期中考试

期中考试将于4.29日晚上7:00-9:00线上进行，请同学相互通知。试卷于7：00前在思源学堂发布，同学们9:00前在思源学堂提交答卷。考试期间将使用腾讯会议全程录制，请同学们7：00前进入会议 ID：331 694 810并开启视频，考试期间每位同学必须全程开启视频。期中考试内容为前9周所学，占总成绩的35%。

开卷考试（可查阅自己的笔记，不能上网搜索），独立完成，严禁相互交流！请同学们自觉遵守考试规则，发现违规按0分处理。

空气与气体动力学

张科

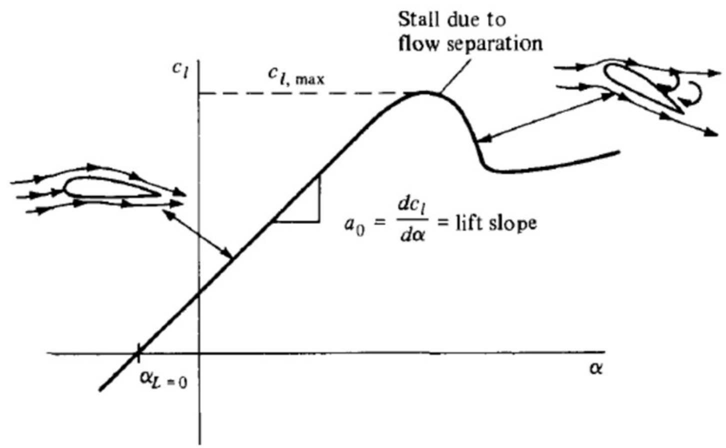
回顾：

1.标准大气；

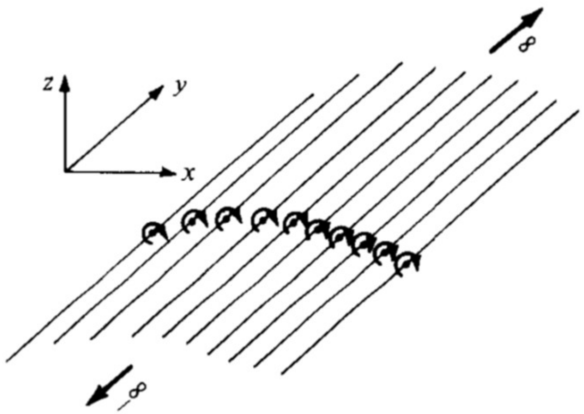
2.翼型几何参数、气动参数；

3.翼型气动特征；

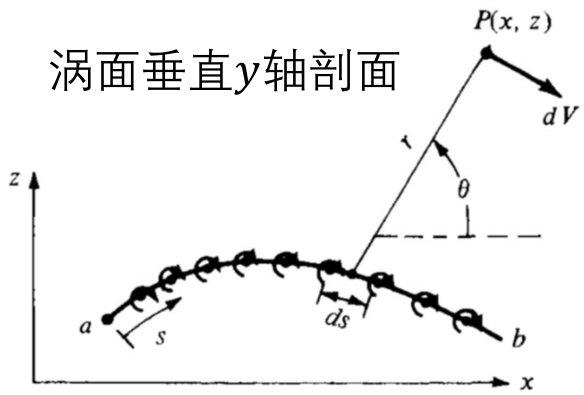
4.涡面。



$$\phi(x,z) = -\frac{1}{2\pi}\int_a^b \gamma(s)\theta ds$$

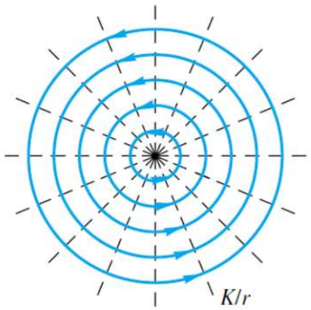


涡面垂直y轴剖面



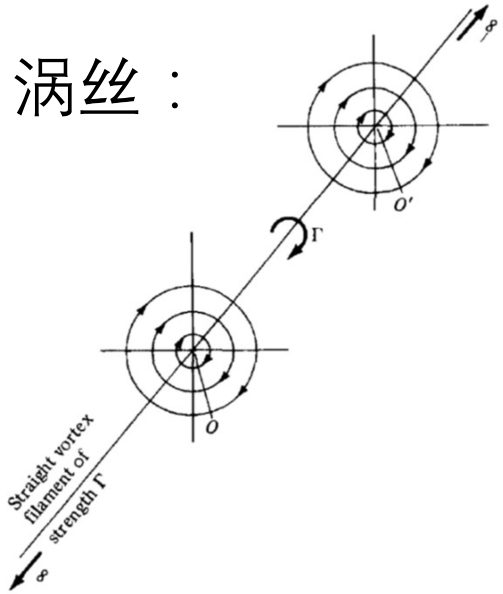
9.4涡面理论（4.4）

① 点涡：



$$V_\theta = \frac{\Gamma}{2\pi r} = \frac{K}{r}$$

② 涡丝：



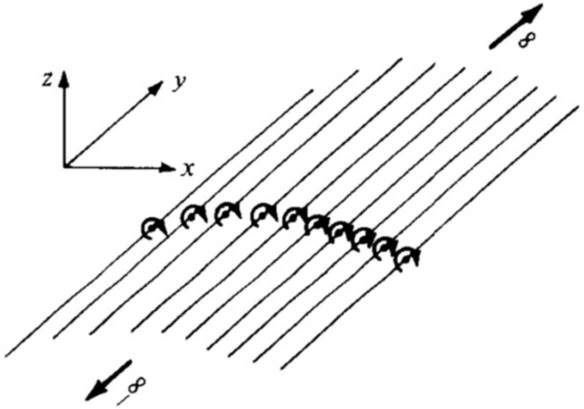
过o点，两端无线延伸，强度为Γ的直线涡丝。

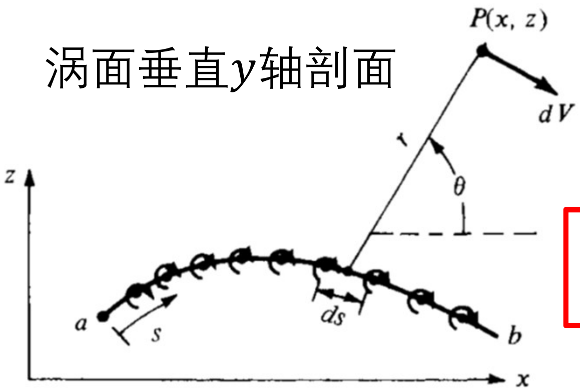
顺时针Γ>0。

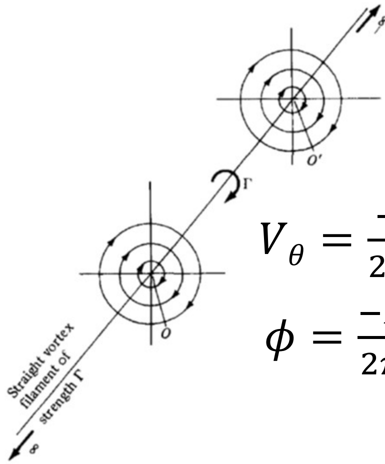
$$V_\theta = \frac{-\Gamma}{2\pi r} \qquad \phi = \frac{-\Gamma}{2\pi}\theta$$

