# 空气与气体动力学

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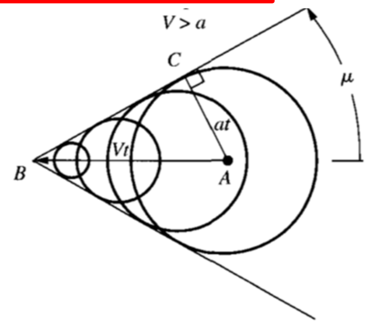
# 回顾：

## 1.高速一维定常无粘流:

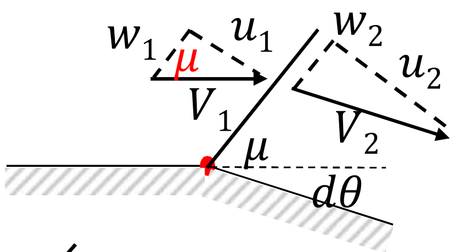
$$\boxed{\frac{T_0}{T}=1+\frac{\gamma-1}{2}Ma^2}\quad \boxed{\frac{p_0}{p}=\left(1+\frac{\gamma-1}{2}Ma^2\right)^{\frac{\gamma}{\gamma-1}}}$$

$$\boxed{s_2-s_1=(\gamma-1)C_v ln\frac{p_{01}}{p_{02}}}\quad Ma^{*2}=\frac{(\gamma+1)Ma^2}{2+(\gamma-1)Ma^2}\quad \boxed{\frac{\rho_0}{\rho}=\left(1+\frac{\gamma-1}{2}Ma^2\right)^{\frac{1}{\gamma-1}}}$$

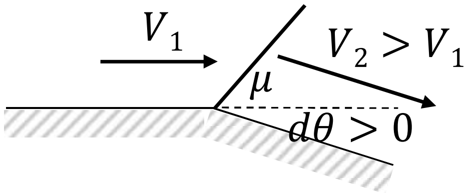
## 2.马赫波：$\mu=arcsin\frac{a}{V}=arcsin\frac{1}{Ma}$



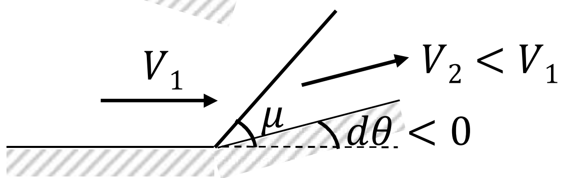
## 3.过马赫波参数变化：$\boxed{w_1=w_2}$



$$\boxed{\frac{dV}{V}=\frac{d\theta}{\sqrt{Ma^2-1}}}$$



$\boxed{d\theta>0(\text{外折})\rightarrow dV>0(\text{加速})}$



$\boxed{d\theta<0(\text{内折})\rightarrow dV<0(\text{减速})}$

# 11.4马赫波与膨胀波(8.2,8.7)

4. 过马赫波参数变化(**弱**扰动传播)：

$$\frac{dV}{V}=\frac{d\theta}{\sqrt{Ma^2-1}}$$

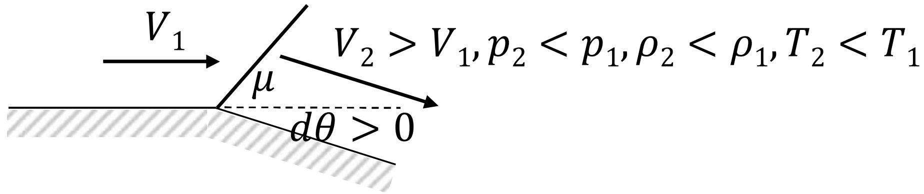
$$\frac{dT}{T}=-(\gamma-1)Ma^2\frac{dV}{V}$$

