空气与气体动力学

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5.8 给定速度场：$u=2y+3z$，$v=3z+x$，$w=2x+4y$，如速度场以 m/s 计，液体的粘度 $\mu=0.008\,Pa\cdot s$，试求切应力。

解：$\tau_{yx}=\tau_{xy}=\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)=0.008\times(2+1)=0.024\,Pa$　　$\tau_{xx}=2\mu\frac{\partial u}{\partial x}=0$

$\tau_{zx}=\tau_{xz}=\mu\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)=0.008\times(3+2)=0.040\,Pa$　　$\tau_{yy}=2\mu\frac{\partial v}{\partial y}=0$

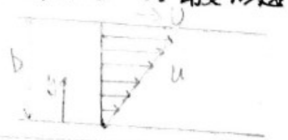
$\tau_{yz}=\tau_{zy}=\mu\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)=0.008\times(3+4)=0.056\,Pa$　　$\tau_{zz}=2\mu\frac{\partial w}{\partial z}=0$

3.20 两无限大平行平板间充满不可压缩粘性流体，上板以恒定速度U向右运动，下板固定不动，此时两平板间的速度分布是线性的，$u=\frac{Uy}{b}$，试确定 1) 相对体积膨胀率 2) 旋转角速度矢量 3) 角变形速率

解：1) $\vec{\nabla}\cdot\vec{V}=0$

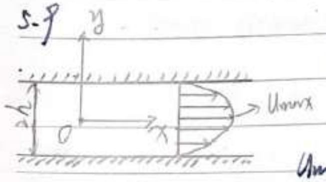
2) $\vec{\omega}=\frac{1}{2}\vec{\nabla}\times\vec{V}=-\frac{1}{2}\frac{U}{b}\vec{k}$

3) $\gamma_{yx}=\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}=\frac{U}{b}$



Campus

5.9 已知：两无限大平板之间层流流场：



$\frac{u}{u_{max}}=1-\left(\frac{y}{h}\right)^2$

$u_{max}=3\,m/s$，$h=40\,mm$，

$\mu=1.002\times10^{-3}\,Pa\cdot S$.

求：沿x方向单位体积流体所受到的切应力。

解：由已知得：

$u=u_{max}\left(1-\left(\frac{y}{h}\right)^2\right)=3-\frac{3}{0.0016}y^2$

则 $\tau=\mu\frac{du}{dy}=-\frac{3.0\ldots}{0.0\ldots}$

则：$F_s=\left(\frac{\partial\tau}{\partial x}+\ldots\right.$

单位体积切应力：$\vec{\nabla}\cdot\tau_{ij}$

3.18 $\vec{\Omega}=\vec{\nabla}\times\vec{V}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ u & v & w\end{vmatrix}$

$\Omega_i=-\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}=0$

$\Omega_j=\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}=\frac{\partial u}{\partial z}=0$

$\Omega_k=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}=-\frac{\partial u}{\partial y}=(a+2by)$

$\vec{\Omega}=(0,0,-a-2by)\neq0$. 有旋

(4)

角变形率 $\varepsilon_{ij}=$ [redacted] $\left(\frac{\partial u_j}{\partial x_i}+\frac{\partial u_i}{\partial x_j}\right)\ (i\neq j)$

$\varepsilon_{12}=$ [redacted] $\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)=$ [redacted] $\frac{\partial u}{\partial y}=$ [redacted] $(a+2by)$

$\varepsilon_{13}=$ [redacted] $\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)=$ [redacted] $\frac{\partial u}{\partial z}=0$

$\varepsilon_{23}=$ [redacted] $\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)=0$

$\varepsilon_{12}=0$ 则 $\frac{1}{2}(a+2by)=0$

$a=b=0$

5.9 单位体积切应力为 $\mu\nabla^2\vec{V}$　(3)

x向单位体积切应力为 $\mu\left(\frac{\partial^2u}{\partial x^2}+\frac{\partial^2u}{\partial y^2}+\frac{\partial^2u}{\partial z^2}\right)$

$=\left[0-\frac{2u_{max}}{h^2}+0\right]\mu$

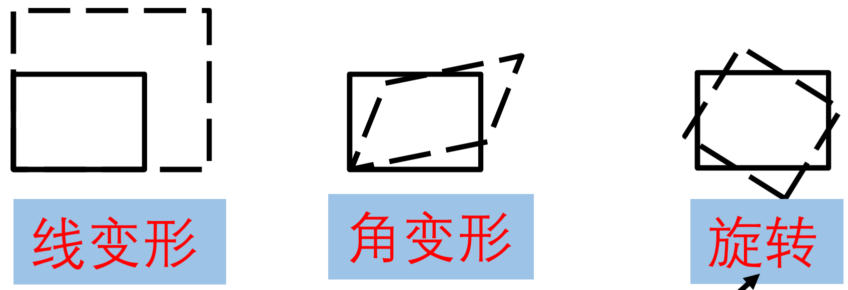
$=1.002\times10^{-3}\times\left(-\frac{2\times3}{0.04^2}\right)$

$=-3.7575\,(N/m^3)$

$$\gamma_{ij}=\frac{\partial u_i}{\partial xj}+\frac{\partial uj}{\partial xi}$$

5.4 流体微团的运动与变形（3.4）

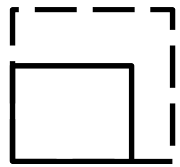
① 流体运动和变形：



$$\frac{\partial u_i}{\partial xj} = \underbrace{\boxed{\frac{1}{2}\left(\frac{\partial u_i}{\partial xj}+\frac{\partial uj}{\partial xi}\right)}}_{\epsilon_{ij}\ (\text{对称})} + \underbrace{\boxed{\frac{1}{2}\left(\frac{\partial ui}{\partial xj}-\frac{\partial uj}{\partial xi}\right)}}_{\omega_{ij}\ (\text{反对称})}$$

ϵ_{11}、ϵ_{22}、ϵ_{33} →线性变形率

ϵ_{12}、ϵ_{23}、ϵ_{13} →角变形率 $\gamma_{ij} = \frac{\partial u_i}{\partial xj} + \frac{\partial uj}{\partial xi}$



$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = \vec{\nabla} \cdot \vec{V}$$

