空气与气体动力学

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回顾：

1.扩压器:

$M_1 > 1$
p_{0_1}
s_1

A_t Weak normal shock

$M \approx 1$ $M < 1$

$M_2 < 1$
$p_{0_2} < p_{0_1}$
$s_2 > s_1$

(b) Actual supersonic diffuser

2.超声速风洞：

3.可压势流线化速度势方程：

Nozzle | Test section | Diffuser

M_e
p_e

p_0

Test model

$M << 1$
p_B

$A_{t1} = A^*$
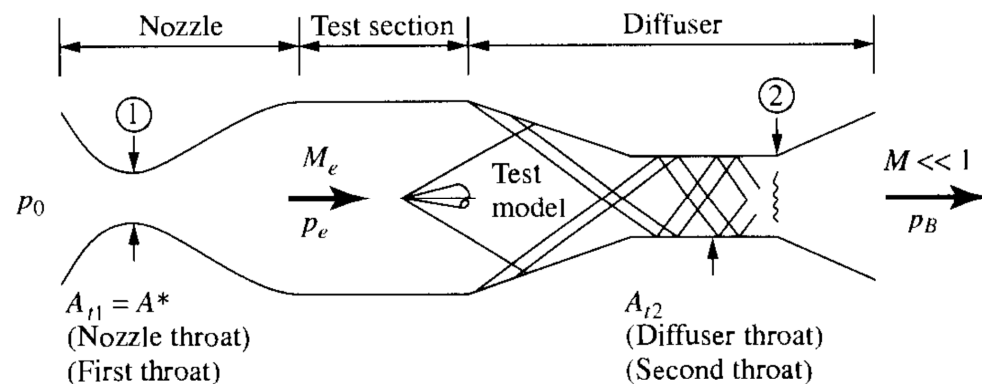(Nozzle throat)
(First throat)

A_{t2}
(Diffuser throat)
(Second throat)

$$\begin{cases} (1-Ma_\infty^2)\dfrac{\partial^2 \widehat{\phi}}{\partial x^2} + \dfrac{\partial^2 \widehat{\phi}}{\partial y^2} = 0 \\ C_p \approx -\dfrac{2\widehat{u}}{V_\infty} \end{cases}$$

适用条件：①小扰动(薄体、小攻角)

②亚、超声速：$Ma_\infty \leq 0.8$，$Ma_\infty > 1.2$，$Ma_\infty < 5$

2

13.3Prandtl-Glauert压缩性修正(10.4)

$$\beta^2 \frac{\partial^2 \hat{\phi}}{\partial x^2} + \frac{\partial^2 \hat{\phi}}{\partial y^2} = 0 \qquad \beta^2 = 1 - Ma_\infty^2$$

仿射变换：$\xi = x, \eta = \beta y$ 引入：$\bar{\phi}(\xi, \eta) = \beta \hat{\phi}(x, y)$

$$\implies \quad \frac{\partial \xi}{\partial x} = 1, \frac{\partial \xi}{\partial y} = 0$$
$$\frac{\partial \eta}{\partial x} = 0, \frac{\partial \eta}{\partial y} = \beta$$

$$\bar{\phi}(\xi, \eta) = \beta \hat{\phi}(x, y) \implies \frac{\partial \hat{\phi}}{\partial x} = \frac{1}{\beta} \frac{\partial \bar{\phi}}{\partial \xi} \frac{\partial \xi}{\partial x} = \frac{1}{\beta} \frac{\partial \bar{\phi}}{\partial \xi} \implies \frac{\partial^2 \hat{\phi}}{\partial x^2} = \frac{1}{\beta} \frac{\partial^2 \bar{\phi}}{\partial \xi^2} \frac{\partial \xi}{\partial x} = \frac{1}{\beta} \frac{\partial^2 \bar{\phi}}{\partial \xi^2}$$

$$\frac{\partial \hat{\phi}}{\partial y} = \frac{1}{\beta} \frac{\partial \bar{\phi}}{\partial \eta} \frac{\partial \eta}{\partial y} = \frac{\partial \bar{\phi}}{\partial \eta} \qquad \frac{\partial^2 \hat{\phi}}{\partial y^2} = \frac{\partial^2 \bar{\phi}}{\partial \eta^2} \frac{\partial \eta}{\partial y} = \beta \frac{\partial^2 \bar{\phi}}{\partial \eta^2}$$

$$\beta^2 \frac{\partial^2 \hat{\phi}}{\partial x^2} + \frac{\partial^2 \hat{\phi}}{\partial y^2} = 0 \implies \beta^2 \frac{1}{\beta} \frac{\partial^2 \bar{\phi}}{\partial \xi^2} + \beta \frac{\partial^2 \bar{\phi}}{\partial \eta^2} = 0$$

$$\frac{\partial^2 \bar{\phi}}{\partial \xi^2} + \frac{\partial^2 \bar{\phi}}{\partial \eta^2} = 0 \qquad \nabla^2 \bar{\phi} = 0 \text{ 不可压势方程！！}$$

13.3Prandtl-Glauert压缩性修正(10.4)

2.　　　(x,y)空间可压流动：

$$\beta^2 \frac{\partial^2 \widehat{\phi}}{\partial x^2} + \frac{\partial^2 \widehat{\phi}}{\partial y^2} = 0$$
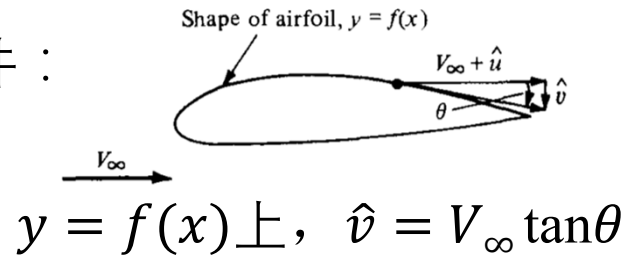
$\widehat{\phi}(x,y), y = f(x)$翼面

$\xi = x, \eta = \beta y$

$\overline{\phi} = \beta \widehat{\phi}$

(ξ,η)空间不可压流动：

$$\frac{\partial^2 \overline{\phi}}{\partial \xi^2} + \frac{\partial^2 \overline{\phi}}{\partial \eta^2} = 0$$

$\overline{\phi}(\xi,\eta), \eta = q(\xi)$翼面

边界条件：

Shape of airfoil, $y = f(x)$

$V_\infty + \widehat{u}$

$\widehat{v}$

θ

V_∞

$y = f(x)$上，$\widehat{v} = V_\infty \tan\theta$

$\eta = q(\xi)$上，$\overline{v} = V_\infty \tan\theta$

$$\frac{\partial \widehat{\phi}}{\partial y} = V_\infty \frac{df}{dx} \qquad \frac{\partial \widehat{\phi}}{\partial y} = \frac{\partial \overline{\phi}}{\partial \eta} \qquad \frac{\partial \overline{\phi}}{\partial \eta} = V_\infty \frac{dq}{d\xi}$$

$$\boxed{\frac{df}{dx} = \frac{dq}{d\xi}}$$ 翼型保持不变！

相同翼型：(x,y)空间可压流动　→　(ξ,η)空间不可压流动!

13.3Prandtl-Glauert压缩性修正(10.4)

$$\beta^2 \frac{\partial^2 \hat{\phi}}{\partial x^2} + \frac{\partial^2 \hat{\phi}}{\partial y^2} = 0 \qquad \begin{array}{c} \xi = x, \eta = \beta y \\ \longrightarrow \\ \bar{\phi} = \beta \hat{\phi} \end{array} \qquad \frac{\partial^2 \bar{\phi}}{\partial \xi^2} + \frac{\partial^2 \bar{\phi}}{\partial \eta^2} = 0 \qquad 两空间 C_p, C_l 关系??$$

3. 压强系数:

$$C_p = -\frac{2\hat{u}}{V_\infty} = -\frac{2}{V_\infty}\frac{\partial \hat{\phi}}{\partial x} = -\frac{2}{V_\infty}\frac{1}{\beta}\frac{\partial \bar{\phi}}{\partial \xi} \qquad \frac{\partial \hat{\phi}}{\partial x} = \frac{1}{\beta}\frac{\partial \bar{\phi}}{\partial \xi}$$

(ξ, η)空间不可压流动: $\frac{\partial \bar{\phi}}{\partial \xi} = \bar{u}$ ($\bar{u}, \bar{v}$为变换空间(ξ, η) 不可压流动扰动速度。)

$$C_p = \frac{1}{\beta}\left(-\frac{2\bar{u}}{V_\infty}\right) \qquad \boxed{C_{p,0} = -\frac{2\bar{u}}{V_\infty}, 不可压流动压强系数} \qquad (小扰动!)$$

相同翼型: $\qquad C_p = \frac{1}{\beta}C_{p,0} \qquad \beta = \sqrt{1 - Ma_\infty^2}$ (压缩性因子)

$$\boxed{C_p = \frac{C_{p,0}}{\sqrt{1 - Ma_\infty^2}}} \qquad Prandtl\text{-}Glauert法则!(亚声速相似律)$$

