空气与气体动力学

张科

回顾：

1. Π原理

2.无量纲化N-S方程

3.无量纲参数

4.相似准则及应用

$$\vec{\nabla}^* \cdot \vec{V}^* = 0$$

$$\frac{D\vec{V}^*}{Dt^*} = -\vec{\nabla}^* p^* + \frac{1}{Re}\nabla^{*2}\vec{V}^* - \frac{1}{Fr^2}\vec{\nabla}^* z^*$$

$$\vec{V}^* = \vec{V}^*(Re) \quad p^* = p^*(Re)$$

$$Re = \frac{\rho U_0 L_0}{\mu} \quad Fr = \frac{U_0}{\sqrt{g L_0}} \quad Ma = \frac{U}{a} \quad Eu = \frac{\Delta p}{0.5\rho U^2}$$

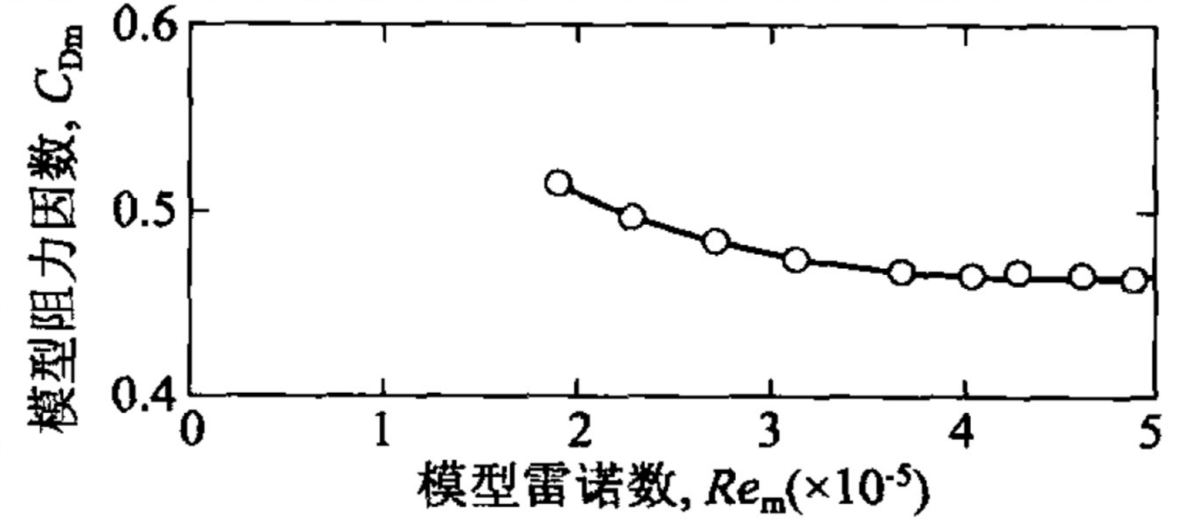
$$St = \frac{fL}{U} \quad We = \frac{\rho U^2 L}{\sigma}$$

6. 流动相似原则（similarity）

设计模型实验，分析实验数据！例7.5~7.10

例 7.8：客车风洞实验，1/16模型(迎风面积$A_m = 0.0305m^2$,模型宽度$W_m = 0.152m$)，得数据如下：$\mu = 1.789\times 10^{-5} Pa\cdot s, \rho = 1.225 kg/m^3$

速度 $U/(m/s)$	18.0	21.8	26.0	30.1	35.0	38.5	40.0	44.1	46.7
阻力 F_D/N	3[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	18.9



求：1.绘制$C_D(\frac{—}{0.}$ … 低气流速度。

2.实物以10(… 率。

解：1. $C_D = \frac{—}{0.5}$

$$Re = \frac{\rho U_\infty W_m}{\mu} = \frac{1.225\times0.152}{1.789\times10^{-5}}\times U_\infty = 1.04\times10^4 U_\infty$$

$Re > 4\times10^5$后，$C_D \approx 0.46$ ➡ $U_\infty > 40m/s$

6. 流动相似原则（similarity）

设计模型实验，分析实验数据！
例7.5~7.10

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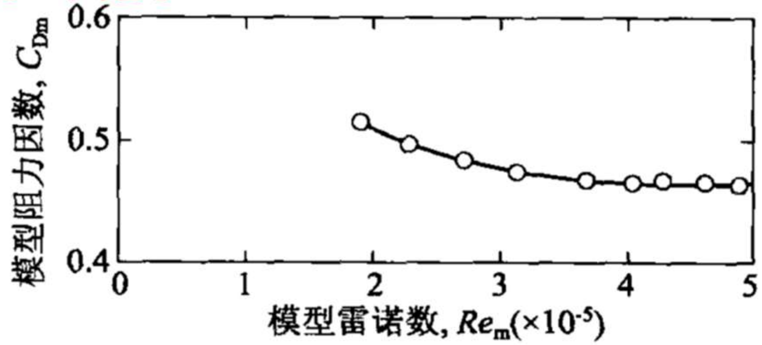
$W_m = 0.152m$)，得数据如下：$\mu = 1.789\times 10^{-5}$



求：1.绘制$C_D(\frac{F_D}{0.5\rho U^2{}_\infty A_m}) - Re(\frac{\rho U_\infty W_m}{\mu})$曲线，$C$

2.实物以$100km/h$行驶时的气动阻力，及克服其所需功率。

解：2. $U_{\infty p} = 100km/h$ =100/3.6=27.78m/s

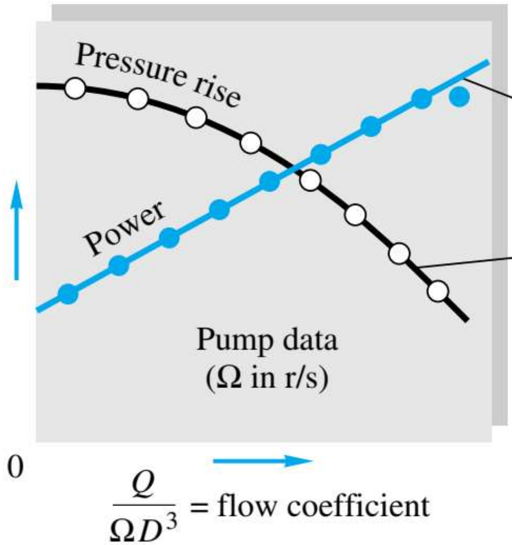
$$Re = \frac{\rho U_{\infty p} W_p}{\mu} = \frac{1.225\times 27.78\times 0.152\times 16}{1.789\times 10^{-5}} = 4.63\times 10^6 \Rightarrow C_D = 0.46$$

$$F_D = C_D \times 0.5\rho U^2{}_{\infty p} A_p = 0.46\times 0.5\times 1.225\times 27.78^2\times 0.0305\times 16^2$$
$$= 1.698\times 10^3 N$$

$$\Rightarrow \dot{W} = F_D \times U_{\infty p} = 47.2\times 10^3 W$$

作业：

P5.61 If viscosity is neglected, typical pump-flow results from Prob. 5.20 are shown in Fig. P5.61 for a model pump tested in water. The pressure rise decreases and the power required increases with the dimensionless flow coefficient. Curve-fit expressions are given for the data. Suppose a similar pump of 12-cm diameter is built to move gasoline at 20°C and a flow rate of 25 m^3/h. If the pump rotation speed is 30 r/s, find (*a*) the pressure rise and (*b*) the power required.



