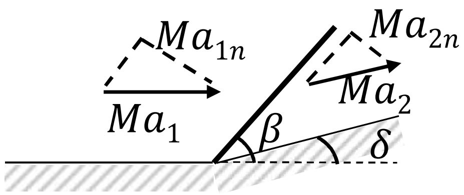
空气与气体动力学

张科

回顾：

1.斜激波:



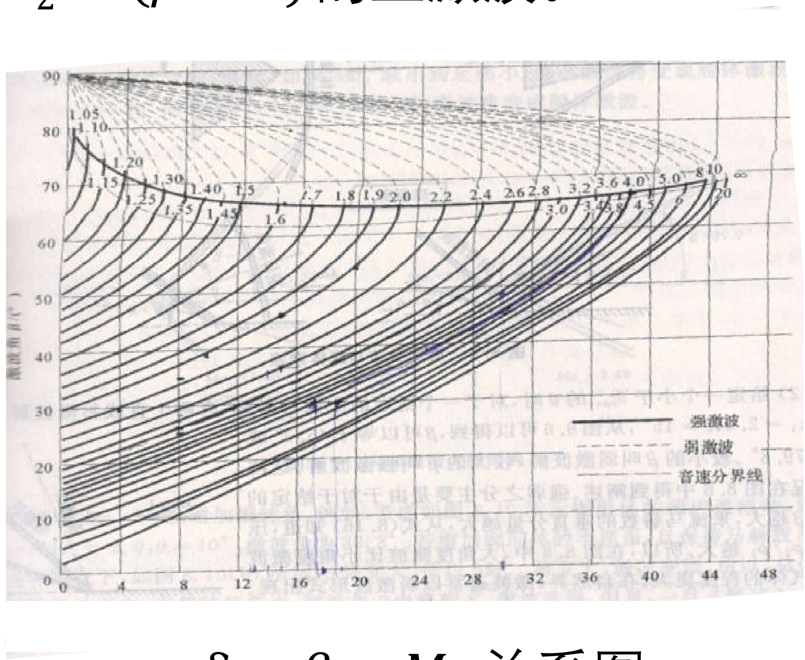
$Ma_{1n} = Ma_1 sin\beta$

$Ma_{2n} = Ma_2 \sin(\beta - \delta)$的正激波。

定量计算，定性分析；

2.激波相交与反射；

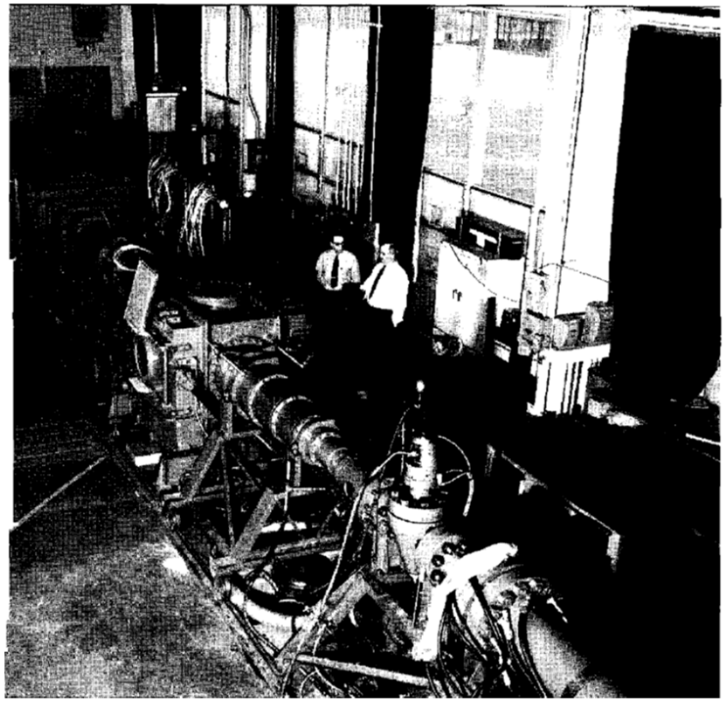
3.激波-膨胀波应用。



$\delta - \beta - Ma$关系图

十二. 准一维可压管内流（空9）

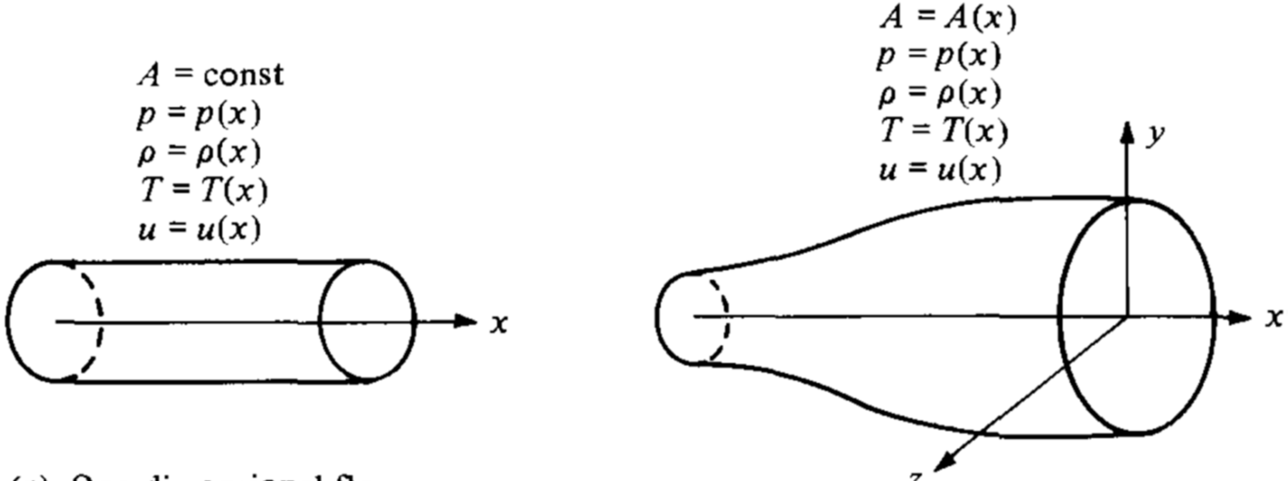
喷管、扩压器、风洞



12.1准一维流控制方程(9.2)

1. 准一维流：

一维流　　　　准一维流：$A(x)$变化小，忽略y,z变化



(a) One-dimensional flow

(b) Quasi-one-dimensional flow

12.1准一维流控制方程(9.2)