13.3Prandtl-Glauert压缩性修正(10.4)

$$C_p = \frac{C_{p,0}}{\sqrt{1 - Ma_\infty^2}}$$

(a) $Ma_\infty = 0.191$

(b) $Ma_\infty = 0.512$

NACA4415

(c) $Ma_\infty = 0.596$
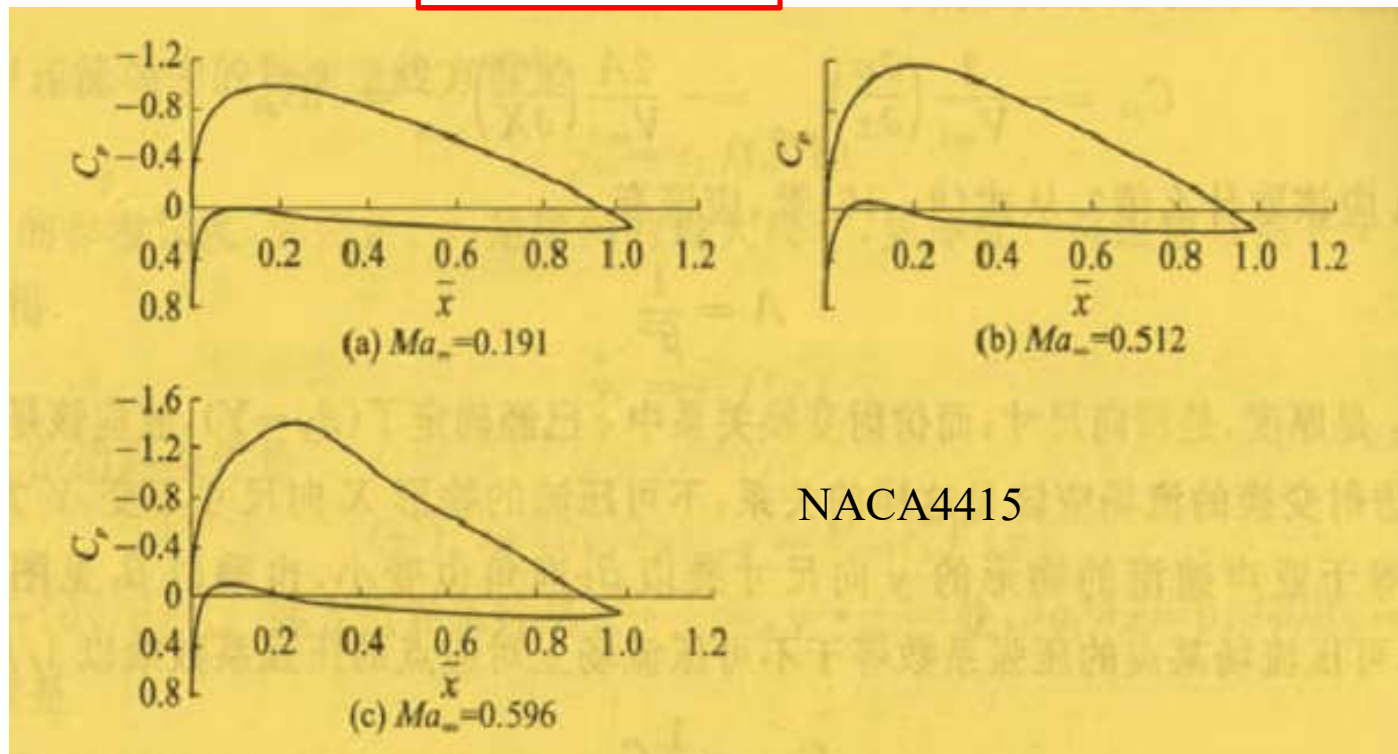
13.3Prandtl-Glauert压缩性修正(10.4)

$$C_p = \frac{C_{p,0}}{\sqrt{1-Ma_\infty^2}}$$

1922,1928

$$C_l = \frac{C_{l,0}}{\sqrt{1-Ma_\infty^2}}$$

$$C_m = \frac{C_{m,0}}{\sqrt{1-Ma_\infty^2}}$$

无粘不可压：$D = 0$

亚声速$(0.3 < Ma < 1)$可压：$D = 0$
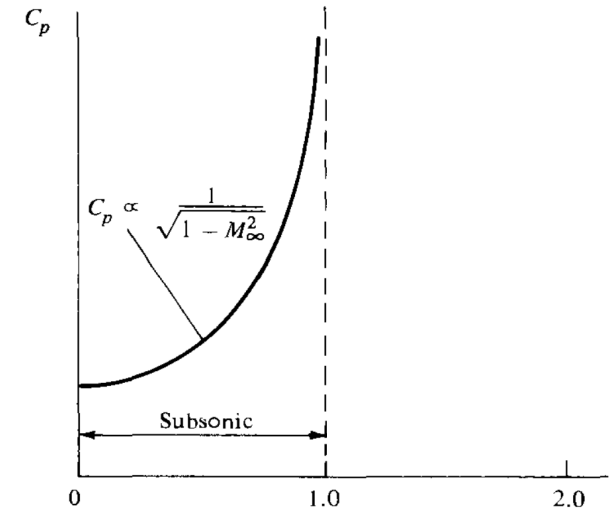
13.3Prandtl-Glauert压缩性修正(10.4)

$$C_p = \frac{C_{p,0}}{\sqrt{1-Ma_\infty^2}}$$

$$C_l = \frac{C_{l,0}}{\sqrt{1-Ma_\infty^2}}$$

$$C_m = \frac{C_{m,0}}{\sqrt{1-Ma_\infty^2}}$$

例：低速 $C_p = -0.3$，求 $Ma = 0.6$ 时 C_p。

解： $C_p = \frac{C_{p,0}}{\sqrt{1-Ma_\infty^2}} = \frac{-0.3}{\sqrt{1-0.6^2}} = -0.375$

例：不可压对称翼型 $C_l = 2\pi\alpha$，求 $Ma = 0.7$ 时 C_l。

解： $C_l = \frac{C_{l,0}}{\sqrt{1-Ma_\infty^2}} = \frac{2\pi\alpha}{\sqrt{1-0.7^2}} = 8.8\alpha$

$\frac{8.8}{2\pi} = 1.4$，升力线斜率增加40%。

$$\boxed{亚声速(0.3 < Ma < 1)：Ma\uparrow\ C_l\uparrow a_0\uparrow}$$

13.3Prandtl-Glauert压缩性修正(10.4)
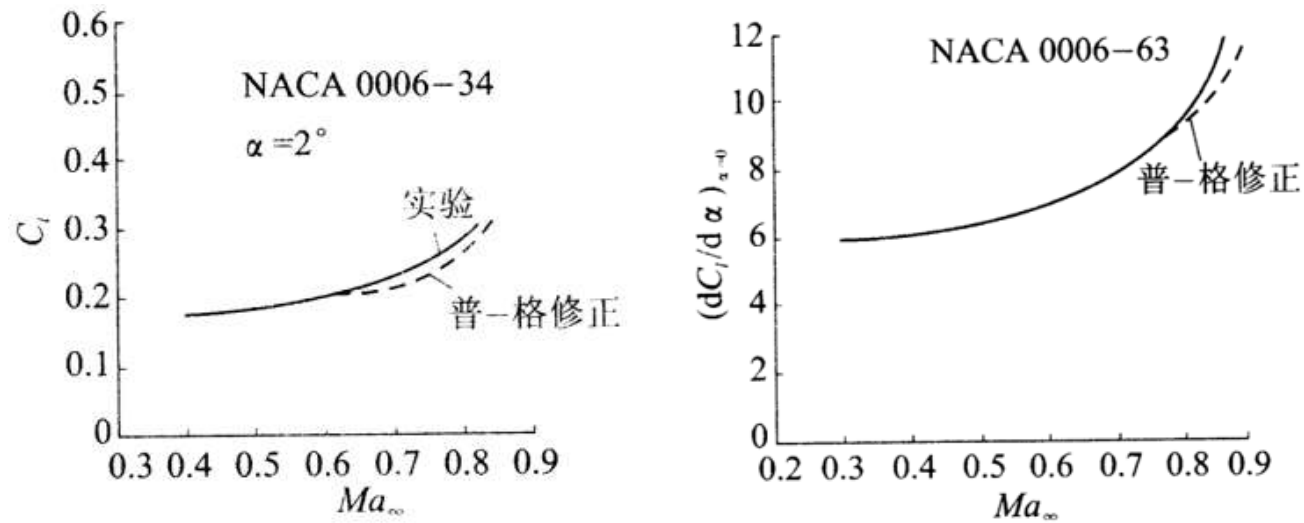
$0.5 < Ma < 0.8$，普朗特-格劳厄特修正与实验差别较大。

13.3Prandtl-Glauert压缩性修正(10.4)

$$C_p = \frac{C_{p0}}{\sqrt{1-Ma_\infty^2}}$$

4. 压缩性修正改进： 二战期间可压气动特性研究

Karman-Tsien 法则：$C_p = \dfrac{C_{p,0}}{\sqrt{1-Ma_\infty^2}+\left(\dfrac{Ma_\infty^2}{1+\sqrt{1-Ma_\infty^2}}\right)C_{p,0}/2}$

Laitone法则： 与本地压强有关

$$C_p = \frac{C_{p,0}}{\sqrt{1-Ma_\infty^2}+\left(\dfrac{Ma_\infty^2\{1+[(\gamma-1)/2]Ma_\infty^2\}}{2\sqrt{1-Ma_\infty^2}}\right)C_{p,0}}$$

非线性影响！

C_p

−1.8
−1.6
−1.4
−1.2
−1.0
−0.8
−0.6 ● Experiment
−0.4
−0.2

Laitone
Karman-Tsien
Prandtl-Glauret

NACA4412翼型，0.3c处

0   0.2   0.4   0.6   0.8 M_∞
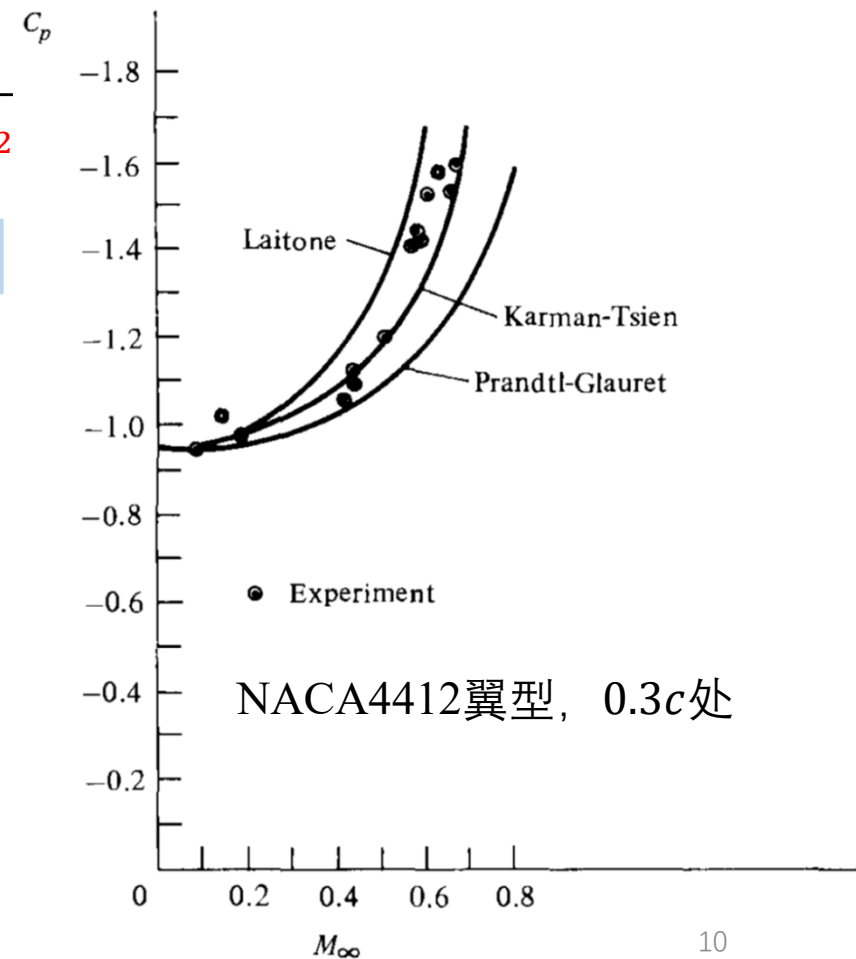
13.3Prandtl-Glauert压缩性修正(10.4)