$$\frac{P}{\rho\Omega^3 D^5} \approx 0.5 + \frac{3Q}{\Omega D^3}$$

$$\frac{\Delta p}{\rho\Omega^2 D^2} \approx 6.0 - 120\left(\frac{Q}{\Omega D^3}\right)^2$$

P5.61

单选题 2分

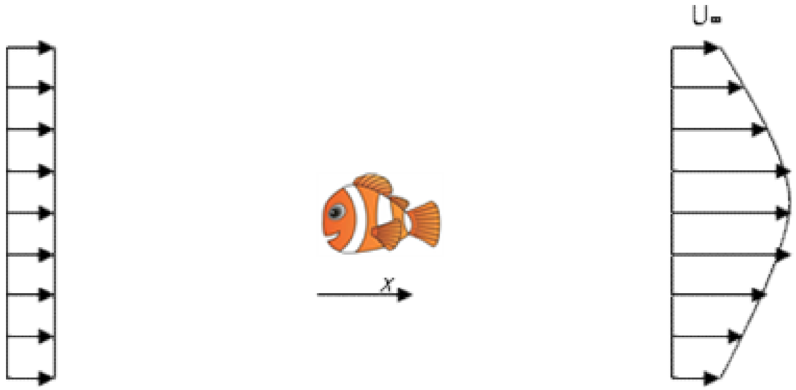
设置

此题未设置答案，请点击右侧设置按钮

鱼在水中运动过程中，前后速度分布如下图所示。

问：（1）鱼受到水对其的作用力沿x_____？

（2）需要给鱼施加沿x_____的力才能使鱼保持匀速运动？



A 正向

B 负向

提交

六. 粘性不可压流动（内流，外流）

粘性不可压内流

通道内流动一般特征（5.6、9.1）、无限大平板间（周向均匀圆管)充分发展层流（5.3、5.4） 、管内流能量损失(9.2-9.4)、

管内流，$\vec{V}$，τ，Q， Δp，h_{LT}

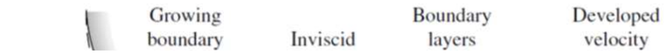
粘性不可压外部扰流

边界层基本概念（10.1）、边界层动量积分方程（10.4）、边界层方程(10.2、10.3)、曲面边界层及边界层分离（10.5）、扰流物体的阻力（10.6）

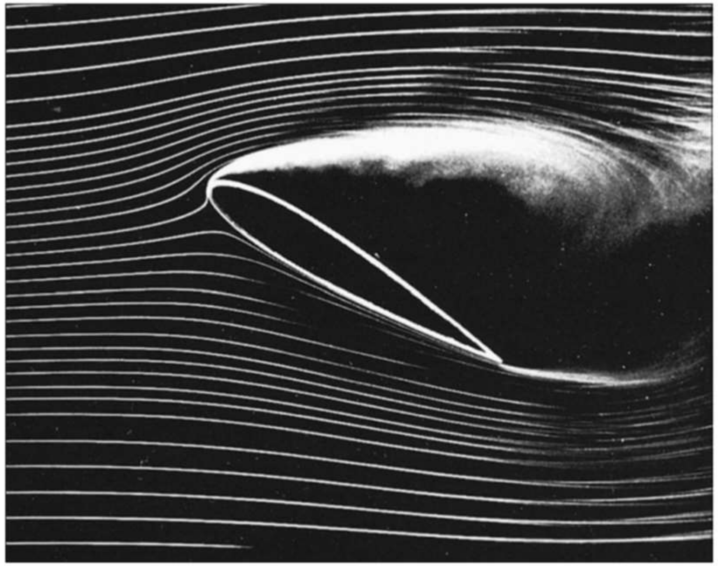
流场分布，u, D, L(飞行器，建筑物，桥梁等)

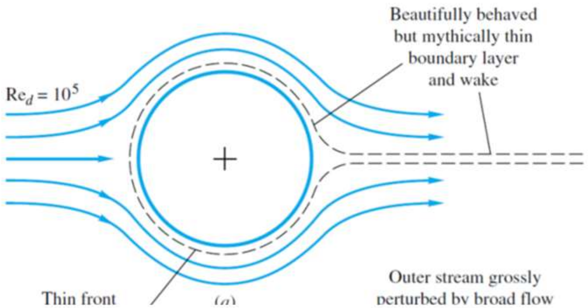
六. 粘性不可压流动（内流，外流）

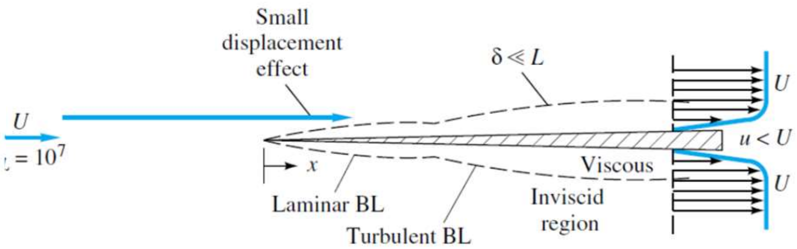
Steam pipe bridge in a geothermal power plant. Pipe flows are everywhere, often occurring in groups or networks. They are designed using the principles outlined in this chapter. *(Courtesy of Dr. E. R. Degginger/Color-Pic Inc.)*

