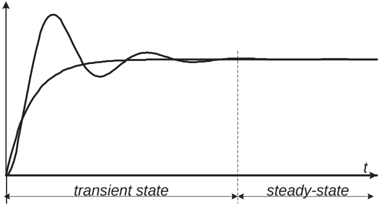
Analiza sistemelor lineare in timp continuu. Sisteme cu timp mort (cu intarzieri). Regulatoare PID

Special thanks to:
A. Bemporad, Automatic Control 1, Lecture Notes, University of Trento, Italy, 2011, http://cse.lab.imtlucca.it/~bemporad/automatic_control_course.html
P. Raica, Systems Theory, Lecture Notes, Technical University of Cluj-Napoca, Cluj-Napoca, 2012, https://cs.utcluj.ro/files/educatie/licenta/2021-2022/22_CALCen_ST.pdf

System analysis

- First step: derive a mathematical model
- Various methods are available for analysis
- Performance is analyzed based on various test signals
- The aim of analysis: study the system behavior in transient and steady-state when the system model and input are known.
- Typical test signals: step, ramp, impulse, sinusoidal



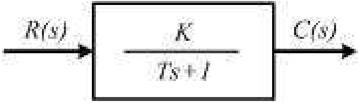
System analysis

- Predict the dynamic behavior of the system from a knowledge of the system model.
- Important characteristics: absolute stability, transient response, steady-state error
  - A stable LTI system: the output comes back to its equilibrium state (system is subjected to a disturbance)
  - An unstable LTI system: either sustained oscillation of the output or the output diverges from equilibrium (system is subjected to a disturbance)
  - If the output of a system at steady state does not exactly agree with the input, the system is said to have *steady-state error*

System analysis

- The system is broken down in simple elements of at most second order and the effects of each element are analyzed
- The behavior of simple elements can be studied using some characteristic parameters:
  - Time constants, T
  - Time delay constant, T_m
  - Damping factor ζ
  - Natural frequency ω_n
  - Gain constant, K

First-order systems



The input-output relationship is given by:

$$\frac{C(s)}{R(s)} = \frac{K}{Ts+1}$$

Analyze the system responses to inputs as the unit step, unit ramp and unit impulse functions. The initial conditions are assumed to be zero.

Unit step response

$$r(t) = 1, \quad R(s) = \frac{1}{s}, \quad C(s) = \frac{K}{Ts+1}\frac{1}{s}$$

$$c(t) = \mathcal{L}^{-1}[C(s)] = \mathcal{L}^{-1}\left[\frac{K}{s} - \frac{KT}{Ts+1}\right] = K(1-e^{-t/T}), \quad (t \geq 0)$$

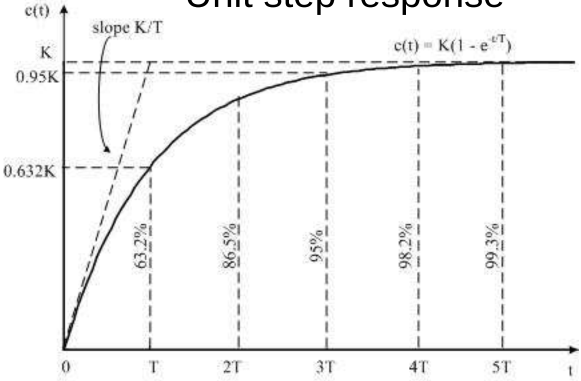
Property: At $t = T$ the value of $c(t)$ is $0.632K$, or the response has reached 63.2% of its total change:

$$c(T) = K(1 - e^{-1}) = 0.632K$$

Property: The slope of the tangent at $t = 0$ is $1/T$:

$$\frac{dc(t)}{dt} = \frac{K}{T}e^{-t/T}\Big|_{t=0} = \frac{K}{T}$$

Unit step response



For $t \geq 4T$ the response remains within 2% of the final value.
The response time is about 4 time constants.

Unit ramp response

$$r(t) = t, \quad R(s) = \frac{1}{s^2}, \quad C(s) = \frac{K}{Ts+1}\frac{1}{s^2}$$

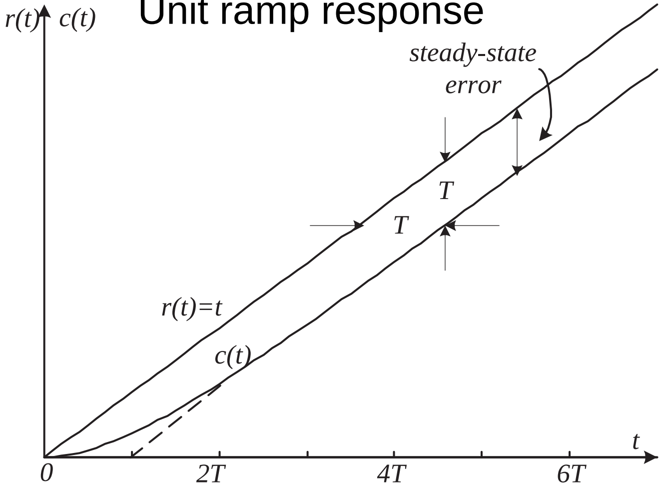
Expanding $C(s)$ into partial fraction gives

$$\begin{aligned} c(t) &= \mathcal{L}^{-1}[C(s)] = \mathcal{L}^{-1}\left[K\left(\frac{1}{s^2} - \frac{T}{s} + \frac{T^2}{Ts+1}\right)\right] \\ &= K(t - T + Te^{-t/T}), \quad (t \geq 0) \end{aligned}$$

