I. Modele matematice intrare-ieșire (MM-ii)

1. MM-ii în timp continuu

$$a_n y^{(n)}(t) + a_{n-1} y^{(n-1)}(t) + \dots + a_1 y^{(1)}(t) + a_0 y(t) = b_m u^{(m)}(t) + \dots + b_1 u^{(1)}(t) + b_0 u(t) \quad / \mathcal{L}$$

$$\Rightarrow a_n s^n y(s) + a_{n-1} s^{n-1} y(s) + \dots + a_1 s y(s) + a_0 y(s) = b_m s^m u(s) + \dots + b_1 s u(s) + b_0 u(s)$$

$$\Rightarrow y(s)(a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0) = u(s)(b_m s^m + \dots + b_1 s + b_0) \Rightarrow$$

$$\boxed{H(s) = \frac{y(s)}{u(s)} = \frac{b_m s^m + \dots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0}}$$

2. MM-ii în timp discret

$$a_n y(k+n) + a_{n-1} y(k+n-1) + \dots + a_1 y(k+1) + a_0 y(k) = b_m u(k+m) + \dots + b_1 u(k+1) + b_0 u(k) \quad / \mathcal{Z}$$

$$a_n z^n y(z) + a_{n-1} z^{n-1} y(z) + \dots + a_1 z y(z) + a_0 y(z) = b_m z^m u(z) + \dots + b_1 z u(z) + b_0 u(z)$$

$$y(z)(a_n z^n + a_{n-1} z^{n-1} + \dots + a_1 z + a_0) = u(z)(b_m z^m + \dots + b_1 z + b_0) \Rightarrow$$

$$\boxed{H(z) = \frac{y(z)}{u(z)} = \frac{b_m z^m + \dots + b_1 z + b_0}{a_n z^n + a_{n-1} z^{n-1} + \dots + a_1 z + a_0}}$$

II. Modele matematice intrare-stare-ieșire (MM-isi)

1. MM-isi în timp continuu

$$\begin{cases} \dot{x}(t) = A x(t) + B u(t) \\ y(t) = C x(t) + D u(t) \end{cases} \quad / \mathcal{L}$$

, pt. sisteme fizic realizabile $D = 0$

$$\Rightarrow \begin{cases} s X(s) = A X(s) + B u(s) \Rightarrow s X(s) - A X(s) = B u(s) \Rightarrow X(s)(sI - A) = B u(s) \\ y(s) = C X(s) \end{cases}$$

$$\Rightarrow X(s) = (sI - A)^{-1} B u(s)$$

$$y(s) = C(sI - A)^{-1} B u(s) \Rightarrow \boxed{H(s) = \frac{y(s)}{u(s)} = C(sI - A)^{-1} B}$$

$\sim 1 \sim$

2. MM-isi în timp discret

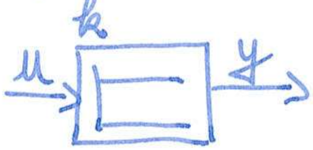
$$\begin{cases} x(k+1) = Ax(k) + Bu(k) \\ y(k) = Cx(k) \end{cases} \Big/ \mathcal{Z} \Rightarrow \begin{matrix} zX(z) = AX(z) + BU(z) \\ Y(z) = CX(z) \end{matrix} \Rightarrow$$

$$X(z)(zI - A)^{-1} = BU(z)$$

$$Y(z) = C(zI - A)^{-1} B\,U(z) \Rightarrow \boxed{H(z) = \frac{Y(z)}{U(z)} = C(zI - A)^{-1} B}$$

III Elemente de transfer (ET)

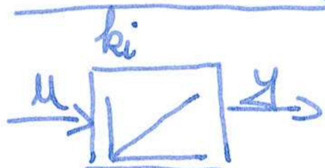
1. Elementul de transfer de tip proporțional (ET-P)



