空气与气体动力学

张科

回顾：

1.热力学基础知识:

2.绝热、可逆、等熵过程：

$$\boxed{\begin{aligned} s_2 - s_1 &= C_v ln\frac{T_2}{T_1} + Rln\frac{\rho_1}{\rho_2} \\ s_2 - s_1 &= C_p ln\frac{T_2}{T_1} - Rln\frac{p_2}{p_1} \end{aligned}}$$

$$\boxed{\frac{p_2}{p_1} = (\frac{\rho_2}{\rho_1})^{\gamma} = (\frac{T_2}{T_1})^{\frac{\gamma}{\gamma-1}}}$$

3.声速：

$$\boxed{a = \sqrt{\gamma\frac{p}{\rho}} = \sqrt{\gamma RT}}$$

4.高速一维定常无粘流：

$$\rho_1 u_1 = \rho_2 u_2$$

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$$

$$h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2}$$

11.3高速一维定常无粘流(绝热dq=0,7.2,7.3)

1. 基本方程：

②	C.V.	①
p_2		p_1
u_2		u_1
ρ_2		ρ_1
T_2		T_1
$\hat{u}_2$		$\hat{u}_1$
h_2		h_1

$$\begin{cases} \rho_1 u_1 = \rho_2 u_2 & \text{连续方程} \\ p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2 & \text{动量方程} \\ h_1 + \dfrac{u_1^2}{2} = h_2 + \dfrac{u_2^2}{2} & \text{绝热能量方程} \end{cases}$$

$$(p_2 - p_1)A = \rho_1 u_1^2\, A - \rho_2 u_2^2\, A$$

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$$

绝热能量方程：

$$\begin{cases} C_p T_1 + \dfrac{u_1^2}{2} = C_p T_2 + \dfrac{u_2^2}{2} = C \\ \dfrac{\gamma}{(\gamma-1)} R T_1 + \dfrac{u_1^2}{2} = \dfrac{\gamma}{(\gamma-1)} R T_2 + \dfrac{u_2^2}{2} \\ \dfrac{a_1^2}{(\gamma-1)} + \dfrac{u_1^2}{2} = \dfrac{a_2^2}{(\gamma-1)} + \dfrac{u_2^2}{2} \end{cases}$$

绝热一维定常可压流动参数变化关系

不可压能量方程：

$$p + \frac{\rho u^2}{2} = C$$

参考点？

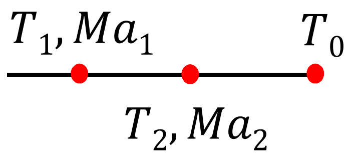
11.3高速一维定常无粘流(绝热dq=0,7.2,7.3)

2\. 参考点：$C_pT_1+\frac{u_1^2}{2}=C_pT_2+\frac{u_2^2}{2}=C$　　$C_vT+\frac{p}{\rho}+\frac{u^2}{2}=C$