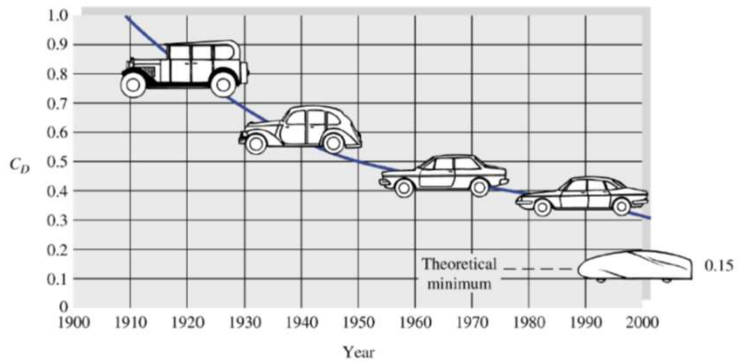


边界层基本概念、边界层动量积分方程、边界层方程、曲面边界层及边界层分离、扰流物体的阻力



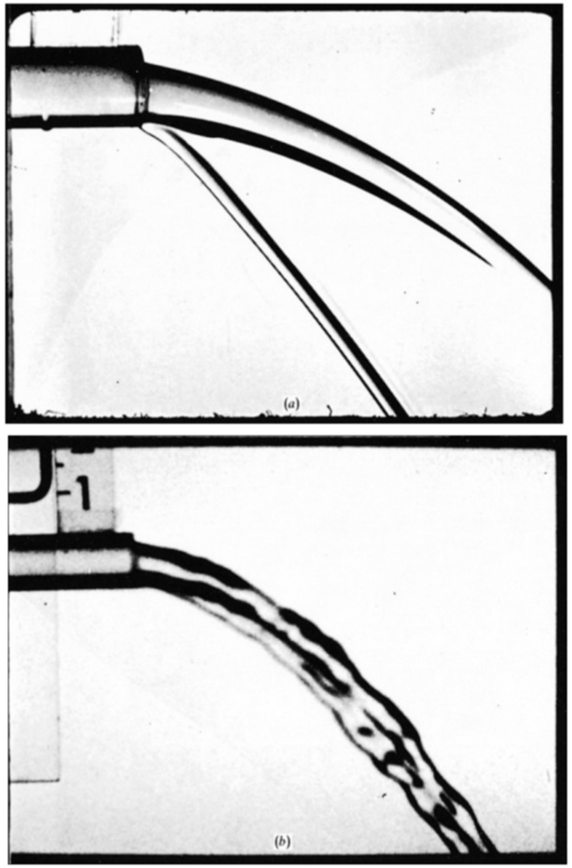




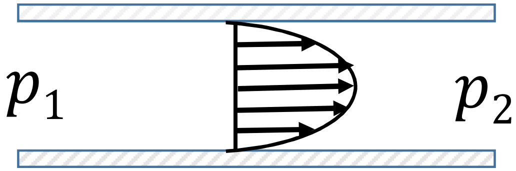


6.1通道内流动一般特征

① 层流、湍流（5.6）



1839年德国工程师G.H.L. Hagen发现粘性流存在两个区域。



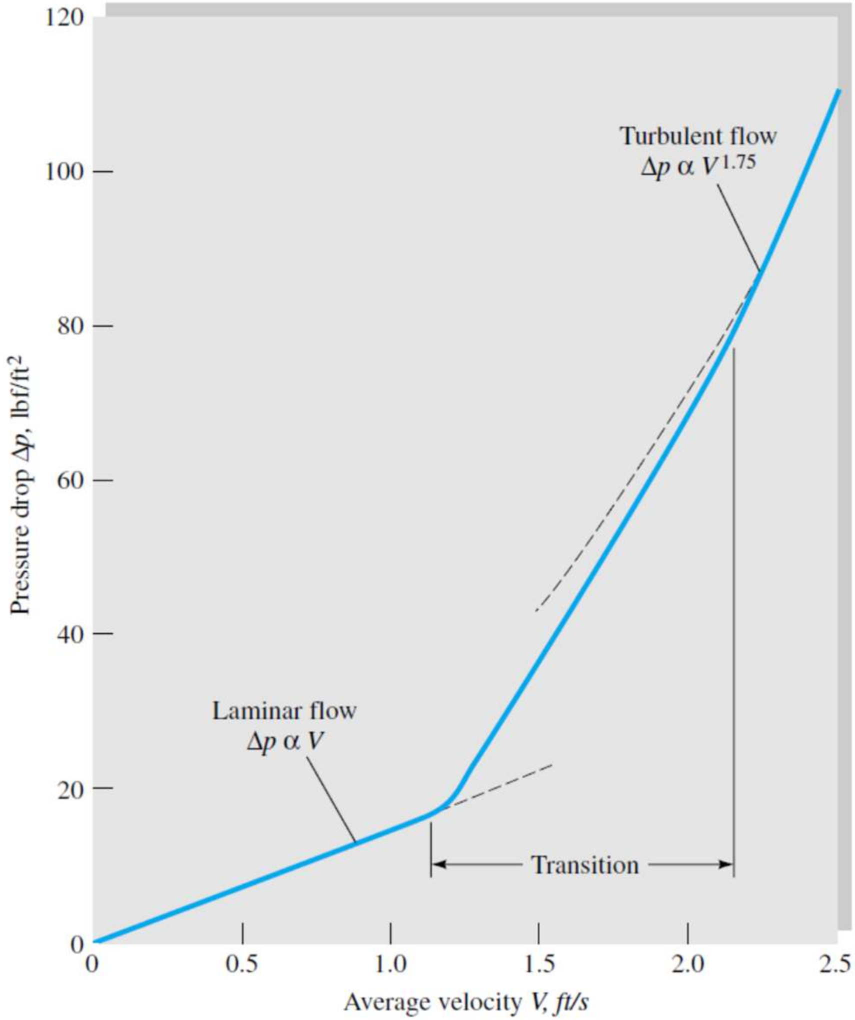
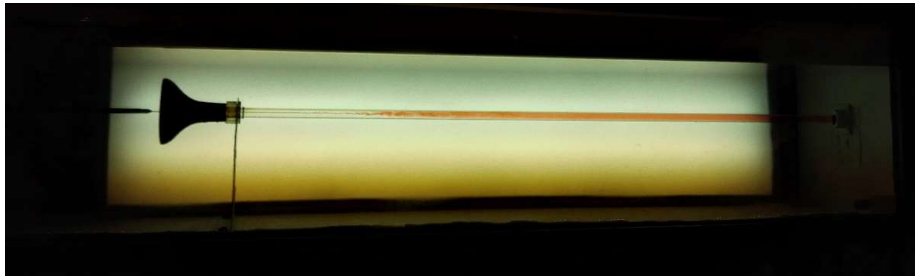


Fig. 6.4 Experimental evidence of transition for water flow in a $\frac{1}{4}$-in smooth pipe 10 ft long.

