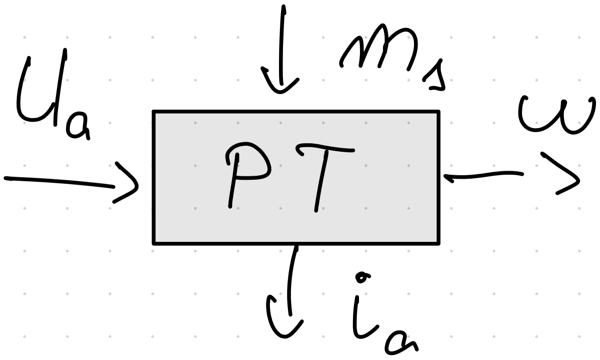

$$H_{\omega,U_a}(s) = \frac{\omega(s)}{U_a(s)} = \frac{1/k_e}{1 + sT_m + s^2 T_m T_a}$$

$$H_{i_a,U_a}(s) = \frac{i_a(s)}{U_a(s)} = \frac{sT_m/R_a}{1 + sT_m + s^2 T_m T_a}$$

$$H_{\omega,m_s}(s) = \frac{\omega(s)}{m_s(s)} = -\frac{R_a}{k_m k_e} \cdot \frac{1 + sT_a}{1 + sT_m + s^2 T_m T_a}$$

$$H_{i_a,m_s}(s) = \frac{i_a(s)}{m_s(s)} = -\frac{1/k_m}{1 + sT_m + s^2 T_m T_a}$$

$$T_a = \frac{L_a}{R_a} \qquad T_m = \frac{J R_a}{k_m k_e}$$

Ecuațiile aferente PC:

$$\begin{cases} \frac{L_a}{R_a} \frac{d\, i_a(t)}{dt} + i_a(t) = \frac{1}{R_a}\left(u_a(t) - e_w(t)\right) \\ J \frac{d\omega(t)}{dt} = m_a(t) - m_f(t) - m_s(t) \end{cases}$$

$m_a(t) = k_m\, i_a(t)$

$m_f(t) = k_f\, \omega(t)$

$e_w(t) = k_e\, \omega(t)$

$T_a = \frac{L_a}{R_a}$

Notăm

$x_1(t) = i_a(t)$

$x_2(t) = \omega(t)$

$$\begin{cases} \frac{L_a}{R_a} \dot{x}_1(t) + x_1(t) = \frac{1}{R_a}\left[u_a(t) - e_w(t)\right] \\ J \dot{x}_2(t) = m_a(t) - m_f(t) - m_s(t) \end{cases}$$

$k_f \approx 0$

$$\begin{cases} L_a \dot{x}_1(t) + R_a x_1(t) = u_a(t) - k_e\, x_2(t) \\ J \dot{x}_2(t) = k_m\, x_1(t) - m_s(t) \end{cases}$$

$$\begin{cases} L_a \, \dot{x}_1(t) = -R_a \, x_1(t) - k_e \, x_2(t) + u_a(t) \\ J \, \dot{x}_2(t) = k_m \, x_1(t) - m_J(t) \end{cases}$$

$$\rightarrow \begin{cases} \dot{x}_1(t) = -\dfrac{1}{T_a} x_1(t) - \dfrac{k_e}{L_a} x_2(t) + \dfrac{1}{L_a} u_a(t) \\ \dot{x}_2(t) = \dfrac{k_m}{J} x_1(t) - \dfrac{1}{J} m_J(t) \end{cases}$$

$\downarrow$ matrice

$$\dot{x}(t) = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \overbrace{\begin{bmatrix} -\dfrac{1}{T_a} & -\dfrac{k_e}{L_a} \\ \dfrac{k_m}{J} & 0 \end{bmatrix}}^{A} \overbrace{\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}}^{x(t)} + \overbrace{\begin{bmatrix} \dfrac{1}{L_a} & 0 \\ 0 & -\dfrac{1}{J} \end{bmatrix}}^{B} \overbrace{\begin{bmatrix} u_a(t) \\ m_J(t) \end{bmatrix}}^{u(t)}$$

$$y(t) = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{C} \underbrace{\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}}_{x(t)}$$

$$\begin{cases} \dot{x}(t) = A\,x(t) + B\,u(t) \\ y(t) = C\,x(t) + \underbrace{D\,u(t)}_{=0} \end{cases}$$

$$A = \begin{bmatrix} -\frac{1}{T_a} & -\frac{k_e}{L_a} \\ \frac{k_m}{J} & 0 \end{bmatrix} \quad B = \begin{bmatrix} \frac{1}{L_a} & 0 \\ 0 & -\frac{1}{J} \end{bmatrix} \quad C = \frac{I}{2}$$

$$H(s) = \frac{Y(t)}{U(t)} = C \underbrace{(sI - A)}^{-1} B$$

$$sI = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} \quad sI - A = \begin{bmatrix} s + \frac{1}{T_a} & \frac{k_e}{L_a} \\ -\frac{k_m}{J} & s \end{bmatrix}$$

