Analiza sistemelor
in regimuri stationare.
Indicatori de performanta (de calitate)
Curs 7

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Introduction

- The behavior of control systems (CSs) in several regimes is determined by the system structure, and by the type and parameter values of the controller
- Both the permanent and the transient regimes are of interest
- The CS performance can be assessed on the basis of criteria to appreciate the performance (the quality) and, as part of these criteria, on the basis of indices to assess the performance (the quality)
- These criteria are also referred to as performance indices and they are employed in the definitions of the performance specifications imposed to CS design

Operating regimes of control systems

- A system is in a steady-state if the variables which define the behavior of the system are unchanging in time. If a system is in a steady-state, then the recently observed behavior of the system will continue in the future
- In many systems, a steady-state is not achieved until some time after the system is started or initiated. This situation is often identified as a transient state (usually viewed as a transition from a permanent regime to another one), start-up or warm-up period
- The second regime in which the system can be characterized is the permanent regime. This regime is a particular operating regime where system’s variables or their variations with respect to time take constant values. The permanent regimes can be installed only in stable systems

- **The steady-state regime** (**SSR**) – when input variable take constant values:

$$w(t) = w_{\infty}\sigma(t)$$

$$v(t) = v_{\infty}\sigma(t)$$

- $w(t)$ – the reference input
- $v(t)$ – the disturbance input
- $\sigma(t)$ – the unit step signal

- The effect of $w(t)$ and $v(t)$ is the annulation of the transient regime in the system
- As a result, all variables of the system take constant values called steady-state values (SSVs)

- **The constant speed regime** (**CSR**) – is installed in a system when one input has a linear variation with respect to time ($t \to \infty$) and the other one being considered constant of even zero:

$$w(t) = t\, w_\infty \sigma(t)$$

$$v(t) = v_\infty \sigma(t)$$

- The **constant acceleration regime** (CAR) – is installed in a system (at $t \to \infty$) when one of the inputs has a parabolic type variation and the other inputs are constant or even zero:

$$w(t) = \frac{1}{2} t^2\, w_\infty \sigma(t)$$

$$v(t) = v_\infty \sigma(t)$$

Conditions to install the SSR

- The steady-state regime can be installed in a physical / dynamical system only if the system is stable, the inputs are constant with respect to time (w_∞ = const and v_∞ = const) and after the transient regimes are annulated
- In case of **continuous-time systems** the following conditions define the system operation in SSR:
- For the state variables: $\mathbf{x}' = \mathbf{0} \Leftrightarrow \mathbf{x}'_\infty = \mathbf{0}$
- For the integral (I) blocks: $u_\infty = 0 \rightarrow y_\infty = \text{const}$
- For the derivative (D) blocks: $\forall u_\infty = \text{const} \rightarrow y_\infty = 0$
- For the proportional (P) blocks: $y_\infty = k\, u_\infty$
- If the continuous-time system is stable and the presence of time delay does not affect the steady-state behavior then:

$$k = \lim_{s \to 0} H(s) = H(0) = \frac{b_0}{a_0} \Rightarrow y_\infty = k\, u_\infty$$

- The case of **discrete-time systems** has the same principles as the continuous-time ones but the with the following modifications:
- For the state variables: $\mathbf{x}_{k+1} = \mathbf{x}_k \Leftrightarrow \Delta \mathbf{x}_k = \mathbf{x}_{k+1} - \mathbf{x}_k = \mathbf{0}$
- For the I blocks: $u_\infty = 0,\ y_\infty = \text{const}$
- For the D blocks: $u_\infty = \text{const},\ y_\infty = 0$
- For the P blocks: $y_\infty = k\, u_\infty$
- If the system is characterized by a stable rational transfer function (t.f.) and the system does not contain pure D and I components, *B*(1) ≠ 0 and *A*(1) ≠ 0, the system

$$H(z) = \frac{B(z)}{A(z)} = \frac{b_m z^m + \ldots + b_1 z + b_0}{a_n z^n + \ldots + a_1 z + a_0}, m \le n$$

