空气与气体动力学

张科

4.1 弦长 0.6096m，来流速度 1.524 m/s，攻角为 4° 时，计算在弦长处的升力和力矩，取单位展长

1. 查表所得数据有误
2. 力矩单位带错

4.1. 解：$c=0.6096$，$V_\infty=1.524$，$\alpha=4^\circ$ ①

查图4.5得 $\alpha=4^\circ$ 时 $C_l=0.6$，$C_{m,\frac{c}{4}}=-0.04$

$$L=\frac{1}{2}\rho V_\infty^2\cdot C_l\cdot c=\frac{1}{2}\times1.2\times1.524^2\times0.6\times0.6096=0.5097N$$

$$M=\frac{1}{2}\rho V_\infty^2\cdot C_{m,\frac{c}{4}}\cdot c^2=\frac{1}{2}\times1.2\times1.524^2\times(-0.04)\times0.6096^2=-0.0207(N\cdot m)$$

4.2 解：对称翼型，$C_l=2\pi\alpha$

$$C_l=2\pi\alpha=0.5 \qquad x_{pc}=x_{ac}=\frac{c}{4}$$

$$\alpha=\frac{0.5}{2\pi}\cdot\frac{180^\circ}{\pi}(^\circ)$$

$$=4.56^\circ$$

2

4.3 中弧线是圆弧形状(恒定曲率半径)的翼型，中弧线的最大值为 kc，此处 k 是常数，c 为翼型的弦长。自由来流速度为 V_∞，攻角为 α。假定 $k \ll 1$，证明 γ 分布的近似表达式为

$$\gamma = 2V_\infty\left(\alpha\frac{1+\cos\theta}{\sin\theta}\right) + 4k\sin\theta$$

4.3. 证明，根据几何关系

$R = \frac{kc}{2} + \frac{c}{8k}$，其中 c 为弦长，k 为常数且 $k \ll 1$

$$\frac{z}{c} = \sqrt{\left(\frac{R}{c}\right)^2 - \left(\frac{x}{c}-\frac{1}{2}\right)^2} - \sqrt{\left(\frac{R}{c}\right)^2 - \frac{1}{4}} \qquad 0 \le \frac{x}{c} \le 1$$

将 $\frac{z}{c}$ 以 $\frac{x}{c}$ 为自变量，在 $\frac{x}{c} = \frac{1}{2}$ 处做泰勒展开

$$\frac{z}{c} = -\sqrt{\frac{R^2}{c^2}-\frac{1}{4}} - r - \frac{c}{2R}\left(\frac{x}{c}-\frac{1}{2}\right)^2 - \frac{c^3}{8R^3}\left(\frac{x}{c}-\frac{1}{2}\right)^4 + \cdots \quad (保留4阶)$$

$$\frac{dz}{dx} = -\frac{1}{16R^3}\left[8c^3\left(\frac{x}{c}\right)^3 - 12c^3\left(\frac{x}{c}\right)^2 + (16cR^2+6c^3)\left(\frac{x}{c}\right) - 8cR^2 - c^3\right] \qquad 0 \le \frac{x}{c} \le 1$$

作变量代换 $\frac{x}{c} = \frac{1}{2} - \frac{1}{2}\cos\theta$

$$A_0 = \alpha - \frac{1}{\pi}\int_0^\pi \frac{dz}{dx}\,d\theta = \alpha - \frac{1}{\pi}\int_0^\pi \frac{1}{16R^3}\left[-(16cR^2+6c^3)\left(\frac{1}{2}-\frac{1}{2}\cos\theta\right) - 8c^3\left(\frac{1}{2}-\frac{1}{2}\cos\theta\right)^3 + 12c^3\left(\frac{1}{2}-\frac{1}{2}\cos\theta\right)^2 + 8cR^2 + c^3\right]d\theta$$

$$A_0 = \alpha - \frac{1}{\pi}\left\{-\frac{1}{48R^3}\left[c^3\sin^3\theta - (24cR^2+3c^3)\sin\theta\right]\right\}\Bigg|_{\theta=0}^{\pi}$$

$$A_0 = \alpha - \frac{1}{\pi}\times 0 = \alpha$$

$$A_1 = \frac{2}{\pi}\int_0^\pi \frac{dz}{dx}\cos\theta\,d\theta = \frac{2}{\pi}\times\left\{\frac{1}{512R^3}\left[c^3\sin4\theta + (64cR^2+8c^3)\sin2\theta + (128cR^2+12c^3)\theta\right]\right\}\Bigg|_{\theta=0}^{\pi}$$

$$A_1 = \frac{2}{\pi}\times\frac{128cR^2+12c^3}{512R^3}\times\pi$$

代入 $R = \frac{kc}{2} + \frac{c}{8k}$ 得 $A_1 = \frac{64k^5+56k^3+4k}{64k^6+48k^4+12k^2+1}$ $\quad\because k \ll 1$

$\therefore A_1 \approx 4k$

$$A_n = \frac{2}{\pi}\int_0^\pi \frac{dz}{dx}\cos n\theta\,d\theta = \frac{2}{\pi}\left[C_1\sin(n+3)\theta + C_2\sin(n+1)\theta + C_3\sin(n-1)\theta\right]\Big|_{\theta=0}^{\pi}$$

式中 C_1，C_2，C_3 均为常数，可见在 $n \ge 2$ 时，A_n 积分表达式恒为零

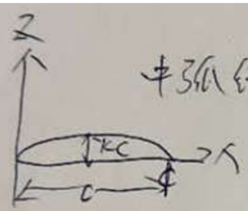
故 $A_n\ (n \ge 2) = 0$

$$\therefore \gamma(\theta) = 2V_\infty A_0\frac{1+\cos\theta}{\sin\theta} + 2V_\infty\sum_{n=1}^{\infty}A_n\sin n\theta$$

$$\gamma(\theta) = 2V_\infty\left(\alpha\frac{1+\cos\theta}{\sin\theta} + 4k\sin\theta\right)$$

$= 4.56°$

4.3解. 中弧线方程为 $\frac{z}{c} = -4K\left(\frac{x}{c} - \frac{1}{2}\right)^2 + K \quad 0 \le x \le c$



$$\frac{dz}{dx} = -4K\left[2\left(\frac{x}{c}\right) - 1\right]$$

若 $x = \frac{c}{2}(1-\cos\theta)$ 则 $\frac{dz}{dx} = 4K\cos\theta \quad 0 \le \theta \le \pi$

$$A_0 = \alpha - \frac{1}{\pi}\int_0^{\pi} \frac{dz}{dx} d\theta_0 = \alpha - \frac{1}{\pi}\int_0^{\pi} 4K\cos\theta_0 \, d\theta_0 = \alpha$$

$$A_n = \frac{2}{\pi}\int_0^{\pi} \frac{dz}{dx}\cos n\theta_0 \, d\theta_0 = \frac{2}{\pi}\int_0^{\pi} 4K\cos\theta_0\cos n\theta_0 \, d\theta_0$$

$$A_1 = \frac{2}{\pi}\int_0^{\pi} 4K\cos\theta_0\cos\theta_0 \, d\theta_0 = \frac{8K}{\pi}\left[\frac{1}{2}\sin\theta\cos\theta + \frac{\theta}{2}\right]\Big|_0^{\pi} = 4K.$$

$$A_2 = \frac{2}{\pi}\int_0^{\pi} 4K\cos 2\theta_0\cos\theta \, d\theta = \frac{8K}{\pi}\left(\sin\theta - \frac{2}{3}\sin^3\theta\right)\Big|_0^{\pi} = 0$$

$$\therefore \gamma(\theta) = 2V_\infty A_0 \frac{1+\cos\theta}{\sin\theta} + 2V_\infty \sum_{n=1}^{\infty} A_n \sin n\theta$$

$$\approx 2V_\infty \alpha \frac{1+\cos\theta}{\sin\theta} + 2V_\infty (4K)\sin\theta$$

$$= 2V_\infty\left[\alpha \frac{1+\cos\theta}{\sin\theta} + 4K\sin n\theta\right)$$

4.4 针对题4.3中的翼型，证明：零升攻角为2k(rad)，气动中心处力矩系数是-kπ

解: 由4.3推知: $A_1 = 4k \quad A_2 = 0$

1. 符号

故 $C_{m,\frac{c}{4}} = \frac{\pi}{4} \cdot 4k = k\pi.$

4.4. 解: $\alpha_{L=0} = -\frac{1}{\pi}\int_0^{\pi} \frac{dz}{dx}(\cos\theta_0 - 1) d\theta_0$

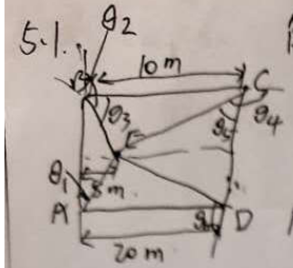
$$= -\frac{1}{\pi}\int_0^{\pi} 4K\cos\theta_0(\cos\theta_0 - 1) d\theta_0$$

$$= -\frac{2K}{\pi}\int_0^{\pi} \frac{\cos 2\theta_0 - 1}{2} d(2\theta_0)$$

$$= 2K.$$

$$C_{m,\frac{c}{4}} = \frac{\pi}{4}(A_2 - A_1) = \frac{\pi}{4}(0 - 4K) = -K\pi.$$

5.1.



