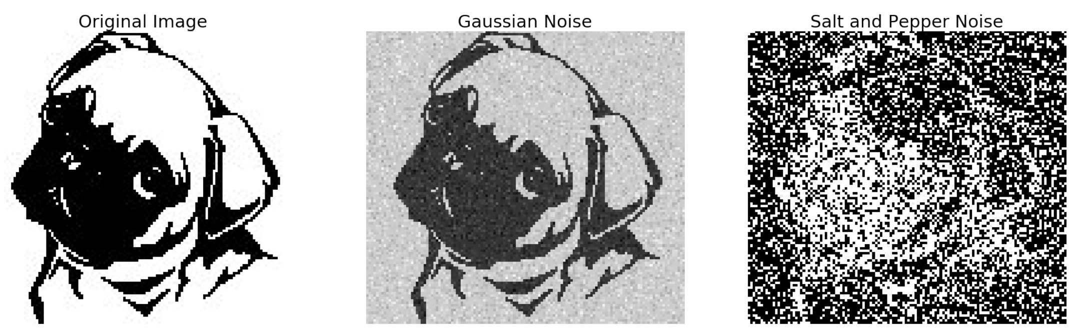
Inference

November 30, 2017

In [1]:
```
import numpy as np
import random
# np.set_printoptions(threshold=np.nan)
import matplotlib.pyplot as plt
import matplotlib.pylab as pylab
from scipy.misc import imread
from scipy.ndimage import convolve, generate_binary_structure
pylab.rcParams['figure.figsize'] = (32.0, 24.0)
pylab.rcParams['font.size'] = 24

def add_gaussian_noise(im, prop, varSigma):
    N = int(np.round(np.prod(im.shape) * prop))
    index = np.unravel_index(np.random.permutation(np.prod(im.shape))[1:N], im.shape)
    e = varSigma*np.random.randn(np.prod(im.shape)).reshape(im.shape)
    im2 = np.copy(im)
    im2[index] += e[index]
    return im2

def add_saltnpeppar_noise(im, prop):
    N = int(np.round(np.prod(im.shape) * prop))
    index = np.unravel_index(np.random.permutation(np.prod(im.shape))[1:N], im.shape)
    im2 = np.copy(im)
    im2[index] = 1 - im2[index]
    return im2

# proportion of pixels to alter
prop = 0.7
varSigma = 0.1
im = imread('image.jpg')
im = im / 255

fig = plt.figure()
ax = fig.add_subplot(131)
ax.set_axis_off()
ax.set_title("Original Image")
ax.imshow(im, cmap='gray')

im2 = add_gaussian_noise(im,prop,varSigma)
```

```
ax2 = fig.add_subplot(132)
ax2.set_axis_off()
ax2.set_title("Gaussian Noise")
ax2.imshow(im2, cmap='gray')

im3 = add_saltnpeppar_noise(im,prop)
ax3 = fig.add_subplot(133)
ax3.set_axis_off()
ax3.set_title("Salt and Pepper Noise")
ax3.imshow(im3,cmap='gray')

plt.show()
```

```
/home/gregory/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:29: DeprecationWarning
`imread` is deprecated in SciPy 1.0.0, and will be removed in 1.2.0.
Use ``imageio.imread`` instead.
```




```
In [2]: def neighbours(i, j, M, N, size = 4):
            if size == 4:
                if (i == 0 and j == 0):
                    n = [(0,1), (1,0)]
                elif (i == 0 and j == N-1):
                    n = [(0,N-2), (1,N-1)]
                elif (i == M-1 and j==0):
                    n = [(M-1,1), (M-2,0)]
                elif (i == M-1 and j == N-1):
                    n = [(M-1,N-2), (M-2,N-1)]
                elif (i == 0):
                    n = [(0,j-1), (0,j+1), (1,j)]
                elif (i == M-1):
                    n = [(M-1,j-1), (M-1,j+1), (M-2,j)]
                elif (j == 0):
                    n = [(i-1,0), (i+1,0), (i,1)]
                elif (j == N-1):
```

```
            n = [(i-1,N-1), (i+1,N-1), (i,N-2)]
        else:
            n = [(i-1,j), (i+1,j), (i,j-1), (i,j+1)]
        return n
    if size == 8:
        if (i == 0 and j == 0):
            n = [(0,1), (1,0), (1,1)]
        elif (i == 0 and j == N-1):
            n = [(0,N-2), (1,N-1), (1,N-2)]
        elif (i == M-1 and j==0):
            n = [(M-1,1), (M-2,0), (M-2,1)]
        elif (i == M-1 and j == N-1):
            n = [(M-1,N-2), (M-2,N-1), (M-2, N-2)]
        elif (i == 0):
            n = [(0,j-1), (0,j+1), (1,j), (1,j-1), (1,j+1)]
        elif (i == M-1):
            n = [(M-1,j-1), (M-1,j+1), (M-2,j), (M-2,j-1), (M-2,j+1)]
        elif (j == 0):
            n = [(i-1,0), (i+1,0), (i,1), (i-1,1), (i+1,1)]
        elif (j == N-1):
            n = [(i-1,N-1), (i+1,N-1), (i,N-2), (i-1,N-2), (i+1,N-2)]
        else:
            n = [(i-1,j), (i+1,j), (i,j-1), (i,j+1), (i-1,j-1), (i+1,j-1), (i-1,j+1), (i
        return n

def neighbours_vec(A, size=4):
    if (size == 4):
        k = np.array([[0, 1, 0], [1, 0, 1], [0, 1, 0]])
    else:
        k = np.array([[1, 1, 1], [1, 0, 1], [1, 1, 1]])
    B = convolve(A, k, mode='constant') * A
    return B
```