For $K = 1$, the error signal $e(t)$:

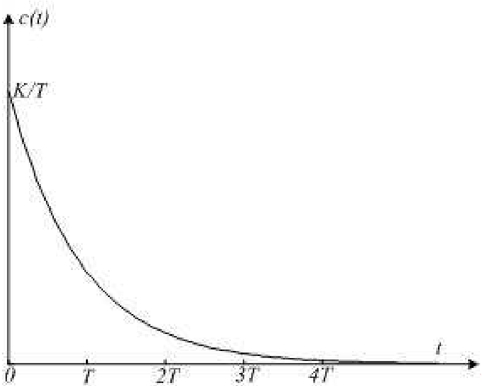
$$e(t) = r(t) - c(t) = T(1 - e^{-t/T}), \quad e(\infty) = T$$

Unit ramp response

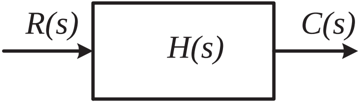


Unit impulse response

$$r(t) = \delta(t), \quad R(s) = 1, \quad C(s) = \frac{K}{Ts+1}, \quad c(t) = \frac{K}{T}e^{-t/T}, \quad (t \geq 0)$$



Second-order systems



$$H(s) = \frac{C(s)}{R(s)} = \frac{1}{\frac{1}{\omega_n^2}s^2 + \frac{2\zeta}{\omega_n}s + 1} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

ω_n - *the natural frequency*, ζ - *the damping factor*

Example.

$$H(s) = \frac{1}{s^2 + s + 1}, \quad \frac{1}{\omega_n^2} = 1; \quad \frac{2\zeta}{\omega_n} = 1; \quad \Rightarrow \omega_n = 1; \quad \zeta = \frac{1}{2}$$

Second-order systems

The roots of the characteristic equation (System poles)

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

are:

$$s_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$

Poles are complex for $0 < \zeta < 1$ *and real for* $\zeta \geq 1$.

$0 < \zeta < 1$: the system is **underdamped** and the transient response is oscillatory.
$\zeta = 1$: the system is **critically damped**
$\zeta > 1$: system is **overdamped** (response does not oscillate)
$\zeta = 0$: the transient response does not die out.

Unit step response

$$r(t) = 1, \quad R(s) = \frac{1}{s}, \quad C(s) = \frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n^2 s + \omega_n^2)}$$

Underdamped case $(0 < \zeta < 1)$

$$C(s) = \frac{1}{s} - \frac{s + \zeta\omega_n}{(s + \zeta\omega_n)^2 + \omega_d^2} - \frac{\zeta\omega_n}{(s + \zeta\omega_n)^2 + \omega_d^2}$$

where $\omega_d = \omega_n\sqrt{1 - \zeta^2}$ - **damped natural frequency**.

$$\mathcal{L}^{-1}[C(s)] = c(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1 - \zeta^2}} \cdot sin\left(\omega_d t + arctan\frac{\sqrt{1 - \zeta^2}}{\zeta}\right)$$

Unit step response

$\zeta = 0$: the response becomes undamped and oscillations continue indefinitely.

$$c(t) = 1 - \cos \omega_n t, \quad (t \geq 0)$$

ω_n = the undamped natural frequency of the system = frequency at which the system would oscillate if the damping were decreased to zero.

Critically damped case, ($\zeta = 1$): poles are equal and real

$$C(s) = \frac{\omega_n^2}{(s + \omega_n)^2 s}, \ \Rightarrow, \ c(t) = 1 - e^{-\omega_n t}(1 - \omega_n t), \quad (t \geq 0)$$

Unit step response

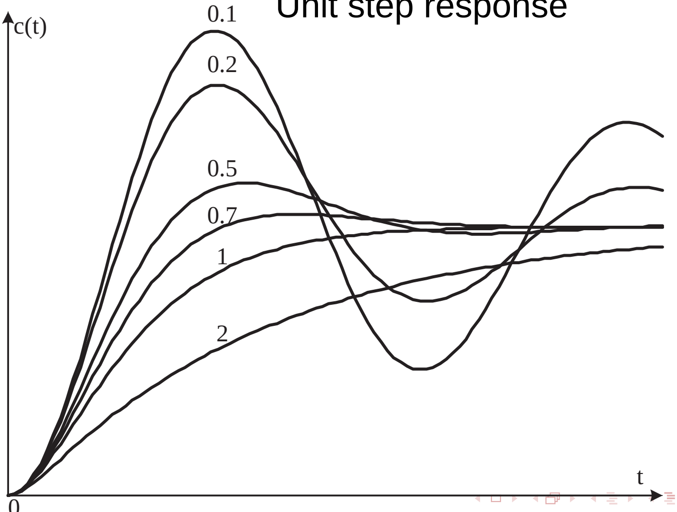
Overdamped case, $(\zeta > 1)$: poles are negative real

$$C(s) = \frac{\omega_n^2}{s(s + \zeta\omega_n + \omega_n\sqrt{\zeta^2 - 1})(s + \zeta\omega_n - \omega_n\sqrt{\zeta^2 - 1})}$$