解：$F=\rho U_\infty \Gamma \cdot L_{AB}$ ①

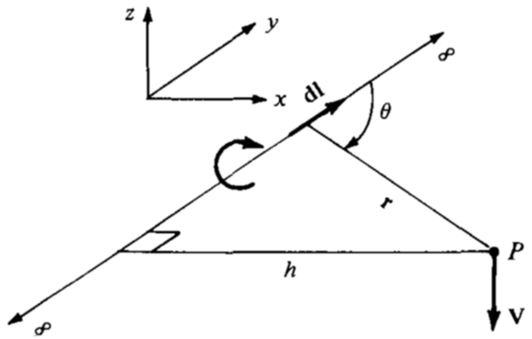
$\Gamma = \frac{F}{\rho U_\infty \cdot L_{AB}} = \frac{10^4}{1.2\times 100\times 20} = \frac{5}{1.2} = 4.167\ (m^2/s)$

AB段在E处诱导速度 $w_1 = -\frac{\Gamma}{4\pi h_1}\int_{\theta_1}^{\theta_2}\sin\theta d\theta$

$= -\frac{\Gamma}{4\pi h_1}(\cos\theta_1-\cos\theta_2)$

$= -\frac{5}{1.2\times4\times3.14\times5}\left(\frac{10}{\sqrt{10^2+5^2}}+\frac{10}{\sqrt{10^2+5^2}}\right)$

$= -\frac{5}{1.2\times4\times3.14}\left(\frac{4}{\sqrt{125}}\right)$



直线：$\vec{V} = \int_{-\infty}^{+\infty}\frac{\Gamma}{4\pi}\frac{d\vec{l}\times\vec{r}}{|\vec{r}|^3} = -\frac{\Gamma}{4\pi h}\int_0^{\pi}\sin\theta d\theta$

BC段在E处诱导速度

$w_2 = -\frac{\Gamma}{4\pi h_2}\int_{\theta_3}^{\theta_4}\sin\theta d\theta = -\frac{\Gamma}{4\pi h_2}(\cos\theta_3-\cos\theta_4)$

$= -\frac{5}{1.2\times4\times3.14\times10}\left(\frac{5}{\sqrt{10^2+5^2}}+\frac{15}{\sqrt{15^2+10^2}}\right)$

$= -\frac{5}{1.2\times4\times3.14}\left(\frac{0.5}{\sqrt{125}}+\frac{1.5}{\sqrt{325}}\right)$

AD段诱导速度 $w_3 = w_2$

CD段诱导速度 $w_4 = -\frac{\Gamma}{4\pi h_4}(\cos\theta_5-\cos\theta_6) = -\frac{5}{1.2\times4\times3.14\times15}\left(\frac{10}{\sqrt{10^2+15^2}}+\frac{10}{\sqrt{10^2+15^2}}\right)$

$= -\frac{5}{1.2\times4\times3.14}\cdot\frac{1.3}{\sqrt{325}}$

∴ $w = w_1+w_2+w_3+w_4$

$= -\frac{5}{1.2\times4\times3.14}\left(\frac{5}{\sqrt{125}}+\frac{4.3}{\sqrt{325}}\right) = 0.227\ m/s$

5-3、俯视图为椭圆形的机翼，以45m/s速度飞越海平面，翼载 $W/S=1000\,N/m^2$，机翼无扭转，从翼梢到翼根截面形状相同，截面上的升力线斜率为5.7，机翼展长10.0m，展弦比为5，分别证明截面上的升力系数为0.806，阻力系数为0.041，有效攻角、下洗角、绝对攻角沿展长方向是常量，分别为2.94°、8.1°、11.04°，证明：克服诱导阻力所需要作的功率为46000 W.

$$C_l = a_0(\alpha - \alpha_{L=0})\ ;$$

5.3解：$U_\infty = 45\,m/s$，$\frac{W}{S} = 1000\,N/m^2$，$a_0 = 5.7$，$b = 10$，$AR = b^2/S = 5$

椭圆机翼 $C_L = C_l = \frac{W}{\frac{1}{2}\rho V_\infty^2 \cdot S} = 0.803$

$$C_D = \frac{C_L^2}{\pi AR} = \frac{0.803^2}{3.14 \times 5} = 0.041$$

$C_l = a_0(\alpha_{eff} - \alpha_{L=0})$ 题目无 $\alpha_{L=0}$ 数据，(设 $\alpha_{L=0} = 0$)

有效迎角：$\alpha_{eff} = \frac{C_L}{a_0} = \frac{0.803}{5.7} = \frac{0.803}{5.7} \times \frac{180°}{3.14} = 8.08°$

下洗角：$\alpha_i = \frac{C_L}{\pi AR} = \frac{0.803}{3.14 \times 5} \times \frac{180°}{3.14} = 2.93°$

绝对迎角：$\alpha = \alpha_{eff} + \alpha_i = 11.01°$

$$P = D \cdot U_\infty = C_{Di} \cdot \frac{1}{2}\rho U_\infty^2 \cdot S \cdot U_\infty = 0.041 \times 0.5 \times 1.225 \times 45^2 \times \frac{10^2}{5} \times 45$$
$$= 45768\ W$$

5-4. 机翼面积为 15.7935 m^2，翼展长度 9.7536m，最大总重量 1111.32kg. 升力线斜率为 0.1033/(°)，$\alpha_{L=0} = -3°$，$\tau = 0.12$，如果飞机在海平面上平飞，飞机总重为最大总重，速度为 53.64333 m/s，计算攻角

解: $C_L = \frac{L}{\frac{1}{2}\rho v_\infty^2 S} = \frac{1111.32\times 9.8}{\frac{1}{2}\times 1.225\times 53.64333^2\times 15.7935} = 0.39$

$C_L = a(\alpha - \alpha_{L=0})$

$\Rightarrow \cancel{\alpha = \frac{C_L}{a} + \alpha_{L=0} = \frac{0.39}{0.1033} - 3 = 0.775°}$

$\Rightarrow \alpha = \frac{C_L}{a} + \alpha_{L=0}$.

$a = \frac{a_0}{1+\frac{a_0}{\pi AR}(1+\tau)} = \frac{0.1033}{1+\frac{0.1033}{\pi\frac{9.7536^2}{15.7935}}(1+0.12)} = 0.1$

$\alpha = \frac{0.39}{0.1027} - 3 = 0.797°$

没换单位导致计算错误

$$a = \frac{a_0}{1+\frac{a_0}{\pi AR}(1+\tau)}$$

公式中a_0单位用$1/rad$！

5.4 解: $S=15.7935m^2$, $b=9.7536m$, $W=1111.32kg$, $a_0=0.1033/(°)$ $= 5.92(1/rad)$

$\alpha_{L=0} = -3°$, $\tau=0.12$, $V_\infty=53.6433 m/s$

$a = \frac{a_0}{1+\frac{a_0}{\pi AR}(1+\tau)} = \frac{5.92}{1+\frac{5.92\times(1+0.12)}{3.14\times 6}}$ $AR = \frac{b^2}{S} = 6$

$= 4.38 (\frac{1}{rad})$

$C_L = \frac{W}{\frac{1}{2}\rho V_\infty^2\cdot S} = 0.39$

$C_L = a(\alpha - \alpha_{L=0})$

$\alpha = \frac{C_L}{a} + \alpha_{L=0}$ $W/\frac{1}{2}\rho V_\infty^2$

$= \frac{0.39}{4.38} + \frac{3}{180}\times 3.14 = 0.141 rad = 8.1°$

6-5 在一个超声速风洞储气罐内，罐内的速度可忽略不计，罐中的温度为900K。若喷管出口温度为500K，假设流动是绝热的，计算出口速度.

6.5 $C_pT_1+\frac{u_1^2}{2}=C_pT_2+\frac{u_2^2}{2}$

$u_1=0$

$C_p=\frac{\gamma}{\gamma-1}R=\frac{1.4}{0.4}\times8.31=29.085$

$29.085\times(900-500)=\frac{u_2^2}{2}$

$u_2=152.54\,m/s$

比热容算错！！

$R=\frac{\bar{R}}{M}=\frac{8312}{29}=287\ J/kgK$

760.65

6.5 解 $T_0=900K$，$T_1=500K$.

绝热. $C_pT_1+\frac{V_1^2}{2}=C_pT_0$

$V=\sqrt{2C_p(T_0-T_1)}=\sqrt{2\times\frac{1.4\times287}{0.4}\times400}=896.44(m/s)$

6-6 一个翼型处于来流压力 $P_\infty = 0.61\ atm$，密度 $\rho_\infty = 0.61\ kg/m^3$，速度 $V_\infty = 300\ m/s$。翼型表面某点的压力 $P_\infty = 0.5\ atm$。在流动是等熵的前提下，计算该点的速度.

6.6. 解，设来流中一点为参考点 1，翼型表面为参考点 2

对于等熵过程 $\left(\frac{P_2}{P_1}\right) = \left(\frac{\rho_2}{\rho_1}\right)^\gamma$

$$\therefore \rho_2 = \rho_1 \left(\frac{P_2}{P_1}\right)^{\frac{1}{\gamma}} = 0.61\left(\frac{0.5}{0.61}\right)^{\frac{1}{1.4}} = 0.53\ kg/m^3$$

$$T_1 = \frac{P_1}{\rho_1 R_g} = \frac{0.61 \times 101325}{0.61 \times 287} = 353.05\ K$$

$$T_2 = \frac{P_2}{\rho_2 R_g} = \frac{0.5 \times 101325}{0.53 \times 287} = 333.06\ k$$

根据一维定常流能量方程

$$V_2 = \sqrt{\frac{V_1^2}{2} + C_p(T_1 - T_2)} = \sqrt{\frac{1}{2} \times 300^2 + 1004 \times (353.05 - 333.06)} = 255.1\ m/s$$

公式错！！

6.6. 解：来流与表面某点间能量方程：

$$C_p T_\infty + \frac{V_\infty^2}{2} = C_p T_s + \frac{V_s^2}{2}$$

等熵：$\frac{P_s}{P_\infty} = \left(\frac{T_s}{T_\infty}\right)^{\frac{\gamma}{\gamma-1}}$ $T_\infty = 352.96K$

$T_s = 333.466K$

6.7 由 $\frac{p}{\rho}+\frac{u^2}{2}=C$ 和 $p=\rho RT$ 可得

$$RT+\frac{u^2}{2}=C.$$

$$\Rightarrow RT_1 = RT_2+\frac{u_2^2}{2}$$

$$\Rightarrow u_2=\sqrt{2[R(T_1-T_2)]}$$

$$=\sqrt{2\times287\times(900-500)}$$

$$=479\ m/s.$$

6.7解：不可压伯努利方程

6.6用

$$P_\infty+\frac{1}{2}\rho_\infty V_\infty^2 = P_s+\frac{1}{2}\rho_\infty V_s'^2$$

