空气与气体动力学

张科

回顾:

1. Prandtl-Glauert压缩性修正: $C_p = \frac{C_{p,0}}{\sqrt{1-Ma_{\infty}^2}}$ $C_l = \frac{C_{l,0}}{\sqrt{1-Ma_{\infty}^2}}$ $C_m = \frac{C_{m,0}}{\sqrt{1-Ma_{\infty}^2}}$

相同翼型: (x,y)空间可压流动 \rightarrow (ξ,η) 空间不可压流动!

$$\beta^2 \frac{\partial^2 \widehat{\phi}}{\partial x^2} + \frac{\partial^2 \widehat{\phi}}{\partial y^2} = 0 \qquad \qquad \frac{\partial^2 \overline{\phi}}{\partial \xi^2} + \frac{\partial^2 \overline{\phi}}{\partial \eta^2} = 0$$

$$\frac{\partial^2 \overline{\phi}}{\partial \xi^2} + \frac{\partial^2 \overline{\phi}}{\partial \eta^2} = 0$$

2.临界马赫数: M∞=Mcr=0.61

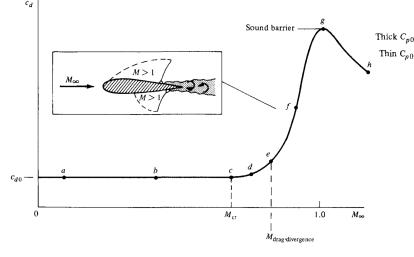
$$M_{\infty} = M_{\rm cr} = 0.61$$
Local $M_A = 1.0$

3.阻力发散,声障

减 C_d , C_{dmax} :

薄翼型;后掠翼。

超临界翼型;面积律。



4.超声速翼型特点

尖头尖尾、对称、薄;激波,膨胀波。

Thick airfoil

 $C_{p,cr} = f(M_{cr})$

$$(1-Ma_{\infty}^2)\frac{\partial^2 \hat{\phi}}{\partial x^2} + \frac{\partial^2 \hat{\phi}}{\partial y^2} = 0$$
 $Ma_{\infty} < 1$ 椭圆型方程 $Ma_{\infty} > 1$ 双曲型方程

1. 线化扰动速度势方程:

$$Ma_{\infty} > 1$$
: $\lambda^2 \frac{\partial^2 \widehat{\phi}}{\partial x^2} - \frac{\partial^2 \widehat{\phi}}{\partial y^2} = 0$ $\lambda = \sqrt{Ma_{\infty}^2 - 1}$

解为: $\hat{\phi} = f(x - \lambda y)$

$$\frac{\partial \widehat{\phi}}{\partial x} = f'(x - \lambda y) \qquad \frac{\partial^2 \widehat{\phi}}{\partial x^2} = f'' \\ \frac{\partial \widehat{\phi}}{\partial y} = (-\lambda)f'(x - \lambda y) \qquad \frac{\partial^2 \widehat{\phi}}{\partial y^2} = \lambda^2 f'' \qquad \lambda^2 \frac{\partial^2 \widehat{\phi}}{\partial x^2} - \frac{\partial^2 \widehat{\phi}}{\partial y^2} = \lambda^2 f'' - \lambda^2 f'' = 0$$

1. 线化扰动速度势方程:

$$Ma_{\infty} > 1$$
: $\lambda^{2} \frac{\partial^{2} \widehat{\phi}}{\partial x^{2}} - \frac{\partial^{2} \widehat{\phi}}{\partial y^{2}} = 0$ $\lambda = \sqrt{Ma_{\infty}^{2} - 1}$ $\widehat{\phi} = f(x - \lambda y)$ 为解!

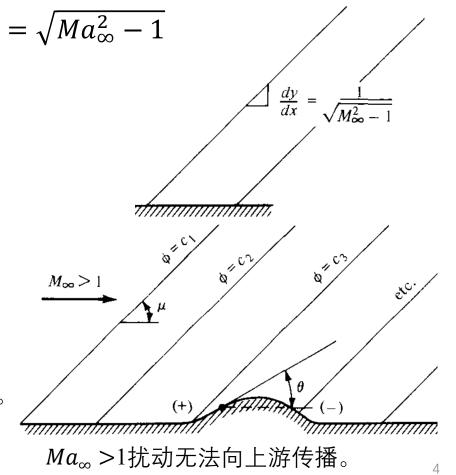
$$\hat{\phi}$$
沿 $x - \lambda y = C$ 为 常数。

$$x - \lambda y = C$$
是斜率 $\frac{dy}{dx} = \frac{1}{\lambda} = \frac{1}{\sqrt{Ma_{\infty}^2 - 1}}$ 的直线。

$$tan\mu = \frac{1}{\sqrt{Ma_{\infty}^2 - 1}}$$
, μ 为马赫角。

 $\hat{\phi}$ 沿马赫线为常数,小扰动沿Ma线传播!