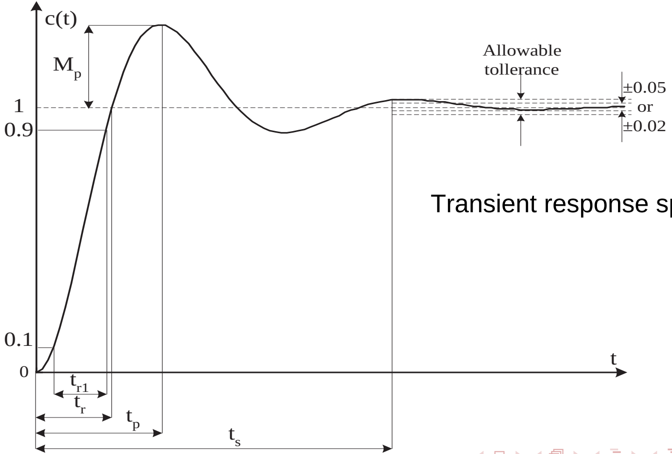
$$c(t) = 1 + \frac{\omega_n}{2\sqrt{\zeta^2 - 1}} \left(\frac{e^{-s_1 t}}{s_1} - \frac{e^{-s_2 t}}{s_2} \right)$$

where $s_1 = (\zeta + \sqrt{\zeta^2 - 1})\omega_n$ and $s_2 = (\zeta - \sqrt{\zeta^2 - 1})\omega_n$ are the system poles.
The response $c(t)$ includes two decaying exponential terms.

Unit step response



Rise time, Peak time, Maximum overshoot, Settling time



Transient response specifications

Transient response specifications

- Rise time, t_r - the time required for the response to rise from 10% to 90%, 5% to 95% or 0% to 100% of its final value.
- Peak time, t_p - the time required for the response to reach the first peak of the overshoot.
- Maximum (percent) overshoot M_p - the maximum peak value of the response curve measured from the steady-state value of the response
- Settling time, t_s - the time required for the response curve to reach and stay within a range of 2% or 5% about the final value.

Rise time

Obtain the rise time t_r by letting $c(t_r) = 1$ or

$$c(t_r) = 1 = 1 - e^{-\zeta\omega_n t_r}(cos\omega_d t_t + \frac{\zeta}{\sqrt{1-\zeta^2}sin\omega_d t_r})$$

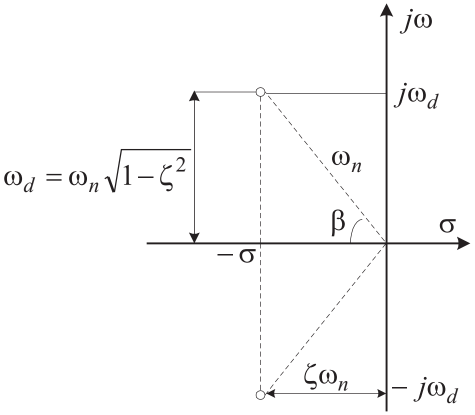
$$cos\omega_d t + \frac{\zeta}{\sqrt{1-\zeta^2}sin\omega_d t_r} = 0, \quad tan\omega_d t_r = -\frac{\zeta}{\sqrt{1-\zeta^2}} = -\frac{\omega_d}{\sigma}$$

$$t_r = \frac{1}{\omega_d} \cdot arctan\left(\frac{\omega_d}{-\sigma}\right) = \frac{\pi - \beta}{\omega_d}$$

β and σ are defined in next Figure.

Important picture !!!

Poles of second-order systems



Obtain the peak time by differentiating $c(t)$ with respect to time and letting the derivative equal zero:

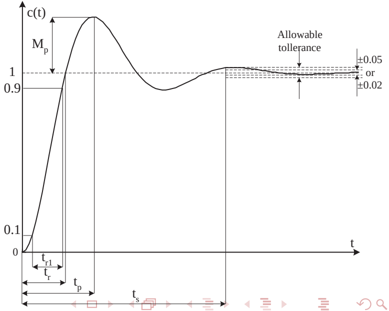
$$\frac{dc(t)}{dt}\Big|_{t=t_p} = sin(\omega_d t_p) \cdot \frac{\omega_n}{\sqrt{1-\zeta^2}} \cdot e^{-\zeta\omega_n t_p} = 0$$

Peak time

$$sin(\omega_d t_p) = 0$$

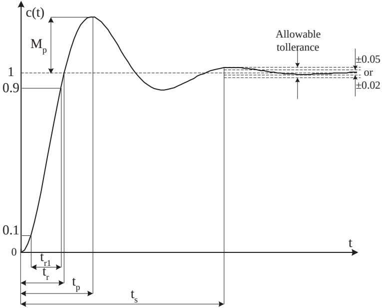
$$\omega_d t_p = 0, \pi, 2\pi, 3\pi, ...,$$

$$t_p = \frac{\pi}{\omega_d}$$



M_p occurs at the peak time or at $t = t_p = \frac{\pi}{\omega_d}$.