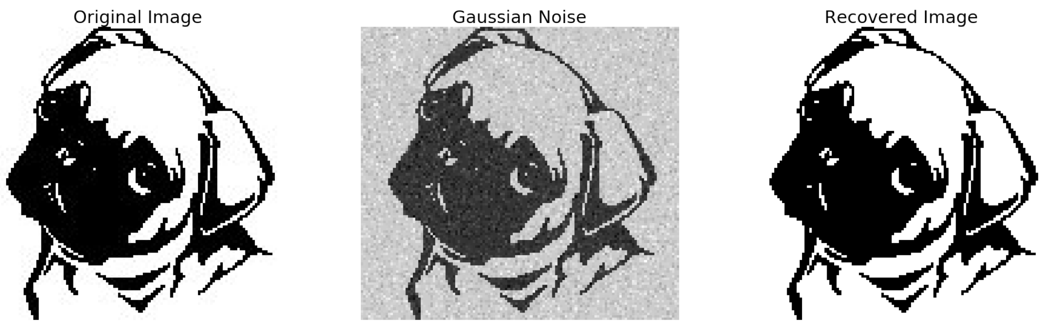
In [3]:
```
def L_i(x, y):
    weight = 2.1
    return np.multiply(np.multiply(x, y), weight)

def E_0(i, j, x, x_i=None):
    weight = 1
    shape = x.shape
    neigh = neighbours(i, j, shape[0], shape[1], size=4)
    neigh = list(map((lambda t: x[t[0]][t[1]]), neigh))
    if x_i == None:
        return np.sum(1 * neigh * weight)
    else:
        neigh = np.reshape(neigh, (-1, 1))
        x_i = np.reshape(x_i, (-1, 1))
        return np.sum(weight * np.dot(x_i, neigh.T), axis=1)
```

```
def L(x, y):
    weight = -2.1
    return np.multiply(np.multiply(x, y), weight)
```