MM-ii: $y(t) = k\,u(t) \;/\; \mathcal{L}$

$$Y(s) = k\,U(s) \Rightarrow \boxed{H(s) = \frac{Y(s)}{U(s)} = k}$$

2. Elementul de transfer de tip integrator (ET-i)

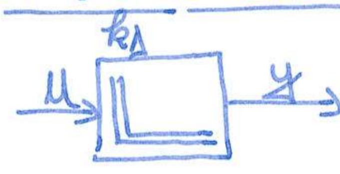


MM-ii: $y(t) = k_i \int_0^t u(\tau)\,d\tau \;/\;$ derivăm

$$y'(t) = k_i\,u(t) \;/\; \mathcal{L} \Rightarrow sY(s) = k_i U(s) \Rightarrow$$

$$Y(s) = \frac{k_i}{s} U(s) \Rightarrow \boxed{H(s) = \frac{Y(s)}{U(s)} = \frac{k_i}{s}}$$

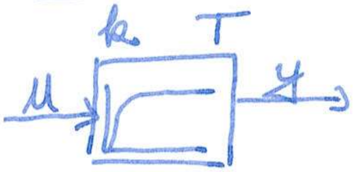
3. Elementul de transfer de tip derivator (ET-Δ)



MM-ii: $y(t) = k_\Delta\,u'(t) \;/\; \mathcal{L} \Rightarrow$

$$Y(s) = k_\Delta s\,U(s) \Rightarrow \boxed{H(s) = \frac{Y(s)}{U(s)} = k_\Delta s}$$

4. Elementul de transfer de tip proporțional cu temporizare de ordinul 1 (ET-PT1) – filtru trece-jos

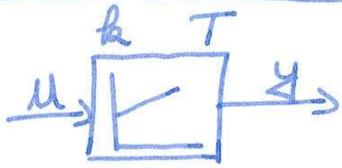


MM-ii: $T y'(t) + y(t) = k\,u(t) \;/\; \mathcal{L}$

$$TsY(s) + Y(s) = kU(s) \Rightarrow Y(s)(Ts+1) = kU(s)$$

$$\Rightarrow \boxed{H(s) = \frac{Y(s)}{U(s)} = \frac{k}{1+sT}}$$

5. Elementul de transfer de tip proporțional-integrator (ET-PI)



MM-ii: $y(t) = k\left[u(t) + \frac{1}{T}\int_0^t u(\tau)d\tau\right] \Rightarrow$

$y(t) = k u(t) + \frac{k}{T}\int_0^t u(\tau)d\tau$ / derivăm

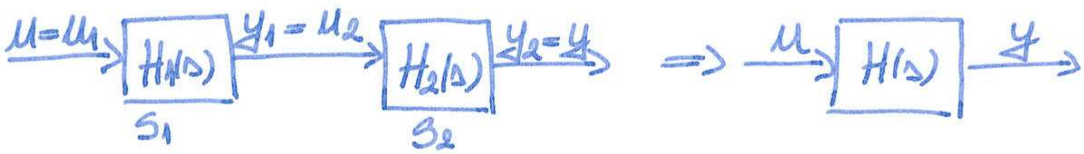
$y'(t) = k u'(t) + \frac{k}{T} u(t)$ / $\mathcal{L} \Rightarrow$

$s y(s) = k s u(s) + \frac{k}{T} u(s) \Rightarrow s y(s) = u(s)\left(k s + \frac{k}{T}\right) = u(s)\frac{k(sT+1)}{T} \Rightarrow$