$$\frac{dp}{p}=\frac{\gamma}{\gamma-1}\frac{dT}{T}=-\gamma Ma^2\frac{dV}{V}$$

$$\frac{d\rho}{\rho}=\frac{1}{\gamma-1}\frac{dT}{T}=-Ma^2\frac{dV}{V}$$

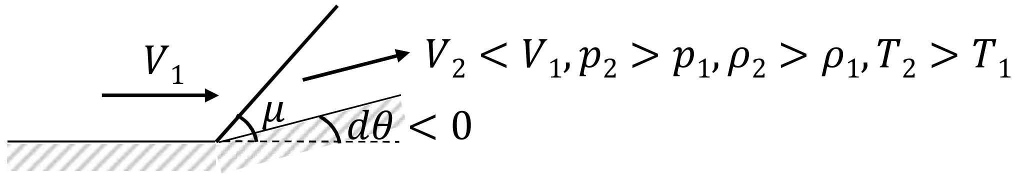
外折: $d\theta>0, dV>0, dp, d\rho, dT<0$

加速膨胀（膨胀波）

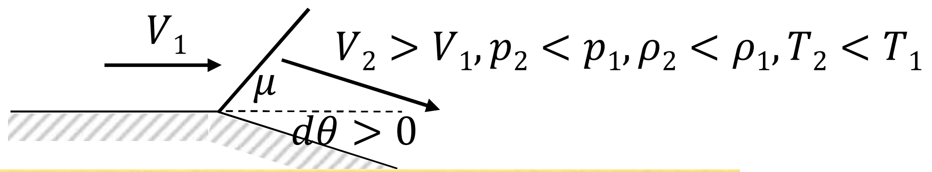



内折: $d\theta<0, dV<0, dp, d\rho, dT>0$

减速压缩（压缩波）



# 11.4马赫波与膨胀波(8.2,8.7)



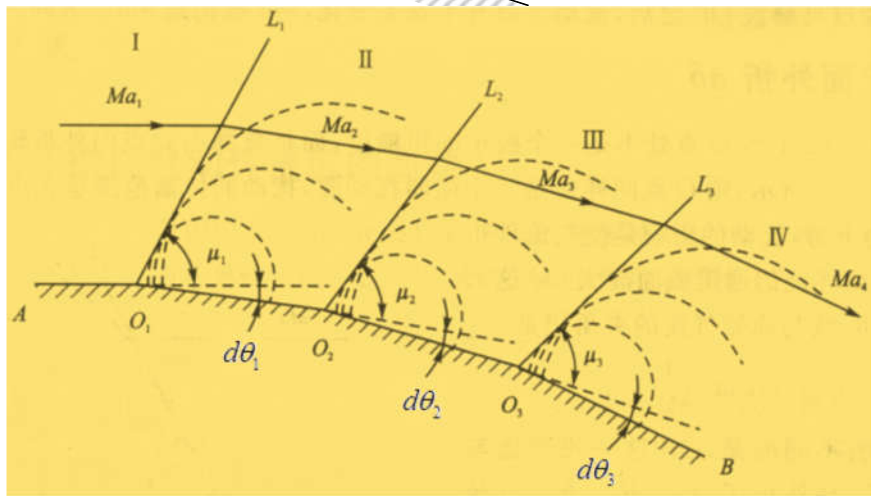

## 5. 膨胀波(外折$V$⬆$\rho$⬇)：

连续小折角（连续膨胀）：

$$\mu = \arcsin\frac{1}{Ma}$$

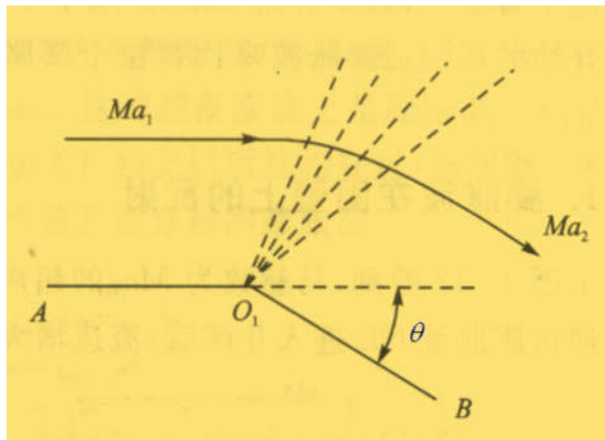
$$Ma_1 < Ma_2 < Ma_3 \ldots$$

$\mu_1 > \mu_2 > \mu_3$ ...膨胀波不相交




$O_1, O_2, O_3$无限靠近$O_1L_1, O_2L_2, O_3L_3$集成扇形波束——膨胀波！

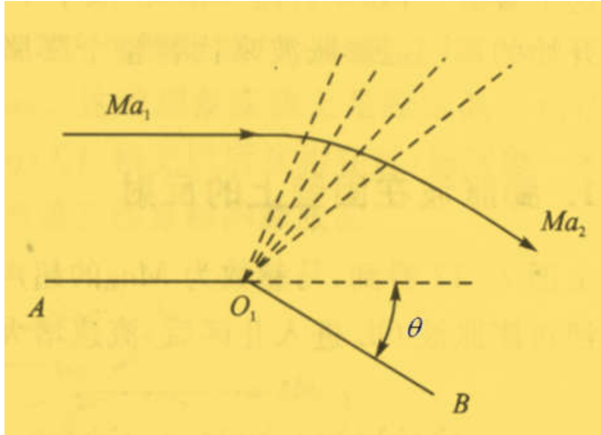
等熵过程！



# 11.4马赫波与膨胀波(8.2,8.7)

5. 膨胀波(外折$V$↑$\rho$↓)：等熵过程！

$Ma_2\sim Ma_1$，$\theta$关系???




$$\frac{dV}{V}=\frac{d\theta}{\sqrt{Ma^2-1}} \quad ①$$

$$V=aMa$$

$$\frac{dV}{V}=\frac{da}{a}+\frac{dMa}{Ma}$$

$$a^2=\gamma RT\rightarrow\frac{da}{a}=\frac{1}{2}\frac{dT}{T}$$

$$\frac{dV}{V}-\frac{1}{2}\frac{dT}{T}=\frac{dMa}{Ma}$$

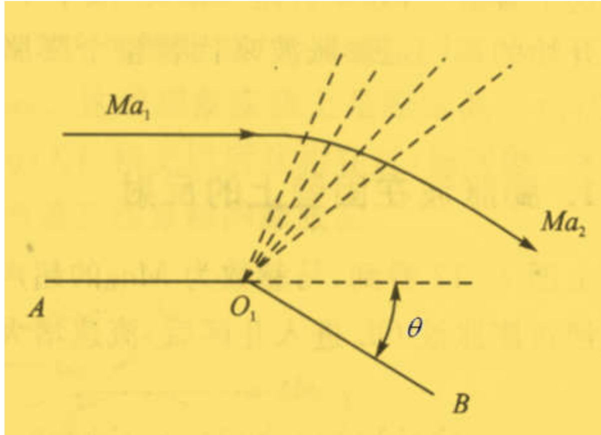
$$\frac{dT}{T}=-(\gamma-1)Ma^2\frac{dV}{V}$$

$$\frac{dV}{V}\left[1+\frac{(\gamma-1)Ma^2}{2}\right]=\frac{dMa}{Ma} \quad ②$$

① +② ⇨ $$\frac{dMa}{Ma}=\frac{1+\frac{(\gamma-1)}{2}Ma^2}{\sqrt{Ma^2-1}}d\theta \qquad d\theta=\frac{\sqrt{Ma^2-1}}{1+\frac{(\gamma-1)}{2}Ma^2}\frac{dMa}{Ma}$$

# 11.4马赫波与膨胀波(8.2,8.7)

5. 膨胀波(外折$V\uparrow\rho\downarrow$)： 等熵过程！




$Ma_2 \sim Ma_1$， $\theta$关系???

$$d\theta = \frac{\sqrt{Ma^2-1}}{1+\frac{(\gamma-1)}{2}Ma^2}\frac{dMa}{Ma}$$

$$\Rightarrow \theta = \int \frac{\sqrt{Ma^2-1}}{1+\frac{(\gamma-1)}{2}Ma^2}\frac{dMa}{Ma}$$

定义$\upsilon = \int \frac{\sqrt{Ma^2-1}}{1+\frac{(\gamma-1)}{2}Ma^2}\frac{dMa}{Ma}$ 普朗特-迈耶函数

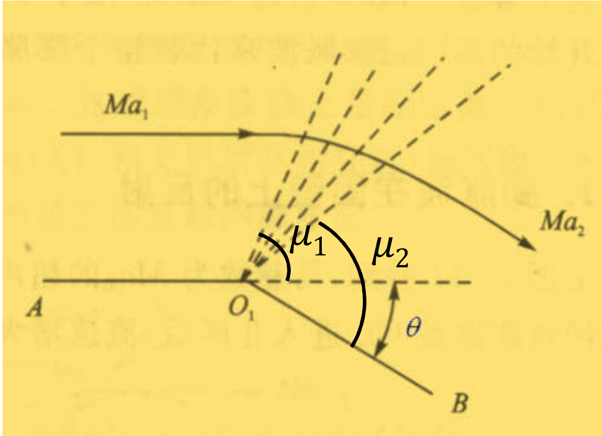
$$\theta = \upsilon(Ma_2) - \upsilon(Ma_1) \qquad \upsilon(Ma) = \sqrt{\frac{\gamma+1}{\gamma-1}}\arctan\sqrt{\frac{\gamma+1}{\gamma-1}(Ma^2-1)} - \sqrt{Ma^2-1}$$

附表C: $Ma \sim \upsilon, \mu$

$$\left.\begin{array}{r} Ma_1 \rightarrow \upsilon(Ma_1) \\ \theta \end{array}\right\} \rightarrow \upsilon(Ma_2) \rightarrow Ma_2 \rightarrow \frac{p_0}{p_2}, \frac{\rho_0}{\rho_2}, \frac{T_0}{T_2}$$

附录A: $\frac{p_0}{p_1}, \frac{\rho_0}{\rho_1}, \frac{T_0}{T_1}$

# 11.4马赫波与膨胀波(8.2,8.7)




$\theta = \upsilon(Ma_2) - \upsilon(Ma_1)$

例：$Ma_1 = 1.5, p_1 = 1atm, T_1 = 288K, \theta = 15°$。
求波后$Ma_2$，$p_2$，$T_2$，$p_{02}$，$T_{02}$，前后波夹角$\alpha$。