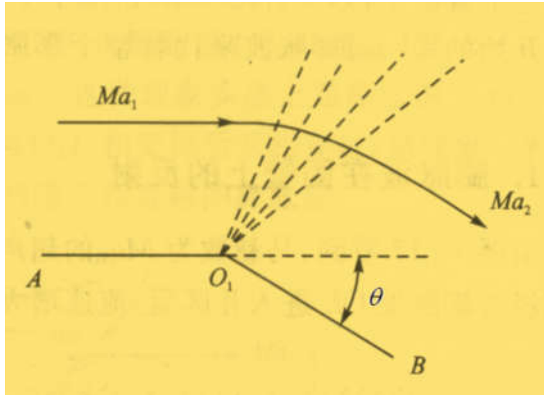
$$V_s'=\sqrt{\frac{2(P_\infty-P_s)}{\rho_\infty}+V_\infty^2}$$

$$=\sqrt{\frac{2\times0.11\times1.013\times10^5}{0.61}+300^2}$$

$$=355.72\ (m/s)$$

$$\Delta V=\frac{V_s-V_s'}{V_s}=\frac{359.41-355.71}{359.41}=1\%$$

回顾：

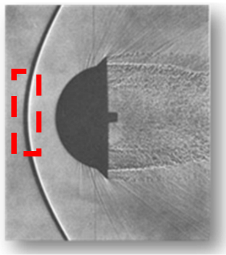


1.膨胀波: $\theta = \upsilon(Ma_2) - \upsilon(Ma_1)$ 等熵过程！

$$\theta = \int \frac{\sqrt{Ma^2-1}}{1+\frac{(\gamma-1)}{2}Ma^2}\frac{dMa}{Ma} \qquad \upsilon = \int \frac{\sqrt{Ma^2-1}}{1+\frac{(\gamma-1)}{2}Ma^2}\frac{dMa}{Ma}$$

普朗特-迈耶函数

2.正激波： $\boxed{a^{*2} = u_1 u_2}$



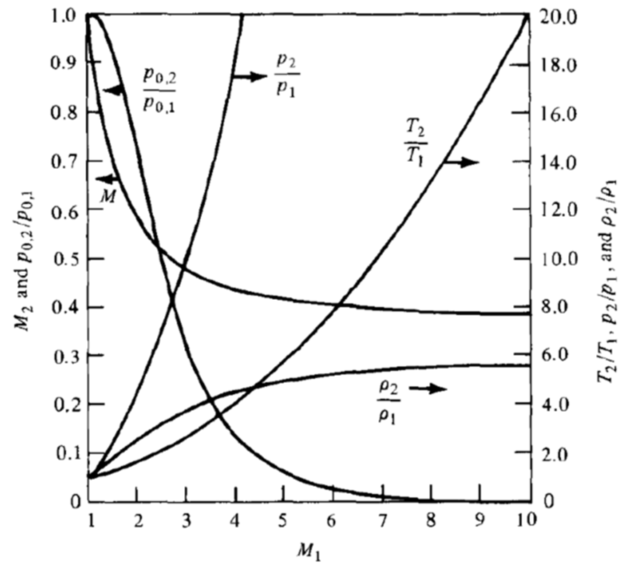
p_1	p_2
u_1	u_2
ρ_1	ρ_2
T_1	T_2
Ma_1	Ma
s_1	s_2
T_{01}	T_{02}
p_{01}	p_{02}

$$Ma_2^2 = \frac{1+\frac{\gamma-1}{2}Ma_1^2}{\gamma Ma_1^2 - \frac{\gamma-1}{2}} \qquad \frac{\rho_2}{\rho_1} = \frac{(\gamma+1)Ma_1^2}{2+(\gamma-1)Ma_1^2}$$

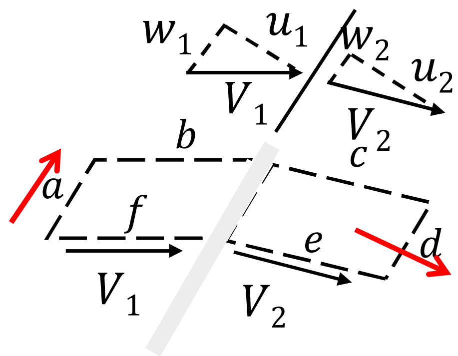
$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma+1}(Ma_1^2 - 1)$$

$$\frac{T_2}{T_1} = \frac{2+(\gamma-1)Ma_1^2}{(\gamma+1)Ma_1^2}\left[1+\frac{2\gamma}{\gamma+1}(Ma_1^2 - 1)\right]$$

$$\sigma = \frac{p_{02}}{p_{01}} = \left[\frac{2+(\gamma-1)Ma_1^2}{(\gamma+1)Ma_1^2}\right]^{\frac{-\gamma}{\gamma-1}}\left(\frac{2\gamma}{\gamma+1}Ma_1^2 - \frac{\gamma-1}{\gamma+1}\right)^{\frac{-1}{\gamma-1}}$$



11.6斜激波(8.3)

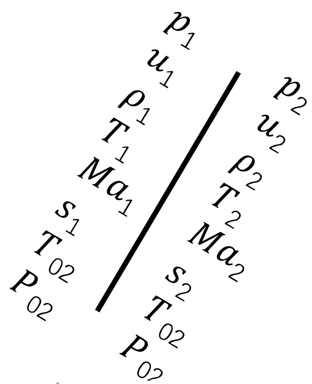


2. 控制方程：

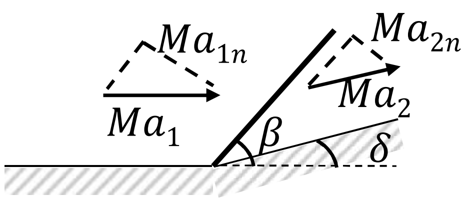
$$\boxed{\rho_1 u_1 = \rho_2 u_2 \quad ①} \qquad \boxed{p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2 \quad ②}$$

$$w_1 = w_2 \qquad h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$$

$$\boxed{h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2} \quad ③}$$



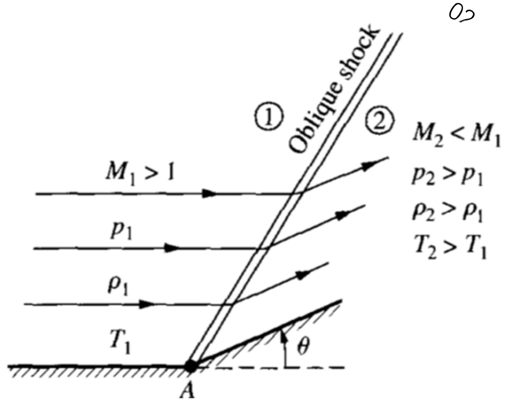
方程①②③可看作波前后速度为u_1, u_2的正激波控制方程！



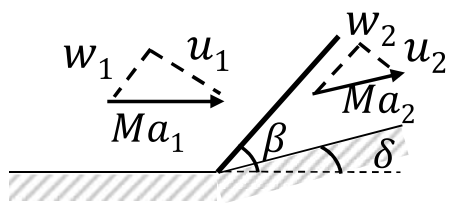
斜激波可视为

$$Ma_{1n} = Ma_1 sin\beta$$

$$Ma_{2n} = Ma_2 \sin(\beta - \delta)$$ 的正激波。



11.6斜激波(8.3)



3. 参数关系：

p_1	p_2
u_1	u_2
ρ_1	ρ_2
T_1	T_2
Ma_1	Ma_2
s_1	s_2
T_{02}	T_{02}
P_{02}	P_{02}

$$\boxed{Ma_2{}^2=\frac{1+\frac{\gamma-1}{2}Ma_1{}^2}{\gamma Ma_1{}^2-\frac{\gamma-1}{2}}}$$

$$\boxed{\frac{\rho_2}{\rho_1}=\frac{(\gamma+1)Ma_1{}^2}{2+(\gamma-1)Ma_1{}^2}}$$

$$\boxed{\frac{p_2}{p_1}=1+\frac{2\gamma}{\gamma+1}(Ma_1{}^2-1)}$$

$$Ma_{2n}{}^2=\frac{1+\frac{\gamma-1}{2}Ma_{1n}{}^2}{\gamma Ma_{1n}{}^2-\frac{\gamma-1}{2}} \qquad \frac{\rho_2}{\rho_1}=\frac{(\gamma+1)Ma_{1n}{}^2}{2+(\gamma-1)Ma_{1n}{}^2}$$

$$\frac{p_2}{p_1}=1+\frac{2\gamma}{\gamma+1}(Ma_{1n}{}^2-1) \qquad \frac{T_2}{T_1}=\frac{p_2}{p_1}\frac{\rho_1}{\rho_2}$$

$$P=\frac{\Delta p_1}{p_1}=\frac{2\gamma}{\gamma+1}(Ma_{1n}{}^2-1) \quad Ma_{1n}=Ma_1 sin\beta \quad Ma_{2n}=Ma_2\sin(\beta-\delta)$$

$Ma_1,\beta \rightarrow Ma_{1n} \rightarrow$斜激波参数关系； β =??

$$tan\beta=\frac{u_1}{w_1},\ \tan(\beta-\delta)=\frac{u_2}{w_2}$$

$$w_1=w_2 \Rightarrow \frac{tan\beta}{\tan(\beta-\delta)}=\frac{u_2}{u_1}=\frac{\rho_1}{\rho_2}=\frac{2+(\gamma-1)Ma_1{}^2sin^2\beta}{(\gamma+1)Ma_1{}^2sin^2\beta}$$

$$\beta=f(Ma_1,\delta)$$

$$\Rightarrow tan\delta=2cot\beta\frac{Ma_1{}^2sin^2\beta-1}{Ma_1{}^2sin^2\beta\,(\gamma+cos2\beta)+2}$$