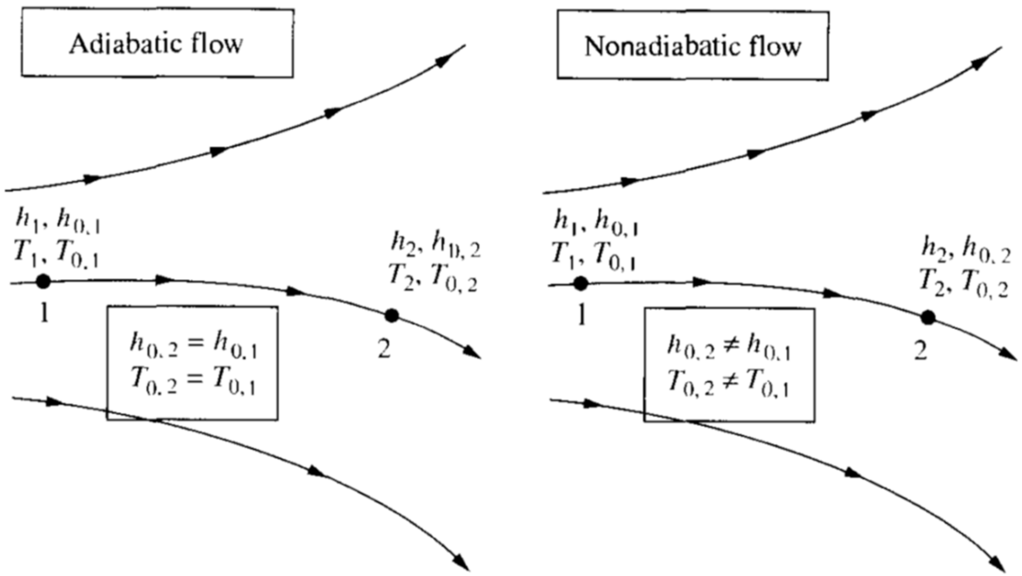
参考驻点：驻点$u=0$

绝热滞止：$C_pT+\frac{u^2}{2}=C_pT_0$　　T：静温, T_0: 滞止点总温, 总焓h_0(总能量)

$$\frac{T_0}{T}=1+\frac{u^2}{2C_pT}=1+\frac{u^2}{\frac{2\gamma}{(\gamma-1)}RT}=\boxed{1+\frac{\gamma-1}{2}Ma^2}$$



$a=\sqrt{\gamma RT}$



不可压：$p=p_0-\frac{\rho V^2}{2}$

绝热可压：$T=T_0-\frac{V^2}{2C_p}$

$$\boxed{\frac{T_0}{T}=1+\frac{\gamma-1}{2}Ma^2}$$

附表A

11.3高速一维定常无粘流(绝热dq=0,7.2,7.3)

2. 参考点： $C_pT_1+\frac{u_1^2}{2}=C_pT_2+\frac{u_2^2}{2}=C$　　$C_vT+\frac{p}{\rho}+\frac{u^2}{2}=C$

参考驻点： 驻点$u=0$　绝热滞止: $\boxed{\frac{T_0}{T}=1+\frac{\gamma-1}{2}Ma^2}$

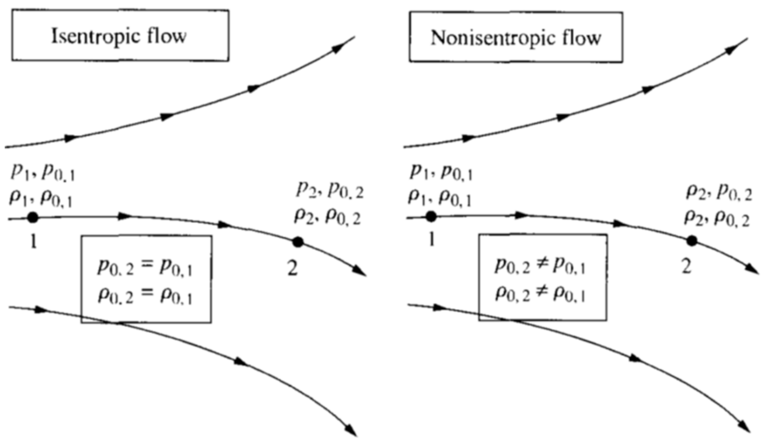
等熵滞止:　驻点$u=0$，对应总压p_0, 总密度ρ_0

（绝热、可逆）

等熵: $\frac{p_0}{p}=(\frac{\rho_0}{\rho})^{\gamma}=(\frac{T_0}{T})^{\frac{\gamma}{\gamma-1}}$ ⇒

$$\boxed{\frac{p_0}{p}=(1+\frac{\gamma-1}{2}Ma^2)^{\frac{\gamma}{\gamma-1}}}$$

$$\boxed{\frac{\rho_0}{\rho}=(1+\frac{\gamma-1}{2}Ma^2)^{\frac{1}{\gamma-1}}}$$

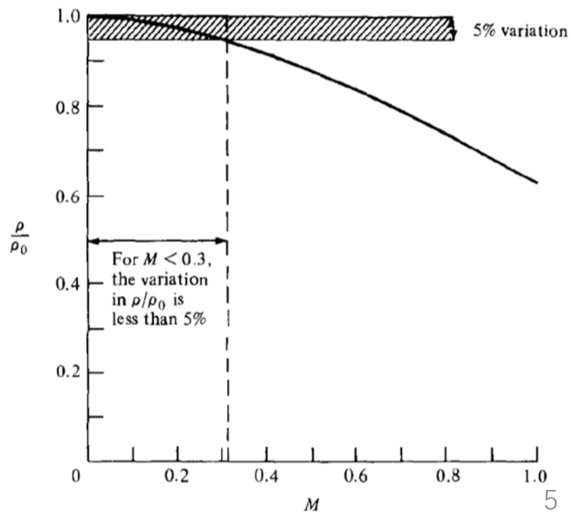


等熵过程：参数~Ma

附表A

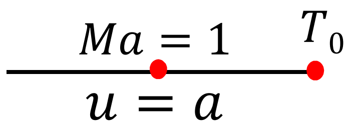
$Ma<0.3$→

$\frac{\rho}{\rho_0}<0.95, \frac{\Delta\rho}{\rho}<5\%$



11.3高速一维定常无粘流(绝热dq=0,7.2,7.3)

