Transformata Z

$$F(z) = Z\{f(k)\} = \sum_{k=0}^{\infty} f(k) z^{-k}$$

① $f(k) = \sigma(k) = \begin{cases} 1, & k \geq 0 \\ 0, & k < 0 \end{cases}$

$$F(k) = Z\{\sigma(k)\} = \sum_{k=0}^{\infty} \sigma(k) z^{-k} =$$

$$= \sum_{k=0}^{\infty} z^{-k} = 1 + z^{-1} + z^{-2} + \dots + z^{-\infty} =$$

$(n \to \infty)$

progresie geometrica: $q = z^{-1}$

$b \dfrac{q^n - 1}{q - 1}, q \neq 1$

$nb_1, q = 1$

$$\sum \dots = 1 \cdot \frac{\cancel{\left(z^{-1}\right)^{n+1}} - 1}{z^{-1} - 1} = \frac{1}{1 - z^{-1}}$$

$$Z\{\sigma(k)\} = \frac{1}{1 - z^{-1}}$$

(2) $f(k) = e^{-ak} \Rightarrow \mathcal{Z}\{f(k)\} = \sum_{k=0}^{\infty} e^{-ak} z^{-k}$

$$= \sum_{k=0}^{\infty} \left(e^{-a} \cdot z^{-1}\right)^{k} = 1 + \left(e^{-a} z^{-1}\right) + \left(e^{-a} z^{-1}\right)^{2} + \dots + \underbrace{\left(e^{-a} \cdot z^{-1}\right)^{n}}_{0}\quad n \to \infty$$

$$= 1 \cdot \frac{\overbrace{\left(e^{-a} z^{-1}\right)^{n+1}}^{0} - 1}{e^{-a} z^{-1} - 1} = \frac{1}{1 - e^{-a} z^{-1}}$$

$$\mathcal{Z}\{f(k)\} = \frac{1}{1 - e^{-a} z^{-1}}$$

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(3) $\mathcal{Z}\{f(k+1)\} = \sum_{k=0}^{\infty} f(k+1)\, z^{-k}$

Notăm $k+1 = \tau$ $\quad k \to \infty \to \tau \to \infty$

$k = \tau - 1$ $\quad k \to 0 \to \tau \to 1$

$$= \sum_{\tau=1}^{\infty} f(\tau)\, z^{-(\tau-1)} = z \sum_{\tau=1}^{\infty} f(\tau)\, z^{-\tau}$$

$$= z\left[F(z) - f(0)\right]$$

$$\mathcal{Z}\{f(k+1)\} = z\left[F(z) - f(0)\right]$$

$$\mathcal{Z}\{f(k+2)\} = \sum_{k=0}^{\infty} f(k+2)\, z^{-k}$$

$$k+2 = \tau \quad \rightarrow$$
$$k = \tau - 2 \quad \rightarrow$$

$$= \sum_{\tau=2}^{\infty} f(\tau) \cdot z^{-k} \cdot z^{2} =$$

$$= z^2 \left[F(z) - f(0) - f(1)\, z^{-1} \right]$$

$$\boxed{\mathcal{Z}\{f(k+n)\} = z^n \left[F(z) - \sum_{k=0}^{n-1} f(k)\, z^{-k} \right]}$$

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(4) $y(k+1) - a\, y(k) = 0, \quad y(0) = y_0$

$$\mathcal{Z}\{y(k+1) - a\, y(k)\} = \mathcal{Z}\{0\}$$

$$\mathcal{Z}\{y(k+1)\} - a\, \mathcal{Z}\{y(k)\} = 0$$

$$z\left[Y(z) - y(0)\right] - a\, Y(z) = 0$$

$$(z-a)\,Y(z) = y_0 \cdot z$$

$$Y(z) = \frac{y_0 \cdot z}{z-a} = y_0 \cdot \frac{\cancel{z}}{\cancel{z}(1-a z^{-1})}$$

$$= \frac{y_0}{1-a z^{-1}}$$

$$\mathcal{Z}^{-1}\{Y(z)\} = y(k)$$

$$y(k) = \mathcal{Z}^{-1}\left\{ y_0 \frac{1}{1-a z^{-1}} \right\}$$

$$\boxed{y(k) = a^k}$$

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(5) $y(k+2) - 18\, y(k+1) + 32\, y(k) = 0$ $\quad y_0 = 0$, $y_1 = 2$

$$\mathcal{Z}\{y(k+2)\} - 18\, \mathcal{Z}\{y(k+1)\} + 32\, \mathcal{Z}\{y(k)\} = 0$$

$$z^2\left[Y(z) - y(0) - y(1) z^{-1}\right] - 18 z\left[Y(z) - y_0\right] +$$

$$+ 32\, Y(z) = Y(z)\left(z^2 - 18z + 32\right) - 2z = 0$$

$$\rightarrow Y(z) = \frac{2z}{z^2 - 18z + 32} = \frac{2z}{(z-2)(z-16)}$$

$$y(k) = Z^{-1}\left\{\frac{2z^{-1}}{(1-2z^{-1})(1-16z^{-1})}\right\}$$

$$\Rightarrow \frac{A}{(1-2z^{-1})} + \frac{B}{(1-16z^{-1})} = \frac{2z^{-1}}{- \; - - -}$$

$$A(1-16z^{-1}) + B(1-2z^{-1}) = 2z^{-1}$$

$$\begin{cases} A + B = 0 \\ -16A - 2B = 2 \end{cases} \qquad -16A + 2A = 2$$

$$-14A = 2 \to \begin{cases} A = -\frac{1}{7} \\ B = +\frac{1}{7} \end{cases}$$

$$y(k) = Z\left\{-\frac{1}{7}\underbrace{\frac{1}{(1-2z^{-1})}}_{a^k}\right\} + Z\left\{+\frac{1}{7}\frac{1}{1-16z^{-1}}\right\}$$

$$= -\frac{1}{7}2^k + \frac{1}{7}16^k = \frac{1}{7}(16^k - 2^k)$$

Tema: 4, 5, 6, 7 pag 9

1 -> timp continuu

Z -> timp discret