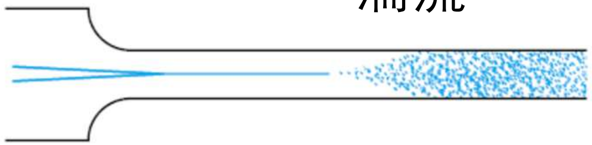
6.1通道内流动一般特征

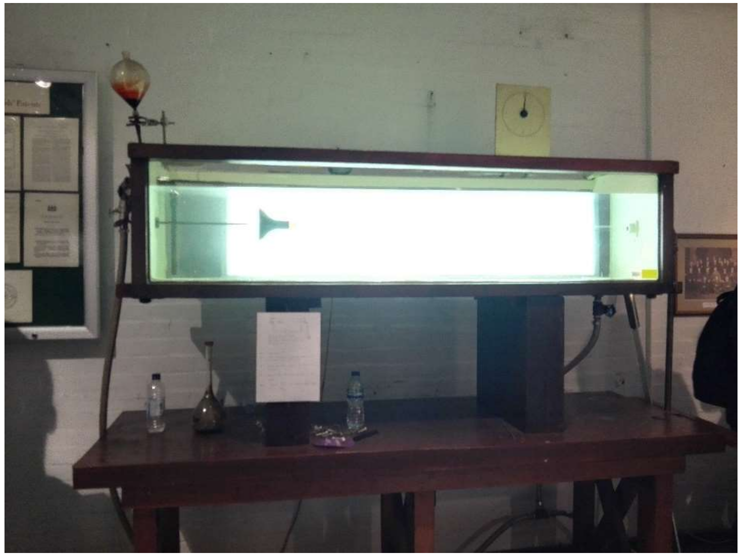


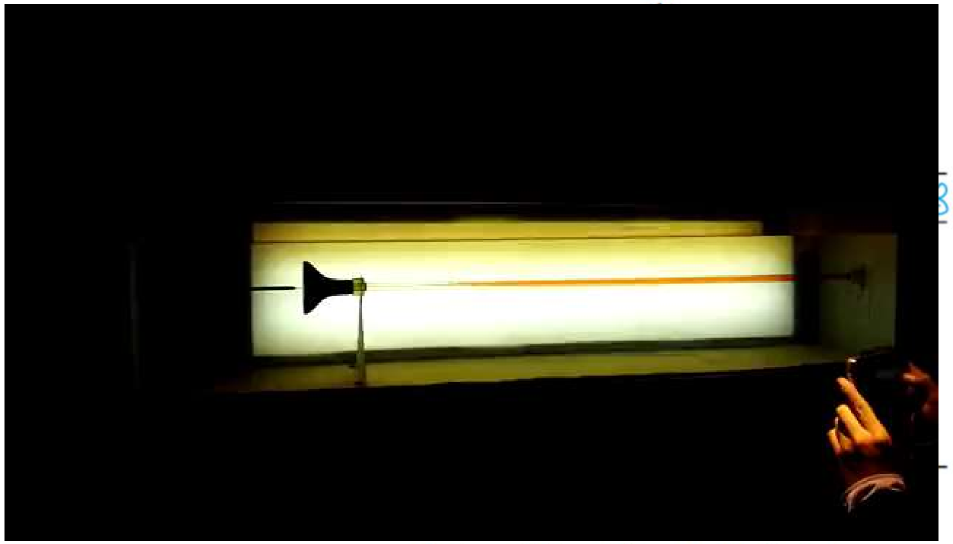
① 层流、湍流（5.6）

1883年英国学者Osborne Reynolds实验表明，粘性流随$\frac{\rho V d}{\mu}$（Re数）变化。

湍流





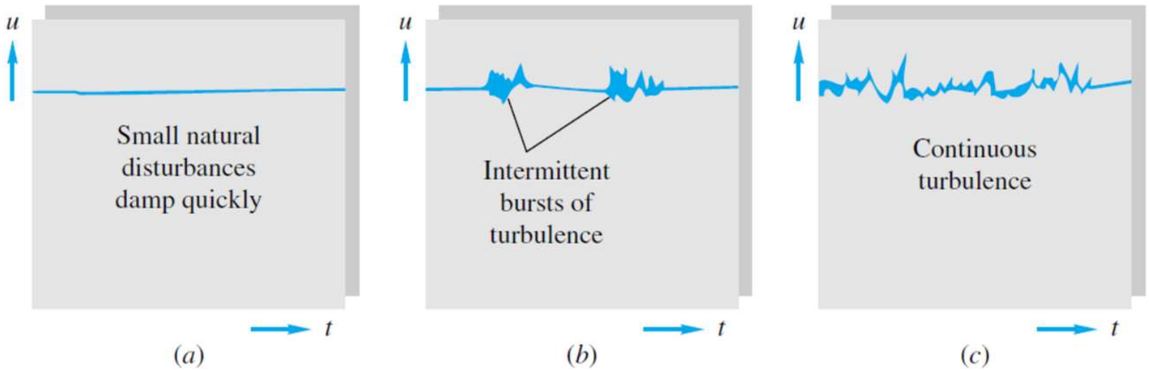


condition (*b*). (*From Ref. 4.*)

6.1通道内流动一般特征

① 层流、湍流（5.6）

1883年英国学者Osborne Reynolds实验表明，粘性流随$\frac{\rho V d}{\mu}$（Re数）变化。



$$\bar{V}_a < \bar{V}_b < \bar{V}_c \qquad Re_a < Re_b < Re_c$$

$$\bar{V} = \frac{1}{A}\int u dA \qquad Re = \frac{\rho \bar{V} D}{\mu}$$

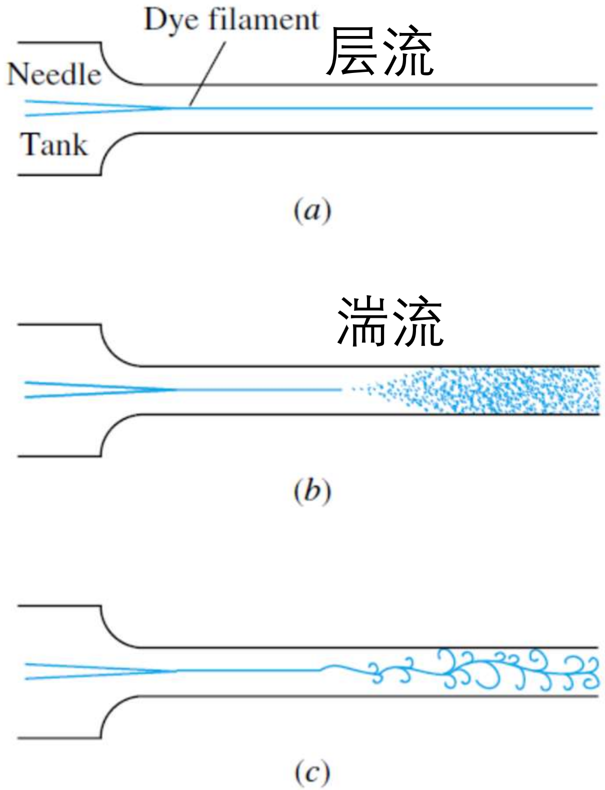


Fig. 6.5 Reynolds' sketches of pipe-flow transition: (*a*) low-speed, laminar flow; (*b*) high-speed, turbulent flow; (*c*) spark photograph of condition (*b*). *(From Ref. 4.)*