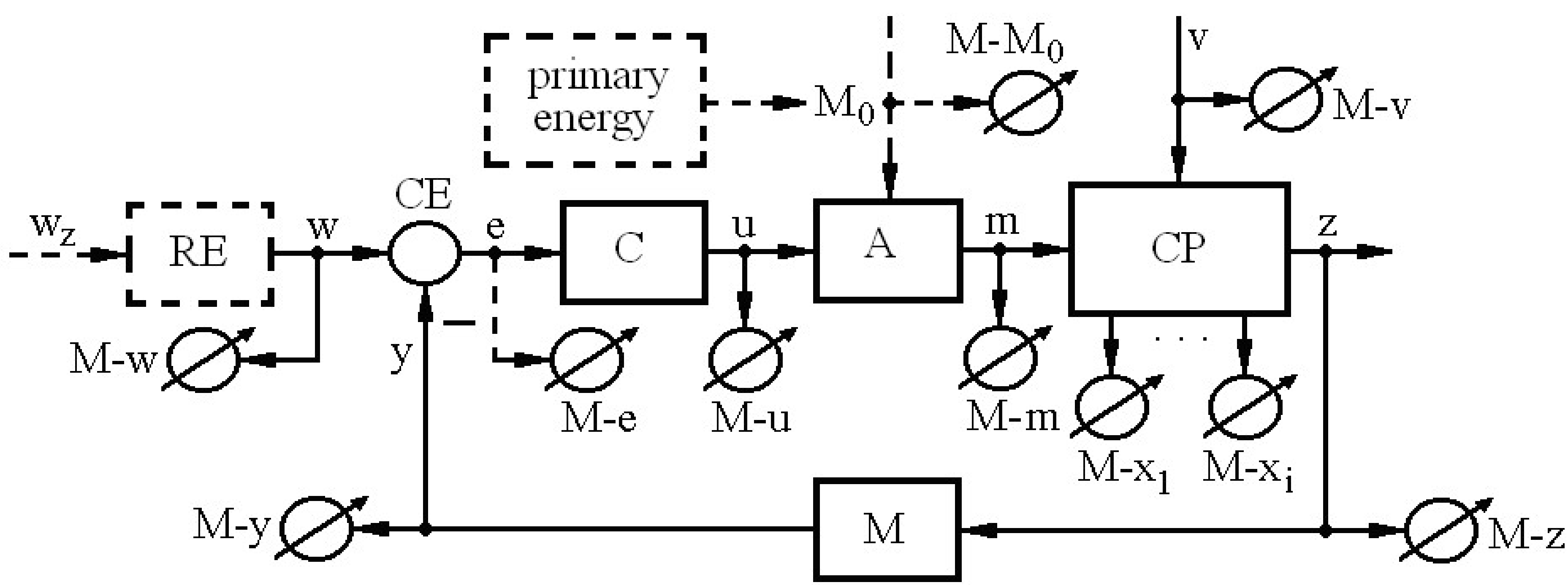
becomes:

$$k = \lim_{z \to 1} H(z) = H(1) \Rightarrow y_\infty = k\, u_\infty$$

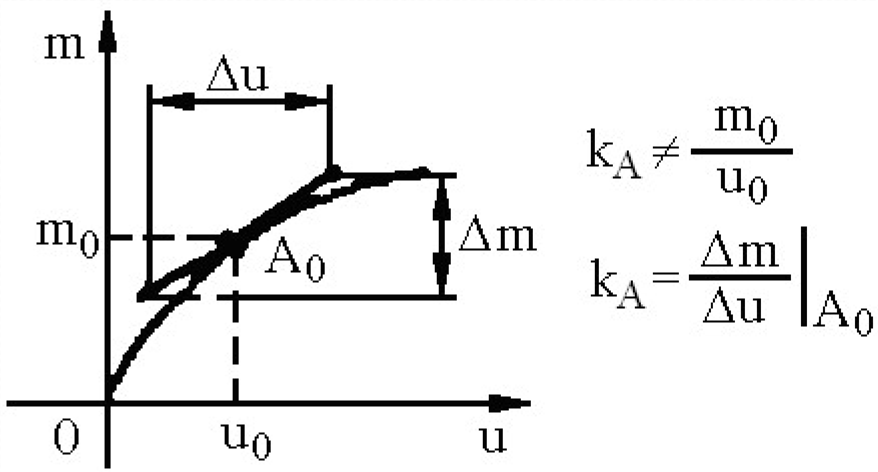
Computation of SSVs of a system

- The graphical characterizations of the dependencies between the steady-state values (SSVs) of different variables are called static characteristics (SCs)
- Computation of SSVs can be carried out a priori or a posteriori with respect to system's construction, but only for limited domains of variations:
  - The experimental computation of SSVs from SSR measurements conducted on system's variables – it is a posteriori with respect to system's construction
  - The analytical computation of SSVs. This computation – based on system's MMs – can be considered as a priori or a posteriori with respect to system's construction
- First, only those variables that are accessible to measurements are measured. The values of other variables are next derived on the basis of MMs if it is possible by computation or by estimation. In this situation, SCs are obtained only for blocks for which their input and output SSVs have been obtained

- For installing the SSR in the system, it is important to have access to the inputs of integral elements (with distinct I component). The access to the control error $e(t)$ is important with this regard. If the controller contains an I component, then the SSR relationship is $e_\infty = 0$, which confirms the installing of an SSR



- The functional block of the CS can contain continuous nonlinearities, which lead to nonlinear SCs of those blocks:



- Since MM is used around of a steady-state operating point (s.s.o.p.), it will require the linearization and the expression of linearized MMs by the difference of the input and output variables of the block with respect to the certain s.s.o.p.
- The linearized MM attached to the s.s.o.p. A_0 is: $\Delta y_\infty = k\,\Delta u_\infty$

where: $k = \left.\dfrac{dy}{du}\right|_{A_0} \approx \left.\dfrac{\Delta y}{\Delta u}\right|_{A_0}$

Analytical computation of SSVs

- Several techniques are used in the analytical computation of SSVs of CSs. These techniques depend on:
  - The expression of system's MM (the MM type), the representation of system's structure, the block diagram
  - The linearity / nonlinearity of the MM
  - The information treatment in time (continuous time, discrete time)
  - The information concerning some SSVs of the system, etc.
- The most frequently used practical situations are synthesized as follows in terms of case studies. The following presentation will be focused on the linear (linearized) situation
- **The continuous-time systems** – in the lecture material

- **The discrete-time systems**. A linear n^{th} order stable system with r inputs and q outputs is considered. The SS-MM of this system is:

$$\mathbf{x}_{k+1} = \mathbf{A}\,\mathbf{x}_k + \mathbf{B}\,\mathbf{u}_k$$
$$\mathbf{y}_k = \mathbf{C}\,\mathbf{x}_k$$

- The SSR relation is:

$$\mathbf{x}_{k+1} = \mathbf{x}_k = \mathbf{x}_\infty$$

- That results in $\mathbf{x}_\infty = \mathbf{A}\,\mathbf{x}_\infty + \mathbf{B}\,\mathbf{u}_\infty \Leftrightarrow (\mathbf{I} - \mathbf{A})\,\mathbf{x}_\infty = \mathbf{B}\,\mathbf{u}_\infty$

  and $\mathbf{y}_\infty = \mathbf{C}\,\mathbf{x}_\infty$

  leading to: $\mathbf{y}_\infty = (\mathbf{I} - \mathbf{A})^{-1}\,\mathbf{B}\,\mathbf{u}_\infty$