5.翼型：

机翼：戈泰特法则（1946）

$$\left(C_{l1}\right)_{Ma_\infty,\ \alpha,\ \bar{i},\ \bar{f}} = \frac{1}{\beta}\left(C_{l3}\right)_{0,\ \alpha,\ \bar{i},\ \bar{f}}$$

$$\left(C_{m1}\right)_{Ma_\infty,\ \alpha,\ \bar{i},\ \bar{f}} = \frac{1}{\beta}\left(C_{m3}\right)_{0,\ \alpha,\ \bar{i},\ \bar{f}}$$

$$\left(C_{l1}^{\alpha}\right)_{Ma_\infty,\ \alpha,\ \bar{i},\ \bar{f}} = \frac{1}{\beta}\left(C_{l3}^{\alpha}\right)_{0,\ \alpha,\ \bar{i},\ \bar{f}}$$

$$\left(C_{p1}\right)_{Ma_\infty,\ \alpha,\ \bar{i},\ \bar{f},\ \lambda,\ \tan\chi,\ \eta} = \frac{1}{\beta}\left(C_{p3}\right)_{0,\ \alpha,\ \bar{i},\ \bar{f},\ \boxed{\beta\lambda,\ \frac{1}{\beta}\tan\chi,\ \eta}}$$

$$\left(C_{M1}\right)_{Ma_\infty,\ \alpha,\ \bar{i},\ \bar{f},\ \lambda,\ \tan\chi,\ \eta} = \frac{1}{\beta}\left(C_{M3}\right)_{0,\ \alpha,\ \bar{i},\ \bar{f},\ \boxed{\beta\lambda,\ \frac{1}{\beta}\tan\chi,\ \eta}}$$

$$\left(C_{L1}^{\alpha}\right)_{Ma_\infty,\ \alpha,\ \bar{i},\ \bar{f},\ \lambda,\ \tan\chi,\ \eta} = \frac{1}{\beta}\left(C_{L3}^{\alpha}\right)_{0,\ \alpha,\ \bar{i},\ \bar{f},\ \boxed{\beta\lambda,\ \frac{1}{\beta}\tan\chi,\ \eta}}$$

机翼平面形状有关

展弦比、后掠角、梢根比
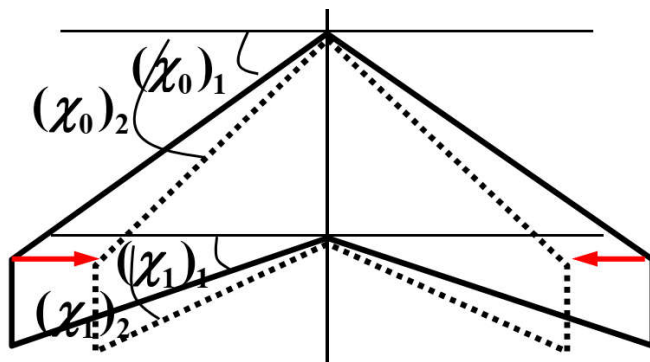
亚声速机翼对应不可压机翼：
展弦比变小、后掠角变大、稍根比不变！

13.4临界马赫数(10.5)

线化理论$Ma < 0.8$，跨声速$(0.8 < Ma < 1.2)$非线性强，理论复杂！

跨声速$(0.8 < Ma < 1.2)$特性：

Local $M_A = 0.435$

$M_\infty = 0.3$

A

A: 最低压强点，$Ma_A > Ma_\infty$

A略早于t_{max}

Local $M_A = 1.0$

$M_\infty = M_{cr} = 0.61$

A

$Ma_A =$1时，$Ma_\infty = Ma_{cr}$临界马赫数！

Local $M_A = 0.772$

$M_\infty = 0.5$

A

$Ma_\infty\uparrow$，$Ma_A\uparrow$

Sonic line where $M = 1$

$M > 1$

$M_\infty = 0.65 > M_{cr}$
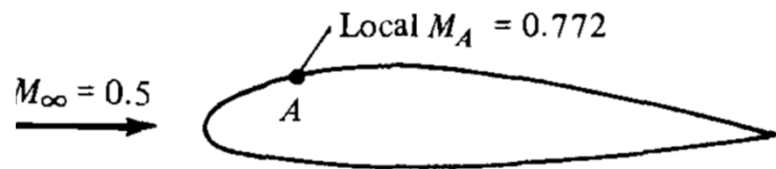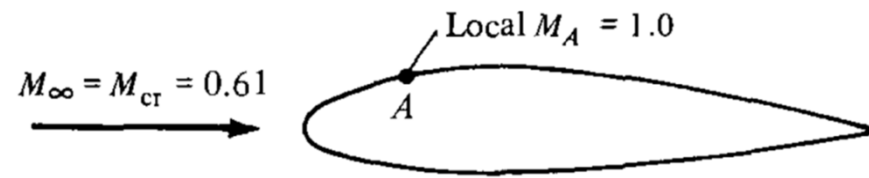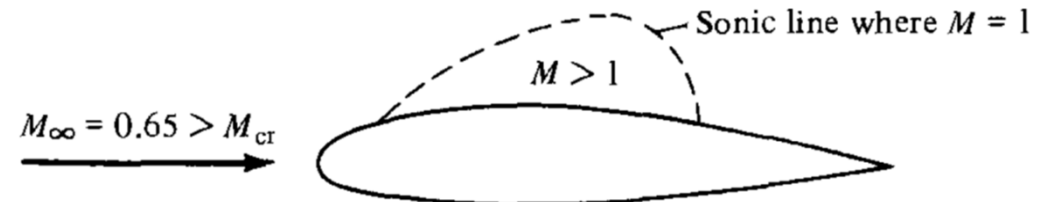
$Ma_\infty > Ma_{cr}$时翼型阻力急剧增加！！

13.4临界马赫数(10.5)

$$M_\infty = M_{cr} = 0.61 \qquad \text{Local } M_A = 1.0$$

临界马赫数Ma_{cr}如何确定？
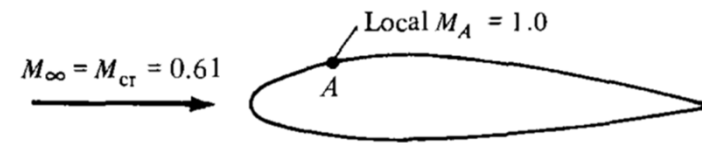
$$C_{p,A} = \frac{p_A - p_\infty}{0.5\rho V_\infty^2} = (\frac{p_A}{p_\infty} - 1)\frac{2}{\gamma Ma_\infty^2} = \frac{2}{\gamma Ma_\infty^2}[(\frac{1+\frac{\gamma-1}{2}Ma_\infty^2}{1+\frac{\gamma-1}{2}Ma_A^2})^{\frac{\gamma}{\gamma-1}} - 1]$$

$$\boxed{\frac{p_0}{p} = (1 + \frac{\gamma-1}{2}Ma^2)^{\frac{\gamma}{\gamma-1}}}$$

临界马赫数$Ma_\infty = Ma_{cr} \rightarrow Ma_A = 1$

$$C_{p,cr} = \frac{2}{\gamma Ma_{cr}^2}[(\frac{1+\frac{\gamma-1}{2}Ma_{cr}^2}{1+\frac{\gamma-1}{2}})^{\frac{\gamma}{\gamma-1}} - 1] \qquad ①$$

等熵流参数关系，与翼型无关！

确定Ma_{cr}需$C_{p,min}$！

压缩性修正：$C_p = \dfrac{C_{p0}}{\sqrt{1-Ma_\infty^2}} \qquad ②$
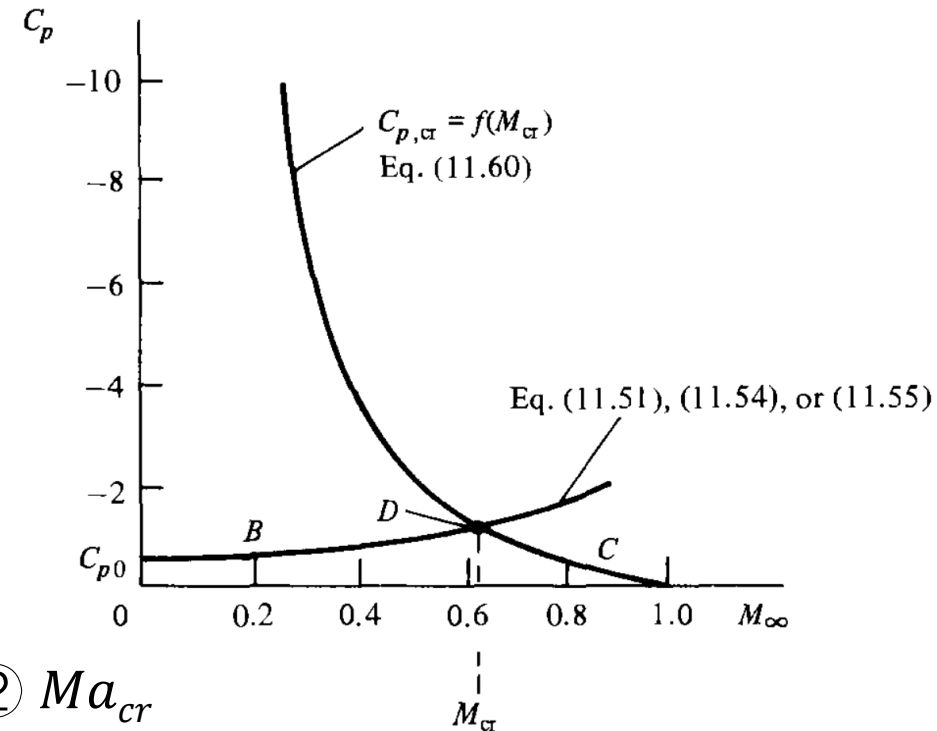
确定Ma_{cr}: 不可压$C_{p,0A} \rightarrow C_{p,0A}(Ma_\infty) \rightarrow ①+② \ Ma_{cr}$