2. 参考点：



参考临界点：　临界点$Ma=1, u^*=a^*$

临界点参数：T^*, p^*, ρ^*, a^*　　$\frac{T_0}{T}=1+\frac{\gamma-1}{2}Ma^2$

$$\frac{T_0}{T^*}=1+\frac{\gamma-1}{2}\quad \frac{p^*}{p_0}=\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}\quad \frac{\rho^*}{\rho_0}=\left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}}$$

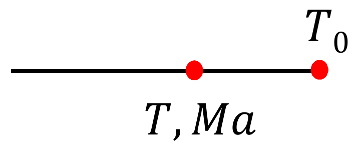
空气：$\frac{T^*}{T_0}=0.833, \frac{p^*}{p_0}=0.528, \frac{\rho^*}{\rho_0}=0.634$

绝热能量方程：$\frac{a^2}{(\gamma-1)}+\frac{u^2}{2}=\frac{a^{*2}}{(\gamma-1)}+\frac{a^{*2}}{2}=\frac{\gamma+1}{2(\gamma-1)}a^{*2}=\frac{a_0^2}{(\gamma-1)}$

a^*, a_0 都能代表总能量。

11.3高速一维定常无粘流(绝热dq=0,7.2,7.3)

3. 特征马赫数（速度系数）：



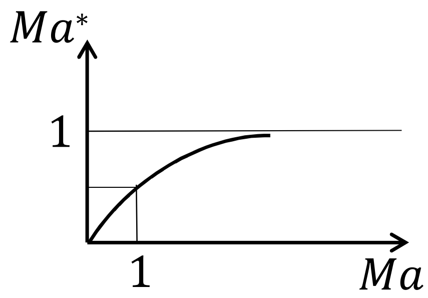
运动过程中，即使V不变，若T变，则a变，Ma变！

选临界点a^*为参考速度，定义特征马赫数：

$$Ma^* = \frac{V}{a^*},\ a^*为V = a时的a^*，非当地a。\quad \frac{T_0}{T} = 1 + \frac{\gamma-1}{2}Ma^2$$

$$Ma^{*2} = \frac{V^2}{a^{*2}} = \frac{V^2}{a^2}\frac{a^2}{a^{*2}} = Ma^2\frac{T}{T^*} = Ma^2\frac{T}{T_0}\frac{T_0}{T^*} = Ma^2/(1+\frac{\gamma-1}{2}Ma^2)(\frac{\gamma+1}{2})$$

$$\Rightarrow\quad Ma^{*2} = \frac{(\gamma+1)Ma^2}{2+(\gamma-1)Ma^2} \qquad Ma^2 = \frac{\frac{2}{\gamma+1}Ma^{*2}}{1-\frac{\gamma-1}{\gamma+1}Ma^{*2}}$$



$Ma = 1 \rightarrow Ma^* = 1$

$Ma > 1 \rightarrow Ma^* > 1$

$Ma < 1 \rightarrow Ma^* < 1$

$Ma \rightarrow \infty \rightarrow Ma^* = \sqrt{\frac{\gamma+1}{\gamma-1}} = \sqrt{6}$

11.3高速一维定常无粘流(绝热dq=0,7.2,7.3)

例：某点Ma, p, T分别为3.5，0.3atm，180K。求T_0，p_0，T^*，a^*，Ma^*。

解：$\frac{T_0}{T} = 1 + \frac{\gamma-1}{2}Ma^2 = 1 + \frac{1.4-1}{2}3.5^2 = 3.45$

$\frac{p_0}{p} = (\frac{T_0}{T})^{\frac{\gamma}{\gamma-1}}$ =76.25

$\Rightarrow$ $T_0 = 621K, p_0 = 2.32Mpa$

$\frac{T^*}{T_0} = 0.833$ $\Rightarrow$ $T^* = 517.5K$，$a^* = \sqrt{\gamma RT^*} = 456m/s$ V =?