适用: 线化超声流(小扰动),不适用激波、膨胀波。



$$\lambda^2 \frac{\partial^2 \widehat{\phi}}{\partial x^2} - \frac{\partial^2 \widehat{\phi}}{\partial y^2} = 0 \qquad \widehat{\phi} = f(x - \lambda y)$$

2. 线化 C_p : $C_p = -\frac{2\hat{u}}{V}$ 势流场中:

$$\hat{u} = \frac{\partial \hat{\phi}}{\partial x} = f'$$

$$\hat{v} = \frac{\partial \hat{\phi}}{\partial y} = -\lambda f'$$

$$\hat{u} = -\frac{\hat{v}}{\lambda}$$

翼面上: $\hat{v} \approx V_{\infty} tan\theta \approx V_{\infty} \theta$

翼面上:
$$\hat{u} = -\frac{V_{\infty}\theta}{\lambda}$$

$$C_p = -\frac{2\hat{u}}{V_{\infty}} = \frac{2\theta}{\lambda} = \frac{2\theta}{\sqrt{Ma_{\infty}^2 - 1}}$$
 超声速线化压强系数!

$$C_p = \frac{2\theta}{\sqrt{Ma_{\infty}^2 - 1}} \quad C_p = \frac{2\theta}{\sqrt{Ma_{\infty}^2 - 1}}$$

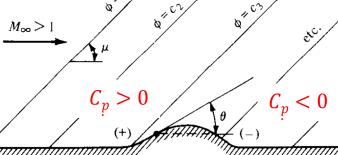
$$C_p = \frac{2\theta}{\sqrt{Ma_\infty^2 - 1}}$$
 C_p 仅与 θ 、 Ma_∞ 有关!一级近似。

2. 线化*C_p*:

$$C_p = \frac{2\theta}{\sqrt{Ma_{\infty}^2 - 1}}$$

 $Ma_{\infty} > 1$ $M_{\infty} > 1$

→波阻

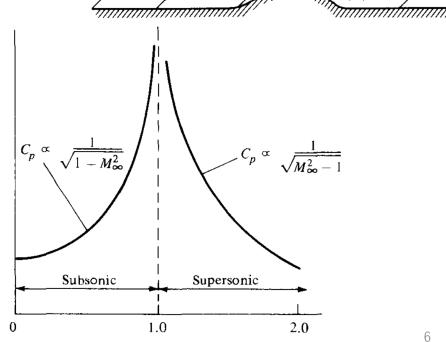


(1) $C_p \propto \theta$: 突起前高压,后低压;

(2)
$$C_p \propto \frac{1}{\sqrt{Ma_{\infty}^2 - 1}}$$
: $Ma_{\infty} \uparrow C_p \downarrow$;

亚声速:
$$C_p = \frac{C_{p,0}}{\sqrt{1-Ma_{\infty}^2}}$$

超声速 C_p 与 θ 、 Ma_{∞} 有关。



13.7超声速线化理论(10.9) $C_p = \frac{2\theta}{\sqrt{Ma_{co}^2 - 1}}$

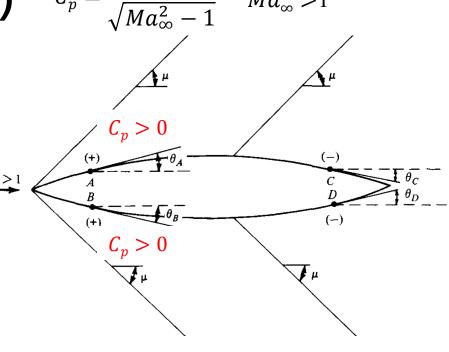
3. 超声速翼型理论:

$$\theta = \frac{dy}{dx}$$
:上表面 $\frac{dy}{dx} > 0$,下表面 $\frac{dy}{dx} < 0$ 。

$$C_p = \frac{\pm 2\theta}{\sqrt{Ma_{\infty}^2 - 1}}$$

+:壁面向气流内折;-:壁面远离气流外折。

$$C_{p_{,} u} = \frac{2\frac{dy}{dx}}{\sqrt{Ma_{\infty}^2 - 1}}$$
 $C_{p_{,} l} = \frac{-2\frac{dy}{dx}}{\sqrt{Ma_{\infty}^2 - 1}}$