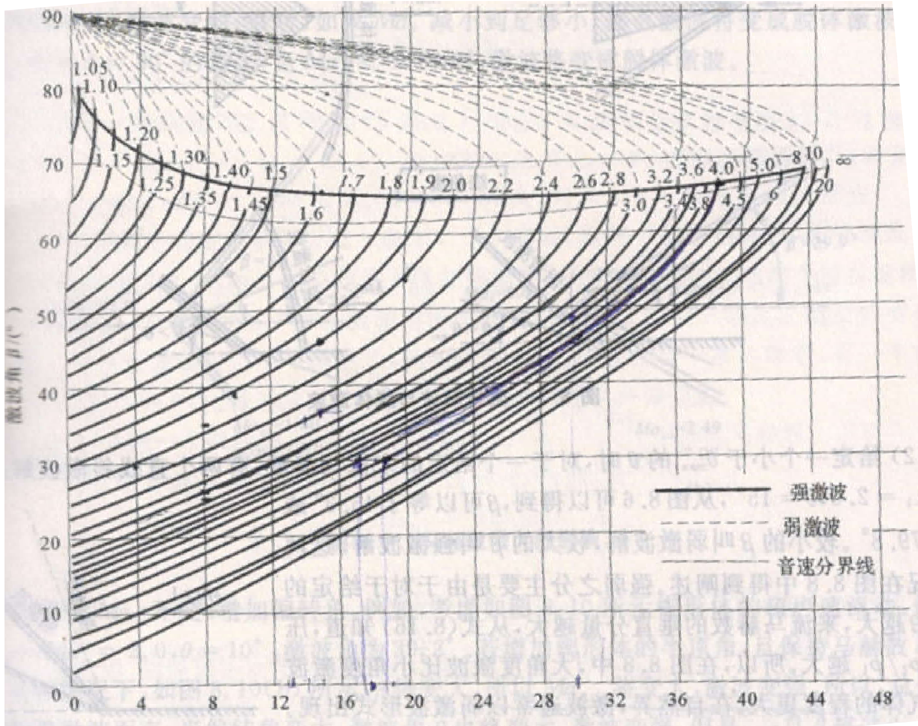
$\boxed{\delta-\beta-Ma\text{关系式！}}$

11.6斜激波(8.3)

4. 激波图线（定性）：

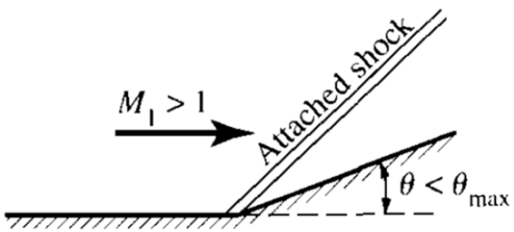
$$tan\delta = 2cot\beta \frac{Ma_1{}^2 sin^2\beta - 1}{Ma_1{}^2 sin^2\beta\ (\gamma + cos2\beta) + 2}$$

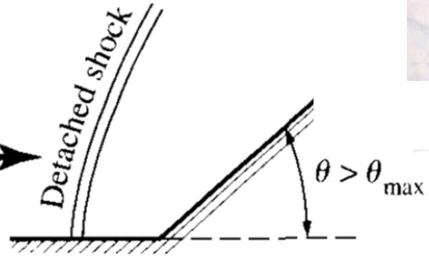
$\boxed{\delta - \beta - Ma\text{关系式！}}$ $\boxed{\text{P199：图8.6}}$



➢ 任一Ma，存在δ_{max}；

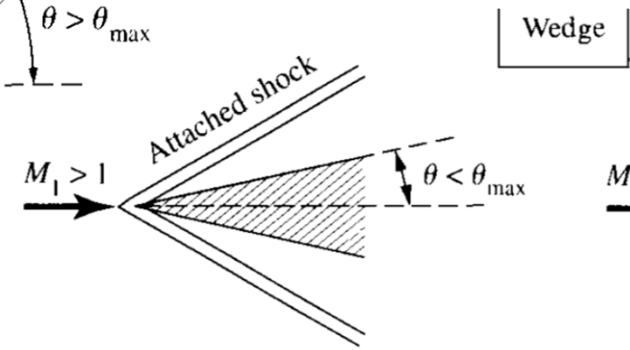
$\delta > \delta_{max}$ 无直线斜激波，脱体激波。

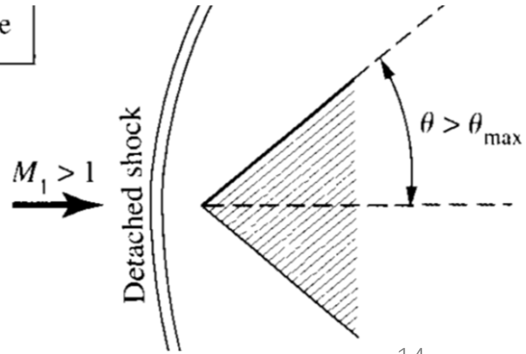




Ma_1⬆，δ_{max}⬆；

$Ma_1 \rightarrow \infty$，$\delta_{max} \rightarrow 45.5°(\gamma = 1.4)$。





11.6斜激波(8.3)

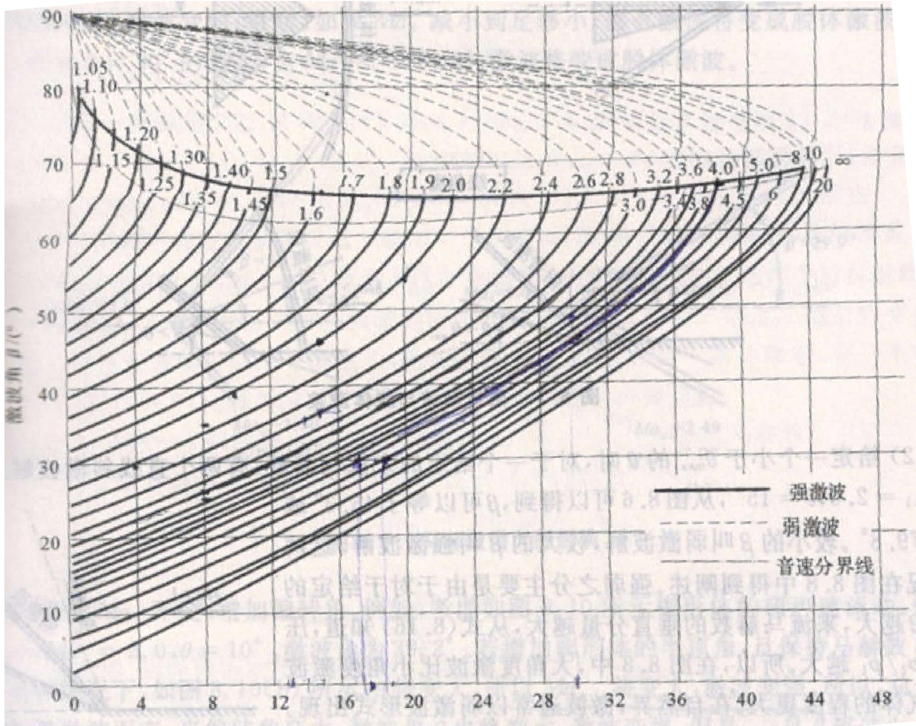
4. 激波图线（定性）：

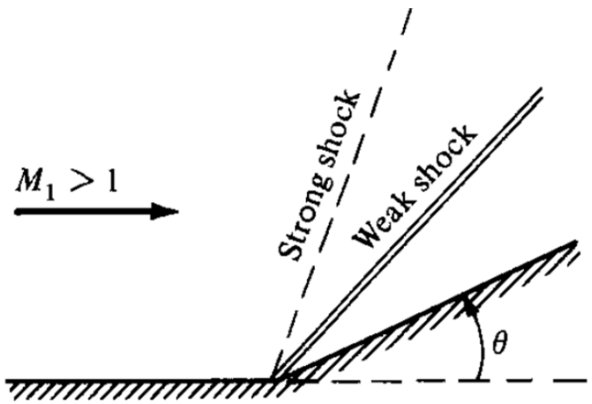
- $\delta < \delta_{\max}$，单一δ，Ma_1对应两解，$\beta_1\,\beta_2$。

β小——弱激波， $Ma_2 > 1$；

β大——强激波， $Ma_2 < 1$；

常见弱激波。





正激波$Ma_2 < 1$！！

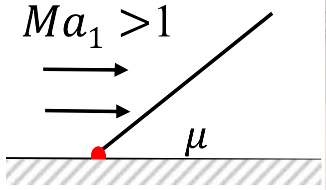
斜激波$Ma_{2n} = Ma_2 \sin(\beta - \delta) < 1$

11.6斜激波(8.3)