9.4涡面理论（4.4）

③ 涡面： 无穷多条涡丝，排列一起，形成涡面。







$$V_\theta = \frac{-\Gamma}{2\pi r}$$

$$\phi = \frac{-\Gamma}{2\pi}\theta$$

$$\boxed{\phi(x,z) = -\frac{1}{2\pi}\int_a^b \gamma(s)\theta ds}$$

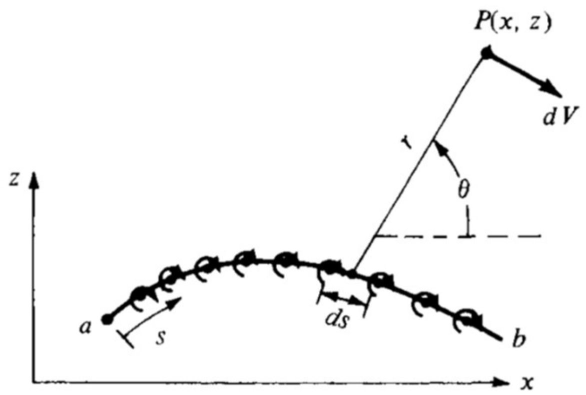
$\gamma(s)$：沿s单位长度涡面强度。

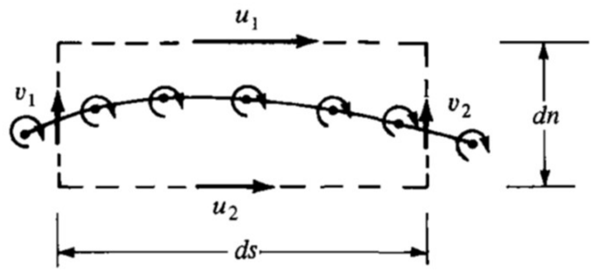
微元ds涡强$\gamma(s)ds$，在P处诱导速度$d\vec{V} = \frac{-\gamma(s)ds}{2\pi r}$，速度势函数$d\phi = \frac{-\gamma(s)ds}{2\pi}\theta$。

整个涡面在某点(x,z)速度势函数$\phi(x,z) = -\int_a^b \frac{\gamma(s)\theta}{2\pi} ds$

9.4涡面理论（4.4）

③ 涡面： 跨涡面速度关系：$\boxed{\text{跨涡面切向速度差=涡面当地强度}}$





涡面上下切向速度不连续。

绕封闭虚线环量：$\Gamma = v_1 dn + u_1 ds - v_2 dn - u_2 ds$

$$= (u_1 - u_2)ds + (v_1 - v_2)dn$$

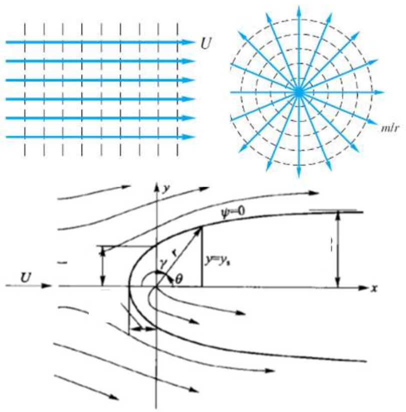
$\Gamma = \gamma(s)ds$ ➡ $\gamma(s)ds = (u_1 - u_2)ds + (v_1 - v_2)dn$

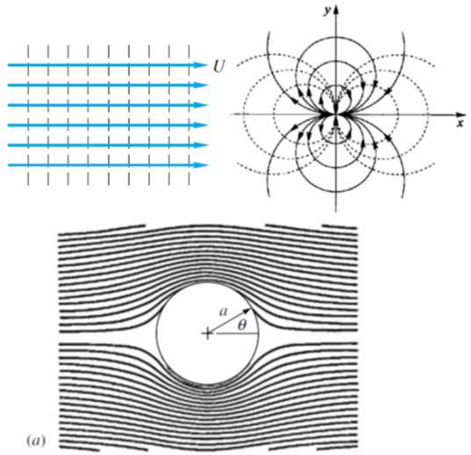
涡面dn→0 ➡ $\gamma(s)ds = (u_1 - u_2)ds$

$$\boxed{\gamma(s) = (u_1 - u_2)}$$

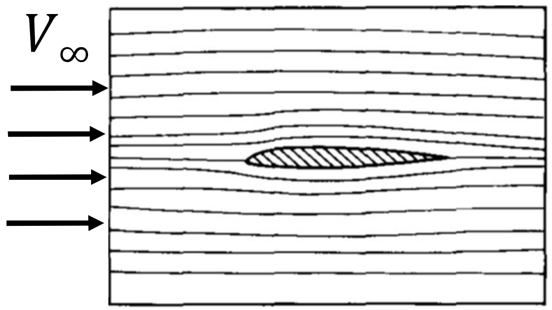
9.4涡面理论（4.4）

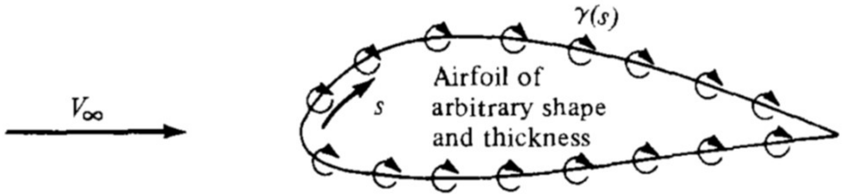
④ 1912~1922 Prandtle提出无粘不可压翼型理论。





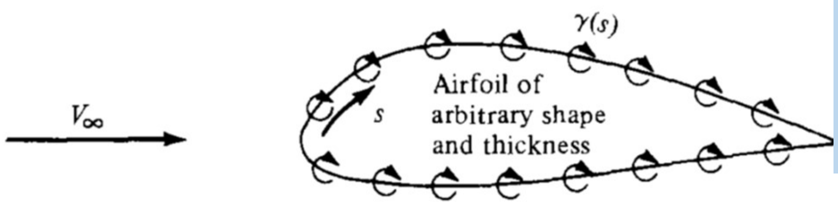
势流解??





9.4涡面理论（4.4）

④ 1912~1922 Prandtle提出无粘不可压翼型理论。

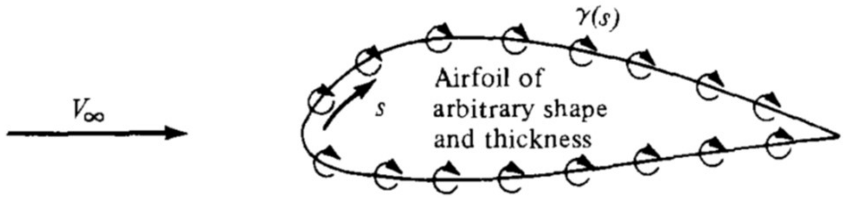


无粘势流模拟翼面边界层之外的流动，
实际粘性流中：
翼面边界层内du/dy大，Ω大。(边界层内产生涡量)

1. 将翼面换做涡面$\gamma(s)$，
2. V_∞+涡面诱导速度使翼面为流线，(V_n =0,无穿透边界条件)。
3. 求解$\gamma(s)$，
4. $\Gamma = \gamma(s)ds$，$L=\rho_\infty V_\infty \Gamma$

