

# Statistical Inference1

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## Statistical Inference for Data Science: Chapter 3 Exercises

1. I pull a card from a deck and do not show you the result. I say that the resulting card is a heart. What is the probability that it is the queen of hearts?
  - **Given that there are 13 cards in each suit, the probability that you drew the queen of hearts is  $\frac{1}{13}$**
2. The odds associated with a probability  $p$  are defined as what?
  - **While probability is a fractional value defining how likely something will happen, odds represent a ratio describing desired outcomes versus all other outcomes and may take on any value from zero to infinity.**
3. What is the probability of getting two sixes when rolling a pair of dice?
  - **Since these are independent events, the probability of getting two sixes may be given as:  $P(6 \cap 6) = P(6)P(6) = \left(\frac{1}{6}\right)\left(\frac{1}{6}\right) = \frac{1}{36}$**
4. The probability that a manuscript gets accepted to a journal is 12%. However, given that a revision is asked for, the probability that it gets accepted is 90%. Is it possible that the probability that a manuscript has a revision asked for is 20%?
  - **The laws of conditional probability of non-independent events state that  $P(A|B) = \frac{P(A \cap B)}{P(B)} \rightarrow P(A \cap B) = P(A|B)P(B)$ . If we let  $A = 0.12$  and  $B = 0.90$ , we get that  $P(A \cap B) = 0.18$ . However, since the probability that ANY manuscript is accepted is 12%, this is impossible, as 18% (the probability that a manuscript will be accepted AND revised)  $>$  12% (the probability that a manuscript will be accepted whether it is revised or not). Therefore, it is not possible for  $P(B)=0.20$**
5. Suppose that 5% of housing projects have issues with asbestos. The sensitivity of a test for asbestos is 93% and the specificity is 88%. What is the probability that a housing project has no asbestos given a negative test expressed as a percentage to the nearest percentage point?
  - **Bayes rule states that  $P(B|A) = \frac{P(A|B)P(B)}{P(A|B)P(B)+P(A|B^c)P(B^c)}$ . Let  $A$  = the probability that a test is negative and  $B$  = the probability that the house has asbestos. To make this more clear, I'll rewrite Bayes's rule as  $P(D^c|-) = \frac{P(-|D^c)P(D^c)}{P(-|D^c)P(D^c)+P(-|D)P(D)}$ . Thus,  $P(-|D^c) = 0.88$  and  $P(D^c) = 1 - 0.05 = 0.95$ .  $P(-|D)$  is a bit trickier - however, it may easily be found by understanding that, if a house has asbestos, it still may only get one of two results on the test: positive or negative. Therefore,  $P(-|D) = 1 - P(+|D) = 1 - 0.93 = 0.07$ . The probability that a house does not have asbestos given a negative test result is thus given by:  $P(D^c|-) = \frac{(0.88)(0.95)}{(0.88)(0.95)+(0.07)(0.05)} = 0.9958 \approx 100\%$**