3. 超声速翼型理论:
$$C_p = \frac{\pm 2\theta}{\sqrt{Ma_{\infty}^2 - 1}}$$

$$C_{p,u} = \frac{-2\alpha}{\sqrt{Ma_{\infty}^2 - 1}}$$

$$C_{p,l} = \frac{2\alpha}{\sqrt{Ma_{\infty}^2 - 1}}$$

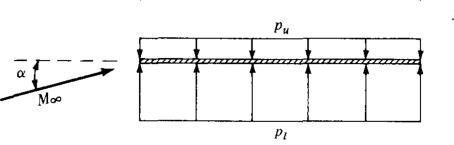
$$\Delta C_{p,\alpha} = C_{p,l} - Cp_{,u} = \frac{4\alpha}{\sqrt{Ma_{\infty}^2 - 1}} = \frac{4\alpha}{B}$$

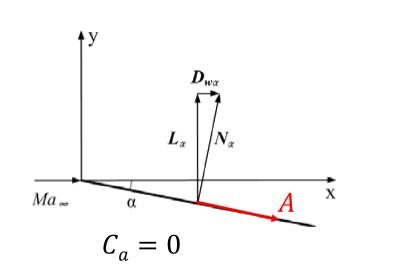
$$C_{0}^{c}(C_{p,l} - C_{p,u}) dx$$

$$B = \frac{1}{\sqrt{Ma_{\infty}^2 - 1}}$$

$$C_n = \frac{1}{c} \int_0^c (C_{p_i l} - C_{p_i u}) \, dx$$

$$= \frac{4\alpha}{\sqrt{Ma_{\infty}^2 - 1}} \frac{1}{c} \int_0^c dx = \frac{4\alpha}{\sqrt{Ma_{\infty}^2 - 1}}$$



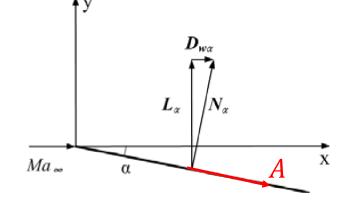


3. 超声速翼型理论:

1)平板:
$$C_n = \frac{4\alpha}{\sqrt{Ma_{\infty}^2 - 1}}$$
 $C_a = 0$

$$C_l = C_n cos\alpha - C_a sin\alpha$$

$$C_d = C_n \sin\alpha + C_a \cos\alpha$$



$$sin\alpha \approx \alpha, cos\alpha \approx 1$$

$$C_l = C_n$$
 $C_d = C_n \alpha$

$$C_d = C_n \alpha$$

$$Ma_{\infty}>1$$
,平板:

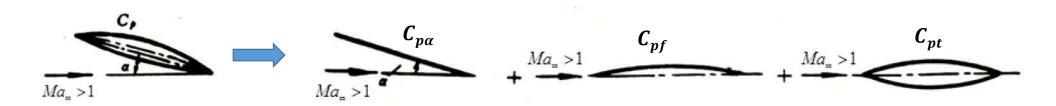
$$C_l = \frac{4\alpha}{\sqrt{Ma_{\infty}^2 - 1}}$$

$$Ma_{\infty}>1$$
,平板: $C_l=\frac{4\alpha}{\sqrt{Ma_{\infty}^2-1}}$ $C_d=\frac{4\alpha^2}{\sqrt{Ma_{\infty}^2-1}}$ 无粘:波阻! $C_d=C_l\alpha$ 升致波阻!

$$Ma_{\infty}$$
<1无粘: C_d =0!!

$$C_{p_{j}u} = \frac{2\theta}{B}$$
 $C_{p_{j}l} = \frac{-2\theta}{B}$ $B = \sqrt{Ma_{\infty}^{2} - 1}$