解：$Ma_1 = 1.5 \rightarrow \upsilon_1 = 11.91°, \mu_1 = 41.8°$ (附表C)

$\upsilon_2 = \upsilon_1 + \theta = 26.91° \rightarrow Ma_2 = 2.01, \mu_2 = 30°$

$\alpha = \mu_1 + \theta - \mu_2 = 26.8°$

附表A：$Ma_1 = 1.5 \rightarrow \frac{p_0}{p_1} = 3.67, \frac{T_0}{T_1} = 1.45$

$Ma_2 = 2.01 \rightarrow \frac{p_0}{p_2} = 7.824, \frac{T_0}{T_2} = 1.8$

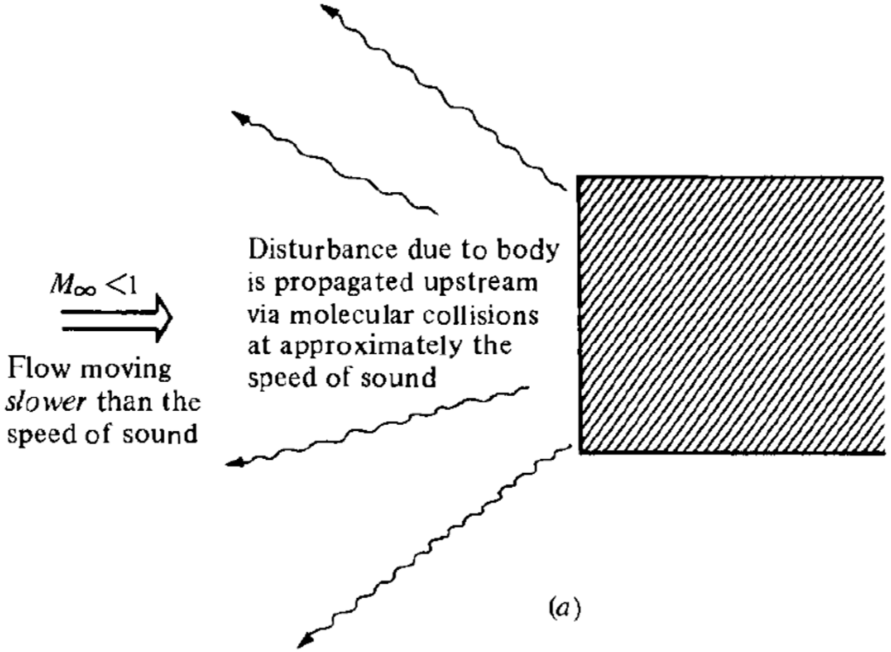
$p_2 = \frac{p_2}{p_0}\frac{p_0}{p_1}p_1 = 0.469atm$

$T_2 = \frac{T_2}{T_0}\frac{T_0}{T_1}T_1 = 232K$

$p_{01} = p_{02} = 3.671atm$

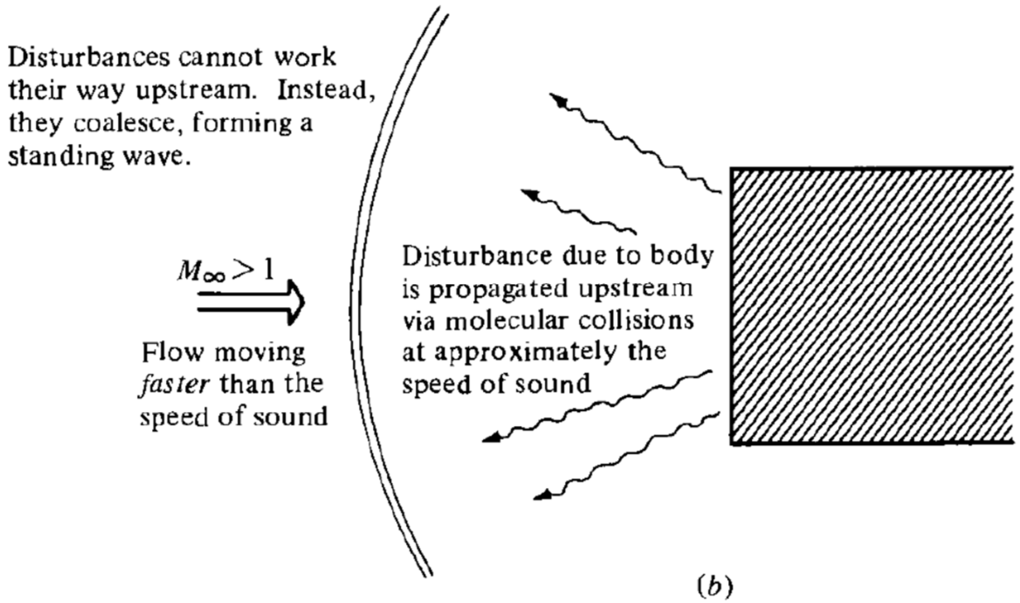
$T_{01} = T_{02} = 417.6K$

# 11.5正激波(7.2,7.5,7.6)



(a)

$Ma < 1$ 扰动向四周传播

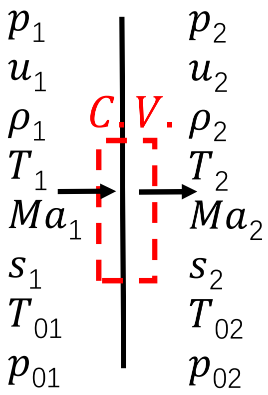


(b)

$Ma > 1$ 扰动限制在固定区域，扰动汇集成激波。

# 11.5正激波(7.2,7.5,7.6)

## 1. 过正激波速度变化：




对C.V.，连续方程：

$$\rho_1 u_1 = \rho_2 u_2 \qquad ①$$

动量方程：

$$(p_1 - p_2)A = \rho_2 u_2^2\, A - \rho_1 u_1^2\, A$$

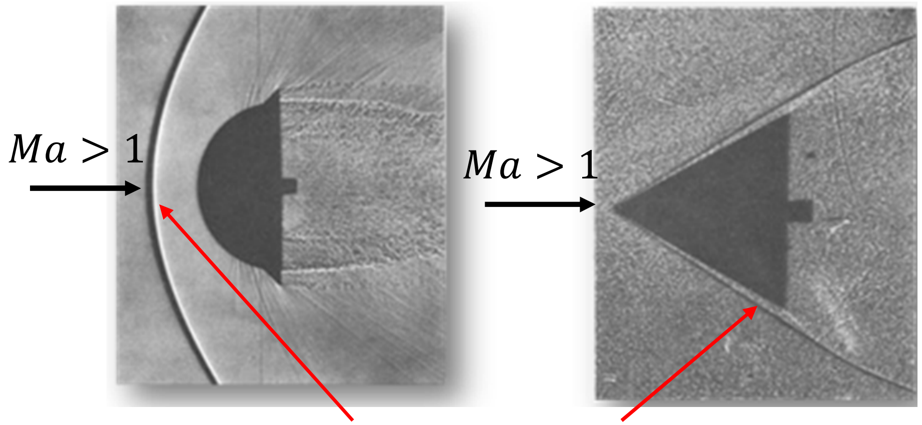
$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2 \qquad ②$$

