空气与气体动力学

张科

回顾：

1.势流、势函数、流函数：

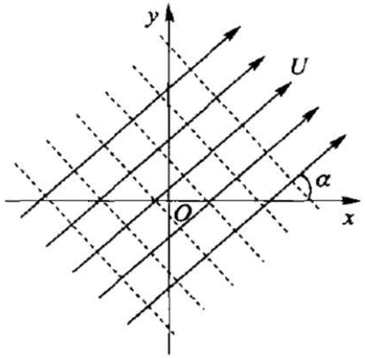
$$u=\frac{\partial\phi}{\partial x}=\frac{\partial\psi}{\partial y} \qquad u_r=\frac{\partial\phi}{\partial r}=\frac{1}{r}\frac{\partial\psi}{\partial\theta}$$

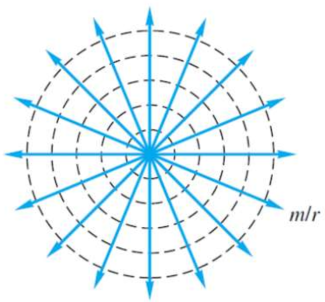
$$v=\frac{\partial\phi}{\partial y}=-\frac{\partial\psi}{\partial x} \qquad u_\theta=\frac{1}{r}\frac{\partial\phi}{\partial\theta}=-\frac{\partial\psi}{\partial r}$$

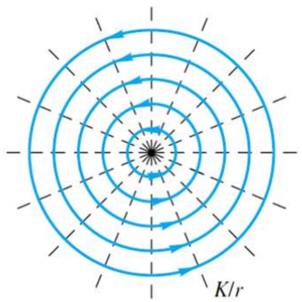
不可压势流：$\nabla^2\phi=0$，$\nabla^2\psi=0$

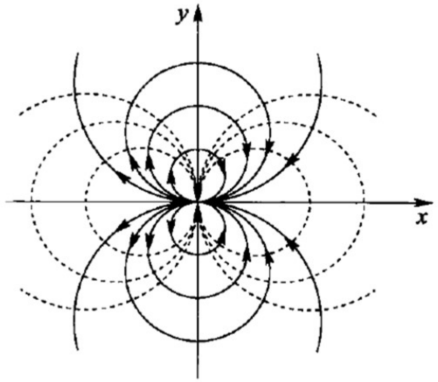
2.基本平面势流：

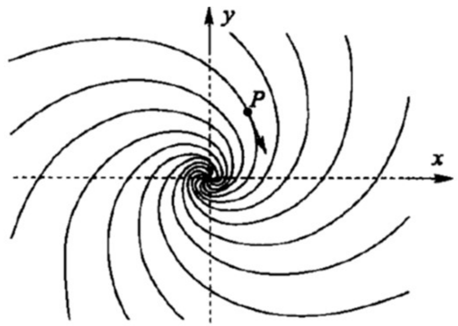
3.基本势流叠加：

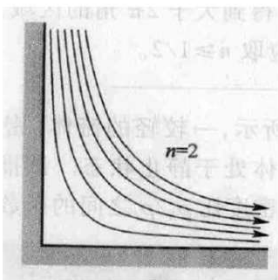








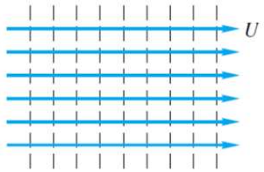


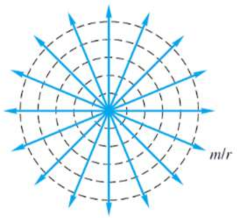


$$\psi=Ur^n\sin n\theta$$

8.4基本流叠加（8.4）

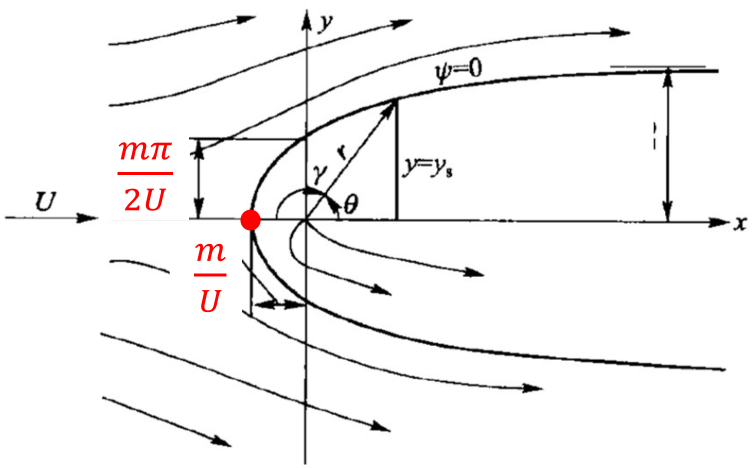
3.半无穷长物体绕流(Rankine half-body,兰金半体扰流)





$\psi = Ursin\theta$　$\psi = m\theta$

$\phi = Urcos\theta$　$\phi = mlnr$



绕流物体轮廓线上有滞止点。

$\psi = \psi_1 + \psi_2 = Ursin\theta + m\theta$　　$m = \frac{Q}{2\pi}$: 点源强度

$\phi = \phi_1 + \phi_2 = Urcos\theta + mlnr$

$u_r = \frac{\partial \phi}{\partial r} = Ucos\theta + \frac{m}{r}$

$u_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta} = -Usin\theta$

滞止点$u_r = u_\theta = 0$

$$\left.\begin{aligned} u_r = Ucos\theta + \frac{m}{r} = 0 \\ u_\theta = -Usin\theta = 0 \end{aligned}\right\} \Rightarrow \theta = \pi,\ \ r = m/U$$

轮廓线上：$\psi_0 = U\frac{m}{U}sin\pi + m\pi = m\pi$

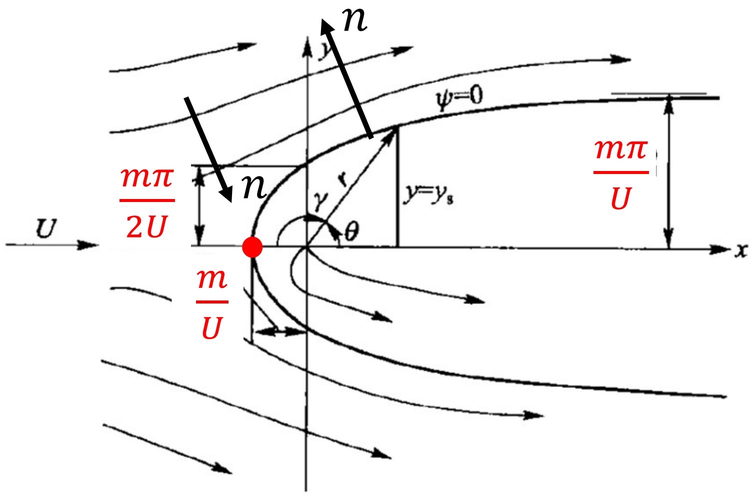
轮廓线为：$\psi = \boxed{Ursin\theta + m\theta = m\pi}$　轮廓线曲线方程