4. 线化压强系数叠加(迎角、弯度、厚度):



$$C_p = C_{p\alpha} + C_{pf} + C_{pt}$$

 α :迎角为 α 的平板绕流;

f:迎角为0,中弧线弯度为f的弯板绕流;

t:迎角、弯度均为0. 厚度为t的翼型绕流。

$$C_{p,u} = C_{p\alpha,u} + C_{pf,u} + C_{pt,u} = \frac{2}{B} \left[\left(\frac{dy_{\alpha}}{dx} \right)_{u} + \left(\frac{dy_{f}}{dx} \right)_{u} + \left(\frac{dy_{t}}{dx} \right)_{u} \right]$$

$$C_{p,l} = C_{p\alpha,l} + C_{pf,l} + C_{pt,l} = \frac{-2}{B} \left[\left(\frac{dy_{\alpha}}{dx} \right)_{l} + \left(\frac{dy_{f}}{dx} \right)_{l} + \left(\frac{dy_{f}}{dx} \right)_{l} \right]$$

$$C_{p_{j}u} = \frac{2\theta}{B}$$
 $C_{p_{j}l} = \frac{-2\theta}{B}$ $B = \sqrt{Ma_{\infty}^{2} - 1}$

4. 线化压强系数叠加(迎角、弯度、厚度):

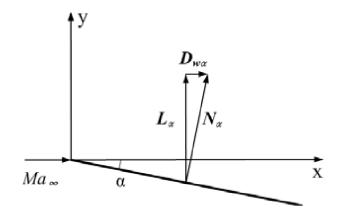
$$C_p = C_{p\alpha} + C_{pf} + C_{pt}$$

1) 平板绕流($C_{p\alpha}$):

$$C_{p\alpha_{\perp}u} = \frac{2}{B} \left(\frac{dy_{\alpha}}{dx} \right)_{u} = \frac{-2\alpha}{B}$$

$$C_{p\alpha, l} = \frac{-2}{B} \left(\frac{dy_{\alpha}}{dx} \right)_{u} = \frac{2\alpha}{B}$$

$$\Delta C_{p\alpha} = \frac{4\alpha}{B} \qquad B = \sqrt{Ma_{\infty}^2 - 1}$$



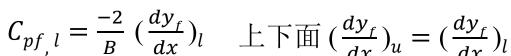
$$C_{p_i u} = \frac{2\theta}{B}$$
 $C_{p_i l} = \frac{-2\theta}{B}$ $B = \sqrt{Ma_\infty^2 - 1}$

4. 线化压强系数叠加(迎角、弯度、厚度):

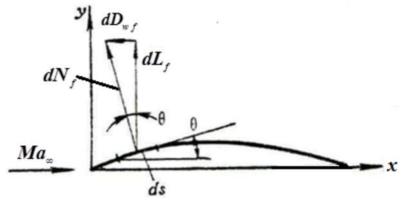
$$C_p = C_{p\alpha} + C_{pf} + C_{pt}$$

2) 弯板绕流(C_{pf}):

$$C_{pf_{,}u} = \frac{2}{B} \left(\frac{dy_{f}}{dx} \right)_{u}$$



$$\Delta C_{pf} = \frac{-4}{B} \left(\frac{dy_f}{dx} \right) \qquad B = \sqrt{Ma_{\infty}^2 - 1}$$



$$C_{p_{j}u} = \frac{2\theta}{B}$$
 $C_{p_{j}l} = \frac{-2\theta}{B}$ $B = \sqrt{Ma_{\infty}^{2} - 1}$

4. 线化压强系数叠加(迎角、弯度、厚度):

$$C_p = C_{p\alpha} + C_{pf} + C_{pt}$$

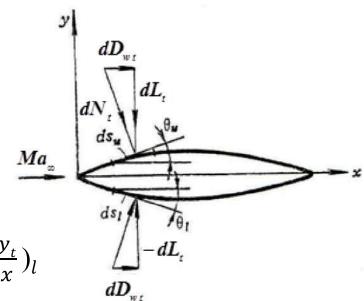
3) 厚度影响(C_{nt}):

$$C_{pt_{,}u} = \frac{2}{B} \left(\frac{dy_{t}}{dx} \right)_{u}$$

$$C_{pt_l} = \frac{-2}{B} \left(\frac{dy_t}{dx} \right)_l$$
 上下面对称 $\left(\frac{dy_t}{dx} \right)_u = -\left(\frac{dy_t}{dx} \right)_l$

$$\Delta C_{pt} = 0$$

厚度引起的压差载荷为0!!