$Ma = \frac{V}{a} = 3.5$ $a = \sqrt{\gamma RT} = 268.9m/s$

$\Rightarrow$ $V = 941m/s$ $Ma^* = \frac{V}{a^*} = 2.06$

或 $Ma^{*2} = \frac{(\gamma+1)Ma^2}{2+(\gamma-1)Ma^2}$ $\Rightarrow$ $Ma^* = 2.06$

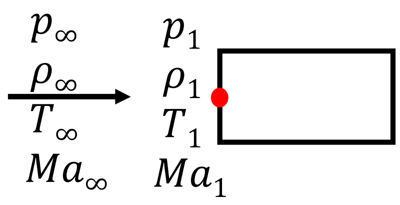
查附表A

$Ma = 3.5 \rightarrow \frac{T_0}{T}, \frac{p_0}{p}$

11.3高速一维定常无粘流(绝热dq=0,7.2,7.3)

例：飞机在$h = 5000m$高空$Ma = 0.8$飞行，发动机进气口：$A_1 = 0.5m^2$
$Ma = 0.4$ 。求来流T_0，p_0，ρ_0，T^*，进气口T_1，p_1，ρ_1，$\dot{m}$。

解：$h = 5km$高空大气参数：$p_\infty = 5.4 \times 10^4 pa, \rho_\infty = 0.737kg/m^3$，$T_\infty = 255.65K$



① $\frac{T_0}{T_\infty} = 1 + \frac{\gamma-1}{2}Ma^2 = 1 + \frac{1.4-1}{2}0.8^2 = 1.128$ $T_0 = 288.37K$

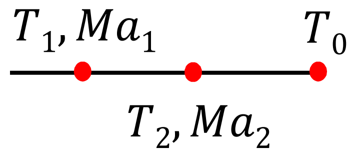
$\frac{p_0}{p_\infty} = (\frac{T_0}{T_\infty})^{\frac{\gamma}{\gamma-1}} = 1.524$ $\frac{\rho_0}{\rho_\infty} = (\frac{T_0}{T_\infty})^{\frac{1}{\gamma-1}} = 1.3514$

$p_0 = 8.2 \times 10^4 pa, \rho_0 = 0.996kg/m^3$

查附表A

$Ma \rightarrow \frac{T_0}{T}, \frac{p_0}{p}, \frac{\rho_0}{\rho}$

② $Ma = 0.4,$ $\frac{p_1}{p_0} = 0.8956$ $\frac{\rho_1}{\rho_0} = 0.9243$ $\frac{T_1}{T_0} = 0.9690$



$p_1 = 7.37 \times 10^4 pa, \rho_1 = 0.919kg/m^3, T_1 = 279.4K$

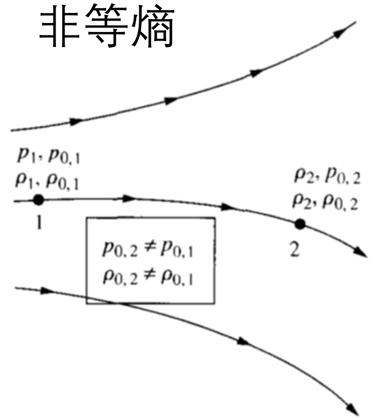
③ $\dot{m} = \rho_1 V_1 A_1 = \rho_1 Ma_1 a_1 A_1$ $a_1 = \sqrt{\gamma R T_1} = 335m/s$

$= 61.6kg/s$

11.3高速一维定常无粘流(绝热dq=0,7.2,7.3)

4. 熵变与总压： p_{02}~s关系？

非等熵



$$s_2 - s_1 = C_p ln\frac{T_2}{T_1} - R ln\frac{p_2}{p_1} \qquad C_p - C_v = R, C_p/C_v = \gamma$$

$$= \gamma C_v ln\frac{T_2}{T_1} - (\gamma - 1)C_v ln\frac{p_2}{p_1}$$

$$= (\gamma - 1)C_v ln[(\frac{T_2}{T_1})^{\frac{\gamma}{\gamma-1}}\frac{p_1}{p_2}] \qquad ①$$

$$\frac{p_1}{p_2} = \frac{p_1}{p_{01}}\frac{p_{01}}{p_{02}}\frac{p_{02}}{p_2} = \frac{p_{01}}{p_{02}}(\frac{T_1}{T_{01}})^{\frac{\gamma}{\gamma-1}}(\frac{T_{02}}{T_2})^{\frac{\gamma}{\gamma-1}}$$