6.1通道内流动一般征

① 层流、湍流（5.6）

1883年英国学者Osborne Reynolds实验表明，粘性流随$\frac{\rho V d}{\mu}$（Re数）变化。

$$\bar{V} = \frac{1}{A}\int u dA \qquad Re = \frac{\rho \bar{V} D}{\mu}$$

圆管：$Re < 2000$~2300为层流。

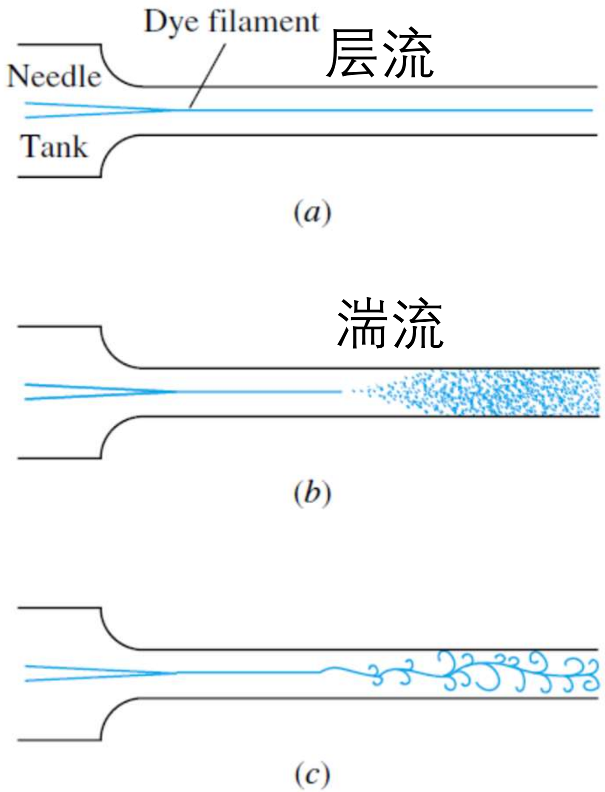
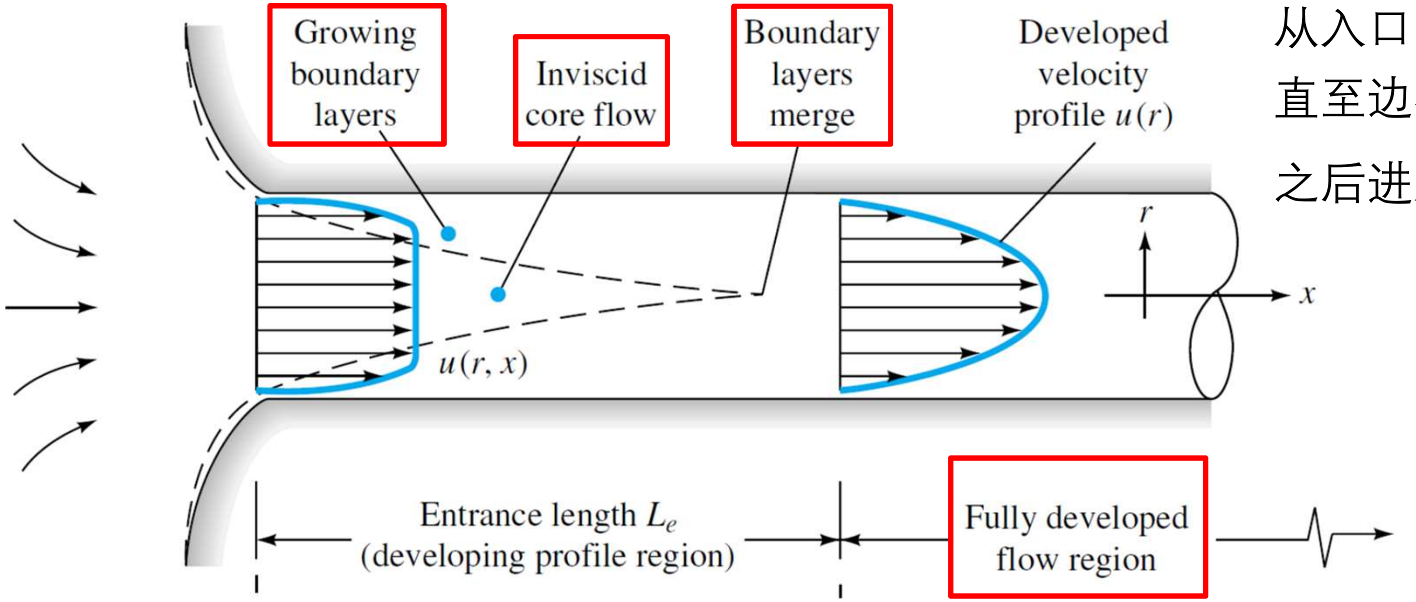


Fig. 6.5 Reynolds' sketches of pipe-flow transition: (*a*) low-speed, laminar flow; (*b*) high-speed, turbulent flow; (*c*) spark photograph of condition (*b*). *(From Ref. 4.)*

6.1通道内流动一般征

② 起始段、充分发展流动（9.1）



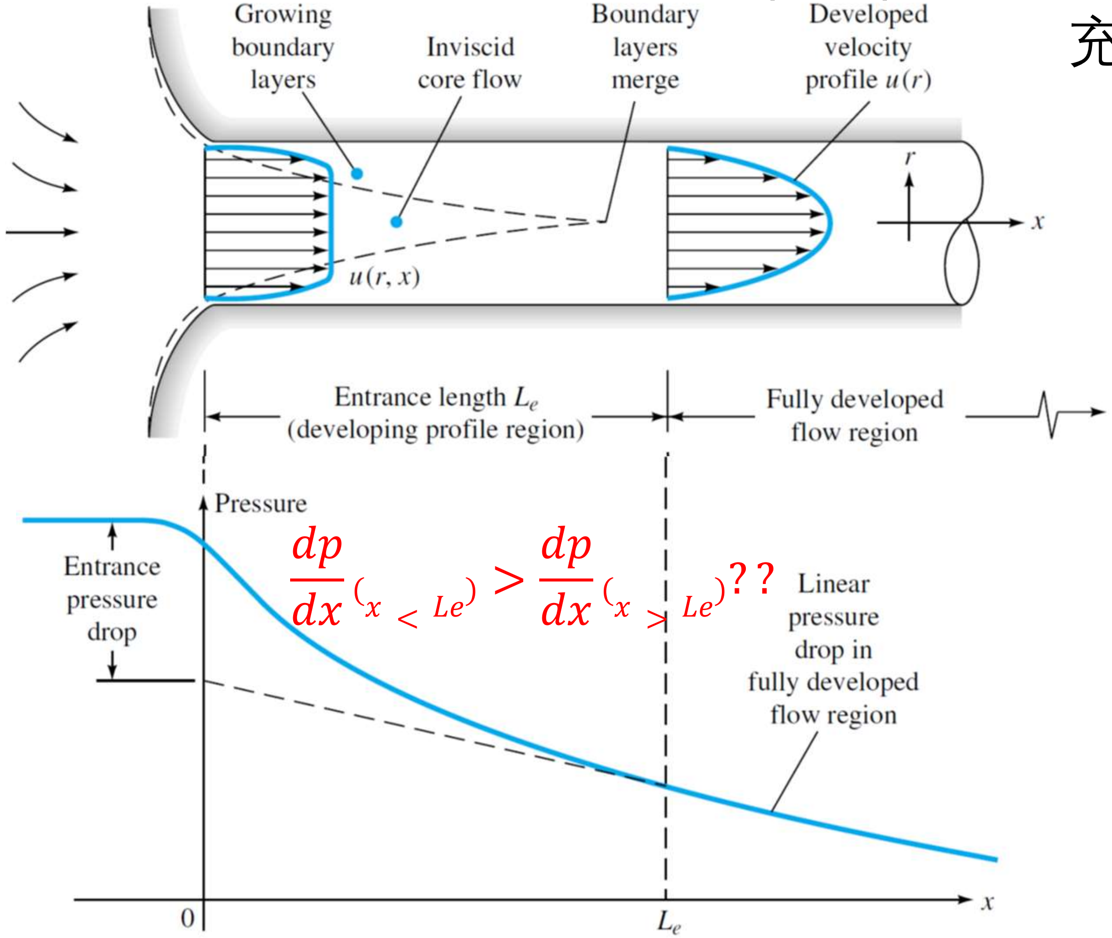
内流发展受壁面限制，
壁面附近为有速度梯度的边界层区，
远离壁面为速度均匀的无粘核心区
从入口向下游壁面粘性影响范围逐渐扩大，
直至边界层在轴线处相交，
之后进入充分发展流动，$\frac{\partial u}{\partial x} = 0$

入口段长度L_e　　$x > L_e$充分发展流动

$u = u(r)$ 不再随x变化

6.1通道内流动一般征

② 起始段、充分发展流动（9.1)



充分发展后 $\frac{\partial u}{\partial x} = 0, u = u(r)$ 不变

$\tau = \frac{du}{dr}$ =constant

$\Delta p \propto x$

入口段长度L_e

层流：$L_e/D \approx 0.06Re$

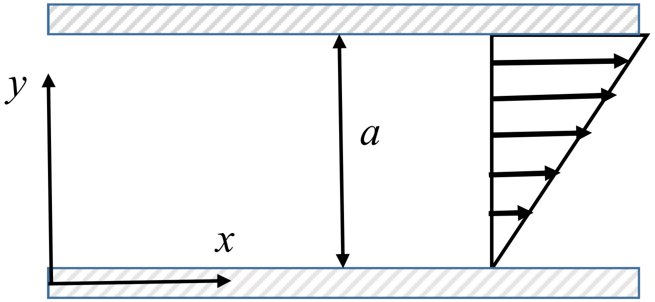
$Re = 2000$，$L_e \approx 120\ D$

湍流：$L_e/D \approx 4.4Re^{1/6}$

$L_e \approx 25\sim40D$

$L_{e层} > L_{e湍}$??

