空气与气体动力学

张科

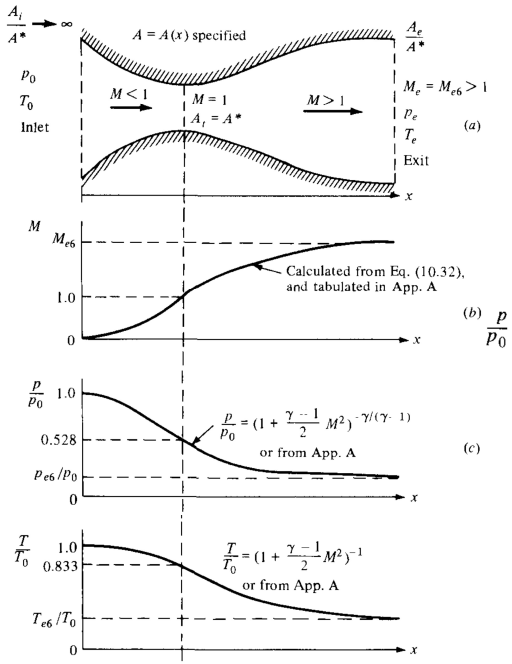
回顾：准一维可压内流 $d(\rho u A)=0$ $dp=-\rho u du$ $dh+udu=0$

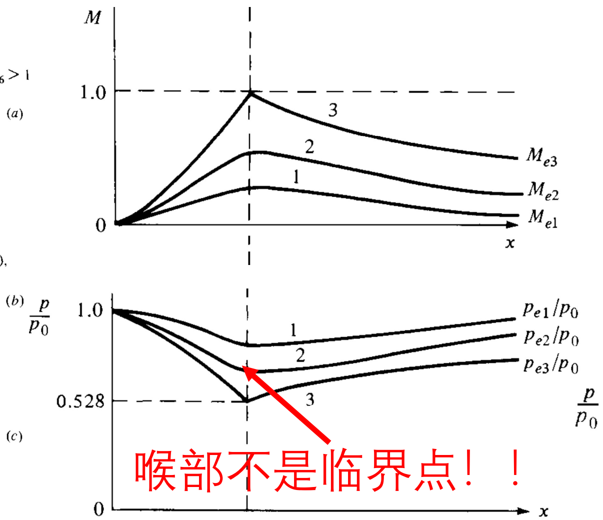
1.速度面积关系式: $\boxed{\dfrac{dA}{A}=(Ma^2-1)\dfrac{du}{u}}$

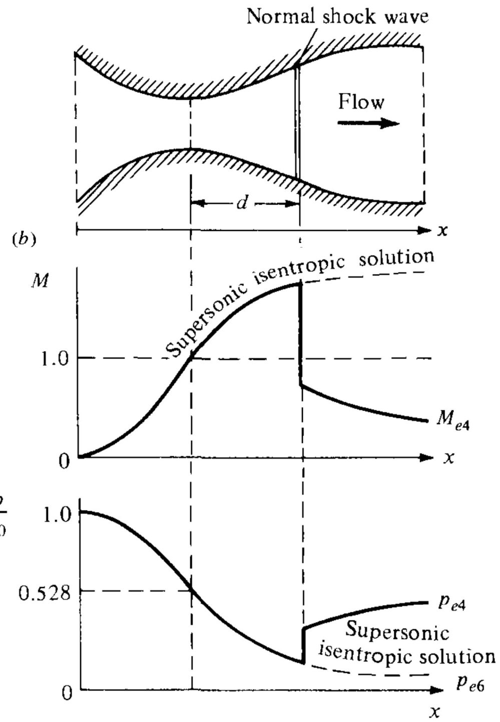
$0<Ma<1$时， $Ma>1$时，

$dA>0$ $du<0$, $A\uparrow$ $u\downarrow$ $dA>0$ $du>0$, $A\uparrow$ $u\uparrow$

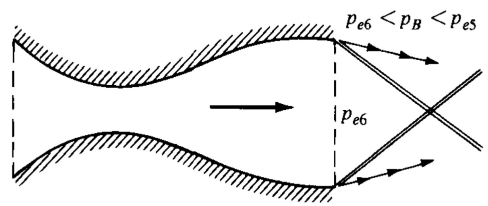
2.喷管(减压、增速):

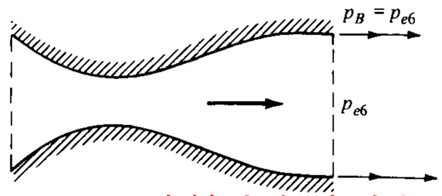




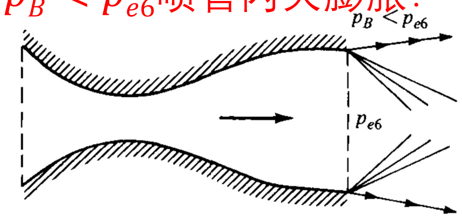


$p_B>p_{e6}$喷管内过膨胀!





$p_B<p_{e6}$喷管内欠膨胀!



12.3扩压器(9.4)

扩压器(diffuser)：减速、增压。

等熵：$s_1 = s_2, p_{02} = p_{01}$, 可用能不变。

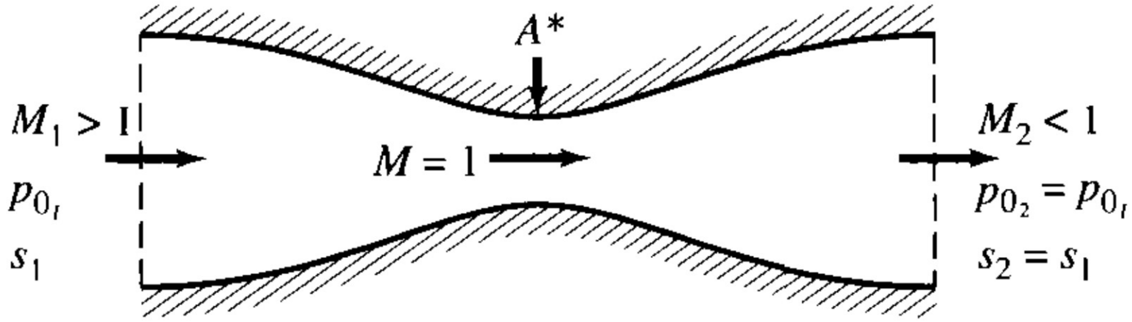
超声速气流等熵减速——极不可能!