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$时：$Ur = m\pi/2$　$\boxed{r = \frac{m\pi}{2U}}$

8.4基本流叠加（8.4）

$$u_r = U\cos\theta + \frac{m}{r}$$
$$u_\theta = -U\sin\theta$$

3.半无穷长物体绕流(Rankine half-body,兰金半体扰流)



绕流物体轮廓线上有滞止点。

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轮廓线为：$\psi = \boxed{Ur\sin\theta + m\theta = m\pi}$ 轮廓线曲线方程

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$时：$Ur = m\pi/2$ $\boxed{r = \frac{m\pi}{2U}}$

$r \to \infty$时，$\theta \to 0$: $Ur\sin\theta = m\pi$

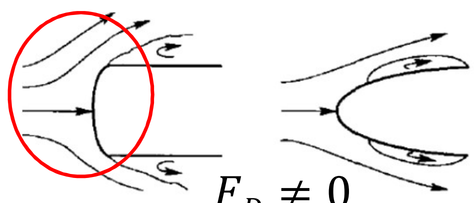
$y = m\pi/U$

半无限长、无粘：

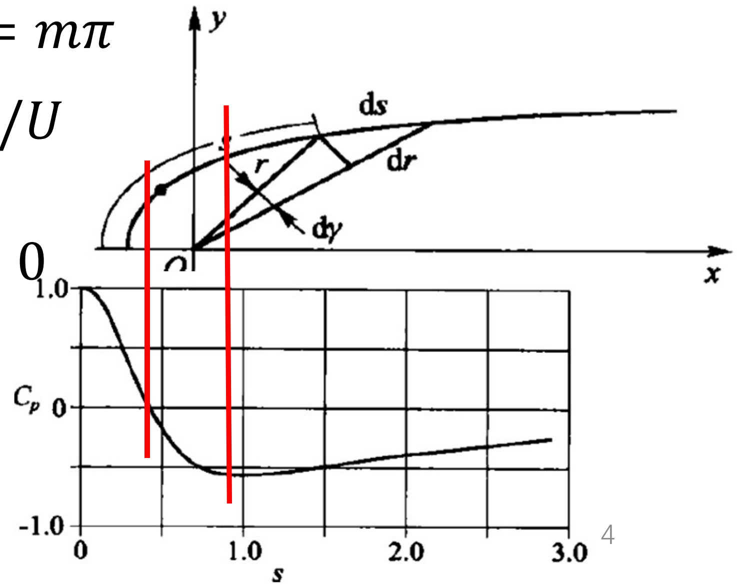
$$F_D = \int_0^\pi (p_s - p_\infty)\, r\cos\theta d\theta = 0$$

$$p_\infty + \frac{1}{2}\rho U^2 = p_s + \frac{1}{2}\rho U_s^2$$

$$C_p = \frac{p_s - p_\infty}{\frac{1}{2}\rho U^2} = \frac{\frac{1}{2}\rho[U^2 - u_r^2 - u_\theta^2]}{\frac{1}{2}\rho U^2}$$
$$= \frac{2}{\gamma}\sin\gamma\cos\gamma - \frac{1}{\gamma^2}\sin^2\gamma$$



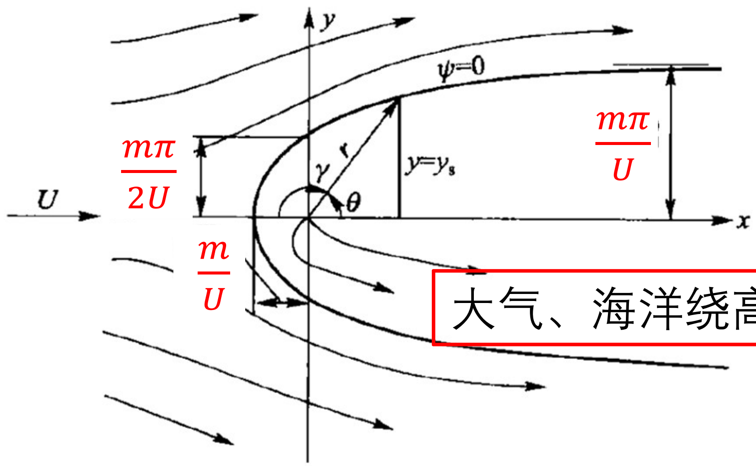
$F_D \neq 0$



8.4基本流叠加（8.4）

$$u_r = U\cos\theta + \frac{m}{r}$$
$$u_\theta = -U\sin\theta$$

3.半无穷长物体绕流(Rankine half-body,兰金半体扰流)



大气、海洋绕高地流动。

例 8.5 如图 8.21 所示，河床有一类似于半体柱的隆起，高 4 m。已知 B 点压强 130 kPa，河流速度 2.5 m/s。试利用势流理论确定高于 B 点 2 m 的 A 点的压强。

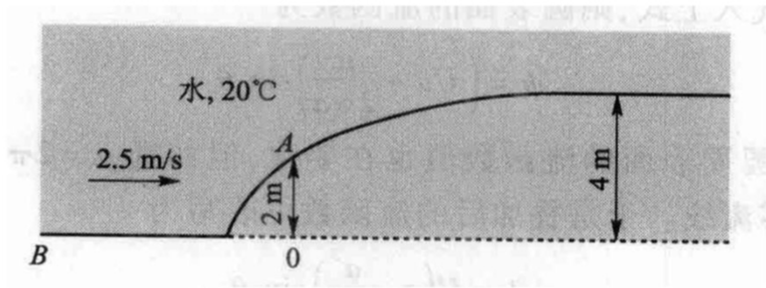


图 8.21 绕河床隆起的流动

P307

解：$\frac{m\pi}{U} = 4\text{m}$，$U = 2.5\text{m}/s$，$p_\infty = 130Kpa$

$$\psi = Ur\sin\theta + m\theta = m\pi$$

A点：$r\sin\theta = 2\text{m}$

$$\theta = \pi/2, r = 2\text{m}$$
$$m = 10/\pi$$

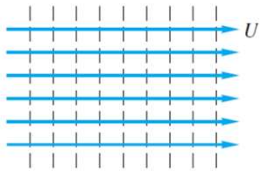
$$u_{Ar} = U\cos\theta + \frac{m}{r} = 5/\pi \quad u_{A\theta} = -U\sin\theta = -2.5$$