能量方程：

$$h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2}$$

$$C_p T_1 + \frac{u_1^2}{2} = C_p T_2 + \frac{u_2^2}{2}$$

$$\frac{a_1^2}{(\gamma-1)} + \frac{u_1^2}{2} = \frac{a_2^2}{(\gamma-1)} + \frac{u_2^2}{2} = \frac{\gamma+1}{\gamma-1}\frac{a^{*2}}{2} \qquad ③$$




正激波；斜激波

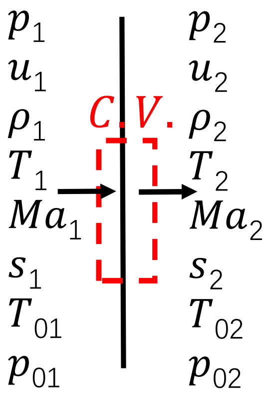
激波：①$10^{-5}$cm厚，相当于分析平均自由程;

②过激波$V$⬇$p$⬆, 强度大, 压缩迅速,

绝热非等熵。参数如何变化？

# 11.5正激波(7.2,7.5,7.6)

1. 过正激波速度变化：




$$\rho_1 u_1 = \rho_2 u_2 \qquad ①$$

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2 \qquad ②$$

$$\frac{a_1^2}{(\gamma-1)} + \frac{u_1^2}{2} = \frac{a_2^2}{(\gamma-1)} + \frac{u_2^2}{2} = \frac{\gamma+1}{\gamma-1}\frac{a^{*2}}{2} \qquad ③$$

$$③ \rightarrow \quad a_1^2 = \frac{\gamma+1}{2}a^{*2} - \frac{\gamma-1}{2}u_1^2$$

$$a_2^2 = \frac{\gamma+1}{2}a^{*2} - \frac{\gamma-1}{2}u_2^2$$

$$② / ① \rightarrow \quad \frac{p_1}{\rho_1 u_1} + u_1 = \frac{p_2}{\rho_2 u_2} + u_2$$

$$\frac{p_1}{\rho_1 u_1} - \frac{p_2}{\rho_2 u_2} = u_2 - u_1$$

$$a^2 = \left(\frac{dp}{d\rho}\right)_s = \gamma\frac{p}{\rho}$$

$$\Rightarrow \quad \frac{a_1^2}{\gamma u_1} - \frac{a_2^2}{\gamma u_2} = u_2 - u_1$$

$$\Rightarrow \quad \frac{\gamma+1}{2}\frac{a^{*2}}{\gamma u_1} - \frac{\gamma-1}{2\gamma}u_1 - \frac{\gamma+1}{2}\frac{a^{*2}}{\gamma u_2} + \frac{\gamma-1}{2\gamma}u_2 = u_2 - u_1$$

$$\frac{(\gamma+1)(u_2 - u_1)}{2\gamma u_1 u_2}a^{*2} + \frac{\gamma-1}{2\gamma}(u_2 - u_1) = u_2 - u_1$$

$$\Rightarrow \quad \frac{\gamma+1}{2\gamma u_1 u_2}a^{*2} + \frac{\gamma-1}{2\gamma} = 1 \quad \Rightarrow \quad \boxed{a^{*2} = u_1 u_2}$$ 普朗特激波关系式

# 11.5正激波(7.2,7.5,7.6)

1. 过正激波速度变化：

| | |
|---|---|
| $p_1$ | $p_2$ |
| $u_1$ | $u_2$ |
| $\rho_1$ | $\rho_2$ |
| $T_1$ | $T_2$ |
| $Ma_1$ | $Ma_2$ |
| $s_1$ | $s_2$ |
| $T_{01}$ | $T_{02}$ |
| $p_{01}$ | $p_{02}$ |

$$\boxed{a^{*2} = u_1 u_2}$$ 普朗特激波关系式

$$Ma_1^* Ma_2^* = 1$$ 正激波前后速度系数关系：

$$Ma_1^*>1, Ma_2^*<1$$

波前超声速，波后亚声速。

$$Ma^{*2} = \frac{(\gamma+1)Ma^2}{2+(\gamma-1)Ma^2}$$

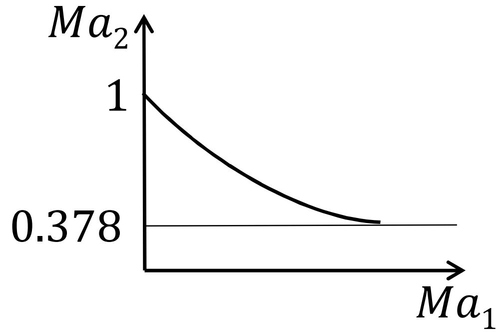
$$\frac{(\gamma+1){Ma_1}^2}{2+(\gamma-1){Ma_1}^2}\frac{(\gamma+1){Ma_2}^2}{2+(\gamma-1){Ma_2}^2}=1$$

正激波前后$Ma$关系 $$\boxed{{Ma_2}^2 = \frac{1+\frac{\gamma-1}{2}{Ma_1}^2}{\gamma {Ma_1}^2-\frac{\gamma-1}{2}}}$$

$Ma_1 = 1 \rightarrow Ma_2 = 1$

$Ma_1\uparrow$，$Ma_2\downarrow$

$Ma_1\rightarrow\infty$，$Ma_2\rightarrow\sqrt{\frac{\gamma-1}{2\gamma}}=0.378$



# 11.5正激波(7.2,7.5,7.6)

2. 过正激波参数变化： $\boxed{a^{*2}=u_1u_2}$

- 正激波前后$Ma$关系 $\boxed{Ma_2^{\ 2}=\dfrac{1+\frac{\gamma-1}{2}Ma_1^{\ 2}}{\gamma Ma_1^{\ 2}-\frac{\gamma-1}{2}}}$ 附录B！

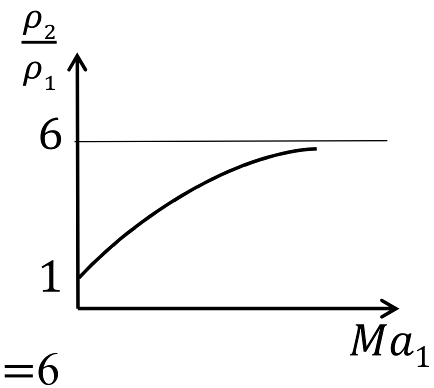
- 正激波前后密度关系

$$\frac{\rho_2}{\rho_1}=\frac{u_1}{u_2}=\frac{u_1^2}{u_1u_2}=\frac{u_1^2}{a^{*2}}=Ma_1^{*}=\frac{(\gamma+1)Ma_1^{\ 2}}{2+(\gamma-1)Ma_1^{\ 2}}$$

$Ma_1>1,\ \frac{\rho_2}{\rho_1}>1$

$Ma_1\uparrow,\quad \frac{\rho_2}{\rho_1}\uparrow$

$Ma_1\rightarrow\infty,\quad \frac{\rho_2}{\rho_1}\rightarrow\frac{\gamma+1}{\gamma-1}=6$