Percent overshoot



$$M_p = c(t_p) - 1 = -e^{-\zeta\omega_n\pi/\omega_d}\left(cos\pi + \frac{\zeta}{\sqrt{1-\zeta^2}}sin\pi\right) = e^{-\pi\frac{\zeta}{\sqrt{1-\zeta^2}}}$$

Settling time

$$c(t) = 1 - e^{-\zeta\omega_n t} \cdot sin\left(\omega_d t + arctan\frac{\sqrt{1-\zeta^2}}{\zeta}\right)$$

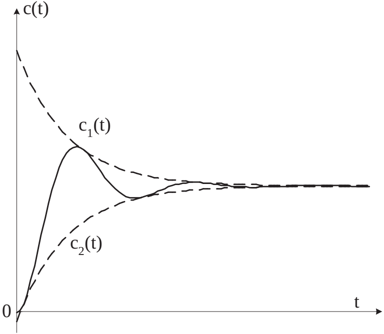
envelope curves:

$$c_{1,2}(t) = \pm e^{-\zeta\omega_n t}/\sqrt{1-\zeta^2}$$

$c_1, c_2, c(t)$ will reach 2% from the final value approximately when

$$e^{-\zeta\omega_n t_s} < 0.02, \text{ or } \zeta\omega_n t_s \cong 4$$

$$t_s = \frac{4}{\zeta\omega_n}$$



Example

Consider the closed-loop system with the transfer function:

$$H(s) = \frac{25}{s^2 + 6s + 25} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n + \omega_n^2}$$

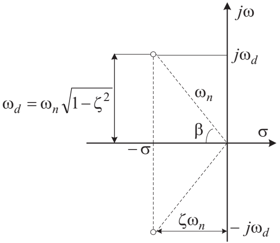
Calculate:

$$\omega_n = 5, \quad \zeta = 0.6, \ \ \omega_d = \omega_n\sqrt{1 - \zeta^2} = 5\sqrt{1 - 0.6^2} = 4$$

The poles negative real part:

$$\sigma = -\zeta\omega_n = -3.$$

Example



$$\beta = arctan\frac{\omega_d}{\sigma} = 0.93$$

$$t_r = \frac{\pi - \beta}{\omega_d} = \frac{3.14 - 0.93}{4} = 0.55sec$$

$$t_p = \frac{\pi}{\omega_d} = \frac{3.14}{4} = 0.78sec$$

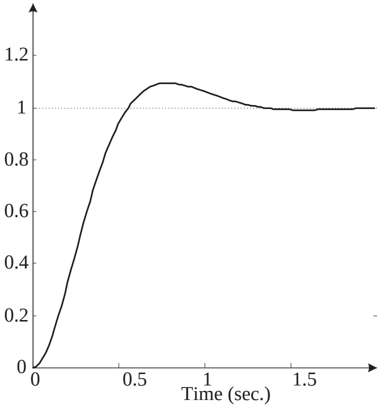
$$M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}} = 0.095$$

$$M_p(\%) = 9.5\%$$

$$t_s = \frac{4}{-\sigma} = \frac{4}{\zeta\omega_n} = \frac{4}{3} = 1.33sec$$

The step response of the system. The values of the system parameters can be seen from the plot.

Example



Note

Consider a 1st or 2nd order system with the transfer function

$$H_k(s) = \frac{k}{Ts+1} = k \cdot H(s),\ \text{or}\ H_k(s) = k \cdot \frac{\omega_n^2}{s^2 + 2\zeta\omega_n + \omega_n^2} = k \cdot H(s)$$

The system response when the input is a unit step $R(s) = 1/s$ is:

$$c(t) = \mathcal{L}^{-1}[H_k(s) \cdot R(s)] = \mathcal{L}^{-1}[\frac{k \cdot H(s)}{s}] = k \cdot \mathcal{L}^{-1}[\frac{H(s)}{s}]$$

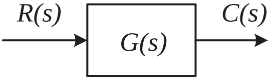
The time response is $k \cdot c(t)$ where $c(t)$ is the response of the system with a unity gain.

Steady-state error

The **steady-state error** $=$ the error between the input $(r(t))$ and the output $(c(t))$ at steady-state.

$$e_{ss} = \lim_{t\to\infty}(r(t) - c(t))$$

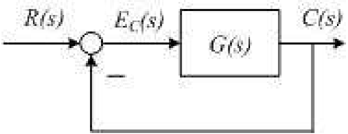
The open-loop system *error* :



$$E(s) = R(s) - C(s) = (1 - G(s))R(s)$$

Steady-state error

For a unity feedback closed-loop system, the *error* is:



$$E(s) = R(s) - C(s) = R(s) - R(s)\frac{G(s)}{1+G(s)} = \frac{1}{1+G(s)}R(s)$$

Final value theorem:

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s)$$

Steady-state error

For a unit step input

Open-loop system:

$$e_{ss} = \lim_{s \to 0} s(1 - G(s))(\frac{1}{s}) = \lim_{s \to 0}(1 - G(s)) = 1 - G(0)$$

Unity feedback closed-loop system:

$$e_{ss} = \lim_{s \to 0} s \left(\frac{1}{1 + G(s)} \right) \left(\frac{1}{s} \right) = \frac{1}{1 + G(0)}$$