  $\rightarrow$ the DC gain matrix

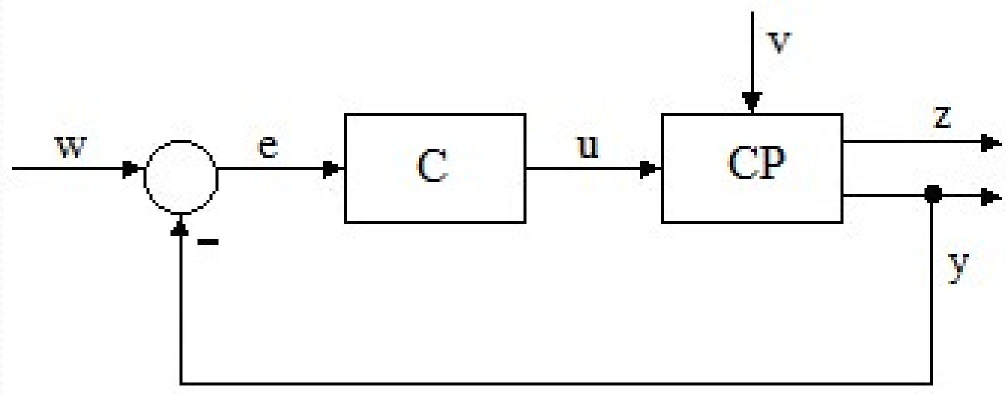
- **The computation of system's SSVs** is carried out as follows. The continuous-time and discrete-time cases are treated in similar manner, and the differences appear only in the SSR conditions
- The presence of an I block in the block diagram of the CS (CP) leads to the zero steady-state input of that block, $u_\infty = 0$. The presence of a D block in the block diagram of the CS (CP) leads to the zero steady-state output of this block, $y_\infty = 0$. Using these conditions, the constant output and input of these blocks are next computed using backward calculations
- For each typical block (I, D, P, ...) the SSR conditions are expressed along with the equations for the computation of SSVs
- These equations lead to an algebraic system with the dimension depending on system's complexity. The usual dimension is related to (n+q) equations with (n+q+r) SSVs
- If a sufficient number of SSVs is known for which the algebraic system is compatible (for example, r SSVs but not any), the rest of SSVs can be computed

Effects of controller type on steady-state behavior of control systems

- Only aspects concerning the properties caused by the type of controller on the steady-state behavior of CS will be discussed as follows. The analysis is related to the classical CS structure, for which the following operational relationships can be expressed:

$$H_v(\lambda) = \frac{H_N(\lambda)}{1 + H_R(\lambda) H_P(\lambda)}$$

$$H_w(\lambda) = \frac{H_R(\lambda) H_P(\lambda)}{1 + H_R(\lambda) H_P(\lambda)}$$



- *w* is the reference input
- $\lambda = s$ or z
- $H_N(\lambda)$ is part of $H_P(\lambda)$ passed by the disturbance input *v*

- **The CS behavior with respect to the reference input. Properties induced by the type of controller**. This case concerns mainly the SSR and CSR with respect to w
- $q_0 \in \{0, 1, 2\}$ is the multiplicity order of the pole in origin (the number of I components) of the open-loop system
- Stationary dependences with respect to the reference input:

w	y_∞			e_∞		
	$q_0=0$	$q_0=1$	$q_0=2$	$q_0=0$	$q_0=1$	$q_0=2$
$w(s)=\frac{1}{s}w_\infty$ (SSR)	$\frac{k_0}{1+k_0}w_\infty$	$1\cdot w_\infty$	$1\cdot w_\infty$	$\frac{1}{1+k_0}w_\infty$	$0\cdot w_\infty$	$0\cdot w_\infty$
$w(s)=\frac{1}{s^2}w_\infty$ (CSR)	∞	∞	∞	∞	$\frac{1}{k_0}\cdot w_\infty$	$0\cdot w_\infty$

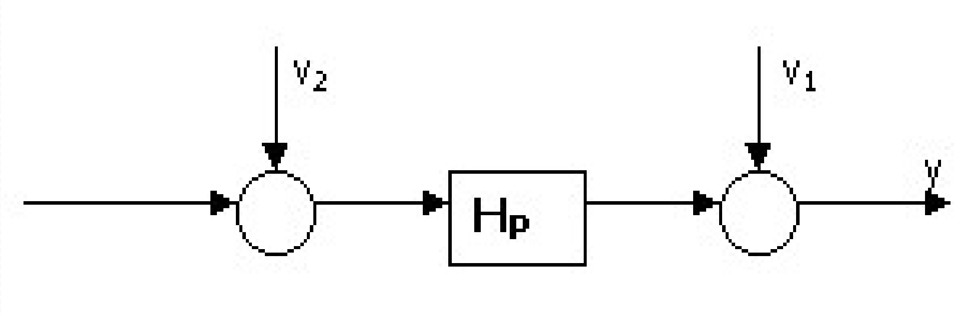
- Aspects concerning the SSR and CSR:
  - Ensuring the condition of zero steady-state control error ($e_\infty = 0$) requires the existence of I component in the controller structure in SSR and two I components in CSR
  - The systems with controllers without I component operate with nonzero control error in SSR (they do not ensure the output exactly equal to the reference input). This does not indicate that those systems operate wrongly, but that they operate in a specific operating mode, which can be desired in certain situations
  - Even the systems with controller with I component operate in CSR with nonzero control error (they do not ensure the output exactly equal to the reference input). In other words, the tracking systems should be characterized by $q_0 > 1$, with serious difficulties in their stabilization and their sensitivity with respect to parametric variations