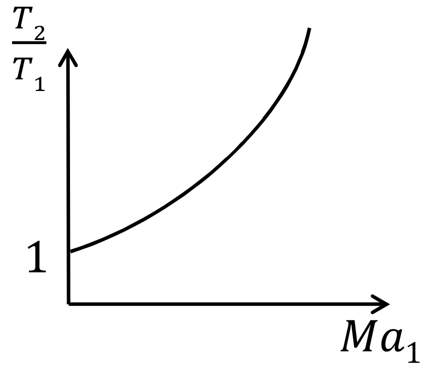



- 正激波前后温度关系

$$\frac{T_2}{T_1}=\frac{T_2}{T_{02}}\frac{T_{02}}{T_1}=\frac{1}{1+\frac{\gamma-1}{2}Ma_2^{\ 2}}\left(1+\frac{\gamma-1}{2}Ma_1^{\ 2}\right)$$

$$=\frac{2+(\gamma-1)Ma_1^{\ 2}}{(\gamma+1)Ma_1^{\ 2}}\left[1+\frac{2\gamma}{\gamma+1}\left(Ma_1^{\ 2}-1\right)\right]$$

$Ma_1\rightarrow\infty,\quad \frac{T_2}{T_1}\rightarrow\infty$




$Ma_1\uparrow,\quad \frac{p_2}{p_1}\uparrow,\ \frac{T_2}{T_1}\uparrow$

- 正激波前后压强关系

$$\frac{p_2}{p_1}=\frac{\rho_2}{\rho_1}\frac{T_2}{T_1}=1+\frac{2\gamma}{\gamma+1}\left(Ma_1^{\ 2}-1\right)$$

$Ma_1\rightarrow\infty,\quad \frac{p_2}{p_1}\rightarrow\infty$

# 11.5正激波(7.2,7.5,7.6)

2. 过正激波参数变化：

- 激波强度： $P=\frac{\Delta p}{p_1}=\frac{p_2-p_1}{p_1}=\frac{2\gamma}{\gamma+1}(Ma_1{}^2-1)$　　$Ma_1>1, \Delta p>0$
  $Ma_1$⬆，$P$⬆(激波增强)
- $\frac{\rho_2}{\rho_1},\frac{p_2}{p_1},\frac{T_2}{T_1}, Ma_2 \sim f(Ma_1)$，$Ma_1<1$有解，无激波？？

$$s_2-s_1=C_p ln\frac{T_2}{T_1}-Rln\frac{p_2}{p_1}$$

$$=\gamma C_v ln\frac{T_2}{T_1}-(\gamma-1)C_v ln\frac{p_2}{p_1}$$

$$=\gamma C_v ln\frac{T_2}{T_1}\frac{p_1}{p_2}+C_v ln\frac{p_2}{p_1} \qquad \frac{T}{p}=\frac{1}{R\rho}$$

$$=\gamma C_v ln\frac{\rho_1}{\rho_2}+C_v ln\frac{p_2}{p_1}$$

$$=C_v ln[(\frac{\rho_1}{\rho_2})^\gamma\frac{p_2}{p_1}]=C_v ln\{[\frac{2+(\gamma-1)Ma_1{}^2}{(\gamma+1)Ma_1{}^2}]^\gamma[1+\frac{2\gamma}{\gamma+1}(Ma_1{}^2-1)]\}$$

$Ma_1>1$→$\Delta s>0$: 激波前后熵增，激波薄，V、T梯度大，剧烈压缩，能量耗散

$Ma_1<1$→$\Delta s<0$：不可能!!

# 11.5正激波(7.2,7.5,7.6)

## 2. 过正激波参数变化：

- 总压变化：

$$s_2 - s_1 = C_v ln\{[\frac{2+(\gamma-1)Ma_1^{\ 2}}{(\gamma+1)Ma_1^{\ 2}}]^{\gamma}[1+\frac{2\gamma}{\gamma+1}(Ma_1^{\ 2}-1)]\}$$

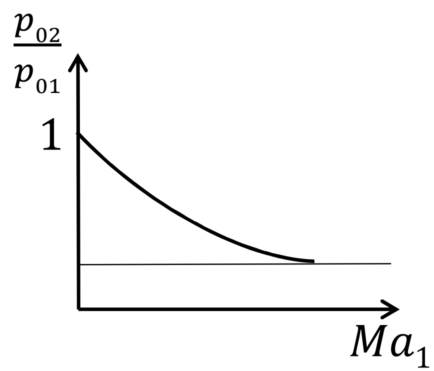
$$s_2 - s_1 = (\gamma-1)C_v ln\frac{p_{01}}{p_{02}}$$

$$\sigma = \frac{p_{02}}{p_{01}} = [\frac{2+(\gamma-1)Ma_1^{\ 2}}{(\gamma+1)Ma_1^{\ 2}}]^{\frac{-\gamma}{\gamma-1}}(\frac{2\gamma}{\gamma+1}Ma_1^{\ 2}-\frac{\gamma-1}{\gamma+1})^{\frac{-1}{\gamma-1}}$$ 附录B！

