

C. Tenemos que

$$x^2(\vec{\theta}) = \sum_{i=1}^N \left( \frac{y_i - M(x_i, \vec{\theta})}{\sigma_i} \right)^2$$

$$\sigma = 1$$

Por tanto

$$= \sum_{i=1}^N (y_i - M(x_i, \theta))^2$$

Ahora si derivamos tenemos lo siguiente

$$\frac{\partial x^2(\vec{\theta})}{\partial \theta_i} = \frac{d}{d\theta_i} (y_i - M(x_i, \vec{\theta}))^2$$
$$= -2(y_i - M(x_i, \theta)) \cdot \left( \frac{\partial M(x_i, \vec{\theta})}{\partial \theta_i} \right)$$

$$\frac{\partial x^2(\vec{\theta})}{\partial \theta_i} = -2 \sum_{i=1}^N (y_i - M(x_i, \vec{\theta})) \frac{\partial M(x_i, \vec{\theta})}{\partial \theta_i} \quad // Rta$$

D. Segun sabemos el descenso del gradiente se define como:

$$\vec{X}_{j+1} = \vec{X}_j - \gamma \nabla F(\vec{X}_j) \quad \text{y} \quad \nabla F(x_j) = \frac{\partial x^2(\vec{\theta})}{\partial \theta_i}$$

Sabiendo lo anterior, solo queda calcular la derivada de:

$$\frac{\partial M(x_i, \vec{\theta})}{\partial \theta_i} = \nabla_{\theta} M(x_i, \vec{\theta})$$

Por tanto:

$$\vec{\theta}_{j+1} = \vec{\theta}_j - \gamma \left( -2 \sum_{i=1}^N (y_i - M(x_i, \vec{\theta}_j)) \nabla M(x_i, \vec{\theta}_j) \right) \quad // Rta$$