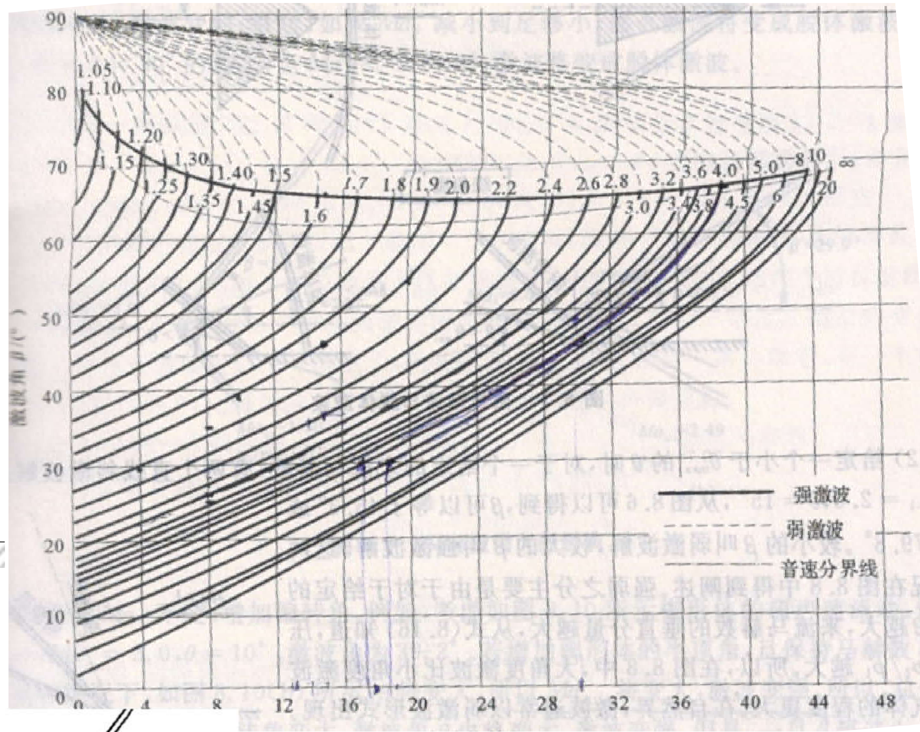
4. 激波图线（定性）：

- $\delta = 0 \rightarrow \beta = 90°$(正激波)或$\mu$(马赫波)。

气流无偏转！ $Ma_1 \mid Ma_2$



粗糙元



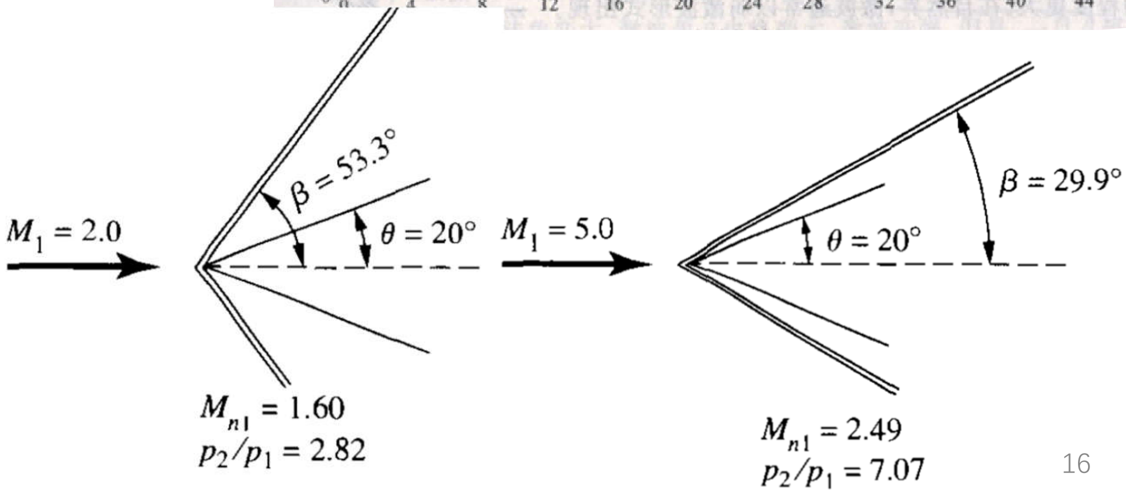
对弱激波：

- 定δ：Ma_1↑，β↓；Ma_1↓，β↑；

Ma_1↑, 激波贴近壁面，强度变大；

Ma_1↓, 激波远离壁面，强度变小；

$Ma_1 < Ma_{min}$, 脱体激波。



11.6斜激波(8.3)

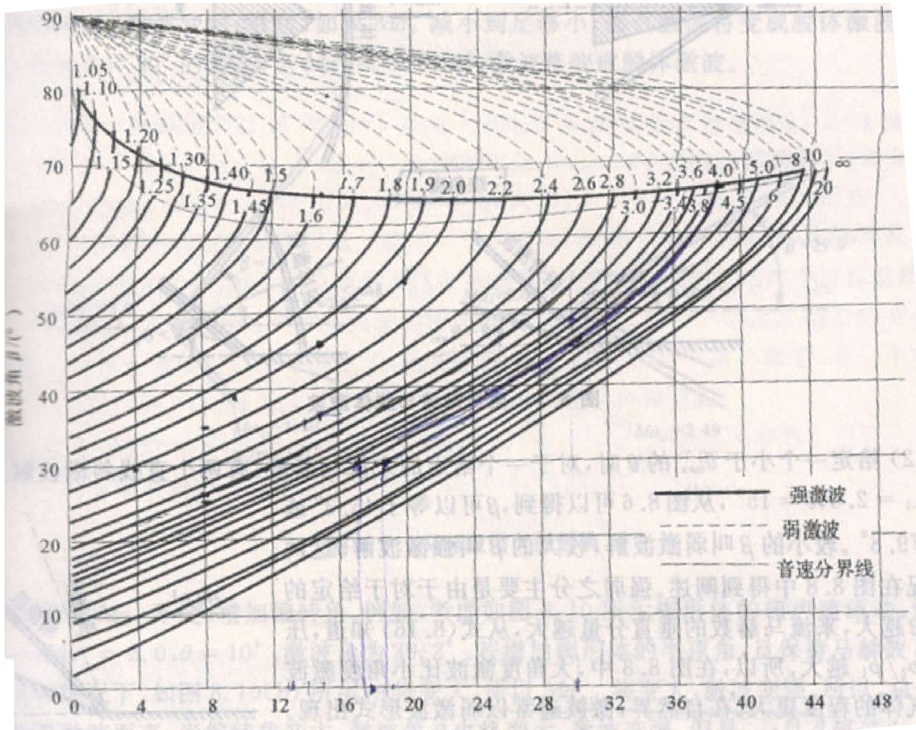
4. 激波图线（定性）：

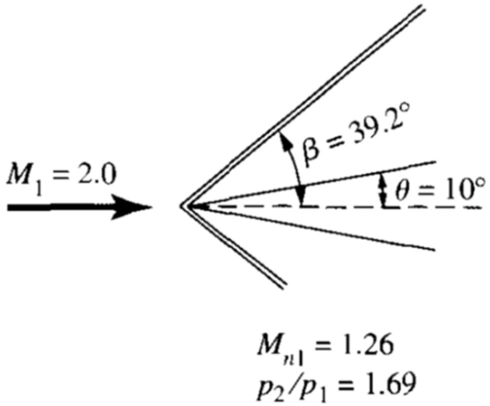
- 定Ma：δ⬆，β⬆；δ⬇，β⬇；

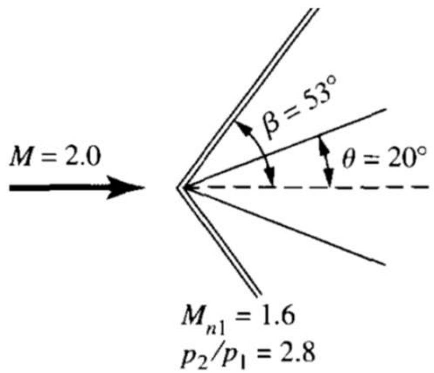
  δ⬆, 激波远离壁面，强度变大；

  δ⬇, 激波贴近壁面，强度变小；

  $\delta > \delta_{max}$, 脱体激波。



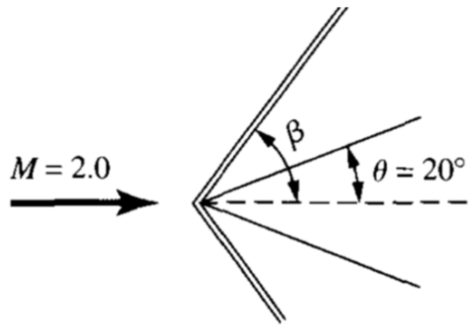




11.6斜激波(8.3)

例：$Ma_1 = 2, p_1 = 1atm, T_1 = 288K, \delta = 20°$。求$Ma_2, p_2, T_2, p_{02}. T_{02}$ 。

解：图8.6, $Ma_1 = 2\ , \delta = 20° \rightarrow \ \beta = 53.4°$



$$Ma_{1n} = Ma_1 sin\beta = 1.6$$

附表B：$Ma_{1n} = 1.6 \rightarrow Ma_{2n} = 0.6684$

$$Ma_2 = \frac{Ma_{2n}}{\sin(\beta-\delta)} = \frac{0.6684}{\sin(53.4° - 20°)} = 1.21$$

$$\frac{p_2}{p_1} = 2.82 \quad \frac{\rho_2}{\rho_1} = 2.032 \quad \frac{T_2}{T_1} = 1.388 \quad \frac{p_{02}}{p_{01}} = 0.8952$$

$$p_2 = 2.82atm \quad T_2 = 399.7atm$$

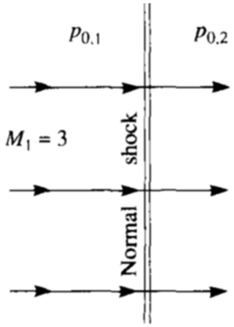
附表A：$Ma_1 = 2 \rightarrow \frac{p_{01}}{p_1} = 7.824 \quad p_{02} = \frac{p_{02}}{p_{01}} \frac{p_{01}}{p_1} p_1 = 7.0atm$

$$\frac{T_{01}}{T_1} = 1.8 \qquad T_{02} = T_{01} = \frac{T_{01}}{T_1} T_1 = 518.4K$$

11.6斜激波(8.3)

例：$Ma_1 = 3$, 过激波减速。

(1)过正激波，p_{02}；(2)先过$\beta = 40°$斜激波，再过正激波p_{03}；求p_{03}/p_{02}。



解：(1)附表B：$Ma_1 = 3 \rightarrow \frac{p_{02}}{p_{01}} = 0.3283$, $Ma_2 = 0.4752$

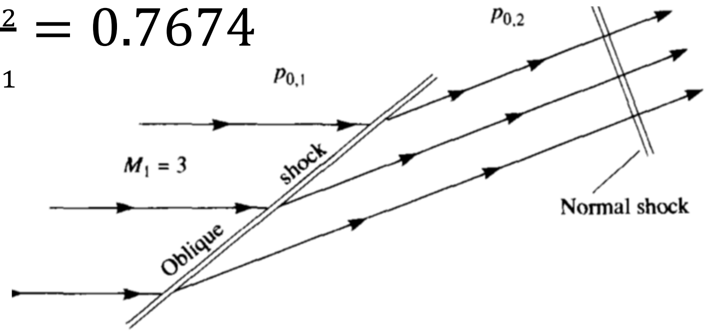
(2) $Ma_1 = 3$, $\beta = 40° \rightarrow Ma_{1n} = Ma_1 \sin\beta = 1.93$

图8.6 $\rightarrow$ $\delta = 22°$

附表B：$Ma_{2n} = 0.5899$, $Ma_2 = \frac{Ma_{2n}}{\sin(\beta - \delta)} = 1.9$ $\frac{p_{02}}{p_{01}} = 0.7674$

正激波：$Ma_2 = 1.9$, 附表B $\rightarrow \frac{p_{03}}{p_{02}} = 0.7535$

$$\frac{p_{03}}{p_{01}} = \frac{p_{03}}{p_{02}} \frac{p_{02}}{p_{01}} = 0.7535 \times 0.7674 = 0.578$$