9.4涡面理论（4.4）

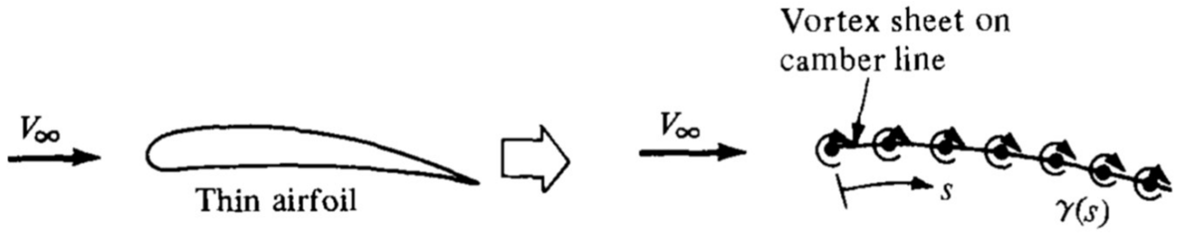
④ 1912~1922 Prandtle提出无粘不可压翼型理论。



V_n =0,无穿透边界条件。

任意形状翼型无通用解析解，有数值解（面元法）；

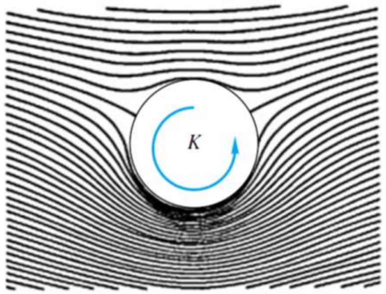
1922Max Munk提出若翼型厚度小，上下面重合，用中弧线模拟翼面。

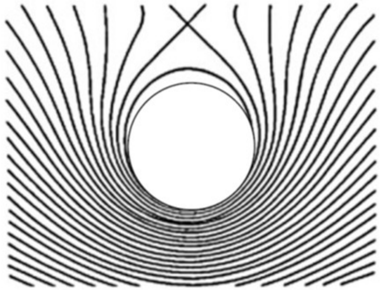


薄翼理论

9.5库塔条件（4.5）

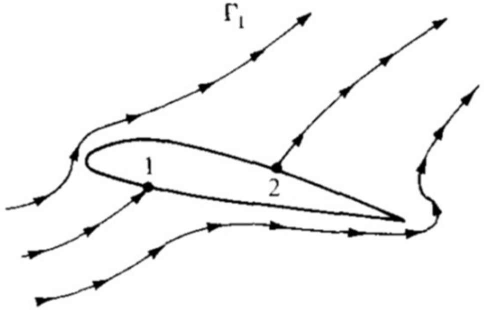
$\nabla^2\phi = 0$，势流解无限多。



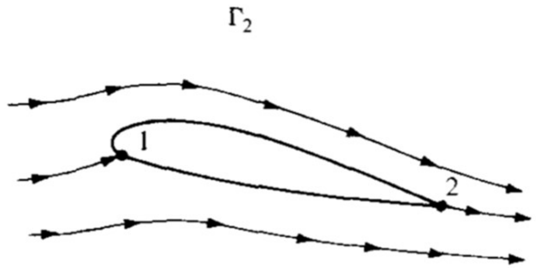


Γ不同，流动(势流解)不同。

启动时



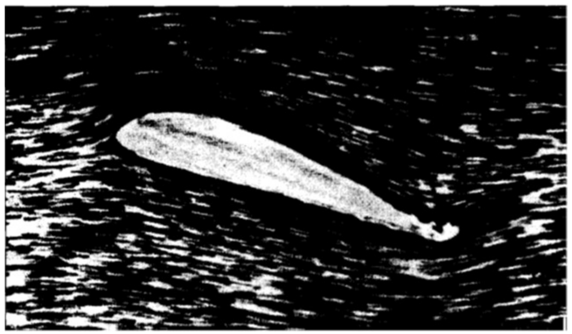
稳定后(定常)



同一α，不同Γ不同势流解。

实际粘性流，$C_l \sim \alpha$单一对应！

边界条件(粘性)决定定常下Γ！



差异？

9.5库塔条件（4.5）

实际粘性流，$C_l \sim \alpha$单一对应！　边界条件(粘性)决定定常下Γ！

库塔条件(1902)：

① 给定α，定常后Γ使流体光滑离开后缘；

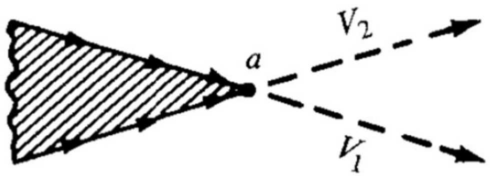
② 后缘角$\tau \neq 0$，$V_1 = V_2 = 0$，后缘为滞止点；