Steady-state error - example

Consider a first-order system:

$$G(s) = \frac{k}{Ts+1}$$

Input: $R(s) = 1/s$. The steady-state error of the open-loop system:

$$e_{ss} = 1 - G(0) = 1 - k$$

If $G(s)$ is the open-loop transfer function for a unity feedback closed-loop system:

$$e_{ss} = \frac{1}{1+G(0)} = \frac{1}{1+k}$$

Additional zero

Consider a system with the transfer function $H(s)$. The system step response:

$$y(t) = \mathcal{L}^{-1}[Y(s)] = \mathcal{L}^{-1}\left[\frac{H(s)}{s}\right]$$

Add a zero at $-a$ and divide the transfer function with a (the gain of the new system is unchanged):

$$H_z(s) = \frac{s+a}{a}H(s) = \frac{s}{a}H(s) + H(s)$$

The step response of the system $H_z(s)$ result:

$$y_z(t) = \mathcal{L}^{-1}\left[\frac{1}{s}\left(\frac{s}{a}H(s) + H(s)\right)\right] = \frac{1}{a}\dot{y}(t) + y(t)$$

If a is small (the zero is close to the imaginary axis) $1/a$ is large $\Rightarrow$ The step response of $H_z(s)$ will increase with the quantity $1/a \cdot \dot{y}(t)$.

The effect of addition of a zero is the increase of the overshoot.

Example. Consider the system with the transfer function:

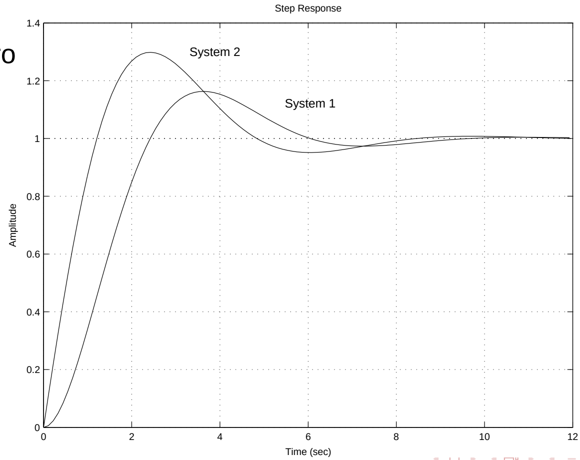
$$\text{System 1:} \quad H_1(s) = \frac{1}{s^2 + s + 1}$$

We add a zero at -1 and obtain:

$$\text{System 2:} \quad H_2(s) = \frac{s + 1}{s^2 + s + 1}$$

System1 : no zeros; System 2: a zero at -1

Additional zero - example

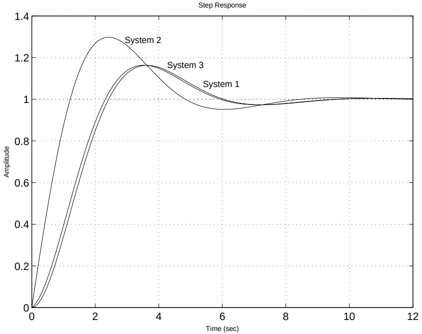


If the zero was added at -10 (the gain divided by 10)

$$\text{System 3:} \quad H_3(s) = \frac{0.1(s+10)}{s^2+s+1}$$

System 1; System 2: zero at -1; System 3: zero at -10

Additional zero
- example



Higher-order systems

Consider a system $H(s)$, with unit step input $R(s) = 1/s$ an output $C(s)$

$$C(s) = H(s) \cdot R(s) = \frac{a^m s^m + \ldots + a_1 s + a_0}{s(b^n s^n + \ldots + b_1 s + b_0)}, \quad (m \leq n)$$

$$C(s) = \frac{K \prod_{i=1}^{m} (s + z_i)}{s \prod_{j=1}^{q} (s + p_j) \prod_{k=1}^{r} (s^2 + 2\zeta_k \omega_k s + \omega_k^2)}$$

where $q + 2r = n$.

$$C(s) = \frac{a}{s} + \sum_{j=1}^{q} \frac{a_j}{s + p_j} + \sum_{k=1}^{r} \frac{b_k (s + \zeta_k \omega_k) + c_k \omega_k \sqrt{1 - \zeta_k^2}}{s^2 + 2\zeta_k \omega_k s + \omega_k^2}$$

Higher-order systems

$$
\begin{aligned}
c(t) &= a + \sum_{j=1}^{q} a_j e^{-p_j t} + \sum_{k=1}^{r} b_k e^{-\zeta_k \omega_k t} cos\omega_k \sqrt{1-\zeta_k^2}\, t \\
&+ \sum_{k=1}^{r} b_k e^{-\zeta_k \omega_k t} sin\omega_k \sqrt{1-\zeta_k^2}\, t
\end{aligned}
$$

If the poles lie in the left-half s-plane: $c(\infty) = a$.

The poles located far from the $j\omega$ (imaginary) axis have large negative real parts. The exponential terms that correspond to these poles decay very rapidly to zero.

The poles located nearest the $j\omega$ axis correspond to transient response terms that decay slowly: **dominant poles**

System approximation using the concept of dominant poles

Consider a system with a transfer function:

$$H(s) = \frac{k(s+a)}{(\frac{1}{\omega_n^2}s^2 + \frac{2\zeta}{\omega_n}s + 1)(Ts+1)}$$

$z_1 = -a$, $p_{1,2} = -\zeta\omega_n \pm j\sqrt{1-\zeta^2}$, $p_3 = -1/T$.