- **The CS behavior with respect to the disturbance input. Properties induced by the type of controller**. The computations are carried out considering $w(t) = 0$ ($w(s)=0$) and

$$y(s)=\frac{k_N B_N(s)/A_N(s)}{1+\frac{k_0}{s^{q_0}}\frac{B_0(s)}{A_0'(s)}}v(s)=\frac{s^{q_0}k_N B_N(s)/A_N(s)}{s^{q_0}+k_0\frac{B_0(s)}{A_0'(s)}}v(s)=-e(s),$$

$$\frac{B_N(0)}{A_N(0)}=1,\ \frac{B_0(0)}{A_0'(0)}=1$$

- $H_N(s) = k_P B_P(s)/A_P(s)$, if the disturbance is applied to the process input (v_2), and it is called load disturbance
- $H_N(s) = 1$, if the disturbance is applied to the process output (v_1), and it is called additive disturbance on the process output or noise (illustrated in the next slide)



- Steady-state dependencies with respect to the disturbance input:

v	$y_\infty = -e_\infty \quad (e_\infty = -y_\infty)$		
	$q_0=0$	$q_0=1$	$q_0=2$
$v(s) = \frac{1}{s} v_\infty$	$\frac{k_N}{1+k_0} v_\infty$	$0 \cdot v_\infty$	$0 \cdot v_\infty$

- Conclusions:
  - The use of controllers with I (2 I) component(s) ensures the rejection of constant disturbances (and the zero steady-control error is also ensured), $y_\infty = 0 \cdot v_\infty = \gamma_n \cdot v_\infty$
  - The use of controllers without I component, namely, P-xy controllers (the case of systems with P, PDT1, PD2T2, …, controllers) ensures the following steady-state relationship:

$$y_\infty = \frac{k_N}{1+k_0} \cdot v_\infty = \gamma_n \cdot v_\infty$$

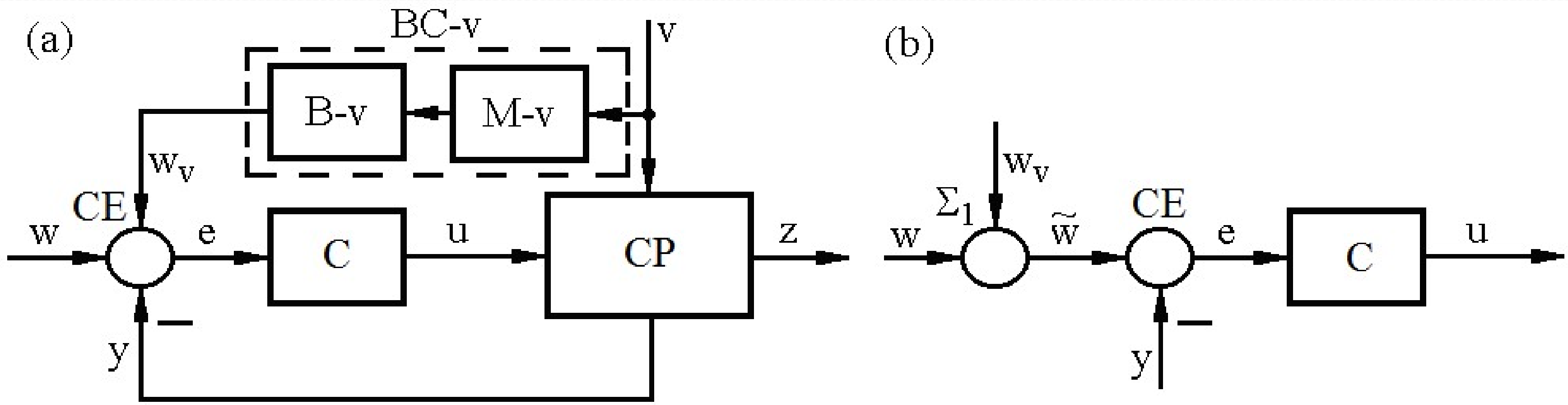
- γ_n – **the natural static coefficient**