③ 后缘角$\tau = 0$，$V_1 = V_2 \neq 0$。

$$\boxed{\text{后缘TE：}\gamma(TE) = V_1 - V_2 = 0}$$

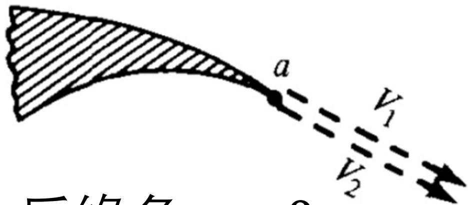
粘性→库塔条件

无粘势流解$\gamma(s)$→C_p→C_l；粘性流τ_w→C_d



后缘角$\tau \neq 0$：

$V_1 = V_2 = 0$



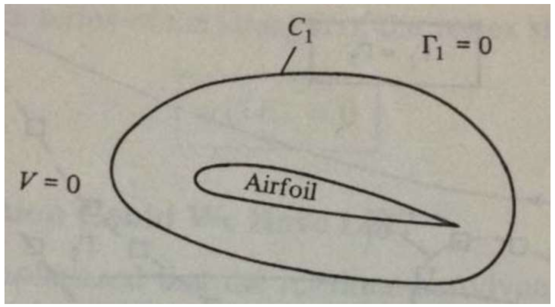
后缘角$\tau = 0$：

$V_1 = V_2 \neq 0$

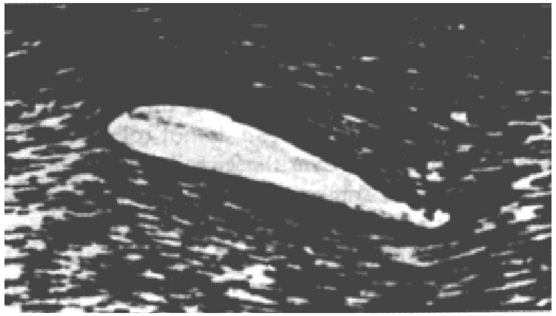
9.6起动涡、开尔文环量定理（4.5）

开尔文环量定理： $\boxed{\frac{D\Gamma}{Dt}=0}$ 无粘、不可压时，相同流体微团组成的封闭曲线上，环量对时间变化率为0。

翼型在静止空气中

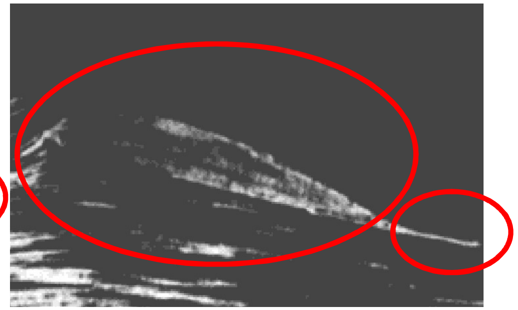


翼型刚起动



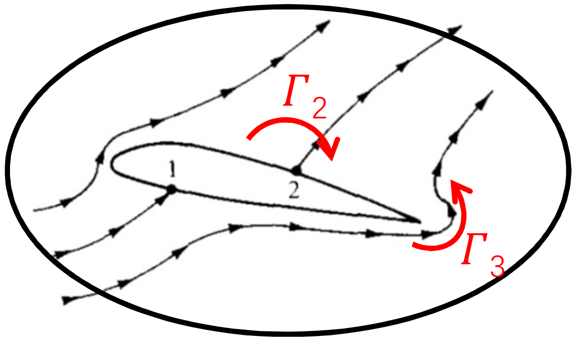
翼型起动中

稳定后



$\Gamma_1=\Gamma_2+\Gamma_3$=0

$\boxed{\text{附着涡}}$ $\Gamma_2=-\Gamma_3$



尖后缘，大速度梯度→Γ_3 $\boxed{\text{起动涡}}$

后缘涡不断向下游脱落

Γ_3⬆ Γ_2⬆

直至$\gamma(TE)$ =0

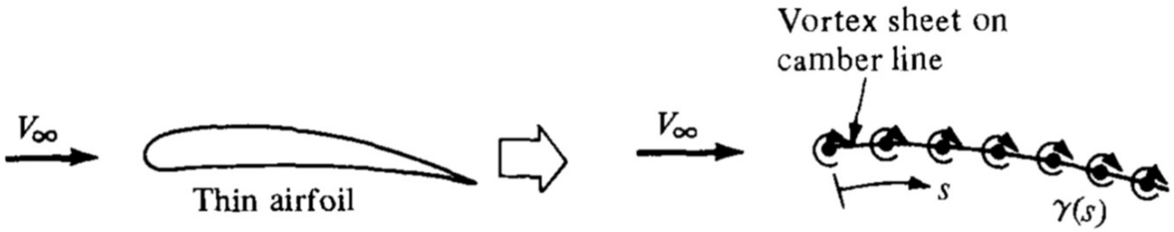
Γ_3不再增加

Γ_2=C

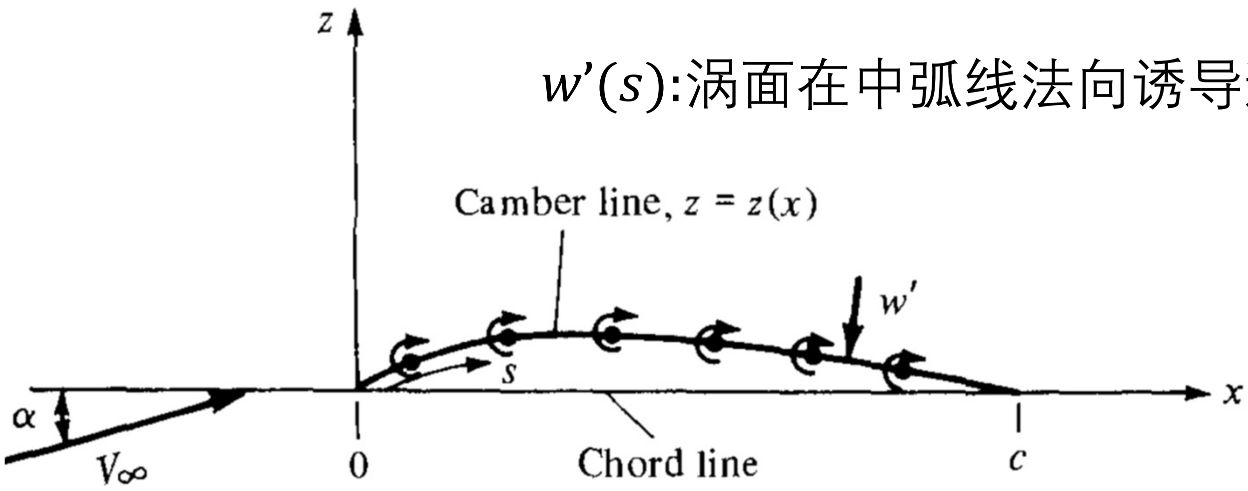
$L=\rho U\Gamma_2$

9.7经典薄翼理论（对称翼型4.6）

1. 在薄翼中弧线布涡面$\gamma(s)$, 使中弧线为流线(V_n=0)&$\gamma(TE)$=0。



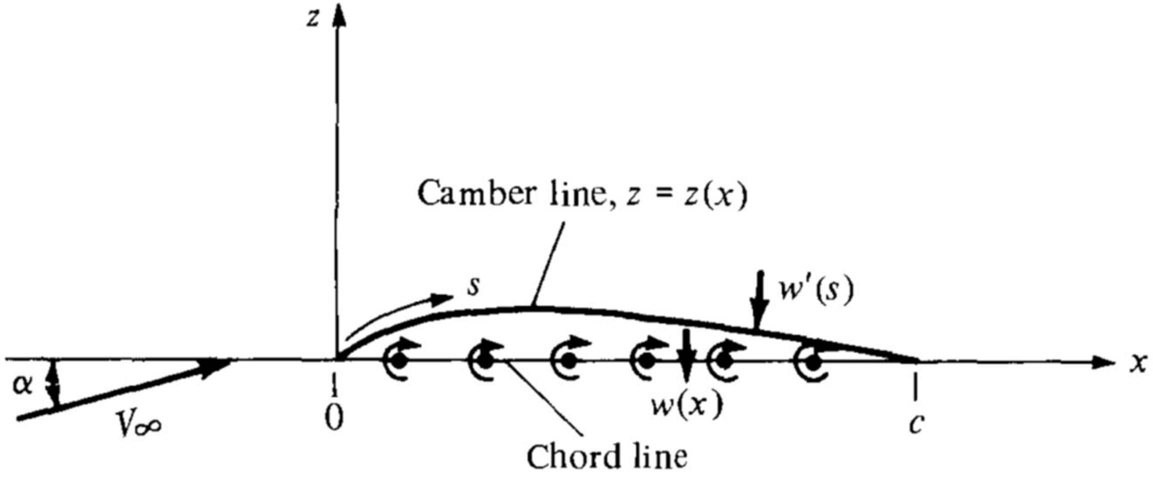
$w'(s)$:涡面在中弧线法向诱导速度

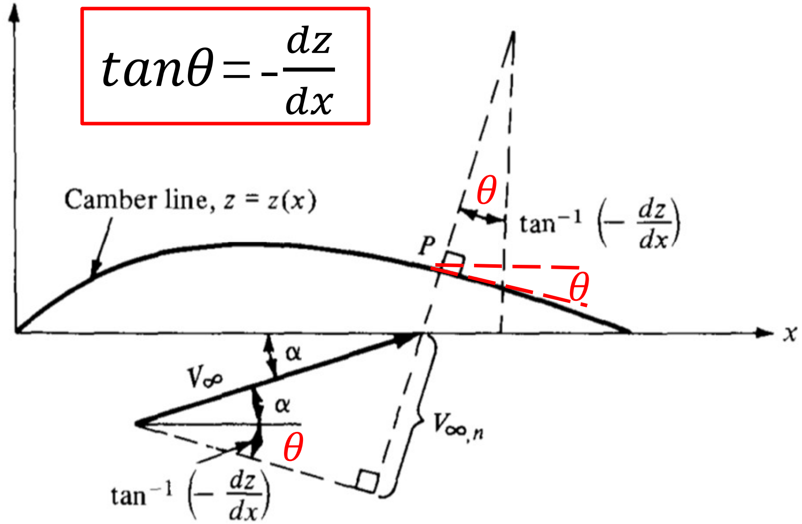


9.7经典薄翼理论（对称翼型4.6）

1. 在薄翼中弧线布涡面$\gamma(s)$，使中弧线为流线(V_n=0)&$\gamma(TE)$=0。

$\boxed{\text{涡面布在弦线上，中弧线为流线}(V_n=0)\&\gamma(c)=0\text{。}}$





中弧线上：$w(s) = V_{\infty,n} + w'(s)$

$V_{\infty,n} = V_\infty \sin(\alpha+\theta) \;= V_\infty \sin[\alpha + tan^{-1}(-\frac{dz}{dx})]$

$\boxed{\text{薄翼}\theta\text{小，小迎角}\alpha\text{小}}$ $\approx V_\infty\left(\alpha - \frac{dz}{dx}\right)$