流体微团的相对体膨胀率

$$\vec{\nabla} \cdot \vec{V} = 0$$

流体微团体积不变，

即不可压！

4.3 某平面不可压缩流动沿 x 方向的速度分量为 $u = Ax$，式中 $A = 2\ \mathrm{s}^{-1}$，坐标长度单位为 m，已知(0,0)点的表压强 $p_0 = 190\ \mathrm{kPa}$，流体密度 $\rho = 1.50\ \mathrm{kg/m^3}$，$z$ 轴铅垂向上。试确定最简单的 y 方向的速度分量表示式，计算(2,1)点的流体加速度和压强梯度，并求压强沿 x 方向的变化。

4.3.(1) 不可压连续性方程：$\nabla\cdot\vec{V}=0$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$$A+\frac{\partial v}{\partial y}=0$$

$$\frac{\partial v}{\partial y}=-2$$

简单 $v=-2y$

$\vec{V}=2x-2y$

(2) $\vec{a}=\frac{D\vec{V}}{Dt}=(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y})\vec{i}+(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y})\vec{j}$

$=(2x\cdot2-2y\cdot0)\vec{i}+[2x\cdot0-2y\cdot(-2)]\vec{j}$

$=4x\vec{i}+4y\vec{j}$

$\vec{a}(2,1)=8\vec{i}+4\vec{j}$

(3) $\boxed{\text{欧拉方程：}}$ 沿x方向：$\rho(u\frac{\partial u}{\partial x}+\underbrace{v\frac{\partial u}{\partial y}}_{0})=-\frac{\partial p}{\partial x}+\underbrace{\cancel{\rho g_x}}_{0}$　沿y方向：$\rho(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y})=-\frac{\partial p}{\partial y}+\underbrace{\cancel{\rho g_y}}_{0}$

$$\frac{\partial p}{\partial x}=-6x \qquad \frac{\partial p}{\partial y}=-6y$$

沿z向：$\cancel{\rho(u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y})}\ 0=-\frac{\partial p}{\partial z}+\rho g_z$

$\frac{\partial p}{\partial z}=\rho g_z=14.7$

$\nabla p=-6x\vec{i}-6y\vec{j}+14.7\vec{k}$

$\nabla p(2,1)=-12\vec{i}-6\vec{j}+14.7\vec{k}$

沿x方向(y恒定) $\frac{dp}{dx}=-6x$

$p=-3x^2+C$

$p(0,0)=C=190\mathrm{kPa}$

$\therefore p(x)=(190000-3x^2)\ \mathrm{Pa}$

5.10 已知 $\vec{V}=kx\vec{i}+ky\vec{j}-2kz\vec{k}$

DV/Dt求错！！

解：(1) 连续性方程：$\nabla\cdot\vec{V}=0$.

本题中 $\nabla\cdot\vec{V}=\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=k+k-2k=0$. 满足连续方程.

N-S 方程：$\rho\frac{D\vec{V}}{Dt}=\rho\vec{g}-\nabla p+\mu\nabla^2\vec{V}$ 流体不可压，且为牛顿流体.

∴满足 N-S 方程

(2) 由 $\rho\frac{D\vec{V}}{Dt}=\rho\vec{g}-\nabla p+\mu\nabla^2\vec{V}$

$\therefore \rho k\vec{i}=-\frac{\partial p}{\partial x}$

$\rho k\vec{j}=-\frac{\partial p}{\partial y}$

$-2\rho k\vec{k}=-\frac{\partial p}{\partial z}-\rho g\vec{k}$

$\Rightarrow$

$P(x)=(-\rho kx\vec{i}+C_1)\vec{i}$

$P(x)=(-\rho ky+C_2)\vec{j}$

~~$P(z)=(-2\rho kz+C_3)\vec{k}$~~

$P(z)=(2\rho kz-\rho g+C_3)\vec{k}$

C_1,C_2,C_3 为常数.

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(2) $\rho\cdot\frac{D\vec{v}}{Dt}=-\nabla p+\rho\vec{g}+\mu\nabla^2\vec{v}$ 定常.

x方向：$\rho\left(\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)$

代入，$\rho\cdot(k+0+0)=-\frac{\partial p}{\partial x}+\mu\cdot 0$，$\frac{\partial p}{\partial x}=-\rho k$

同理，y方向：$\frac{\partial p}{\partial y}=-\rho k$

z方向：$\frac{\partial p}{\partial z}=2\rho k-g$

~~故 $p(x,y,z)=-\rho kx-\rho ky+(2\rho k-g)z+C$~~

5.10 解：(1) $u=kx$，$v=ky$，$w=-2kz$

$\nabla\cdot\vec{V}=\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}$

$=k+k-2k$

$=0$

∴这一速度场满足不可压连续方程.

因此满足 N-S方程.

(2) N-S方程：

x方向：$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)+\rho g_x$

$\rho(kx\cdot k)=-\frac{\partial p}{\partial x}\Rightarrow\frac{\partial p}{\partial x}=-\rho k^2x$ —— ①

y方向：$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)+\rho g_y$

$\rho(ky\cdot k)=-\frac{\partial p}{\partial y}\Rightarrow\frac{\partial p}{\partial y}=-\rho k^2y$ —— ②

z方向：$\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)-\rho g$

$\rho[-2kz\cdot(-2k)]=-\frac{\partial p}{\partial z}-\rho g\Rightarrow\frac{\partial p}{\partial z}=-\rho(4k^2z+g)$ —— ③

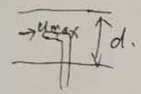
①+②+③ $\Rightarrow$ $dP=-\rho[k^2xdx+k^2ydy+(4k^2z+g)dz]$

选(0,0,0)为参数：$P(x,y,z)-P_{(0,0,0)}=-\rho\left[\frac{k^2x^2}{2}+k^2\frac{y^2}{2}+2k^2z^2+gz\right]$

$P_{(x,y,z)}=P_{(0,0,0)}-\frac{\rho}{2}(k^2x^2+k^2y^2+4k^2z^2+2gz)$

$\cdots(190000-3x^2)$ Pa.

4-8. 解.



