空气与气体动力学

张科

7.17 利用一个 1/10 的货车模型在风洞内作测试，货车的迎风面积 $A_m = 0.1\ m^2$，当风速 $V_m = 75$ m/s 时测得气动阻力 $F_D = 350$ N。计算实验条件下的阻力因数；如果假设模型与原型货车的阻力因数相同，试推算原型货车在高速公路上以 90 km/h 的速度行驶时的气动阻力。为保证动力相似，与 90 km/h 速度相应的风洞内模型实验速度应为多少？这样的速度是否合适，为什么？

1.动力相似？
不完全相似：
Re相同，Ma<0.3
Ma相同，Ma>0.3
完全相似：Re, Ma均相等
2.速度是否使Ma>0.3？

解：$C_D = \dfrac{F_D}{\frac{1}{2}\rho V_m^2 A_m} = \dfrac{350}{\frac{1}{2}\times 1.2\times 75^2\times 0.1} = 1.037$

$F_{D,p} = C_D \cdot \frac{1}{2}\rho V_p^2 \cdot A_p = 1.037\times\frac{1}{2}\times 1.2\times(90\times 10^3/3600)^2\times 10 = 3888.75(N)$

V_p：动力相似：$Re_m = Re_p$

$$\frac{\rho L_m V_m}{\mu} = \frac{\rho L_p V_p}{\mu}$$

$$V_m = V_p \cdot \frac{L_p}{L_m} = 90\times 10^3/3600 \times 10 = 250(m/s)$$

不合适，$Ma = \dfrac{V_m}{a} = \dfrac{250}{350} > 0.3$ 变为可压缩

7.22 ...

7.22 一船体长 200 m，航行速度为 25 km/h。若用船模以 2.5 m/s 的速度在水池中拖动，试确定两种流动的弗劳德数和模型的长度。

7.22. 解：$F_r=\frac{U}{\sqrt{gL}}=\frac{25\times10^3/3600}{\sqrt{9.8\times200}}=0.155$

$$\frac{U_m}{\sqrt{gL_m}}=\frac{U_p}{\sqrt{gL_p}}$$

$$L_m=\left(\frac{U_m}{U_p}\right)^2L_p=\cancel{\left(\frac{25\times10^3/3600}{2.5}\right)^2\times200}$$

$$=\left(\frac{2.5}{25\times10^3/3600}\right)^2\times200$$

$$=25.94\,m$$

计算错误

补充：$\rho\frac{D\vec{V}}{Dt}=-\vec{\nabla}P-\rho g\vec{\nabla}z+\mu\nabla^2\vec{V}$

无量纲化：

解：设特征长度 L_0，速度 U_0，特征压力 $P_0=\rho U_0^2$

时间 $t_0=\frac{L_0}{U_0}$

无量纲变量：$x^*=\frac{x}{L_0}$，$y^*=\frac{y}{L_0}$，$z^*=\frac{z}{L_0}$

$\vec{V}^*=\frac{\vec{V}}{U_0}$，$P^*=\frac{P}{\rho U_0^2}$，$t^*=\frac{tU_0}{L_0}$

$\vec{\nabla}^*=L_0\vec{\nabla}$　$\nabla^{*2}=L_0^2\nabla^2$

$$\vec{\nabla}P=\frac{\vec{\nabla}^*}{L_0}\cdot P^*(\rho U_0^2)=\vec{\nabla}^*P^*\left(\frac{\rho U_0^2}{L_0}\right)$$

$$\rho g\vec{\nabla}z=\rho g\frac{\vec{\nabla}^*}{L_0}(z^*L_0)=\rho g\vec{\nabla}^*z^*$$

$$\rho\frac{D\vec{V}}{Dt}=\rho\left[\frac{\partial\vec{V}}{\partial t}+(\vec{V}\cdot\vec{\nabla})\vec{V}\right]$$

$$=\rho\left[\frac{\partial\vec{V}^*}{\partial t^*}\cdot\frac{U_0}{L_0/U_0}+\left(\vec{V}^*\cdot\nabla^*\frac{U_0}{L_0}\right)\cdot(\vec{V}^*U_0)\right]$$

$$=\frac{\rho U_0^2}{L_0}\left[\frac{\partial\vec{V}^*}{\partial t^*}+(\vec{V}^*\cdot\nabla^*)\vec{V}^*\right]$$

$$\mu\nabla^2\vec{V}=\mu\frac{\nabla^{*2}}{L_0^2}\cdot\vec{V}^*\cdot U_0=\frac{\mu U_0}{L_0^2}\nabla^{*2}\vec{V}^*$$

$\Rightarrow$ N-S方程变为：

$$\frac{\rho U_0^2}{L_0}\left[\frac{D\vec{V}^*}{Dt^*}\right]=-\vec{\nabla}^*P^*\left(\frac{\rho U_0^2}{L_0}\right)-\rho g\vec{\nabla}^*z^*+\frac{\mu U_0}{L_0^2}\nabla^{*2}\vec{V}^*$$

$$\boxed{\frac{D\vec{V}^*}{Dt^*}=-\vec{\nabla}^*P^*-\frac{gL_0}{U_0^2}\vec{\nabla}^*z^*+\frac{\mu}{\rho U_0L_0}\nabla^{*2}\vec{V}^*}$$

P5.61 If viscosity is neglected, typical pump-flow results from Prob. 5.20 are shown in Fig. P5.61 for a model pump tested in water. The pressure rise decreases and the power required increases with the dimensionless flow coefficient. Curve-fit expressions are given for the data. Suppose a similar pump of 12-cm diameter is built to move gasoline at 20°C and a flow rate of 25 m^3/h. If the pump rotation speed is 30 r/s, find (*a*) the pressure rise and (*b*) the power required.