2. 准一维流控制方程：

连续方程： $\rho_1 u_1 A_1 = \rho_2 u_2 A_2$ ①

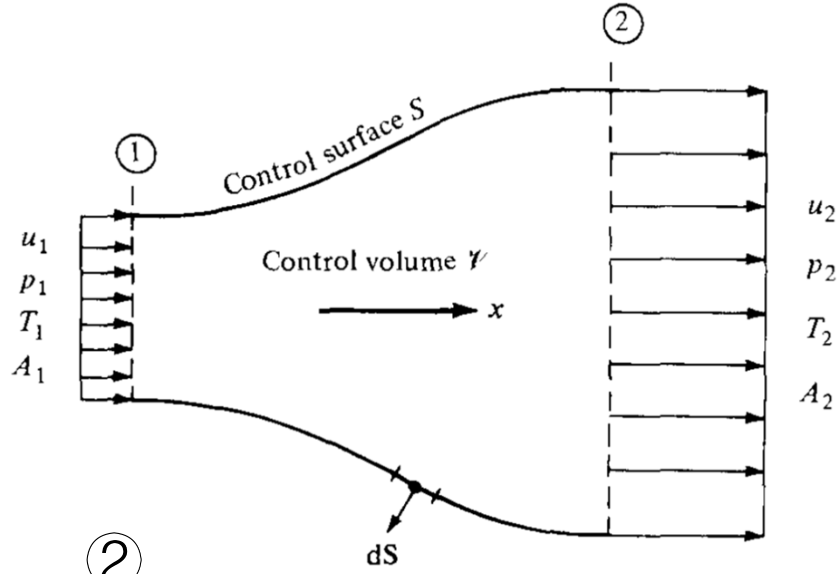
动量方程：

$$p_1 A_1 - p_2 A_2 + \int_{A_1}^{A_2} p dA = \rho_2 u_2^2 A_2 - \rho_1 u_1^2 A_1$$

$$p_1 A_1 + \rho_1 u_1^2 A_1 + \int_{A_1}^{A_2} p dA = p_2 A_2 + \rho_2 u_2^2 A_2$$ ②

能量方程： $h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2} = h_0$ ③

状态方程： $P = \rho RT \quad h = C_p T$



12.1准一维流控制方程(9.2)

$$\rho_1 u_1 A_1 = \rho_2 u_2 A_2$$

$$p_1 A_1 + \rho_1 u_1^2\ A_1 + \int_{A_1}^{A_2} p dA = p_2 A_2 + \rho_2 u_2^2\ A_2$$

$$h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2} = h_0$$

2. 准一维流控制方程：

微分方程：连续： $\rho u A = C$

$$d(\rho u A) = 0 \qquad ①$$

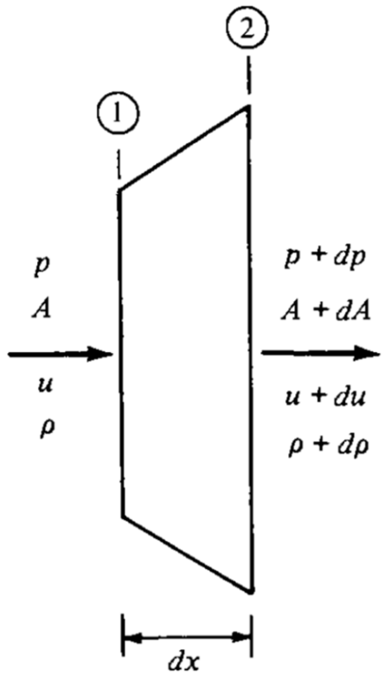
动量： $pA + \rho u^2\ A + pdA = (p+dp)(A+dA) + (\rho + d\rho)(u+du)^2\ (A+dA)$

$$\rightarrow Adp + Au^2 d\rho + \rho u^2 dA + 2\rho u A du = 0$$

$$① \rightarrow \rho u dA + \rho A du + u A d\rho = 0$$

$$Adp = -\rho u A du \quad \rightarrow \quad dp = -\rho u du \qquad ②$$

欧拉方程！准一维动量方程



能量： $h + \dfrac{u^2}{2} = C \quad \rightarrow \quad dh + udu = 0 \qquad ③$

12.1准一维流控制方程(9.2)

2. 准一维流控制方程：

$$d(\rho u A) = 0 \quad ① \quad \rightarrow \quad \frac{d\rho}{\rho} + \frac{du}{u} + \frac{dA}{A} = 0 \quad ④$$

$$dp = -\rho u du \quad ② \quad \rightarrow \quad \frac{dp}{\rho} = -udu$$

$$dh + udu = 0 \quad ③$$

$$\frac{dp}{d\rho}\frac{d\rho}{\rho} = a^2\frac{d\rho}{\rho} = -udu$$

$$\rightarrow \quad \frac{d\rho}{\rho} = -Ma^2\frac{du}{u} \quad ⑤$$

$$④+⑤ \quad \rightarrow \quad -Ma^2\frac{du}{u} + \frac{du}{u} + \frac{dA}{A} = 0$$

$$\boxed{\frac{dA}{A} = (Ma^2 - 1)\frac{du}{u}}$$

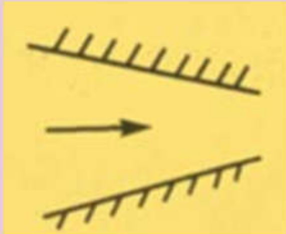
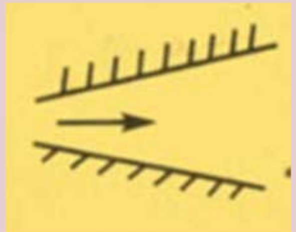
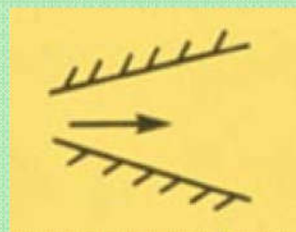
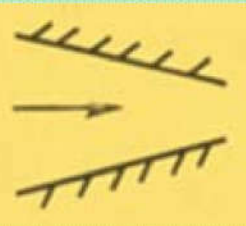
面积速度关系式！

12.1准一维流控制方程(9.2)