13.4临界马赫数(10.5)
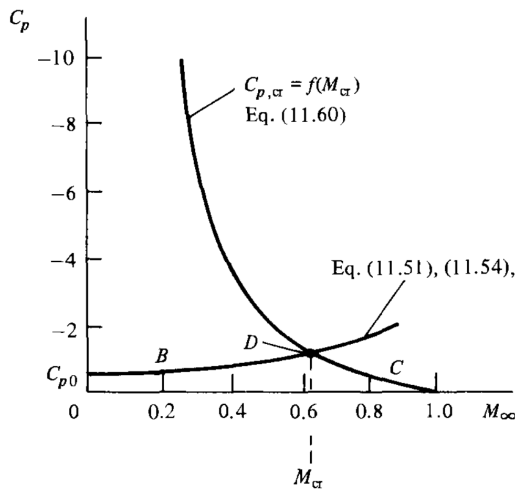
$$C_{p,cr} = \frac{2}{\gamma Ma_{cr}^2}\left[\left(\frac{1+\frac{\gamma-1}{2}Ma_{cr}^2}{1+\frac{\gamma-1}{2}}\right)^{\frac{\gamma}{\gamma-1}} - 1\right] \quad ①$$

压缩性修正：$C_p = \dfrac{C_{p0}}{\sqrt{1-Ma_\infty^2}}$ ②

$M_\infty = M_{cr} = 0.61$ Local $M_A = 1.0$

临界马赫数Ma_∞与$C_{p,0min}$有关，
受厚度、弯度、迎角影响！
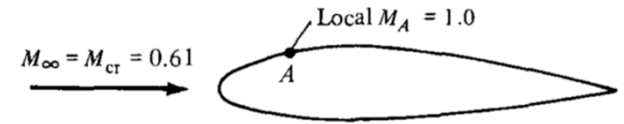
厚度影响: ??

$\boxed{\bar{t}\uparrow, |C_{p,0min}|\uparrow}$
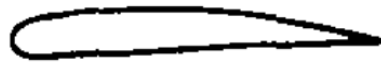
Thick airfoil

$C_p \approx -\dfrac{2\hat{u}}{V_\infty} \rightarrow \bar{t}\uparrow, \hat{u}\uparrow, |C_p|\uparrow$

$\boxed{\bar{t}\uparrow, |C_{p,0min}|\uparrow, Ma_{cr}\downarrow}$

Thin airfoil

$\boxed{Ma_{cr厚} < Ma_{cr薄}}$

$C_{p,cr} = f(M_{cr})$

Thin airfoil

Thick airfoil

Thick C_{p0}

Thin C_{p0}

Thick airfoil

Thin airfoil

$(M_{cr})_{thin}$

$(M_{cr})_{thick}$

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13.4临界马赫数(10.5)

$M_\infty = M_{cr} = 0.61$ Local $M_A = 1.0$ A
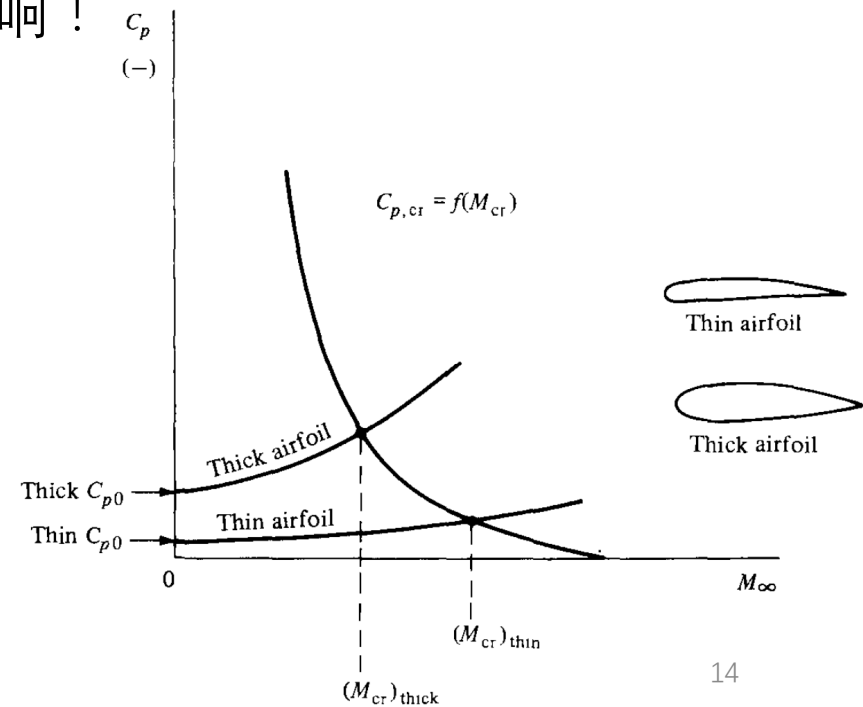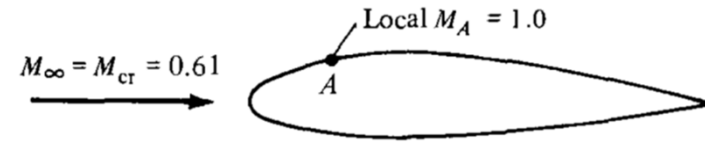
下临界马赫数Ma_{cr}：上翼面开始出现$Ma_{max} = 1$时来流Ma；

上临界马赫数Ma：翼面上全为超声速流时来流Ma；

上下临界马赫数Ma间为跨声速区间!!　与厚度、弯度、迎角有关！

$\bar{t} = 0.01,\ 0.05,\ 0.10$ 对应跨声速区间：

$Ma_\infty = 0.946 \sim 1.051,\ 0.833 \sim 1.143,\ 0.717 \sim 1.219$

13.5阻力发散Ma，声障(10.6)

1. $C_d \sim Ma$:

①$a-c$: $Ma_\infty < Ma_{cr}$, C_d不随Ma_∞变化！

$Ma_\infty \uparrow, Re \uparrow$，摩擦阻力$\downarrow$,

$\frac{dC_p}{dx} \uparrow$，分离区变大，压差阻力$\uparrow$

→C_d变化小；

②$c-d$: $Ma_\infty > Ma_{cr}$, 上表面超声速区

→产生激波→阻力增加

③$c-d-e$: $Ma_\infty \uparrow$, 超声速区$\uparrow$,

→激波$\uparrow$ →阻力$\uparrow$
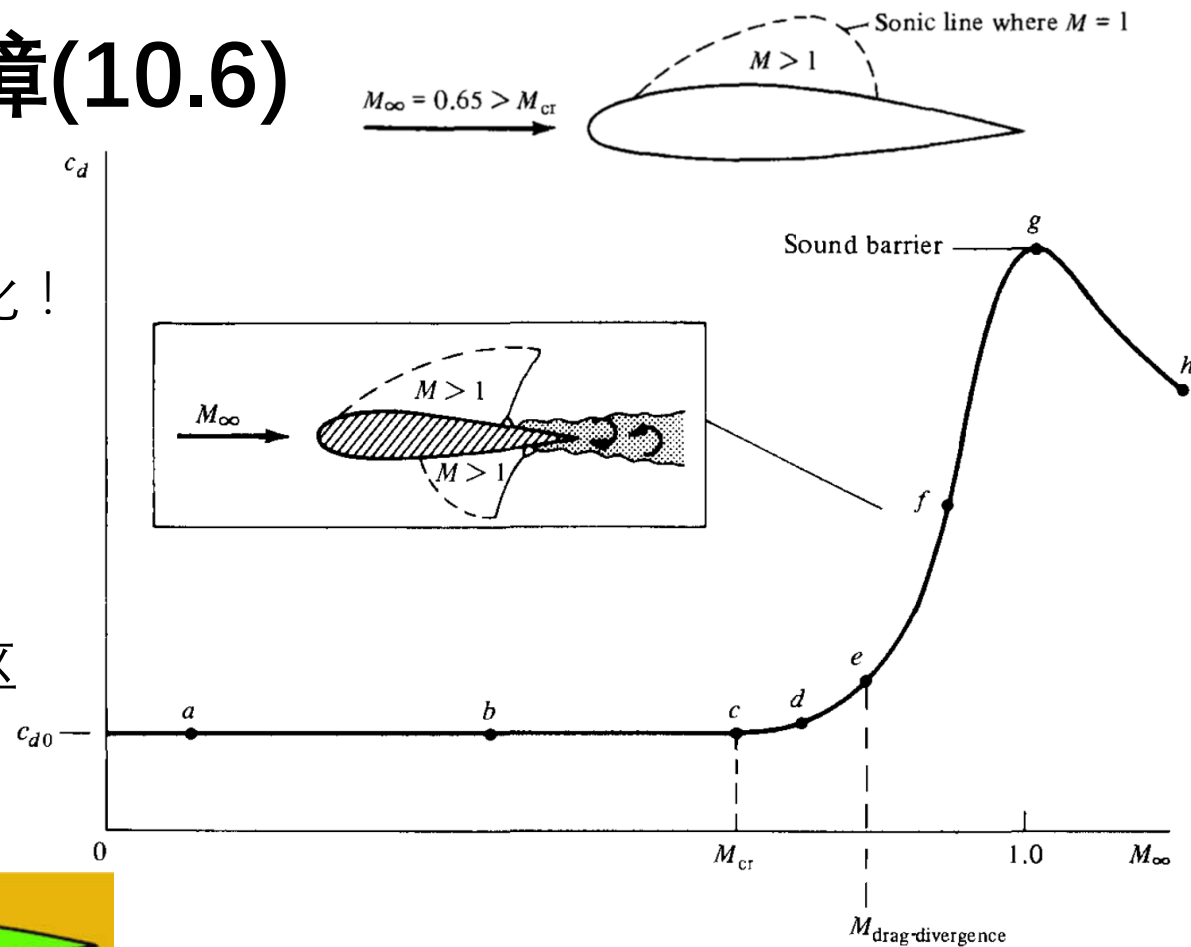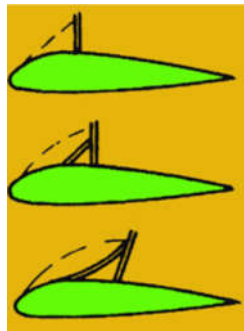
13.5阻力发散Ma，声障(10.6)

1. $C_d \sim Ma$:

④e: Ma_∞↑, 上下翼面超声速区扩大，
激波→逆压梯度↑,流动分离↑
→阻力开始急剧增加(阻力发散)。

阻力发散Ma:
$$dC_d \Big/ dMa_\infty = 0.1$$

g：C_d可增加到10 $C_{d,0}$。

⑤g: Ma_∞→1, 上下翼面激波移至后缘，
→阻力达最大C_{dmax}。

⑥$g-f$: Ma_∞ >1, 进入超声速流，
Ma_∞↑C_d↓
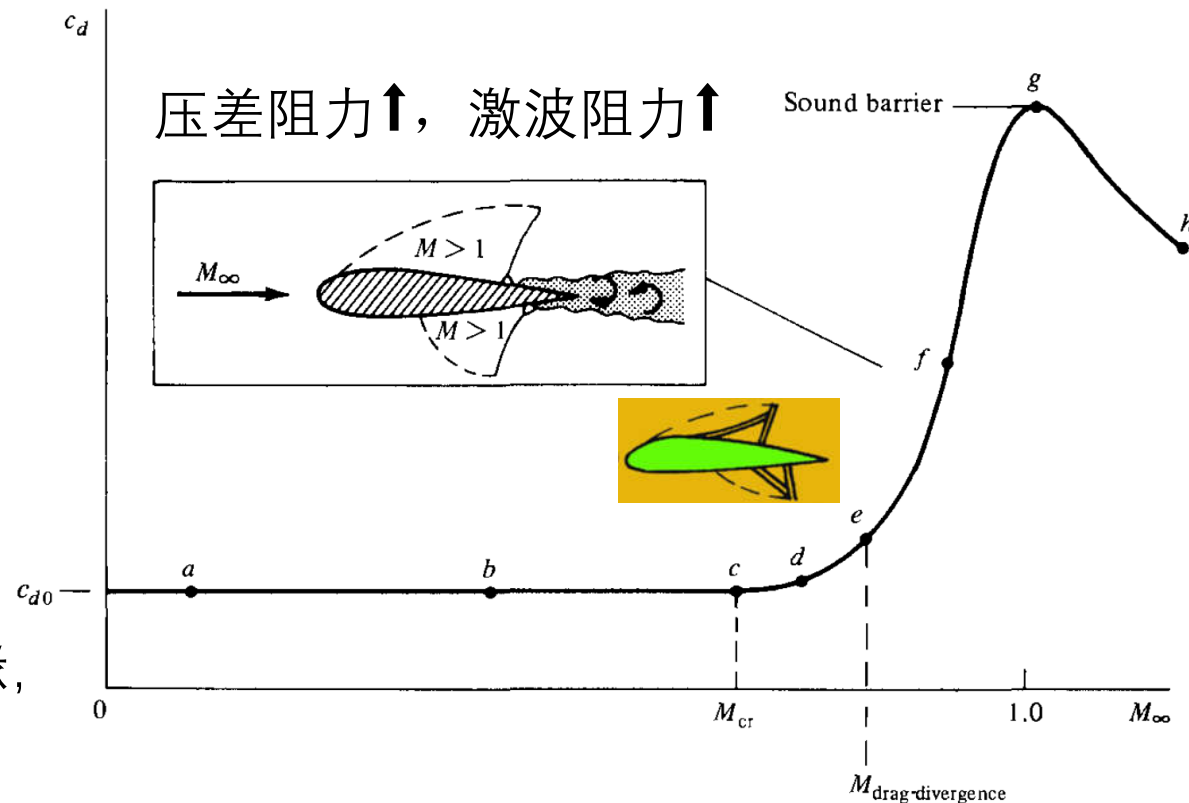
压差阻力↑，激波阻力↑

进入超声速流之前，必须跨越极大阻力！！

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13.5阻力发散Ma，声障(10.6)

1. $C_d \sim Ma$:

$Ma_\infty \to 1, C_d \uparrow\uparrow$：声障

需足够动力，才能跨越声障，进入超声速区！
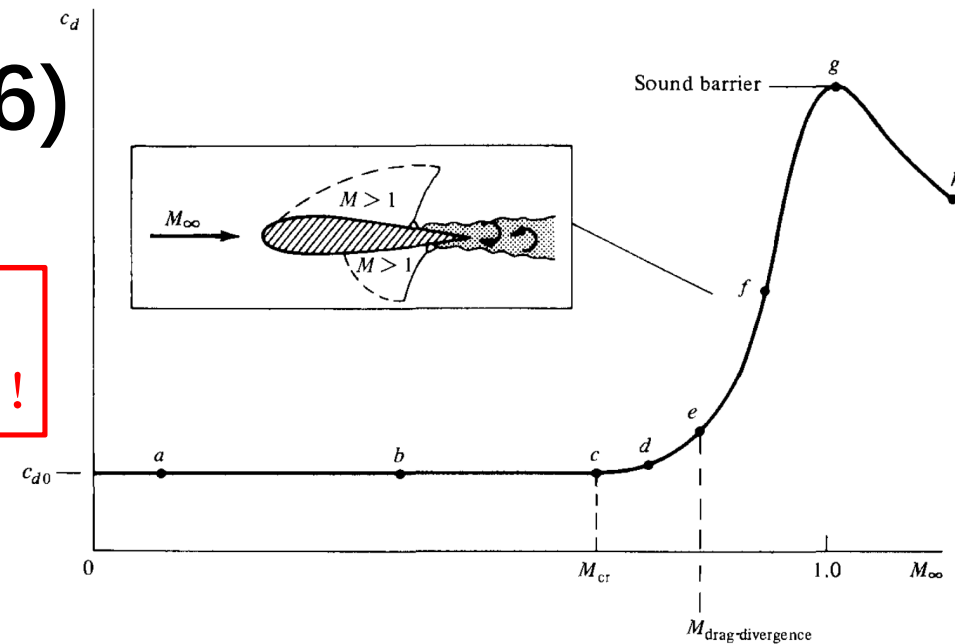
$$C_p = \frac{C_{p,0}}{\sqrt{1 - Ma_\infty^2}} \quad Ma_\infty \to 1, C_p \to \infty ??$$

$Ma_\infty \to 1,$ 上式不成立！！

(d) $C_d - Ma_\infty$ (Ma_∞的范围较大)
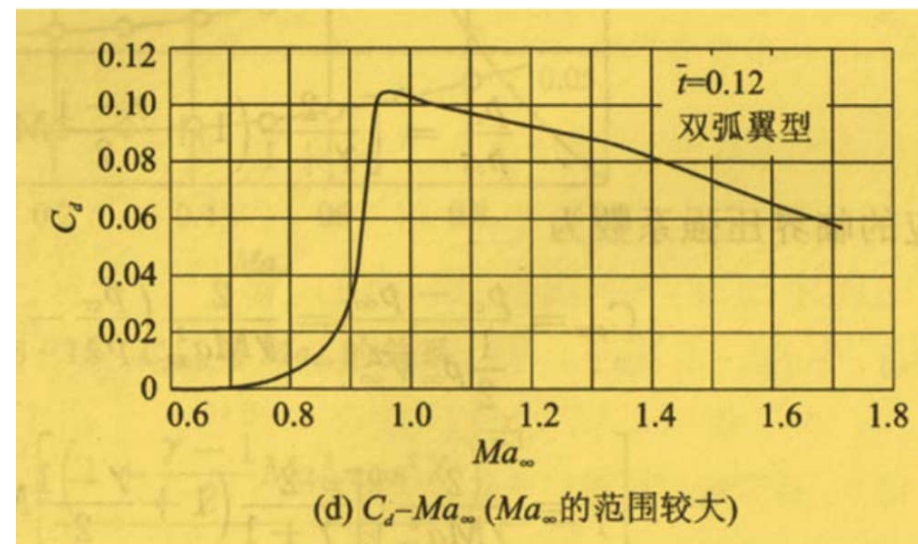
13.5 阻力发散Ma，声障(10.6)

2. 如何减小C_d, C_{dmax}：　$C_{dmax} \rightarrow 2{\sim}3C_{d0}$

① 增大Ma_{cr}：　减小$\bar{t}$!!

薄翼型；后掠翼。
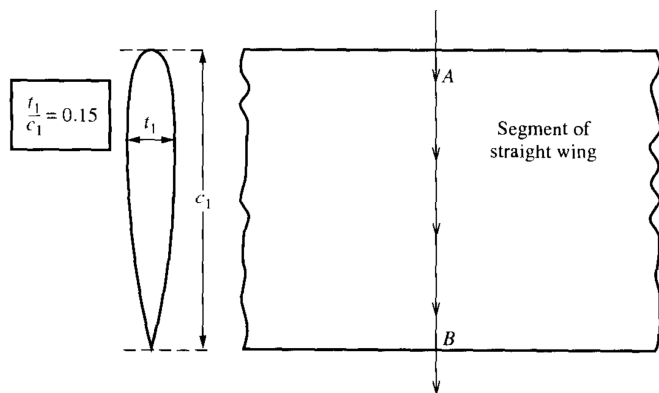
后掠翼$\bar{t}\downarrow$，$Ma_{cr}\uparrow$。
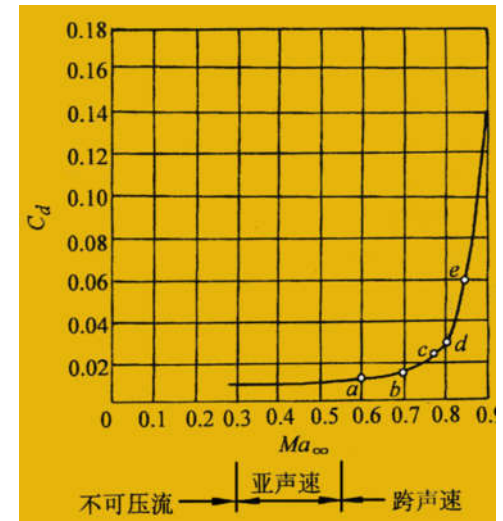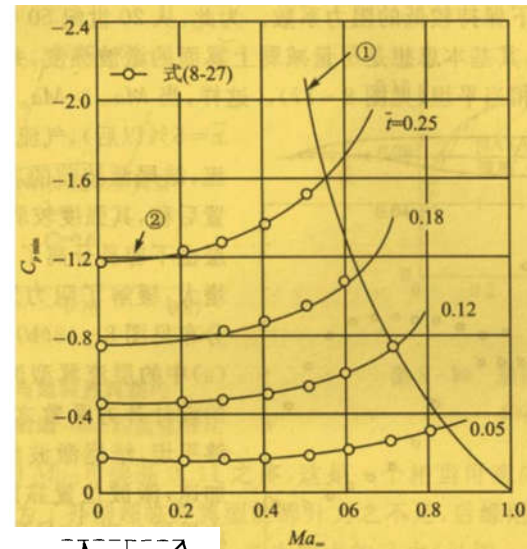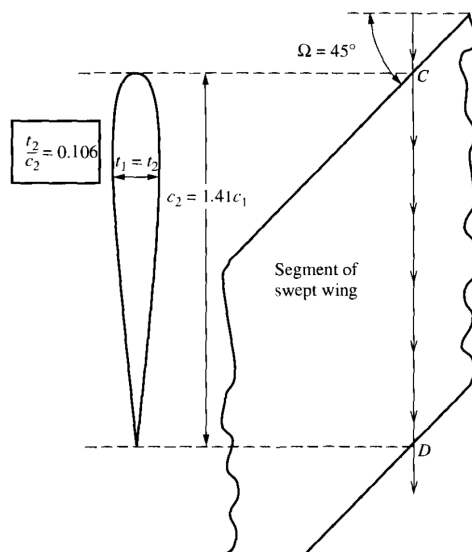
13.5阻力发散Ma，声障(10.7)