中轴线上: $\frac{1}{2}\rho_{气}u_{max}^2=\Delta P$

$$u_{max}=\sqrt{\frac{2\Delta P}{\rho_{气}}}$$

$$\Delta P=\rho_{酒精}g\,l\sin\alpha$$

$$=800\times9.8\times0.075\times0.2$$

$$=117.6\ \text{Pa}$$

$$u_{max}=\sqrt{\frac{2\times117.6}{1.66}}\approx12\ \text{m/s}$$

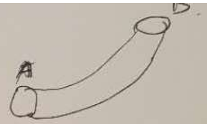
$$\dot m=\rho\bar UA$$

$$=\rho_{气}0.8u_{max}\frac{\pi}{4}d^2$$

$$=1.66\times0.8\times12\times\frac{3.14}{4}\times0.2^2$$

$$=0.5\ \text{kg/s}$$

4.21



解: 已知: $A_A=0.3m^2$, $U_A=1.8m/s$

$P_a=117kPa$

$A_B=0.15m^2$ $h_B-h_A=6m$

求: P_B. 流体为水

解: A-B两点. 伯努利方程

$$P_A+\frac{1}{2}\rho U_A^2+\rho gh_A=P_B+\frac{1}{2}\rho U_B^2+\rho gh_B$$

$$P_B=P_A+\frac{1}{2}\rho(U_A^2-U_B^2)+\rho g(h_A-h_B)$$

$\frac{P}{PV}$

由连续方程: $U_AA_A=U_BA_B$

$$U_B=3.6m/s$$

$$P_B=P_A-\frac{1}{2}\rho(U_B^2-U_A^2)-\rho g(h_B-h_A)$$

$$=[117-\frac{1}{2}(3.6^2-1.8^2)-9.8\times6]kPa$$

$$=53.34\ kPa.$$

回顾：

1. 层流、湍流（Re）

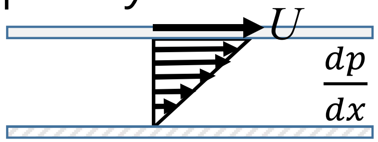
2.入口段，充分发展流动 $\frac{\partial \vec{V}}{\partial x} = 0$

3.无限大平板间充分发展层流（N-S方程简化、求解）

$$\frac{dp}{dx} = \mu \frac{d^2u}{dy^2} = C$$

(a).平板库埃特流动(couette flow, purely shear driven flow)

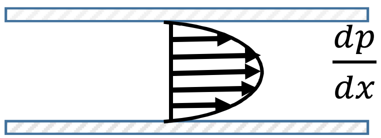
- $u(y) = \frac{U}{a} y$



$\frac{dp}{dx} = 0$

(b).平板泊肃叶流动$\frac{dp}{dx} \neq 0$ (purely pressure driven flow)

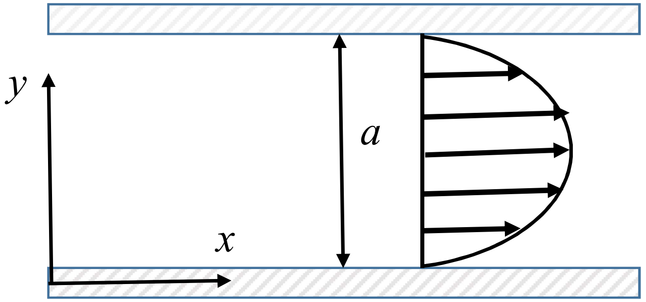
- $u(y) = \frac{a^2}{2\mu} \left(-\frac{dp}{dx}\right) \frac{y}{a} \left(1 - \frac{y}{a}\right)$



$\frac{dp}{dx} < 0$

6.2无限大平板间充发展层流（5.3）

求：$\vec{V}$，τ，Q，Δp

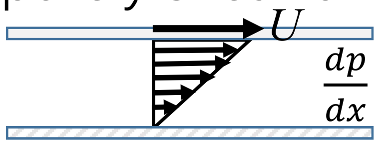


$u_{wall} = 0$

解: $\dfrac{dp}{dx} = \mu \dfrac{d^2u}{dy^2} = C$

(a).平板库埃特流动(couette flow, purely shear driven flow)

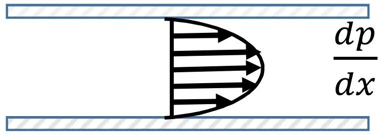
➢ $u(y) = \dfrac{U}{a} y$



$\dfrac{dp}{dx} = 0$

(b).平板泊肃叶流动$\dfrac{dp}{dx} \neq 0$ (purely pressure driven flow)

➢ $u(y) = \dfrac{a^2}{2\mu}\left(-\dfrac{dp}{dx}\right)\dfrac{y}{a}\left(1 - \dfrac{y}{a}\right)$



$\dfrac{dp}{dx} < 0$

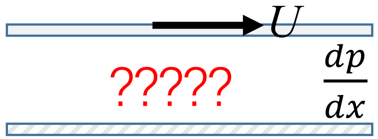
Note:

1.仅适用层流.

Re<2000层流；Re>7700湍流

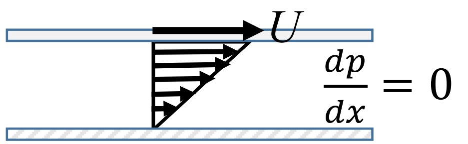
2.入口效应，$L_e/D \approx 0.06Re$

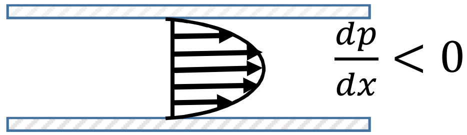
(c).一般(a+b)???



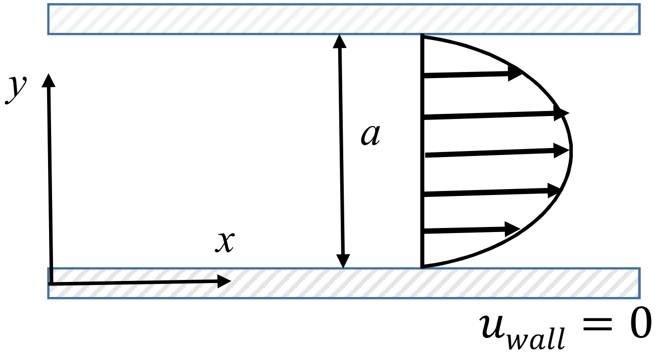
$\dfrac{dp}{dx} < 0$

6.2无限大平板间充发展层流（5.3）





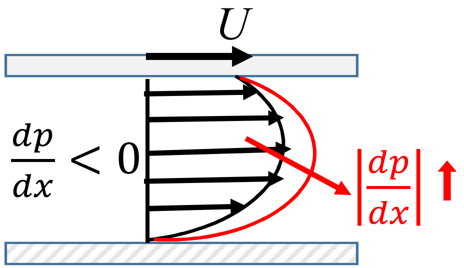
求：$\vec{V}$，τ，Q，Δp

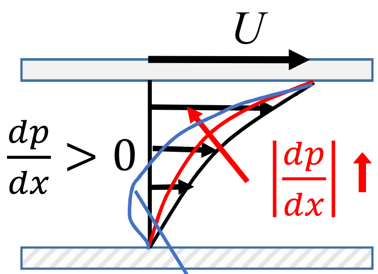


解: $\frac{dp}{dx}=\mu\frac{d^2u}{dy^2}=C$

(c).一般(a+b)