2. 准一维流控制方程：$\boxed{\frac{dA}{A} = (Ma^2 - 1)\frac{du}{u}}$

$0 < Ma < 1$时，
$dA > 0$ $du < 0$, $A\uparrow$ $u\downarrow$

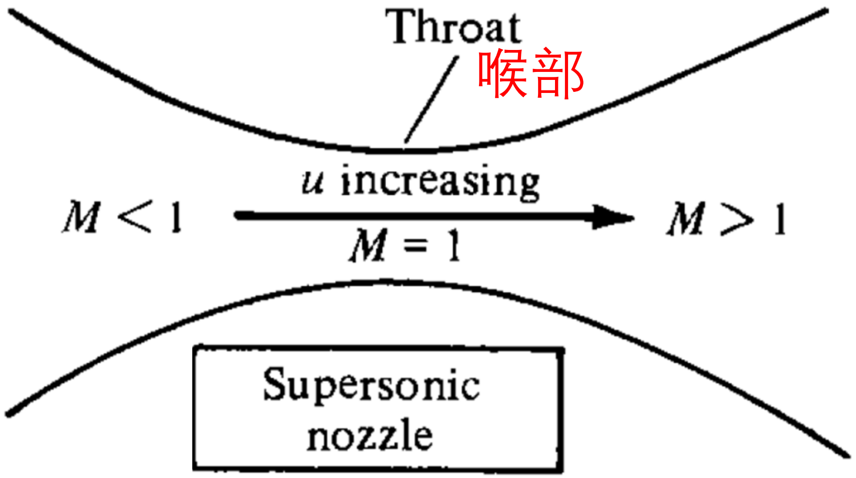
$Ma > 1$时，
$dA > 0$ $du > 0$, $A\uparrow$ $u\uparrow$

马赫数	加速	减速扩压
Ma<1(亚声速)	 渐缩喷管	 渐扩扩压管
Ma>1(超声速)	 渐扩喷管	 渐缩扩压管

12.1准一维流控制方程(9.2)

2. 准一维流控制方程： $\boxed{\frac{dA}{A} = (Ma^2 - 1)\frac{du}{u}}$

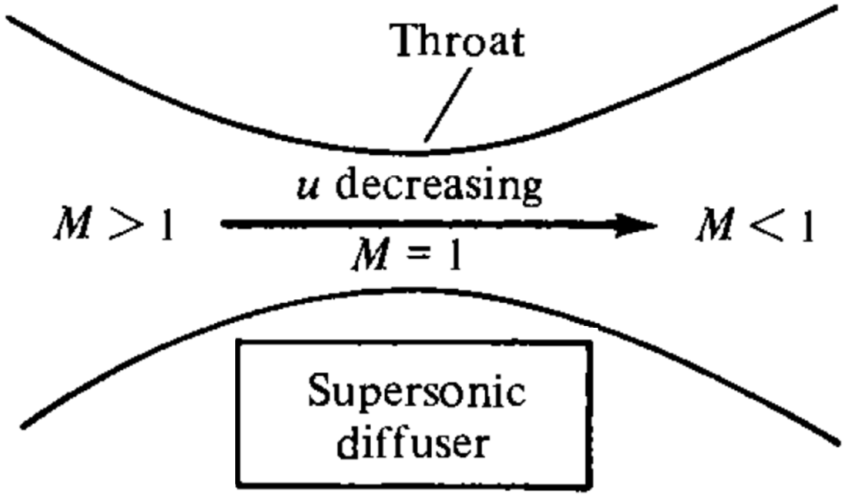
静止气体等熵加速至$Ma > 1$:



超声速喷管(拉瓦尔喷管)

$Ma\uparrow\ p\downarrow$

s气体等熵减速至$Ma < 1$:



超声速扩压器

$Ma\downarrow\ p\uparrow$

12.2喷管(9.3) (nozzle):减压、增速；喷气式飞机、火箭发动机等重要部件！

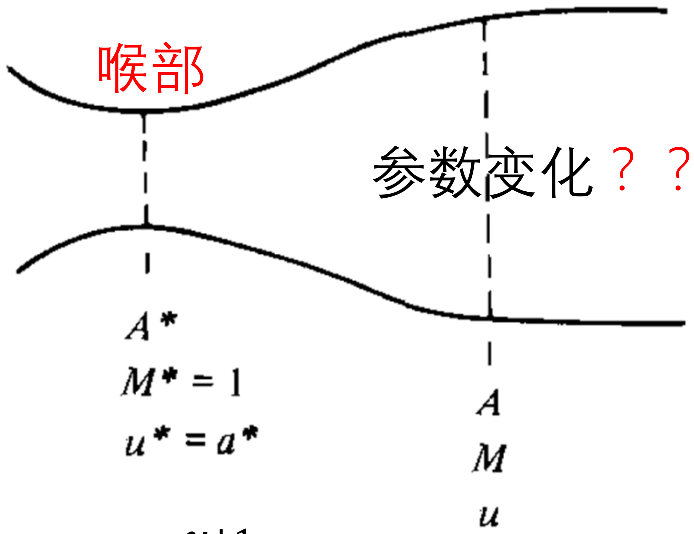
1. 控制方程：　$\rho u A = C$

$$\frac{A}{A^*} = \frac{\rho^* u^*}{\rho u} = \frac{\rho^*}{\rho_0}\frac{\rho_0}{\rho^*}\frac{a^*}{u}$$

等熵：$\frac{\rho^*}{\rho_0} = \left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}}$　$\frac{\rho_0}{\rho} = \left(1 + \frac{\gamma-1}{2}Ma^2\right)^{\frac{1}{\gamma-1}}$

$$\left(\frac{u}{a^*}\right)^2 = Ma^{*2} = \frac{(\gamma+1)Ma^2}{2+(\gamma-1)Ma^2}$$

$$\rightarrow \left(\frac{A}{A^*}\right)^2 = \left(\frac{\rho^*}{\rho_0}\right)^2\left(\frac{\rho_0}{\rho^*}\right)^2\frac{1}{Ma^{*2}} = \frac{1}{Ma^2}\left[\frac{2}{\gamma+1}\left(1 + \frac{\gamma-1}{2}Ma^2\right)\right]^{\frac{\gamma+1}{\gamma-1}}$$



面积-马赫数关系式！　附表A

- $Ma = f\left(\frac{A}{A^*}\right)$，$A \geq A^*$　$A < A^*$？？ 无等熵流动解！
- 给定$\frac{A}{A^*}$，$(Ma>1, Ma<1)$两解。$\frac{A}{A^*} = 2 \rightarrow Ma = 0.31$或$Ma = 2.2$