2. 如何减小C_d, C_{dmax}:

②增大Ma_{cr}与$Ma_{阻力发散}$间隔：1965，超临界翼型！
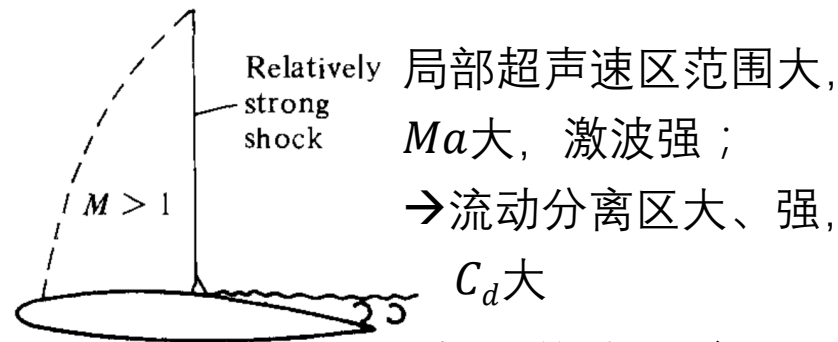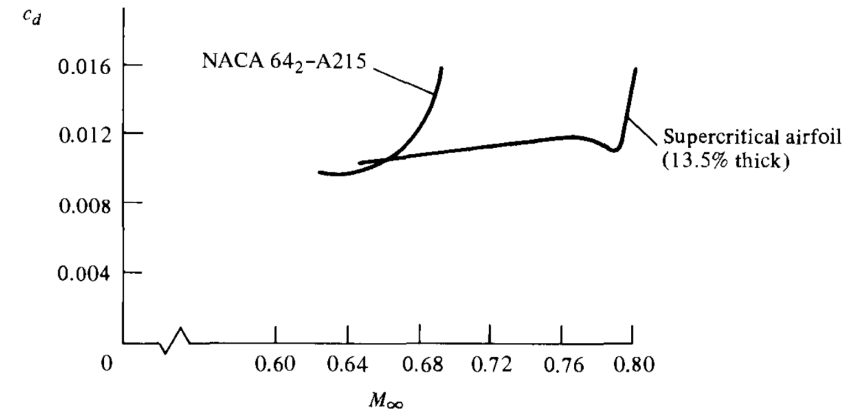
局部超声速区范围大，Ma大，激波强；
→流动分离区大、强，C_d大

减小激波强度！??
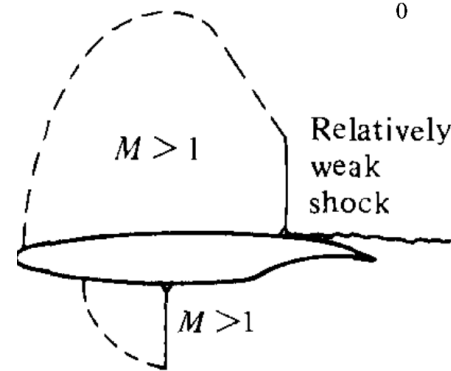
超临界翼型：上表面平坦，C_p变化小；
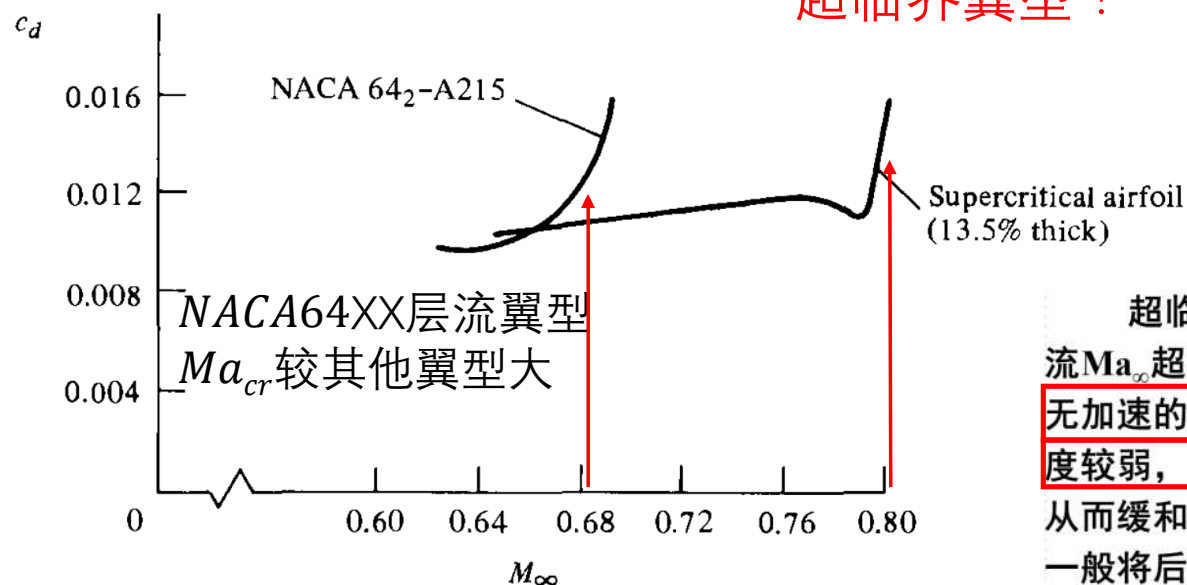Ma小，激波较弱；
局部超声速区范围小，
→分离区小，弱；
C_d增加缓慢，$Ma_{阻力发散}$大
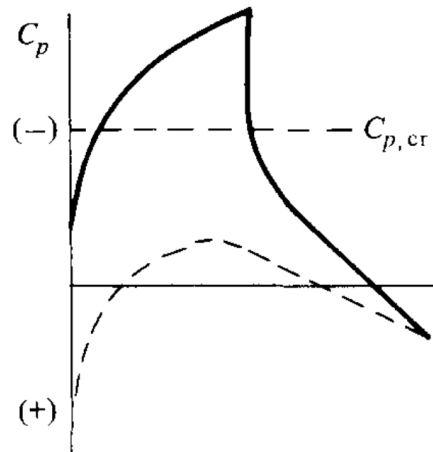
上表面C_p变化→
→波前Ma大小→
局部超声速区强度
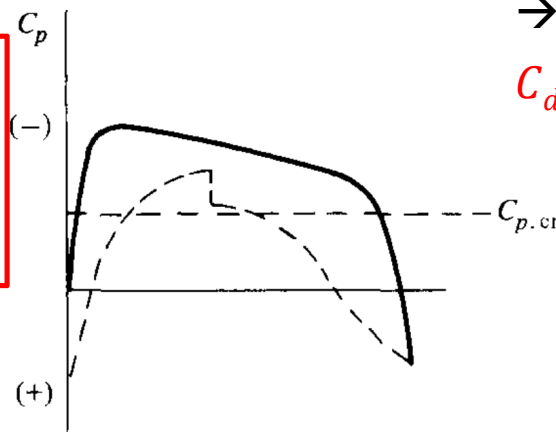
13.5阻力发散Ma，声障(10.6)

2. 如何减小C_d, C_{dmax}：

②增大Ma_{cr}与$Ma_{阻力发散}$间隔： 1965，Whitecomb 超临界翼型！

尖后缘

NACA64XX层流翼型
Ma_{cr}较其他翼型大

超临界翼型的几何特点是，上翼面曲率较小比较平坦，使来流Ma_∞超过临界马赫数后，大约从距前缘5%弦长处沿上翼面为一无加速的均匀超声速，这样尾激波前的超声速马赫数较低，激波强度较弱，且伸展范围不大，波后逆压梯度较小，边界层不易分离，从而缓和了阻力发散现象。为了补偿超临界翼型前段升力的不足，一般将后缘附近的下表面做成内凹形以增大翼型后段弯度使后段能产生较大升力，称为后缘加载。
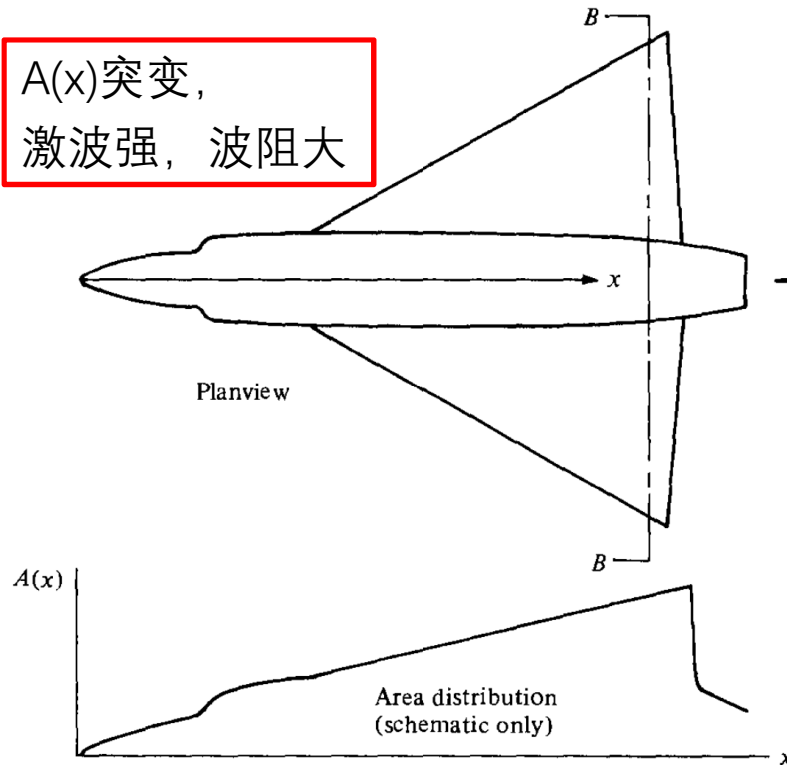
13.5阻力发散Ma，声障(11.4)