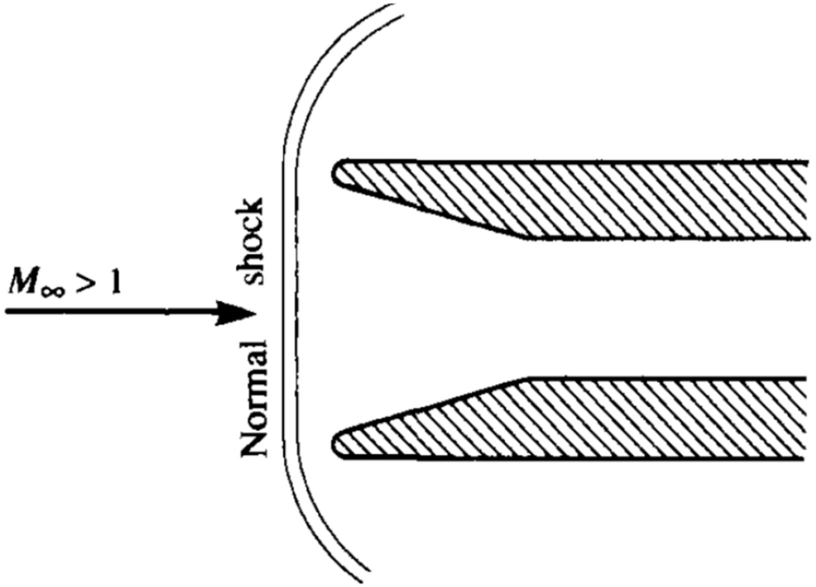


p_0⬆，Δp_0⬇；流动效率⬆

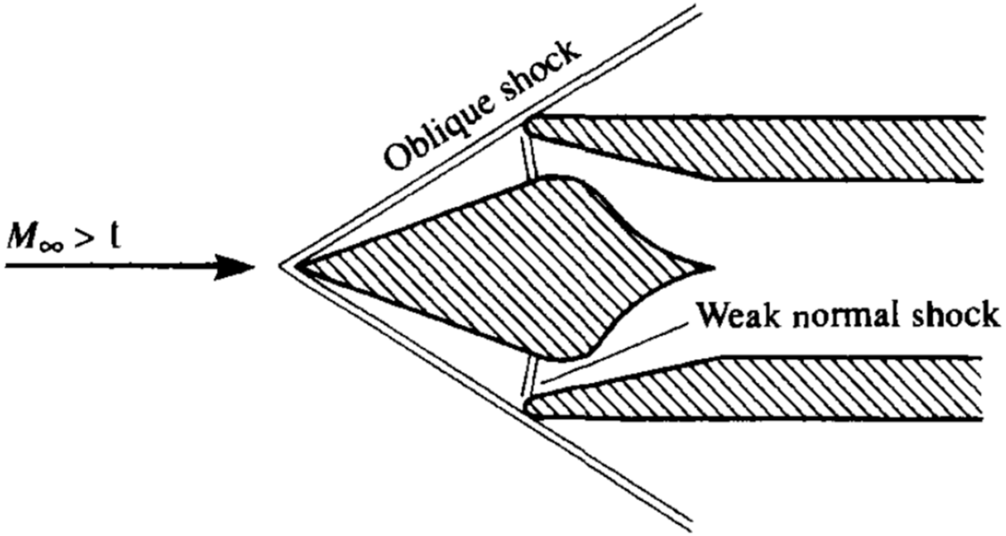
$\boxed{\text{减小正激波前}Ma \rightarrow \Delta p_0\downarrow}$

11.6斜激波(8.3)

减小正激波前$Ma \rightarrow \Delta p_0 \downarrow$



(a) Normal shock inlet



(b) Oblique shock inlet

11.7激波的相交与反射

设计分析超声速飞行器、发动机、风洞时需注意！

1960s，X-15发动机飞行实验$Ma = 4\sim7$，发动机激波烧坏机身。

1.反射：

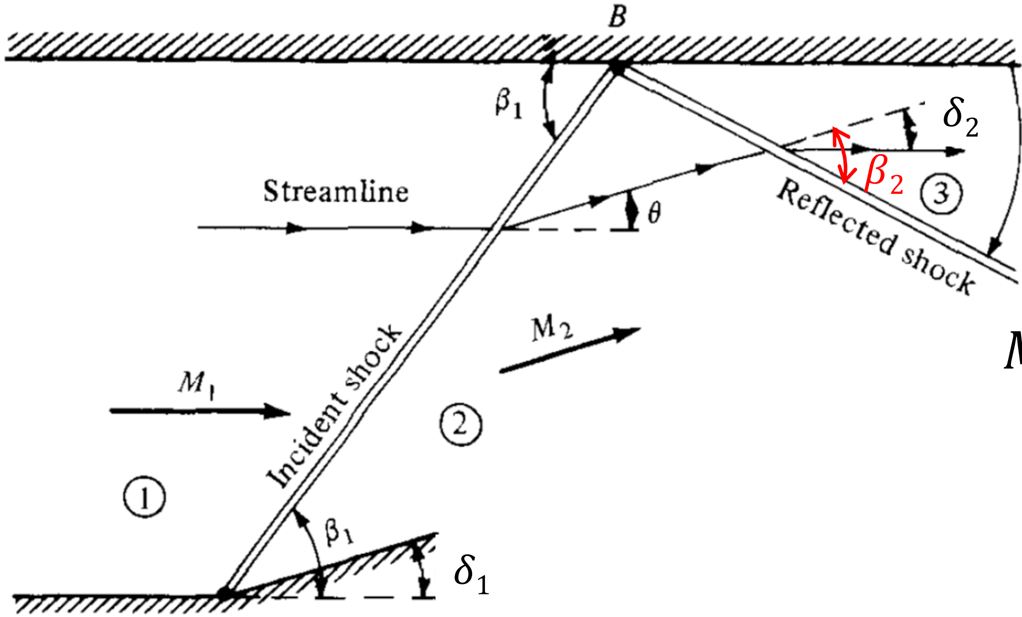


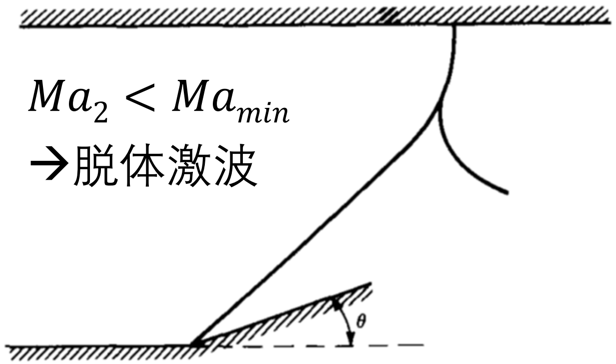
Figure 9.17 Regular reflection of a shock wave from a solid boundary.

$$Ma_2 < Ma_1$$

$$\delta_2 = \delta_1$$

$$\rightarrow \quad \beta_2 > \beta_1$$

$$Ma_1 + \delta \rightarrow Ma_2 + \delta \rightarrow Ma_2, \ \beta_2$$



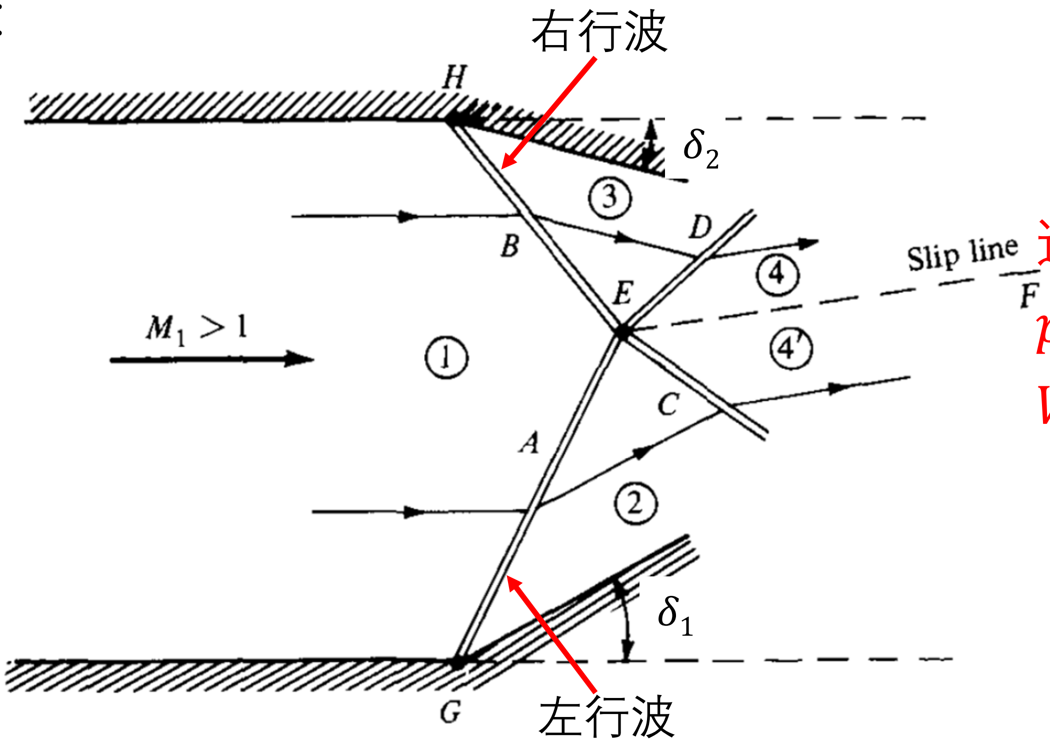
$Ma_2 < Ma_{min}$

→脱体激波

Figure 9.18 Mach reflectian.

11.7激波的相交与反射

2.异侧相交：



速度方向相同

$p_4 = p_4{}',$

$V_4 \neq V_4{}', s_4 \neq s_4{}'$

Figure 9.19 Intersection of right- and left-running shock waves.