- $u(y)=\frac{U}{a}+y\frac{a^2}{2\mu}(-\frac{dp}{dx})\frac{y}{a}(1-\frac{y}{a})$





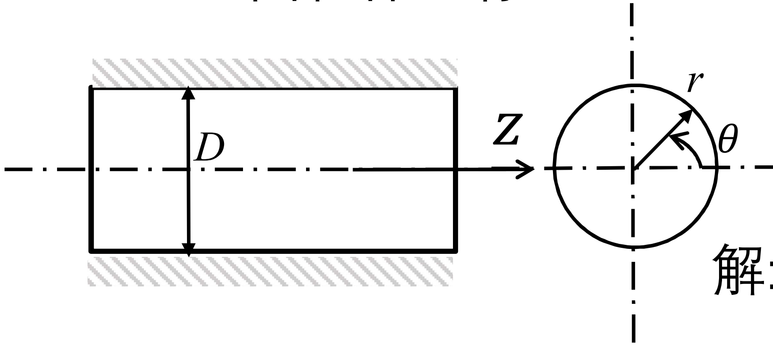
$\frac{dp}{dx}>0$,
逆压可能发生
流动分离 ！！

压强沿流动方向降低→顺压　压强沿流动方向升高→逆压

有回流，产生漩涡，流动分离

6.3圆管内充分发展层流（周向均匀泊肃叶流动5.4）

圆柱体坐标



求：$\vec{V}$，τ，Q，Δp　解微分连续性方程、N-S方程。

假设：牛顿流体、不可压，流动沿轴向（仅有u_z）

充分发展流动($\frac{\partial \vec{V}}{\partial z}=0$)、沿周向均匀($\frac{\partial}{\partial \theta}=0$)、定常($\frac{\partial}{\partial t}=0$)

解：定常、不可压连续性方程：$\vec{\nabla}\cdot\vec{V}=0$

$$\frac{1}{r}\cancel{\frac{\partial(rur)}{\partial r}}+\frac{1}{r}\cancel{\frac{\partial(u_\theta)}{\partial \theta}}+\cancel{\frac{\partial(u_z)}{\partial z}}=0$$

$$\frac{\partial uz}{\partial z}=0 \quad \frac{\partial uz}{\partial \theta}=0 \Rightarrow u_z=u_z(r)$$

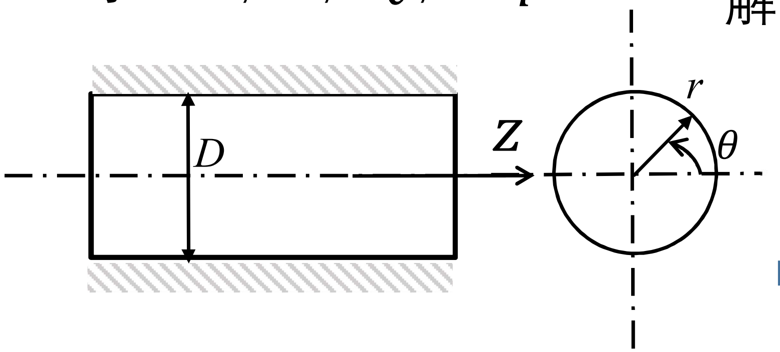
N-S方程：r向：$\rho\left(\cancel{\frac{DV_r}{Dt}-\frac{V_\theta^2}{r}}\right)=-\frac{\partial p^*}{\partial r}+\mu\left(\cancel{\nabla^2 V_r-\frac{V_r}{r^2}-\frac{2}{r^2}\frac{\partial V_\theta}{\partial \theta}}\right)$

$$\frac{D}{Dt}=\frac{\partial}{\partial t}+u_r\frac{\partial}{\partial r}+\frac{u_\theta}{r}\frac{\partial}{\partial \theta}+u_z\frac{\partial}{\partial z}$$

$$\Rightarrow \frac{\partial p}{\partial r}=0$$

6.3圆管内充分发展层流（周向均匀泊肃叶流动5.4）

求：$\vec{V}$，τ，Q，Δp



解: N-S方程：θ向：

$$\frac{\partial p}{\partial r}=0$$

$$\rho\left(\cancel{\frac{\mathrm{D}V_\theta}{\mathrm{D}t}+\frac{V_rV_\theta}{r}}\right)=-\frac{1}{r}\frac{\partial p^*}{\partial\theta}+\mu\left(\cancel{\nabla^2V_\theta-\frac{V_\theta}{r^2}+\frac{2}{r^2}\frac{\partial V_r}{\partial\theta}}\right)$$

$$\Rightarrow\ \frac{\partial p}{\partial\theta}=0\ \Rightarrow\ p=p(z)\qquad p\text{为名义压强}p^*$$

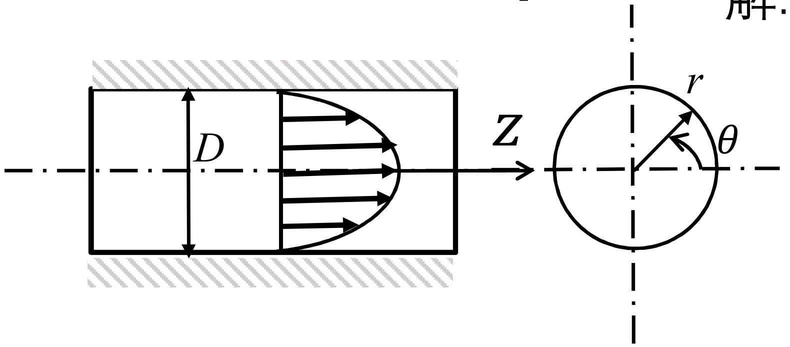
N-S方程：z向：$\rho\cancel{\frac{\mathrm{D}V_z}{\mathrm{D}t}}=-\frac{\partial p^*}{\partial z}+\mu\nabla^2V_z$ $\qquad\frac{\mathrm{D}}{\mathrm{D}t}=\cancel{\frac{\partial}{\partial t}}+u_r\cancel{\frac{\partial}{\partial r}}+\cancel{\frac{u_\theta}{r}\frac{\partial}{\partial\theta}}+u_z\cancel{\frac{\partial}{\partial z}}$

$$\frac{1}{\mu}\frac{dp}{dz}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right)+\frac{1}{r^2}\cancel{\frac{\partial^2u_z}{\partial\theta^2}}+\cancel{\frac{\partial^2u_z}{\partial z^2}}\qquad\frac{\partial\vec{V}}{\partial z}=0$$

$$\frac{1}{\mu}\frac{dp}{dz}=\frac{1}{r}\frac{d}{dr}\left(r\frac{du_z}{dr}\right)$$