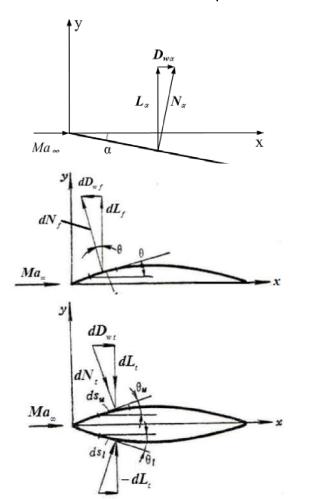


4. 线化压强系数叠加(迎角、弯度、厚度):

$$C_p = C_{p\alpha} + C_{pf} + C_{pt}$$

$$\Delta C_{p\alpha} = \frac{4\alpha}{B}$$
 $\Delta C_{pf} = \frac{-4}{B} \left(\frac{dy_f}{dx} \right)$ $\Delta C_{pt} = 0$

$$\begin{split} \Delta C_p &= \Delta C_{p\alpha} + \Delta C_{pf} + \Delta C_{pt} \\ &= \frac{4\alpha}{B} - \frac{4}{B} \left(\frac{dy_f}{dx} \right) \end{split}$$



13.7超声速线化理论(10.9) $\Delta C_p = \Delta C_{p\alpha} + \Delta C_{pf} + \Delta C_{pt} = \frac{4\alpha}{B} - \frac{4}{B} \left(\frac{dy_f}{dx} \right)$

5.
$$C_l$$
: $C_l = C_{l\alpha} + C_{lf} + C_{lt}$

$$C_{l\alpha} = \frac{1}{c} \int_0^c \Delta C_{p\alpha} \, dx / cos\alpha \cdot cos\alpha$$

$$= \frac{1}{c} \int_0^c \Delta C_{p\alpha} \, dx$$

$$= \frac{4\alpha}{B}$$

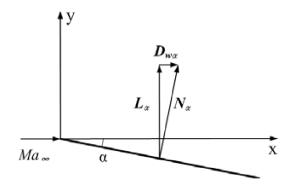
$$C_{lf} = \frac{1}{c} \int_0^c \Delta C_{pf} \, ds cos\theta$$

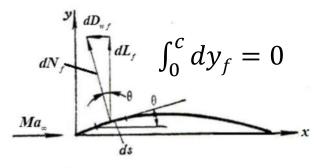
$$= \frac{1}{c} \int_0^c -\frac{4}{B} \left(\frac{dy_f}{dx}\right) \, dx$$

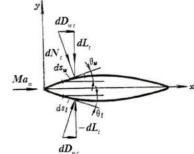
$$= \frac{1}{c} \int_0^c -\frac{4}{B} dy_f$$

$$= -\frac{4}{Bc} \int_0^c dy_f = 0$$

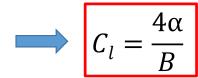
$$\Delta C_{pt} = 0 \rightarrow C_{lt} = 0$$



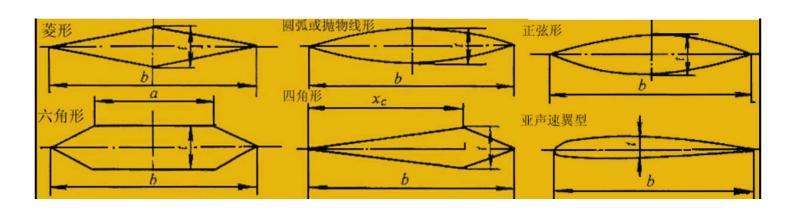




5.
$$C_l$$
: $C_l = C_{l\alpha} + C_{lf} + C_{lt}$ $C_{l\alpha} = \frac{4\alpha}{B}$ $C_{lf} = 0$ $C_{lt} = 0$ 弯度、厚度对升力无贡献!



超声速翼型:升力由 α 决定,与f,t无关!



超声速翼型:无弯度对称、厚度小!