绝热： $T_{02} = T_{01}$ ⟹ $\frac{p_1}{p_2} = \frac{p_{01}}{p_{02}}(\frac{T_1}{T_2})^{\frac{\gamma}{\gamma-1}}$ ② T_0总能量， p_0有用能

②→① $\boxed{s_2 - s_1 = (\gamma - 1)C_v ln\frac{p_{01}}{p_{02}}}$

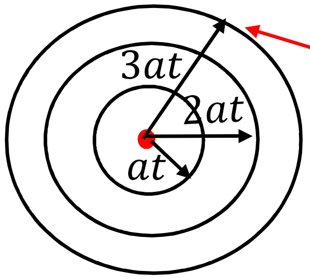
绝热不可逆：s↑p_0↓($s_2 > s_1$， $p_{01} < p_{02}$)

等熵：$ds = 0, p_0$不变

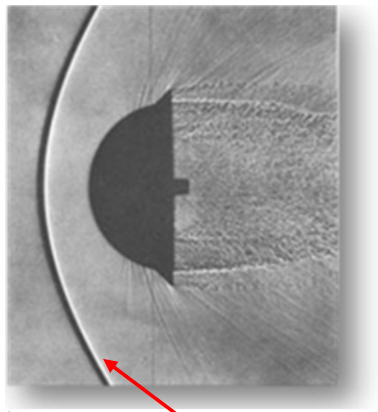
11.4马赫波与膨胀波(8.2,8.7)

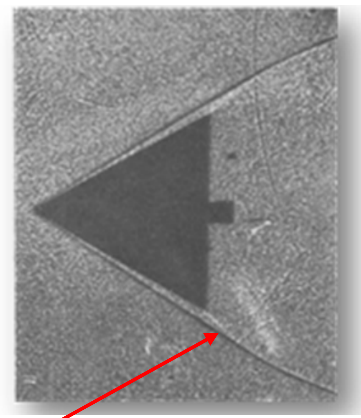
1. 小扰动传播：

扰动源$V=0$



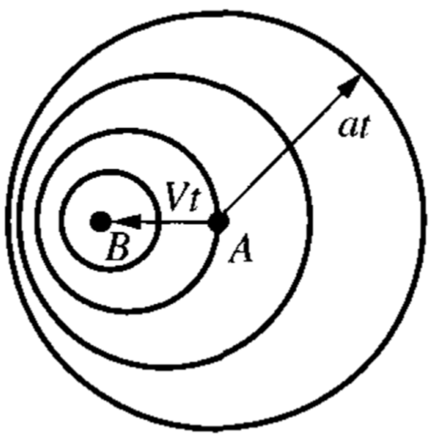
扰动 波波前, 扰动向全场传播





脱体激波、斜激波； 产生原因？

$V<a$



扰动源从A到B,

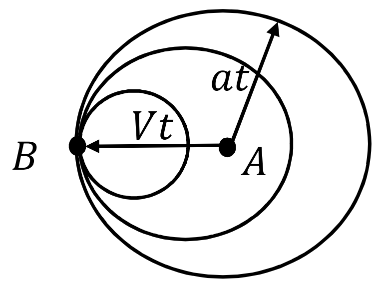
扰动源运动速度V <扰动波速度a,

扰动源在波前之内,

扰动向全场传播,

扰源未到已受干扰，参数提前改变。

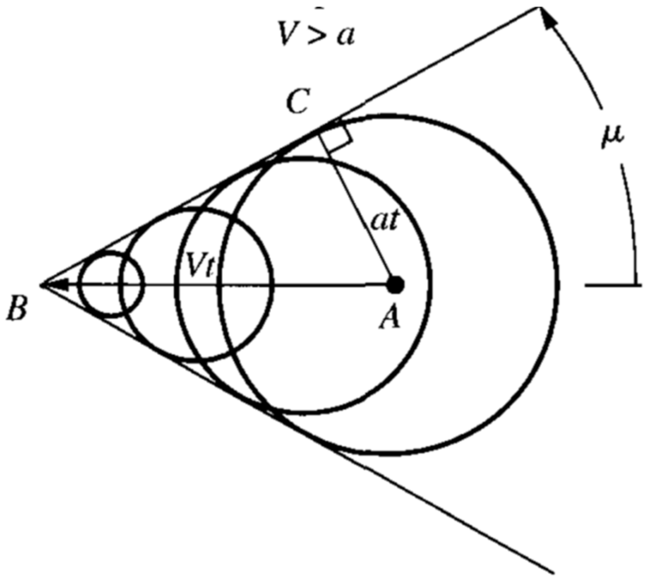
$V=a$



扰动源在波前上。

11.4马赫波与膨胀波(8.2,8.7)