$$\boxed{H(s) = \frac{y(s)}{u(s)} = \frac{k(sT+1)}{sT}}$$

IV. Principalele conexiuni de sisteme

1. Conexiunea serie



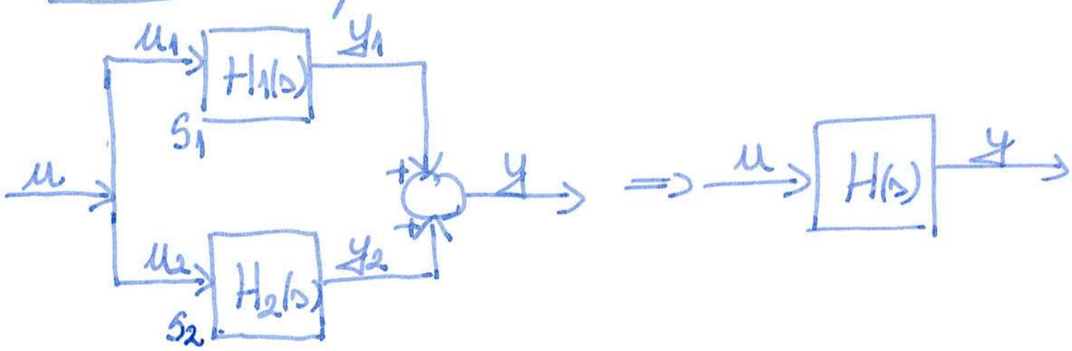
S_1: $\left.\begin{array}{l} y_1(s) = H_1(s)u_1(s) \\ y_1(s) = u_2(s) \\ u_1(s) = u(s) \end{array}\right\} \Rightarrow u_2(s) = H_1(s)u(s)$

S_2: $\left.\begin{array}{l} y_2(s) = H_2(s)u_2(s) \\ y_2(s) = y(s) \end{array}\right\} \Rightarrow y(s) = H_2(s)u_2(s)$

$\Rightarrow y(s) = H_1(s)H_2(s)\cdot u(s)$

$\Rightarrow \boxed{H(s) = \frac{y(s)}{u(s)} = H_1(s)H_2(s)}$

2. Conexiunea paralel



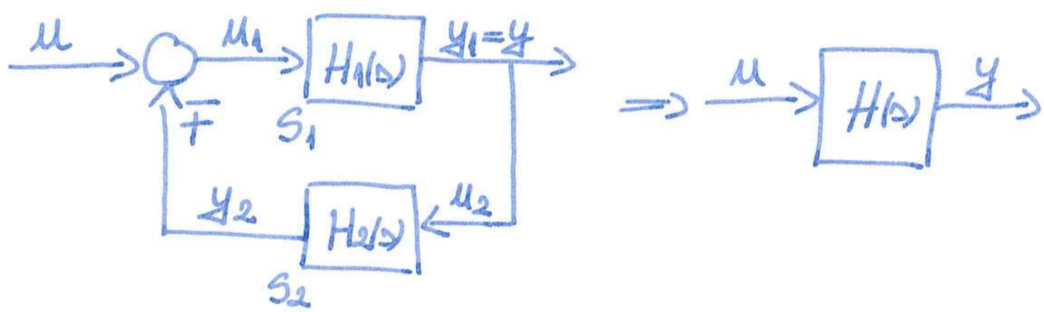
S_1: $\left.\begin{array}{l} y_1(s) = H_1(s)u_1(s) \\ u_1(s) = u(s) \end{array}\right\} \Rightarrow y_1(s) = H_1(s)u(s)$

S_2: $\left.\begin{array}{l} y_2(s) = H_2(s)u_2(s) \\ u_2(s) = u(s) \end{array}\right\} \Rightarrow y_2(s) = H_2(s)u(s)$

$\Rightarrow y(s) = y_1(s) + y_2(s) = [H_1(s) + H_2(s)]\cdot u(s)$

$\Rightarrow \boxed{H(s) = \frac{y(s)}{u(s)} = H_1(s) + H_2(s)}$

3. Conexiunea cu reacție



$$S_1: \left.\begin{array}{l} y_1(s)=H_1(s)u_1(s) \\ y_1(s)=y(s) \\ u_1(s)=u(s)\mp y_2(s) \end{array}\right\} \Rightarrow y(s)=H_1(s)[u(s)\mp y_2(s)]$$

$$S_2: \left.\begin{array}{l} y_2(s)=H_2(s)u_2(s) \\ u_2(s)=y(s) \end{array}\right\} \Rightarrow y_2(s)=H_2(s)y(s)$$