中弧线法向与z轴夹角θ

中弧线切向与x轴夹角θ

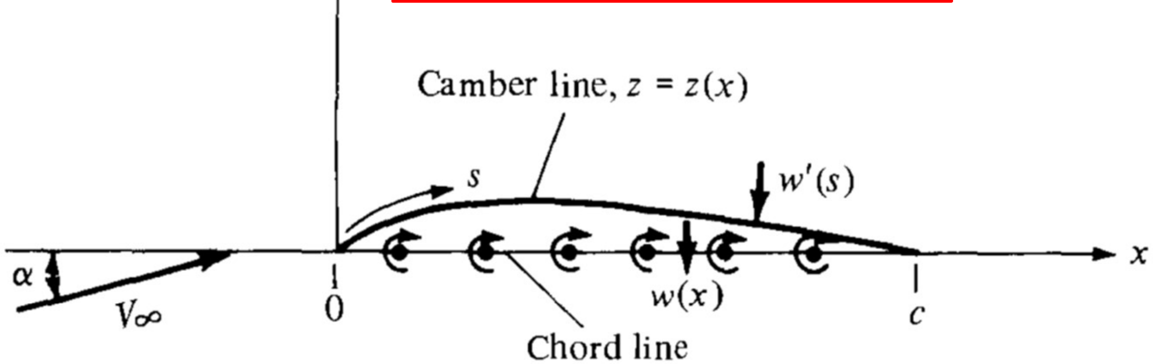
9.7经典薄翼理论（对称翼型4.6）

$$V_\theta = \frac{-\Gamma}{2\pi r}$$

1. 在薄翼中弧线布涡面$\gamma(s)$，使中弧线为流线(V_n =0)&$\gamma(TE)$=0。

涡面布在弦线上，中弧线为流线(V_n =0)&$\gamma(c)$=0。

薄翼θ小，小迎角α小



中弧线上： $w(s) = V_{\infty,n} + w'(s)$ ①

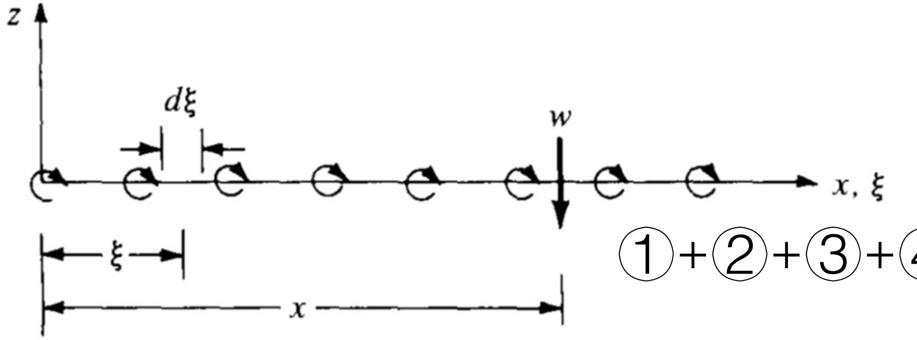
$V_{\infty,n} \approx V_\infty(\alpha - \frac{dz}{dx})$ ②

$w'(s) \approx w'(x)$ ③

微元$\gamma(\xi)\mathrm{d}\xi$在x处诱导速度dw：

$$dw = \frac{-\gamma(\xi)\mathrm{d}\xi}{2\pi(x-\xi)}$$

$$w'(x) = -\int_0^c \frac{\gamma(\xi)\mathrm{d}\xi}{2\pi(x-\xi)}$$ ④

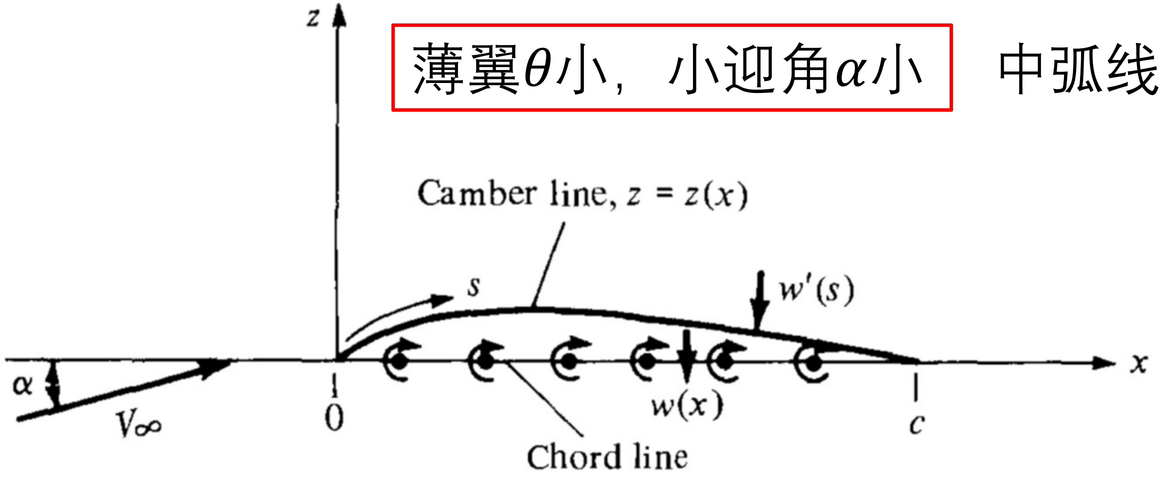


①+②+③+④ ⇒ $V_\infty(\alpha - \frac{dz}{dx}) - \int_0^c \frac{\gamma(\xi)\mathrm{d}\xi}{2\pi(x-\xi)} = 0$

9.7经典薄翼理论（对称翼型4.6）

1. 在薄翼中弧线布涡面$\gamma(s)$，使中弧线为流线(V_n =0)&$\gamma(TE)$=0。

涡面布在弦线上，中弧线为流线(V_n =0)&$\gamma(c)$=0。



薄翼θ小，小迎角α小

中弧线上： $w(s) = V_{\infty,n} + w'(s)$

$$V_\infty\left(\alpha - \frac{dz}{dx}\right) - \int_0^c \frac{\gamma(\xi)\mathrm{d}\xi}{2\pi(x-\xi)} = 0$$

$$\frac{1}{2\pi}\int_0^c \frac{\gamma(\xi)\mathrm{d}\xi}{2\pi(x-\xi)} = V_\infty\left(\alpha - \frac{dz}{dx}\right)$$

薄翼理论基本方程

求解$\gamma(s)$且$\gamma(c)$=0!

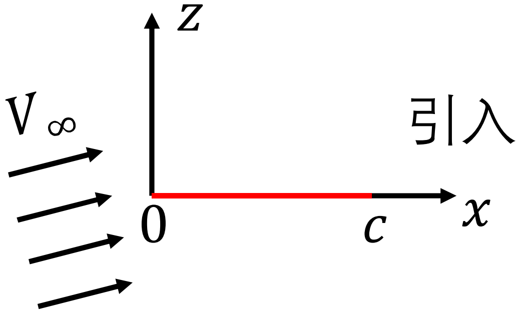
9.7经典薄翼理论（对称翼型4.6）

$$\boxed{\frac{1}{2\pi}\int_0^c \frac{\gamma(\xi)\mathrm{d}\xi}{2\pi(x-\xi)} = V_\infty\left(\alpha - \frac{dz}{dx}\right)}$$

2. 对称翼型(无弯度)，$\frac{dz}{dx}=0$（平板扰流）

$$\frac{1}{2\pi}\int_0^c \frac{\gamma(\xi)\mathrm{d}\xi}{2\pi(x-\xi)} = V_\infty\alpha$$

小迎角平板无粘不可压势流基本方程！



引入变量变换：$\xi=\frac{c}{2}(1-cos\theta)$　　$d\xi=\frac{c}{2}sin\theta d\theta$

$x=\frac{c}{2}(1-cos\theta_0)$　　$0\le\theta\le\pi$　　$0\le\xi\le c$

$$\Rightarrow \frac{1}{2\pi}\int_0^\pi \frac{\gamma(\theta)sin\theta d\theta}{cos\theta-cos\theta_0} = V_\infty\alpha$$

有精确解：$\boxed{\gamma(\theta) = 2V_\infty\alpha\frac{1+cos\theta}{sin\theta}}$

$$\Gamma=\int_0^c \gamma(\xi)\mathrm{d}\xi = \frac{c}{2}\int_0^\pi \gamma(\theta)sin\theta d\theta$$
$$= \frac{c}{2}\int_0^\pi 2V_\infty\alpha\frac{1+cos\theta}{sin\theta}sin\theta d\theta$$
$$=\pi V_\infty\alpha c$$