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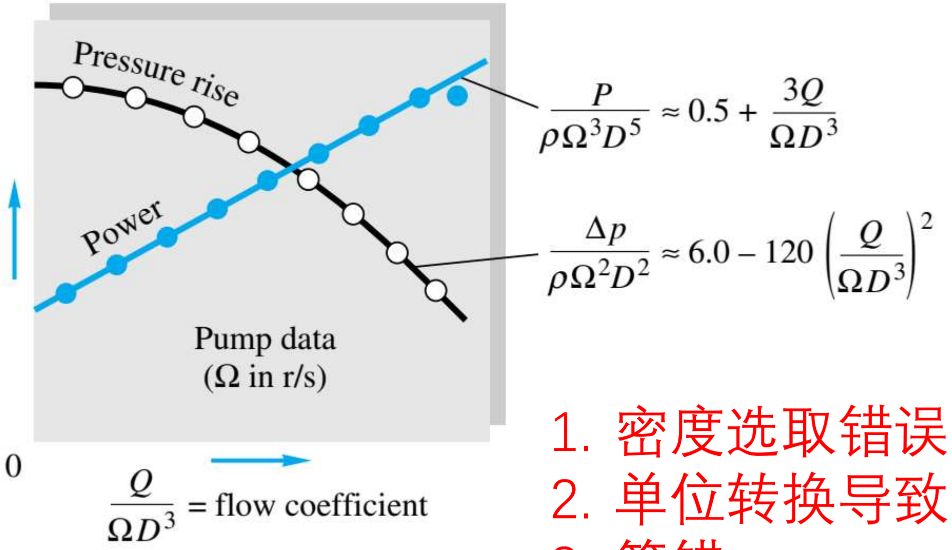


P5.61

1. 密度选取错误
2. 单位转换导致混乱
3. 算错

Solution: For gasoline at 20°C, take $\rho \approx 680$ kg/m^3 and $\mu \approx$ 2.92E–4 kg/m·s. Convert Q = 25 m^3/hr = 0.00694 m^3/s. Then we can evaluate the "flow coefficient":

$$\frac{Q}{\Omega D^3} = \frac{0.00694}{(30)(0.12)^3} \approx 0.134, \quad \text{whence} \quad \frac{\Delta p}{\rho\Omega^2 D^2} \approx 6 - 120(0.134)^2 \approx 3.85$$

$$\text{and} \quad \frac{P}{\rho\Omega^3 D^5} \approx 0.5 + 3(0.134) \approx 0.902$$

With the dimensionless pressure rise and dimensionless power known, we thus find

$$\Delta p = (3.85)(680)(30)^2(0.12)^2 \approx \mathbf{34000\ Pa} \quad \textit{Ans. (a)}$$

$$P = (0.902)(680)(30)^3(0.12)^5 \approx \mathbf{410\ W} \quad \textit{Ans. (b)}$$

表 B3 标准大气压下常见液体的物理性质(20℃)

液体	密度 ρ/(kg/m^3)	动力粘度 μ/[kg/(m·s)]	表面张力 σ/(N/m)*	蒸气压强 p_v/(N/m^2)	体积弹性模量 E_v/(N/m^2)
氨	608	2.20 E-4	2.13 E-2	9.10 E+5	—
苯	881	6.51 E-4	2.88 E-2	1.01 E+4	1.4 E+9
四氯化碳	1 590	9.67 E-4	2.70 E-2	1.20 E+4	9.65 E+8
酒精	789	1.20 E-3	2.28 E-2	5.7 E+3	9.0 E+8
乙二醇	1 117	2.14 E-2	4.84 E-2	1.2 E+1	—
氟利昂 12	1 327	2.62 E-4	—	—	—
汽油	680	2.92 E-4	2.16 E-2	5.51 E+4	9.58 E+8
甘油	1 260	1.49	6.33 E-2	1.4 E-2	4.34 E+9
煤油	804	1.92 E-3	2.8 E-2	3.11 E+3	1.6 E+9
水银	13,550	1.56 E-3	4.84 E-1	1.1 E-3	2.55 E+10
甲醇	791	5.98 E-4	2.25 E-2	1.34 E+4	8.3 E+8
SAE 10W 油	870	1.04 E-1	3.6 E-2	—	1.31 E+9
SAE 10W30 油	876	1.7 E-1	—	—	—
SAE 30W 油	891	2.9 E-1	3.5 E-2	—	1.38 E+9
SAE 50W 油	902	8.6 E-1	—	—	—
水	998	1.00 E-3	7.28 E-2	2.34 E+3	2.19 E+9
海水(30%)	1 025	1.07 E-3	7.28 E-2	2.34 E+3	2.33 E+9

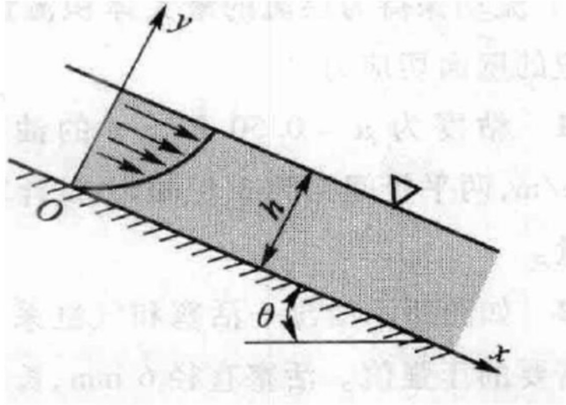
5.2 试分别确定水($\nu = 1.13 \times 10^{-6}\ m^2/s$)和重质柴油($\nu = 205 \times 10^{-6}\ m^2/s$)以 1.067 m/s 的速度在直径为 305 mm 的管道中流动时的流动状态。

$$Re = \frac{\rho \bar{V} D}{\mu}$$

5.4 已知一内径为 10 mm 圆管内流动雷诺数 $Re = 1\ 500$,试求进口段长度。

层流：$L_e/D \approx 0.06Re$

5.14 考虑图示具有自由面的薄层粘性流体的定常层流：(1) 试证明该流动的速度分布 $u=\dfrac{\rho g}{\mu}y\left(h-\dfrac{y}{2}\right)\sin\theta$；(2) 计算通过单位宽度的流量（垂直于纸面为单位宽度）；(3) 试求在自由面下具有与平均流速相同速度点之深度。



5.14. 由 $\nabla\cdot\vec{V}=0$ 且 $v=w=0$ $\Rightarrow$ $\dfrac{\partial u}{\partial x}=0$ $\Rightarrow$ $u=u(y)$