13.7超声速线化理论(10.9) $c_l = \frac{4\alpha}{B}$ 超声速翼型:升力由 α 决定,与f,t无关! 弯度、厚度对升力无贡献!

$$C_l = \frac{4\alpha}{B}$$

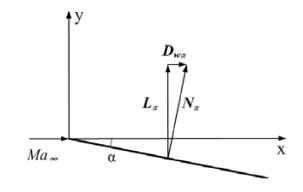
6.
$$C_{dw}$$
: $C_{dw} = C_{dw\alpha} + C_{dwf} + C_{dwt}$ (波阻)

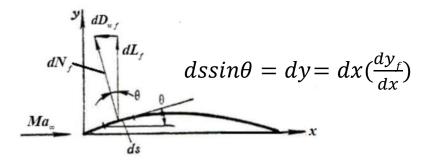
$$C_{dw\alpha} = C_{l\alpha}\alpha = \frac{4\alpha^2}{B}$$
 升致波阻!
$$C_{dwf} = \frac{1}{c} \int_0^c -\Delta C_{pf} \, dssin\theta$$

$$= \frac{1}{c} \int_0^c \frac{4}{B} \left(\frac{dy_f}{dx} \right) \left(\frac{dy_f}{dx} \right) dx$$

$$= \frac{4}{cB} \int_0^c \left(\frac{dy_f}{dx} \right)^2 dx$$

弯度产生波阻!





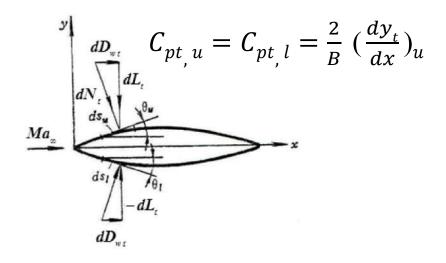
6.
$$C_{dw}$$
: $C_{dw} = C_{dw\alpha} + C_{dwf} + C_{dwt}$

$$C_{dwt} = \frac{1}{c} \int_0^c \frac{4}{B} \left(\frac{dy_t}{dx} \right) ds sin\theta$$

$$= \frac{1}{c} \int_0^c \frac{4}{B} \left(\frac{dy_t}{dx} \right) \left(\frac{dy_t}{dx} \right) dx$$

$$= \frac{4}{cB} \int_0^c \left(\frac{dy_t}{dx} \right)^2 dx$$
 弯度产生波阻!

$$C_{dw\alpha} = \frac{4\alpha^2}{B} \quad C_{dwf} = \frac{4}{cB} \int_0^c \left(\frac{dy_f}{dx}\right)^2 dx$$



$$C_{dw} = C_{dw\alpha} + C_{dwf} + C_{dwt}$$

$$C_{dw} = \frac{4\alpha^2}{B} + \frac{4}{cB} \int_0^c \left[\left(\frac{dy_f}{dx} \right)^2 + \left(\frac{dy_t}{dx} \right)^2 \right] dx = \frac{4}{B} (\alpha^2 + g_f^2 + g_t^2)$$

升致波阻

零升波阻 f,t仅产生波阻,不产生升力!

 $(\alpha$ 引起,与f,t无关)! (与f,t有关)!

$$C_l = \frac{4\alpha}{B}$$

$$C_{dw} = \frac{4\alpha^2}{B} + \frac{4}{cB} \int_0^c \left[\left(\frac{dy_f}{dx} \right)^2 + \left(\frac{dy_t}{dx} \right)^2 \right] dx$$

$$C_l \propto \alpha, C_{dw} \propto \alpha^2 \quad \alpha \downarrow C_{dw} \downarrow$$
 超声速:小 α 飞行!

可见:薄翼型的波阻系数由两部分组成,一部分与升力有关,另一部分与弯度和厚度有关。

升致波阻:由迎角引起的波阻,与翼型的形状无关

零升波阻:与升力无关,只与翼剖面有关,特别是与翼型厚度有关。

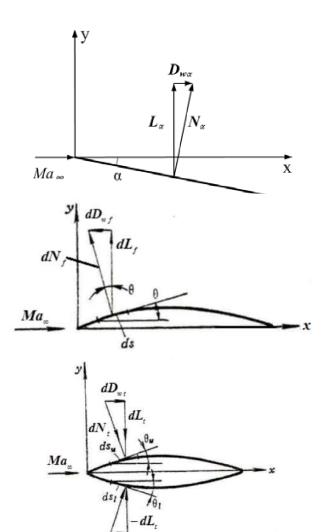
7. C_m : $1C_{m\alpha}$: $\Delta C_{p\alpha}$ 分布均匀,作用点在 $\frac{c}{2}$ 处 $C_{m\alpha,0} = -\frac{C_{l\alpha}}{2}$