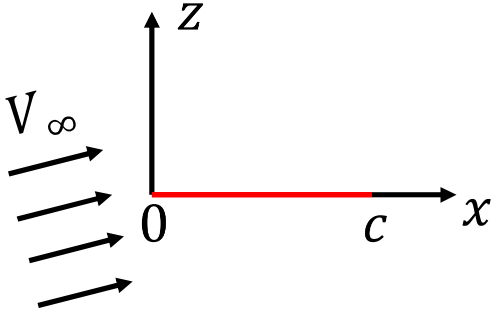
单位展长升力：$L' = \rho V_\infty\Gamma = \pi\alpha c\rho V^2{}_\infty$

升力系数：$\boxed{C_l = \frac{L'}{\frac{1}{2}\rho V^2{}_\infty c} = 2\pi\alpha}$

9.7经典薄翼理论（对称翼型4.6）

$$\boxed{\frac{1}{2\pi}\int_0^c \frac{\gamma(\xi)d\xi}{2\pi(x-\xi)} = V_\infty\left(\alpha - \frac{dz}{dx}\right)}$$

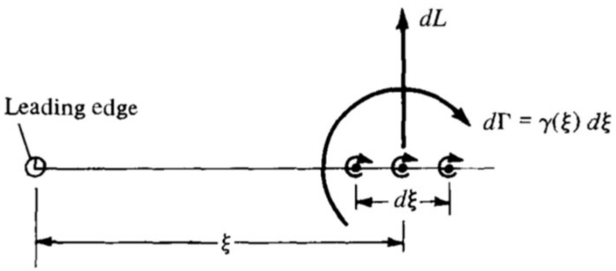
2. 对称翼型(无弯度)，$\frac{dz}{dx}=0$ 有精确解： $\gamma(\theta) = 2V_\infty\alpha\frac{1+cos\theta}{sin\theta}$



$$\boxed{C_l = 2\pi\alpha}$$

$$C_l = a_0(\alpha - \alpha_{L=0})$$

$$\boxed{a_0 = \frac{dC_l}{d\alpha} = 2\pi} \quad \boxed{\alpha_{L=0}=0}$$



$$dL = \rho V_\infty d\Gamma$$

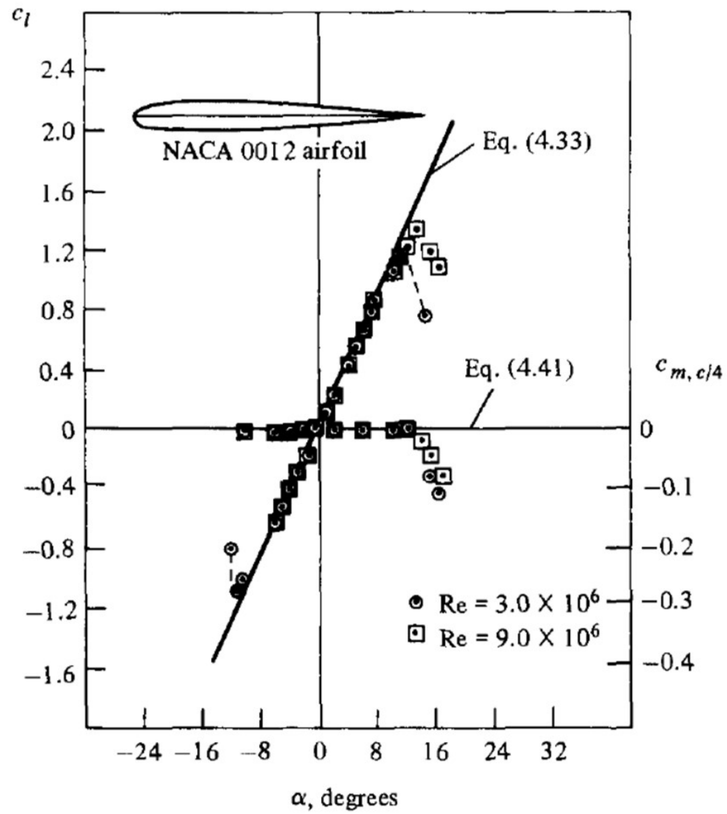
$$d\Gamma = \gamma(\xi)d\xi$$

$$M'_{LE} = -\int_0^c \xi\, dL$$

$$= -\int_0^c \xi\,\rho V_\infty \gamma(\xi)d\xi$$

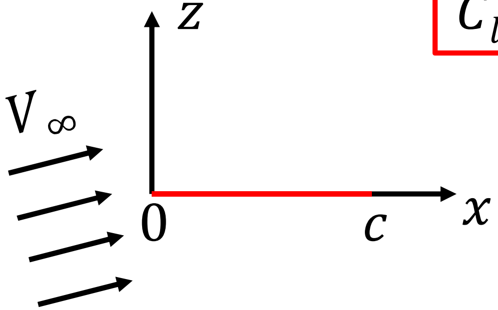
$$= -\frac{\pi\alpha}{2}\left(\frac{1}{2}\rho V^2{}_\infty\right)c^2$$

$$C_{m,LE} = \frac{M'_{LE}}{\frac{1}{2}\rho V^2{}_\infty c^2} = -\frac{\pi\alpha}{2}$$



9.7经典薄翼理论（对称翼型4.6）

2\. 对称翼型(无弯度)，$\frac{dz}{dx}$=0 有精确解： $\gamma(\theta) = 2V_{\infty}\alpha\frac{1+cos}{sin\theta}$



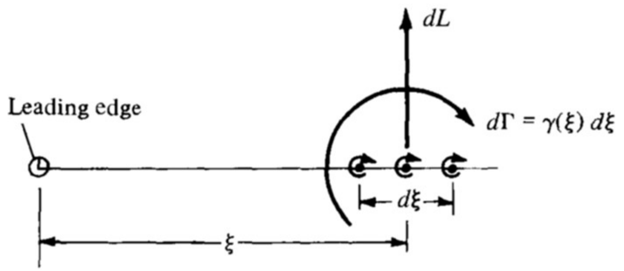
$\boxed{C_l = 2\pi\alpha}$ $\boxed{a_0 = \frac{dC_l}{d\alpha} = 2\pi}$ $\boxed{\alpha_{L=0}=0}$

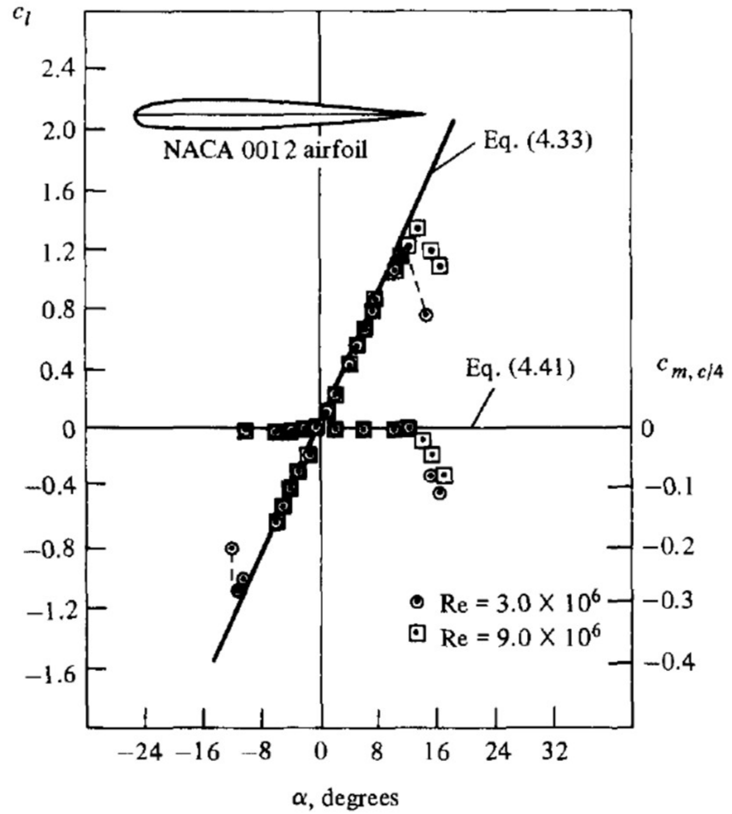
$C_{m,LE} = -\frac{\pi\alpha}{2} = -\frac{C_l}{4}$ 随α变化