6.3圆管内充分发展层流（周向均匀泊肃叶流动5.4）

求：$\vec{V}$，τ，Q，Δp



解:　N-S方程：z向：

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{du_z}{dr}\right)=\frac{1}{\mu}\frac{dp}{dz} \qquad f(r)=g(z)=C$$

$$r\frac{du_z}{dr}=\frac{1}{\mu}\frac{dp}{dz}\frac{r^2}{2}+C_1$$

$$\frac{du_z}{dr}=\frac{1}{\mu}\frac{dp}{dz}\frac{r}{2}+\cancel{C_1/r}$$

$$\left.\frac{du_z}{dr}\right|_{r=0}=0 \;\Rightarrow\; C_1=0$$

（轴对称）

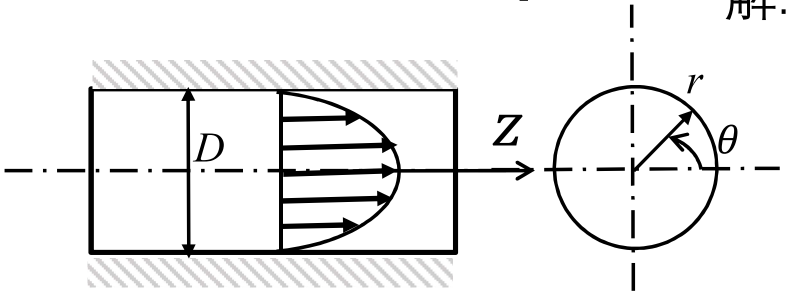
$$u_z=\frac{1}{\mu}\frac{dp}{dz}\frac{r^2}{4}+C_2$$

$$u_z(r=R)=0 \;\Rightarrow\; C_2=-\frac{R^2}{4\mu}\frac{dp}{dz}$$

$$u_z=\frac{R^2}{4\mu}\left(-\frac{dp}{dz}\right)\left(1-\frac{r^2}{R^2}\right)$$

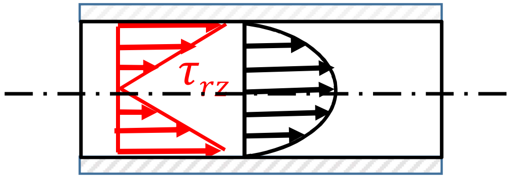
6.3圆管内充分发展层流（周向均匀泊肃叶流动5.4）

求：$\vec{V}$，τ，Q，Δp



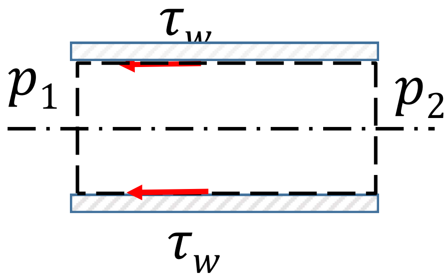
解: $\frac{1}{r}\frac{d}{dr}\left(r\frac{du_z}{dr}\right)=\frac{1}{\mu}\frac{dp}{dz}$

- $u_z=\frac{R^2}{4\mu}\left(-\frac{dp}{dz}\right)\left(1-\frac{r^2}{R^2}\right)$



- Shear stress： $\tau_{rz}=\mu\frac{du_z}{dr}=\frac{dp}{dz}\frac{r}{2}$ $\quad\tau_{wall}=\frac{dp}{dz}\frac{R}{2}$

用积分方程求τ_w??



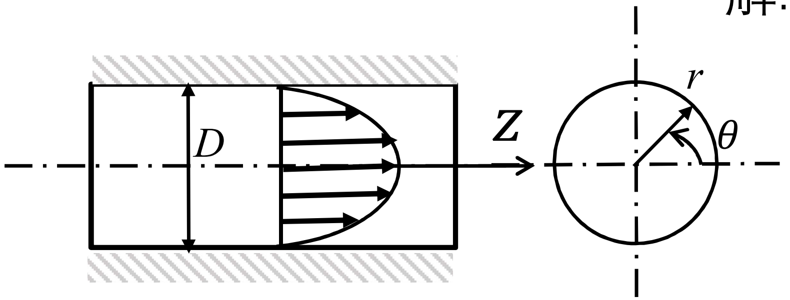
- Volume flow rate:

$$Q=\int_0^R u_z\,2\pi r\,dr=\frac{\pi R^4}{8\mu}\left(-\frac{dp}{dz}\right)=\frac{\pi R^4}{8\mu}\frac{\Delta p}{L}$$

- $\bar{V}=\frac{Q}{A}=\frac{R^2}{8\mu}\left(-\frac{dp}{dz}\right)=\frac{1}{2}u_{max}\quad u_{max}=\frac{R^2}{4\mu}\left(-\frac{dp}{dz}\right)$

6.3圆管内充分发展层流（周向均泊肃叶流动5.4）

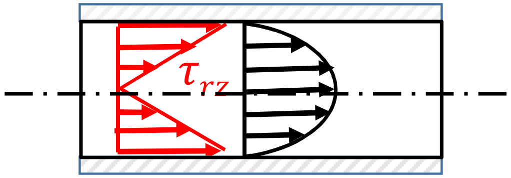
求：$\vec{V}$，τ，Q，　Δp



解:　$\dfrac{1}{r}\dfrac{d}{dr}\left(r\dfrac{du_z}{dr}\right)=\dfrac{1}{\mu}\dfrac{dp}{dz}$

$\tau_{wall}=\dfrac{dp}{dz}\dfrac{R}{2}$, $\bar{V}=\dfrac{R^2}{8\mu}\left(-\dfrac{dp}{dz}\right)$

➢ $u_z=\dfrac{R^2}{4\mu}\left(-\dfrac{dp}{dz}\right)\left(1-\dfrac{r^2}{R^2}\right)$



➢ friction coefficient: $c_f=\dfrac{\tau_w}{0.5\rho\,\bar{V}^2}=\dfrac{16}{Re}$　$Re=\dfrac{\rho\bar{V}D}{\mu}$　自己推导下$c_f\sim Re$ ！