$$\begin{aligned}
& 2C_{mf}: \quad C_{mf,0} = \frac{1}{c^2} \int_0^c \frac{4}{B} \left(\frac{dy_f}{dx} \right) \quad dx \cdot \mathbf{x} \\
& = \frac{-4}{Bc^2} \left(\int_0^c y_f \, dx - y_f x \, \Big|_0^c \right) \\
& = \frac{-4}{Bc^2} \int_0^c y_f \, dx
\end{aligned}$$

$$\Im C_{mt}$$
: C_{mt} =0

$$C_{m_{j}0} = -\frac{C_{l}}{2} - \frac{4}{Bc^{2}} \int_{0}^{c} y_{f} dx$$

$$C_{m,0} + \frac{C_l}{2} = -\frac{4}{Bc^2} \int_0^c y_f \, dx$$
 与 α 无关!



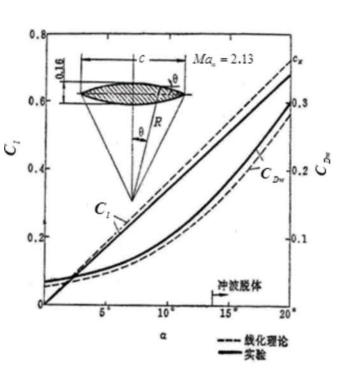
 dD_{w}

7.
$$C_m$$
: $C_{m,0} = -\frac{c_l}{2} - \frac{4}{Bc^2} \int_0^c y_f dx$
$$C_{m,0} + \frac{c_l}{2} = -\frac{4}{Bc^2} \int_0^c y_f dx \qquad 与 \alpha$$
 与 α 无 关 !
$$C_{m,1/2} = C_{m,0} + \frac{c_l}{2} = -\frac{4}{Bc^2} \int_0^c y_f dx \qquad \qquad$$
 与 α 无 关 !

$$rac{c}{2}$$
 为气动中心! $Ma_{\infty} > 1$ 气动中心后移!

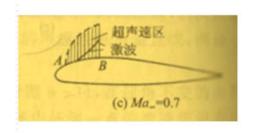
$$Ma_{\infty} < 1, \frac{c}{4}$$
 为气动中心!

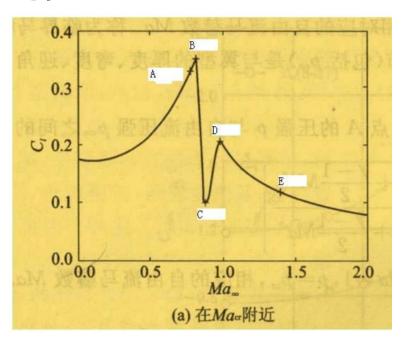
- 8. 一级线化理论的适用性:
 - ▶ 升力线斜率较实验较高: 线化理论未考虑边界层及其与激波相互干扰。
 - ➢ 波阻较实验较低: 线化理论为无粘阻力,未考虑粘性压差、 摩擦阻力。



- 1. $C_l \sim Ma_{\infty}$
 - $ightharpoonup Ma_{\infty} < Ma_{cr} : Ma_{\infty} \uparrow \rightarrow C_l \uparrow$

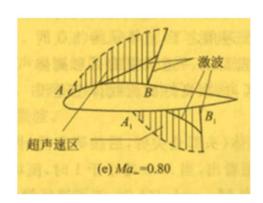
亚声速:
$$C_l = \frac{C_{l,0}}{\sqrt{1-Ma_{\infty}^2}}$$

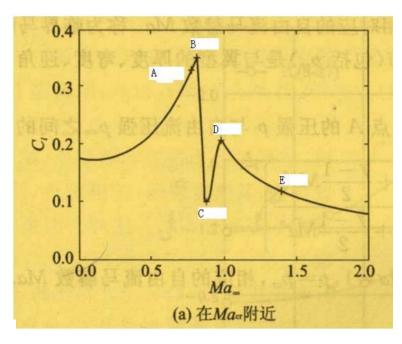




- $1.C_l \sim Ma_{\infty}$
 - 》 B后: Ma_{∞} **1**, 上翼面超声速区扩大, 激波后移, 强度增大, 波后边界层内逆压梯度剧增

 - $ightharpoonup C前: Ma_{\infty}$ **f**, 下翼面也出现超声速区,
 - 激波快速移至后缘,
 - →下翼面压强降低,
 - →升力 C_l ↓!