由 N-S方程 $\rho\dfrac{D\vec{V}}{Dt}=-\nabla p-\rho g\nabla z+\mu\nabla^2\vec{V}$

$$\therefore\begin{cases}0=\mu\dfrac{d^2u}{dy^2}-\dfrac{\partial p}{\partial x}+\rho g\sin\theta\\[2mm] 0=-\dfrac{\partial p}{\partial y}-\rho g\cos\theta\\[2mm] 0=-\dfrac{\partial p}{\partial z}\end{cases}\qquad\Rightarrow\ p=-\rho g\cos\theta\cdot y+f(x)$$

由 $y=h$ 时 $p=p_0$

$\therefore f(x)=p_0+\rho g\cos\theta\cdot h$ $\Rightarrow$ $\dfrac{\partial p}{\partial x}=0$ $\quad\because\begin{cases}y=0,\ u=0\\ y=h,\ \dfrac{du}{dy}=0\end{cases}$

$\therefore u=-\dfrac{\rho g}{2\mu}\sin\theta\cdot y^2+C_1y+C_2$ $\Rightarrow\begin{cases}C_2=0\\ C_1=\dfrac{\rho g\sin\theta}{\mu}\cdot h\end{cases}$

故 $u=\dfrac{\rho g}{\mu}y\left(h-\dfrac{y}{2}\right)\sin\theta$，命题得证

(2) $Q=\int_0^h u\,dy=\int_0^h\dfrac{\rho g}{\mu}\left(h-\dfrac{y}{2}\right)\sin\theta\cdot y\,dx=\dfrac{\rho g\sin\theta}{3\mu}\cdot h^3$

(3) 平均流速 $\vec{V}=\dfrac{Q}{A}=\dfrac{Q}{h}=\dfrac{\rho g\sin\theta}{3\mu}h^2$，令 $\vec{V}=u$ $\Rightarrow$ $y=\dfrac{3-\sqrt{3}}{3}h$.

1. 压力、重力项混淆不清
2. 压力沿x方向变化来历不明

回顾：

1.管内能量损失： $h_{LT}=h_L+h_m$　$Losses=-\int_{CS}\left(\frac{V^2}{2}+gz+\frac{p}{\rho}\right)\rho(\vec{V}\cdot\vec{n})dS$

沿程损失： $h_L=f\left(\frac{L}{D}\right)\frac{\overline{V}^2}{2g}$　层流：$f=\frac{64}{Re}$ 湍流：$f=f\left(Re,\ \frac{e}{D}\right)$ 穆迪图(5.28)

局部损失： $h_m=K\frac{\overline{V}^2}{2g}$

通道面积

2.非圆管： 当量直径$D_h=\frac{4A}{P_r}$

润湿周长

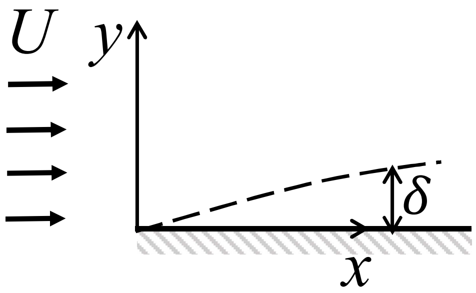
3.边界层概念：

4.边界层参数： $\delta=y\big|_{u=0.99U}$　$\delta^*=\int_0^\infty\left[1-\frac{u(y)}{U}\right]dy$　$\theta=\int_0^\delta\frac{u(y)}{U}\left(1-\frac{u(y)}{U}\right)dy$

5.平板边界层动量积分方程： $\tau_w=\rho U^2\frac{d\theta}{dx}$

6.7边界层方程（10.2，10.3）

2D，不可压，定常，不计重力。



$$\begin{cases}\dfrac{\partial u}{\partial x}+\dfrac{\partial v}{\partial y}=0\\ \rho\left(u\dfrac{\partial u}{\partial x}+v\dfrac{\partial u}{\partial y}\right)=-\dfrac{\partial p}{\partial x}+\mu\left(\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial y^2}\right)\\ \rho\left(u\dfrac{\partial v}{\partial x}+v\dfrac{\partial v}{\partial y}\right)=-\dfrac{\partial p}{\partial y}+\mu\left(\dfrac{\partial^2 v}{\partial x^2}+\dfrac{\partial^2 v}{\partial y^2}\right)\end{cases}$$

无量纲方程：

$$\boxed{\begin{aligned}&\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0\\ &u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\frac{1}{Re}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)\\ &u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial p}{\partial y}+\frac{1}{Re}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)\end{aligned}}$$

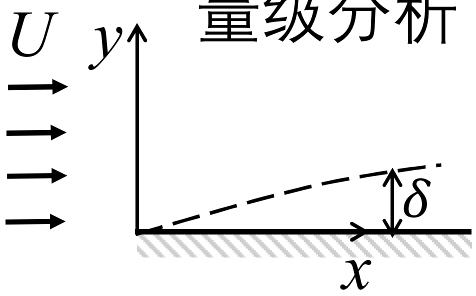
边界层假设(Prandtl 1904)：如果Re很大($Re\gg 1$)，则δ很小($\delta\ll 1$）。

量级分析：$u\sim o(1)$, $x\sim o(1)$, $y\sim o(\delta)$, $\dfrac{\partial u}{\partial x}\sim\dfrac{o(1)}{o(1)}\sim o(1)$, $\dfrac{\partial p}{\partial x}\sim\dfrac{o(1)}{o(1)}\sim o(1)$

$$\left.\begin{aligned}&\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0\\ &\frac{\partial u}{\partial x}\sim o(1)\end{aligned}\right\}\Rightarrow \left.\begin{aligned}&\frac{\partial v}{\partial y}\sim o(1)\\ &y\sim o(\delta)\end{aligned}\right\}\Rightarrow v\sim o(\delta)\Rightarrow {\color{red}v\ll u}$$