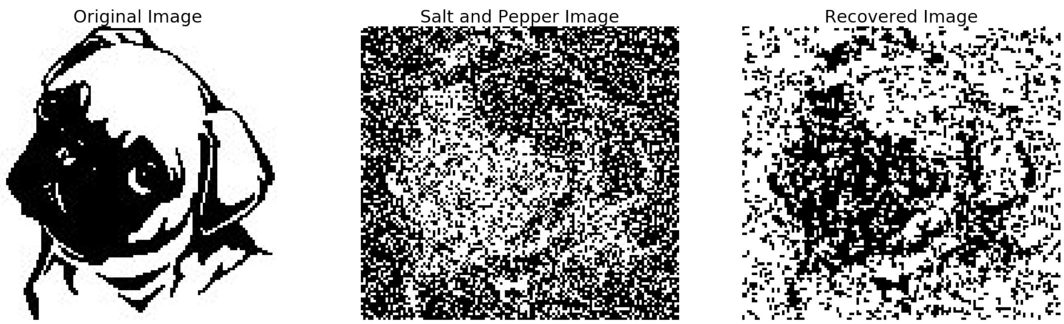
In [4]:
```
def compute_joint(i, j, x_i, x, y, L=None):
    if L == None:
        Z1 = np.sum(np.exp(L_i([-1, 1], y[i][j])))
        likelihood = np.divide(np.exp(L_i(x_i, y[i][j])), Z1)
    else:
        Z1 = np.sum(np.exp(L([-1, 1], y[i][j])))
        likelihood = np.divide(np.exp(L(x_i, y[i][j])), Z1)
    Z0 = np.sum(np.exp(E_0(i, j, x, x_i=[-1, 1])))
    prior = np.divide(np.exp(E_0(i, j, x, x_i)), Z0)[0]
    return (likelihood * prior)

def icm(img, iterations):
    img = img + (img - 1)
    shape = img.shape
    NUM_OF_ITER = iterations
    xPrev = np.copy(img)

    x = np.copy(img)
    for t in range(NUM_OF_ITER):
        for (i, j), x_i in np.ndenumerate(x):
            energy1 = compute_joint(i, j, 1, x, img)
            energy2 = compute_joint(i, j, -1, x, img)
            if energy1 > energy2:
                x[i][j] = 1
            else:
                x[i][j] = -1
        if (np.array_equal(x, xPrev)):
            print(t + 1)
            break;
        xPrev = np.copy(x)

    return (x + 1) / 2

x = icm((im2 > 0.5).astype(float), 20)

fig = plt.figure()
ax = fig.add_subplot(131)
ax.set_axis_off()
ax.set_title("Original Image")
ax.imshow(im, cmap='gray')

ax2 = fig.add_subplot(132)
ax2.set_axis_off()
```

```python
ax2.set_title("Gaussian Noise")
ax2.imshow(im2, cmap='gray')

ax3 = fig.add_subplot(133)
ax3.set_axis_off()
ax3.set_title("Recovered Image")
ax3.imshow(x, cmap='gray')

plt.show()

x2 = icm(im3, 20) * -1

fig = plt.figure()
ax = fig.add_subplot(131)
ax.set_axis_off()
ax.set_title("Original Image")
ax.imshow(im, cmap='gray')

ax2 = fig.add_subplot(132)
ax2.set_axis_off()
ax2.set_title("Salt and Pepper Image")
ax2.imshow(im3, cmap='gray')

ax3 = fig.add_subplot(133)
ax3.set_axis_off()
ax3.set_title("Recovered Image")
ax3.imshow(x2, cmap='gray')

plt.show()
```

2



4



It takes 2 and 5 iterations respectively. For the salt and pepper image, this clearly still isn't a "decent" image, but for the gaussian, the image is very close to the original.

0.0.1 Q2

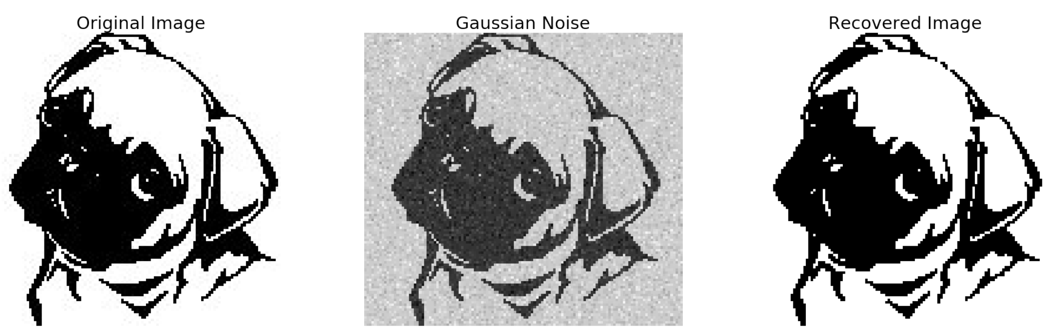
```
In [5]: def compute_posterior(i, j, x, y, L=None):
            if L == None:
                Z1 = np.sum(np.exp(L_i([-1, 1], y[i][j])))
                likelihoodPos = np.divide(np.exp(L_i(1, y[i][j])), Z1)
                likelihoodNeg = np.divide(np.exp(L_i(-1, y[i][j])), Z1)
            else:
                Z1 = np.sum(np.exp(L([-1, 1], y[i][j])))
                likelihoodPos = np.divide(np.exp(L(1, y[i][j])), Z1)
                likelihoodNeg = np.divide(np.exp(L(-1, y[i][j])), Z1)
            Z0 = np.sum(np.exp(E_0(i, j, x, x_i=[-1, 1])))
            priorPos = np.divide(np.exp(E_0(i, j, x, x_i=1)), Z0)[0]
            priorNeg = np.divide(np.exp(E_0(i, j, x, x_i=-1)), Z0)[0]
            return (likelihoodPos * priorPos) / ((likelihoodPos * priorPos) + (likelihoodNeg * p

        def gibbs_sampling(img, iterations, L=None):
            img = img + (img - 1)
            NUM_OF_ITER = iterations
            xPrev = np.copy(img)

            x = np.copy(img)
            for _ in range(NUM_OF_ITER):
                for (i, j), x_i in np.ndenumerate(x):
                    energy = compute_posterior(i, j, xPrev, img, L)
                    t = np.random.uniform(0, 1)
                    if energy > t:
                        x[i][j] = 1
                    else:
                        x[i][j] = -1
                xPrev = np.copy(x)

            return (x + 1) / 2
```