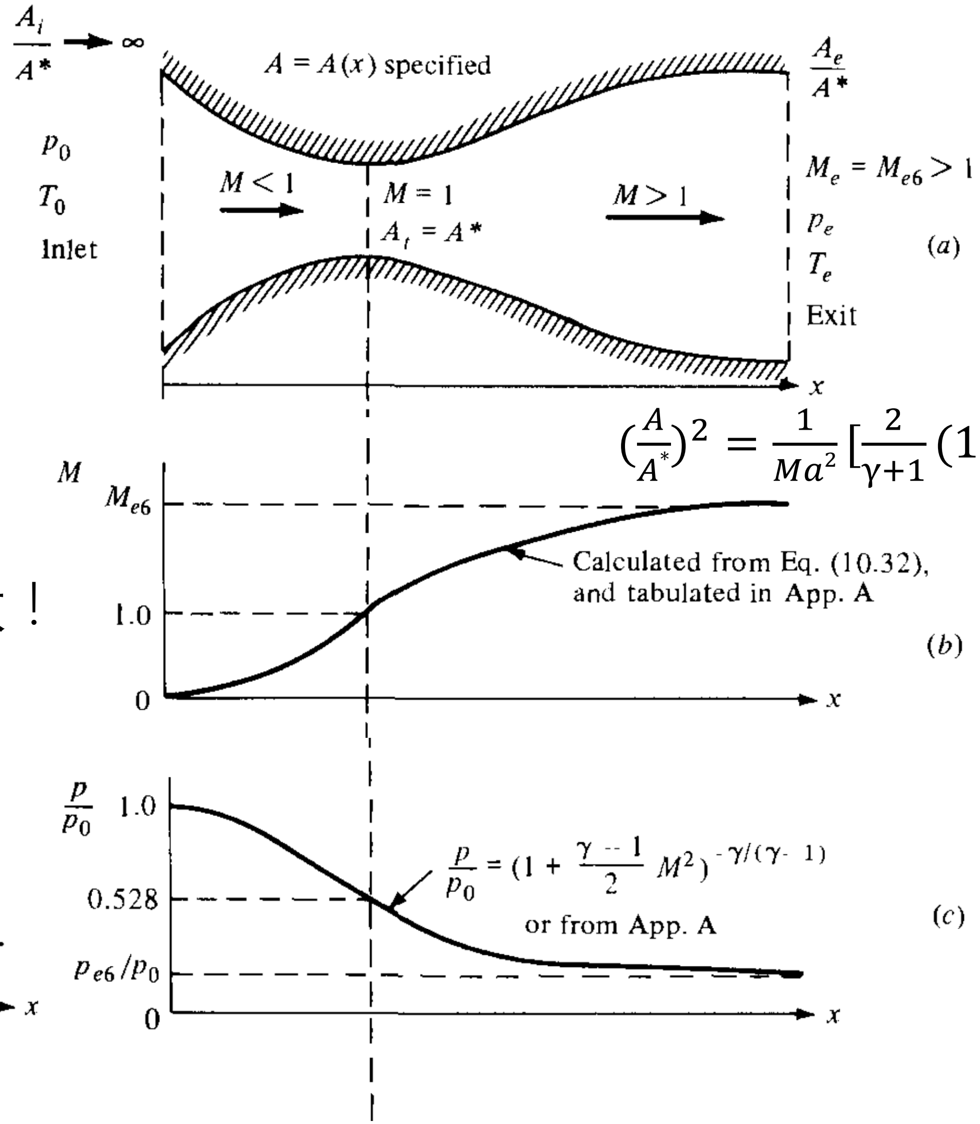
12.2喷管(9.3)

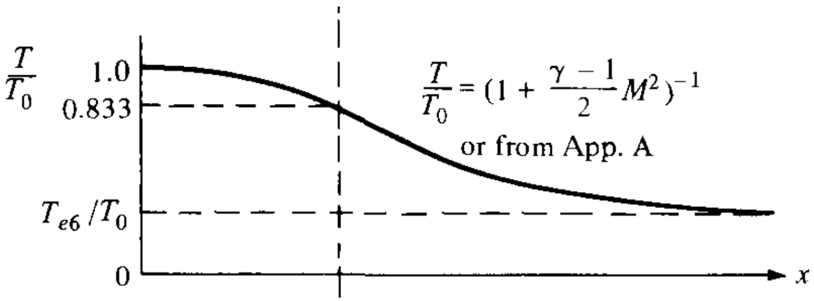
2. 等熵参数变化：

$$\frac{A}{A^*} \rightarrow Ma \rightarrow p, \rho, T$$

$$\left(\frac{A}{A^*}\right)^2 = \frac{1}{Ma^2}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}Ma^2\right)\right]^{\frac{\gamma+1}{\gamma-1}}$$

等熵流动，$\frac{A}{A^*}$决定流动参数！





12.2喷管(9.3)