6.7边界层方程（10.2，10.3）

2D，不可压，定常，不计重力。



量级分析：$u \sim o(1)$, $x \sim o(1)$, $y \sim o(\delta)$,

$$\frac{\partial u}{\partial x} \sim \frac{o(1)}{o(1)} \sim o(1),\quad \frac{\partial p}{\partial x} \sim \frac{o(1)}{o(1)} \sim o(1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{1}{Re}\left(\cancel{\frac{\partial^2 u}{\partial x^2}} + \frac{\partial^2 u}{\partial y^2}\right)$$

量级：$o(1)\frac{o(1)}{o(1)} + o(\delta)\frac{o(1)}{o(\delta)} = \frac{o(1)}{o(1)} + \boxed{\frac{1}{Re}\left(\underbrace{\cancel{\frac{o(1)}{o(1)o(1)}}}_{o(1)} + \underbrace{\boxed{\frac{o(1)}{o(\delta)o(\delta)}}}_{o(\frac{1}{\delta^2})}\right)}$ $\quad \frac{o(1)}{o(1)o(1)} \ll \frac{o(1)}{o(\delta)o(\delta)}$ $\quad \frac{\partial^2 u}{\partial x^2} \ll \frac{\partial^2 u}{\partial y^2}$

$\Rightarrow \frac{1}{Re} \sim o(\delta^2) \Rightarrow \frac{\delta}{x} \sim \frac{1}{\sqrt{Re}}$

$Re \sim o\left(\frac{1}{\delta^2}\right)$

无量纲方程：

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad v \ll u$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{1}{Re}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \frac{1}{Re}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

6.7边界层方程（10.2，10.3）

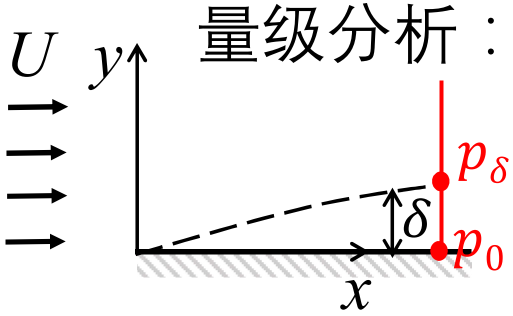
2D，不可压，定常，不计重力。

无量纲方程：

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad v \ll u$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\frac{1}{Re}\left(\cancel{\frac{\partial^2 u}{\partial x^2}}+\frac{\partial^2 u}{\partial y^2}\right)$$

$$\cancel{u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial p}{\partial y}+\frac{1}{Re}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)}$$



量级分析：$u\sim o(1)$, $x\sim o(1)$, $y\sim o(\delta)$,

$$\frac{\partial u}{\partial x}\sim\frac{o(1)}{o(1)}\sim o(1),\quad \frac{\partial p}{\partial x}\sim\frac{o(1)}{o(1)}\sim o(1)$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial p}{\partial y}+\frac{1}{Re}\left(\cancel{\frac{\partial^2 v}{\partial x^2}}+\frac{\partial^2 v}{\partial y^2}\right) \qquad \frac{1}{Re}\sim o(\delta^2)$$

量级：$o(1)\frac{o(\delta)}{o(1)}+o(\delta)\frac{o(\delta)}{o(\delta)}=o(\delta)+o(\delta^2)\left(\underbrace{\boxed{\cancel{\frac{o(\delta)}{o(1)o(1)}}}}_{o(\delta)}+\underbrace{\boxed{\frac{o(\delta)}{o(\delta)o(\delta)}}}_{o(\frac{1}{\delta})}\right)$ $\qquad \frac{\partial^2 v}{\partial x^2}\ll\frac{\partial^2 v}{\partial y^2}$

$$\Rightarrow \frac{\partial p}{\partial y}\sim o(\delta) << \frac{\partial p}{\partial x}$$

$$\frac{\partial p}{\partial y}\approx 0 \qquad p_0=p_\delta \qquad p=p(x)\ only\ for\ \delta\ll 1$$

6.7边界层方程（10.2，10.3）

无量纲方程：

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad v \ll u$$

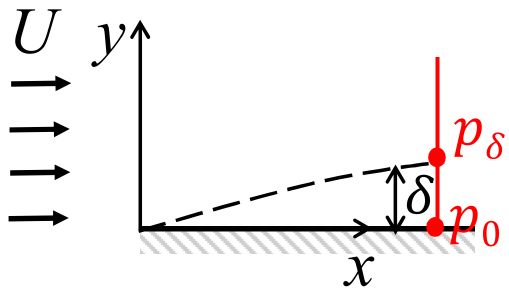
$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\frac{1}{Re}\left(\cancel{\frac{\partial^2 u}{\partial x^2}}+\frac{\partial^2 u}{\partial y^2}\right)$$

$$\cancel{u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial p}{\partial y}+\frac{1}{Re}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)}$$

① 边界层方程(Prandtl's B.L. equation):

$$\begin{cases}\dfrac{\partial u}{\partial x}+\dfrac{\partial v}{\partial y}=0\\[2ex] u\dfrac{\partial u}{\partial x}+v\dfrac{\partial u}{\partial y}=-\dfrac{1}{\rho}\boxed{\dfrac{dp}{dx}}+\nu\dfrac{\partial^2 u}{\partial y^2}\end{cases}$$

由主流区$\frac{dp}{dx}$决定



$$\frac{\partial p}{\partial y}=0$$

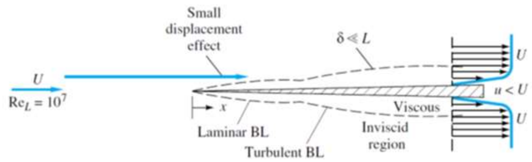
$B.C.:\quad @y=0, u=v=0$

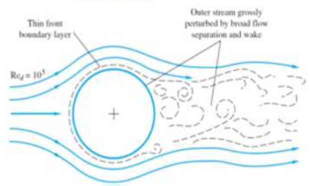
$\qquad @y\to\infty, u=U(x)\quad U(x)$: 主流区流动