壁面内折——斜激波;

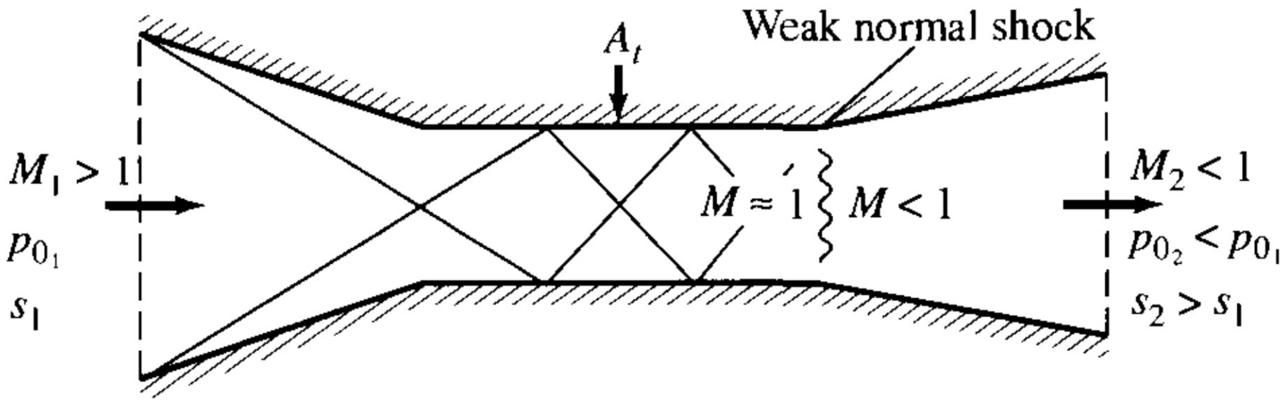
边界层——粘性损失。

p_0⬇, 可用功减小!

扩压器设计：提高$\frac{p_{02}}{p_{01}}$。



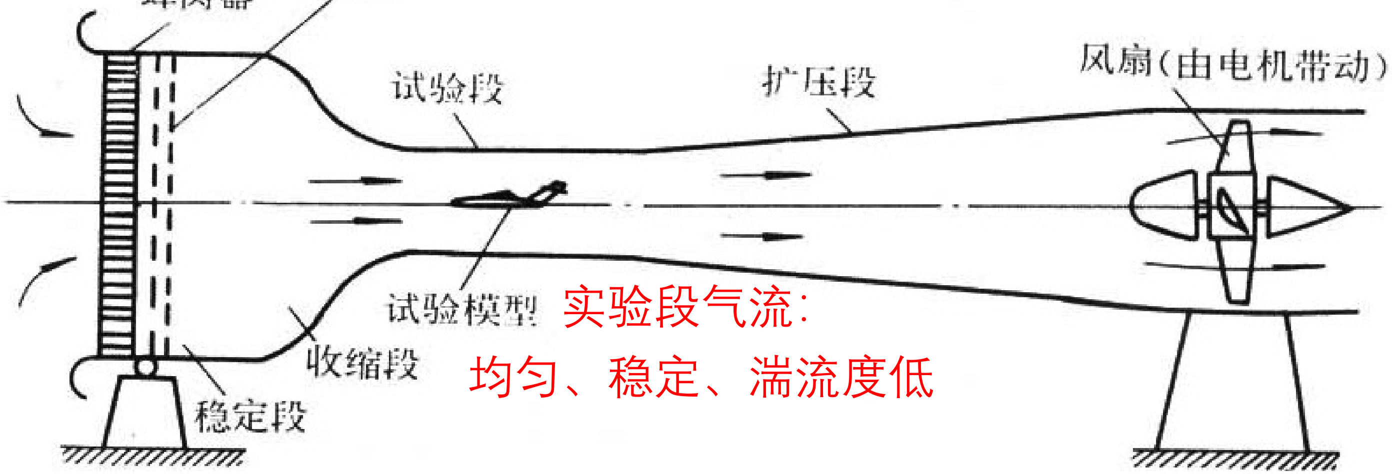
(a) Ideal (isentropic) supersonic diffuser



(b) Actual supersonic diffuser

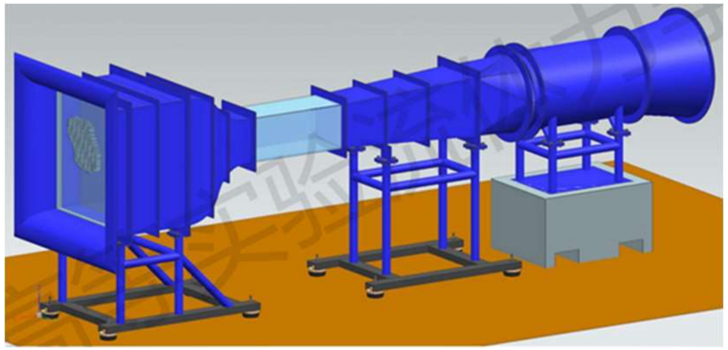
12.4超声速风洞(9.5)

亚声速风洞:

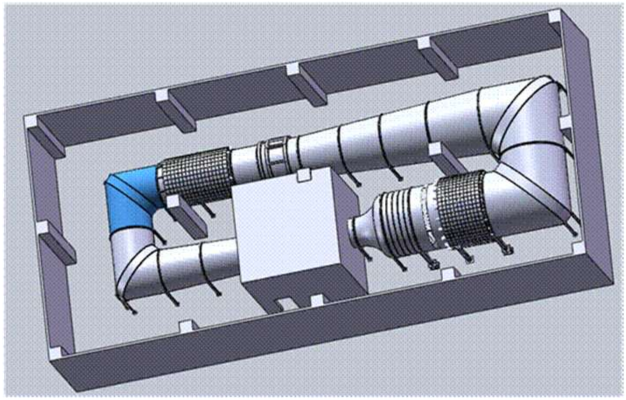


实验段气流:

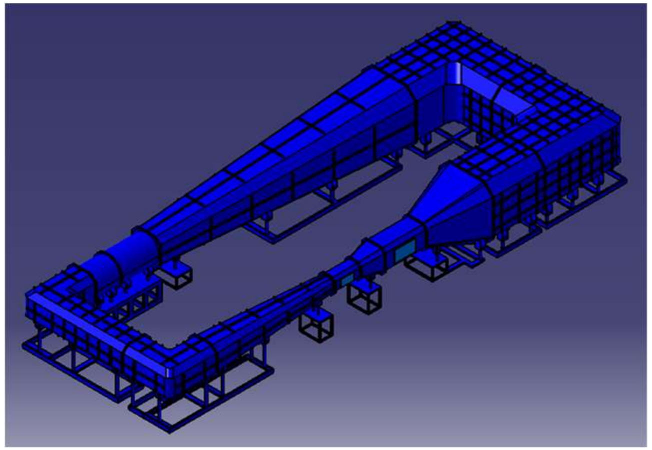
均匀、稳定、湍流度低



直流式



回流式



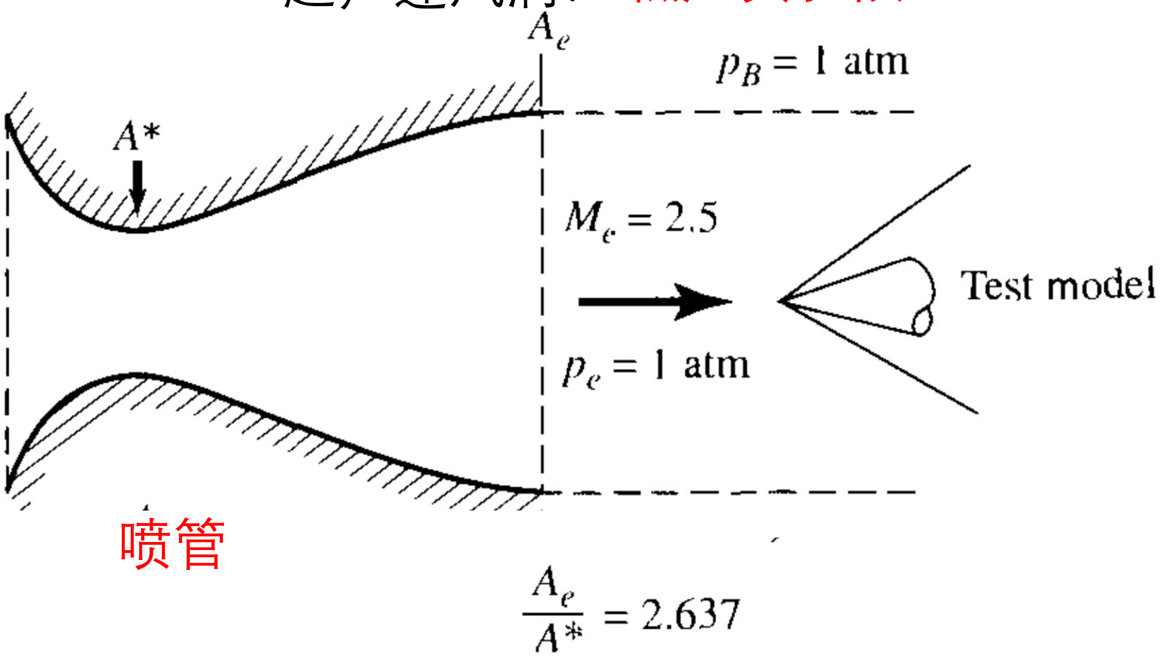
12.4超声速风洞(9.5)

超声速风洞： 低压实验段！

$$\frac{p_0}{p_e} = 17.09$$

$$p_0 = 17.09atm$$

压气机、储气罐



喷管

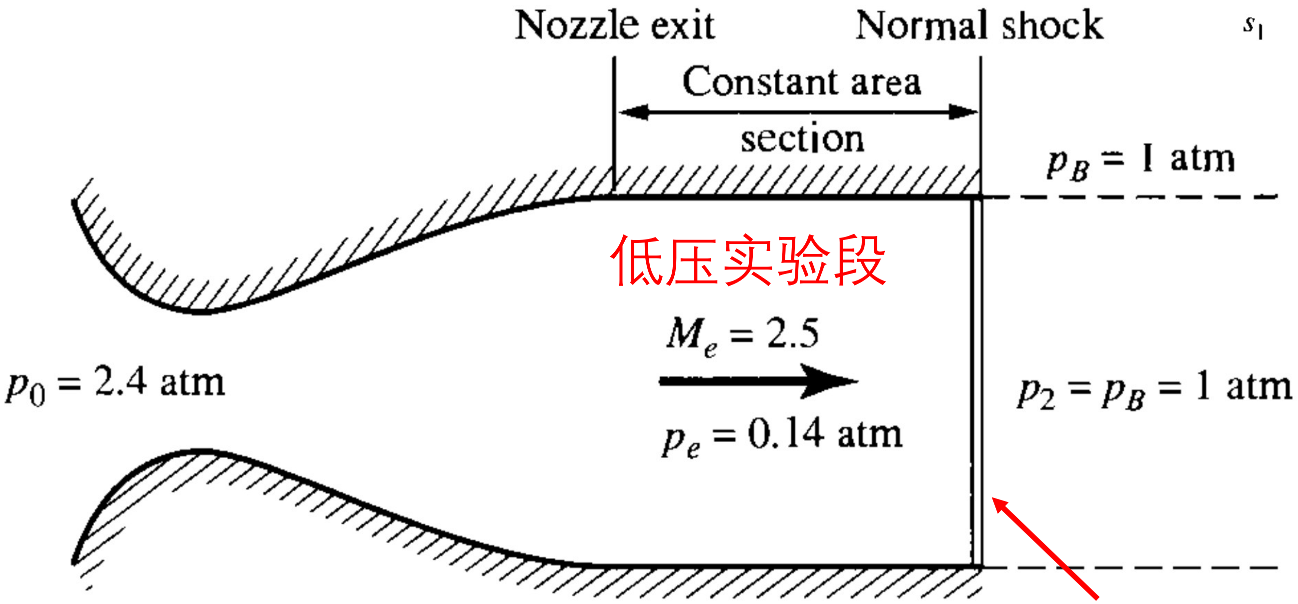
p_e⬇ p_0⬇

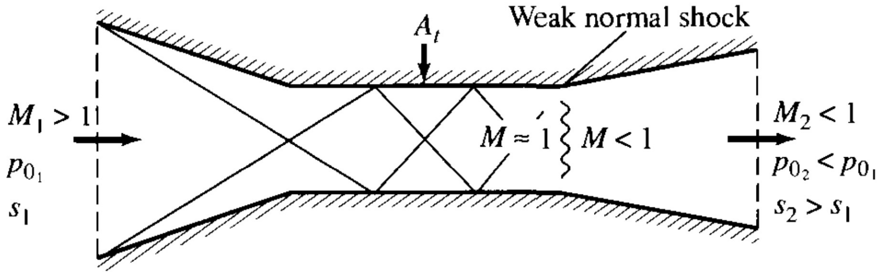
$p_e = 0.14atm$

→$p_0 = 2.4atm$

更易实现！

12.4超声速风洞(9.5)