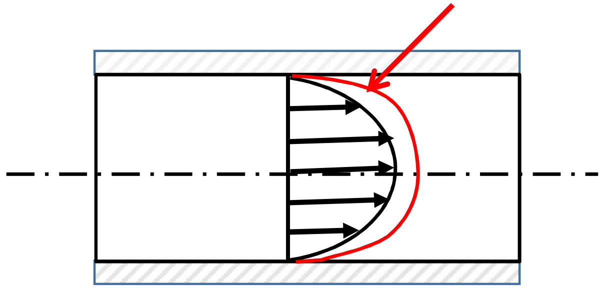
Note:

1.仅适用层流.　Re<Re_{cr}≈ 2000~2300

Re≈ 10^5可维持层流

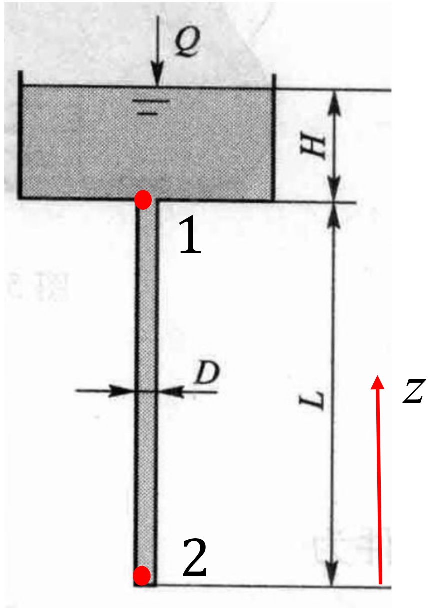
湍流见书5.6~5.8

湍流速度分布，更贴近壁面



6.3圆管内充分发展层流（周向均泊肃叶流动5.4）

毛细管粘度计



圆管层流：$Q=\frac{\pi R^4}{8\mu}\frac{\Delta p}{L}$

$$\mu=\frac{\pi R^4}{8Q}\frac{\Delta p}{L}$$

$$\Delta p=p_{k1}-p_{k2} \qquad p_k=p+\rho g z$$

$$p_{k1}=p_{atm}+\rho g H+\rho g L$$

$$p_{k2}=p_{atm}$$

$$\frac{\Delta p}{L}=\frac{\rho g(H+L)}{L}$$

$$\mu=\frac{\pi R^4}{8Q}\frac{\Delta p}{L}=\frac{\pi R^4}{8Q}\rho g\left(1+\frac{H}{L}\right)$$

Note:

1.忽略入口效应，$L>>D$；

2. 定常，层流，$Re=\frac{\rho\bar{V}D}{\mu}$足够小。

→ D足够小。

毛细管。

单选题 1分

设置

此题未设置答案，请点击右侧设置按钮

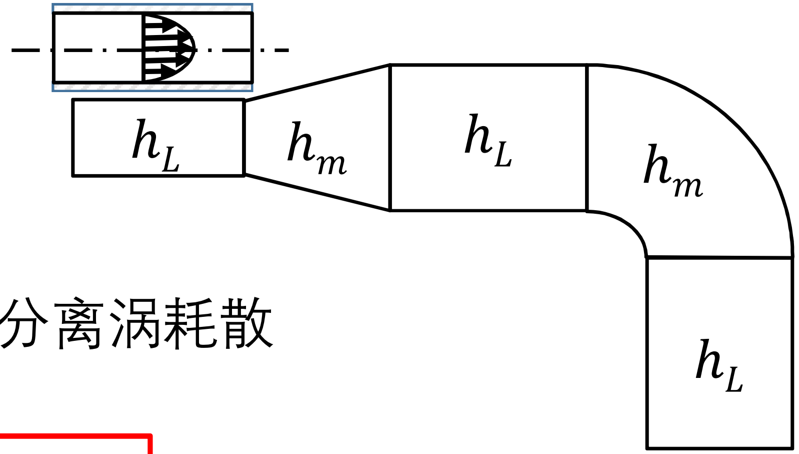
圆管内粘性流动，伯努利方程是否成立？

A 成立

B 不成立

提交

6.4圆管内能量损失（9.2~9.4）



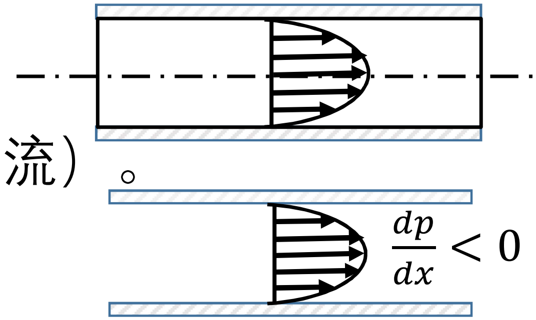
① 水力损失：沿程损失+局部损失

均直管粘性摩擦 连接件流动分离涡耗散

损失水头高度 $h_{LT} = h_L + h_m$

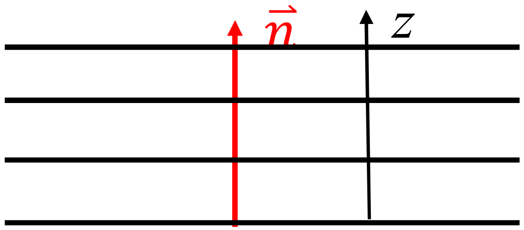
$$Losses = -\boxed{\int_{CS}\left(\frac{V^2}{2} + gz + \frac{p}{\rho}\right)\rho(\vec{V}\cdot\vec{n})dS}$$

◆ 平行剪切流，缓变流



平形剪切流：流体质点沿同一方向运动（等截面平板、圆管层流）。

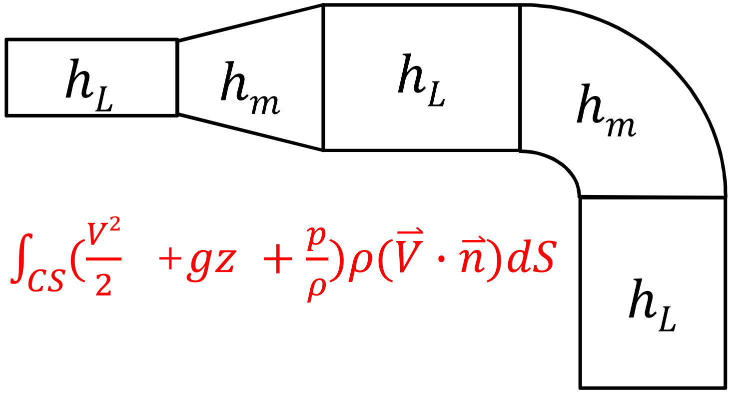
缓变流：流线近似为平行线（夹角很小）。



$$\frac{\partial p_k}{\partial z} = \frac{\rho V^2}{R} \approx 0$$

沿z向：$p_k = p + \rho g z = \text{constant}$

6.4圆管内能量损失（9.2~9.4）

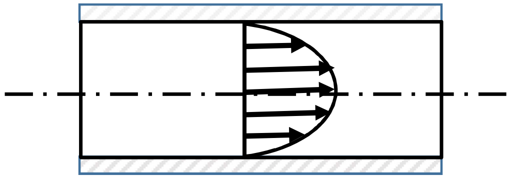


① 水力损失：沿程损失+局部损失

h_{LT} $\quad h_L \quad h_m$

$Losses = -\int_{CS}(\frac{V^2}{2} + gz + \frac{p}{\rho})\rho(\vec{V}\cdot\vec{n})dS$