主流区无粘 :$U\dfrac{dU}{dx}=-\dfrac{1}{\rho}\dfrac{dp}{dx}$ (欧拉方程) ⇨

$$\begin{cases}\dfrac{\partial u}{\partial x}+\dfrac{\partial v}{\partial y}=0\\[2ex] u\dfrac{\partial u}{\partial x}+v\dfrac{\partial u}{\partial y}=U\dfrac{dU}{dx}+\nu\dfrac{\partial^2 u}{\partial y^2}\end{cases}$$

6.7边界层方程（10.2，10.3）





① 边界层方程(Prandtl’s B.L. equation):

$$\begin{cases} \dfrac{\partial u}{\partial x}+\dfrac{\partial v}{\partial y}=0 \\ u\dfrac{\partial u}{\partial x}+v\dfrac{\partial u}{\partial y}=U\dfrac{dU}{dx}+\nu\dfrac{\partial^2 u}{\partial y^2} \end{cases}$$

抛物型方程： 解从前缘向下游推进。

$$\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{dp}{dx}+\nu\frac{\partial^2 u}{\partial y^2}\right)$$

$$B.C.:\quad @y=0, u=v=0$$

$$@y\to\infty, u=U(x)\quad \frac{\partial^n u}{\partial yn}=0$$

壁面处$\dfrac{\partial^2 u}{\partial y^2}$由主流$\dfrac{dU}{dx}$决定,

而主流 $\dfrac{dU}{dx}$ 由壁面形状决定！

壁面形状对速度分布影响：

$$@y=0, u=v=0 \Rightarrow \cancel{u\frac{\partial u}{\partial x}}+\cancel{v\frac{\partial u}{\partial y}}=-\frac{1}{\rho}\frac{dp}{dx}+\nu\frac{\partial^2 u}{\partial y^2}$$

$$\Rightarrow \boxed{\frac{\partial^2 u}{\partial y^2}}=\frac{1}{\mu}\frac{dp}{dx}=\boxed{-\frac{U}{\nu}\frac{dU}{dx}}$$

顺流平板：

$\dfrac{dU}{dx}=0$， $\dfrac{dp}{dx}=0$

$\dfrac{\partial^2 u}{\partial y^2}=0$!

6.7边界层方程（10.2，10.3）

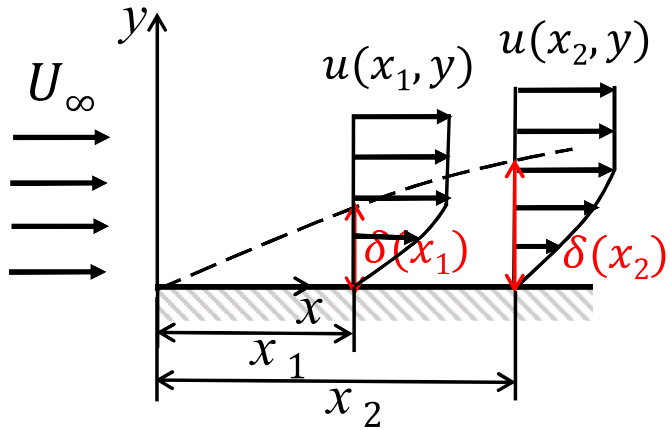
$$\begin{cases} \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \\ u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{dp}{dx}+\nu\frac{\partial^2 u}{\partial y^2} \end{cases}$$

② 平板边界层(Blasius精确解)： $U(x)=U_\infty$，$\frac{dp}{dx}=0$

$$\begin{cases} \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \\ u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2} \end{cases}$$

$B.C.:\quad @y=0, u=v=0$

$\qquad @y\to\infty, u=U_\infty$



不同x处速度分布

$u(x_1,y)$和$u(x_2,y)$有相似性！

若用$\delta(x)$缩放，

两速度分布会重合。

（相似解）！

无固定特征长度的问题，有相似解！

$\delta(x)$为系数，$\frac{\delta}{x}\sim\frac{1}{\sqrt{Re}}=\sqrt{\frac{\nu}{U_\infty x}}$

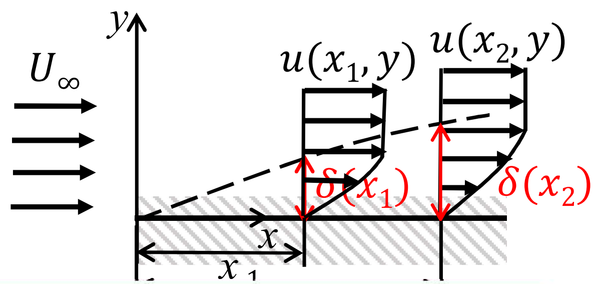
引入无量纲变量： $\eta=\frac{y}{\sqrt{\frac{\nu x}{U_\infty}}}$ $\quad \frac{u}{U_\infty}=f'(\eta)\quad P394.$

$\delta\sim\sqrt{\frac{\nu x}{U_\infty}}$

PDE→ODE：$f'''+\frac{1}{2}ff'=0$ 有数值精确解(Blasius 1908)

（y向粘性扩散的距离）

6.7边界层方程（10.2，10.3）



② 平板边界层(Blasius精确解)：

$$\begin{cases} \dfrac{\partial u}{\partial x}+\dfrac{\partial v}{\partial y}=0 \\ u\dfrac{\partial u}{\partial x}+v\dfrac{\partial u}{\partial y}=\nu\dfrac{\partial^2 u}{\partial y^2} \end{cases}$$

$$\eta=\frac{y}{\sqrt{\frac{\nu x}{U_\infty}}}$$

$y[U/(\nu x)]^{1/2}$	u/U	$y[U/(\nu x)]^{1/2}$	u/U
0.0	0.0	2.8	0.81152
0.2	0.06641	3.0	0.84605
0.4	0.13277	3.2	0.87609
0.6	0.19894	3.4	0.90177
0.8	0.26471	3.6	0.92333
1.0	0.32979	3.8	0.94112
1.2	0.39378	4.0	0.95552
1.4	0.45627	4.2	0.96696
1.6	0.51676	4.4	0.97587
1.8	0.57477	4.6	0.98269
2.0	0.62977	4.8	0.98779
2.2	0.68132	5.0	0.99155
2.4	0.72899	∞	1.00000
2.6	0.77246		

