Determining the Volume of a Surface Defined by Tomographic Scattering Points

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Problem Statement

Much of medical imaging is done by passing a beam of electromagnetic radiation through a region and observing the interference as it passes through. If it travels through a dense material (tumor; bone) individual photons will scatter. Scattering points (SPs) may then be located through backtracking of scattered photons. Using these SPs, we attempted to determine the total volume of the object that gave rise to them. This information could be useful for the imaging of tumors.

Scattering and Beam Strength

The strength of the beam as it passes through a material is given by:

$$S(x) = Ae^{-\propto x}$$

where A is the cross-sectional area of the beam, $\propto$ the attenuation coefficient, and x the depth of the beam as it travels through a region.

Generation of Control Surfaces

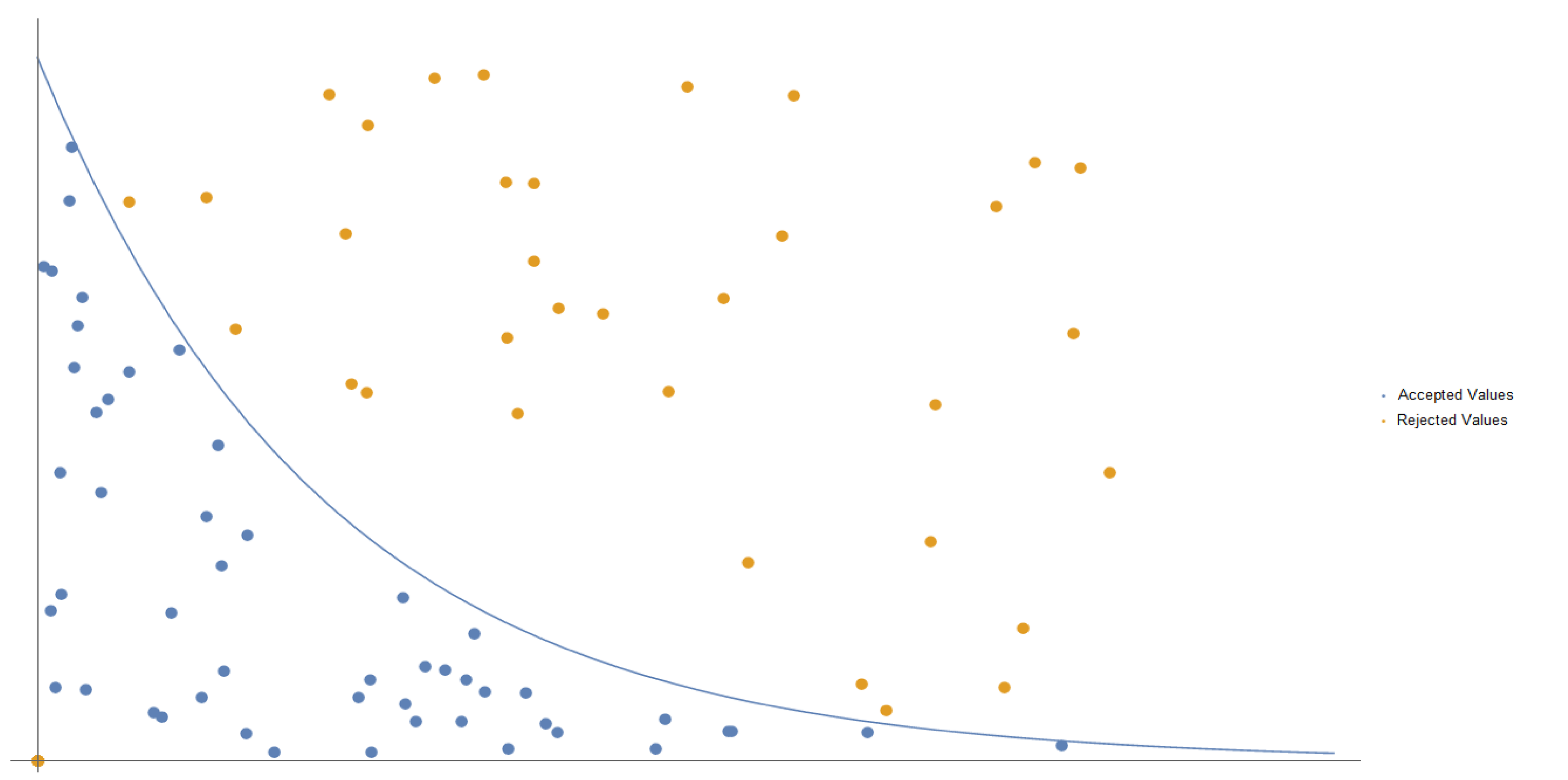
In order to test our volume algorithms, we first had to create surfaces with known volume and surface area. For this project, we created a sphere, a cylinder, and an ellipsoid. The surface points of a sphere of radius r were generated to lie along the vector