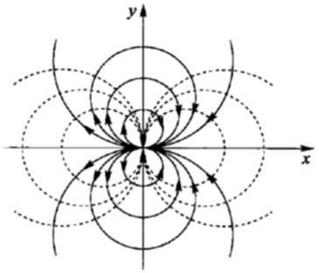
$$p_A + \frac{1}{2}\rho U_A^2 + \rho g z_A = p_\infty + \frac{1}{2}\rho U^2 + \rho g z_B$$

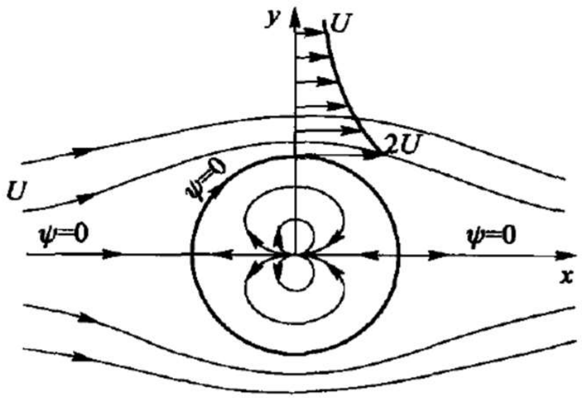
$$p_A = 1.09 \times 10^5 Pa$$

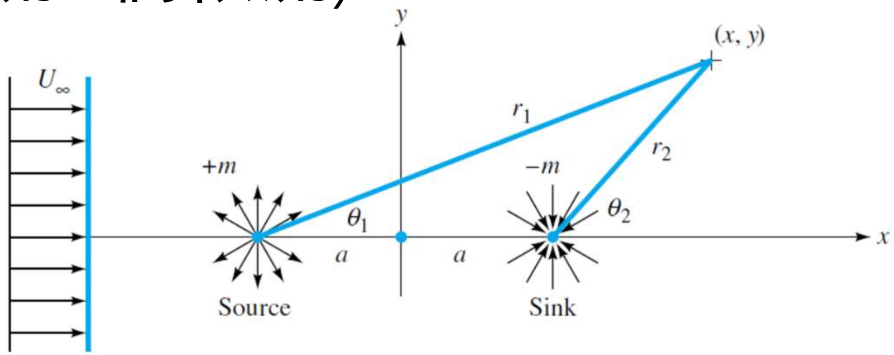
8.4基本流叠加（8.4）

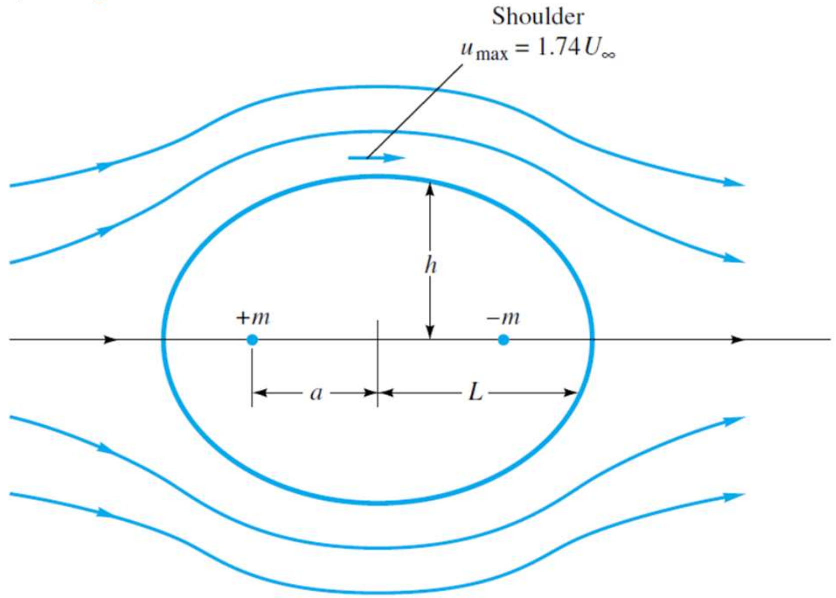
4.绕圆柱无环量流动(均匀流+偶极流)：



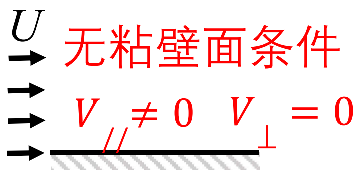




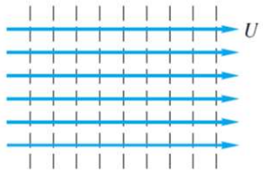


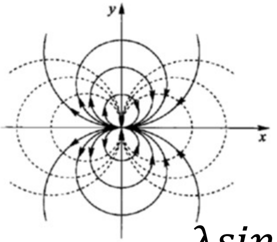


8.4基本流叠加（8.4）



4.绕圆柱无环量流动(均匀流+偶极流)：





$$\psi = Ursin\theta \qquad \phi = Urcos\theta$$

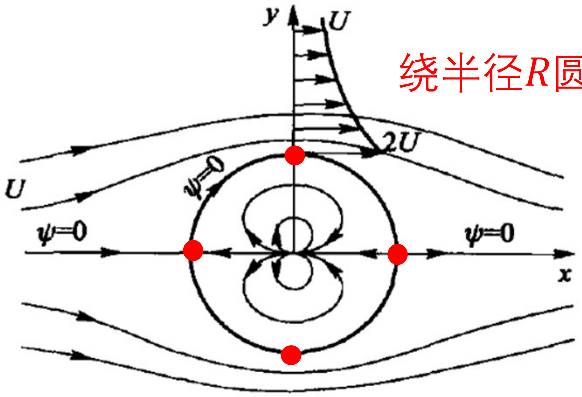
$$\psi = -\frac{\lambda sin\theta}{r} \qquad \phi = \frac{\lambda cos}{r}$$

$$\psi = Ursin\theta - \frac{\lambda sin\theta}{r} \qquad \phi = Urcos\theta + \frac{\lambda cos\theta}{r}$$

封闭流线： $\psi_0 = 0$

绕流体壁面：$\psi = Ursin\theta - \frac{\lambda sin\theta}{r} = 0$

$$r = \sqrt{\frac{\lambda}{U}} = R \ \text{或} \theta = 0, \ \pi$$



绕半径R圆柱势流： $\lambda = R^2 U$

$$\boxed{\begin{aligned}\psi &= U\left(r - \frac{R^2}{r}\right) sin\theta \\ \phi &= U\left(r + \frac{R^2}{r}\right) cos\theta\end{aligned}}$$

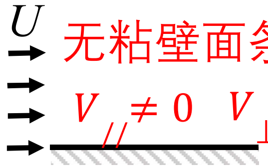
$$u_r = \frac{\partial \phi}{\partial r} = U\left(1 - \frac{R^2}{r^2}\right) cos\theta$$

$$u_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta} = -U\left(1 + \frac{R^2}{r^2}\right) sin\theta$$