Artificial static coefficients and output coupled systems

- The automatic systems that operate with a coupled output, namely with the same output and also called systems that operate in parallel, are frequently used in many industrial applications. For example, the control systems of the synchronous generators coupled to the power systems operate with the same outputs, the power system frequency or voltage
- These systems often require the well stated distribution of the load (the same disturbance applied to these systems) on each of the subsystems that operate with a coupled output. This requirement can be ensured only if the systems that operate with a coupled output are with static coefficient. Moreover, each of the subsystems has its own well computed value of the static coefficient. The requirement can be fulfilled in a convenient manner if the control systems are extended with additional feed forward channels which create artificial static coefficients

- Many particular control systems are coupled by their output variables. It is usual for such systems to impose a well stated distribution of the common load on each of the systems according to certain requirements. The lack of a rigorous control of the distribution of the common load on the coupled systems leads to the risk to make some of the systems to be over-loaded with respect to other ones. The control of the distribution of the load can be solved by the controllers which control such systems
- If the control loops which are coupled by their output variables would have zero static coefficient, trying to ensure the zero steady-state control error, this will lead to taking over a part of the common load which cannot be computed. Therefore, the output coupling of control systems requires that the CS should have nonzero (natural or artificial) static coefficients in order to ensure a well stated distribution of the common load. The distribution algorithm can be computed in terms of several points of view as, for example, proportional to the nominal load of each system which is coupled

- Structure of principle of a CS with artificial static coefficient:

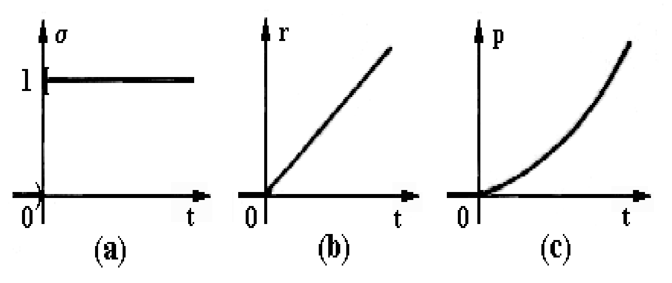


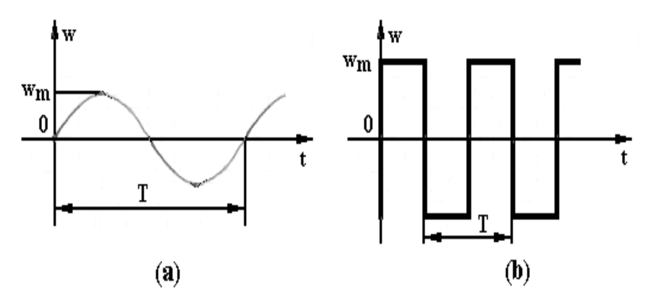
- Calculation of artificial static coefficient – in the lecture material