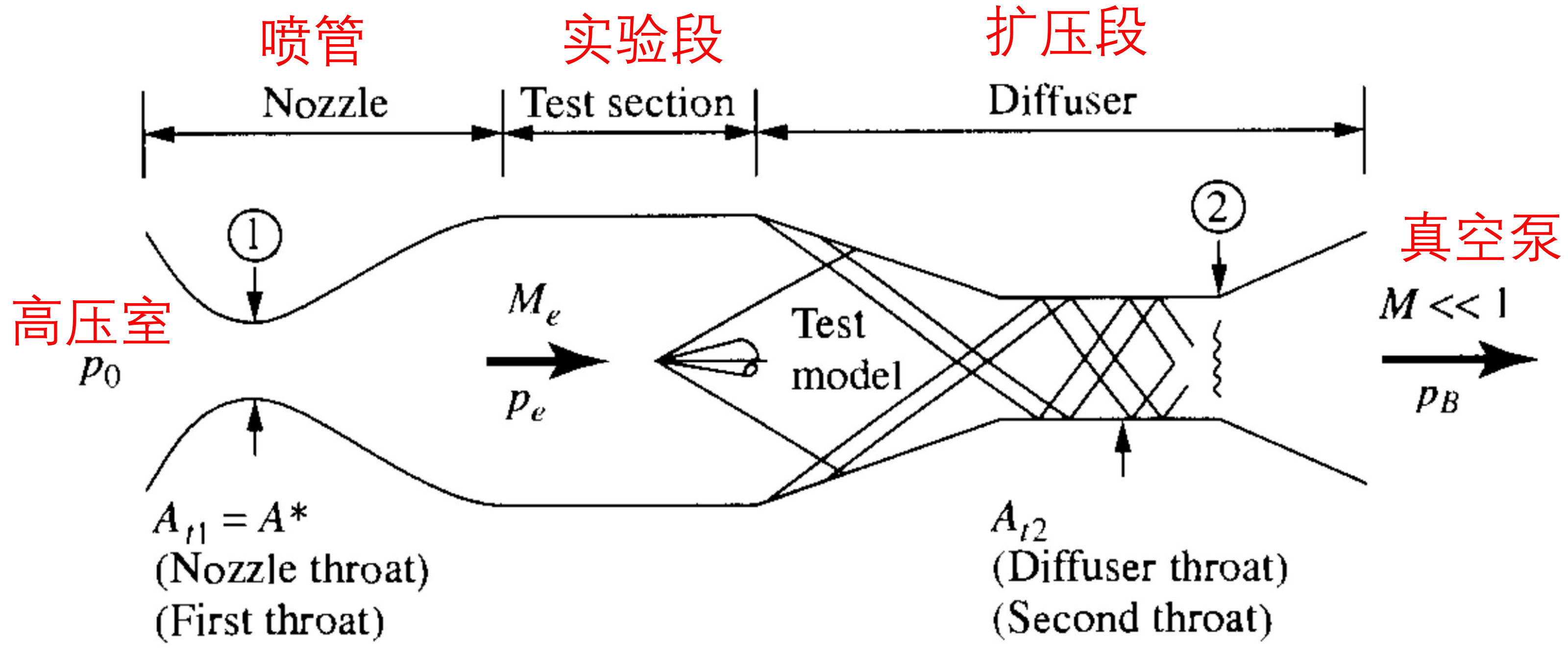
(b) Actual supersonic diffuser

接扩压器!

正激波，增压、减速（扩压器）！

正激波耗散大，不稳定。

12.4超声速风洞(9.5)



损失：模型表面、扩压段——激波；

边界层粘性流动。

7.1 超声速风洞的储气罐内的温度为288K，实验段速度为450m/s。假定风洞中气流为绝热流动，计算实验段Ma.

解：$a = \sqrt{\gamma R T} = \sqrt{1.4 \times 287 \times 288} = 340.17\ m/s$

$\therefore Ma = \frac{V}{a} = \frac{450}{340.17} = 1.32$

储气罐内为滞止参数！！

总温

7.1解，$T_0 = 288K$，$V = 450m/s$

能量方程：$c_p T_0 = c_p T + \frac{V^2}{2}$

$T = T_0 - \frac{V^2}{2c_p} = 187.2K$

实验段 $a = \sqrt{\gamma R T} = 274.2\ m/s$

$Ma = \frac{V}{a} = 1.64$

7.2 一给定点气流温度为300K、压力为1.2atm、速度为250m/s.
计算该点处的总压、总温、临界压力、临界温度和特征马赫数

解: $a=\sqrt{\gamma RT}=\sqrt{1.4\times287\times300}=347.2\ m/s$.

$Ma=\frac{V}{a}=\frac{250}{347.2}=0.72$.

$\frac{T_0}{T}=1+\frac{\gamma-1}{2}Ma^2 \Rightarrow T_0=(1+\frac{\gamma-1}{2}Ma^2)\cdot T=(1-\frac{0.4}{2}0.72^2)300$

$=268.9K$

$\frac{p_0}{p}=(1+\frac{\gamma-1}{2}Ma^2)^{\frac{\gamma}{\gamma-1}} \Rightarrow p_0=(1+\frac{\gamma-1}{2}Ma^2)^{\frac{\gamma}{\gamma-1}}p=(1-\frac{0.4}{2}0.72^2)^{\frac{1.4}{0.4}}1.2$

$=0.818\ atm$

$\frac{T^*}{T_0}=0.833 \Rightarrow T^*=0.833\times268.9=223.99K$.