$C_{m,LE} + \frac{C_l}{4} = 0$ 不随α变化

$C_{m,LE} + \frac{C_l}{4} = C_{m,c/4}$

$\boxed{C_{m,c/4} = 0}$ 不随α变化





$\boxed{c/4\text{为气动中心，也是压力中心！}}$

对称薄翼，小迎角！

9.8有弯度翼型（4.7）

$$\boxed{\frac{1}{2\pi}\int_0^c \frac{\gamma(\xi)\mathrm{d}\xi}{2\pi(x-\xi)} = V_\infty\left(\alpha-\frac{dz}{dx}\right)}$$

有弯度翼型，$\frac{dz}{dx} \neq 0$

$$\frac{1}{2\pi}\int_0^c \frac{\gamma(\xi)\mathrm{d}\xi}{2\pi(x-\xi)} = V_\infty\left(\alpha-\frac{dz}{dx}\right)$$

引入变量变换：

$\xi=\frac{c}{2}(1-cos\theta)$，$x=\frac{c}{2}(1-cos\theta_0)$，$d\xi=\frac{c}{2}sin\theta d\theta$

$$\frac{1}{2\pi}\int_0^\pi \frac{\gamma(\theta)sin\theta d\theta}{cos\theta-cos\theta_0} = V_\infty\left(\alpha-\frac{dz}{dx}\right) \quad ①$$

$$\boxed{\frac{dz}{dx}=0,\ \gamma(\theta)=2V_\infty\alpha\frac{1+cos\theta}{sin\theta}}$$

精确解：$\boxed{\gamma(\theta)=2V_\infty A_0\frac{1+cos}{sin\theta}+2V_\infty\sum_{n=1}^{\infty}A_n sinn\theta \quad ②}$

A_0取决于α和$\frac{dz}{dx}$(弯度)，A_n取决于$\frac{dz}{dx}$。

②代入①

$$A_0-\sum_{n=1}^{\infty}A_n cosn\theta_0=\alpha-\frac{dz}{dx}$$

$$\frac{dz}{dx}=\alpha-A_0+\sum_{n=1}^{\infty}A_n cosn\theta_0$$

为$\frac{dz}{dx}$的傅里叶余弦级数展开式。

$$\alpha-A_0=\frac{1}{\pi}\int_0^\pi\frac{dz}{dx}d\theta_0$$

$$\boxed{f(\theta)=B_0+\sum_{n=1}^{\infty}B_n cosn\theta_0 \qquad B_0=\frac{1}{\pi}\int_0^\pi f(\theta)d\theta,\ B_n=\frac{2}{\pi}\int_0^\pi f(\theta)cosn\theta d\theta}$$

$$\boxed{A_0=\alpha-\frac{1}{\pi}\int_0^\pi\frac{dz}{dx}d\theta_0} \quad \boxed{A_n=\frac{2}{\pi}\int_0^\pi\frac{dz}{dx}cosn\theta_0 d\theta_0}$$

9.8有弯度翼型（4.7）

$$\boxed{\frac{1}{2\pi}\int_0^c \frac{\gamma(\xi)\mathrm{d}\xi}{2\pi(x-\xi)} = V_\infty\left(\alpha - \frac{dz}{dx}\right)}$$

有弯度翼型，$\frac{dz}{dx} \neq 0$

$$\frac{1}{2\pi}\int_0^c \frac{\gamma(\xi)\mathrm{d}\xi}{2\pi(x-\xi)} = V_\infty\left(\alpha - \frac{dz}{dx}\right)$$

$$\boxed{\frac{dz}{dx}=0,\quad \gamma(\theta) = 2V_\infty\alpha\frac{1+co}{sin\theta}}$$

$$\frac{1}{2\pi}\int_0^\pi \frac{\gamma(\theta)sin\theta d}{cos\theta - cos\theta_0} = V_\infty\left(\alpha - \frac{dz}{dx}\right)$$

$$\boxed{\gamma(\theta) = 2V_\infty A_0\frac{1+cos}{sin\theta} + 2V_\infty\sum_{n=1}^{\infty} A_n sinn\theta}$$

$$\boxed{A_0 = \alpha - \frac{1}{\pi}\int_0^\pi \frac{dz}{dx}d\theta_0} \quad \boxed{A_n = \frac{2}{\pi}\int_0^\pi \frac{dz}{dx}cosn\theta_0 d\theta_0}$$

A_0取决于α和$\frac{dz}{dx}$(弯度)，A_n取决于$\frac{dz}{dx}$。

$$\alpha和\frac{dz}{dx} \rightarrow A_0, A_n \rightarrow \gamma(\theta) \rightarrow \Gamma \rightarrow C_l, C_m$$

9.8有弯度翼型（4.7）

$$\boxed{\frac{1}{2\pi}\int_0^c \frac{\gamma(\xi)d\xi}{2\pi(x-\xi)} = V_\infty\left(\alpha - \frac{dz}{dx}\right)}$$

有弯度翼型，$\frac{dz}{dx} \neq 0$ $\quad \frac{1}{2\pi}\int_0^\pi \frac{\gamma(\theta)sin\theta d\theta}{cos\theta - cos\theta_0} = V_\infty\left(\alpha - \frac{dz}{dx}\right)$

$$\boxed{\gamma(\theta) = 2V_\infty A_0 \frac{1+cos}{sin\theta} + 2V_\infty \sum_{n=1}^{\infty} A_n sinn\theta}$$

$$\boxed{A_0 = \alpha - \frac{1}{\pi}\int_0^\pi \frac{dz}{dx} d\theta_0}$$

$$\boxed{A_n = \frac{2}{\pi}\int_0^\pi \frac{dz}{dx} cosn\theta_0 d\theta_0}$$

α和$\frac{dz}{dx} \rightarrow A_0, A_n \rightarrow \gamma(\theta) \rightarrow \Gamma \rightarrow C_l, C_m$

$$\Gamma = \int_0^c \gamma(\xi)d\xi = \frac{c}{2}\int_0^\pi \gamma(\theta)sin\theta d\theta$$

$$= cV_\infty[A_0 \underbrace{\int_0^\pi (1 + cos\theta)\, d\theta}_{=\pi} + \sum_{n=1}^{\infty} A_n \underbrace{\int_0^\pi sinn\theta sin\theta d\theta}_{=\begin{cases}\frac{\pi}{2} & n = 1\\ 0 & n \neq 1\end{cases}}$$

$$= cV_\infty(\pi A_0 + \frac{\pi}{2}A_1)$$

单位展长升力：$L' = \rho V_\infty \Gamma = \rho V^2{}_\infty c(\pi A_0 + \frac{\pi}{2}A_1)$

升力系数：$C_l = \frac{L'}{\frac{1}{2}\rho V^2{}_\infty c} = \pi(2A_0 + A_1) = 2\pi[\alpha + \frac{1}{\pi}\int_0^\pi \frac{dz}{dx}(cos\theta_0 - 1)d\theta_0]$

9.8有弯度翼型（4.7）

$$\frac{1}{2\pi}\int_0^c \frac{\gamma(\xi)d\xi}{2\pi(x-\xi)} = V_\infty\left(\alpha - \frac{dz}{dx}\right)$$