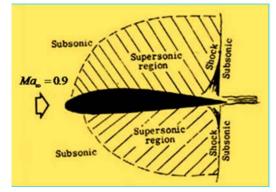


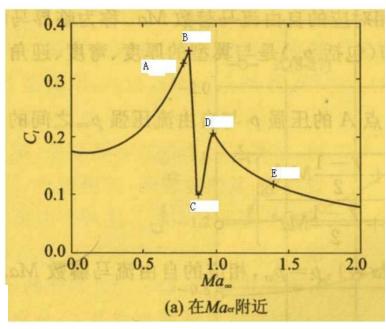
1. $C_l \sim Ma_{\infty}$

 $\succ C \sim D$:

 Ma_{∞} **f**, 下翼面超声速区不变,

- 上翼面激波移至后缘,
- 上翼面边界层分离点后移
- →上翼面 C_p ↓ → C_l ↑

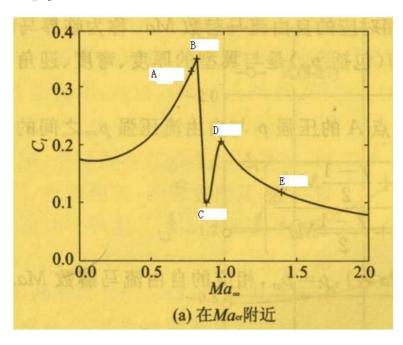




 $1.C_l \sim Ma_{\infty}$

$$C_l = \frac{4\alpha}{\sqrt{Ma_{\infty}^2 - 1}}$$

跨声速飞行中, C_l 几上几下,波动大。



2. λ形激波系:

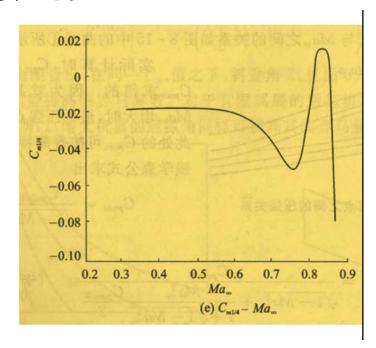
激波→逆压梯度↑

- →边界层厚度↑
- →增厚的边界层使外流形成系列压缩波
- → λ形激波系



 $3.C_m \sim Ma_\infty$:

跨声速飞行中,纵向力矩变化剧烈, 飞行器操作困难,跨越声障易出事故。



例:F104战斗机平面面积S=18.21m²,在11km高空(ρ_{∞} =0.3648kg/m³, T_{∞} =216.78K,R=287J/kgK)以 Ma_{∞} =2飞行。飞机重量为9400kgf。假设飞机重量全由机翼承担,机翼升力系数近似等于翼型升力系数。求飞机机翼的迎角 α 。

解: 11km高空的当地声速为:

$$c = \sqrt{\lambda RT_{\infty}} = \sqrt{1.4 \times 287 \times 216.78} = 295 m / s$$

 $C_l = \frac{4\alpha}{\sqrt{Ma_{\infty}^2 - 1}}$

飞机飞行速度为:

$$V_{\infty} = Ma_{\infty} \cdot c = 2 \times 295 = 590m / s$$

飞机升力为:

$$L = 9400 \times 9.8 = 9.212 \times 10^4 N$$

机翼升力系数为:

$$C_L = \frac{L}{\frac{1}{2}\rho_{\infty}V_{\infty}^2 \cdot S} = \frac{9.212 \times 10^4}{\frac{1}{2} \times 0.3648 \times 950^2 \times 18.21} = 0.08$$

例:F104战斗机平面面积S=18.21m²,在11km高空(ρ_{∞} =0.3648kg/m³, T_{∞} =216.78K,R=287J/kgK)以 Ma_{∞} =2飞行。飞机重量为9400kgf。假设飞机重量全由机翼承担,机翼升力系数近似等于翼型升力系数。求飞机机翼的迎角 α 。

解: 翼型升力系数为: $C_I = C_L = 0.08$

超声速翼型升力系数与迎角、来流马赫数的关系为:

$$C_{l} = \frac{4\alpha}{\sqrt{Ma_{\infty}^{2} - 1}}$$

则, 机翼的迎角为:

迎角很小,符合小扰动假设。

$$\alpha = \frac{C_1}{4} \sqrt{Ma_{\infty}^2 - 1} = \frac{0.08}{4} \sqrt{2^2 - 1} = 0.035 rad = 1.98^{\circ}$$

作业:

复习笔记!

空气动力学书10.1, 10.2, 10.3