$\frac{p^*}{p_0}=0.528 \Rightarrow p^*=0.528\times0.818=0.432\ atm$

$\cancel{Ma^*=\sqrt{\gamma RT^*}=\sqrt{1.4\times287\times223.99}=300}$ $a^*=\sqrt{\gamma RT^*}=$

$Ma^*=\frac{V}{a^*}=\frac{250}{300}=0.833$.

滞止压力、温度求错!

7.2 解: $a=\sqrt{\gamma RT}=\sqrt{1.4\times287\times300}=347.2(m/s)$

$Ma=\frac{V}{a}=0.72$

$Ma=0.72$ 附表A. $\frac{p_0}{p}=1.412$ $\frac{T_0}{T}=1.104$

$p_0=1.694\ atm$ $T_0=331.2K$.

$\frac{T^*}{T_0}=0.833$. $T^*=275.9K$ $a^*=\sqrt{\gamma RT}=332.95\ m/s$

$\frac{p^*}{p_0}=0.528$ $p^*=0.894\ atm$

$Ma^*=\frac{V}{a^*}=\frac{250}{332.95}=0.75$

7.6 如果通过激波的熵增为199.5 J/(kg·K)，问来流马赫数为多大？

$$\Delta s = s_2 - s_1 = c_v \ln\left\{\left[\frac{2+(\gamma-1)Ma_1^2}{(\gamma+1)Ma_1^2}\right]^\gamma \left[1+\frac{2\gamma}{\gamma+1}(Ma_1^2-1)\right]\right\}$$

$$Ma = 1.09$$

计算过程？？

7.6 解：$s_2 - s_1 = -R\ln\frac{p_{02}}{p_{01}} = 199.5\ J/kg\cdot K$

$$\ln\left(\frac{p_{02}}{p_{01}}\right) = \left(\frac{-199.5}{287}\right) = -0.6951$$

$$\frac{p_{02}}{p_{01}} = 0.499 \xRightarrow{\text{附表B}} Ma_1 = 2.5$$

7.8、根据不可压缩流体假设：

$$\frac{p_0}{\rho_0} + \frac{V_0^2}{2} = \frac{p}{\rho} + \frac{V^2}{2} \Rightarrow \frac{1.555atm}{\rho_0} = \frac{1atm}{\rho} + \frac{V^2}{2}$$

$$\frac{\rho_0}{\rho} = 1.371 \quad \rho = \frac{p}{RT} = \frac{1\times1.01325\times10^5}{287\times 288} = 1.226 kg/m^3$$

∴ $\rho_0 = 1.371\cdot\rho = 1.681 kg/m^3$

∴ $V = 148.884 m/s$

∴ 误差为46.62%

不可压？？

7.7. $T_\infty = 288K$, $P_\infty = 1atm$

若 $Ma_\infty = 1$, 则 $P_0/P_\infty = 1.893$

$P_{pitot} = 1.555atm < 1.893atm$, ⇒ 亚声速流

$P_{pitot} = P_0$, $\frac{P_0}{P_\infty} = 1.555 \Rightarrow Ma_\infty = 0.82$

$$a_\infty = \sqrt{\gamma RT} = \sqrt{1.4\times 286.6\times 288} = 340m/s$$

$$V_\infty = Ma_\infty \cdot a_\infty = 278.8m/s$$

7.8、解、不可压：$P_0 = 1.555atm$ $P_\infty = 1atm$

$$V_\infty' = \sqrt{\frac{2\Delta P}{\rho}} = \sqrt{\frac{2\times 0.555\times 1.013\times 10^5}{1.22}} = 303.6 (m/s)$$

$$\frac{|V_\infty' - V_\infty|}{V_\infty} = 8.9\%$$

十三. 绕翼型可压缩流动（空10）

b：79.8m　　S：845m²　　χ=33.5°

$Ma_{巡航}$=0.85　巡航时气动特性如何？？

$0.3 < Ma < 1$:

C_l, C_d ??

$Ma > 1$ 激波产生??

$Ma < 0.3$:

不可压势流

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=0$$

可压势流ϕ？？

13.1速度势方程(10.2)

无粘可压无旋流——存在势函数ϕ。$\vec{\Omega}=\vec{\nabla}\times\vec{V}=0$ 若：$\vec{\nabla}\times\vec{V}=0$，则$\vec{V}=\vec{\nabla}\phi$

定常、无旋、等熵、2D： $\vec{V}=\vec{\nabla}\phi$， $u=\frac{\partial\phi}{\partial x},v=\frac{\partial\phi}{\partial y}$

连续性方程： $\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}=0$ 不可压：$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$

$$\rho\frac{\partial u}{\partial x}+u\frac{\partial\rho}{\partial x}+\rho\frac{\partial v}{\partial y}+v\frac{\partial\rho}{\partial y}=0$$

$$\rho\frac{\partial^2\phi}{\partial x^2}+\frac{\partial\phi}{\partial x}\frac{\partial\rho}{\partial x}+\rho\frac{\partial^2\phi}{\partial y^2}+\frac{\partial\phi}{\partial y}\frac{\partial\rho}{\partial y}=0 \quad ①$$

欧拉方程： $dp=-\rho VdV=-\frac{\rho}{2}dV^2=-\frac{\rho}{2}d(u^2+v^2)$

等熵： $\frac{dp}{d\rho}=a^2 \rightarrow dp=a^2\,d\rho$

$$d\rho=-\frac{\rho}{2a^2}d(u^2+v^2)=-\frac{\rho}{2a^2}d[(\frac{\partial\phi}{\partial x})^2+(\frac{\partial\phi}{\partial y})^2] \quad ②$$

13.1速度势方程(10.2)

$$d\rho = -\frac{\rho}{2a^2} d[(\frac{\partial\phi}{\partial x})^2 + (\frac{\partial\phi}{\partial y})^2] \qquad ②$$

$$\frac{\partial ②}{\partial x}: \quad \frac{\partial\rho}{\partial x} = -\frac{\rho}{2a^2}\frac{\partial}{\partial x}[(\frac{\partial\phi}{\partial x})^2 + (\frac{\partial\phi}{\partial y})^2] = -\frac{\rho}{a^2}(\frac{\partial\phi}{\partial x}\frac{\partial^2\phi}{\partial x^2} + \frac{\partial\phi}{\partial y}\frac{\partial^2\phi}{\partial y\partial x}) \qquad ③$$

$$\frac{\partial ②}{\partial y}: \quad \frac{\partial\rho}{\partial y} = -\frac{\rho}{2a^2}\frac{\partial}{\partial y}[(\frac{\partial\phi}{\partial x})^2 + (\frac{\partial\phi}{\partial y})^2] = -\frac{\rho}{a^2}(\frac{\partial\phi}{\partial x}\frac{\partial^2\phi}{\partial x\partial y} + \frac{\partial\phi}{\partial y}\frac{\partial^2\phi}{\partial y^2}) \qquad ④$$