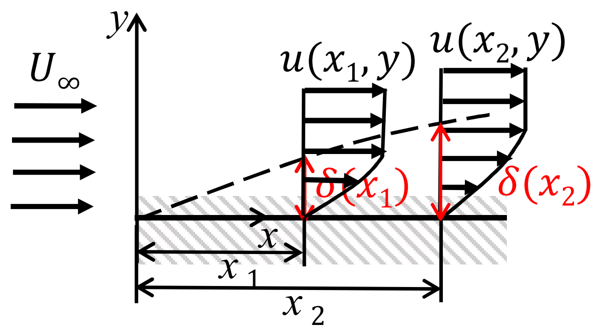
数值精确解：

➢ $\eta=5$，$\dfrac{u}{U_\infty}=0.991$

$$\eta=\frac{y}{\sqrt{\frac{\nu x}{U_\infty}}}=\frac{\delta}{\sqrt{\frac{\nu x}{U_\infty}}}=5$$

$$\frac{\delta}{x}=5\sqrt{\frac{\nu}{U_\infty x}}=\frac{5}{\sqrt{Re_x}}$$

6.7边界层方程（10.2，10.3）



② 平板边界层(Blasius精确解)：

$$\begin{cases} \dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = 0 \\ u\dfrac{\partial u}{\partial x} + v\dfrac{\partial u}{\partial y} = \nu\dfrac{\partial^2 u}{\partial y^2} \end{cases}$$

$$\eta = \frac{y}{\sqrt{\frac{\nu x}{U_\infty}}} \quad \frac{u}{U_\infty} = f'(\eta)$$

$$f''' + \frac{1}{2}ff' = 0$$

Blasius 方程（1908）

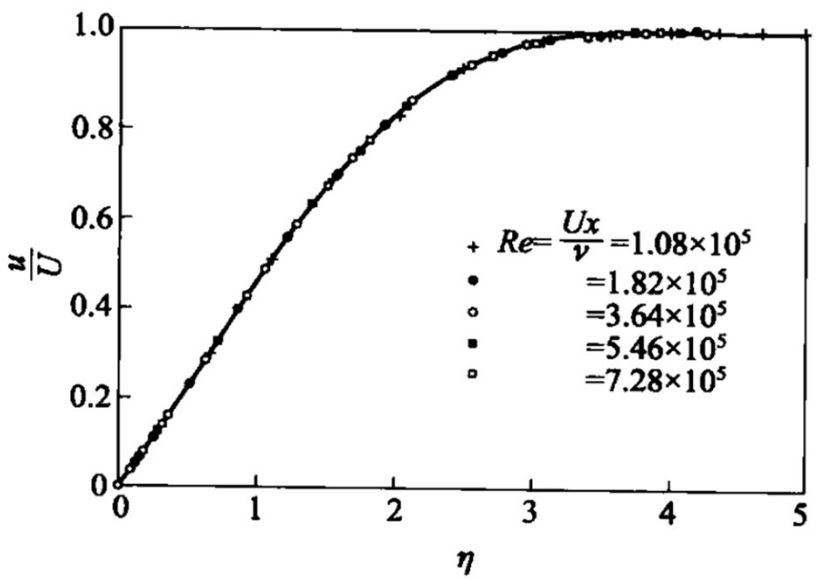
$B.C.: f(0)=0,\ \ f'(0)=0,\ f'(\infty)=1$

数值精确解：

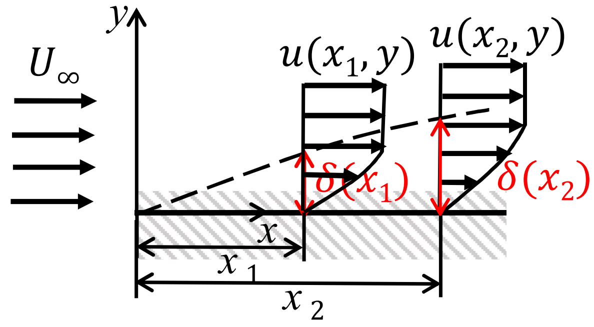
- η =5，$\dfrac{u}{U_\infty} = 0.991$

$$\eta = \frac{y}{\sqrt{\frac{\nu x}{U_\infty}}} = \frac{\delta}{\sqrt{\frac{\nu x}{U_\infty}}} = 5$$

$$\frac{\delta}{x} = 5\sqrt{\frac{\nu}{U_\infty x}} = \frac{5}{\sqrt{Re_x}}$$



6.7边界层方程（10.2，10.3）



② 平板边界层(Blasius精确解)：

$$\begin{cases} \dfrac{\partial u}{\partial x}+\dfrac{\partial v}{\partial y}=0 \\ u\dfrac{\partial u}{\partial x}+v\dfrac{\partial u}{\partial y}=\nu\dfrac{\partial^2 u}{\partial y^2} \end{cases} \xrightarrow{\eta=\frac{y}{\sqrt{\frac{\nu x}{U_\infty}}}\quad \frac{u}{U_\infty}=f'(\eta)} f'''+\frac{1}{2}ff'=0$$

Blasius 方程（1908）

数值精确解：

➢ $\eta=5$，$\dfrac{u}{U_\infty}=0.991$

$$\Rightarrow \eta=\frac{y}{\sqrt{\frac{\nu x}{U_\infty}}}=\frac{\delta}{\sqrt{\frac{\nu x}{U_\infty}}}=5$$

$$\frac{\delta}{x}=5\sqrt{\frac{\nu}{U_\infty x}}=\frac{5}{\sqrt{Re_x}}$$

➢ $$C_f=\frac{\tau_w}{\frac{1}{2}\rho U^2_\infty}=\frac{\mu\frac{\partial u}{\partial y}\big|_{y=0}}{\frac{1}{2}\rho U^2_\infty}=\frac{\mu\sqrt{\frac{U^3_\infty}{\nu x}}f''(0)}{\frac{1}{2}\rho U^2_\infty}=\frac{0.664}{\sqrt{Re_x}}$$