激波后静压增大($\frac{p_2}{p_1}>1$)，总压减小($\frac{p_{02}}{p_{01}}<1$)！

$\Delta p_0 = p_{01} - p_{02}$，激波阻力，

有用能→无用摩擦热



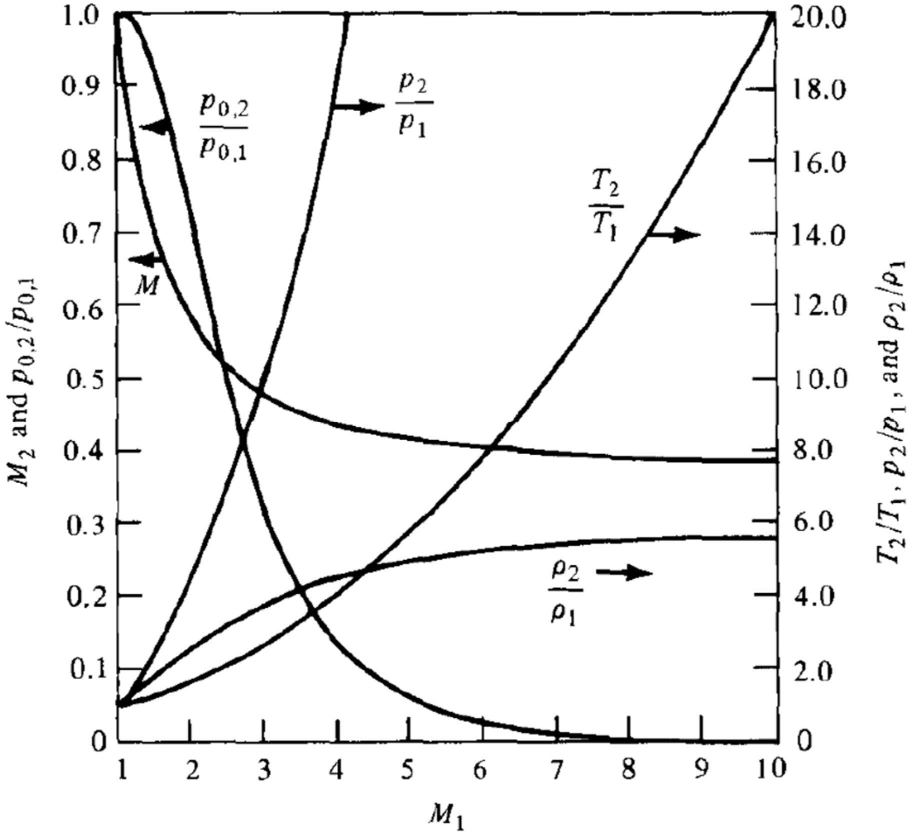

$Ma_1$↑，$\frac{p_{02}}{p_{01}}$↓，$\Delta p_0$↑

$Ma_1 \to \infty$，$\frac{p_{02}}{p_{01}} \to C$

# 11.5正激波(7.2,7.5,7.6)

2. 过正激波参数变化：

| | |
|---|---|
| $p_1$ | $p_2>p_1$ |
| $\rho_1$ | $\rho_2>\rho_1$ |
| $T_1$ | $T_2>T_1$ |
| $Ma_1$ | $Ma_2<Ma_1$ |
| $s_1$ | $s_2>ps_1$ |
| $T_{01}$ | $T_{02}=T_{01}$ |
| $p_{01}$ | $p_{02}<p_{01}$ |



# 11.5正激波(7.2,7.5,7.6)

## 2. 过正激波参数变化：

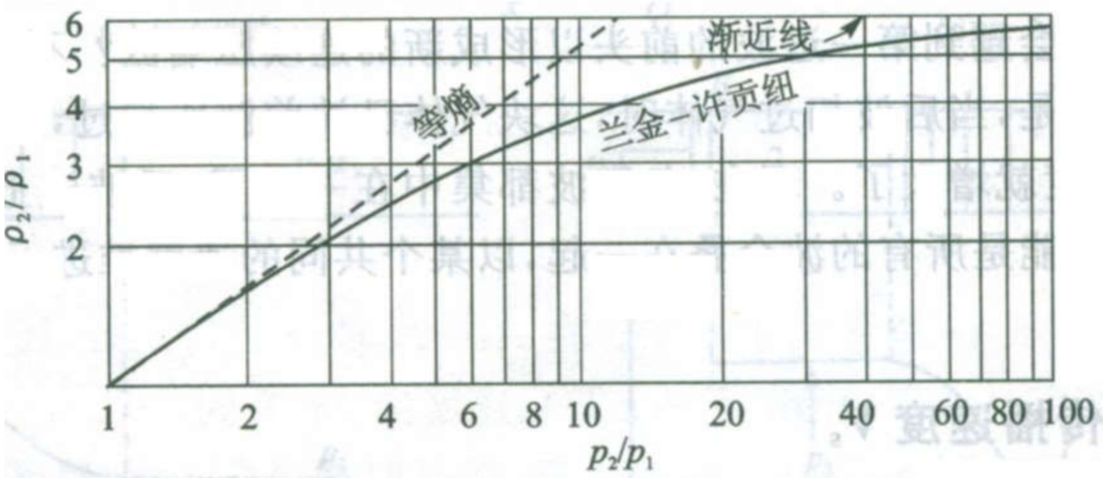
- 兰金-许贡纽方程：

等熵：$\frac{p_2}{p_1}=(\frac{\rho_2}{\rho_1})^{\gamma}$，$\frac{\rho_2}{\rho_1}=(\frac{p_2}{p_1})^{1/\gamma}$

激波：$\frac{p_2}{p_1}=1+\frac{2\gamma}{\gamma+1}(Ma_1{}^2-1)$，$\frac{\rho_2}{\rho_1}=\frac{(\gamma+1)Ma_1{}^2}{2+(\gamma-1)Ma_1{}^2}$

$$\rightarrow \frac{\rho_2}{\rho_1}=\frac{\frac{p_2}{p_1}+\frac{\gamma-1}{\gamma+1}}{\frac{\gamma-1}{\gamma+1}\cdot\frac{p_2}{p_1}+1}$$

绝热突跃(有耗散)关系式！



突跃绝热及等熵关系曲线，$\gamma=1.4$

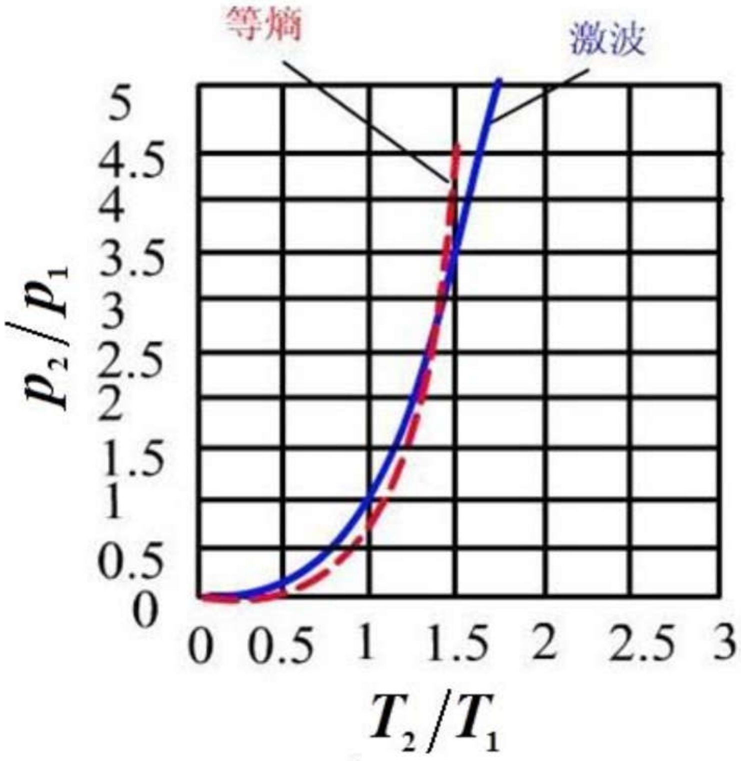
- 弱激波$\frac{p_2}{p_1}$小，近似等熵；
- $\frac{p_2}{p_1}\rightarrow\infty$，$\frac{\rho_2}{\rho_1}\rightarrow 6$

# 11.5正激波(7.2,7.5,7.6)

2. 过正激波参数变化：

$$\frac{\rho_2}{\rho_1}=\frac{\frac{p_2}{p_1}+\frac{\gamma-1}{\gamma+1}}{\frac{\gamma-1}{\gamma+1}\cdot\frac{p_2}{p_1}+1},\quad \frac{T_2}{T_1}=\frac{p_2}{p_1}\frac{\rho_1}{\rho_2}$$