$$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \rightarrow M^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\det(sI - A) = \left(s^2 + \frac{s}{T_a}\right) + \frac{k_e k_m}{L_a J}$$

$$T_a = \frac{L_a}{R_a} \qquad T_m = \frac{J R_a}{k_e k_m}$$

$$\frac{k_e k_m}{L_a J} = \frac{1}{T_m T_a} \qquad \det = s^2 + \frac{s}{T_a} + \frac{1}{T_m T_a} \Rightarrow$$

$$\frac{s^2 T_m T_a + s T_m + 1}{T_m T_a}$$

$$(sI - A)^{-1} = \frac{1}{\det} \cdot \begin{bmatrix} s & -\frac{k_e}{L_a} \\ \frac{k_m}{J} & s + \frac{1}{T_a} \end{bmatrix}$$

$$C \cdot (sI - A)^{-1} = I_2 \cdot (sI - A)^{-1} = (sI - A)^{-1}$$

$$(sI - A)^{-1} \cdot B = \frac{1}{\det} \begin{bmatrix} s & -\frac{k_e}{L_a} \\ \frac{k_m}{J} & s + \frac{1}{T_a} \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{L_a} & 0 \\ 0 & -\frac{1}{J} \end{bmatrix}$$

$$= \frac{1}{\det} \begin{bmatrix} \frac{s}{L_a} \ {\scriptstyle x_1, u_a} & \frac{k_e}{L_a J} \ {\scriptstyle \cancel{x}_1, m_s} \\ \frac{k_m}{L_a J} \ {\scriptstyle x_2, u_a} & -\frac{s T_a + 1}{T_a J} \ {\scriptstyle x_2, m_s} \end{bmatrix}$$

$$H_{i_a u_a} = \frac{s}{L_a} \cdot \frac{1}{\det} = \frac{s}{L_a} \cdot \frac{1}{\left(s^2 + \frac{s}{T_a}\right) + \frac{k_e k_m}{L_a J}}$$

$$= \frac{s}{L_a} \cdot \frac{T_m T_a}{1 + T_m s + T_m T_a s^2} = \boxed{\frac{s \cdot T_m / R_a}{1 + T_m s + T_m T_a s^2}}$$

$$H_{i_a m_s} = \frac{1}{\det} \frac{k_e}{L_a J} = \frac{k_e}{L_a J} \cdot \frac{T_m T_a}{1 + T_m s + T_m T_a s^2}$$

$$\frac{T_a}{L_a} = \frac{1}{R_a} \qquad \frac{k_e k_m}{J R_a} = \frac{1}{T_m} \qquad \rightarrow \frac{k_e T_m}{J R_a} = \frac{1}{k_m}$$

$$\frac{k_e \, T_m T_a}{L_a J} = \frac{\cancel{k_e} \cdot \frac{\cancel{J R_a}}{k_m \cancel{k_e}} \cdot \frac{\cancel{L_a}}{\cancel{R_a}}}{\cancel{L_a J}} = \frac{1}{k_m}$$

$$\boxed{H_{i_a, m_s} = \frac{1/k_m}{1 + T_m s + T_m T_a s^2}}$$

$$H_{\omega, u_a} = \frac{1}{\det} \frac{k_m}{L_a J} : \frac{k_m}{L_a J} \frac{T_m T_a}{1 + T_m s + T_m T_a s^2}$$

$$\frac{\cancel{k_m}}{\cancel{L_a J}} \cdot \frac{\cancel{J R_a}}{\cancel{k_m} k_e} \cdot \frac{\cancel{L_a}}{\cancel{R_a}} = \frac{1}{k_e}$$

$$\boxed{H_{\omega, u_a} = \frac{1/k_e}{1 + T_m s + T_m T_a s^2}}$$

$$H_{\omega,m_s} = \frac{1}{\det} \cdot -\frac{sT_a + 1}{T_a J} =$$

$$= \frac{T_m T_a}{1 + T_m s + T_m T_a s^2} \cdot -\left(\frac{sT_a + 1}{T_a J}\right)$$

$$\frac{\cancel{J} R_a}{k_m k_e} \cdot \frac{\cancel{L_a}}{\cancel{R_a}} \cdot \frac{-(sT_a + 1)}{\cancel{J}} \cdot \frac{\cancel{R_a}}{\cancel{L_a}}$$

$$\boxed{H_{\omega,m_s} = -\frac{R_a}{k_m k_e} \frac{sT_a + 1}{1 + T_m s + T_m T_a s^2}}$$

$$\underbrace{\begin{bmatrix} \dot{\vartheta}_p \\ \dot{\vartheta}_c \end{bmatrix}}_{\dot{x}} = \underbrace{\begin{bmatrix} -\frac{1}{T_p} & \frac{1}{T_p} \\ \frac{k_p}{k_c T_p} & -\frac{1+k_p/k_c}{T_c} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} \vartheta_p \\ \vartheta_c \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} \frac{k_E}{k_p T_p} & 0 \\ 0 & \frac{1}{T_c} \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} u_c \\ \vartheta_e \end{bmatrix}}_{u}$$