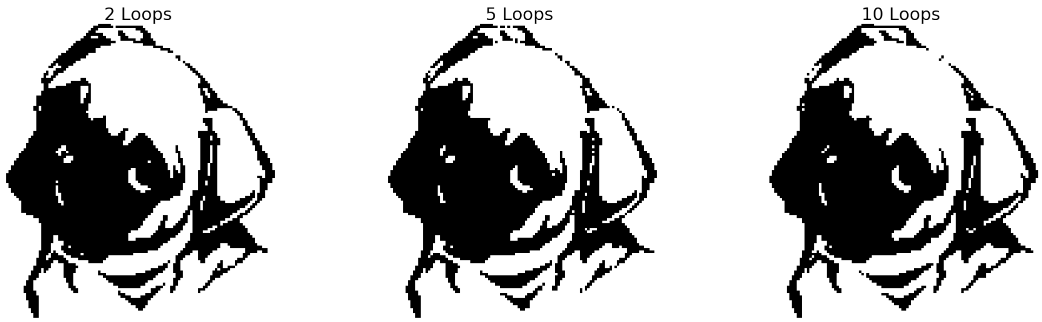
```
def gibbs_random(img, iterations, L=None):
    img = img + (img - 1)
    NUM_OF_ITER = iterations
    print("Number of iterations: ", NUM_OF_ITER)
    xPrev = np.copy(img)

    x = np.copy(img)
    x_i = len(x)
    x_j = len(x[0])
    np.random.seed(42)
    for _ in range(NUM_OF_ITER):
        i = np.random.randint(0, x_i - 1)
        j = np.random.randint(0, x_j - 1)
        energy = compute_posterior(i, j, xPrev, img, L)
        t = np.random.uniform(0, 1)
        if energy > t:
            x[i][j] = 1
        else:
            x[i][j] = -1
        xPrev = np.copy(x)

    return (x + 1) / 2
```

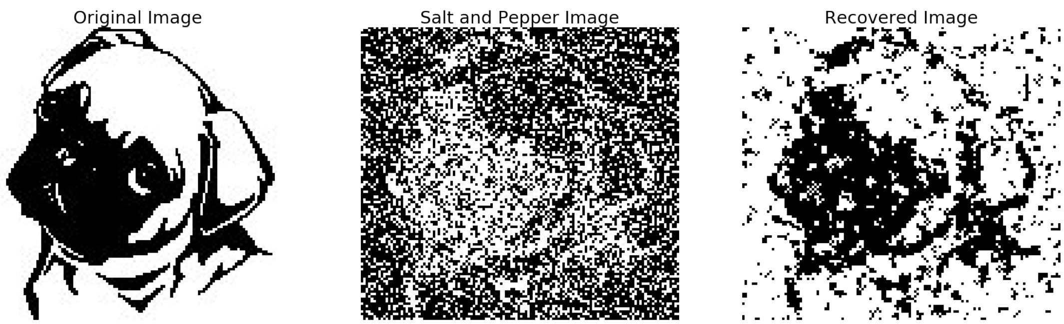
In [6]:

```
x = gibbs_sampling((im2 > 0.3).astype(float), 1)

fig = plt.figure()
ax = fig.add_subplot(131)
ax.set_axis_off()
ax.set_title("Original Image")
ax.imshow(im, cmap='gray')

ax2 = fig.add_subplot(132)
ax2.set_axis_off()
ax2.set_title("Gaussian Noise")
ax2.imshow(im2, cmap='gray')

ax3 = fig.add_subplot(133)
ax3.set_axis_off()
ax3.set_title("Recovered Image")
ax3.imshow(x, cmap='gray')

plt.show()

x3_1 = gibbs_sampling((im2 > 0.3).astype(float), 2)
x3_2 = gibbs_sampling((im2 > 0.3).astype(float), 5)
x3_3 = gibbs_sampling((im2 > 0.3).astype(float), 10)
```

```
fig = plt.figure()
ax = fig.add_subplot(131)
ax.set_axis_off()
ax.set_title("2 Loops")
ax.imshow(x3_1, cmap='gray')

ax2 = fig.add_subplot(132)
ax2.set_axis_off()
ax2.set_title("5 Loops")
ax2.imshow(x3_2, cmap='gray')

ax3 = fig.add_subplot(133)
ax3.set_axis_off()
ax3.set_title("10 Loops")
ax3.imshow(x3_3, cmap='gray')

plt.show()

x2 = gibbs_sampling(im3, 20, L)

fig = plt.figure()
ax = fig.add_subplot(131)
ax.set_axis_off()
ax.set_title("Original Image")
ax.imshow(im, cmap='gray')

ax2 = fig.add_subplot(132)
ax2.set_axis_off()
ax2.set_title("Salt and Pepper Image")
ax2.imshow(im3, cmap='gray')

ax3 = fig.add_subplot(133)
ax3.set_axis_off()
ax3.set_title("Recovered Image")
ax3.imshow(x2, cmap='gray')

plt.show()
```



