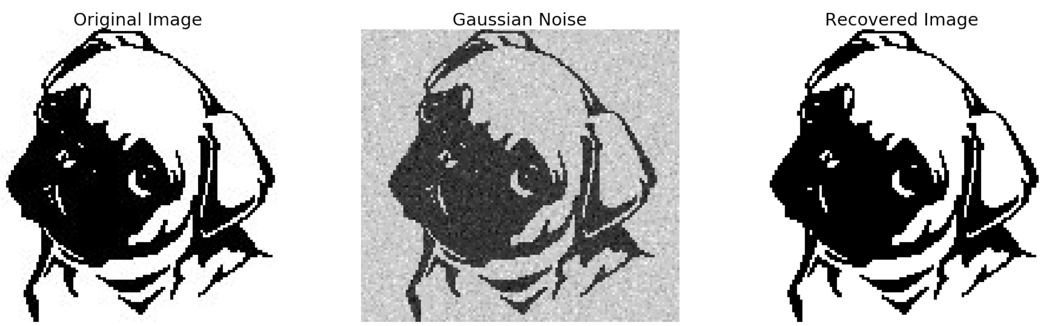
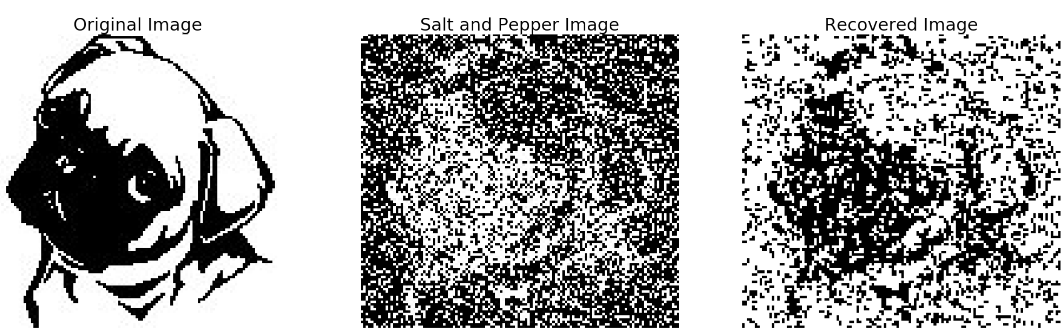
Inference

