CS 6050/7050 Computational Geometry

Spring Semester, 2018

Final Exam

8:00-10:00 p.m., May 2, 2018

NOTE: This exam consists of **five** problems for a total of **100** points (20 points for each problem). If you register the class as 6050, then you have the chance to pick any four problems to work on and your total score will finally be scaled to 100 (for example, if you lose x points for the four problems, then your final score will be $(80 - x) \times \frac{100}{80}$). If you choose to give answers to all five problems, then I will grade all of them and your score will be normal and will not be scaled. So if you choose to work on four problems, please indicate them in the beginning; otherwise, I will assume that you want me to grade all five problems.

1. Let P be a set of points in the plane and q be another point that is outside the convex hull of P. Let l be a line that does NOT intersect the convex hull of P.

   For each of the statements below, tell whether it is true or not. You only need to give the answer and do NOT need to provide any proof. **(20 points)**

   (a) The farthest point of q in P must be on the boundary of the convex hull of P.

   (b) The closest point of q in P must be on the boundary of the convex hull of P.

   (c) The farthest point of l in P must be on the boundary of the convex hull of P.

   (d) The closest point of l in P must be on the boundary of the convex hull of P.

2. A simple polygon P is called *x-monotone* if for any vertical line l, the intersection of P with l is connected. In other words, the intersection should be a line segment, a point, or empty. For example, the polygon in Fig. 1 is x-monotone while the one in Fig. 2 is not. Please answer the following questions.

   (a) Is it true that every convex polygon is x-monotone? You only need to give the answer and the proof is not needed. **(5 points)**

   (b) Let P be a simply polygon that is x-monotone and has n vertices. Suppose P is represented by an array storing all its vertices in a cyclic order along its boundary. Design an $O(n)$ time algorithm to compute the convex hull of P. **(15 points)**

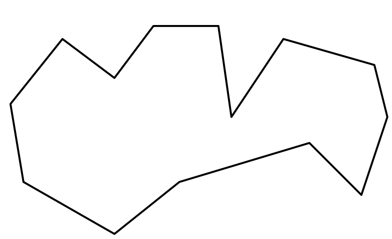

Figure 1: An x-monotone polygon.

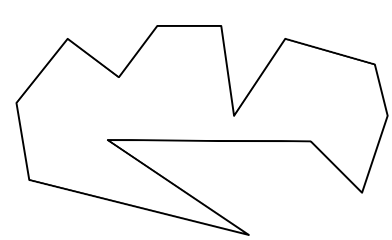

Figure 2: The polygon is not x-monotone.

3. Let S be a set of n pairwise-disjoint vertical line segments and let P be a set of n points in the plane. Imagine that each point q of P shoots a horizontal ray $\rho(q)$ to its right (e.g., see Fig. 3).

   Design an $O(n \log n)$ time algorithm to solve the following *ray-shooting* problem: for each point $q \in P$, report the segment of S that is hit FIRST by the ray $\rho(q)$, and if $\rho(q)$ does not hit any segment of S, report "NULL". **(20 points)**

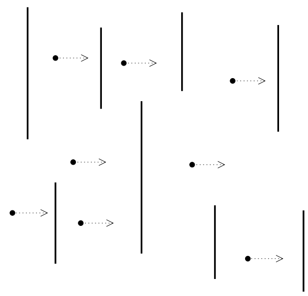

Figure 3: Each point of P shoots a horizontal ray to its right.

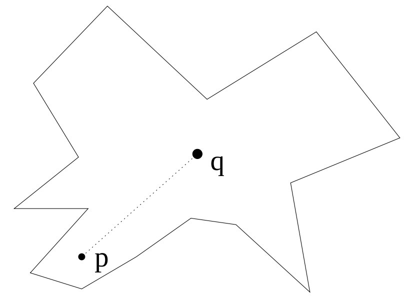

Figure 4: Illustrating a star-shaped simple polygon.

4. A simple polygon P is called *star-shaped* if it contains a point q such that for any point p in P the line segment $\overline{pq}$ is contained in P (e.g., see Fig. 4). Give an $O(n)$ time algorithm to determine whether a simple polygon P is star-shaped, where n is the number of vertices of P. (Hint: Reduce the problem to the half-plane intersection problem.) **(20 points)**

5. We consider the range queries on *weighted points*. Let P be a set of n points in the plane. Each point $p \in P$ has a weight $w(p)$, which is a real number.

   Design a data structure to quickly answer the following queries: Given any query rectangle $R = [x_l, x_r; y_b, y_t]$, return the total sum of the weights of all points in $R \cap P$ (e.g., see Fig. 5). For simplicity, you may assume that no two points in P have the same x-coordinate or y-coordinate.

   Please describe your data structure and the preprocessing time and space. Please also describe how your query algorithm works and the query time. **(20 points)**

   Note: You will receive full points for this problem if the preprocessing time and space of your data structure are both $O(n \log n)$ and the query time is $O(\log^2 n)$.

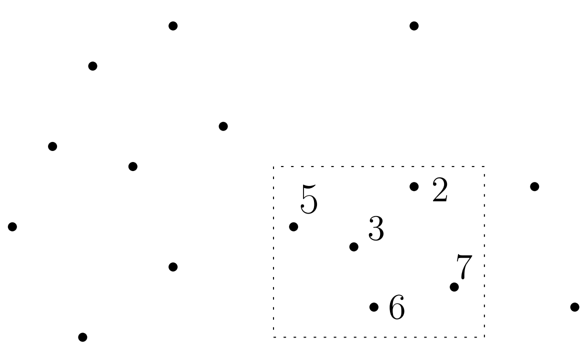

Figure 5: R is the dotted rectangle. The numbers are the weights of the points in R. The answer to this query should be 23 (which is $5 + 3 + 2 + 6 + 7$).