- 相同$\frac{p_2}{p_1}$下，激波后温升更高；



# 11.5正激波(7.2,7.5,7.6)

## 2. 过正激波参数变化：

例：正激波，波前$u_1 = 680 m/s, p_1 = 1 atm, T_1 = 288K$。求波后$u_2$，$p_2$，$T_2$。

解： $a_1 = \sqrt{\gamma RT} = \sqrt{1.4 \times 287 \times 288} = 340\ m/s$

$Ma_1 = u_1/a_1 = 680/340 = 2$ 查附录B:

$\frac{p_2}{p_1} = 4.5 \quad \frac{\rho_2}{\rho_1} = 2.667 \quad \frac{T_2}{T_1} = 1.688 \quad Ma_2 = 0.5773$

$p_2 = 4.5 atm \quad T_2 = 486K \quad a_2 = \sqrt{\gamma RT_2} = 442\ m/s$

$u_2 = Ma_2 a_2 = 255\ m/s$

# 11.5正激波(7.2,7.5,7.6)

2. 过正激波参数变化：

例：$p_1 = 1atm$, 求$\Delta p_0$，若$(a)Ma_1 = 2$， $(b)Ma_1 = 4$ 。

解： $Ma_1 = 2$

查附录A: $\frac{p_1}{p_{01}} = 0.1278 \quad p_{01} = 7.824atm$

查附录B: $\frac{p_{02}}{p_{01}} = 0.7209 \quad p_{02} = 5.64atm$

$\Delta p_0 = 2.184atm$

$Ma_2 = 4$

查附录A: $\frac{p_1}{p_{01}} = 6.586 \times 10^{-3} \quad p_{01} = 151.8atm$

查附录B: $\frac{p_{02}}{p_{01}} = 0.3188 \quad p_{02} = 21.07atm$

$\Delta p_0 = 130.7atm$

$\boxed{Ma_1\downarrow，\Delta p_0\downarrow}$ 减小波前$Ma$，降低耗散！

# 11.5正激波(7.2,7.5,7.6)

3. 可压流速度测量：

- 不可压皮托管测速：$V=\sqrt{\frac{2(P_0-P)}{\rho}}$　　可压皮托管测速$V=$ ？ ？
- 亚声速$Ma_1<1$:

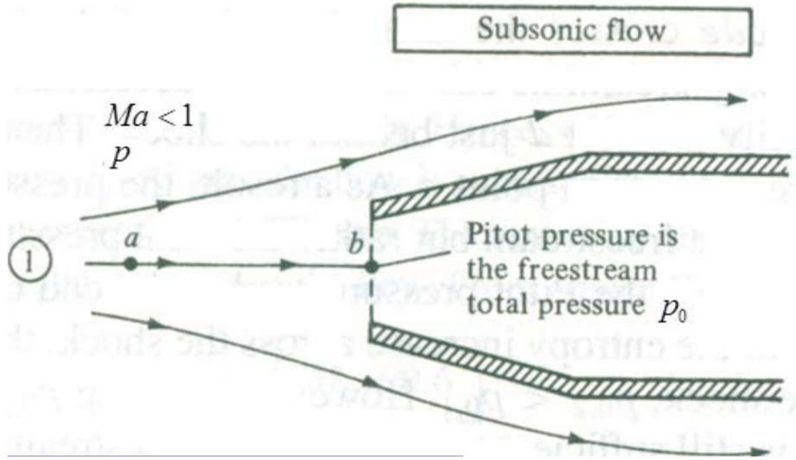
$a-b$等熵过程:

$$P_b=P_{01}$$

$$\frac{p_{01}}{p_1}=(1+\frac{\gamma-1}{2}Ma_1{}^2)^{\frac{\gamma}{\gamma-1}}$$

$$Ma_1{}^2=\frac{2}{\gamma-1}[(\frac{p_{01}}{p_1})^{\frac{\gamma-1}{\gamma}}-1]$$

$$\boxed{u_1{}^2=\frac{2a_1{}^2}{\gamma-1}[(\frac{p_{01}}{p_1})^{\frac{\gamma-1}{\gamma}}-1]}$$

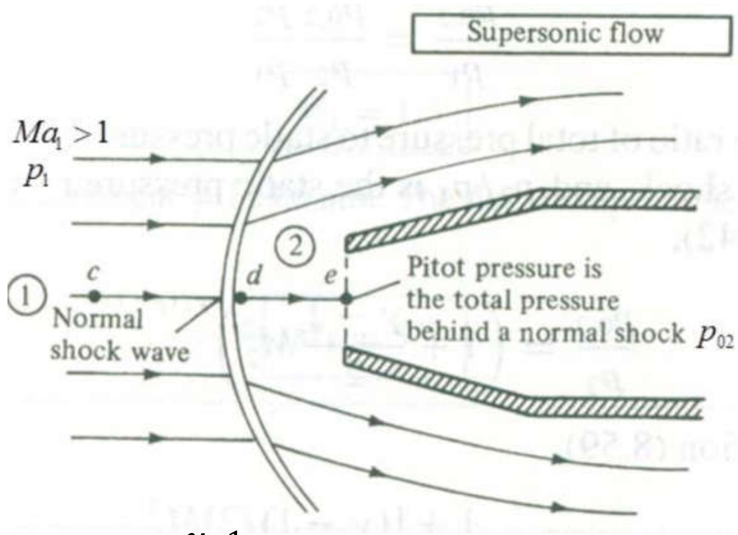



$$\frac{p_{01}}{p_1}\rightarrow Ma_1\rightarrow u_1$$

查附录A

# 11.5正激波(7.2,7.5,7.6)

## 3. 可压流速度测量：




➢ 超声速$Ma_1 > 1$:  $P_e = P_{02} < P_{01}$

$$\frac{p_{02}}{p_1} = \frac{p_{02}}{p_2}\frac{p_2}{p_1}$$

$$= \left(1 + \frac{\gamma-1}{2}Ma_2^2\right)^{\frac{\gamma}{\gamma-1}}\left(\frac{2\gamma}{\gamma+1}Ma_1^2 - \frac{\gamma-1}{\gamma+1}\right) \qquad Ma_2^2 = \frac{1+\frac{\gamma-1}{2}Ma_1^2}{\gamma Ma_1^2 - \frac{\gamma-1}{2}}$$

$$\boxed{\frac{p_{02}}{p_1} = \left[\frac{(\gamma+1)^2 Ma_1^2}{4\gamma Ma_1^2 - 2(\gamma-1)}\right]^{\frac{\gamma}{\gamma-1}}\left(\frac{2\gamma}{\gamma+1}Ma_1^2 - \frac{\gamma-1}{\gamma+1}\right)} = f(Ma_1) \qquad \text{附录B!}$$