2. 如何减小C_d, C_{dmax}：

③面积律： 1950s，Whitecomb

外壳截面变化光滑的超声速子弹比截面变化不连续的，飞行速度大。

改变A(x)分布，使其更连续➔减小C_{Dmax}。

A(x)突变，
激波强，波阻大

A(x)分布光滑，
激波弱，波阻小

Planview

Planview

$A(x)$

Area distribution
(schematic only)

$A(x)$

Area distribution
(schematic only)
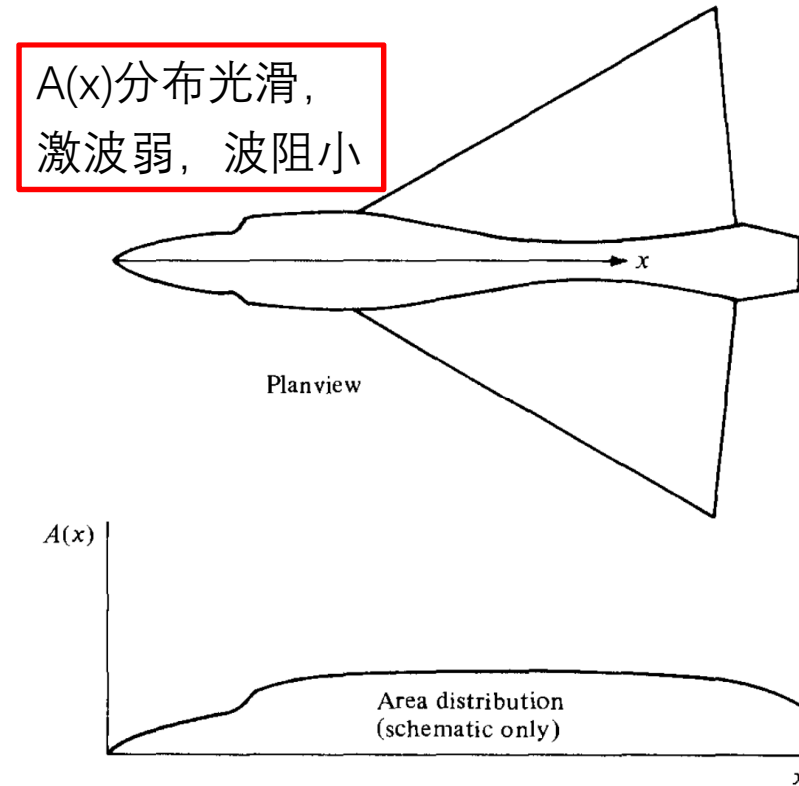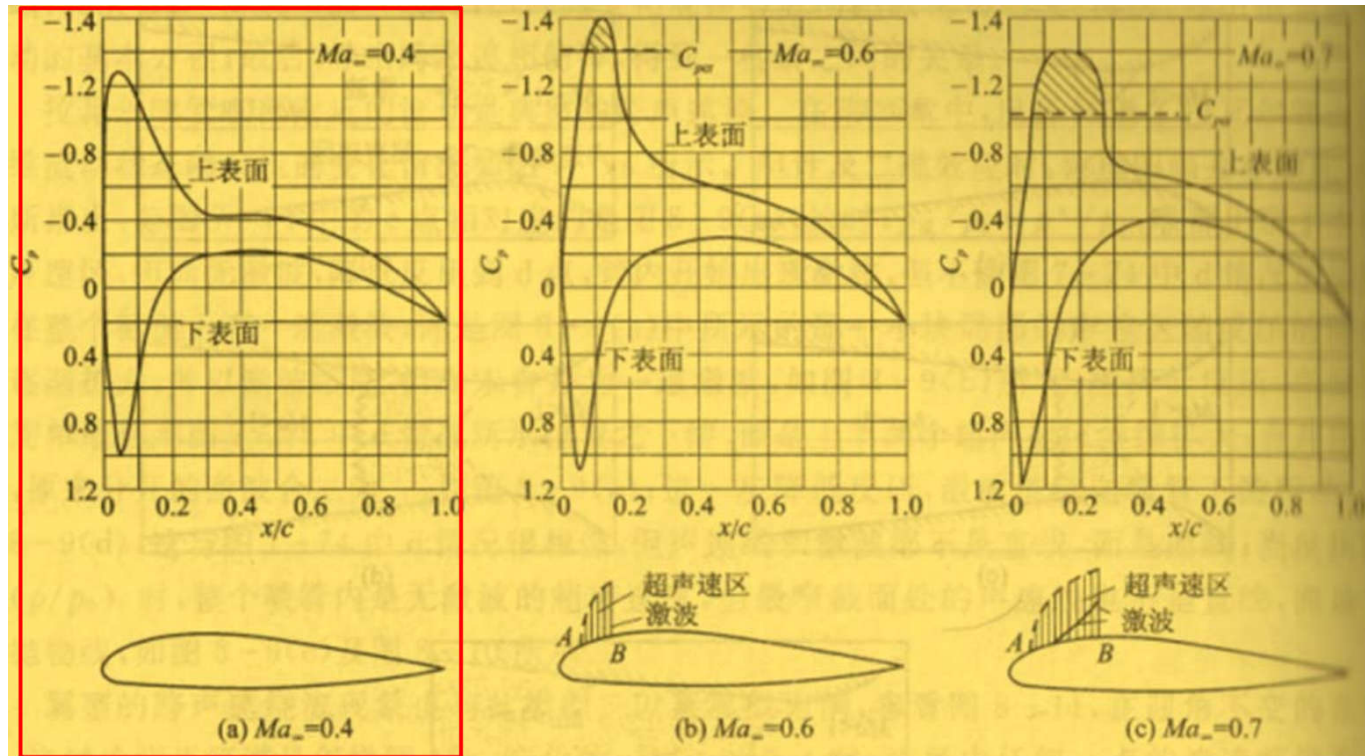
13.5阻力发散Ma，声障(11.4)

3. 跨声翼型绕流特征:

13.5阻力发散Ma，声障(11.4)

3. 跨声翼型绕流特征:

(d) $Ma_\infty = 0.77$　　(e) $Ma_\infty = 0.80$　　(f) $Ma_\infty = 0.85$
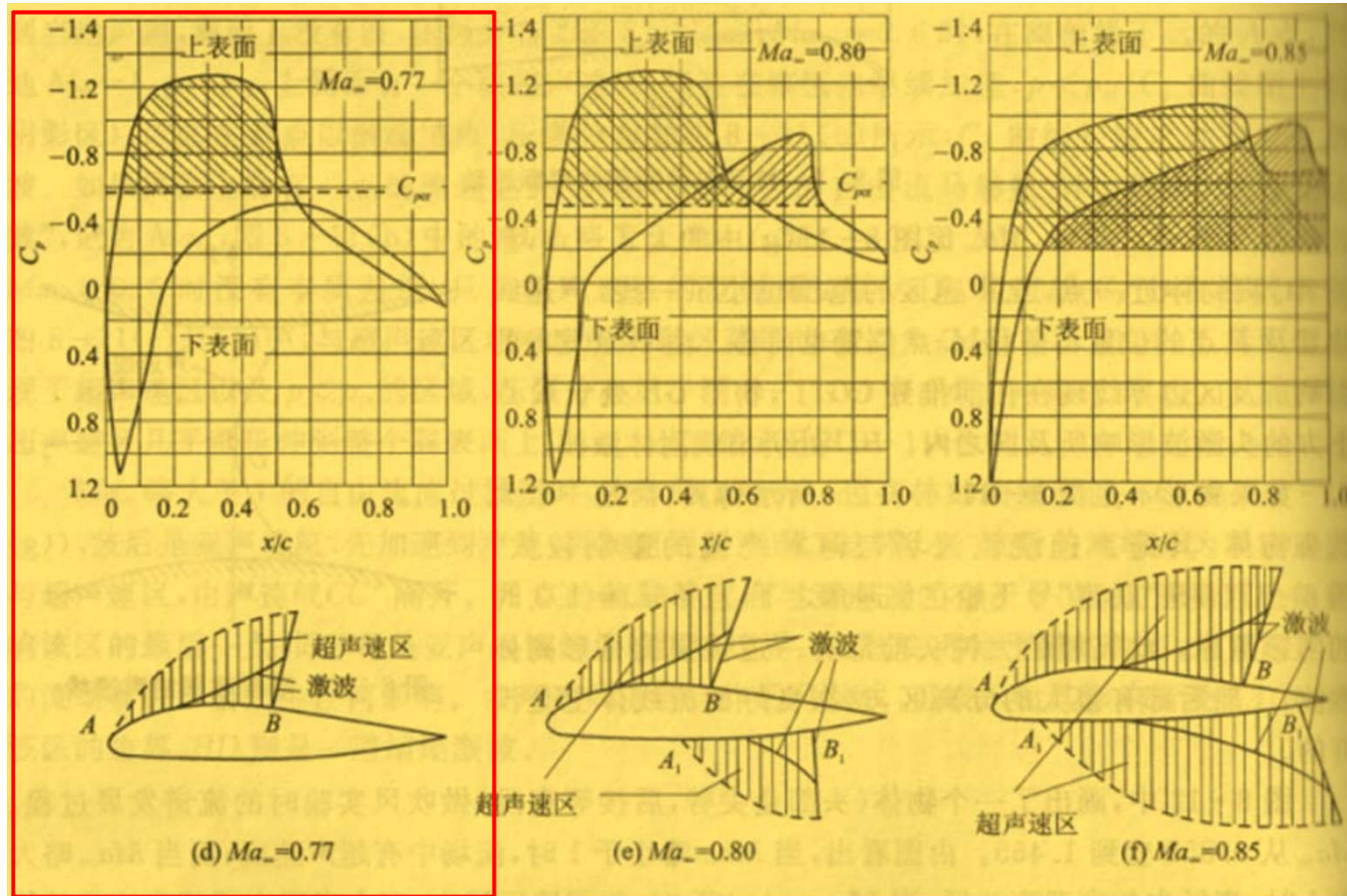
13.5阻力发散Ma，声障(11.4)