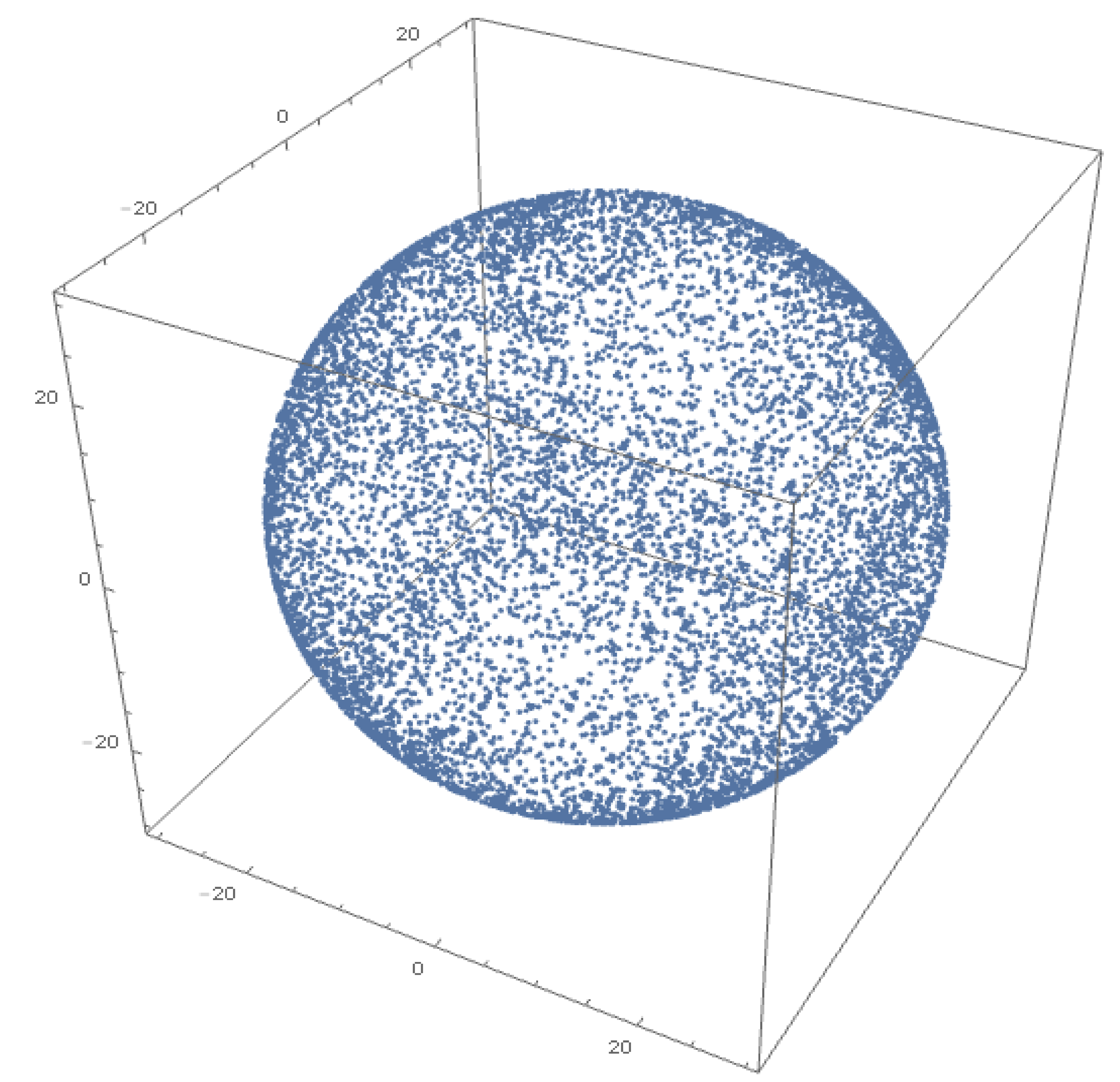
$$\vec{p}(x_0, y_0, z_0) = \frac{r}{\sqrt{{x_0}^2 + {y_0}^2 + {z_0}^2}} \langle x_0, y_0, z_0 \rangle$$