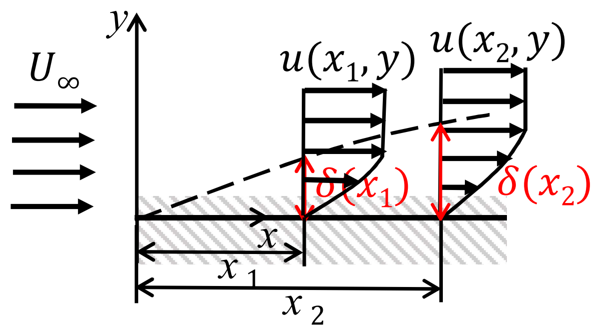
$$D=\int_0^L \tau_w\,dx$$

➢ $$C_D=\frac{D}{\frac{1}{2}\rho U^2_\infty L}=\frac{\int_0^L C_f dx}{L}=\frac{1.328}{\sqrt{Re_L}}$$

➢ $$\delta^*=\int_0^\infty \left(1-\frac{u}{U_\infty}\right)dy=\frac{1.72x}{\sqrt{Re_x}} \qquad \frac{\delta^*}{x}=\frac{1.72}{\sqrt{Re_x}}$$

➢ $$\frac{\theta}{x}=\frac{0.664}{\sqrt{Re_x}}$$

6.7边界层方程（10.2，10.3）



② 平板边界层(Blasius精确解)：

$$\begin{cases} \dfrac{\partial u}{\partial x}+\dfrac{\partial v}{\partial y}=0 \\ u\dfrac{\partial u}{\partial x}+v\dfrac{\partial u}{\partial y}=\nu\dfrac{\partial^2 u}{\partial y^2} \end{cases} \xrightarrow{\eta=\frac{y}{\sqrt{\frac{\nu x}{U_\infty}}} \quad \frac{u}{U_\infty}=f'(\eta)} f'''+\frac{1}{2}ff'=0$$

Blasius 方程（1908）

➢ $C_f=\dfrac{\tau_w}{\frac{1}{2}\rho U^2{}_\infty}=\dfrac{\mu\frac{\partial u}{\partial y}\big|_{y=0}}{\frac{1}{2}\rho U^2{}_\infty}=\dfrac{\mu\sqrt{\frac{U^3{}_\infty}{\nu x}}f''(0)}{\frac{1}{2}\rho U^2{}_\infty}=\dfrac{0.664}{\sqrt{Re_x}}$

$D=\int_0^L \tau_w\,dx$

➢ $C_D=\dfrac{D}{\frac{1}{2}\rho U^2{}_\infty L}=\dfrac{\int_0^L C_f dx}{L}=\dfrac{1.328}{\sqrt{Re_L}}$

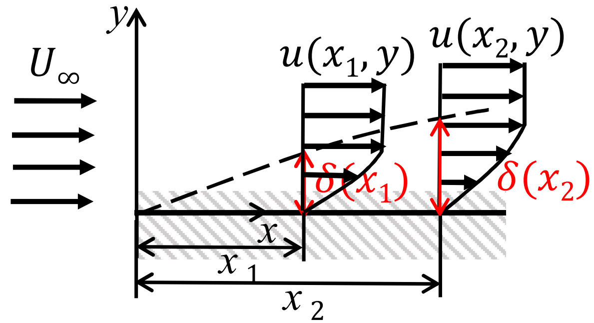
➢ $\delta^*=\int_0^\infty\left(1-\dfrac{u}{U_\infty}\right)dy=\dfrac{1.72x}{\sqrt{Re_x}}$ $\quad \dfrac{\delta^*}{x}=\dfrac{1.721}{\sqrt{Re_x}}$

➢ $\dfrac{\theta}{x}=\dfrac{0.664}{\sqrt{Re_x}}$

➢ 形状因子：

$H=\dfrac{\delta^*}{\theta}=\dfrac{1.721}{0.664}=2.59$

6.7边界层方程（10.2，10.3）



② 平板边界层(Blasius精确解)：

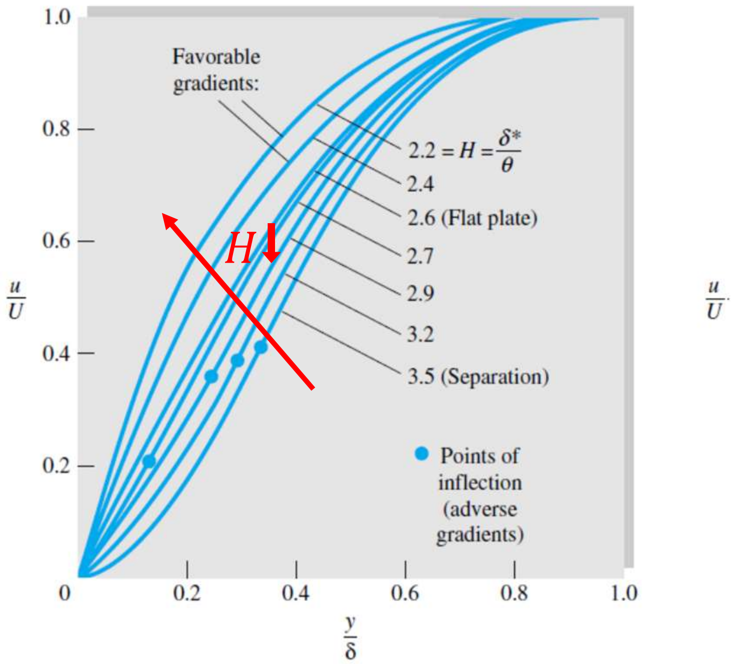
$$\begin{cases} \dfrac{\partial u}{\partial x}+\dfrac{\partial v}{\partial y}=0 \\ u\dfrac{\partial u}{\partial x}+v\dfrac{\partial u}{\partial y}=\nu\dfrac{\partial^2 u}{\partial y^2} \end{cases} \xrightarrow{\eta=\frac{y}{\sqrt{\frac{\nu x}{U_\infty}}} \quad \frac{u}{U_\infty}=f'(\eta)} f'''+\frac{1}{2}ff'=0$$

