Statistical_Inference

Laura Bennett

July 13, 2017

Statistical Inference for Data Science: Chapter 9 Exercises

1. Which hypothesis is typically assumed to be true in hypothesis testing?

- **The null hypothesis**

2. The type I error rate controls what?

- **The probability that the null hypothesis will be incorrectly rejected.**

3. Load the dataset mtcars in the datasets R package. Assume that the data set mtcars is a random sample. Compute the mean MPG, $\bar{x}$, of this sample. You want to test whether the true MPG is μ_0 or smaller using a one-sided 5% level test ($H_0 : \mu = \mu_0$ versus $H_a : \mu < \mu_0$). Using that data set and a Z test: Based on the mean MPG of the sample $\bar{x}$, and by using a Z test: what is the smallest value of μ_0 that you would reject for (to two decimal places)?

- **Remember that a z-score defines the number of standard deviations from the mean a data point is and is calculated as $Z = \frac{\bar{X}-\mu_0}{\frac{s}{\sqrt{n}}}$. We may calculate the Z-score that we want given our desired p-value (0.05); from there, we can solve the Z-score equation to calculate the value of μ_0 that would serve as the cutoff for rejection of the null hypothesis (the null hypothesis being that μ_0 is equal to $\bar{X}$ while the alternative is that it is smaller).**

```
r   library(datasets); data(mtcars)   xbar <- mean(mtcars$mpg)   z <- qnorm(0.05)   sd <-
sd(mtcars$mpg)   n <- length(mtcars$mpg)   cutoff <- xbar-(z*sd/sqrt(n))   round(cutoff,digits=2)
```

`## [1] 21.84` 4. Consider again the mtcars dataset. Use a two group t-test to test the hypothesis that the 4 and 6 cylinder cars have the same mpg. Use a two sided test with unequal variances. Do you reject? +

```
r   mpg4 <- mtcars$mpg[mtcars$cyl==4]   mpg6 <- mtcars$mpg[mtcars$cyl==6]   test<- t.test(mpg4,mpg6,alte
round(test$p.value,digits=4)
```

`## [1] 4e-04` **Since the p-value is less than 0.05, we may reject the null hypothesis in favor of the alterantive (the two different cylinders have different average mpgs)**

5. A sample of 100 men yielded an average PSA level of 3.0 with an sd of 1.1. What are the complete set of values that a 5% two sided Z test of $H_0 : \mu = \mu_0$ would fail to reject the null hypothesis for?

- **This requires the same method as in number 3 - however, we are solving for those values at both the upper and lower cutoffs s.t. 5% of the probability density is outside the interval.**

```
r   xbar <- 3   sd <- 1.1   n <- 100   round(3+c(-1,1)*qnorm(0.975)*sd/sqrt(n),digits=2)
```

```
## [1] 2.78 3.22
```

6. You believe the coin that you're flipping is biased towards heads. You get 55 heads out of 100 flips. Do you reject at the 5% level that the coin is fair?

- **In order to solve this problem, we must find the probability of getting 55 heads out of 100 flips if we had a FAIR coin. We may do that using the cumulative distribution function for a binomial distribution as follows:**

```
r   xbar = 55   n = 100   sd = sqrt(n*0.5*(1-0.5))   pbinom(xbar-1,size=100,prob=0.5,lower.tail
= FALSE)
```

[1] 0.1841008

7. Suppose that in an AB test, one advertising scheme led to an average of 10 purchases per day for a sample of 100 days, while the other led to 11 purchases per day (also for a sample of 100 days). Assuming a common standard deviation of 4 purchases per day and assuming that the groups are independent and their days are iid, perform a Z test of equivalence. Do you reject at the 5% level?

- **The Z statistic for two groups may be calculated as** $Z = \frac{\bar{X_1} - \bar{X_2} - (\mu_1 - \mu_2)}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$**, where in this case** $\bar{X_1} = 10$**,** $\bar{X_2} = 11$**,** $n_1 = n_2 = 100$**,** $s_p = 4$**, and** $(\mu_1 - \mu_2) = 0$ **(since we are testing whether the two population means could be considered "equal," the difference between them would hypothetically be zero). Given a z-statistic, we may then calculate the probability of getting that particular statistic on a standard normal distribution; if the probability is less than 5%, then we may reject the null hypothesis and say that the two groups have statistically significant differences in their means. Thus:**

```
r   n1 <- 100; n2 <- 100   xbar1 <- 10 ;xbar2 <- 11   sp <- 4   z <- (xbar1-xbar2-0)/(sp*sqrt((1/n1)+(1/
pnorm(z)
```

[1] 0.03854994 **Since** $p \approx 0.038 < 0.05$**, we may reject the null hypothesis and support the second advertising scheme with confidence.**

8. A confidence interval for the mean contains:

- **All of the values of the hypothesized mean for which we would fail to reject with** $\alpha = 1 - Conf.Level$

9. In a court of all, all things being equal, if via policy you require a lower standard of evidence ot convict people then:

- **More innocent people will be convicted.**