Performance indices for control systems design

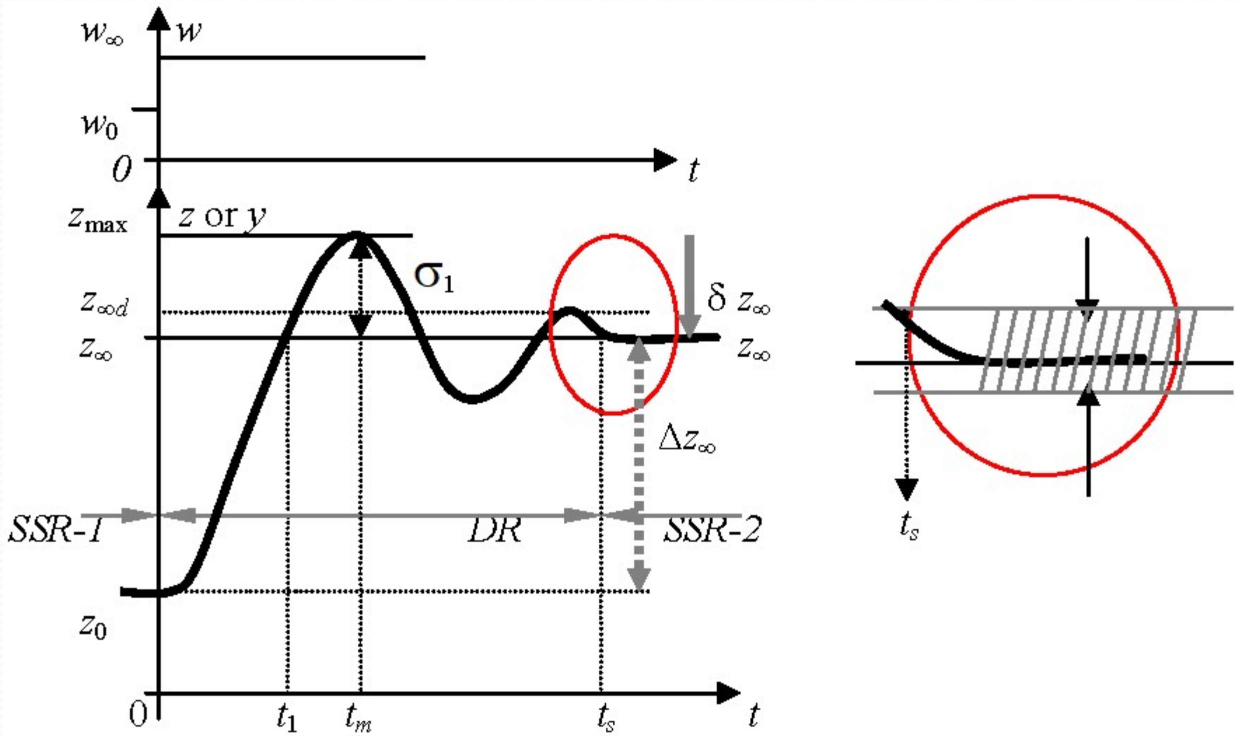
- The properties (the quality) of a CS can be evaluated on the basis of quality assessment criteria and, as part of them, on the basis of quality indices also called performance indices. The quality assessment criteria are divided in two categories:
  - Local criteria
  - Global criteria
- Each type of criterion is associated by the definition of specific performance indices

- **Performance indices defined in particular operating regimes of the CS**. These indices characterize the system behavior but the conclusions can be extended to the frequency domain as well
- The performance indices that can be defined in the CS response with respect to the step signal variation of the reference input or of the disturbance input can be determined:





- Indices that characterize the dynamic regime behavior:
  - t_s – the settling time
  - t_1 – the 0% to 100% rise time
  - t_m – the peak time corresponding to z_{max}
  - σ_1 – the overshoot also called the maximum overshoot



- **Performance indices defined in the closed-loop and open-loop frequency plots** – in the lecture material
- **Global quality assessment criteria and indices**. The process of setting a certain integral index (with a certain structure of the integral) should be related an (indirect) connection between the expression of the integral, its minimum and the quality of the CS. This connection is usually reflected by the empirical performance indices or by some energetic indices (for example, the minimum power consumption)
- Details – in the lecture material

Conclusions

- Since the performance requirements imposed to a CS can be very restrictive and often hardly achievable, the design will often accept reasonable tradeoffs to certain indices. Besides, the main relations that characterize a CS illustrate that the design with respect to the reference input and the design with respect to the disturbance input lead to different results
- The design based on performance indices defined in the system response with respect to particular variations of the reference input employs the possibility for simple setting the correspondences between the time behavior of a system and the pole-zero representation of the t.f. Such correspondences can be expressed as relatively simple relations between the system performance and the pole-zero representation of the system

- The low order systems are approximators for the real behavior of control systems, with results often obtained because of the following reasons:
  - Neglecting the dominated poles / zeros (i.e., the very small time constants) in the process t.f. or the possible application of the small time constants theorem, this approximation also leads to benchmark-type models
  - Accepting the time invariance of the parameters and of the process structure
  - The application of the pole-zero cancellation principle. Knowing the performance of such systems and the effects that result by the extension of the basic configuration with additional poles and/or zeros leads to a suggestive image on the controller design

Thank you very much for your attention!