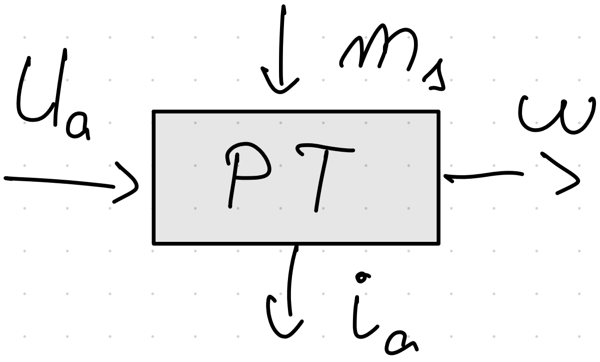

$$H_{\omega,U_a}(s) = \frac{\omega(s)}{U_a(s)} = \frac{1/k_e}{1 + sT_m + s^2 T_m T_a}$$

$$H_{i_a,U_a}(s) = \frac{i_a(s)}{U_a(s)} = \frac{sT_m/R_a}{1 + sT_m + s^2 T_m T_a}$$

$$H_{\omega,m_s}(s) = \frac{\omega(s)}{m_s(s)} = -\frac{R_a}{k_m k_e} \cdot \frac{1 + sT_a}{1 + sT_m + s^2 T_m T_a}$$

$$H_{i_a,m_s}(s) = \frac{i_a(s)}{m_s(s)} = -\frac{1/k_m}{1 + sT_m + s^2 T_m T_a}$$

$$T_a = \frac{L_a}{R_a} \qquad T_m = \frac{J R_a}{k_m k_e}$$

Ecuațiile aferente PC:

$$\begin{cases} \frac{L_a}{R_a} \frac{d\, i_a(t)}{dt} + i_a(t) = \frac{1}{R_a}\left(u_a(t) - e_w(t)\right) \\ J \frac{d\omega(t)}{dt} = m_a(t) - m_f(t) - m_s(t) \end{cases}$$

$m_a(t) = k_m\, i_a(t)$

$m_f(t) = k_f\, \omega(t)$

$e_w(t) = k_e\, \omega(t)$

$T_a = \frac{L_a}{R_a}$

Notăm

$x_1(t) = i_a(t)$

$x_2(t) = \omega(t)$

$$\begin{cases} \frac{L_a}{R_a} \dot{x}_1(t) + x_1(t) = \frac{1}{R_a}\left[u_a(t) - e_w(t)\right] \\ J\, \dot{x}_2(t) = m_a(t) - m_f(t) - m_s(t) \end{cases}$$

$k_f \approx 0$

$$\begin{cases} L_a\, \dot{x}_1(t) + R_a\, x_1(t) = u_a(t) - k_e\, x_2(t) \\ J\, \dot{x}_2(t) = k_m\, x_1(t) - m_s(t) \end{cases}$$

$$\begin{cases} L_a \, \dot{x}_1(t) = -R_a \, x_1(t) - k_e \, x_2(t) + u_a(t) \\ J \, \dot{x}_2(t) = k_m \, x_1(t) - m_s(t) \end{cases}$$

$$\rightarrow \begin{cases} \dot{x}_1(t) = -\frac{1}{T_a} x_1(t) - \frac{k_e}{L_a} x_2(t) + \frac{1}{L_a} u_a(t) \\ \dot{x}_2(t) = \frac{k_m}{J} x_1(t) - \frac{1}{J} m_s(t) \end{cases}$$

$\downarrow$ matrice

$$\dot{x}(t) = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \overbrace{\begin{bmatrix} -\frac{1}{T_a} & -\frac{k_e}{L_a} \\ \frac{k_m}{J} & 0 \end{bmatrix}}^{A} \overbrace{\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}}^{x(t)} + \overbrace{\begin{bmatrix} \frac{1}{L_a} & 0 \\ 0 & -\frac{1}{J} \end{bmatrix}}^{B} \overbrace{\begin{bmatrix} u_a(t) \\ m_s(t) \end{bmatrix}}^{u(t)}$$

$$y(t) = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{C} \underbrace{\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}}_{x(t)}$$

$$\begin{cases} \dot{x}(t) = A x(t) + B u(t) \\ y(t) = C x(t) + \underbrace{D u(t)}_{=0} \end{cases}$$

$$A = \begin{bmatrix} -\frac{1}{T_a} & -\frac{k_e}{L_a} \\ \frac{k_m}{J} & 0 \end{bmatrix} \quad B = \begin{bmatrix} \frac{1}{L_a} & 0 \\ 0 & -\frac{1}{J} \end{bmatrix} \quad C = I_2$$