$\Rightarrow$

$$y(s)=H_1(s)[u(s)\mp H_2(s)y(s)]=H_1(s)u(s)\mp H_1(s)H_2(s)y(s) \Rightarrow$$

$$y(s)\pm H_1(s)H_2(s)y(s)=H_1(s)u(s) \Rightarrow y(s)[1\pm H_1(s)H_2(s)]=H_1(s)u(s)$$

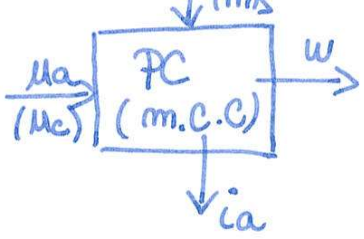
$$\Rightarrow \boxed{H(s)=\frac{y(s)}{u(s)}=\frac{H_1(s)}{1\pm H_1(s)H_2(s)}}$$

Aplicație: Modelarea unui motor de curent continuu (m.c.c)

① Să se găsească MM-isi-ul aferent PC

② Să se calculeze matricea de transfer folosind MM-isi-ul

③ Să se calculeze funcțiile de transfer utilizând SBi din fig.1.5



Ecuațiile primare aferente PC:

$$\begin{cases} \frac{L_a}{R_a}\frac{di_a(t)}{dt}+i_a(t)=\frac{1}{R_a}\left[u_a(t)-e_w(t)\right] \\ J\frac{dw(t)}{dt}=m_a(t)-m_f(t)-m_s(t) \end{cases}$$

știind că $m_a(t)=k_m i_a(t)$, $m_f(t)=k_f w(t)$, $e_w(t)=k_e w(t)$, $T_a=\frac{L_a}{R_a}$

① Notăm $x_1(t)=i_a(t)$, $x_2(t)=w(t)$ $\Rightarrow \begin{cases} \frac{L_a}{R_a}\dot{x}_1(t)+x_1(t)=\frac{1}{R_a}[u_a(t)-e_w(t)] \\ J\dot{x}_2(t)=m_a(t)-m_f(t)-m_s(t) \end{cases}$

$$\boxed{k_1 \approx 0} \Rightarrow \begin{cases} L_a \dot{x}_1(t) + R_a x_1(t) = u_a(t) - k_e x_2(t) \\ J \dot{x}_2(t) = k_m x_1(t) - m_s(t) \end{cases} \Rightarrow$$

$$\begin{cases} L_a \dot{x}_1(t) = -R_a x_1(t) - k_e x_2(t) + u_a(t) \\ J \dot{x}_2(t) = k_m x_1(t) - m_s(t) \end{cases} \Rightarrow \begin{cases} \dot{x}_1(t) = -\underbrace{\frac{R_a}{L_a}}_{1/T_a} x_1(t) - \frac{k_e}{L_a} x_2(t) + \frac{1}{L_a} u_a \\ \dot{x}_2(t) = \frac{k_m}{J} x_1(t) - \frac{1}{J} m_s(t) \end{cases}$$

$$\begin{cases} \dot{x}(t) = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -\frac{1}{T_a} & -\frac{k_e}{L_a} \\ \frac{k_m}{J} & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{L_a} & 0 \\ 0 & -\frac{1}{J} \end{bmatrix} \begin{bmatrix} u_a(t) \\ m_s(t) \end{bmatrix} \\ y(t) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \end{cases}$$

2) $A = \begin{bmatrix} -\frac{1}{T_a} & -\frac{k_e}{L_a} \\ \frac{k_m}{J} & 0 \end{bmatrix}$; $B = \begin{bmatrix} \frac{1}{L_a} & 0 \\ 0 & -\frac{1}{J} \end{bmatrix}$; $C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$H(s) = C \underbrace{(sI - A)}_{M}{}^{-1} B = \begin{bmatrix} H_{i_a u_a}(s) & H_{i_a m_s}(s) \\ H_{\omega u_a}(s) & H_{\omega m_s}(s) \end{bmatrix}$$