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0 \rightarrow \rho\frac{\partial^2\phi}{\partial x^2} + \frac{\partial\phi}{\partial x}\frac{\partial\rho}{\partial x} + \rho\frac{\partial^2\phi}{\partial y^2} + \frac{\partial\phi}{\partial y}\frac{\partial\rho}{\partial y} = 0 \qquad ①$$

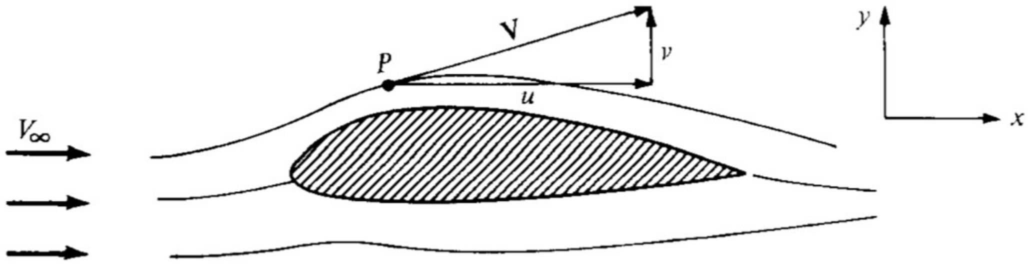
③+④→①:

$$\left.\begin{aligned} &\left[1 - \frac{1}{a^2}\left(\frac{\partial\phi}{\partial x}\right)^2\right]\frac{\partial^2\phi}{\partial x^2} + \left[1 - \frac{1}{a^2}\left(\frac{\partial\phi}{\partial y}\right)^2\right]\frac{\partial^2\phi}{\partial y^2} - \frac{2}{a^2}(\frac{\partial\phi}{\partial x})(\frac{\partial\phi}{\partial y})\frac{\partial^2\phi}{\partial x\partial y} = 0 \\ &a^2 = a_0^2 - \frac{\gamma-1}{2}V^2 = a_0^2 - \frac{\gamma-1}{2}[[(\frac{\partial\phi}{\partial x})^2 + (\frac{\partial\phi}{\partial y})^2]^2 \end{aligned}\right\} ⑤$$

关于ϕ的非线性偏微分方程！！ $(Ma > 0.3)$

$$\boxed{Ma < 0.3 : \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0}$$

13.2线化速度势方程(10.3)



t、f、α较小时，翼型周围=来流+小扰动

$u = V_\infty + \hat{u}, v = \hat{v}$　　$\hat{u}, \hat{v}$ 扰动速度

$\phi = V_\infty x + \hat{\phi}$，　$\hat{\phi}$ 扰动速度势　　$\hat{u} = \frac{\partial \hat{\phi}}{\partial x}, \hat{v} = \frac{\partial \hat{\phi}}{\partial y}$

$$\frac{\partial \phi}{\partial x} = V_\infty + \frac{\partial \hat{\phi}}{\partial x}$$
$$\frac{\partial \phi}{\partial y} = \frac{\partial \hat{\phi}}{\partial y}$$

$$\Rightarrow \quad \frac{\partial^2 \phi}{\partial x^2} = \frac{\partial^2 \hat{\phi}}{\partial x^2}$$
$$\frac{\partial^2 \phi}{\partial y^2} = \frac{\partial^2 \hat{\phi}}{\partial y^2}, \quad \frac{\partial^2 \phi}{\partial x \partial y} = \frac{\partial^2 \hat{\phi}}{\partial x \partial y}$$

代入速度势方程⑤ →

$$\left[a^2 - (V_\infty + \frac{\partial \hat{\phi}}{\partial x})^2\right] \frac{\partial^2 \hat{\phi}}{\partial x^2} + \left[a^2 - \left(\frac{\partial \hat{\phi}}{\partial y}\right)^2\right] \frac{\partial^2 \hat{\phi}}{\partial y^2} - 2(V_\infty + \frac{\partial \hat{\phi}}{\partial x})(\frac{\partial \hat{\phi}}{\partial y}) \frac{\partial^2 \hat{\phi}}{\partial x \partial y} = 0$$

$$\left.\begin{aligned} &[a^2 - (V_\infty + \hat{u})^2] \frac{\partial \hat{u}}{\partial x} + (a^2 - \hat{v}^2) \frac{\partial \hat{v}}{\partial y} - 2(V_\infty + \hat{u}) \hat{v} \frac{\partial \hat{u}}{\partial y} = 0 \\ &\text{能量：} \frac{a_\infty^2}{\gamma - 1} + \frac{V_\infty^2}{2} = \frac{a^2}{\gamma - 1} + \frac{(V_\infty + \hat{u})^2 + \hat{v}^2}{2} \end{aligned}\right\} \Rightarrow$$

13.2线化速度势方程(10.3)

$$(1-M_\infty^2)\frac{\partial\hat{u}}{\partial x}+\frac{\partial\hat{v}}{\partial y}=\boxed{M_\infty^2\left[(\gamma+1)\frac{\hat{u}}{V_\infty}+\frac{\gamma+1}{2}\frac{\hat{u}^2}{V_\infty^2}+\frac{\gamma-1}{2}\frac{\hat{v}^2}{V_\infty^2}\right]\frac{\partial\hat{u}}{\partial x}}$$ A

$$+\boxed{M_\infty^2\left[(\gamma-1)\frac{\hat{u}}{V_\infty}+\frac{\gamma+1}{2}\frac{\hat{v}^2}{V_\infty^2}+\frac{\gamma-1}{2}\frac{\hat{u}^2}{V_\infty^2}\right]\frac{\partial\hat{v}}{\partial y}}$$ B ⑥

$$+\boxed{M_\infty^2\left[\frac{\hat{v}}{V_\infty}\left(1+\frac{\hat{u}}{V_\infty}\right)\left(\frac{\partial\hat{u}}{\partial y}+\frac{\partial\hat{v}}{\partial x}\right)\right]}$$ C 非线性偏微分方程!!