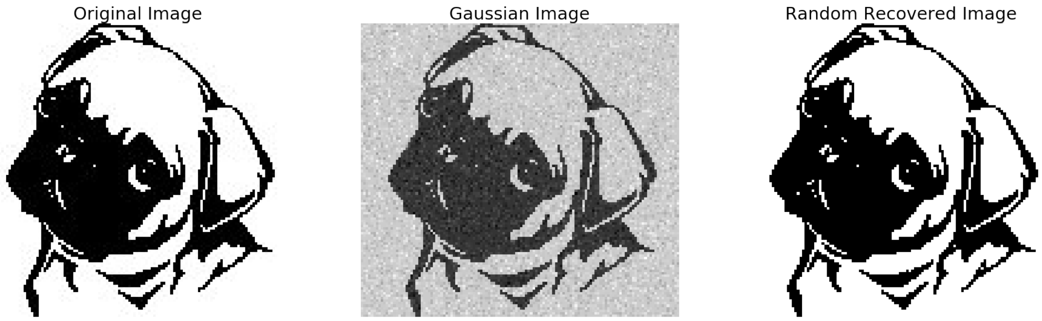




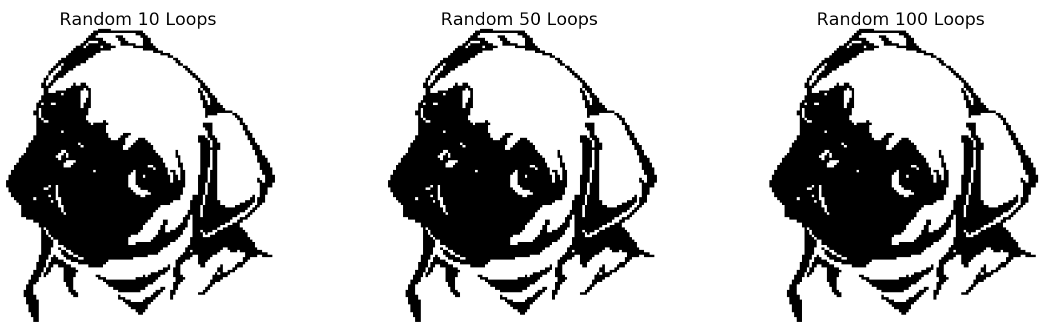

```
In [7]: x3 = gibbs_random((im2 > 0.3).astype(float), 1)

        fig = plt.figure()
        ax = fig.add_subplot(131)
        ax.set_axis_off()
        ax.set_title("Original Image")
        ax.imshow(im, cmap='gray')

        ax2 = fig.add_subplot(132)
        ax2.set_axis_off()
        ax2.set_title("Gaussian Image")
        ax2.imshow(im2, cmap='gray')

        ax3 = fig.add_subplot(133)
        ax3.set_axis_off()
        ax3.set_title("Random Recovered Image")
        ax3.imshow(x3, cmap='gray')

        plt.show()
```

```python
x3_1 = gibbs_random((im2 > 0.3).astype(float), 10)
x3_2 = gibbs_random((im2 > 0.3).astype(float), 50)
x3_3 = gibbs_random((im2 > 0.3).astype(float), 100)

fig = plt.figure()
ax = fig.add_subplot(131)
ax.set_axis_off()
ax.set_title("Random 10 Loops")
ax.imshow(x3_1, cmap='gray')

ax2 = fig.add_subplot(132)
ax2.set_axis_off()
ax2.set_title("Random 50 Loops")
ax2.imshow(x3_2, cmap='gray')

ax3 = fig.add_subplot(133)
ax3.set_axis_off()
ax3.set_title("Random 100 Loops")
ax3.imshow(x3_3, cmap='gray')

plt.show()

x4 = gibbs_random(im3, 500000, L)

fig = plt.figure()
ax = fig.add_subplot(131)
ax.set_axis_off()
ax.set_title("Original Image")
ax.imshow(im, cmap='gray')

ax2 = fig.add_subplot(132)
ax2.set_axis_off()
ax2.set_title("Salt and Pepper Image")
ax2.imshow(im3, cmap='gray')

ax3 = fig.add_subplot(133)
ax3.set_axis_off()
ax3.set_title("Recovered Random Image")
ax3.imshow(x4, cmap='gray')

plt.show()
```