壁面上$r = R$： $u_r = 0, u_\theta = -2Usin\theta$

$\theta = 0,\pi$: $u_r = 0, u_\theta = 0$， 前后滞止点 $\qquad \theta = \pm\frac{\pi}{2}$: $u_\theta = 2U,$ 舷点

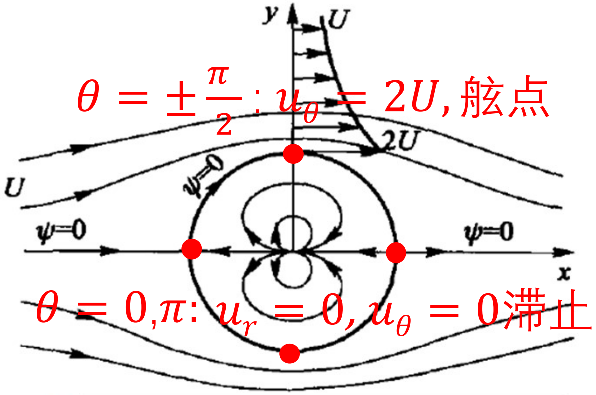
8.4基本流叠加（8.4）



无粘壁面条件

$V_{//} \neq 0 \quad V_{\perp} = 0$

4.绕圆柱无环量流动(均匀流+偶极流)：



$\theta = \pm\frac{\pi}{2}: u_\theta = 2U$, 舷点

$\theta = 0,\pi: u_r = 0, u_\theta = 0$滞止点

$$u_r = \frac{\partial\phi}{\partial r} = U(1 - \frac{R^2}{r^2})\,cos\theta$$

$$u_\theta = \frac{1}{r}\frac{\partial\phi}{\partial\theta} = -U(1 + \frac{R^2}{r^2})\,sin\theta$$

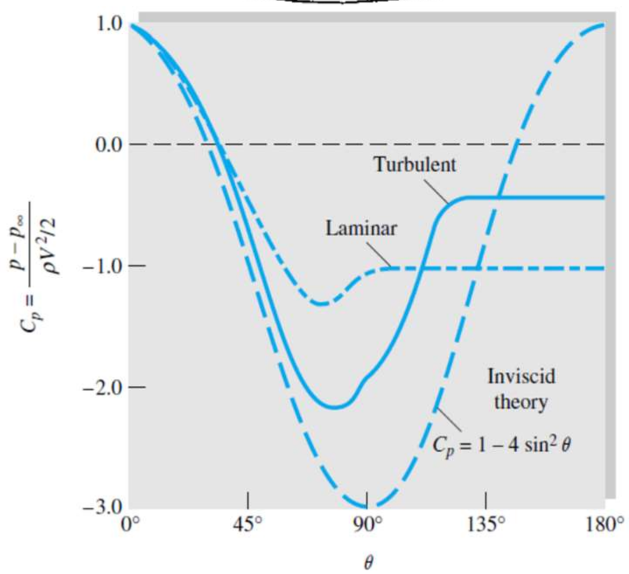
壁面上$r = R$： $u_r = 0, u_\theta = -2Usin\theta$

$$p_\infty + \frac{1}{2}\rho U^2 = p_s + \frac{1}{2}\rho U_s^2$$

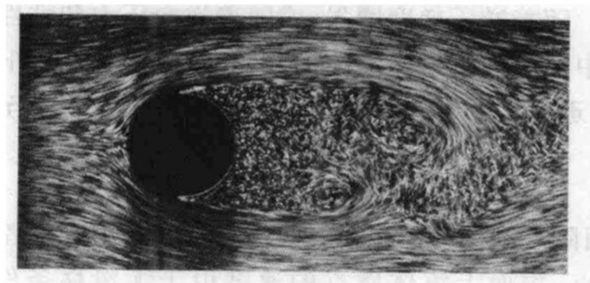
圆柱上$U_s = u_\theta = -2Usin\theta$

$$C_p = \frac{p_s - p_\infty}{\frac{1}{2}\rho U^2} = \frac{\frac{1}{2}\rho[U^2 - 4U^2 sin^2\theta\,]}{\frac{1}{2}\rho U^2} = 1 - 4sin^2\theta$$