11.7激波的相交与反射

2.同侧相交：

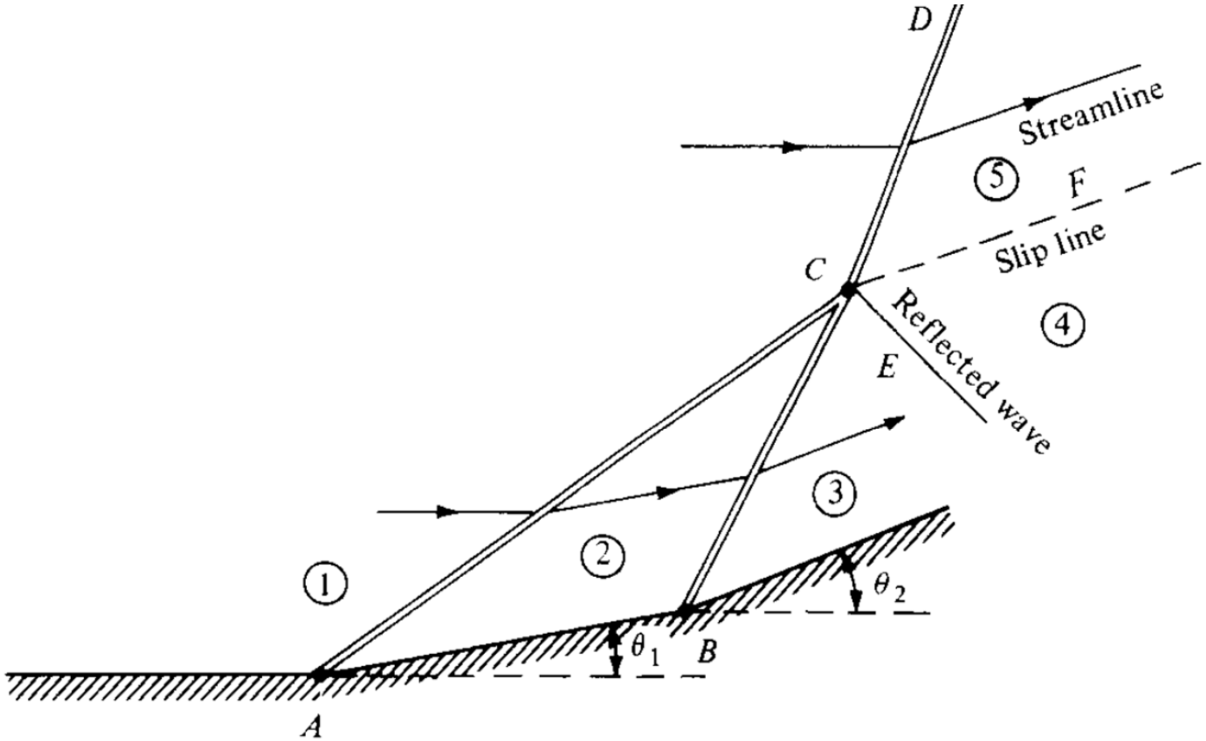


Figure 9.20 Intersection of two left-running shock waves.

11.7激波的相交与反射(8.5)

例：如图$Ma_1 = 3.6$，$\delta = 10°$。

求：ϕ, p_3, T_3, Ma_3

解：① $Ma_1 = 3.6$，$\delta = 10°$ → $\beta_1 = 24°$

$$Ma_{1n} = Ma_1 \sin\beta_1 = 1.464$$

附表B：$Ma_{2n} = 0.7157,$

$$\frac{p_2}{p_1} = 2.32\ , \frac{T_2}{T_1} = 1.294$$

$$\rightarrow Ma_2 = \frac{Ma_{2n}}{\sin(\beta-\delta)} = \frac{0.7157}{\sin(24-10)} = 2.96$$

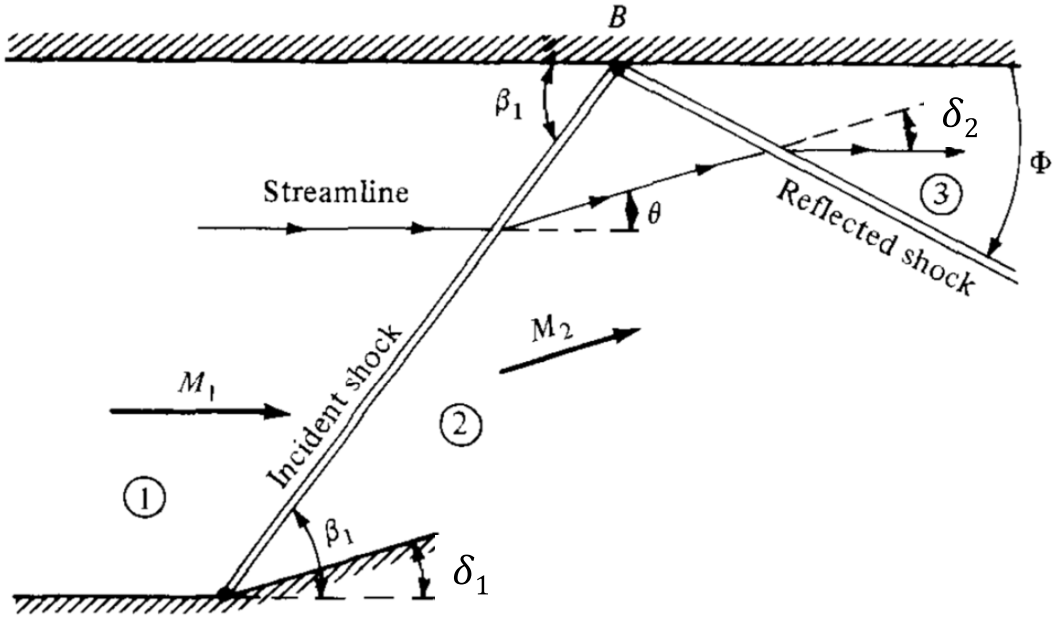


Figure 9.17 Regular reflection of a shock wave from a solid boundary.

11.7激波的相交与反射

例：如图$Ma_1 = 3.6$，$\delta = 10°$。

求：ϕ, p_3, T_3, Ma_3

解：②$Ma_2 = 2.96$，$\delta = 10° \rightarrow \beta_2 = 27.3°$

$$\phi = \beta_2 - \delta = 17.3°$$

$$Ma_{2n} = Ma_2 \sin\beta_2 = 1.357$$

附表B：$Ma_{3n} = 0.7572,$

$$\frac{p_3}{p_2} = 1.991\,, \frac{T_3}{T_2} = 1.229$$

$$\rightarrow Ma_3 = \frac{Ma_{3n}}{\sin(\beta_2 - \delta)} = 2.55$$

$$p_3 = \frac{p_3}{p_2}\frac{p_2}{p_1}p_1 = \cdots, \qquad T_3 = \frac{T_3}{T_2}\frac{T_2}{T_1}T_1 = \cdots$$

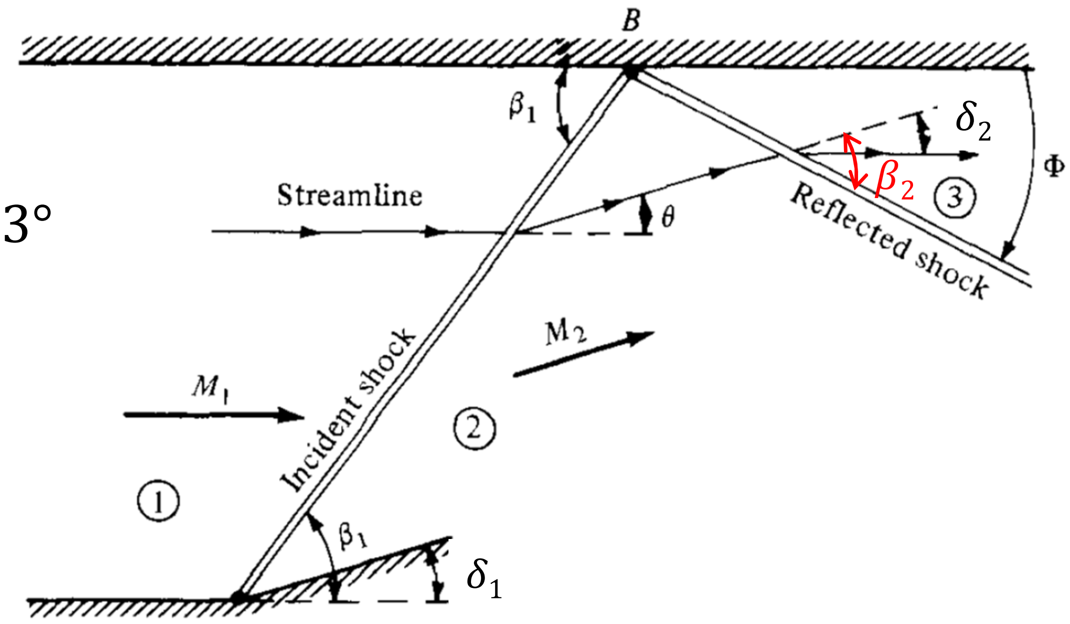


Figure 9.17 Regular reflection of a shock wave from a solid boundary.