Where x_0, y_0, z_0 were random values ranging from -1 to 1. For each shape, 10,000 surface points were generated.

Beam Simulation

To simulate the generation of the SPs as the beam passes through the shapes, we used the formula for beam strength S, and a random number generator to create coordinates (x, y), retaining only the points that fell below the curve of S. Since it was more likely that low x values would be retained, we associated each x coordinate retained with the depth an SP would occur under the surface of each shape.





Volume Approximation

We found the percentage of volume captured by the beam simulation, $P(\propto, r) = \frac{V_c(\propto)}{V_0(r)}$ (where $V_c(\propto)$ is the volume of the convex hull created by the simulated SPs and $V_0(r)$ is the volume of the control sphere of radius r) to linearly correlate with α and r with a correlation coefficient of 0.965. Over a series of trials varying r and $\propto$, the linear trend for $P(\propto, r)$ was found to be:

$$P'(\propto, r_{est}) = 0.213 r_{est} + 1.486 \propto + 93.491$$

Where r_{est} is the sum of the average radius of the SPs and the average penetration depth. Using the equation for $P'(\propto, r_{est})$, the volume of the sphere $V_0(r)$ could be approximated by

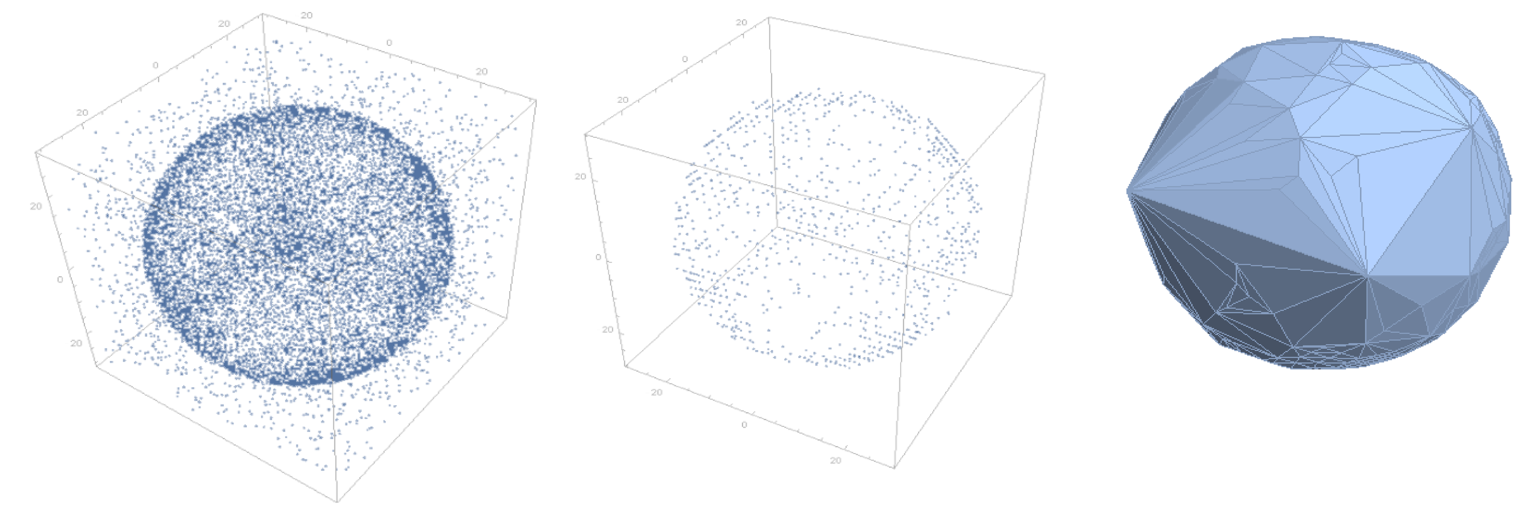
$$\frac{V_c(\propto)}{P'(\propto, r_{est})}$$