$$H(s) = \frac{Y(t)}{U(t)} = C\,\underbrace{(sI - A)}^{-1} B$$

$$sI = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} \quad sI - A = \begin{bmatrix} s + \frac{1}{T_a} & \frac{k_e}{L_a} \\ -\frac{k_m}{J} & s \end{bmatrix}$$

$$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \rightarrow M^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\det(sI - A) = \left(s^2 + \frac{s}{T_a}\right) + \frac{k_e k_m}{L_a J}$$

$$T_a = \frac{L_a}{R_a} \qquad T_m = \frac{J R_a}{k_e k_m}$$

$$\frac{k_e k_m}{L_a J} = \frac{1}{T_m T_a} \qquad \det: s^2 + \frac{s}{T_a} + \frac{1}{T_m T_a} \Rightarrow$$

$$\frac{s^2 T_m T_a + s T_m + 1}{T_m T_a}$$

$$(sI - A)^{-1} = \frac{1}{\det} \cdot \begin{bmatrix} s & -\frac{k_e}{L_a} \\ \frac{k_m}{J} & s+\frac{1}{T_a} \end{bmatrix}$$

$$C \cdot (sI - A)^{-1} = I_2 \cdot (sI - A)^{-1} = (sI - A)^{-1}$$

$$(sI - A)^{-1} \cdot B = \frac{1}{\det} \begin{bmatrix} s & -\frac{k_e}{L_a} \\ \frac{k_m}{J} & s+\frac{1}{T_a} \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{L_a} & 0 \\ 0 & -\frac{1}{J} \end{bmatrix}$$

$$= \frac{1}{\det} \begin{bmatrix} \frac{s}{L_a} \; {\scriptstyle x_1, u_a} & \frac{k_e}{L_a J} \; {\scriptstyle x_1, m_s} \\ \frac{k_m}{L_a J} \; {\scriptstyle x_2, u_a} & -\frac{s T_a + 1}{T_a J} \; {\scriptstyle x_2, m_s} \end{bmatrix}$$

$$H_{i_a u_a} = \frac{s}{L_a} \cdot \frac{1}{\det} = \frac{s}{L_a} \cdot \frac{1}{\left(s^2 + \frac{s}{T_a}\right) + \frac{k_e k_m}{L_a J}}$$

$$= \frac{s}{L_a} \cdot \frac{T_m T_a}{1 + T_m s + T_m T_a s^2} = \boxed{\frac{s \cdot T_m / R_a}{1 + T_m s + T_m T_a s^2}}$$

$$H_{i_a m_s} = \frac{1}{\det} \frac{k_e}{L_a J} = \frac{k_e}{L_a J} \cdot \frac{T_m T_a}{1 + T_m s + T_m T_a s^2}$$

$$\frac{T_a}{L_a} = \frac{1}{R_a} \qquad \frac{k_e k_m}{J R_a} = \frac{1}{T_m} \quad \rightarrow \frac{k_e T_m}{J R_a} = \frac{1}{k_m}$$

$$\frac{k_e \, T_m T_a}{L_a J} = \frac{\cancel{k_e} \cdot \frac{\cancel{J} \cancel{R_a}}{k_m \cancel{k_e}} \cdot \frac{\cancel{L_a}}{\cancel{R_a}}}{\cancel{L_a} \cancel{J}} = \frac{1}{k_m}$$

$$H_{i_a, m_s} = \frac{1/k_m}{1 + T_m s + T_m T_a s^2}$$

$$H_{\omega, u_a} = \frac{1}{\det} \frac{k_m}{L_a J} = \frac{k_m}{L_a J} \frac{T_m T_a}{1 + T_m s + T_m T_a s^2}$$

$$\frac{\cancel{k_m}}{\cancel{L_a} \cancel{J}} \cdot \frac{\cancel{J} \cancel{R_a}}{\cancel{k_m} k_e} \cdot \frac{\cancel{L_a}}{\cancel{R_a}} = \frac{1}{k_e}$$

$$H_{\omega, u_a} = \frac{1/k_e}{1 + T_m s + T_m T_a s^2}$$

$$H_{\omega, m_s} = \frac{1}{\det} \cdot -\frac{sT_a+1}{T_a J} =$$

$$= \frac{T_m T_a}{1+T_m s+T_m T_a s^2} \cdot -\left(\frac{sT_a+1}{T_a J}\right)$$