Consider the real pole far from the $j\omega$ axis; complex poles are dominant. The system order can be reduced by *neglecting the real pole*.

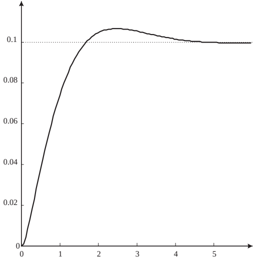
!!! The gain factor must be multiplied by the absolute value of the time constant or $1/pole$ (==¿ same steady-state value).

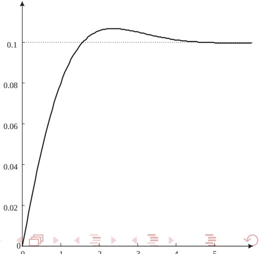
Dominant poles - examples

$$H_1(s) = \frac{s+2}{(s^2+2s+2)(s+10)}$$

$p_1, 2 = -1 \pm j$: dominant, $p_3 = -10$: can be neglected.
Divide the system gain by $|p_3| = 10$ and obtain:

$$H_2(s) = \frac{0.1(s+2)}{s^2+2s+2}$$



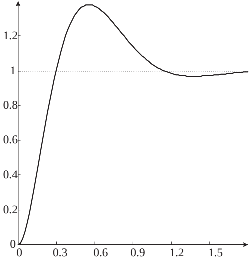


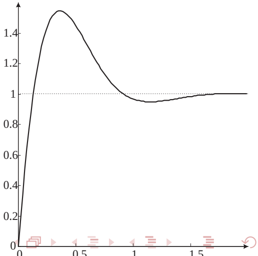
Dominant poles - examples

$$H_1(s) = \frac{62.5(s+2.5)}{(s^2+6s+25)(s+6.25)}$$

$p_1, 2 = -3 \pm 4 \cdot j,\ p_3 = -6.25$. Neglect the real pole and obtain:

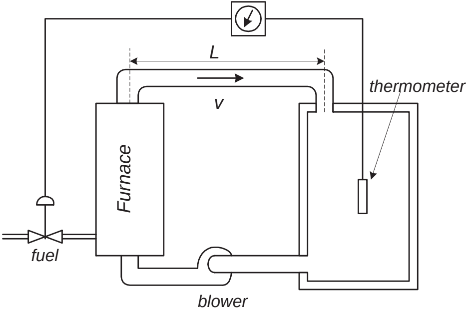
$$H_2(s) = \frac{10(s+2.5)}{s^2+6s+25}$$





Systems with transport lag (dead time)

Transport lag (dead time)



A thermal system. Hot water is circulated to keep the temperature of a chamber constant.

L - distance between the furnace and measure element,

v - the air velocity,

$T = L/v$ **transport lag** or **dead time**

A **dead time** is the time interval between the start of an event at one point in a system and its resulting action at another point in the system.

Transport lag (dead time)

The input $x(t)$ and the output $y(t)$ of a transport lag or dead time element are related by

$$y(t) = x(t - T), \text{where } T \text{ is the dead time.}$$

The transfer function of transport lag or dead time is given by:

$$H(s) = \frac{\mathcal{L}[x(t-T)]}{\mathcal{L}[x(t)]} = \frac{X(s)e^{-sT}}{X(s)} = e^{-sT}$$

A linear system with dead time:

$$\sum_{j=0}^{m} a_j \frac{d^j x(t-T)}{dt^j} = \sum_{j=0}^{n} b_j \frac{d^j y(t)}{dt^j}$$

where $x(t)$ is the input signal, $y(t)$ is the output

Transport lag (dead time)

The Laplace transform of the differential equation:

$$e^{-sT} \cdot \sum_{j=0}^{m} a_j s^j \mathcal{L}[x(t)] = \sum_{j=0}^{n} b_j s^j \mathcal{L}[y(t)]$$

and the transfer function is:

$$H(s) = e^{-sT} \frac{\sum_{j=0}^{m} a_j s^j}{\sum_{j=0}^{n} b_j s^j} = \frac{Y(s)}{X(s)}$$

If we use a Taylor series expansion for e^{-sT}:

Pade approximation

$$e^{-sT} = 1 - Ts + \frac{1}{2!}T^2s^2 - \frac{1}{3!}T^3s^3 + ...$$

and the truncated Taylor series expansion of a ratio of two polynomials:

$$\frac{1 + \alpha Ts}{1 + \beta Ts} = 1 + (\alpha + \beta)Ts - \beta(\alpha - \beta)T^2s^2 + \beta^2(\alpha - \beta)T^3s^3,$$

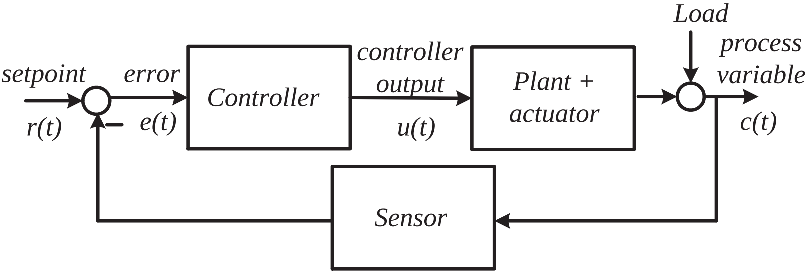
$\Rightarrow$ *Pade approximation*:

$$a)\ e^{-sT} = \frac{1 - \frac{1}{2}Ts}{1 + \frac{1}{2}Ts}, \quad b)\ e^{-sT} = \frac{1 - \frac{1}{2}Ts + \frac{1}{12}T^2s^2}{1 + \frac{1}{2}Ts + \frac{1}{12}T^2s^2}$$

PID controllers

A feedback controller is designed to generate an *output* that causes some corrective effort to be applied to a *process* so as to drive a measurable *process variable* towards a desired value known as the *setpoint*.

