CS 6050/7050 Computational Geometry

Spring Semester, 2018

Assignment 2: Line Segment Intersections and the Sweeping Technique

Due Day: 8:00 p.m., Wednesday, February 21, 2018 (at the beginning of the class)

1. **(20 points)** In this exercise, we study a special *point location* problem.

   Let S be a set of n disjoint line segments whose upper endpoints lie on the horizontal line $y = 1$ and whose lower endpoints lie on the horizontal line $y = 0$ (e.g., see Fig. 1). These segments partition the horizontal strip $[-\infty, +\infty] \times [0 : 1]$ into $n + 1$ regions.

   Give an $O(n \log n)$ time algorithm to build a binary search tree on the segments of S such that given any query point p in the strip, the region containing the query point p can be found in $O(\log n)$ time. Please also describe the query algorithm.

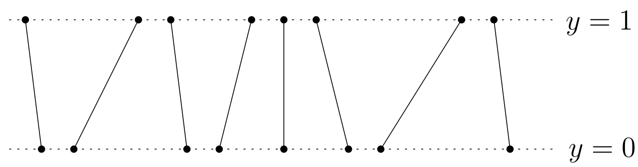

Figure 1: An example for Question 1.

2. **(20 points)** Let S be a set of n triangles in the plane. The triangles are pairwise disjoint, i.e., the boundaries of the triangles do not intersect and no triangle lies completely inside another triangle. Let P be a set of n points in the plane.

   Design an $O(n \log n)$ time algorithm to solve the following problem: For each point q in P, determine whether q is contained in a triangle, and if yes, report that triangle.

   Hint: Use the plane sweeping technique.

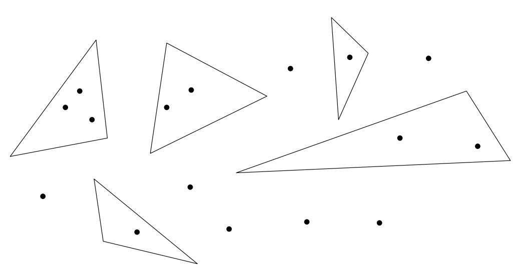

Figure 2: An example for Question 2.

3. **(20 points)** A **disk** consists of a circle plus its interior and is represented by its center point and radius. Two disks intersect if they have a point in common. Given a set of n disks in the plane, design an $O(n \log n)$ time algorithm to determine whether there are two disks that intersect.

   Note that if one disk completely contains another, these two disks also intersect. Also note that the radii of these disks may be different.

4. **(20 points)** Let S be a set of n disjoint line segments in the plane, and let p be a point not on any of the line segments of S. We wish to determine all line segments of S that p can see, that is, all line segments of S that contain some point q so that the open segment $\overline{pq}$ doesn't intersect any line segment of S. Refer to Fig. 3 as an example.

   Design an $O(n \log n)$ time algorithm for this problem. (Hint: use a rotating half-line with its endpoint at p to sweep the plane.)

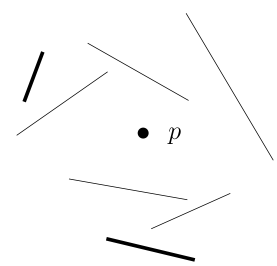

Figure 3: An example for Question 4: p can see all segments except the two thick segments.

Total Points: 80