$$F_x = -\int_0^{2\pi}(p_s - p_\infty)\,cos\theta R d\theta = 0$$

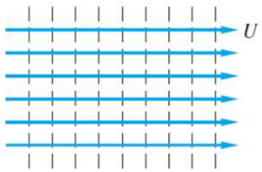


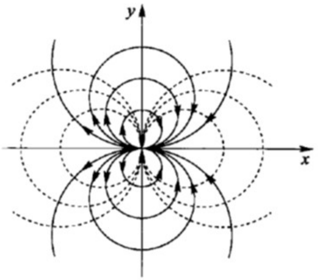
实际粘性流：流动分离，
阻力$F_x \neq 0$

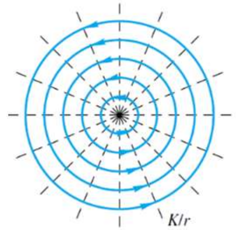


8.4基本流叠加（8.4）

5.绕圆柱有环量流动(均匀流+偶极流+点涡)：

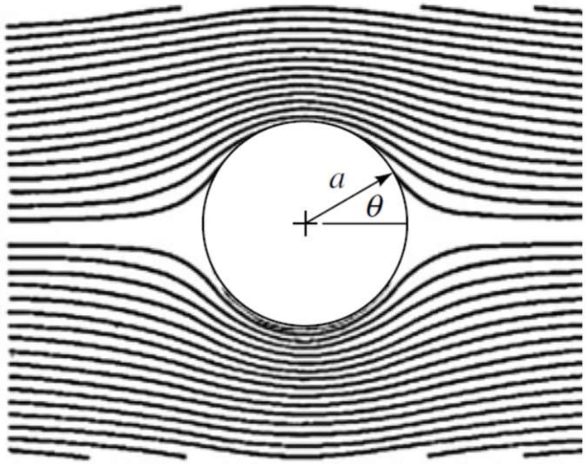




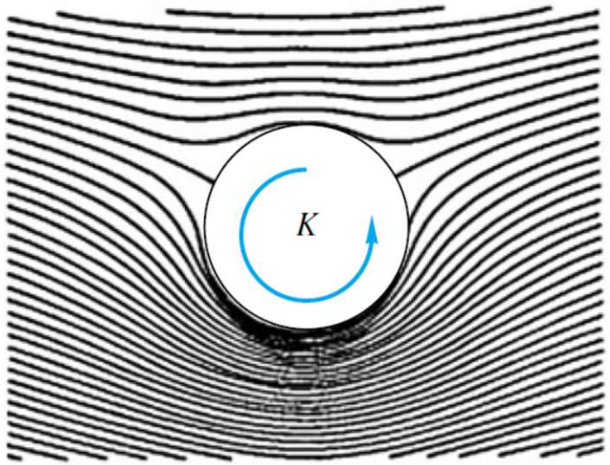


$K=\frac{\Gamma}{2\pi}$: 点涡强度

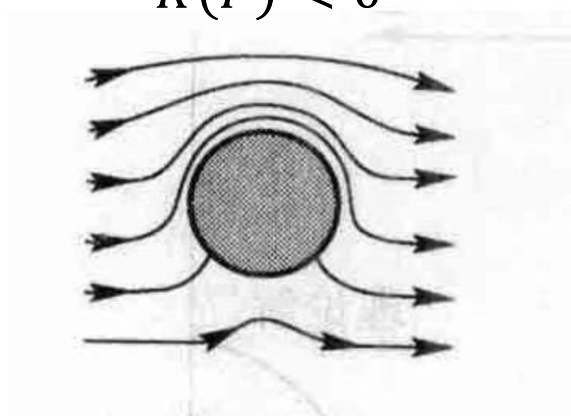
(a)



(b)

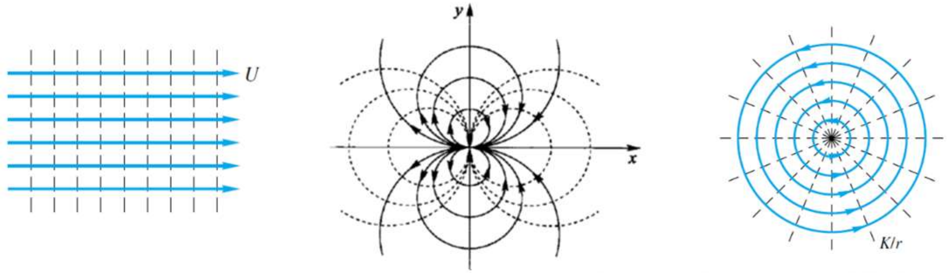


$K(\Gamma)<0$



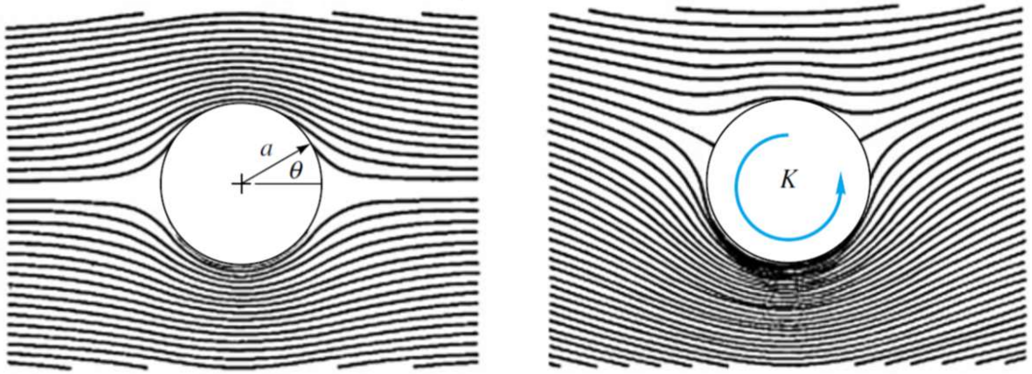
8.4基本流叠加（8.4）

5.绕圆柱有环量流动(均匀流+偶极流+点涡)：



$$\psi = -Klnr \qquad K=\frac{\Gamma}{2\pi}\text{: 点涡强度}$$

$$\phi = K\theta \qquad (\Gamma>0\text{逆时针}，\Gamma<0\text{顺时针})$$



$$\psi = U(r-\frac{R^2}{r})\,sin\theta - Klnr$$

$$\phi = U(r+\frac{R^2}{r})\,cos\theta + K\theta$$

$$u_r = \frac{\partial\phi}{\partial r} = Ucos\theta(1-\frac{R^2}{r^2})$$

$$u_\theta = \frac{1}{r}\frac{\partial\phi}{\partial\theta} = -Usin\theta(1+\frac{R^2}{r^2})+\frac{K}{r}$$

$$\phi = U(r+\frac{R^2}{r})\,cos\theta$$

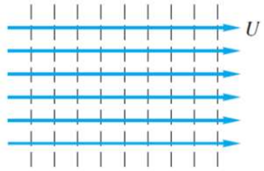
$$\psi = U(r-\frac{R^2}{r})\,sin\theta$$

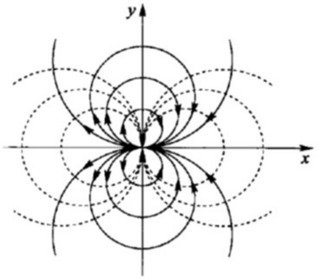
壁面上$r=R$： $u_r=0, u_\theta = -2Usin\theta+\frac{K}{R}$

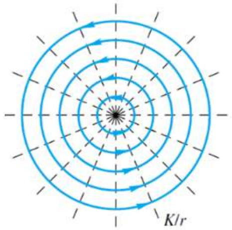
滞止点: $u_\theta = -2Usin\theta+\frac{K}{R}=0 \Rightarrow sin\theta = \frac{K}{2UR}$

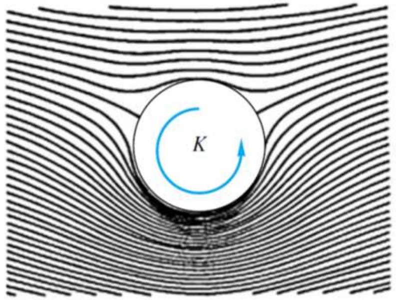
8.4基本流叠加（8.4）

5.绕圆柱有环量流动(均匀流+偶极流+点涡)：









$$\psi = U(r - \frac{R^2}{r})\sin\theta - K\ln r$$

$$\phi = U(r + \frac{R^2}{r})\cos\theta + K\theta$$

$$u_r = \frac{\partial\phi}{\partial r} = U\cos\theta(1 - \frac{R^2}{r^2})$$

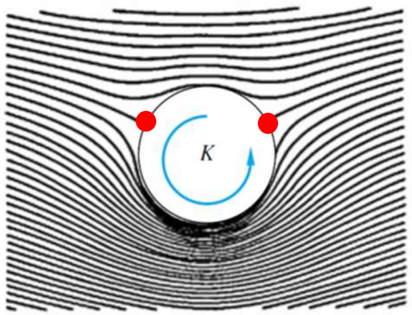
$$u_\theta = \frac{1}{r}\frac{\partial\phi}{\partial\theta} = -U\sin\theta(1 + \frac{R^2}{r^2}) + \frac{K}{r}$$