11.7激波的相交与反射

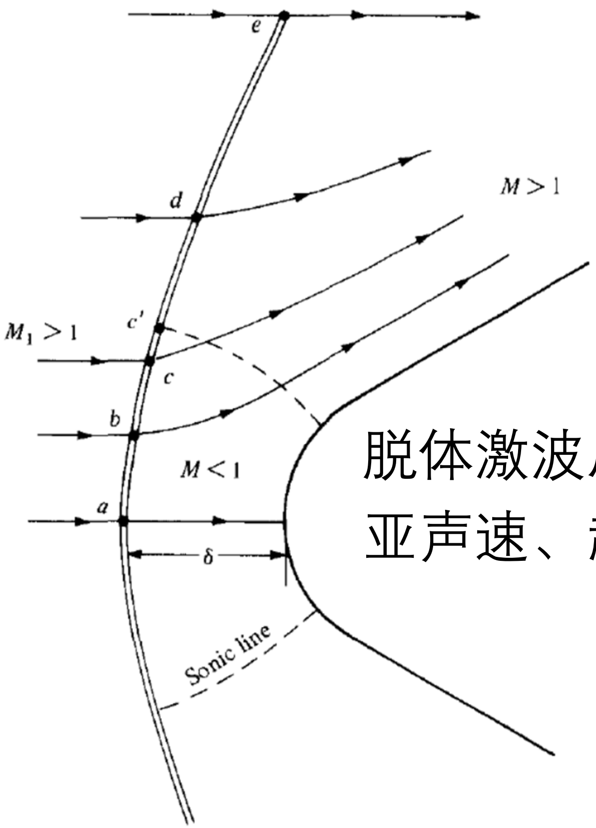
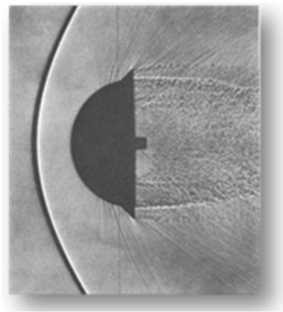
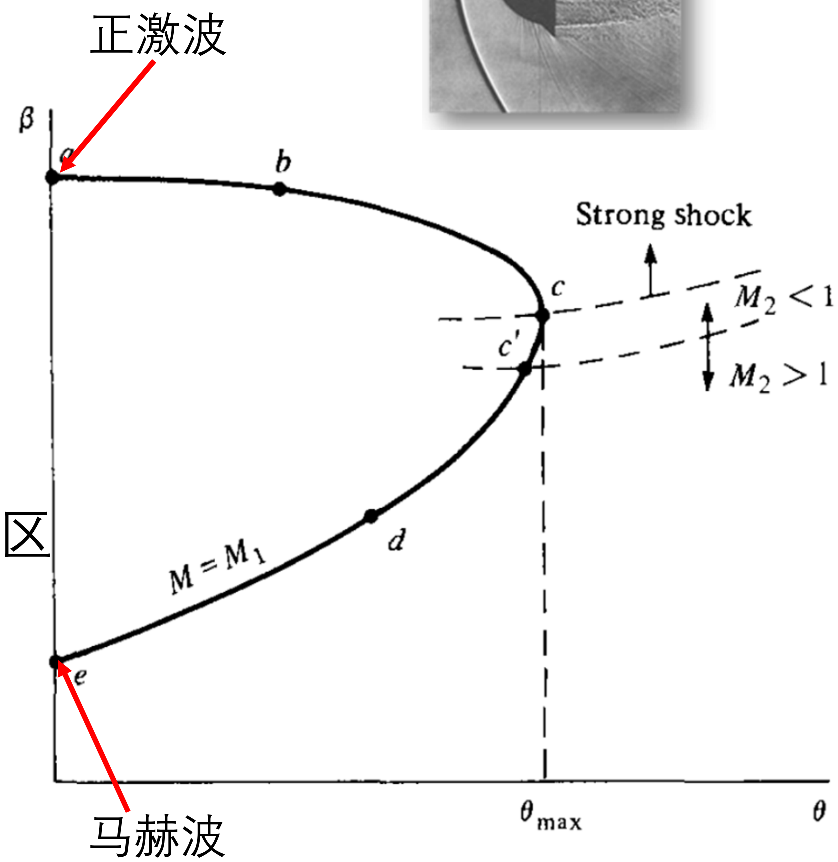


Figure 9.21 Flow over a supersonic blunt body.

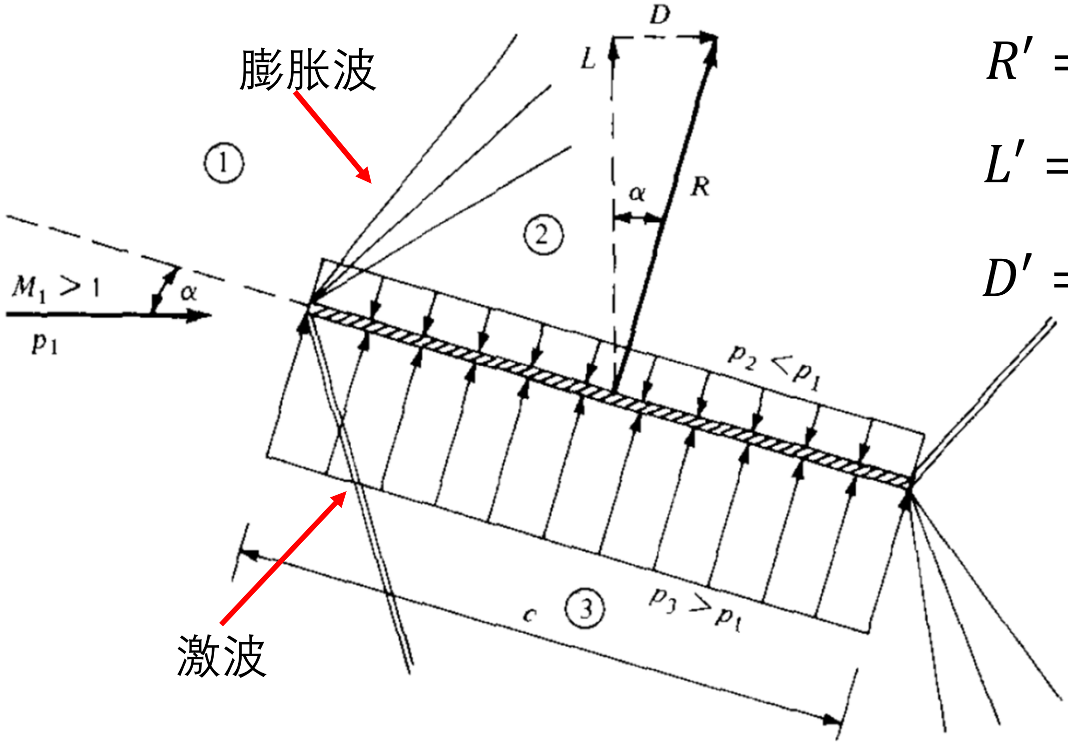
脱体激波后：

亚声速、超声速混合区





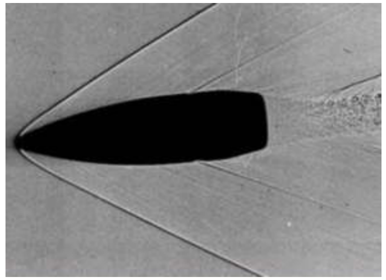
11.8激波-膨胀波应用

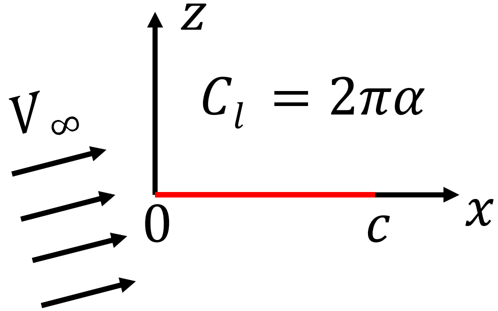


$$R' = (p_3 - p_2)c$$

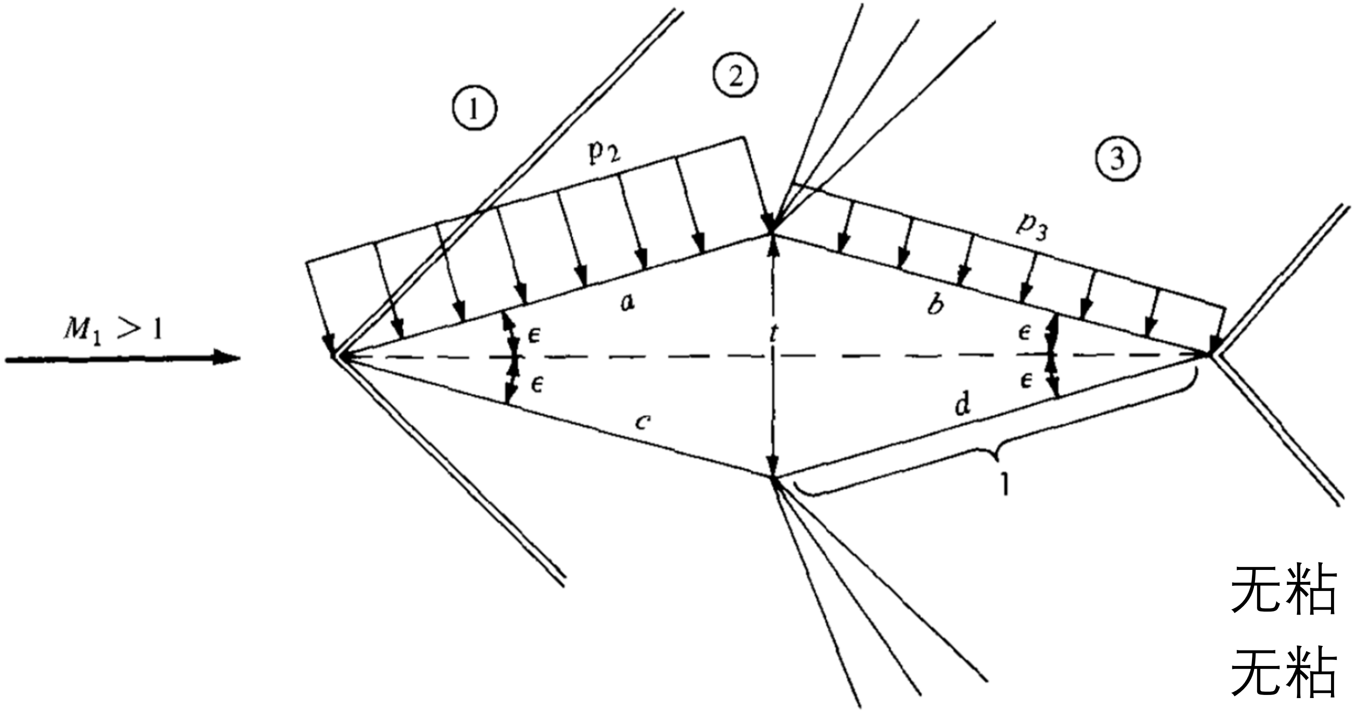
$$L' = (p_3 - p_2)c\cos\alpha$$

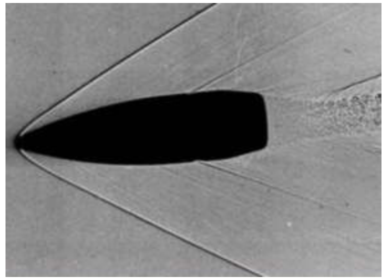
$$D' = (p_3 - p_2)c\sin\alpha$$





11.8激波-膨胀波应用





无粘：$Ma < 1$时$D = 0$

无粘：$Ma > 1$时$D > 0$——波阻！

$$D' = 2(p_2 l - p_3 l)sin\varepsilon$$

$$D' = 2(p_2 - p_3)lsin\varepsilon$$

$$D' = (p_2 - p_3)t$$

作业:

复习笔记!

空气动力学书8.2，8.4，8.7，8.13(2)