November 30, 2017

2

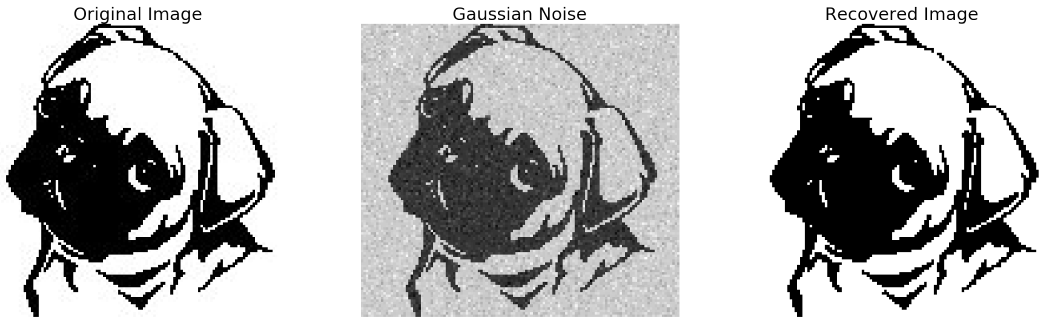


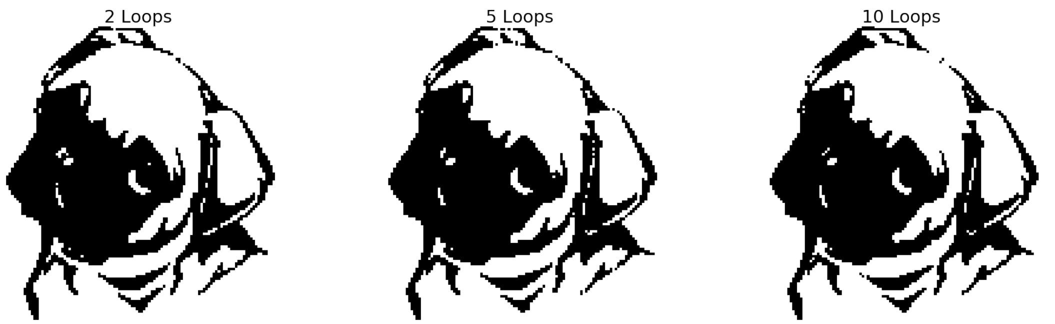
4

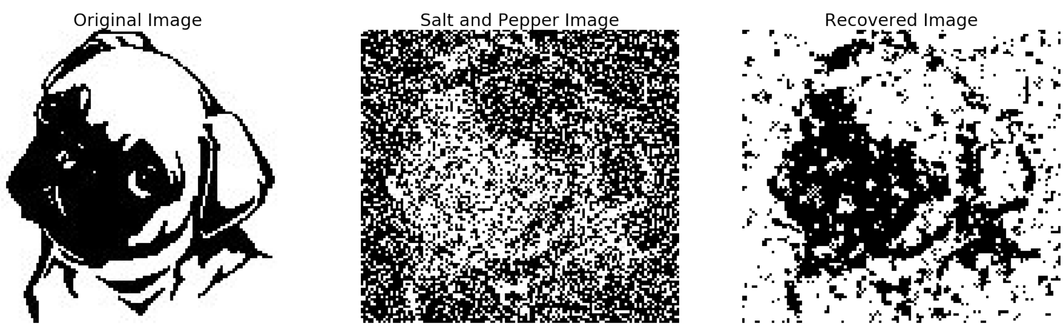


It takes 2 and 5 iterations respectively. For the salt and pepper image, this clearly still isn't a "decent" image, but for the gaussian, the image is very close to the original.

Q2





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Number of iterations:  1
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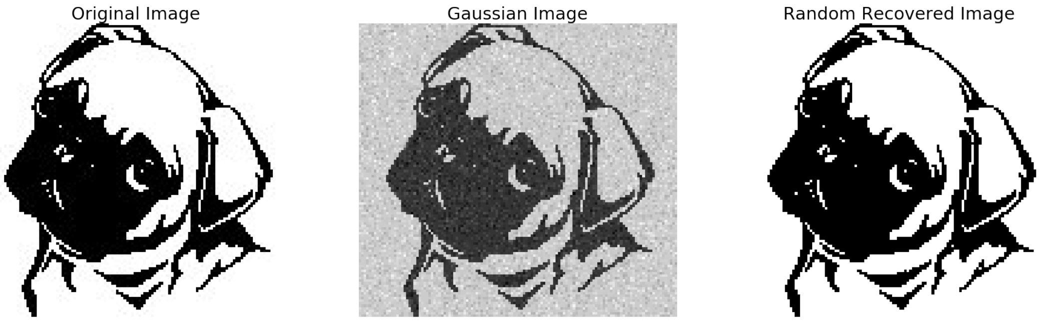


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Number of iterations:  10
Number of iterations:  50
Number of iterations:  100
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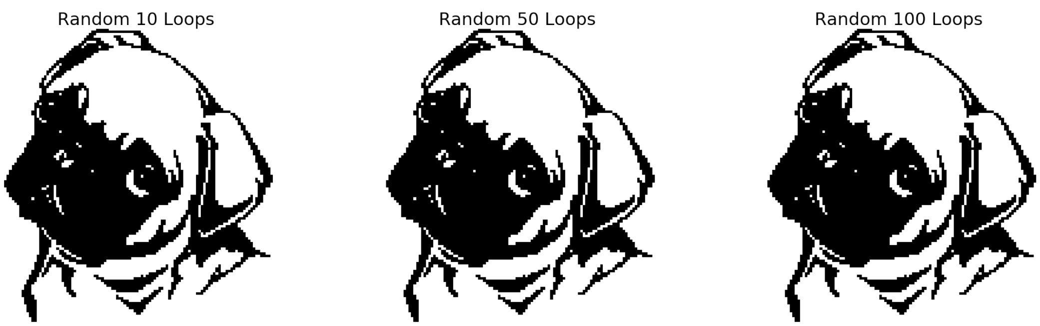


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Number of iterations:  500000
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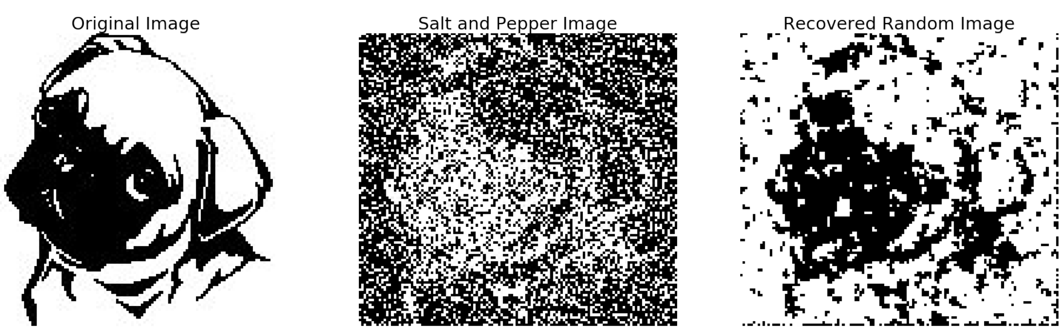


3

When looping randomly, the gaussian noise image seems to near enough recover the image perfectly in what appears to be one iteration, whilst the S&P image seems to peak in quality at around the 500,000 iteration points. In this case, iterations are how many pixels are being updated.

4

As stated above, the image does not always improve, particularly with the salt and pepper image, after an optimal number of iterations, the image declines back into a less-recognisable mess. The gaussian example shows detrimental effects from looping more than a nice amount of times, as the image quality decreases the more loops are applied.