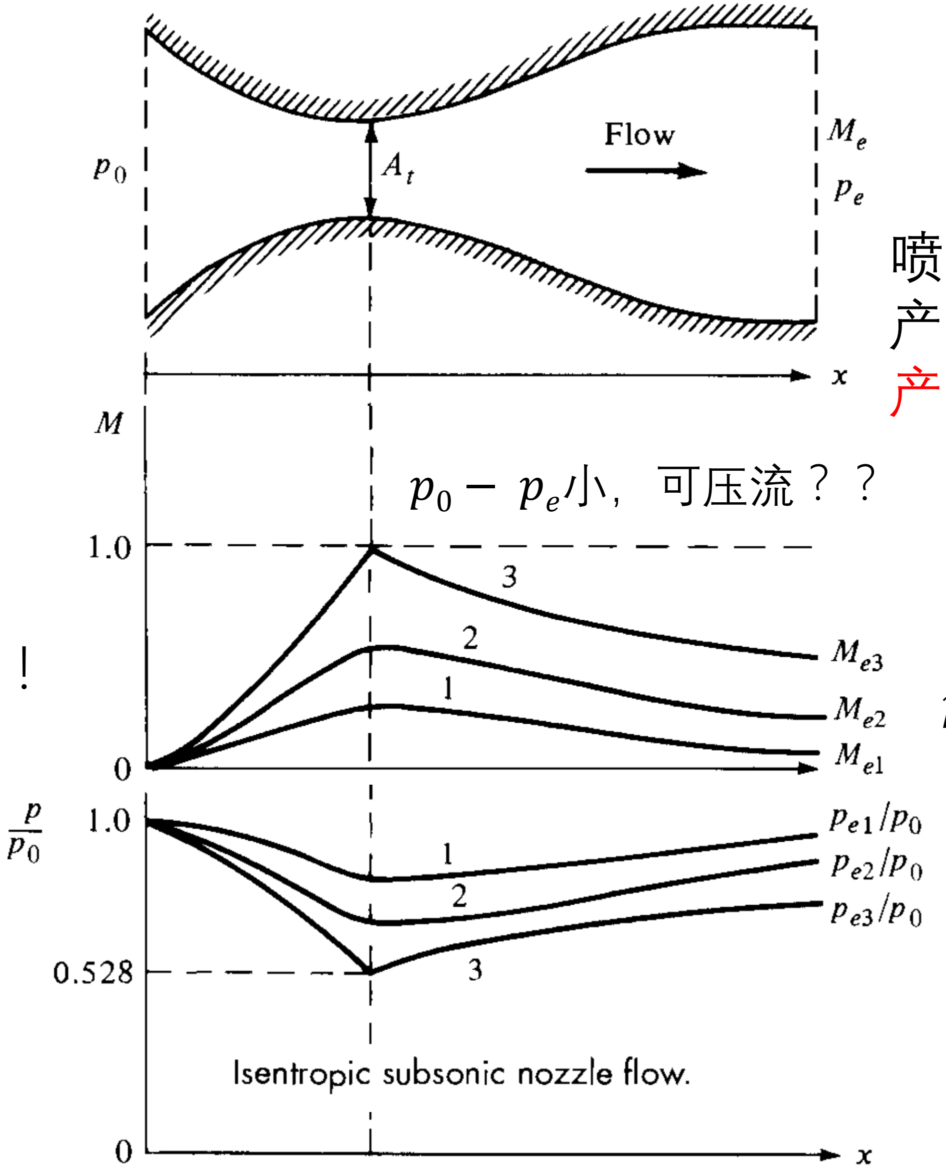
3. 实际流动：

$p_e < p_0$产生流动

- $p_{e3} < pe < p_0$
  亚声速等熵流动,
  $\frac{A}{A^*}$，p_e 决定流动参数！

$p_e >\ p_{e3}$ 喉部非临界参数！

$u_t = a^*$时,
p_e⬇收缩段流动不变,
$\dot{m}$=$\rho^* a^* A_t$ 不再变,
→流动“壅塞”



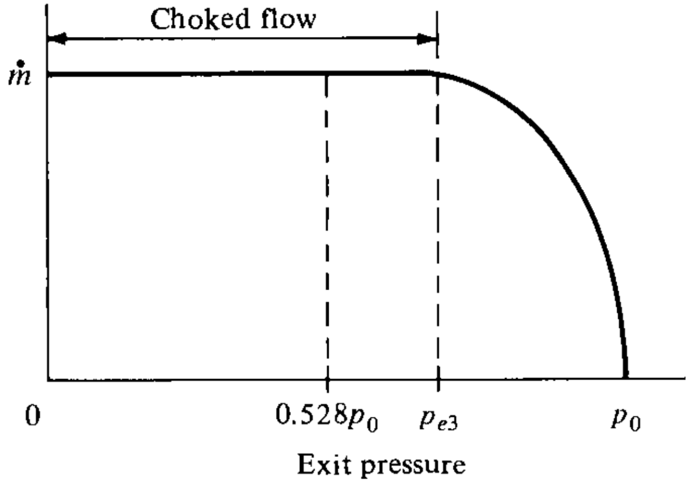
$p_0 - p_e$小，可压流？？

喷管置于大气中，
产生流动？？
产生流动条件？？

$p_0 - p_e$⬆$, Ma$⬆

12.2喷管(9.3)