◆ 动能修正系数：



$$\int_{CS}\frac{V^2}{2}\rho(\vec{V}\cdot\vec{n})dS = \alpha\frac{\bar{V}^2}{2}\rho\bar{V}A = \alpha\dot{m}\frac{\bar{V}^2}{2}$$

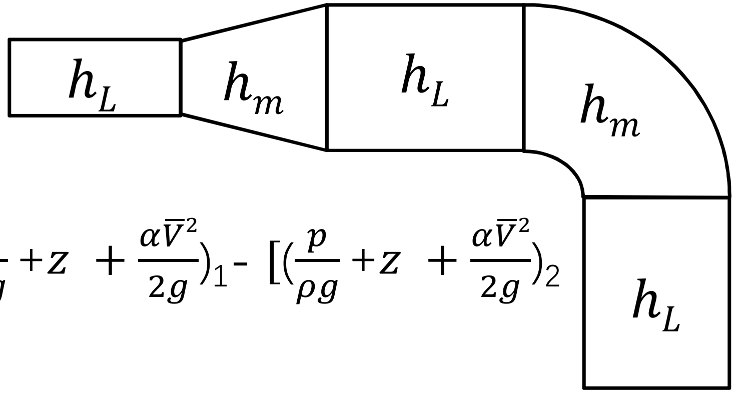
α: 动能修正系数，由$u(r)$定。

$$\begin{cases}\alpha = 1.0 & \text{理想流体} \\ \alpha = 2.0 & \text{充分发展层流}\end{cases}$$

$$Losses = \dot{m}[(\frac{p}{\rho} + gz + \frac{\alpha\bar{V}^2}{2})_1 - [(\frac{p}{\rho} + gz + \frac{\alpha\bar{V}^2}{2})_2] = \dot{m}gh_{LT}$$

$$h_{LT} = (\frac{p}{\rho g} + z + \frac{\alpha\bar{V}^2}{2g})_1 - (\frac{p}{\rho g} + z + \frac{\alpha\bar{V}^2}{2g})_2 = h_L + h_m$$

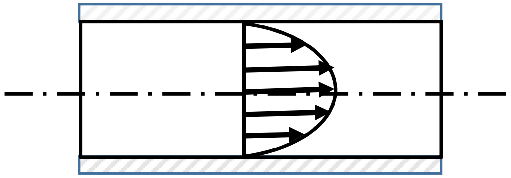
6.4圆管内能量损失（9.2~9.4）



① 水力损失：沿程损失+局部损失

h_{LT} $\qquad$ h_L $\qquad$ h_m $\qquad$ $h_{LT}=(\frac{p}{\rho g}+z+\frac{\alpha\overline{V}^2}{2g})_1-[(\frac{p}{\rho g}+z+\frac{\alpha\overline{V}^2}{2g})_2$

◆ 均直管沿程损失h_L：



$$h_L=(\frac{p}{\rho g}+z+\frac{\alpha\overline{V}^2}{2g})_1-[(\frac{p}{\rho g}+z+\frac{\alpha\overline{V}^2}{2g})_2$$

$$\overline{V}_1=\overline{V}_2 \qquad \alpha_1=\alpha_2 \qquad z_1=z_2$$

$$\Rightarrow h_L=\frac{p_1}{\rho g}-\frac{p_2}{\rho g}=\frac{\Delta p}{\rho g} \qquad ①$$

1)层流：

$$u=\frac{R^2}{4\mu}\frac{\Delta p}{L}(1-\frac{r^2}{R^2}) \qquad \overline{V}=\frac{R^2}{8\mu}\frac{\Delta p}{L}=\frac{D^2}{32}\frac{\Delta p}{L}$$

$$\Rightarrow \Delta p=\frac{32\mu L\overline{V}}{D^2}=32\frac{L}{D}\frac{\mu\overline{V}}{D} \qquad ②$$

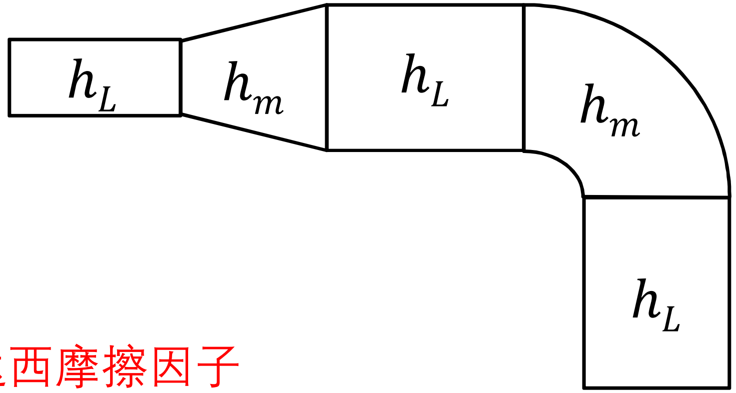
①+② $\Rightarrow$

$$h_L=\frac{\Delta p}{\rho g}$$

$$=32\frac{L}{D}\frac{\mu}{\rho D\overline{V}}\frac{\overline{V}^2}{g}$$

$$=\frac{64}{Re}(\frac{L}{D})\frac{\overline{V}^2}{2g}$$

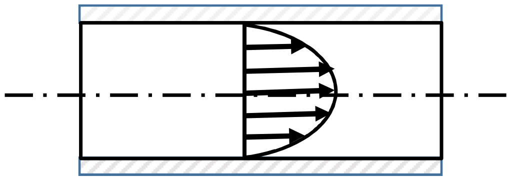
6.4圆管内能量损失（9.2~9.4）



① 水力损失：沿程损失+局部损失

h_{LT} $\quad$ h_L $\quad$ h_m

◆ 均直管沿程损失h_L：



$$u = \frac{R^2}{4\mu}\frac{\Delta p}{L}\left(1-\frac{r^2}{R^2}\right)$$

$$\bar{V} = \frac{R^2}{8\mu}\frac{\Delta p}{L}$$

1)层流：

f:达西摩擦因子

$$h_L = \frac{\Delta p}{\rho g} = \boxed{\frac{64}{Re}}\left(\frac{L}{D}\right)\frac{\bar{V}^2}{2g} = f\left(\frac{L}{D}\right)\frac{\bar{V}^2}{2g}$$