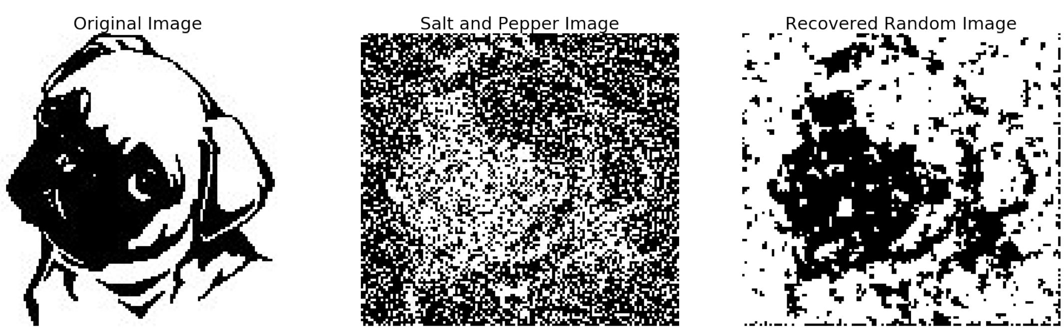
```
Number of iterations:  1
```




```
Number of iterations:  10
Number of iterations:  50
Number of iterations:  100
```




```
Number of iterations:  500000
```



0.0.2 3

When looping randomly, the gaussian noise image seems to near enough recover the image perfectly in what appears to be one iteration, whilst the S&P image seems to peak in quality at around the 500,000 iteration points. In this case, iterations are how many pixels are being updated.

0.0.3 4

As stated above, the image does not always improve, particularly with the salt and pepper image, after an optimal number of iterations, the image declines back into a less-recognisable mess. The gaussian example shows detrimental effects from looping more than a nice amount of times, as the image quality decreases the more loops are applied.

0.0.4 5

The difference between the two different forms is that in a case where our $p(x)$ is very unlikely, such as, using a very unfair coin, yet our approximation is $q(x)$, a normal, fair coin, our $q(x)/p(x)$ will produce a very large output (a large entropy) and therefore need lots of bits to model. On the other hand, with $p(x)/q(x)$, you get a significantly smaller number, therefore a small entropy. As an example, the two representations differ as in one, we are seeing how suitable $p(x)$ is to represent $q(x)$, whilst vice versa in the other example.

To reason about the difference between $KL(q(x)||p(x))$ and $KL(p(x)||q(x))$, we'll consider the specific case where we fix $p(x) = 0, q(x) > 0$. Using the definition of the KL Divergence, $KL(q(x)||p(x)) = \int q(x) log \frac{p(x)}{q(x)}$ we can compute that

$$lim_{p(x)\to 0}\, q(x) log \frac{p(x)}{q(x)} = -\infty$$

This would mean that if there is some region where we assign probability mass in $q(x)$ that $p(x)$ assigns no probability mass to, then the KL divergence will be of magnitude ∞.