有弯度翼型， $\frac{dz}{dx} \neq 0$ $\quad \frac{1}{2\pi}\int_0^\pi \frac{\gamma(\theta)sin\theta d\theta}{cos\theta - cos\theta_0} = V_\infty\left(\alpha - \frac{dz}{dx}\right)$

$$\gamma(\theta) = 2V_\infty A_0 \frac{1+cos}{sin\theta} + 2V_\infty \sum_{n=1}^{\infty} A_n sinn\theta$$

$$A_0 = \alpha - \frac{1}{\pi}\int_0^\pi \frac{dz}{dx} d\theta_0$$

$$A_n = \frac{2}{\pi}\int_0^\pi \frac{dz}{dx} cosn\theta_0 d\theta_0$$

α和$\frac{dz}{dx} \rightarrow A_0, A_n \rightarrow \gamma(\theta) \rightarrow \Gamma \rightarrow C_l, C_m$

升力系数： $C_l = \frac{L'}{\frac{1}{2}\rho V^2_\infty c} = \pi(2A_0 + A_1) = 2\pi[\alpha + \frac{1}{\pi}\int_0^\pi \frac{dz}{dx}(cos\theta_0 - 1)d\theta_0]$

$$a_0 = \frac{dC_l}{d\alpha} = 2\pi$$

$$\alpha_{L=0} = -\frac{1}{\pi}\int_0^\pi \frac{dz}{dx}(cos\theta_0 - 1)d\theta_0$$

零升迎角由弯度决定

9.8有弯度翼型（4.7）

$$\boxed{\frac{1}{2\pi}\int_0^c \frac{\gamma(\xi)\mathrm{d}\xi}{2\pi(x-\xi)} = V_\infty\left(\alpha-\frac{dz}{dx}\right)}$$

有弯度翼型，$\frac{dz}{dx} \neq 0$ $\quad \frac{1}{2\pi}\int_0^\pi \frac{\gamma(\theta)sin\theta d\theta}{cos\theta - cos\theta_0} = V_\infty\left(\alpha-\frac{dz}{dx}\right)$

$$\boxed{\gamma(\theta) = 2V_\infty A_0\frac{1+co}{sin\theta} + 2V_\infty\sum_{n=1}^{\infty} A_n sinn\theta}$$

$$\boxed{A_0 = \alpha - \frac{1}{\pi}\int_0^\pi \frac{dz}{dx}d\theta_0}$$

$$\boxed{A_n = \frac{2}{\pi}\int_0^\pi \frac{dz}{dx}cosn\theta_0 d\theta_0}$$

α和$\frac{dz}{dx} \rightarrow A_0, A_n \rightarrow \gamma(\theta) \rightarrow \Gamma \rightarrow C_l, C_m$

$$\boxed{C_l = \pi(2A_0 + A_1)}$$

$$C_{m,\,LE} = \frac{M'_{LE}}{\frac{1}{2}\rho V^2{}_\infty C^2} = \frac{-\rho V_\infty\int_0^c \xi\gamma(\xi)\mathrm{d}\xi}{\frac{1}{2}\rho V^2{}_\infty C^2} = -\frac{\pi}{2}\left(A_0 + A_1 - \frac{A_2}{2}\right) \boxed{= -\left[\frac{C_l}{4} + \frac{\pi}{4}(A_1 - A_2)\right]}$$

$$\boxed{C_{m,\,c/4} = C_{m,\,LE} + \frac{C_l}{4} = \frac{\pi}{4}(A_2 - A_1)}$$

$c/4$为气动中心，不是压力中心！

有弯度翼型，$C_{m,\,c/4}$ = 常数 $\neq 0$，不随α 变化！

$$\boxed{\overline{x_{cp}} = \frac{-C_{m,LE}}{C_l} = \frac{1}{4}\left[1 + \frac{\pi}{C_l}(A_1 - A_2)\right]}$$

9.8有弯度翼型（4.7）

例：NACA23012翼型，中弧线方程：$\begin{cases} \frac{z}{c} = 2.6595[(\frac{x}{c})^3 - 0.6075(\frac{x}{c})^2 + 0.1147(\frac{x}{c})] & 0 \le \frac{x}{c} \le 0.2025 \\ \frac{z}{c} = 0.02208\ (1 - \frac{x}{c}) & 0.2025 \le \frac{x}{c} \le 1 \end{cases}$

求: (a)$\alpha_{L=0}$，(b)$C_l@\alpha=4°$，(c)$C_{m,c/4}$，(d)$\overline{x_{cp}}@\alpha=4°$

$$\boxed{\alpha_{L=0} = -\frac{1}{\pi}\int_0^{\pi}\frac{dz}{dx}(cos\theta - 1)d\theta}$$

$$\boxed{x = \frac{c}{2}(1 - cos\theta)}$$

解：(a) $\begin{cases} \frac{dz}{dx} = 2.6595[3(\frac{x}{c})^2 - 1.215(\frac{x}{c}) + 0.1147] & 0 \le \frac{x}{c} \le 0.2025 \\ \frac{dz}{dx} = -0.02208 & 0.2025 \le \frac{x}{c} \le 1 \end{cases}$

$\Rightarrow$ $\begin{cases} \frac{dz}{dx} = 0.6840 - 2.3736\cos\theta + 1.995\cos^2\theta & 0 \le \theta \le 0.9335 \\ \frac{dz}{dx} = -0.02208 & 0.9335 \le \frac{x}{c} \le 1 \end{cases}$

$$\boxed{\alpha_{L=0} = -\frac{1}{\pi}\int_0^{\pi}\frac{dz}{dx}(cos\theta - 1)d\theta}$$

$$\alpha_{L=0} = -\frac{1}{\pi}\int_0^{\pi}\frac{dz}{dx}(cos\theta - 1)d\theta$$

$$= -\frac{1}{\pi}\int_0^{0.9335}(0.6840 - 2.3736\cos\theta + 1.995\cos^2\theta)(cos\theta - 1)d\theta$$

$$-\frac{1}{\pi}\int_{0.9335}^{\pi}(0.02208 - 0.02208cos\theta)d\theta$$

$$= -0.0191 \quad = -1.09°$$

$\int cos\theta d\theta = sin\theta$，

$\int cos^2\theta d\theta = \frac{1}{2}sin\theta cos\theta + \frac{1}{2}\theta$

$\int cos^3\theta d\theta = \frac{1}{3}sin\theta(cos^2\theta + 2)$

9.8有弯度翼型（4.7）

例：NACA23012翼型，中弧线方程：

$$\begin{cases} \frac{z}{c} = 2.6595[(\frac{x}{c})^3 - 0.6075(\frac{x}{c})^2 + 0.1147(\frac{x}{c})] & 0 \le \frac{x}{c} \le 0.2025 \\ \frac{z}{c} = 0.02208\,(1 - \frac{x}{c}) & 0.2025 \le \frac{x}{c} \le 1 \end{cases}$$

求: (a)$\alpha_{L=0}$，(b)C_l@α=4°，(c)$C_{m,\,c/4}$，(d)$\overline{x_{cp}}$@α=4°

解：(b) $C_l = a_0(\alpha - \alpha_{L=0})$ $\quad \alpha = 4° = \frac{4}{180}\pi = 0.0698\text{rad} \quad \alpha_{L=0} = -0.0191$

$$= 2\pi(0.0698 + 0.0191)$$

$$= 0.559$$

(c) $C_{m,\,c/4} = \frac{\pi}{4}(A_2 - A_1)$ $\quad A_1 = \frac{2}{\pi}\int_0^{\pi}\frac{dz}{dx}\cos\theta_0 d\theta_0 = 0.0954$ $\quad \boxed{A_n = \frac{2}{\pi}\int_0^{\pi}\frac{dz}{dx}\cos n\theta_0 d\theta_0}$

$= -0.0127$ $\quad A_2 = \frac{2}{\pi}\int_0^{\pi}\frac{dz}{dx}\cos 2\theta_0 d\theta_0 = 0.0794$

(c) $\overline{x_{cp}} = \frac{1}{4}[1 + \frac{\pi}{C_l}(A_1 - A_2)]$

$$= \frac{1}{4}[1 + \frac{\pi}{0.559}(0.0954 - 0.0794)\,]$$

$$= 0.273$$

作业：

复习笔记！

空气动力学书4.2，4.3，4.4