1. 小扰动传播：



扰动源从A到B,

扰动源运动速度V >扰动波速度a,

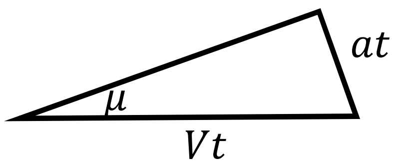
扰动源在波前之外,

扰动传播在一定范围内,

扰源已到才知，参数未能提前改变-突变。

声波倒叙传播：“特朗普” ——“普朗特”

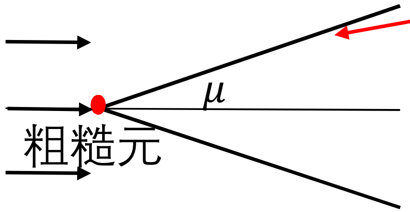
2. 波前包络面：马赫面（马赫锥）：超声速流中扰动传播的包络面。



马赫线与来流夹角μ：$\mu = arcsin\frac{a}{V} = arcsin\frac{1}{Ma}$

11.4马赫波与膨胀波(8.2,8.7)

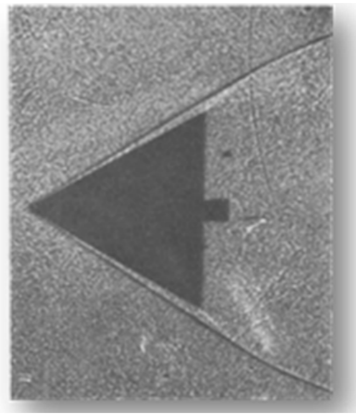
2. 波前包络面：

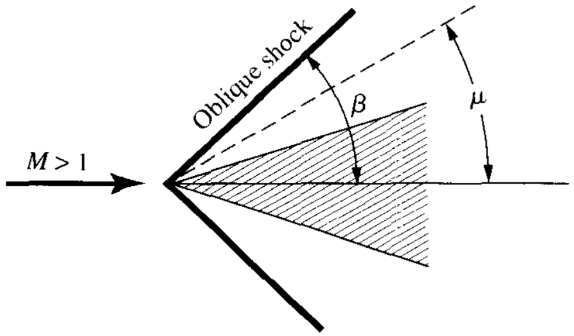


马赫Ma线——特征线（参数导数突跃）

Ma波：弱扰动波（无限弱的斜激波）

斜激波：强扰动波

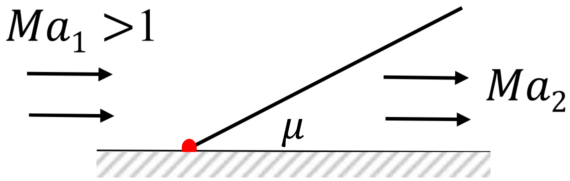




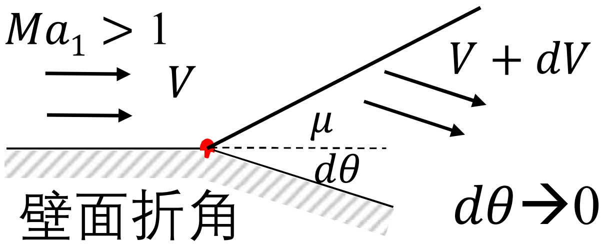
激波斜角$\beta >$ Ma角μ

11.4马赫波与膨胀波(8.2,8.7)