5

To reason about the difference between $KL(q(x)||p(x))$ and $KL(p(x)||q(x))$, we'll consider the specific case where we fix $p(x) = 0, q(x) > 0$. Using the definition of the KL Divergence, $KL(q(x)||p(x)) = \int q(x) log \frac{p(x)}{q(x)}$ we can compute that

$$\lim_{p(x)\to 0} q(x) log \frac{p(x)}{q(x)} = -\infty$$

This would mean that if there is some region where we assign probability mass in $q(x)$ that $p(x)$ assigns no probability mass to, then the KL divergence will be of magnitude ∞.

On the otherhand, if we fix $p(x), q(x)$ as before, and calculate $KL(p(x)||q(x))$ we get a very different result.

$$\lim_{p(x)\to 0} p(x) log \frac{q(x)}{p(x)} = 0$$

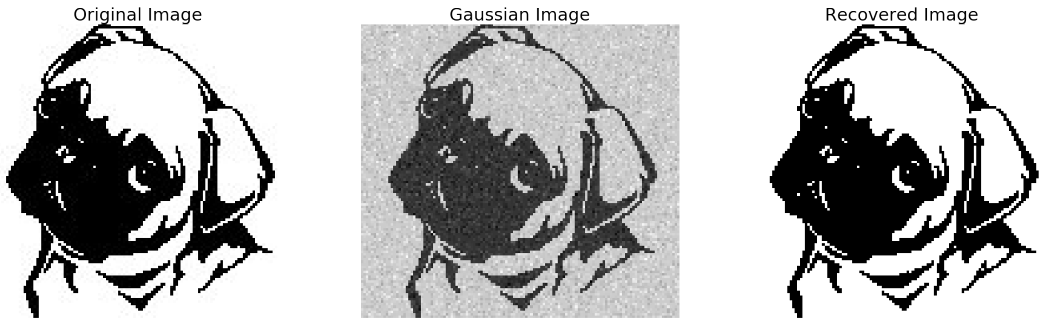
What this suggests is that if there is probability mass at q(x), but none at p(x), it doesn't make a difference to the disimilarity. Clearly we should only be interested in either $KL(q(x)||p(x))$ or $KL(p(x)||q(x))$ since the two (at least in this extreme case) tell us different information.

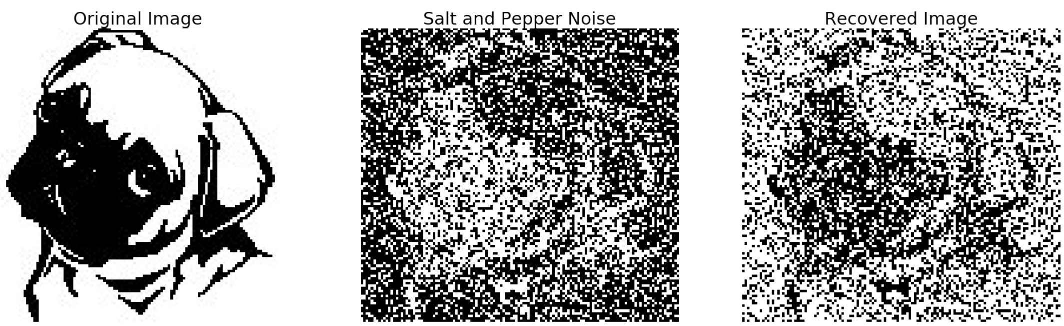
Let's say $q(x)$ is a distribution based off emperical evidence and $p(x)$ is our model and we fix the values as before, then $KL(q(x)||p(x)) = \infty$. Intuitively this makes sense, if a region in the model has 0 probability mass and yet the evidence places probability mass there, it seems sensible to say the model is disimilar to the evidence and that it should be rejected. The alternative, makes little sense when treating $p(x)$ as a model and q(x) as evidence. We saw before that the evidence $q(x)$ placing probability mass where there is none in the model p(x) doesn't increase (or change) the KL divergence which seems wrong.

Taking that $KL(q(x)||p(x))$ is more useful for our purposes, we can also look at the example where $p(x) > 0, q(x) = 0$. In that case, if we took that case then we have $\lim_{q(x)\to 0} q(x) log \frac{p(x)}{q(x)} = 0$. This also makes sense, just because none of our evidence resulted in probability mass at that point, it doesn't mean our model is wrong.

A way you can think of this is if we made the statement "all pugs have red collars", the observation of just one pug without a red collar refutes that statement (ie the case where $KL(q(x)||p(x)) = \infty$). On the otherhand, if we made the statement that "some pugs wear red collars" and didn't see any wearing red collars, we can't refute that statement (ie the case where $KL(q(x)||p(x)) = 0$).

6





7

The variational bayes method, as with the others, produces a good quality retrieval of the gaussian image, but unlike the others, produces a fairly cleaner S&P image.

8

9

10

Images are plt0, plt5 and plt10 respectively