3. 实际流动：



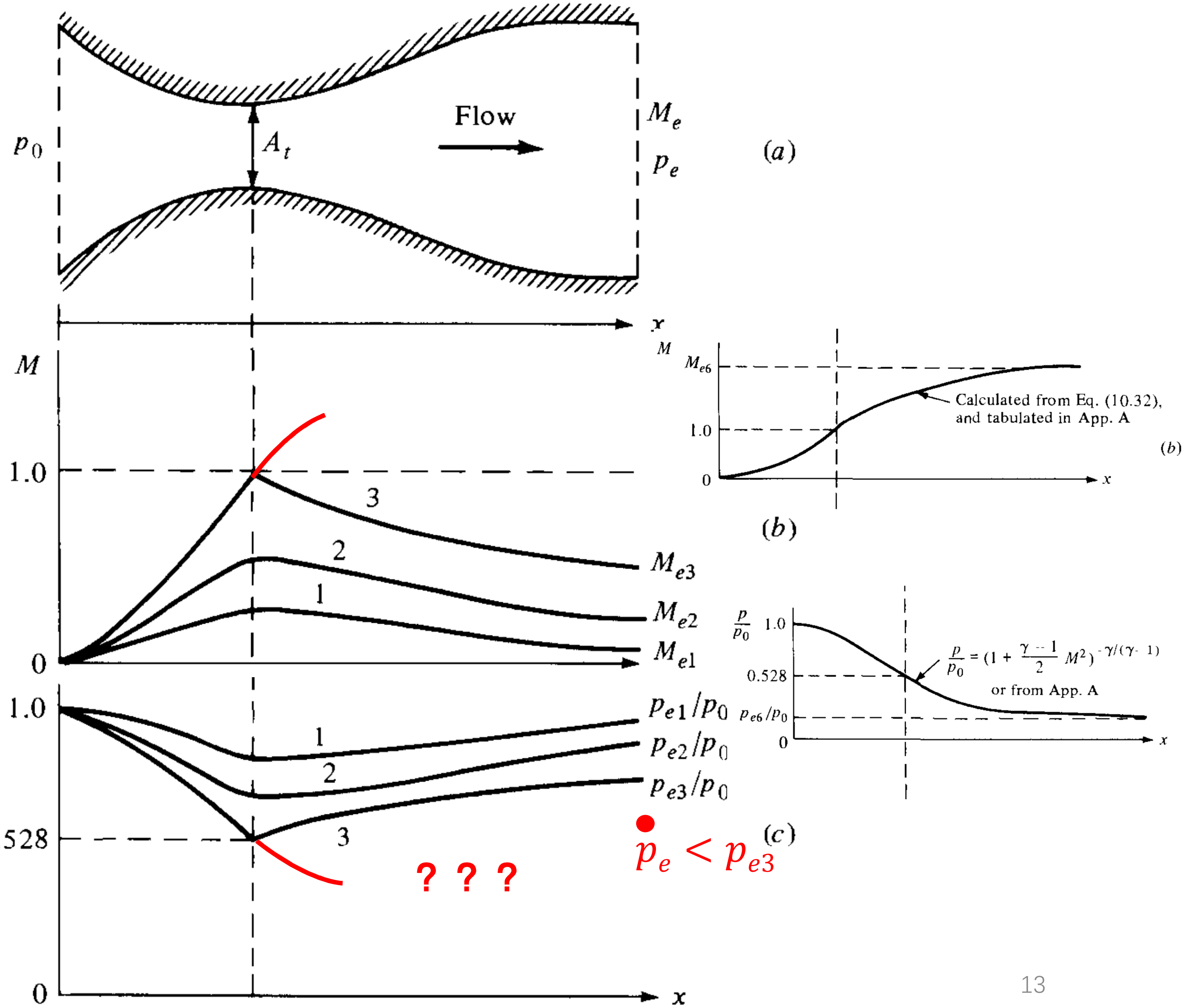
p_e↓$\dot{m}$↑，直至$p_e = pe_3$，

$u_t = a^*$，$\dot{m}$不变

➢ $p_e < p_{e3}$

收缩段不变，

扩张段出现$Ma > 1$！

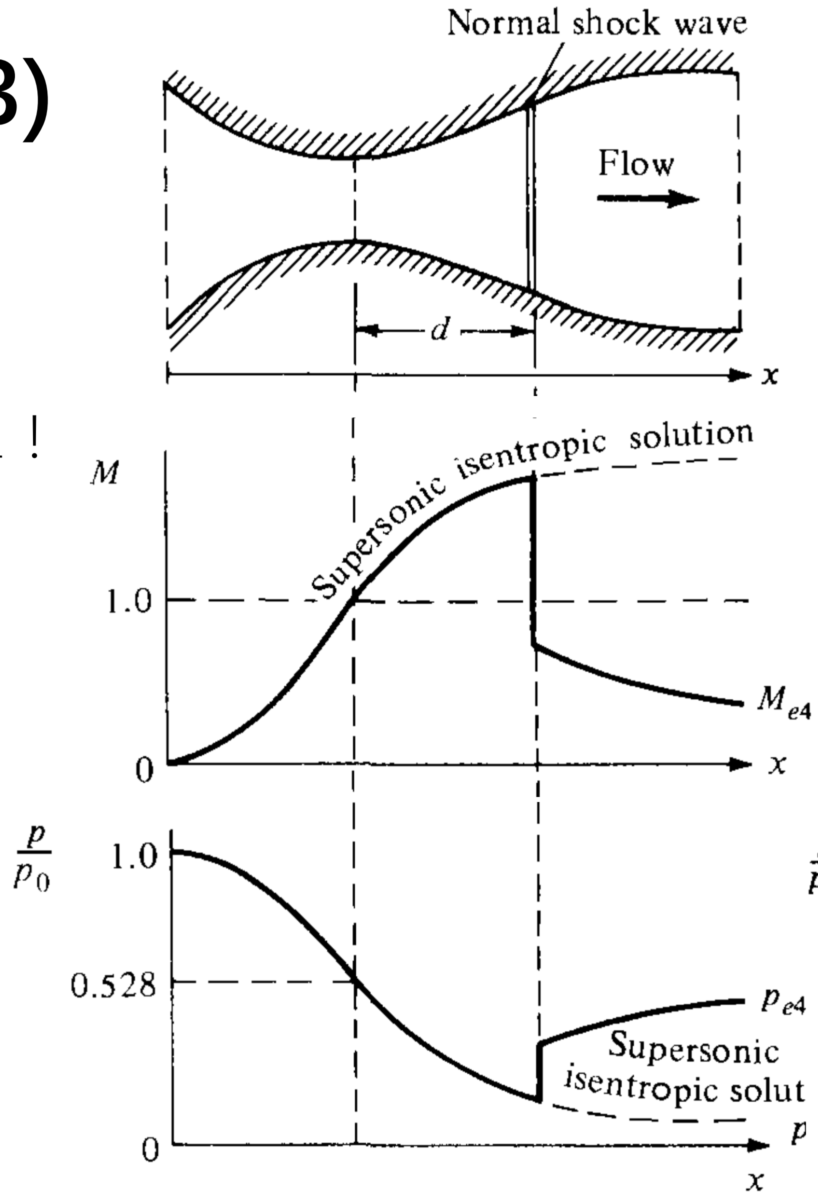


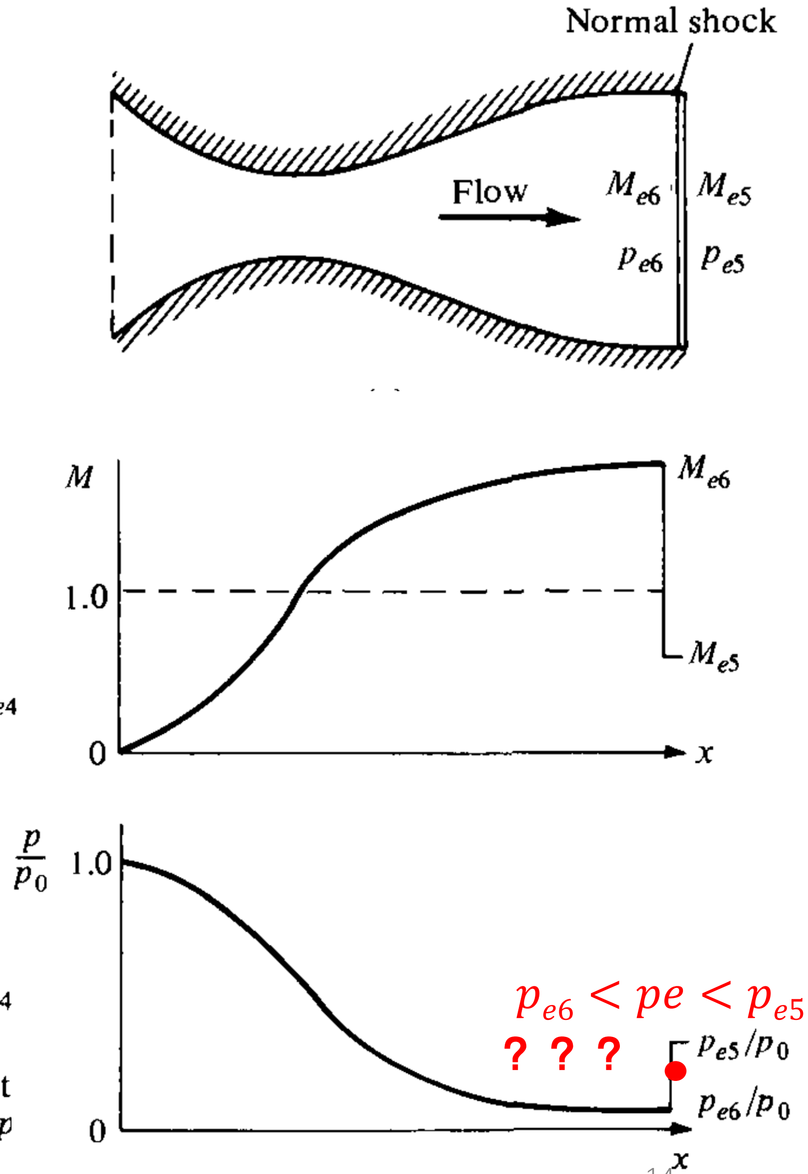
12.2喷管(9.3)