6.2无限大平板间充分发展层流（5.3）



求：$\vec{V}$，τ，Q，Δp　解微分连续性方程、N-S方程。

假设：牛顿流体、不可压，x方向无重力、

充分发展流动($\frac{\partial \vec{V}}{\partial x}=0$)、Z向无限大(2D)、定常($\frac{\partial}{\partial t}=0$)

$$\frac{L_e}{a}<\frac{x}{a} \qquad \frac{a}{W}\ll 1$$

解:　定常、不可压连续性方程：　$\vec{\nabla}\cdot\vec{V}=0$

$$\cancel{\frac{\partial u}{\partial x}}+\frac{\partial v}{\partial y}+\cancel{\frac{\partial w}{\partial z}}=0 \quad \text{2D}$$

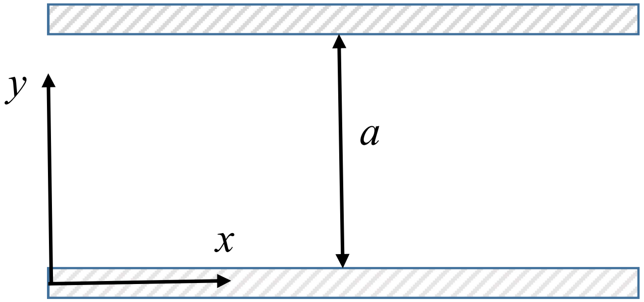
$$\frac{\partial \vec{V}}{\partial x}=0 \Rightarrow \frac{\partial u}{\partial x}=0 \quad \frac{\partial v}{\partial x}=0$$

$$\left.\begin{aligned}&\frac{\partial v}{\partial y}=0,\\&\frac{\partial v}{\partial x}=0\end{aligned}\right\} \Rightarrow v=C$$

$$\left.\begin{aligned}&v=C\\&v=0\\&@y=0,a\end{aligned}\right\} v=0$$

$$\vec{V}=(u(y),0)$$

6.2无限大平板间充分发展层流（5.3）



求：$\vec{V}$，τ，Q，Δp　解微分连续性方程、N-S方程。

假设：牛顿流体、不可压，x方向无重力、

充分发展流动($\frac{\partial \vec{V}}{\partial x}=0$)、Z向无限大(2D)、定常($\frac{\partial}{\partial t}=0$)

$\frac{L_e}{a}<\frac{x}{a}$　　$\frac{a}{W}\ll 1$

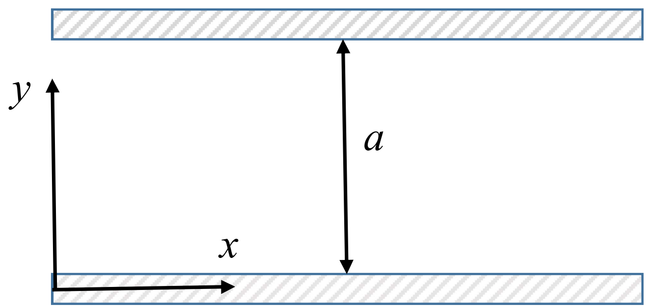
解:　N-S方程：$\rho\frac{D\vec{V}}{Dt}=-\vec{\nabla}p_k+\mu\nabla^2\vec{V}$　　$p_k=p+\rho g y$

x方向:　$\cancel{u\frac{\partial u}{\partial x}}+\cancel{v\frac{\partial u}{\partial y}}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu(\cancel{\frac{\partial^2 u}{\partial x^2}}+\frac{\partial^2 u}{\partial y^2})$

$\frac{\partial u}{\partial x}=0$　$v=0$　$\frac{\partial u}{\partial x}=0$

$$\Rightarrow\quad \nu\frac{\partial^2 u}{\partial y^2}=\frac{1}{\rho}\frac{\partial p}{\partial x}$$

6.2无限大平板间充分发展层流（5.3）



求：$\vec{V}$，τ，Q，Δp　解微分连续性方程、N-S方程。

假设：牛顿流体、不可压，x方向无重力、

充分发展流动($\frac{\partial \vec{V}}{\partial x}=0$)、Z向无限大(2D)、定常($\frac{\partial}{\partial t}=0$)

$$\frac{L_e}{a}<\frac{x}{a} \qquad \frac{a}{W}\ll 1$$

解:　N-S方程：$\rho\frac{D\vec{V}}{Dt}=-\vec{\nabla}p_k+\mu\nabla^2\vec{V}$　　$p_k=p+\rho g y$

y方向:　$\cancel{u\frac{\partial v}{\partial x}}+\cancel{v\frac{\partial v}{\partial y}}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu(\cancel{\frac{\partial^2 v}{\partial x^2}}+\cancel{\frac{\partial^2 v}{\partial y^2}})$

v=0

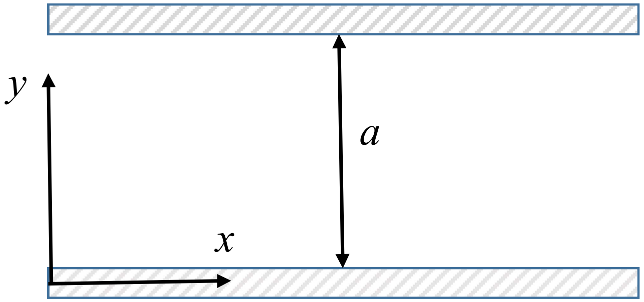
$\Rightarrow \frac{\partial p}{\partial y}=0$

z方向：$w=0$

$\frac{\partial p}{\partial z}=0$

$\Rightarrow p=p(x)$only

6.2无限大平板间充分发展层流（5.3）



求：$\vec{V}$，τ，Q，　Δp　解微分连续性方程、N-S方程。

假设：牛顿流体、不可压，x方向无重力、

充分发展流动($\frac{\partial \vec{V}}{\partial x}=0$)、Z向无限大(2D)、定常($\frac{\partial}{\partial t}=0$)

$$\frac{L_e}{a}<\frac{x}{a} \qquad \frac{a}{W}\ll 1$$

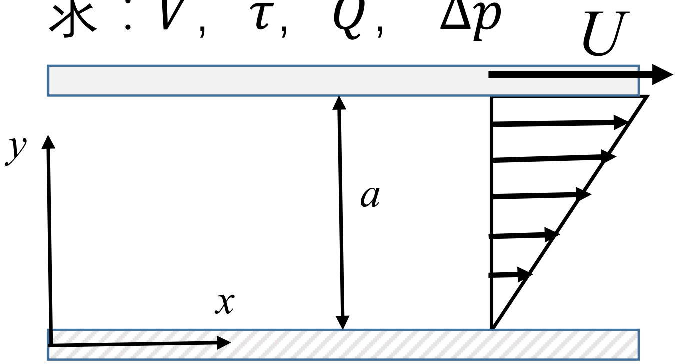
解:

$$\left.\begin{aligned} \nu\frac{\partial^2 u}{\partial y^2}&=\frac{1}{\rho}\frac{\partial p}{\partial x} \\ u=u(y),p&=p(x) \end{aligned}\right\} \frac{dp}{dx}=\mu\frac{d^2u}{dy^2} \qquad ODE\text{常微分方程}$$

$$f(x)=f(y)=constant$$

6.2无限大平板间充分发展层流（5.3）

求：$\vec{V}$，τ，Q，Δp

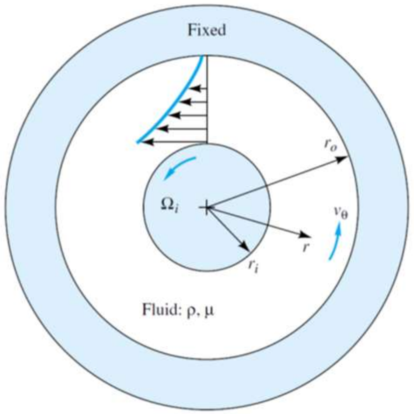


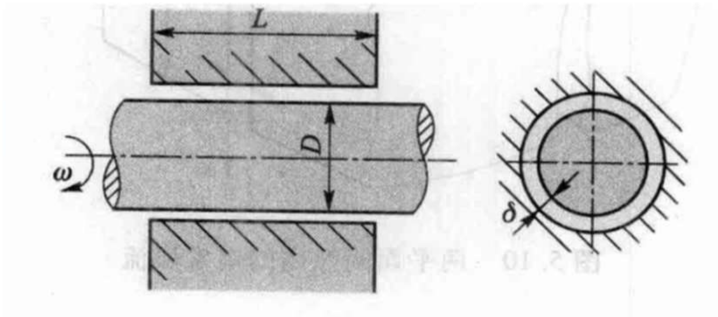
解: $$\frac{dp}{dx} = \mu \frac{d^2u}{dy^2} = C$$

(a).平板库埃特流动(couette flow, purely shear driven flow)

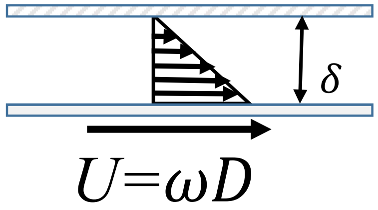
$$\frac{dp}{dx} = 0, u(0) = 0, u(a) = U \quad \text{（边界条件）}$$

$$\frac{d^2u}{dy^2} = 0 \Rightarrow \left.\begin{aligned} u(y) &= ky + B \\ u(0) &= 0 \\ u(a) &= U \end{aligned}\right\} u(y) = \frac{U}{a}y$$



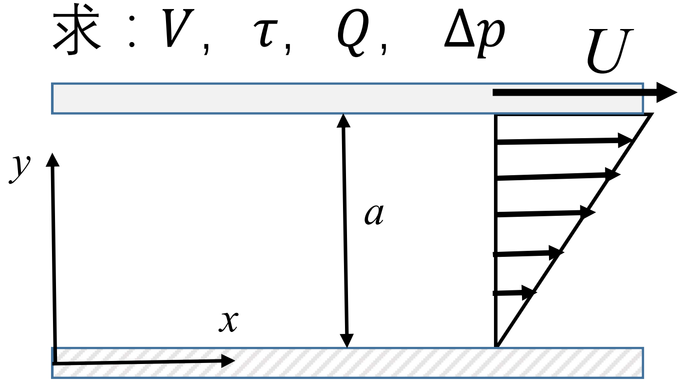


$\delta \ll D$时可看作



6.2无限大平板间充分发展层流（5.3）

求：$\vec{V}$，τ，Q，Δp



解: $$\frac{dp}{dx} = \mu\frac{d^2u}{dy^2} = C$$

(a).平板库埃特流动(couette flow, purely shear driven flow)

$$\frac{dp}{dx} = 0, \quad ➢ \; u(y) = \frac{U}{a}y$$

➢ Shear stress： $$\tau_{yx} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = \mu\frac{du}{dy} = \mu\frac{U}{a} = C$$

$$\tau_{wall} = \mu\frac{U}{a}$$

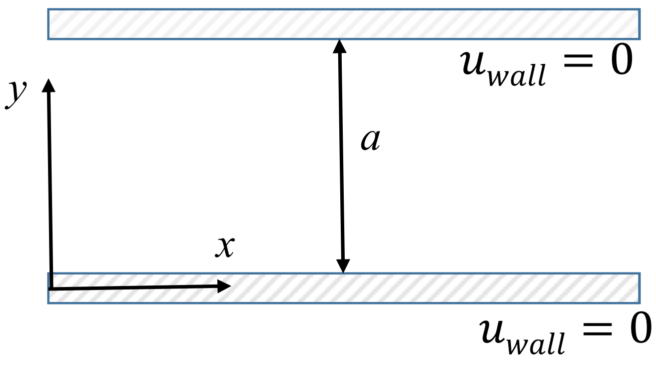
➢ Volume flow rate: $Q = \int u(y)dydz = b\int \frac{U}{a}ydy$ (宽度为b)

$$q = \frac{Q}{b} = \int_0^a \frac{U}{a}ydy = \frac{Ua}{2}$$

➢ $$\bar{V} = \frac{q}{a} = \frac{U}{2}$$

6.2无限大平板间充分发展层流（5.3）

求：$\vec{V}$, τ, Q, Δp



解: $$\frac{dp}{dx} = \mu\frac{d^2u}{dy^2} = C$$

(b).平板泊肃叶流动$\frac{dp}{dx} \neq 0$ (purely pressure driven flow)

$$\frac{dp}{dx} = \mu\frac{d^2u}{dy^2} = C$$

$$\mu\frac{du}{dy} = \frac{dp}{dx}y + C_1$$

$$\mu u(y) = \frac{dp}{dx}\frac{y^2}{2} + C_1 y + C_2$$

$$u(y) = \frac{1}{2\mu}\frac{dp}{dx}y^2 + \frac{C_1}{\mu}y + \frac{C_2}{\mu}$$

$$u(y) = \frac{1}{2\mu}\frac{dp}{dx}y^2 - \frac{a}{2\mu}\frac{dp}{dx}y$$

边界条件: $u(0) = 0$

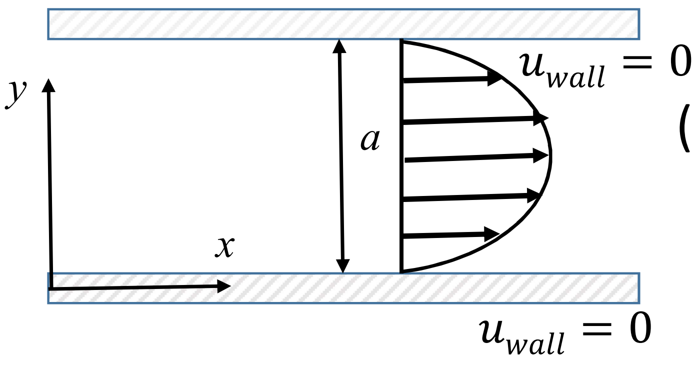
$u(a) = 0$

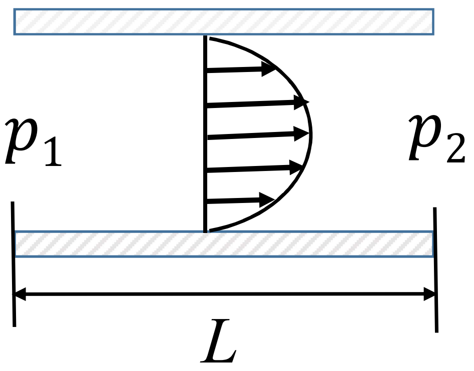
$$u(0) = \frac{C_2}{\mu} = 0 \Rightarrow C_2 = 0$$

$$u(a) = \frac{1}{2\mu}\frac{dp}{dx}a^2 + \frac{C_1}{\mu}a = 0 \Rightarrow C_1 = -\frac{a}{2}\frac{dp}{dx}$$