Conventional control system (control loop)



Example

Temperature control in a room:

- Process - the room,
- Process output - temperature
- Setpoint - desired room temperature,
- Transducer/Sensor - thermocouple
- Controller - thermostat
- Controller output - the activation signal to the air conditioner
- Actuator - the air conditioner
- Load(s) - random heat sources (such as sunshine and warm bodies)

PID controller (PID control algorithm)

- PID (Proportional-Integral-Derivative) is the control algorithm most often used in industrial control (95 %).

$$u(t) = K_P e(t) + K_I \int_0^t e(\tau) d\tau + K_D \frac{de(t)}{dt}$$

  - $u(t)$ - the current controller output
  - $e(t)$ - error = setpoint - measured output
  - K_P - *proportional tuning constant*
  - K_I - *integral tuning constant*
  - K_D - *derivative tuning constant*

Tuning = setting the K_P, K_I, and K_D so that the weighted sum of the P, I and D terms produces a controller output that drives the process variable in the direction required to eliminate the error.

$$u(t) = K_P e(t) + K_I \int_0^t e(\tau) d\tau + K_D \frac{de(t)}{dt}$$

PID controller tuning

Sluggish process:

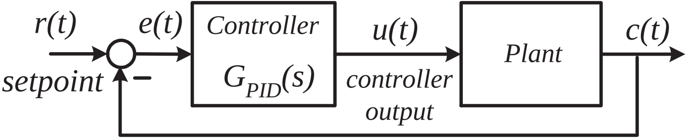
- If an error is introduced abruptly, the controller's initial reaction will be determined primarily by the D term.
- The proportional term will then come in to play: keeps the controller's output going until the error is eliminated.
- The integral term will also begin to contribute to the controller's output as the error accumulates over time. The process variable may overshoot.

PID controller tuning

Three approaches:

1. 'Trial-and-error' (experienced engineers). Example: decrease of $K_I \Rightarrow$ reduces the overshoot but slows the rate of change of the error
2. The analytical approach. If a mathematical model of the process is known, there are various methods for controller tuning.
3. A compromise between purely heuristic trial-and-error techniques and the analytical techniques. It was originally proposed in 1942 by John G. Ziegler and Nathaniel B. Nichols of Taylor Instruments.

PID controller tuning



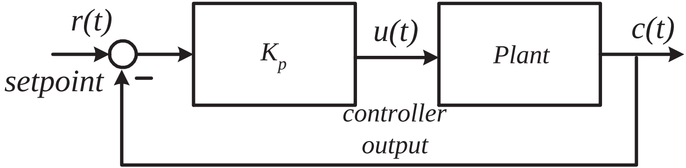
In practice the output of a PID controller is given by:

$$u(t) = K_p \left[e(t) + \frac{1}{T_i} \int_0^t e(\tau) d\tau + T_d \frac{de(t)}{dt} \right]$$

$$G_{PID}(s) = \frac{U(s)}{E(s)} = K_p \left(1 + \frac{1}{T_i s} + T_d s \right)$$

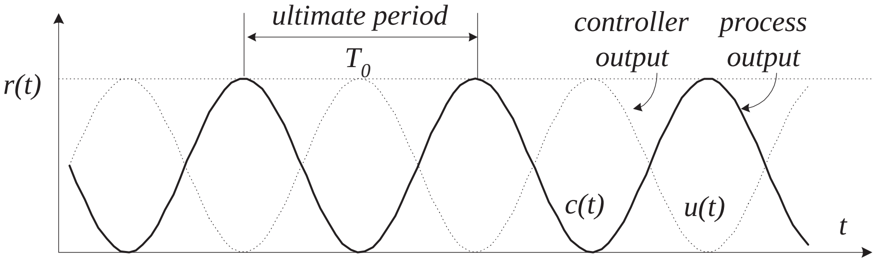
where K_p = proportional gain, T_i = integral time, T_d = derivative time

The Ultimate Sensitivity Method



- Set $T_i = \infty$, and $T_d = 0$.
- Increase K_p from 0 up to a critical value K_0 where the output, $c(t)$, first exhibits sustained oscillations.

The Ultimate Sensitivity Method



Determine experimentally:

- K_0 - the ultimate gain
- T_0 - critical period.