3. 实际流动：

➢ $p_e < p_{e3}$
收缩段不变，
扩张段出现$Ma > 1$！
管内出现正激波！
p_e⬇正激波后移！

➢ $p_e = p_{e5}$
正激波到出口。



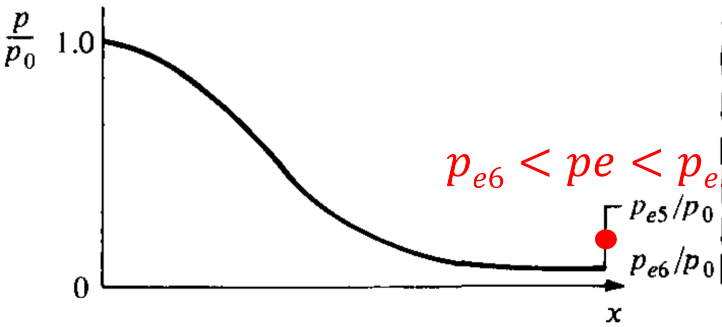


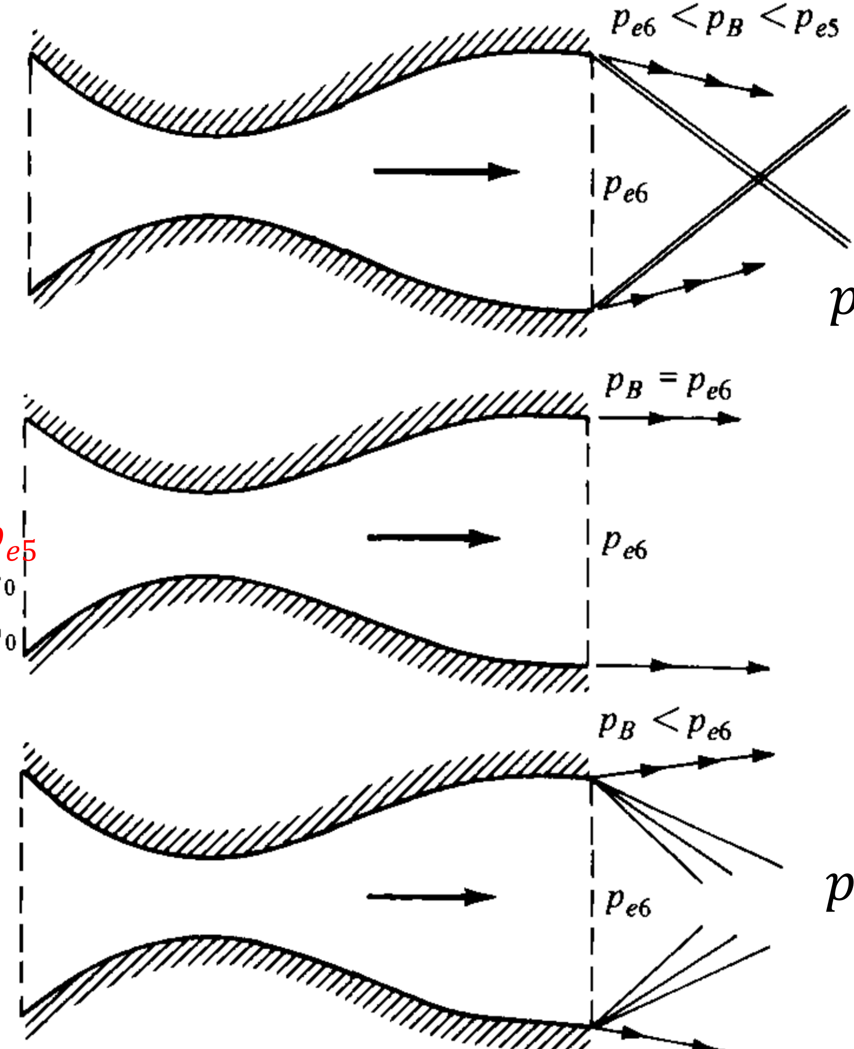
12.2喷管(9.3)

3. 实际流动：

- ➢ $p_{e6} < pe < p_{e5}$

出口处有斜激波!





$p_B > p_{e6}$喷管内过膨胀！

p_B⬇激波⬇

- ➢ $p_e < p_{e6}$

出口处有膨胀波!

已知$A(x)$→$Ma(x)$!

$p_B < p_{e6}$喷管内欠膨胀！

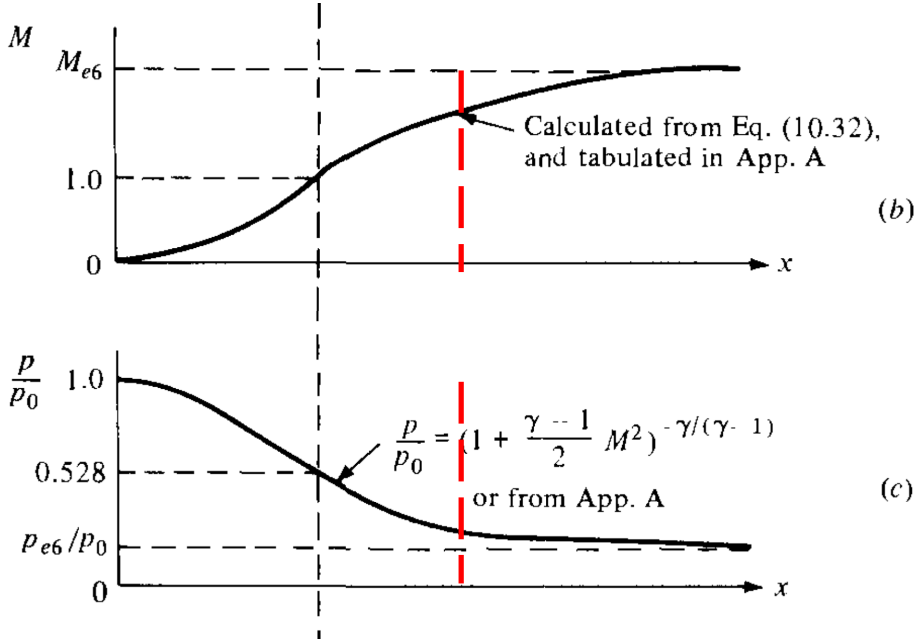
实际喷管设计稍有误差，内部易出现斜激波，三维可压！

12.2喷管(9.3)

例：$p_0 = 1atm, T_0 = 288K, \frac{A_e}{A^*} = 2,$ 等熵流动。求p^*, T^*, Ma_e, pe, T_e。

① $Ma_e > 1$; ② $Ma_e < 1$,但$Ma_t = 1$; ③ $p_e = 0.973atm$，求Ma_t， Ma_e。

解：① $Ma_e > 1$：



$\frac{A_e}{A^*} = 2$， 附表A

$\rightarrow Ma_e = 0.3$, $Ma_e = 2.2$

附表A: $Ma_e = 2.2 \rightarrow$

$\frac{p_0}{p_e} = 10.69, \frac{T_0}{T_e} = 1.968$

$\rightarrow p_e = 0.0935atm, T_e = 145\text{K}$

$\frac{p^*}{p_0} = 0.528, \frac{T^*}{T_0} = 0.833 \rightarrow p^* = 0.528atm, T^* = 240K$

12.2喷管(9.3)

例：$p_0 = 1atm, T_0 = 288K, \frac{A_e}{A^*} = 2,$ 等熵流动。求$Ma^*, p^*, T^*, Ma_e, pe, T_e$。

