

Transformata Z

$$F(z) = Z \{ f(k) \} = \sum_{k=0}^{\infty} f(k) z^{-k}$$

$$\textcircled{1} f(k) = \sigma(k) = \begin{cases} 1, & k \geq 0 \\ 0, & k < 0 \end{cases}$$

$$F(k) = Z \{ \sigma(k) \} = \sum_{k=0}^{\infty} \sigma(k) z^{-k} =$$

$$= \sum_{k=0}^{\infty} z^{-k} = 1 + z^{-1} + z^{-2} + \dots + z^{-\infty} =$$

(n → ∞)

progresie geometrica: $q = z^{-1}$

$$b \quad \frac{2^m - 1}{2 - 1}, \quad q \neq 1$$

$$nb_1, q = 1$$

$$\sum \dots = 1 \cdot \frac{(z^{-1})^{n+1} \xrightarrow{0} -1}{z^{-1} - 1} =$$

$$\underline{\underline{\frac{1}{1 - z^{-1}}}}}$$

$$Z \{ \sigma(k) \} = \frac{1}{1 - z^{-1}}$$

$$(2) f(k) = e^{-ak} \Rightarrow \mathcal{Z}\{f(k)\} = \sum_{k=0}^{\infty} e^{-ak} z^{-k}$$

$$= \sum_{k=0}^{\infty} (e^{-a} \cdot z^{-1})^k = 1 + (e^{-a} z^{-1}) + (e^{-a} z^{-1})^2 + \dots + (e^{-a} z^{-1})^n \quad n \rightarrow \infty$$

$$= 1 \cdot \frac{(e^{-a} z^{-1})^{n+1} - 1}{e^{-a} z^{-1} - 1} = \frac{1}{1 - e^{-a} z^{-1}}$$

$$\mathcal{Z}\{f(k)\} = \frac{1}{1 - e^{-a} z^{-1}}$$

$$(3) \mathcal{Z}\{f(k+1)\} = \sum_{k=0}^{\infty} f(k+1) z^{-k}$$

Notation $k+1 = \tau$ $k \rightarrow \infty \rightarrow \tau \rightarrow \infty$
 $k = \tau - 1$ $k \rightarrow 0 \rightarrow \tau \rightarrow 1$

$$= \sum_{\tau=1}^{\infty} f(\tau) z^{-(\tau-1)} = z \sum_{\tau=1}^{\infty} f(\tau) z^{-\tau}$$

$$= z[F(z) - f(0)]$$

$$\mathcal{Z}\{f(k+1)\} = z[F(z) - f(0)]$$

$$\mathcal{Z} \{ f(k+2) \} = \sum_{k=0}^{\infty} f(k+2) z^{-k}$$

$$k+2 = \tau \quad \rightarrow$$

$$k = \tau - 2 \quad \rightarrow$$

$$= \sum_{\tau=2}^{\infty} f(\tau) \cdot z^{-k} \cdot z^2 =$$

$$= z^2 \left[F(z) - f(0) - f(1)z^{-1} \right]$$

$$\mathcal{Z} \{ f(k+n) \} = z^n \left[F(z) - \sum_{k=0}^{n-1} f(k) z^{-k} \right]$$

$$(4) \quad y(k+1) - a y(k) = 0, \quad y(0) = y_0$$

$$\mathcal{Z} \{ y(k+1) - a y(k) \} = \mathcal{Z} \{ 0 \}$$

$$\mathcal{Z} \{ y(k+1) \} - a \mathcal{Z} \{ y(k) \} = 0$$

$$\mathcal{Z} [y(z) - y(0)] - a y(z) = 0$$

$$(z-a)Y(z) = y_0 \cdot z$$

$$Y(z) = \frac{y_0 \cdot z}{z-a} = y_0 \cdot \frac{\cancel{z}}{\cancel{z}(1-a z^{-1})}$$

$$= \frac{y_0}{1-a z^{-1}}$$

$$z^{-1} \{ Y(z) \} = y(k)$$

$$y(k) = z^{-1} \left\{ y_0 \frac{1}{1-a z^{-1}} \right\}$$

$$\rightarrow \boxed{y(k) = a^k}$$

$$(5) \quad y(k+2) - 18y(k+1) + 32y(k) = 0 \quad \begin{matrix} y_0 = 0 \\ y_1 = 2 \end{matrix}$$

$$z \{ y(k+2) \} - 18z \{ y(k+1) \} + 32z \{ y(k) \} = 0$$

$$z^2 [Y(z) - y(0) - y(1)z^{-1}] - 18z [Y(z) - y_0] + 32Y(z) = Y(z)(z^2 - 18z + 32) - 2z = 0$$

$$\rightarrow Y(z) = \frac{2z}{z^2 - 18z + 32} = \frac{2z}{(z-2)(z-16)}$$

$$Y(k) = Z^{-1} \left\{ \frac{2z^{-1}}{(1-2z^{-1})(1-16z^{-1})} \right\}$$

$$\Rightarrow \frac{A}{(1-2z^{-1})} + \frac{B}{(1-16z^{-1})} = \frac{2z^{-1}}{\dots}$$

$$A(1-16z^{-1}) + B(1-2z^{-1}) = 2z^{-1}$$

$$\begin{cases} A + B = 0 \\ -16A - 2B = 2 \end{cases}$$

$$-16A + 2A = 2$$

$$-14A = 2 \rightarrow$$

$$\begin{cases} A = -\frac{1}{7} \\ B = +\frac{1}{7} \end{cases}$$

$$Y(k) = Z^{-1} \left\{ -\frac{1}{7} \underbrace{\frac{1}{(1-2z^{-1})}}_{a^k} \right\} + Z^{-1} \left\{ +\frac{1}{7} \frac{1}{1-16z^{-1}} \right\}$$

$$= -\frac{1}{7} 2^k + \frac{1}{7} 16^k = \frac{1}{7} (16^k - 2^k)$$

Tema : 4, 5, 6, 7 pag 9

$s \rightarrow$ timp continuu

$z \rightarrow$ timp discret