小扰动(细长体、小攻角): $\frac{\hat{u}}{V_\infty},\frac{\hat{v}}{V_\infty}<<1 \qquad \frac{\hat{u}^2}{V_\infty^2},\frac{\hat{v}^2}{V_\infty^2}<<1$

1. $Ma_\infty\le 0.8$或$Ma_\infty>1.2$时: $A<<(1-Ma_\infty^2)\frac{\partial\hat{u}}{\partial x}$

2. $Ma_\infty<5$: $B<<\frac{\partial\hat{v}}{\partial y} \qquad C\approx 0$

$$\Rightarrow\quad (1-Ma_\infty^2)\frac{\partial\hat{u}}{\partial x}+\frac{\partial\hat{v}}{\partial y}=0$$ 线性偏微分方程!!

$$\boxed{(1-Ma_\infty^2)\frac{\partial^2\hat{\phi}}{\partial x^2}+\frac{\partial^2\hat{\phi}}{\partial y^2}=0}$$

13.2线化速度势方程(10.3)

$$\boxed{(1-Ma_{\infty}^{2})\frac{\partial^{2}\widehat{\phi}}{\partial x^{2}}+\frac{\partial^{2}\widehat{\phi}}{\partial y^{2}}=0}$$

适用条件：①小扰动(薄体、小攻角)

②亚、超声速：$Ma_{\infty}\leq 0.8$，$Ma_{\infty}>1.2$， $Ma_{\infty}<5$

不适用：①厚体、跨声速$(0.8<Ma_{\infty}<1.2)$

②高超声速$(Ma_{\infty}>5)$

13.2线化速度势方程(10.3)

$C_p \sim \hat{u}, \hat{v}, \hat{\phi}$??

➢ 压力系数线化： $C_p = \frac{p-p_\infty}{\frac{1}{2}\rho_\infty V_\infty^2}$

$$\frac{1}{2}\rho_\infty V_\infty^2 = \frac{1}{2}\frac{\gamma p_\infty}{\gamma p_\infty}\rho_\infty V_\infty^2$$

$$= \frac{\gamma}{2}p_\infty \frac{\rho_\infty}{\gamma p_\infty} V_\infty^2 \qquad a_\infty^2 = \frac{\gamma p_\infty}{\rho_\infty}$$

$$= \frac{\gamma}{2}p_\infty Ma_\infty^2$$

$$\Rightarrow \quad C_p = (p - p_\infty)\frac{2}{\gamma p_\infty Ma_\infty^2} = (\frac{p}{p_\infty} - 1)\frac{2}{\gamma Ma_\infty^2} \qquad ①$$

能量： $T + \frac{V^2}{2C_p} = T_\infty + \frac{V_\infty^2}{2C_p}$

$$T - T_\infty = \frac{V_\infty^2 - V^2}{\frac{2\gamma}{(\gamma-1)}R}$$

$$\frac{T}{T_\infty} - 1 = \frac{\gamma-1}{2}\frac{V_\infty^2 - V^2}{\gamma R T_\infty} = \frac{\gamma-1}{2}\frac{V_\infty^2 - V^2}{a_\infty^2} = -\frac{\gamma-1}{2}\frac{(\hat{u}^2 + \hat{v}^2 + 2\hat{u}V_\infty)}{a_\infty^2}$$

$$V^2 = (V_\infty + \hat{u})^2 + \hat{v}^2$$

13.2线化速度势方程(10.3)

$$C_p=(\frac{p}{p_\infty}-1)\frac{2}{\gamma Ma_\infty^2} \qquad ①$$

$$\frac{T}{T_\infty}=1-\frac{\gamma-1}{2}\frac{(\hat{u}^2+\hat{v}^2+2\hat{u}V_\infty)}{a_\infty^2}$$

等熵: $$\frac{p}{p_\infty}=(\frac{T}{T_\infty})^{\frac{\gamma}{\gamma-1}}=[1-\frac{\gamma-1}{2}\frac{(\hat{u}^2+\hat{v}^2+2\hat{u}V_\infty)}{a_\infty^2}]^{\frac{\gamma}{\gamma-1}}$$

$$=[1-\boxed{\frac{\gamma-1}{2}Ma_\infty^2(\frac{2\hat{u}}{V_\infty}+\frac{\hat{u}^2+\hat{v}^2}{V_\infty^2})}]^{\frac{\gamma}{\gamma-1}}$$

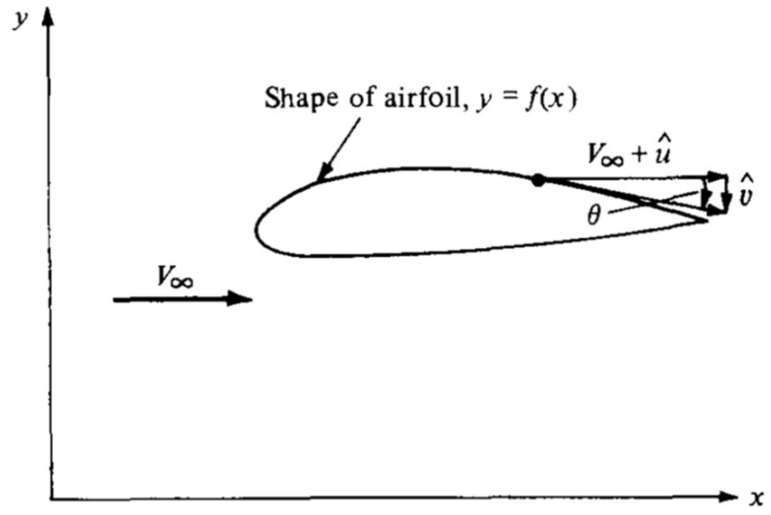
$$\varepsilon=\frac{\gamma-1}{2}Ma_\infty^2(\frac{2\hat{u}}{V_\infty}+\frac{\hat{u}^2+\hat{v}^2}{V_\infty^2})$$

小扰动: $$\frac{\hat{u}}{V_\infty}<<1 \quad \frac{\hat{u}^2}{V_\infty^2},\frac{\hat{v}^2}{V_\infty^2}<<1$$

$$\frac{p}{p_\infty}=(1-\varepsilon)^{\frac{\gamma}{\gamma-1}}\approx 1-\frac{\gamma}{\gamma-1}\varepsilon+\ldots=1-\frac{\gamma}{2}Ma_\infty^2(\frac{2\hat{u}}{V_\infty}+\frac{\hat{u}^2+\hat{v}^2}{V_\infty^2}) \qquad ②$$

①+② → $$C_p=-\frac{2\hat{u}}{V_\infty}-\frac{\hat{u}^2+\hat{v}^2}{V_\infty^2}\approx-\frac{2\hat{u}}{V_\infty}$$ 线化压力系数!