① $Ma_e > 1$; ② $Ma_e < 1$,但$Ma_t = 1$; ③ $p_e = 0.973atm$，求Ma_t， Ma_e。

解： ② $Ma_e < 1$, $Ma_t = 1$ ：

$\frac{A_e}{A^*} = 2$， 附表A

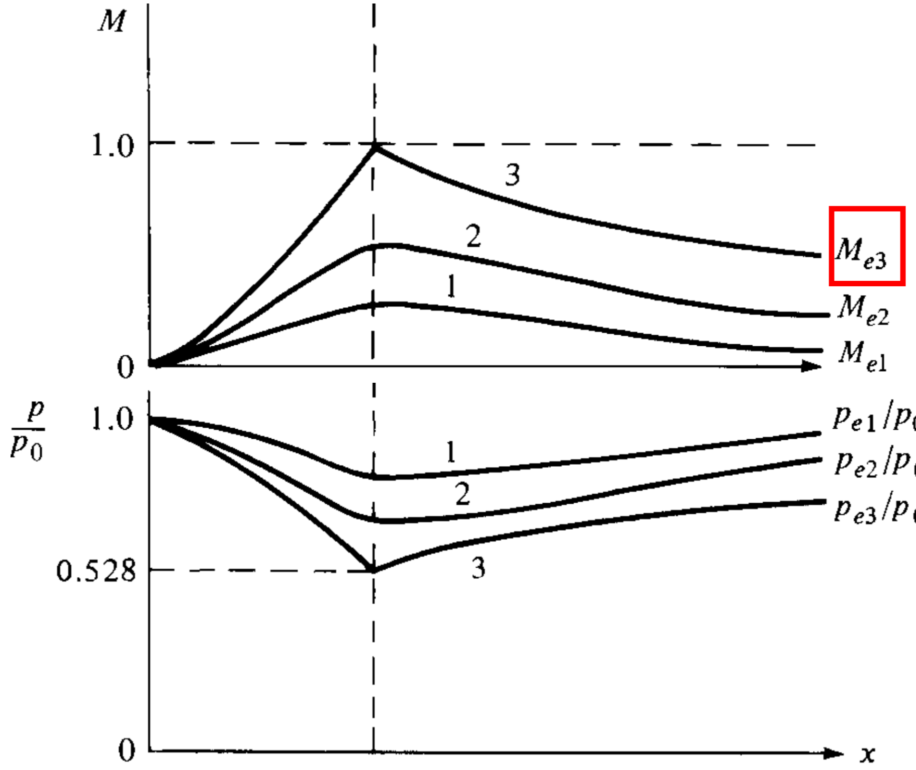
→ $Ma_e = 0.3$, $Ma_e = 2.2$

附表A: $Ma_e = 0.3$ →

$\frac{p_e}{p_0} = 0.9395, \frac{T_e}{T_0} = 0.9823$

→ $p_e = 0.9395atm, T_e = 282.9\text{K}$

$p^* = 0.528atm$， $T^* = 240K$



12.2喷管(9.3)

例：$p_0 = 1atm, T_0 = 288K, \frac{A_e}{A^*} = 2,$ 等熵流动。求$Ma^*, p^*, T^*, Ma_e, pe, T_e$。

① $Ma_e > 1$; ② $Ma_e < 1$,但$Ma_t = 1$; ③ $p_e = 0.973atm$，求Ma_t，Ma_e。

解：③ $p_e = 0.973atm \quad > pe_3 = 0.9395atm$

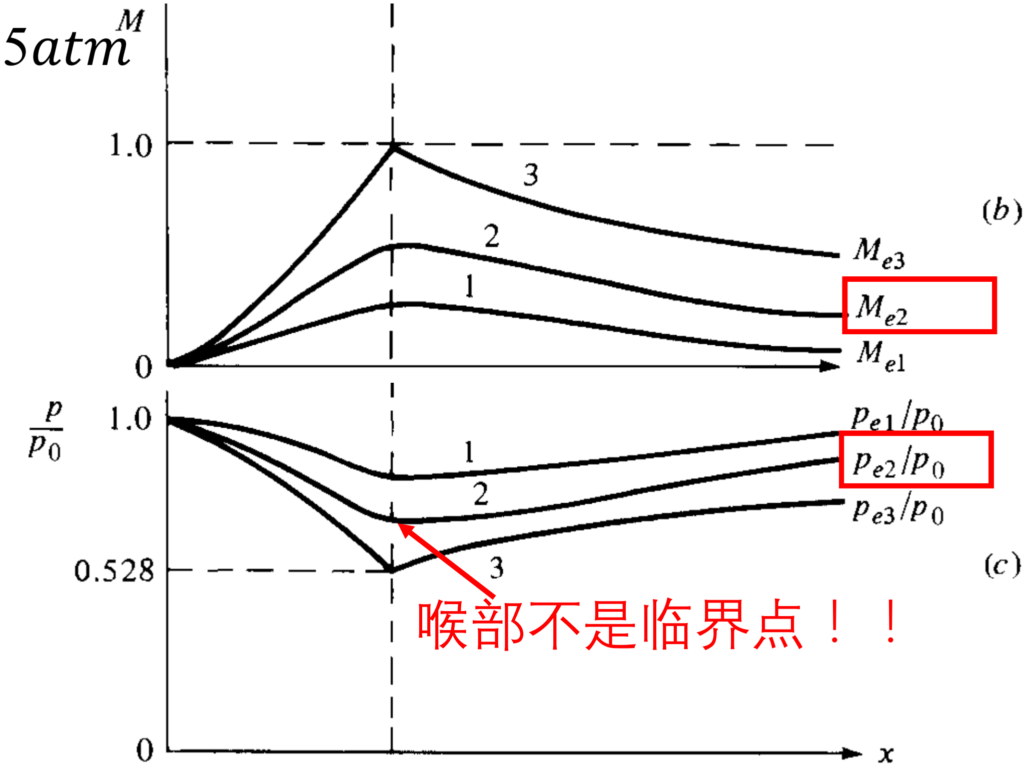
附表A: $\frac{p_e}{p_0} = 0.973 \rightarrow Ma_e = 0.2$

$\frac{A_e}{A^*} = 2.964 \quad \frac{A_e}{A^*} = 2$???

$\frac{A_e}{A_t} = 2 \quad \frac{A_t}{A^*} \rightarrow Ma_t$

$$\frac{A_t}{A^*} = \frac{A_t}{A_e}\frac{A_e}{A^*} = \frac{1}{2_e} \times 2.964 = 1.482$$

附表A: $\rightarrow Ma_e = 0.44$



作业：

复习笔记！

空气动力学书9.1, 9.7, 9.8