3. 跨声翼型绕流特征:
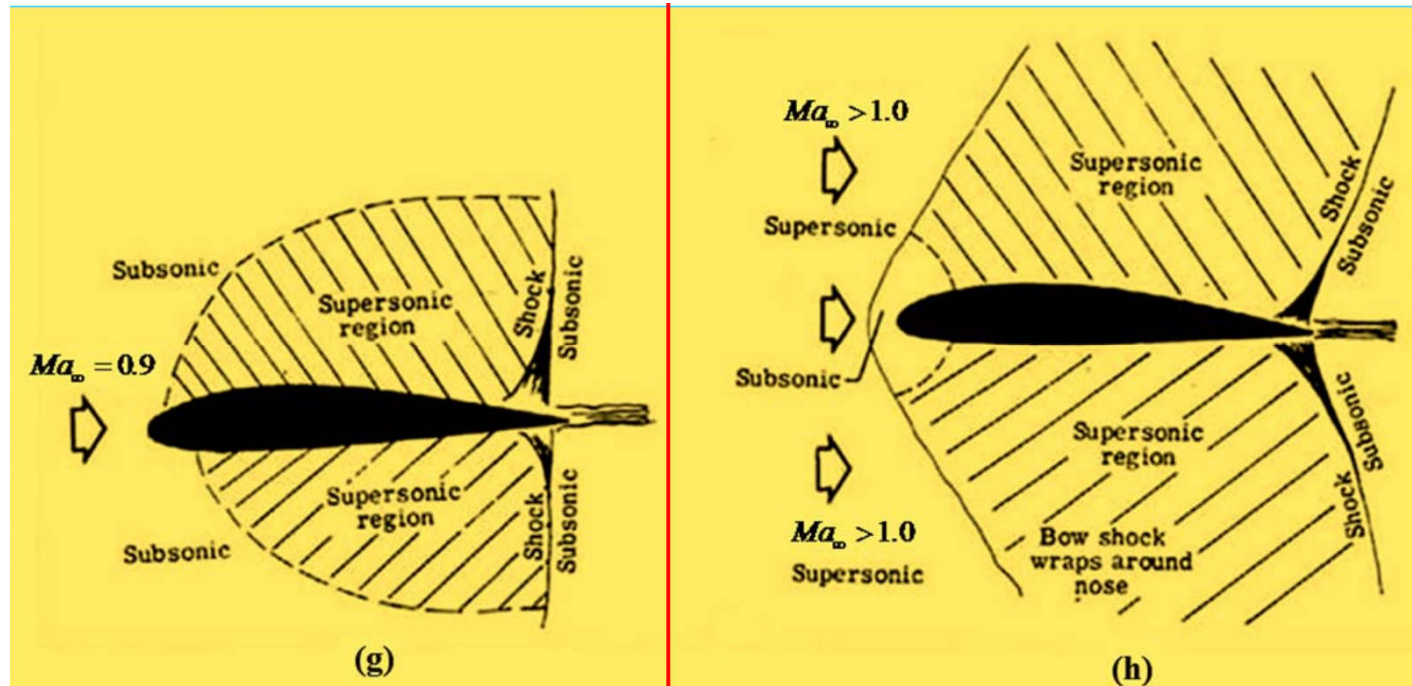
13.6超声速翼型、绕流特点(10.8)

1947, X-1首次超声速飞行，跨越声障后，飞行平顺！

1.超声速翼型特点:
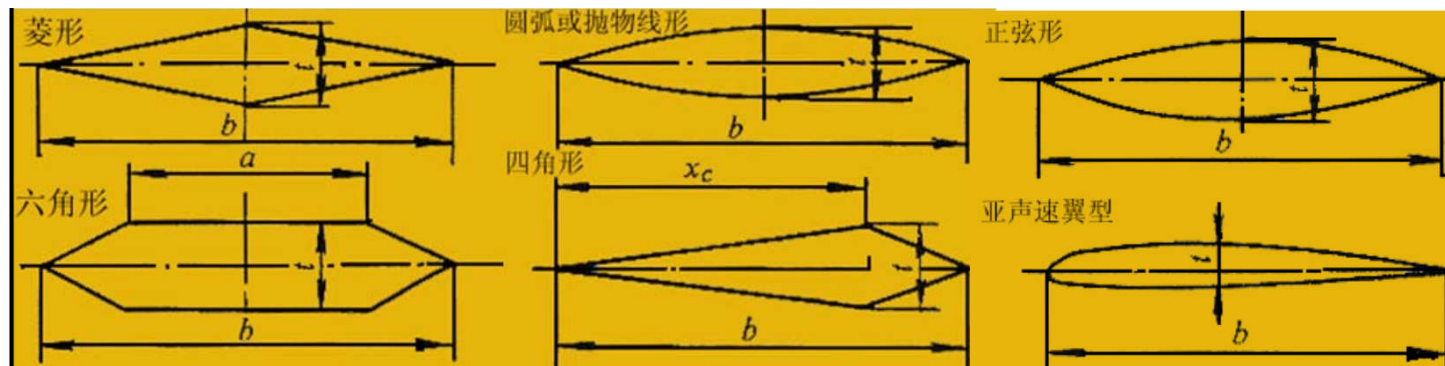
1）尖头尖尾；

2）对称翼型；→减小波阻！

3）厚度小。

低超声速飞行器，小圆头，兼顾低速性能（起飞和着陆）。

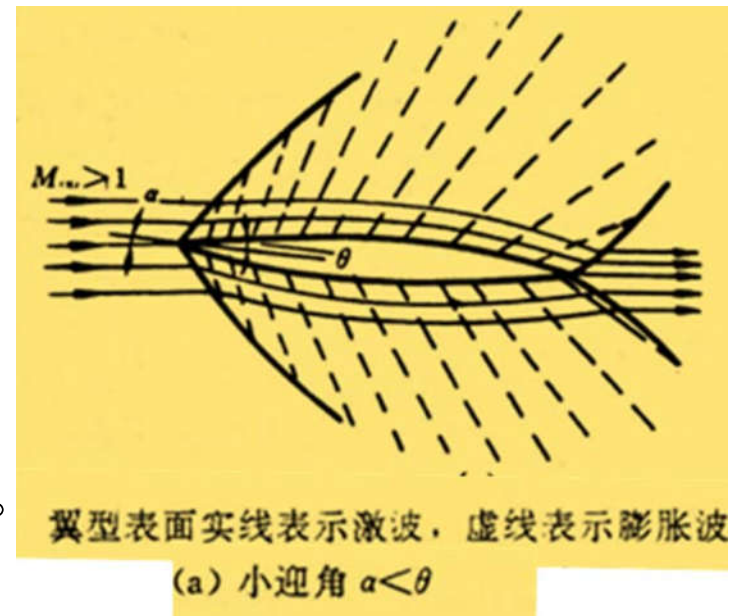# 13.6超声速翼型、绕流特点(10.8)
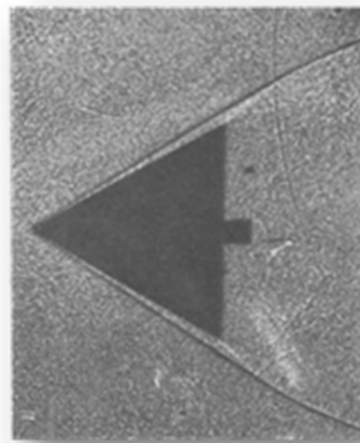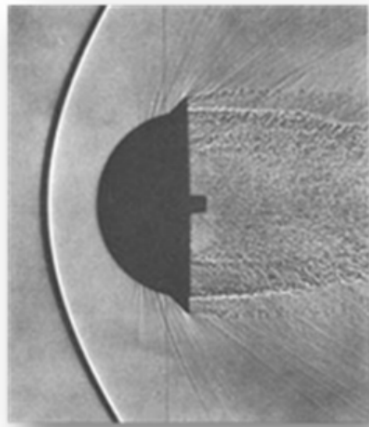
2.超声速翼型绕流特点：

1）钝头体：脱体激波；
2）尖头体：斜激波   →波阻！

13.6超声速翼型、绕流特点(10.8)

2.超声速翼型绕流特点：

（1） $0 < \alpha < \theta$:

上下翼面前缘均有斜激波；
上翼面激波较弱，下翼面较强；
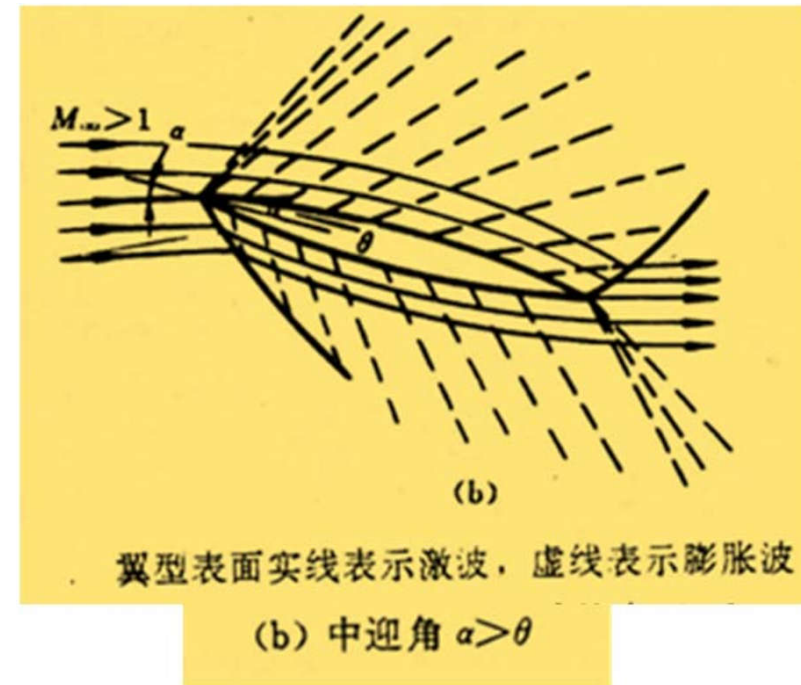激波后加速膨胀；
后缘上下翼面均有斜激波；
后缘汇聚后，气流方向相同，压强相等。

双弧翼，前缘半顶角θ

翼型表面实线表示激波，虚线表示膨胀波
(a) 小迎角 $\alpha < \theta$

13.6超声速翼型、绕流特点(10.8)

2.超声速翼型绕流特点：

（2） $\alpha > \theta$:

前缘上翼面膨胀波，下翼面斜激波；
上下翼面前后缘间膨胀波；
后缘上翼面斜激波，下翼面膨胀波；
后缘汇聚后，气流方向相同，压强相等。

双弧翼，前缘半顶角θ

翼型表面实线表示激波，虚线表示膨胀波

(b) 中迎角 $\alpha > \theta$

作业:

复习笔记!

空气动力学书10.1, 10.2, 10.3