13.2线化速度势方程(10.3)



$$\begin{cases} (1-Ma_\infty^2)\dfrac{\partial^2\hat{\phi}}{\partial x^2}+\dfrac{\partial^2\hat{\phi}}{\partial y^2}=0 \quad \rightarrow \hat{\phi},\hat{u} \rightarrow C_p \\ C_p \approx -\dfrac{2\hat{u}}{V_\infty} \end{cases}$$

边界条件：$@\infty\ \hat{\phi}=\text{constant}, \hat{u}=\hat{v}=0$

@物面 $v_\perp=0$

$$\tan\theta=\frac{v}{u}=\frac{\hat{v}}{V_\infty+\hat{u}}$$

$$\hat{u} \ll V_\infty \rightarrow \hat{v}=V_\infty\tan\theta \quad =V_\infty\left(\frac{dy}{dx}\right)_s$$

$$\frac{\partial\hat{\phi}}{\partial y}=V_\infty\tan\theta$$ 物面为流线近似方程

13.3Prandtl-Glauert压缩性修正(10.4)

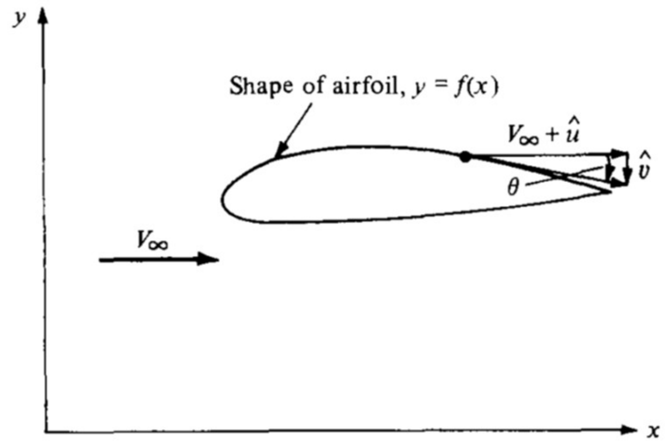
1904-1940，低Ma飞行，不可压空气动力学，低速翼型理论、实验；

二战期间，$Ma\uparrow$，高速翼型理论，可压流动，压缩性修正。

1. Prandtl-Glauert方法

（基于线化扰动速度势方程，薄翼、小攻角、$Ma<0.7$）

亚声速、可压、无粘流：



$$(1-Ma_\infty^2)\frac{\partial^2\hat{\phi}}{\partial x^2}+\frac{\partial^2\hat{\phi}}{\partial y^2}=0 \qquad Ma<1\text{椭圆型方程}$$

$$\boxed{\beta^2\frac{\partial^2\hat{\phi}}{\partial x^2}+\frac{\partial^2\hat{\phi}}{\partial y^2}=0} \qquad \beta^2=1-Ma_\infty^2$$

$$Ma<1: \quad \boxed{\frac{\partial^2\hat{\phi}}{\partial x^2}+\frac{\partial^2\hat{\phi}}{\partial y^2}=0}$$ 关联？？

13.3Prandtl-Glauert压缩性修正(10.4)

$$\beta^2\frac{\partial^2\hat{\phi}}{\partial x^2}+\frac{\partial^2\hat{\phi}}{\partial y^2}=0$$

仿射变换：$\xi=x,\eta=\beta y$　　引入：$\bar{\phi}(\xi,\eta)=\beta\hat{\phi}(x,y)$

$$\Rightarrow\quad \frac{\partial\xi}{\partial x}=1,\frac{\partial\xi}{\partial y}=0$$

$$\frac{\partial\eta}{\partial x}=0,\frac{\partial\eta}{\partial y}=\beta$$

$$\bar{\phi}(\xi,\eta)=\beta\hat{\phi}(x,y)\quad\Rightarrow\quad \frac{\partial\hat{\phi}}{\partial x}=\frac{1}{\beta}\frac{\partial\bar{\phi}}{\partial\xi}\frac{\partial\xi}{\partial x}=\frac{1}{\beta}\frac{\partial\bar{\phi}}{\partial\xi}\quad\Rightarrow\quad \frac{\partial^2\hat{\phi}}{\partial x^2}=\frac{1}{\beta}\frac{\partial^2\bar{\phi}}{\partial\xi^2}\frac{\partial\xi}{\partial x}=\frac{1}{\beta}\frac{\partial^2\bar{\phi}}{\partial\xi^2}$$

$$\frac{\partial\hat{\phi}}{\partial y}=\frac{1}{\beta}\frac{\partial\bar{\phi}}{\partial\eta}\frac{\partial\eta}{\partial y}=\frac{\partial\bar{\phi}}{\partial\eta}\qquad \frac{\partial^2\hat{\phi}}{\partial y^2}=\frac{\partial^2\bar{\phi}}{\partial\eta^2}\frac{\partial\eta}{\partial y}=\beta\frac{\partial^2\bar{\phi}}{\partial\eta^2}$$

$$\beta^2\frac{\partial^2\hat{\phi}}{\partial x^2}+\frac{\partial^2\hat{\phi}}{\partial y^2}=0\quad\Rightarrow\quad \beta^2\frac{1}{\beta}\frac{\partial^2\bar{\phi}}{\partial\xi^2}+\beta\frac{\partial^2\bar{\phi}}{\partial\eta^2}=0$$

$$\frac{\partial^2\bar{\phi}}{\partial\xi^2}+\frac{\partial^2\bar{\phi}}{\partial\eta^2}=0$$

$\nabla^2\bar{\phi}=0$ 不可压势方程！！