3. 过马赫波速度变化(**弱**扰动传播)：



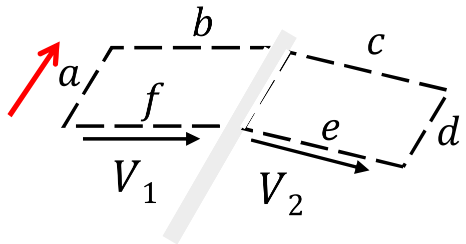
粗糙元

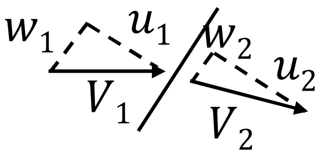


壁面折角

波前后参数关系？

$d\theta \rightarrow 0$时马赫波





$abcdef$组成控制体C.V.：

质量守恒：

$$-\rho_1 u_1 A+\rho_2 u_2 A=0 \qquad ①$$

切向动量方程：

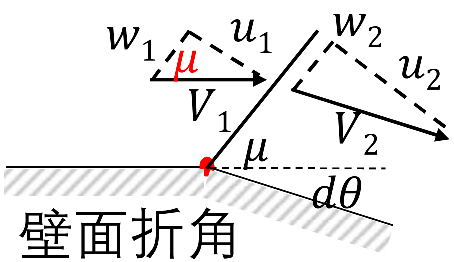
$\sum F_\tau=0$ 无粘，切向应力为0。

$$\sum F_\tau=0=-\rho_1 u_1 A w_1+\rho_2 u_2 A w_2 \qquad ②$$

①+ ②→ $\boxed{w_1=w_2}$ 过马赫波切向速度不变。（与马赫波相切方向）

11.4马赫波与膨胀波(8.2,8.7)

3. 过马赫波速度变化(**弱**扰动传播)：



壁面折角

$\boxed{w_1 = w_2}$ 过马赫波切向速度不变。

$$V\cos\mu = (V + dV)\cos(\mu + d\theta)$$

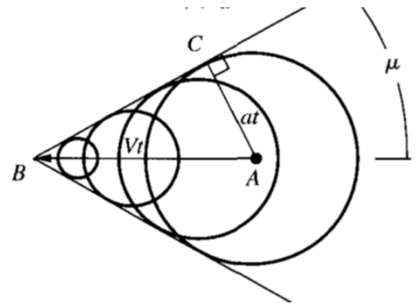
$$V\cos\mu = (V + dV)(\cos\mu\cos d\theta - \sin\mu\sin d\theta)$$

$$1 + \frac{dV}{V} = \frac{\cos\mu}{\cos\mu\cos d\theta - \sin\mu\sin d\theta} \qquad d\theta \to 0,\ \sin d\theta \to d\theta,\ \cos d\theta \to 1$$

$$1 + \frac{dV}{V} \approx \frac{1}{1 - \tan\mu d\theta} \qquad \frac{1}{1-x} = 1 + x + x^2 + \cdots + x^n$$

$$1 + \frac{dV}{V} \approx 1 + \tan\mu d\theta + o()$$

$$\Rightarrow \frac{dV}{V} = \tan\mu d\theta \qquad \tan\mu = \frac{a}{\sqrt{V^2 - a^2}} = \frac{1}{\sqrt{Ma^2 - 1}}$$



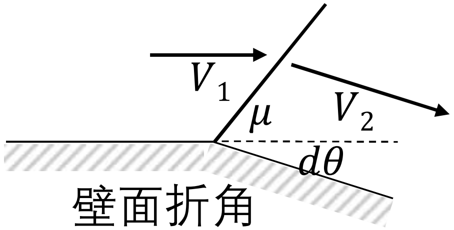
$$\Rightarrow \boxed{\frac{dV}{V} = \frac{d\theta}{\sqrt{Ma^2 - 1}}}$$

过马赫波V变化关系！

$$\mu = \arcsin\frac{a}{V} = \arcsin\frac{1}{Ma}$$

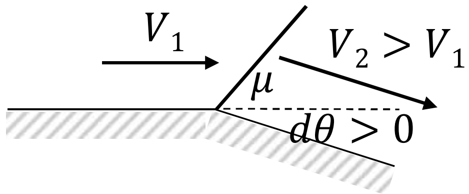
11.4马赫波与膨胀波(8.2,8.7)

