}} = \frac{1}{k_e}$$

$$H_{u, u_a} = \frac{1/k_e}{1 + T_m s + T_m T_a s^2}$$

$$H_{w, m_s} = \frac{1}{\det} \cdot - \frac{sT_a + 1}{T_a J}$$

$$= \frac{T_m T_a}{1 + T_m s + T_m T_a s^2} \cdot - \left( \frac{sT_a + 1}{T_a J} \right)$$

$$\frac{\cancel{J} R_a}{k_m k_e} \cdot \frac{\cancel{L_a}}{\cancel{R_a}} \cdot \frac{-(sT_a + 1)}{J} \cdot \frac{\cancel{R_a}}{\cancel{L_a}}$$

$$H_{w, m_s} = - \frac{R_a}{k_m k_e} \frac{sT_a + 1}{1 + T_m s + T_m T_a s^2}$$

$$\underbrace{\begin{bmatrix} \dot{\theta}_p \\ \dot{\theta}_c \end{bmatrix}}_X = \underbrace{\begin{bmatrix} -\frac{1}{T_p} & \frac{1}{T_p} \\ \frac{k_p}{k_c T_c} & -\frac{1+k_p/k_c}{T_c} \end{bmatrix}}_A \underbrace{\begin{bmatrix} \theta_p \\ \theta_c \end{bmatrix}}_X + \underbrace{\begin{bmatrix} \frac{k_E}{k_p T_p} & 0 \\ 0 & \frac{1}{T_c} \end{bmatrix}}_B \underbrace{\begin{bmatrix} u_c \\ \theta_e \end{bmatrix}}_U$$

$$y = I x$$

$$\rightarrow H(s) = C (sI - A)^{-1} B$$

$$sI - A = \begin{bmatrix} s + \frac{1}{T_p} & -\frac{1}{T_p} \\ -\frac{k_p}{k_c T_c} & s + \frac{1+k_p/k_c}{T_c} \end{bmatrix}$$

$$\det(sI - A) = \left(s + \frac{1}{T_p}\right) \left(s + \frac{1+k_p/k_c}{T_c}\right) - \frac{k_p}{k_c T_c} \cdot \frac{1}{T_p} \Rightarrow$$

$$= s^2 + \frac{s + s k_p/k_c}{T_c} + \frac{s}{T_p} + \frac{1+k_p/k_c}{T_p T_c} - \frac{k_p}{k_c T_c T_p}$$



$$\frac{s^2 + \frac{s + s k_p/k_c}{T_c} + \frac{1}{T_p} + \frac{1 + k_p/k_c}{T_p T_c} - \frac{k_p}{k_c T_c T_p}}{s^2 T_p T_c + s T_p + s k_p T_p/k_c + s T_c + 1 + k_p/k_c} =$$

$$- \frac{k_p}{k_c T_c T_p} =$$

$$\det = \frac{s^2 k_c T_p T_c + s k_c T_p + s k_p T_p + s k_c T_c + k_c}{k_c T_c T_p}$$

$$(sI - A)^{-1} B = \frac{1}{\det} \begin{bmatrix} \frac{s T_c + 1 + k_p/k_c}{T_c} & \frac{1}{T_p} \\ \frac{k_p}{k_c T_c} & \frac{s T_p + 1}{T_p} \end{bmatrix} \begin{bmatrix} \frac{k_c}{k_p T_p} & 0 \\ 0 & \frac{1}{T_c} \end{bmatrix} =$$

$$= \frac{1}{\det} \begin{bmatrix} \frac{s T_c + 1 + k_p/k_c}{T_c} \cdot \frac{k_c}{k_p T_p} & \frac{1}{T_p T_c} \\ \frac{k_p}{k_c T_c} \cdot \frac{k_c}{k_p T_p} & \frac{s T_p + 1}{T_p} \cdot \frac{1}{T_c} \end{bmatrix}$$

$$H(s) = \begin{bmatrix} H_{\theta_p u_c} & H_{\theta_p \theta_e} \\ H_{\theta_c u_c} & H_{\theta_c \theta_e} \end{bmatrix}$$

$$H_{\theta_c u_c} = \frac{1}{\det} \cdot \frac{\cancel{k_p}}{k_c T_c} \cdot \frac{k_c}{\cancel{k_p} T_p} = \frac{1}{\det} \frac{k_e}{k_c T_c T_p}$$

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$$\frac{s^2 k_c T_p T_c + s k_c T_p + s k_p T_p + s k_c T_c + k_c}{k_c T_c T_p} \cdot \frac{k_e}{k_c T_c T_p}$$

$$H_{\theta_c u_c} = \frac{k_e}{s^2 k_c T_p T_c + s(k_c T_p + k_p T_p + k_c T_c) + k_c}$$

$$H_{\theta_c \theta_e} = \frac{1}{\det} \cdot \frac{s T_p + 1}{T_p} \cdot \frac{1}{T_c} = \frac{1}{\det} \frac{s T_p + 1}{T_p T_c}$$

$$H_{\theta_c \theta_e} = \frac{s T_p k_c + k_c}{s^2 k_c T_p T_c + s(k_c T_p + k_p T_p + k_c T_c) + k_c}$$

$$H_{\theta_p u_c} = \frac{1}{\det} \cdot \frac{s T_c + 1 + k_p/k_c}{T_c} \cdot \frac{k_e}{k_p T_p} = \frac{k_c T_p \cancel{k_c} k_e (s T_c + 1 + k_p/k_c)}{\dots \cancel{k_p} T_p \cancel{k_c}}$$

$$H_{\theta_p u_c} = \frac{s k_c k_e T_c + k_e k_e + k_e k_p}{s^2 k_c T_p T_c + s(k_c T_p + k_p T_p + k_c T_c) + k_c}$$

$$H_{\theta_p \theta_e} = \frac{1}{\det} \cdot \frac{1}{T_p T_c} = \frac{\cancel{k_c} T_p \cancel{T_c}}{\dots} \cdot \frac{1}{T_p T_c}$$

$$H_{\theta_p \theta_e} = \frac{k_c}{s^2 k_c T_p T_c + s(k_c T_p + k_p T_p + k_c T_c) + k_c}$$