Ziegler-Nichols methods

The Ultimate Sensitivity Method

Type of controller	K_p	T_i	T_d
P	$0.5K_0$	∞	0
PI	$0.45K_0$	$1/1.2T_0$	0
PID	$0.6K_0$	$0.5T_0$	$0.125T_0$

Ziegler-Nichols methods

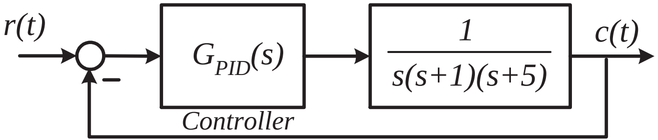
The PID controller tuned by the ultimate sensitivity method of Ziegler-Nichols gives:

$$
\begin{aligned}
G_{PID}(s) &= K_p\left(1+\frac{1}{T_i s}+T_d s\right)=0.6K_0\left(1+\frac{1}{0.5T_0 s}+0.125T_0 s\right)\\
&= 0.075K_0T_0\frac{(s+4/T_o)^2}{s}
\end{aligned}
$$

Thus, the PID controller has a pole at the origin and double zeros at $s=-4/T_0$.

Derivative cautions. If the process output rate of change is caused by noise, the D term may cause over-corrections. (ex. in pressure and level control)

Ziegler-Nichols tuning rules can be applied to plants where the model is known.



$T_i = \infty$, $T_d = 0$, closed-loop transfer function:

$$\frac{C(s)}{R(s)} = \frac{K_p}{s(s+1)(s+5) + K_p}$$

The value of K_p that makes the system marginally stable so that sustained oscillation occurs can be obtained by use of Routh's stability criterion.

Ziegler-Nichols methods

The characteristic equation:

$$s^3 + 6s^2 + 5s + K_p = 0$$

$\Rightarrow \quad K_0 = 30$

$$(s+1)(s^2+5) = (s+1)(s+\omega_n^2) = 0$$

$\Rightarrow \quad \omega_n = \sqrt{5},$

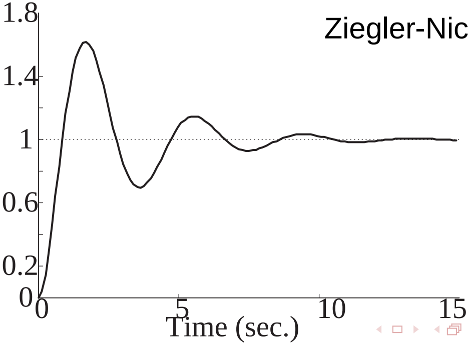
$$T_0 = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{5}} = 2.81$$

$$K_p = 0.6K_0 = 18, \quad T_i = 0.5T_0 = 1.405, \quad T_d = 0.125T_0 = 0.35$$

The transfer function of the PID controller is:

$$G_{PID}(s) = 18\left(1 + \frac{1}{1.405s} + 0.35s\right) = \frac{6.32(s+1.42)^2}{s}$$

The unit step response of this system (computer simulation):



Ziegler-Nichols methods

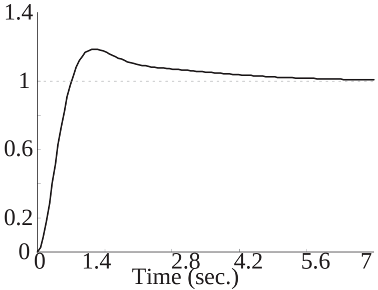
Move zero to -0.65:

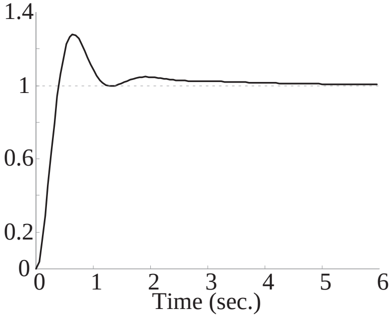
$$G_{PID}(s) = \frac{13.84(s+0.65)^2}{s}$$

Increase K_p to 39.42:

$$G_{PID}(s) = \frac{30.322(s+0.65)^2}{s}$$

Ziegler-Nichols methods

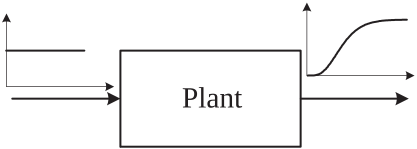




Ziegler-Nichols methods

Transient response method or *reaction curve (open-loop) method*

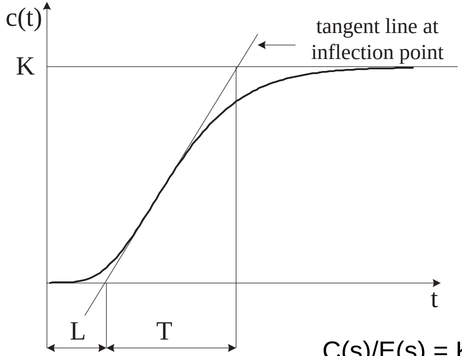
Make a step change to the process and record the process output:



$\Rightarrow$ open-loop gain K, the loop apparent deadtime L, the loop time constant T.

Transient response method

Ziegler-Nichols methods



C(s)/E(s) = K exp(-L s) / (1 + T s)

The transfer function of the plant = first order with time delay:

Type of controller	K_p	T_i	T_d
P	T/L	∞	0
PI	$0.9T/L$	$L/0.3$	0
PID	$1.2T/L$	$2L$	$0.5L$

PID controller:

$$G_{PID}(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right) = 0.6T\frac{(s+1/L)^2}{s}$$

a pole at the origin and double zeros at $s = -1/L$.

Ziegler-Nichols methods

Zeigler-Nichols tuning methods tend to produce systems whose transient response is rather oscillatory and so will need to be tuned further prior to putting the system into closed-loop operation.

Thank you very much for your attention!