6.2无限大平板间充发展层流（5.3）

求：$\vec{V}$，τ，Q，Δp





$p_1 > p_2$

压强沿流动方向降低→顺压流动

解: $$\frac{dp}{dx}=\mu\frac{d^2u}{dy^2}=C$$

(b).平板泊肃叶流动$\frac{dp}{dx}\neq 0$ (purely pressure driven flow)

$$u(y)=\frac{1}{2\mu}\frac{dp}{dx}y^2-\frac{a}{2\mu}\frac{dp}{dx}y$$

$$=\frac{a^2}{2\mu}\frac{dp}{dx}[(\frac{y}{a})^2-(\frac{y}{a})]$$

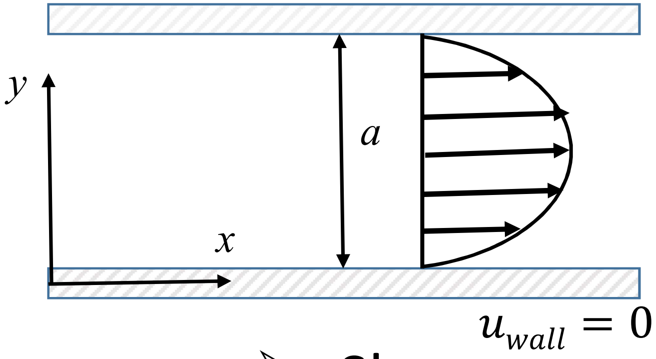
$$=\frac{a^2}{2\mu}\underbrace{\boxed{(-\frac{dp}{dx})}}_{>0}\frac{y}{a}\underbrace{\boxed{(1-\frac{y}{a})}}_{>0}>0$$

$u(y)>0$, 则$\frac{dp}{dx}<0$ $\quad \frac{dp}{dx}=C \quad p=kx+B$

$$-\frac{dp}{dx}=\frac{p_1-p_2}{L}=\frac{\Delta p}{L} \qquad u(y)=\frac{a^2}{2\mu}(\frac{\Delta p}{L})\frac{y}{a}(1-\frac{y}{a})$$

6.2无限大平板间充发展层流（5.3）

求：$\vec{V}$， τ， Q， Δp



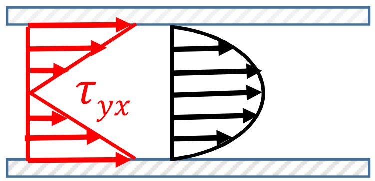
解: $$\frac{dp}{dx} = \mu \frac{d^2u}{dy^2} = C$$

(b).平板泊肃叶流动$\frac{dp}{dx} \neq 0$ (purely pressure driven flow)

➢ $u(y) = \frac{a^2}{2\mu}\left(-\frac{dp}{dx}\right)\frac{y}{a}\left(1-\frac{y}{a}\right)$

➢ Shear stress : $\tau_{yx} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = \mu\frac{du}{dy} = a\frac{dp}{dx}\left(\frac{y}{a} - \frac{1}{2}\right)$

$$\tau_{wall} = \pm a\frac{dp}{dx}$$



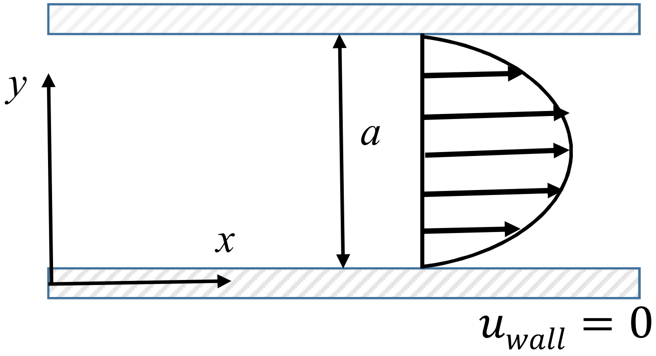
➢ Volume flow rate: $q = \frac{Q}{b} = \int_0^a u(y)dy$

$$= \frac{a^3}{12\mu}\left(-\frac{dp}{dx}\right)$$

➢ $\bar{V} = \frac{q}{a} = \frac{a^2}{12\mu}\left(-\frac{dp}{dx}\right)$ ➢ @$y = a/2, u_{max} = \frac{a^2}{8\mu}\left(-\frac{dp}{dx}\right)$=1.5$\bar{V}$

6.2无限大平板间充发展层流（5.3）

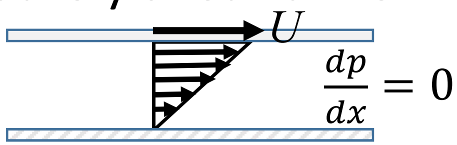
求：$\vec{V}$，τ，Q，Δp



解: $\dfrac{dp}{dx} = \mu \dfrac{d^2u}{dy^2} = C$

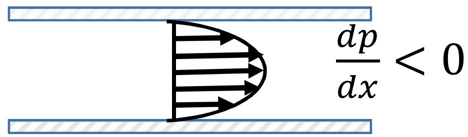
(a).平板库埃特流动(couette flow, purely shear driven flow)

➢ $u(y) = \dfrac{U}{a} y$



(b).平板泊肃叶流动$\dfrac{dp}{dx} \neq 0$ (purely pressure driven flow)

➢ $u(y) = \dfrac{a^2}{2\mu}(-\dfrac{dp}{dx})\dfrac{y}{a}(1 - \dfrac{y}{a})$



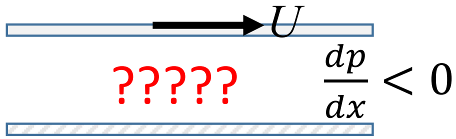
Note:

1.仅适用层流.

Re<2000层流；Re>7700湍流

2.入口效应，$L_e/D \approx 0.06Re$

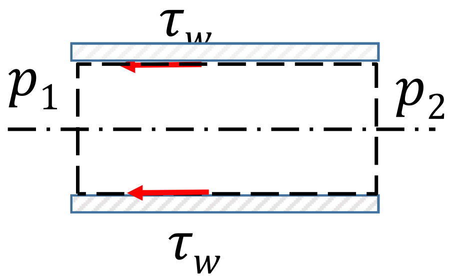
(c).一般(a+b)???



作业：

复习笔记！

1.用积分方程求τ_w。



5.2，5.4，5.14

看例5.3~5.7