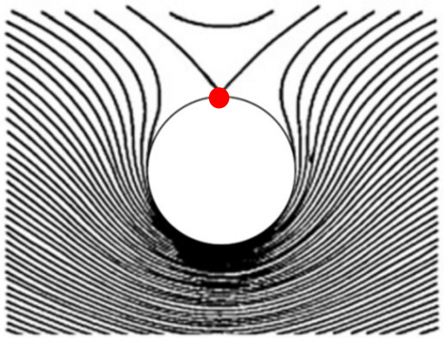
滞止点: $u_\theta = -2U\sin\theta + \frac{K}{R} = 0$ $\quad \sin\theta = \frac{K}{2UR}$

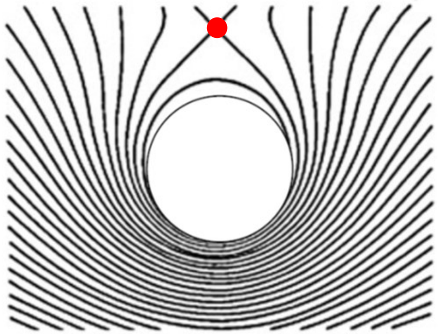
$\left|\frac{K}{2UR}\right| < 1$: $\theta_{s1} = \arcsin\frac{K}{2UR}$, $\theta_{s2} = \pi - \theta_{s1}$

$\left|\frac{K}{2UR}\right| = 1$: $\theta_s = \pm\frac{\pi}{2}$

$\left|\frac{K}{2UR}\right| > 1$: 滞止点不在圆柱上





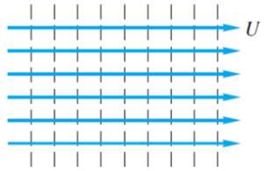


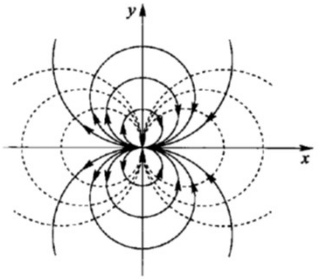
8.4基本流叠加（8.4）

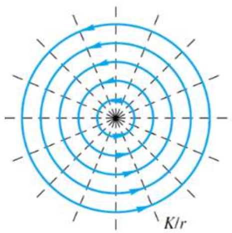
$K=\frac{\Gamma}{2\pi}$: 点涡强度

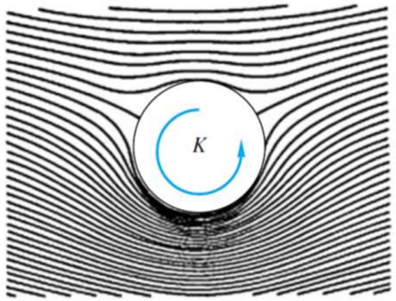
($\Gamma>0$逆时针，$\Gamma<0$顺时针)

5.绕圆柱有环量流动(均匀流+偶极流+点涡)：









$$\psi=U(r-\frac{R^2}{r})\,sin\theta-Klnr$$

$$\phi=U(r+\frac{R^2}{r})\,cos\theta+K\theta$$

$$u_r=\frac{\partial\phi}{\partial r}\quad=Ucos\theta(1-\frac{R^2}{r^2})$$

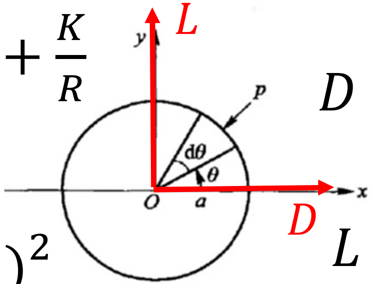
$$u_\theta=\frac{1}{r}\frac{\partial\phi}{\partial\theta}=-Usin\theta(1+\frac{R^2}{r^2})+\frac{K}{r}$$

圆柱表面$r=R$：$U_s=u_\theta=-2Usin\theta+\frac{K}{R}$

$$p_\infty+\frac{1}{2}\rho U^2=p_s+\frac{1}{2}\rho U_s^2$$

$$p_s=p_\infty+\frac{1}{2}\rho U^2-\frac{1}{2}\rho(-2Usin\theta+\frac{K}{R})^2$$

$$=p_\infty+\frac{1}{2}\rho[U^2-(2Usin\theta-\frac{K}{R})^2]$$



$$D=Fx=-\int_0^{2\pi}(p_s-p_\infty)\,cos\theta Rd\theta$$
$$=0$$

$$L=Fy=-\int_0^{2\pi}(p_s-p_\infty)\,sin\theta Rd\theta$$
$$=-\int_0^{2\pi}2\rho Usin^2\theta K\,d\theta$$
$$=-2\rho UK\pi$$

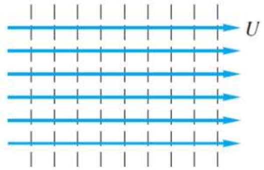
Kutta-Joukowski Lift Theorem：$\boxed{L\;=-\rho U\Gamma}$ 升力理论

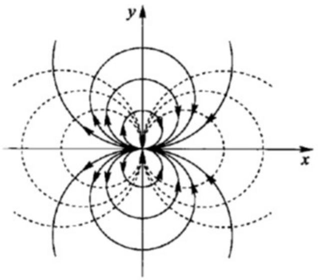
8.4基本流叠加（8.4）

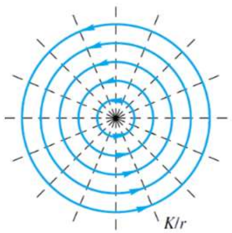
$K=\frac{\Gamma}{2\pi}$: 点涡强度

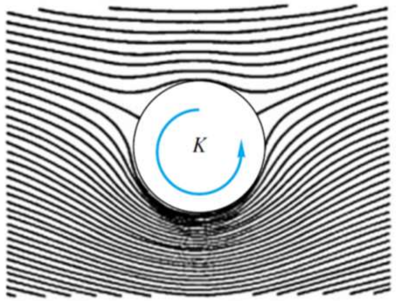
($\Gamma>0$逆时针，$\Gamma<0$顺时针)