3. 过马赫波速度变化(弱扰动传播)：

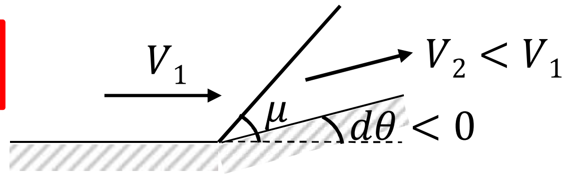


过马赫波V变化关系：$\boxed{\dfrac{dV}{V}=\dfrac{d\theta}{\sqrt{Ma^2-1}}}$ 弱扰动(等熵流动)。

$\boxed{d\theta>0\text{(外折)} \rightarrow dV>0\text{(加速)}}$



$\boxed{d\theta<0\text{(内折)} \rightarrow dV<0\text{(减速)}}$



11.4马赫波与膨胀波(8.2,8.7)

过马赫波V变化关系：$\boxed{\frac{dV}{V}=\frac{d\theta}{\sqrt{Ma^2-1}}}$

4. 过马赫波参数变化(**弱**扰动传播)：

$$\frac{dV}{V}\rightarrow\frac{dp}{p},\frac{d\rho}{\rho},\frac{dT}{T}???$$

绝热：

$$C_pT+\frac{V^2}{2}=C$$

$$T=C'-\frac{V^2}{2C_p}$$

$$dT=-\frac{VdV}{C_p}$$

$$\frac{dT}{T}=-\frac{V}{C_p}\frac{dV}{T}\qquad C_p=\frac{\gamma}{\gamma-1}R$$

$$\frac{dT}{T}=-\frac{V}{\frac{\gamma}{\gamma-1}R}\frac{dV}{T}=-\frac{(\gamma-1)V^2}{\gamma RT}\frac{dV}{V}=-\frac{(\gamma-1)V^2}{a^2}\frac{dV}{V}$$

$$\boxed{\frac{dT}{T}=-(\gamma-1)Ma^2\frac{dV}{V}}$$

等熵：$\frac{p}{\rho^\gamma}=C\quad\frac{p}{T^{\frac{\gamma}{\gamma-1}}}=C$

$$\boxed{\frac{dp}{p}=\frac{\gamma}{\gamma-1}\frac{dT}{T}=-\gamma Ma^2\frac{dV}{V}}\qquad\boxed{\frac{d\rho}{\rho}=\frac{1}{\gamma-1}\frac{dT}{T}=-Ma^2\frac{dV}{V}}$$

11.4马赫波与膨胀波(8.2,8.7)

4. 过马赫波参数变化(**弱**扰动传播)：

$$\boxed{\frac{dV}{V}=\frac{d\theta}{\sqrt{Ma^2-1}}}$$

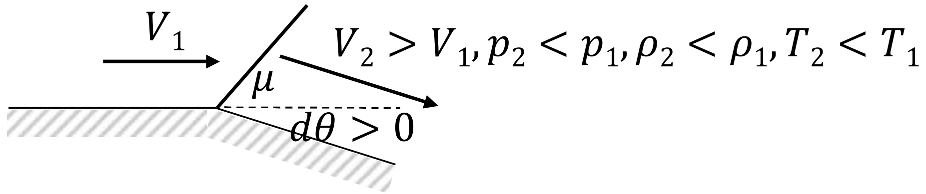
$$\boxed{\frac{dT}{T}=-(\gamma-1)Ma^2\frac{dV}{V}}$$

$$\boxed{\frac{dp}{p}=\frac{\gamma}{\gamma-1}\frac{dT}{T}=-\gamma Ma^2\frac{dV}{V}}$$

$$\boxed{\frac{d\rho}{\rho}=\frac{1}{\gamma-1}\frac{dT}{T}=-Ma^2\frac{dV}{V}}$$

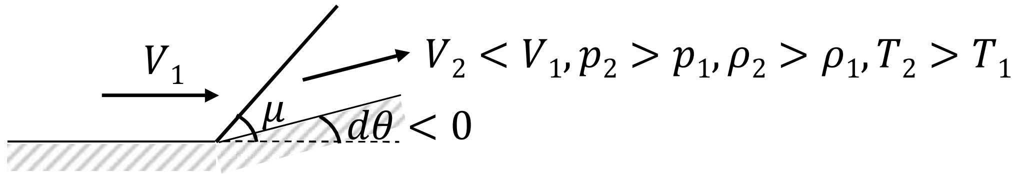
外折: $d\theta>0, dV>0, dp, d\rho, dT<0$

加速膨胀（膨胀波）



内折: $d\theta<0, dV<0, dp, d\rho, dT>0$

减速压缩（压缩波）



作业：

复习笔记！

空气动力学书7.1~7.6