雷利空速管公式

$$\frac{p_{02}}{p_1} \rightarrow Ma_1 \rightarrow u_1$$

查附录B

# 11.5正激波(7.2,7.5,7.6)

## 3. 可压流速度测量：

例：$p_1 = 1atm$, 求$Ma$，若$p_{pitot} = 1.276atm, 2.714atm$ , $12.06atm$ 。

解： 亚声速、超声速？？

若$Ma_1 = 1$： $\frac{p_{01}}{p_1} = 1.893$

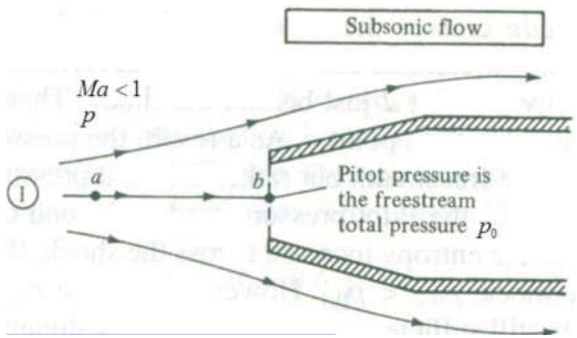
$(a)$ $p_{01} = 1.276 < 1.893$ $Ma_1 < 1$(亚声速)

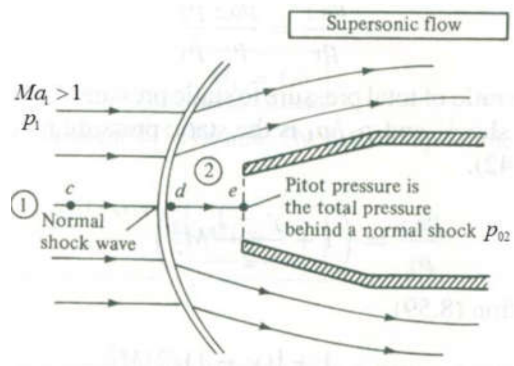
$\frac{p_{01}}{p_1} = 1.276$ 查附录A→ $Ma_1 = 0.6$

$(b)$ $p_{pitot} = 2.714 > 1.893$ $Ma_1 > 1$(超声速)

$\frac{p_{02}}{p_1} = 2.714$ 查附录B→ $Ma_1 = 1.3$

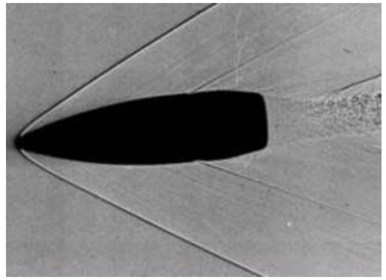
$(c)$ $\frac{p_{02}}{p_1} = 12.06$ 查附录B→ $Ma_1 = 3$



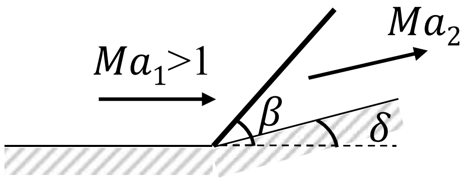



# 11.6斜激波(8.3)

超声速气流过凹壁或楔形体时，转折点产生强压缩波——斜激波！



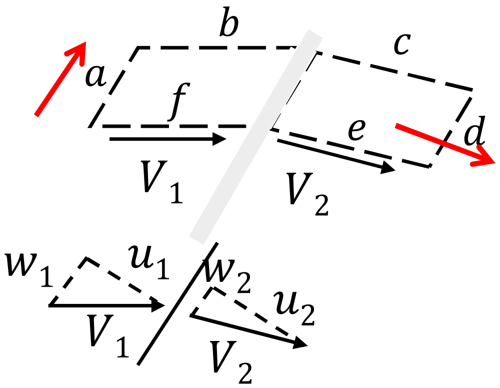
## 1. 参数：




$\delta$：气流折角(波前后气流夹角)

$\beta$：激波倾斜角(波面与来流夹角)

## 2. 控制方程：




质量守恒： $-\rho_1 u_1 A + \rho_2 u_2 A = 0 \rightarrow \rho_1 u_1 = \rho_2 u_2$ ①

切向动量方程：$\sum F_\tau = 0 = -\rho_1 u_1 A w_1 + \rho_2 u_2 A w_2$ ②

①+ ②→ $\boxed{w_1 = w_2}$

法向动量方程：$p_1 A - p_2 A = -\rho_1 u_1 A u_1 + \rho_2 u_2 A u_2$

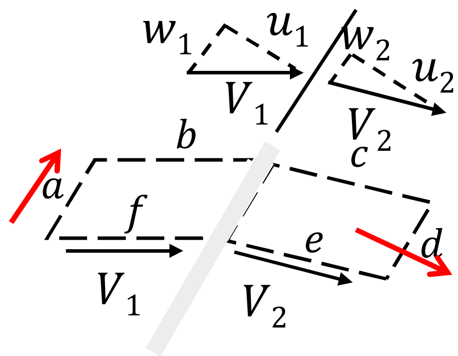
$$p_1 + \rho_1 {u_1}^2 = p_2 + \rho_2 {u_2}^2 \quad ③$$

能量方程： $0 = -\rho_1 u_1 A\left(h_1 + \frac{{V_1}^2}{2}\right) + \rho_2 u_2 A\left(h_2 + \frac{{V_2}^2}{2}\right)$

$$h_1 + \frac{{V_1}^2}{2} = h_2 + \frac{{V_2}^2}{2}$$

# 11.6斜激波(8.3)

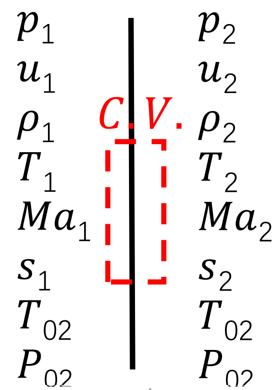
## 2. 控制方程：



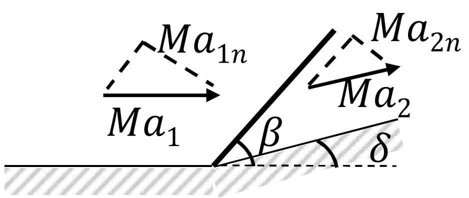

$$\boxed{\rho_1 u_1 = \rho_2 u_2 \quad ①}$$ $$\boxed{p_1 + \rho_1 {u_1}^2 = p_2 + \rho_2 {u_2}^2 \quad ②}$$

$$w_1 = w_2 \qquad h_1 + \frac{{V_1}^2}{2} = h_2 + \frac{{V_2}^2}{2}$$

$$\boxed{h_1 + \frac{{u_1}^2}{2} = h_2 + \frac{{u_2}^2}{2} \quad ③}$$



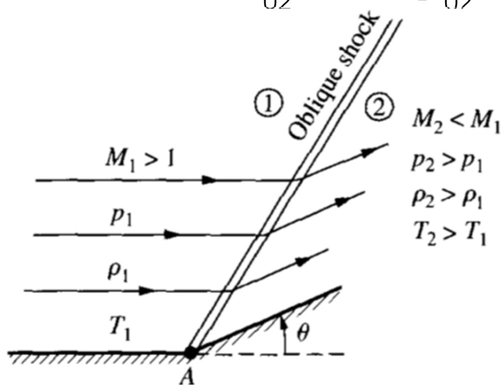

方程①②③可看作波前后速度为$u_1, u_2$的正激波控制方程！




斜激波可视为

$$Ma_{1n} = Ma_1 sin\beta$$

$$Ma_{2n} = Ma_2 \sin(\beta - \delta)$$ 的正激波。



作业:

复习笔记!

空气动力学书7.7~7.8