This consistently resulted in more accurate volume approximations of $V_0(r)$ than $V_c(\propto)$.

α \ r	2	6	10	14
0.6	94.34%	95.95%	96.83%	97.31%
0.8	94.55%	95.92%	97.29%	97.68%
1	95.37%	96.49%	97.50%	97.94%
1.2	95.52%	96.88%	97.73%	98.11%
1.4	95.66%	96.97%	97.94%	98.27%

Noise

Noise is defined as irregularities in the data that could obscure the signal. To account for this, we divided the scanned region into a voxel grid and counted the SPs in each voxel. If a voxel contained a significant number of SPs, the average of these points was retained and plotted; a second convex hull was then formed out of these points.

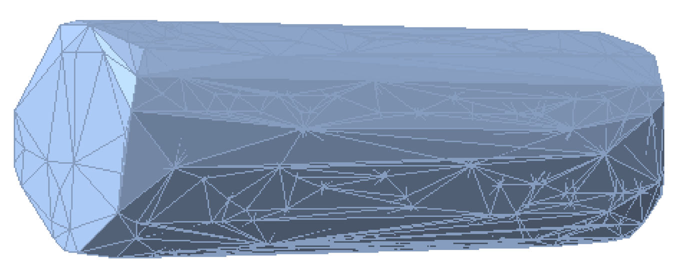


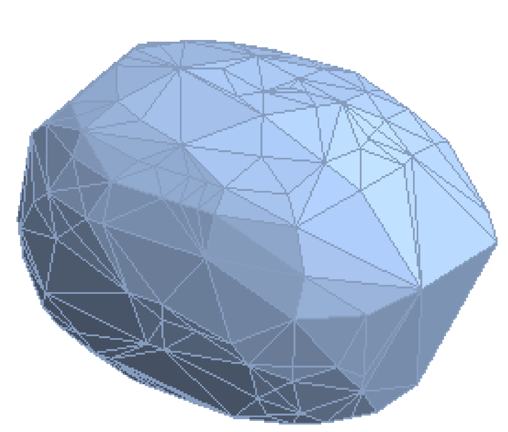
Results

The sphere, using the volume approximation algorithm with the voxels, was the most accurate.

α \ r	2	6	10	14
0.6	92.69%	106.84%	102.13%	92.05%
0.8	94.37%	107.59%	103.14%	94.86%
1	91.75%	106.63%	102.73%	96.12%
1.2	94.00%	106.62%	102.97%	96.74%
1.4	91.39%	105.93%	102.16%	96.43%

The cylinder had an average accuracy of 87%. The ellipsoid 83%.





Future Work

Future work that could be done with this project would be to study a greater variety of shapes; to differentiate between materials with varying attenuation constants; and to alter our data acquisition to model how various modern medical devices collect data.

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