5.绕圆柱有环量流动(均匀流+偶极流+点涡)：









$$\psi=U\left(r-\frac{R^2}{r}\right)\sin\theta-K\ln r$$

$$\phi=U\left(r+\frac{R^2}{r}\right)\cos\theta+K\theta$$

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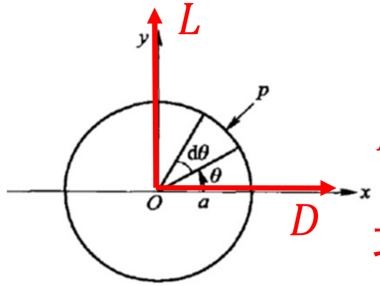

$$D=0$$

$L=-\rho U\Gamma$　升力理论

均匀流中任意形状柱体，单位长度受升力为$\rho U\Gamma$，方向为流动沿Γ反向旋转90°

8.4基本流叠加（8.4）

$K = \frac{\Gamma}{2\pi}$: 点涡强度

$(\Gamma > 0$逆时针，$\Gamma < 0$顺时针$)$

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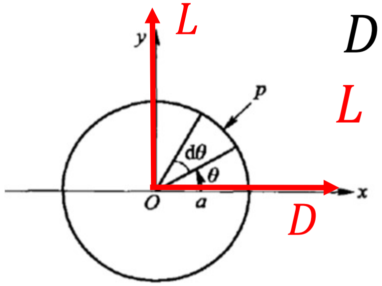
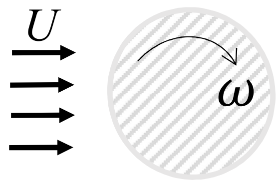


$D = 0$

$L = -\rho U \Gamma$ 升力理论

均匀流中任意形状柱体，单位长度受升力为$\rho U \Gamma$，方向为流动沿Γ反向旋转90°

例：圆柱$D = 0.5m$,长$L = 3m$，以$\omega = 180r/\min$顺时针旋转，来流空气$10m/s$。求：所受升力$F_L = ?$



解： $\Gamma = (\omega R)(2\pi R) = 2\pi\omega R^2$

$$F_L = \rho U \Gamma L$$

$$= \rho U 2\pi\omega R^2 L$$

$$= 266.5N$$

作业：

复习笔记！

8.18，8.22

大纲

流体力学基础部分

1. 基本概念（2.5）
2. 流体静力学（3.5）
3. 流体运动学基础（2）
4. 流体动力学积分方程（6）
5. 流体动力学微分方程（4）
6. 粘性不可压流动（7）
7. 相似原理（3）
8. 无粘不可压势流理论（4）

空气动力学部分

1. 绕翼型不可压流动（7）
2. 绕机翼不可压流动（7）
3. 高速可压流动基础（8）
4. 一维定常可压管内流（3）
5. 绕翼型亚声速流动（3）
6. 绕翼型超、跨声速流动（4）

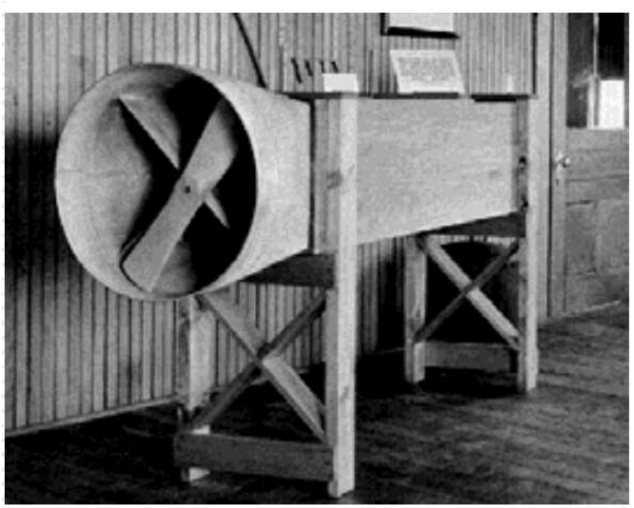
作业：

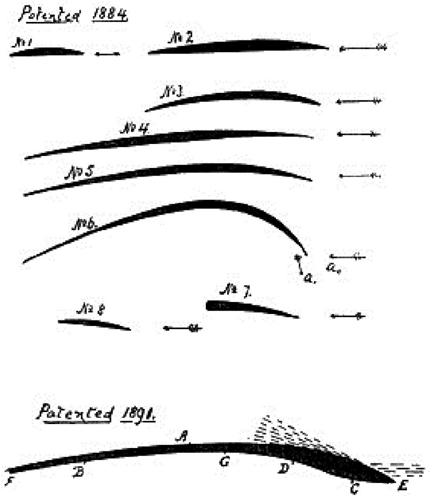
1.阅读1.1(p.1-4)“空气动力学发展”，总结发展史。

九. 不可压翼型理论（空4，Aero4）

9.1 标准大气
9.2 翼型几何、气动参数
9.3 翼型气动特征
9.4 面涡理论
9.5 库塔条件
9.6 开尔文环量定理、启动涡
9.7 经典薄翼理论
9.8 有弯度翼型扰流
9.9 面元法、涡板块法
9.10 粘性流动、翼型阻力