$$y = I x$$

$$\rightarrow H(s) = C\,(sI - A)^{-1} B$$

$$sI - A = \begin{bmatrix} s + \frac{1}{T_p} & -\frac{1}{T_p} \\ -\frac{k_p}{k_c T_c} & s + \frac{1+k_p/k_c}{T_c} \end{bmatrix}$$

$$\det(sI - A) = \left(s + \frac{1}{T_p}\right)\left(s + \frac{1+k_p/k_c}{T_c}\right) - \frac{k_p}{k_c T_c} \cdot \frac{1}{T_p} \rightarrow$$

$$= s^2 + \frac{s + s\,k_p/k_c}{T_c} + \frac{s}{T_p} + \frac{1+k_p/k_c}{T_p T_c} - \frac{k_p}{k_c T_c T_p}$$

$$s^2 + \frac{s + s\,k_p/k_c}{T_c} + \frac{s}{T_p} + \frac{1 + k_p/k_c}{T_p T_c} - \frac{k_p}{k_c T_c T_p}$$

$$\frac{s^2 T_p T_c + s T_p + s\, k_p T_p/k_c + s T_c + 1 + k_p/k_c}{T_p T_c}$$

$$- \frac{k_p}{k_c T_c T_p} =$$

$$\det = \frac{s^2 k_c T_p T_c + s k_c T_p + s k_p T_p + s k_c T_c + k_c}{k_c T_c T_p}$$

$$(sI - A)^{-1} B = \frac{1}{\det} \begin{bmatrix} \frac{sT_c + 1 + k_p/k_c}{T_c} & \frac{1}{T_p} \\ \frac{k_p}{k_c T_c} & \frac{sT_p + 1}{T_p} \end{bmatrix} \begin{bmatrix} \frac{k_e}{k_p T_p} & 0 \\ 0 & \frac{1}{T_c} \end{bmatrix} =$$

$$= \frac{1}{\det} \begin{bmatrix} \frac{sT_c + 1 + k_p/k_c}{T_c} \cdot \frac{k_e}{k_p T_p} & \frac{1}{T_p T_c} \\ \frac{k_p}{k_c T_c} \cdot \frac{k_e}{k_p T_p} & \frac{sT_p + 1}{T_p} \cdot \frac{1}{T_c} \end{bmatrix}$$

$$H(s) = \begin{bmatrix} H_{\theta_p u_c} & H_{\theta_p \theta_e} \\ H_{\theta_c u_c} & H_{\theta_c \theta_e} \end{bmatrix}$$

$$H_{\vartheta_c u_c} = \frac{1}{\det} \cdot \frac{\cancel{k_p}}{k_c T_c} \cdot \frac{k_e}{\cancel{k_p} T_p} = \frac{1}{\det} \, \frac{k_e}{k_c T_c T_p}$$

$$\frac{}{\frac{s^2 k_c T_p T_c + s k_c T_p + s k_p T_p + s k_c T_c + k_c}{\cancel{k_c T_c T_p}}} \cdot \frac{k_e}{\cancel{k_c T_c T_p}}$$

$$\boxed{H_{\vartheta_c u_c} = \frac{k_e}{s^2 k_c T_p T_c + s\left(k_c T_p + k_p T_p + k_c T_c\right) + k_c}}$$

$$H_{\vartheta_c \vartheta_e} = \frac{1}{\det} \, \frac{s T_p + 1}{T_p} \cdot \frac{1}{T_c} = \frac{1}{\det} \, \frac{s T_p + 1}{T_p T_c}$$

$$\boxed{H_{\vartheta_c \vartheta_e} = \frac{s T_p k_c + k_c}{s^2 k_c T_p T_c + s\left(k_c T_p + k_p T_p + k_c T_c\right) + k_c}}$$

$$H_{\vartheta_p u_c} = \frac{1}{\det} \cdot \frac{s T_c + 1 + k_p/k_c}{T_c} \cdot \frac{k_e}{k_p T_p} = \frac{k_c \cancel{T_p} \cancel{T_c}}{\dots} \, \frac{k_c}{k_p \cancel{T_p} \cancel{T_c}} \left(s T_c + 1 + k_p/k_c\right)$$

$$\boxed{H_{\vartheta_p u_c} = \frac{s k_c k_e T_c + k_c k_e + k_e k_p}{s^2 k_c T_p T_c + s\left(k_c T_p + k_p T_p + k_c T_c\right) + k_c}}$$

$$H_{\vartheta_p \vartheta_e} = \frac{1}{\det} \cdot \frac{1}{T_p T_c} = \frac{k_c \cancel{T_p} \cancel{T_c}}{\dots} \cdot \frac{1}{\cancel{T_p T_c}}$$

$$\boxed{H_{\vartheta_p \vartheta_e} = \frac{k_c}{s^2 k_c T_p T_c + s\left(k_c T_p + k_p T_p + k_c T_c\right) + k_c}}$$