On the otherhand, if we fix $p(x), q(x)$ as before, and calculate $KL(p(x)||q(x))$ we get a very different result.

$$lim_{p(x)\to 0}\, p(x) log \frac{q(x)}{p(x)} = 0$$

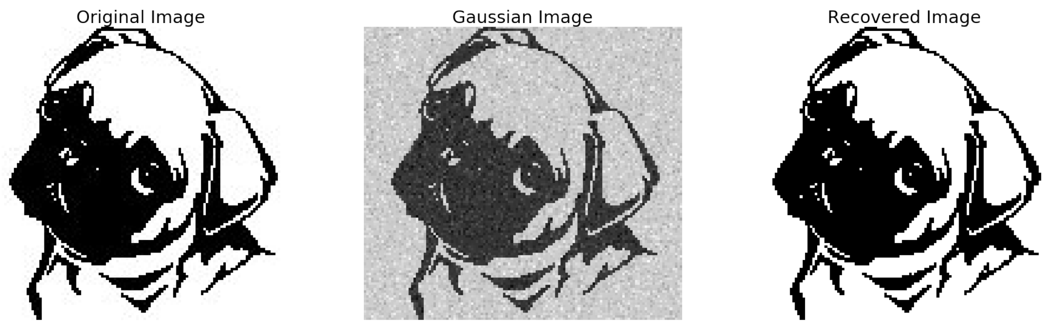
What this suggests is that if there is probability mass at q(x), but none at p(x), it doesn't make the two any less similar. NEEDS FININSHING (WILL DO TOMORROW) https://link.springer.com/content/pdf/10.1007%2F978-3-642-00659-3.pdf <- helpful for this, pg 57

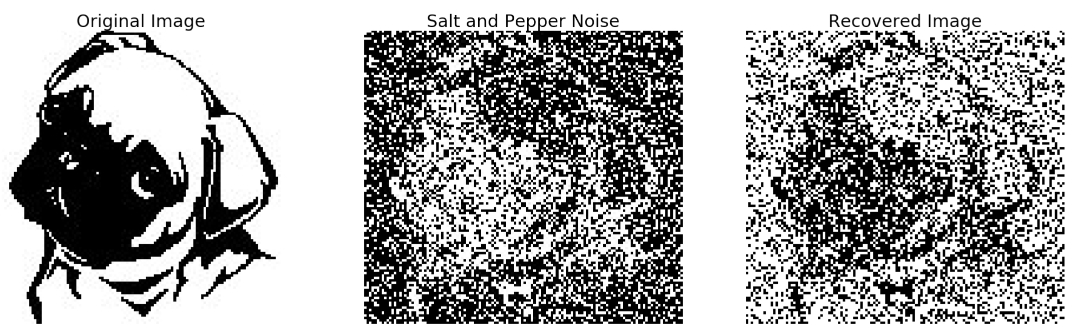
0.0.5 6

```
In [8]: def mean_field(img, iterations, L=None):
            img = img + (img - 1)
            shape = np.shape(img)
            mu = np.copy(img)
            muPrev = np.copy(mu)
            m = np.random.randn(shape[0], shape[1])
            mPrev = np.copy(m)
            NUM_OF_ITER = iterations
```

```
    weight = 1
    if L == None:
        upd = L_i(1, img) - L_i(-1, img)
    else:
        upd = L(1, img) - L(-1, img)
    for _ in range(NUM_OF_ITER):
        for (i, j), _ in np.ndenumerate(mu):
            neigh = neighbours(i, j, shape[0], shape[1], size=4)
            neigh = [muPrev[t[0]][t[1]] for t in neigh]
            m[i, j] = np.sum(weight * neigh)
        mu = np.tanh(mPrev + upd)
        mPrev = np.copy(m)
        muPrev = np.copy(mu)
    return mu;

x = mean_field((im2 > 0.3).astype(float), 5) > 0

fig = plt.figure()
ax = fig.add_subplot(131)
ax.set_axis_off()
ax.set_title("Original Image")
ax.imshow(im, cmap='gray')

ax2 = fig.add_subplot(132)
ax2.set_axis_off()
ax2.set_title("Gaussian Image")
ax2.imshow(im2, cmap='gray')

ax3 = fig.add_subplot(133)
ax3.set_axis_off()
ax3.set_title("Recovered Image")
ax3.imshow(x, cmap='gray')

plt.show()

x2 = mean_field(im3, 100, L) > 0

fig = plt.figure()
ax = fig.add_subplot(131)
ax.set_axis_off()
ax.set_title("Original Image")
ax.imshow(im, cmap='gray')

ax2 = fig.add_subplot(132)
ax2.set_axis_off()
ax2.set_title("Salt and Pepper Noise")
ax2.imshow(im3, cmap='gray')
```

```
ax3 = fig.add_subplot(133)
ax3.set_axis_off()
ax3.set_title("Recovered Image")
ax3.imshow(x2, cmap='gray')

plt.show()
```