九. 不可压翼型理论（空4，Aero4）





1880s英国H.F.菲利普风洞测试翼型，德国奥托利林塔尔测试曲线翼滑翔机；1900~1902美国莱特兄弟测试200余种翼型，1903人类第一次动力飞行。

早期局限于解释与估算，1910s空气动力学理论精确计算低速翼型气动力。

1912~1918，Prandtle：

机翼气动分析
- 翼剖面（翼型）
- 翼剖面修正（三维机翼）

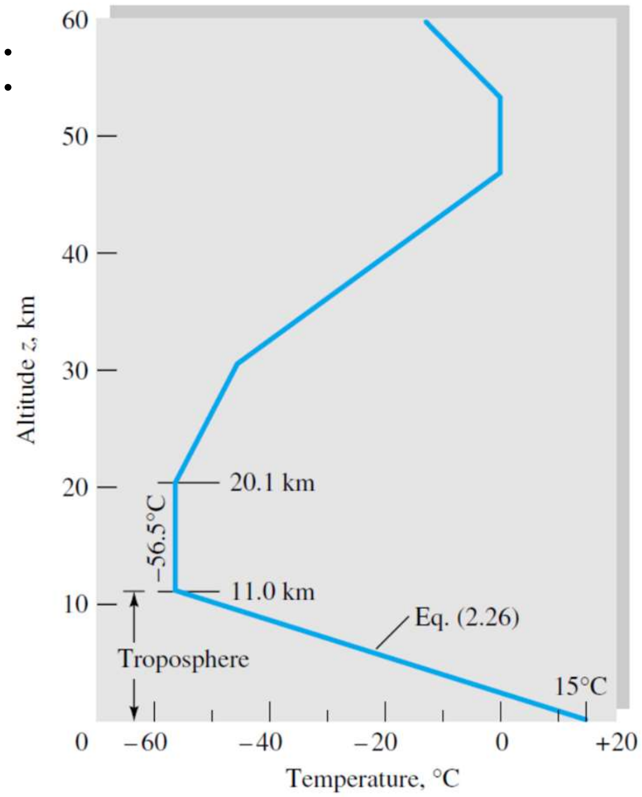
9.1标准大气（p.10~13）

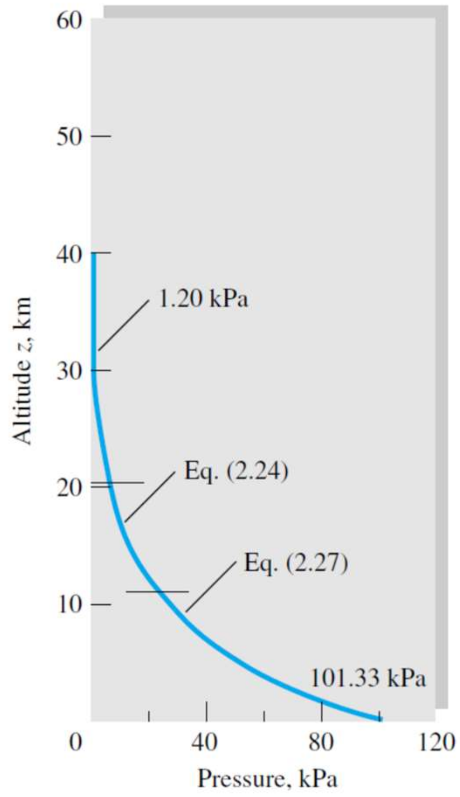
大气温度、压强分布：

$$\frac{dp}{p} = -\frac{g}{R}\frac{dz}{T}$$

$$T = T_0 - \alpha z$$

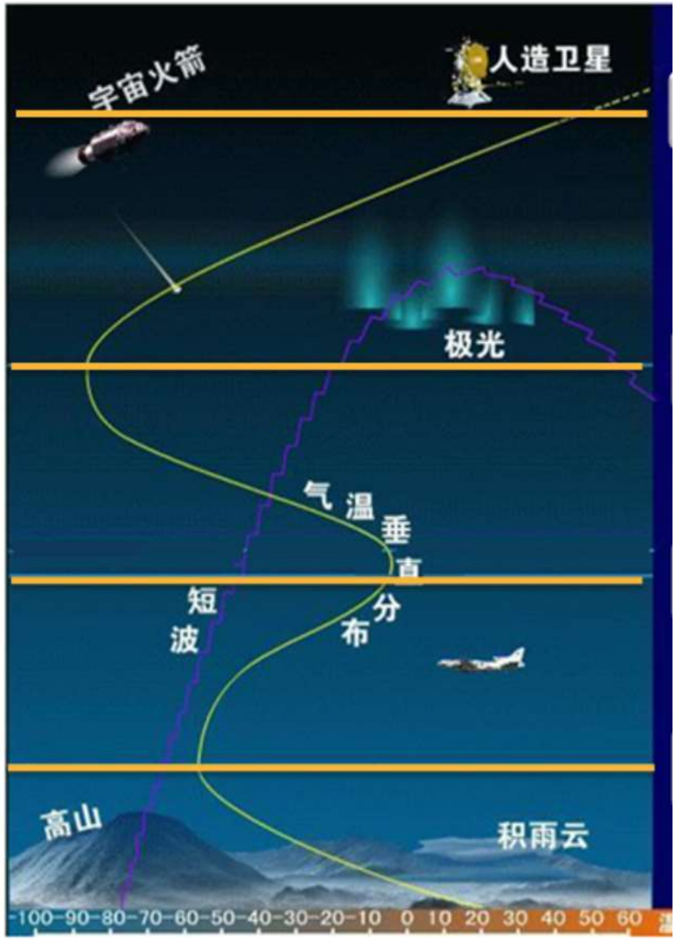
$$p_2 = p_1 \exp[-\frac{g(z_2 - z_1)}{RT_0}]$$





9.1标准大气（p.10~13）

$$\frac{dp}{p} = -\frac{g}{R}\frac{dz}{T}$$



大气温度、成分随高度变化；

大气压强、密度不断降低。

9.1标准大气（p.10~13）

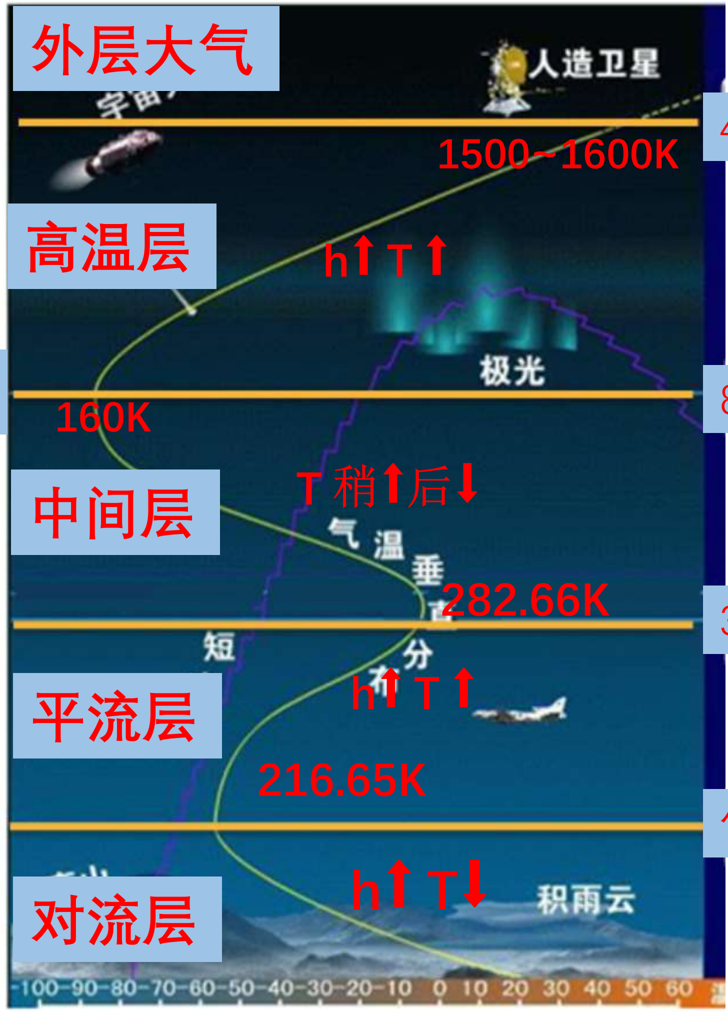
高层大气：
组分不均匀，
受紫外线辐射。

85km

低层大气：
N_2 78.1%，O_2 21%

平流层底层：
能见度高，受力稳，
噪声小，安全系数高。
大飞机巡航高度

小型飞机、农用、军用飞机



逃逸层，粒子溢入太空

400km

受太阳短波辐射，电离层

人造卫星轨道
近地点~100km

85km

1/3000稀薄空气

飞机<39km，探测气球<44km

32km

1/4空气，仅水平方向流动，
无风雨雷电等气象；

低纬16~18km，中高10~12km，两级7~10km

3/4空气，有垂直方向流动，
风雨雷电等气象剧烈；

无可闻声

150km

良导体

100km