$$h_L = f\left(\frac{L}{D}\right)\frac{\bar{V}^2}{2g}$$ f：摩擦因子，由流动状态 $u(r)$ 决定。

层流：$f = \dfrac{64}{Re}$

粗糙度

2)湍流：$\Delta p = g(\rho, \mu, D, L, \bar{V}, e)$ （书5.8）

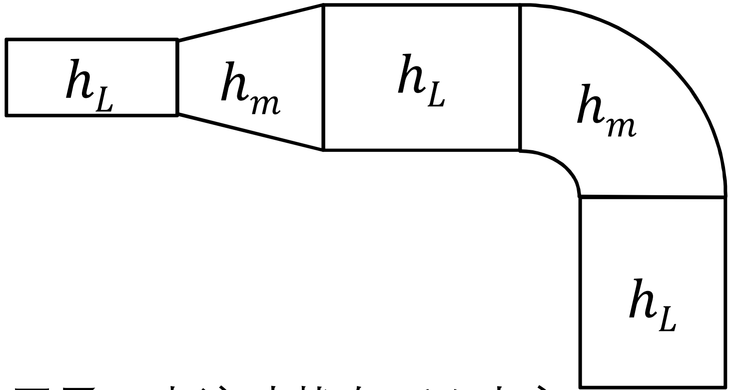
f:达西摩擦因子

莫迪图（图5.28）

Π原理：$\dfrac{\Delta p}{0.5\rho\bar{V}^2} = G\left(\dfrac{\mu}{\rho\bar{V}D}, \dfrac{L}{D}, \dfrac{e}{D}\right) = G\left(Re, \dfrac{L}{D}, \dfrac{e}{D}\right) = \dfrac{L}{D}\boxed{f\left(Re, \dfrac{e}{D}\right)}$

$$\Delta p = f\frac{L}{D}\left(\frac{1}{2}\rho\bar{V}^2\right) \quad h_L = \frac{\Delta p}{\rho g} = f\left(\frac{L}{D}\right)\frac{\bar{V}^2}{2g}$$

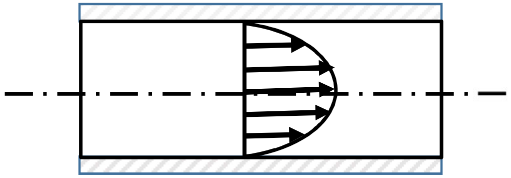
6.4圆管内能量损失（9.2~9.4）



① 水力损失：沿程损失+局部损失

h_{LT}　　h_L　　h_m

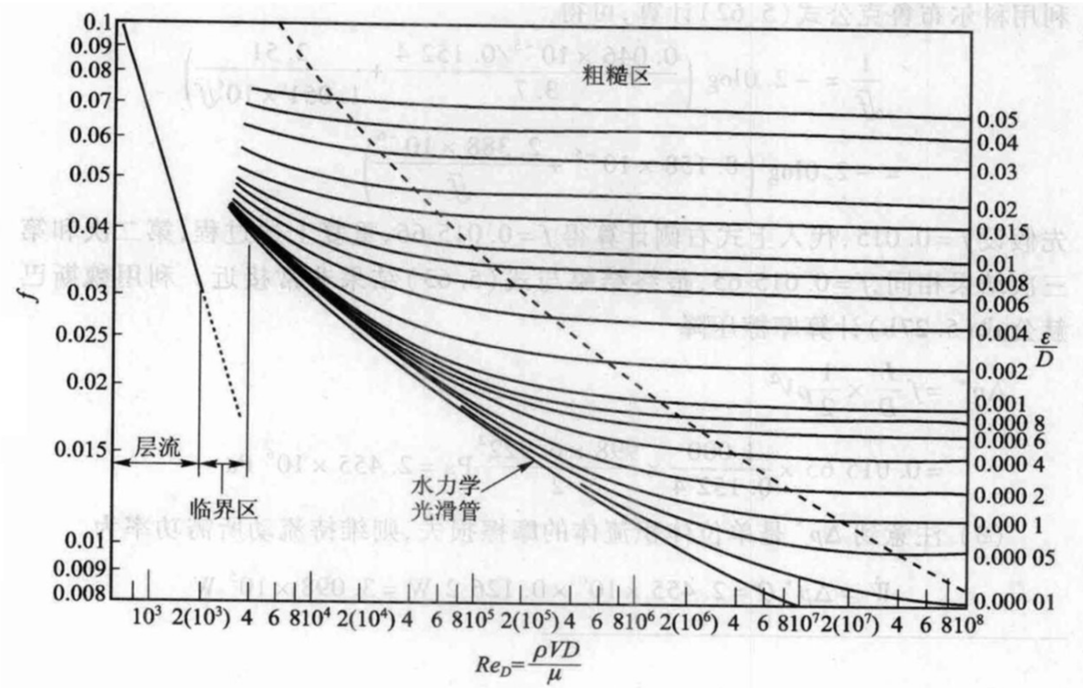
◆ 均直管沿程损失h_L：$h_L = f(\frac{L}{D})\frac{\bar{V}^2}{2g}$　f: 摩擦因子，由流动状态$u(r)$决定。



层流：$f = \frac{64}{Re}$

湍流：$f = f\,(Re, \frac{e}{D})$

（穆迪图5.28



1939科尔布鲁克：$\frac{1}{f^{1/2}} = -2.0\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{Re_D f^{1/2}}\right)$

1983哈兰德：$\frac{1}{f^{1/2}} = -1.8\log\left[\left(\frac{\varepsilon/D}{3.7}\right)^{1.11} + \frac{6.9}{Re_D}\right]$

$\bar{V} \rightarrow Re + \frac{e}{D} \rightarrow f \rightarrow h_L$　例题5.10！

作业:

复习笔记!

5.20, 9.1，9.23

看例5.10，例9.1，9.2（自学章节5.6~5.8)