0.0.6 7

The variational bayes method, as with the others, produces a good quality retrieval of the gaussian image, but unlike the others, produces a fairly cleaner S&P image.

```
In [9]: col = imread('colorImage.jpg')
        mask = imread('colMask.jpg')
        mask = mask / 255

        fig = plt.figure()
        ax = fig.add_subplot(121)
        ax.set_axis_off()
        ax.set_title("Original Image")
        ax.imshow(col)
```

```
ax = fig.add_subplot(122)
ax.set_axis_off()
ax.set_title("Mask")
ax.imshow(mask, cmap='gray')

plt.show()
```

```
/home/gregory/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:1: DeprecationWarning:
`imread` is deprecated in SciPy 1.0.0, and will be removed in 1.2.0.
Use ``imageio.imread`` instead.
  """Entry point for launching an IPython kernel.
/home/gregory/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:2: DeprecationWarning:
`imread` is deprecated in SciPy 1.0.0, and will be removed in 1.2.0.
Use ``imageio.imread`` instead.
```


```
In [10]: foregroundR = ax.hist(col[:, :, 0].flatten(), bins=256, weights=mask.flatten(), density
         backgroundR = ax.hist(col[:, :, 0].flatten(), bins=256, weights=((mask - 1) * -1).flatt
         foregroundG = ax.hist(col[:, :, 1].flatten(), bins=256, weights=mask.flatten(), density
         backgroundG = ax.hist(col[:, :, 1].flatten(), bins=256, weights=((mask - 1) * -1).flatt
         foregroundB = ax.hist(col[:, :, 2].flatten(), bins=256, weights=mask.flatten(), density
         backgroundB = ax.hist(col[:, :, 2].flatten(), bins=256, weights=((mask - 1) * -1).flatt

         foregroundR = foregroundR[0]
         backgroundR = backgroundR[0]
         foregroundG = foregroundG[0]
         backgroundG = backgroundG[0]
         foregroundB = foregroundB[0]
         backgroundB = backgroundB[0]

         def seg_likelihood(img):
             weight = 20
             pos = foregroundR[img[:, :, 0]] + foregroundG[img[:, :, 1]] + foregroundB[img[:, :,
             neg = backgroundR[img[:, :, 0]] + backgroundG[img[:, :, 1]] + backgroundB[img[:, :,
             return weight * (pos - neg)
```

```
def mean_field_segmentation(img, iterations):
    shape = np.shape(img)
    mu = ((np.dot(img, [0.299, 0.587, 0.114]) / 255) < 0.5).astype(float)
    muPrev = np.copy(mu)
    m = np.random.randn(shape[0], shape[1])
    mPrev = np.copy(m)
    NUM_OF_ITER = iterations
    weight = 1.5
    likelihood = seg_likelihood(img)
    for _ in range(NUM_OF_ITER):
        for (i, j), _ in np.ndenumerate(mu):
            neigh = neighbours(i, j, shape[0], shape[1], size=4)
            neigh = np.array([muPrev[t[0]][t[1]] for t in neigh], dtype=np.float32)
            m[i, j] = np.sum(weight * neigh)
        mu = np.tanh(mPrev + likelihood)
        mPrev = np.copy(m)
        muPrev = np.copy(mu)
    return mu;
```

```
In [11]: x = mean_field_segmentation(col, 20) > 0.1

         fig = plt.figure()
         ax = fig.add_subplot(131)
         ax.set_axis_off()
         ax.set_title("Original Image")
         ax.imshow(col)

         newX = np.repeat(x[:, :, np.newaxis], 3, axis=2)

         ax2 = fig.add_subplot(132)
         ax2.set_axis_off()
         ax2.set_title("Foreground")
         ax2.imshow(np.where(newX, col, 255))

         ax2 = fig.add_subplot(133)
         ax2.set_axis_off()
         ax2.set_title("Background")
         ax2.imshow(np.where(newX == False, col, 255))

         plt.show()
```