$$\frac{\cancel{J} R_a}{k_m k_e} \cdot \frac{\cancel{L_a}}{\cancel{R_a}} \cdot \frac{-(sT_a+1)}{\cancel{J}} \cdot \frac{\cancel{R_a}}{\cancel{L_a}}$$

$$H_{\omega, m_s} = -\frac{R_a}{k_m k_e} \frac{sT_a+1}{1+T_m s+T_m T_a s^2}$$

$$\underbrace{\begin{bmatrix}\dot\vartheta_p\\ \dot\vartheta_c\end{bmatrix}}_{\dot x} = \underbrace{\begin{bmatrix} -\frac{1}{T_p} & \frac{1}{T_p}\\ \frac{k_p}{k_c T_p} & -\frac{1+k_p/k_c}{T_c}\end{bmatrix}}_{A} \underbrace{\begin{bmatrix}\vartheta_p\\ \vartheta_c\end{bmatrix}}_{x} + \underbrace{\begin{bmatrix}\frac{k_E}{k_p T_p} & 0\\ 0 & \frac{1}{T_c}\end{bmatrix}}_{B} \underbrace{\begin{bmatrix}u_c\\ \vartheta_e\end{bmatrix}}_{u}$$

$$y = I\, x$$

$$\rightarrow H(s) = C\,(sI - A)^{-1} B$$

$$sI - A = \begin{bmatrix} s + \frac{1}{T_p} & -\frac{1}{T_p}\\ -\frac{k_p}{k_c T_c} & s + \frac{1+k_p/k_c}{T_c}\end{bmatrix}$$

$$\det(sI - A) = \left(s + \frac{1}{T_p}\right)\left(s + \frac{1+k_p/k_c}{T_c}\right) - \frac{k_p}{k_c T_c}\cdot\frac{1}{T_p} \longrightarrow$$

$$= s^2 + \frac{s + s\,k_p/k_c}{T_c} + \frac{s}{T_p} + \frac{1+k_p/k_c}{T_p T_c}$$

$$s^2 + \frac{s + s\,k_p/k_c}{T_c} + \frac{s}{T_p} + \frac{1 + k_p/k_c}{T_p T_c} =$$

$$\frac{s^2 T_p T_c + s T_p + s\,k_p T_p/k_c + s T_c + 1 + k_p/k_c}{T_p T_c}$$

$$(sI - A)^{-1} = \frac{1}{det} \cdot \begin{bmatrix} s + \frac{1 + k_p/k_c}{T_c} & \frac{1}{T_p} \\ \frac{k_p}{k_c T_c} & s + \frac{1}{T_p} \end{bmatrix}$$

$$C = I_2$$

$$\rightarrow (sI - A)^{-1} \cdot B = \frac{1}{det} \begin{bmatrix} s + \frac{1 + k_p/k_c}{T_c} & \frac{1}{T_p} \\ \frac{k_p}{k_c T_c} & s + \frac{1}{T_p} \end{bmatrix} \begin{bmatrix} \frac{k_E}{k_p T_p} & 0 \\ 0 & \frac{1}{T_c} \end{bmatrix}$$

$$= \frac{1}{det} \begin{bmatrix} \overset{\theta_p,\ u_c}{\frac{s\,k_E}{k_p T_p} + \frac{(1 + k_p/k_c)}{T_c} \frac{k_E}{k_p T_p}} & \overset{\theta_p,\ \theta_e}{\frac{1}{T_p T_c}} \\ \overset{\theta_c,\ u_c}{\frac{k_p k_E}{k_c T_c k_p T_p}} & \overset{\theta_c,\ \theta_e}{\frac{s}{T_c} + \frac{1}{T_p T_c}} \end{bmatrix}$$

$$H_{\theta_c u_c}(s) = \frac{1}{\det} \frac{\cancel{k_p} k_E}{k_c T_c \cancel{k_p} T_p} =$$

$$= \frac{T_p \cdot k_e / k_c}{s^2 T_p T_c + s\left(T_p + k_p T_p / k_c + T_c\right) + 1 + k_p / k_c}$$

$$H_{\theta_c, \theta_e} = \frac{1}{\det} \left(\frac{s}{T_c} + \frac{1}{T_c T_p} \right) =$$

$$= \frac{1}{\det} \left(\frac{s T_p + 1}{T_c T_p} \right) =$$

$$= \frac{\cancel{T_p T_c} \quad (s T_p + 1) / \cancel{T_c T_p}}{s^2 T_p T_c + s T_p + s \, k_p T_p / k_c + s T_c + 1 + k_p / k_c}$$