Blasius 方程（1908）

➢ 形状因子：

$$H=\frac{\delta^*}{\theta}=\frac{1.721}{0.664}=2.59$$

H⬇，速度分布更贴近壁面，更饱满



6.7边界层方程（10.2，10.3）

② 平板边界层：

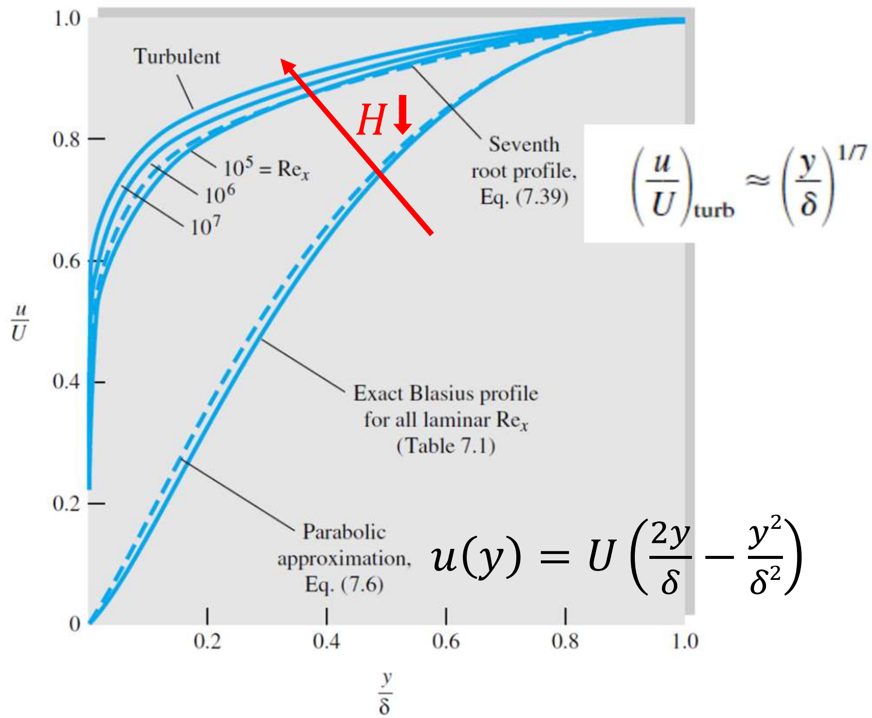
$$\begin{cases} \dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = 0 \\ u\dfrac{\partial u}{\partial x} + v\dfrac{\partial u}{\partial y} = \nu\dfrac{\partial^2 u}{\partial y^2} \end{cases}$$

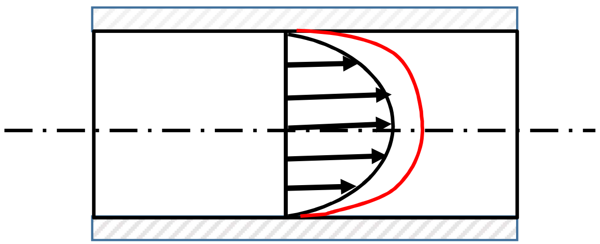
湍流：P402~P404.

$$\frac{\delta}{x} = \frac{0.16}{Re_x^{1/7}} \qquad C_f = \frac{0.027}{Re_x^{1/7}}$$

$$\frac{\delta^*}{x} = \frac{1}{8}\delta \qquad H = 1.3$$

$$C_D = \frac{0.031}{\mathrm{Re}_L^{1/7}} = \frac{7}{6}c_f(L)$$





$H\downarrow$，速度分布更贴近壁面，更饱满

作业：

复习笔记！

10.25，10.28

Secondary dimension	SI unit	BG unit	Conversion factor
Area $\{L^2\}$	m^2	ft^2	$1\ m^2 = 10.764\ ft^2$
Volume $\{L^3\}$	m^3	ft^3	$1\ m^3 = 35.315\ ft^3$
Velocity $\{LT^{-1}\}$	m/s	ft/s	1 ft/s = 0.3048 m/s
Acceleration $\{LT^{-2}\}$	m/s^2	ft/s^2	$1\ ft/s^2 = 0.3048\ m/s^2$
Pressure or stress $\{ML^{-1}T^{-2}\}$	$Pa = N/m^2$	lbf/ft^2	$1\ lbf/ft^2 = 47.88\ Pa$
Angular velocity $\{T^{-1}\}$	s^{-1}	s^{-1}	$1\ s^{-1} = 1\ s^{-1}$
Energy, heat, work $\{ML^2T^{-2}\}$	$J = N \cdot m$	$ft \cdot lbf$	$1\ ft \cdot lbf = 1.3558\ J$
Power $\{ML^2T^{-3}\}$	W = J/s	$ft \cdot lbf/s$	$1\ ft \cdot lbf/s = 1.3558\ W$
Density $\{ML^{-3}\}$	kg/m^3	$slugs/ft^3$	$1\ slug/ft^3 = 515.4\ kg/m^3$
Viscosity $\{ML^{-1}T^{-1}\}$	$kg/(m \cdot s)$	$slugs/(ft \cdot s)$	$1\ slug/(ft \cdot s) = 47.88\ kg/(m \cdot s)$
Specific heat $\{L^2T^{-2}\Theta^{-1}\}$	$m^2/(s^2 \cdot K)$	$ft^2/(s^2 \cdot °R)$	$1\ m^2/(s^2 \cdot K) = 5.980\ ft^2/(s^2 \cdot °R)$

P5.65 In turbulent flow near a flat wall, the local velocity u varies only with distance y from the wall, wall shear stress τ_w, and fluid properties ρ and μ. The following data were taken in the University of Rhode Island wind tunnel for airflow, $\rho = 0.0023\ slug/ft^3$, $\mu = 3.81$ E-7 slug/(ft · s), and $\tau_w = 0.029\ lbf/ft^2$:

y, in	0.021	0.035	0.055	0.080	0.12	0.16
u, ft/s	50.6	54.2	57.6	59.7	63.5	65.9

(*a*) Plot these data in the form of dimensionless u versus dimensionless y, and suggest a suitable power-law curve fit. (*b*) Suppose that the tunnel speed is increased until $u = 90$ ft/s at $y = 0.11$ in. Estimate the new wall shear stress, in lbf/ft^2.