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Taylor's Rule estimation by OLS (1990 – 2020)

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Taylor's Rule

The Taylor rule is is an interest rate forecasting model proposed in 1992 by the american economist John B. Taylor.
It suggests that the Federal Reserve should raise rates when inflation is above target or when GDP is above potential (too fast GDP growth).
It also suggests that the Fed should lower rates when inflation is below the target level or when GDP is below potential (too slow GDP growth).

The analytical expression of the Taylor rule is the following:

$$i = r^* + \pi + \beta_1(\pi - \pi^*) + \beta_2 Y\%$$

- π^* Target Inflation
- r^* Natural Real Interest Rate
- $Y\%$ Output Gap (% of Potential Output)
- i Nominal Interest Rate
- π Rate of Inflation

According to this rule, the *nominal interest rate* is linearly correlated to the *inflation gap,* and the *output gap* expressed as percentage of the potential output, with intercept being the sum of the *inflation rate* and the *natural real interest rate*.

Assumptions & Data

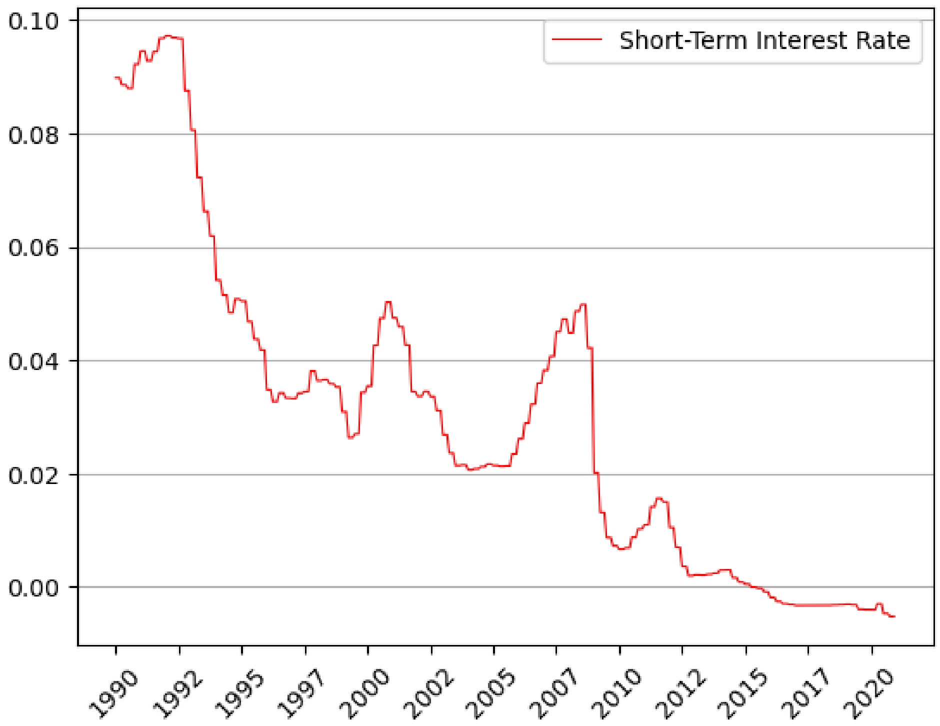
- We gathered monthly data relative to Austria's economy, from Jan-1990 to Dec-2020;
- The data relative to the *short term interest rate*, *nominal GDP*, *potential output*, *rate of inflation* and *unemployment rate* has been retrieved from OECD.Stats ;
- The data relative to the *residential property price index* (*RPPI*) has been retrieved from OENB.at ;
- All the data processing, included the interpolation, have been done using Python (the code is provided in the notebook);
- Austria's Inflation Target is set by the European Central Bank (ECB), as Austria is a member of the Eurozone and uses the euro as its currency. The ECB's primary objective is to maintain price stability, defined as a year-on-year increase in the Harmonised Index of Consumer Prices (HICP) for the euro area of 2% over the medium term. This means that the ECB aims to keep the annual inflation rate in Austria and in the other Eurozone countries at around 2%. There are several reasons why a 2% inflation target is considered appropriate for the Eurozone. One is that a low and stable inflation rate helps to provide price stability, which is important for consumers and businesses to make long-term decisions and investments. A moderate level of inflation also helps to avoid the risk of deflation, which can lead to a spiral of falling prices and economic stagnation. Another reason is that a 2% inflation target is believed to provide sufficient flexibility for central banks to respond to economic shocks and fluctuations. It is seen, therefore, as a balance between the benefits of price stability and the need for flexibility in monetary policy. To keep the annual inflation rate at around 2%, the Oesterreichische Nationalbank (OeNB) uses a range of monetary policy instruments, such as adjusting interest rates, providing liquidity to the banking system, and buying or selling government bonds. Target rates may vary in response to changing economic conditions and circumstances; the ECB's monetary policy stance and inflation target have been adjusted as needed to support economic growth and maintain price stability in the Eurozone, for example during the COVID-19 pandemic. Nevertheless Austria's inflation target has not changed significantly over time, therefore we assumed it to be constant in our model.

Interest Rate

On OECD.Stats, we found quarterly time series for the Short Term Interest Rate and decided to do interpolation by assuming constant values in each quarter to obtain monthly values (Jan = Feb = Mar = Q1)

	year	quarter	month	i
0	1990	1	1	0.089833
1	1990	1	2	0.089833
2	1990	1	3	0.089833
3	1990	2	4	0.088600
4	1990	2	5	0.088600
...	...	...	...	...
367	2020	3	8	-0.004717
368	2020	3	9	-0.004717
369	2020	4	10	-0.005227
370	2020	4	11	-0.005227
371	2020	4	12	-0.005227

372 rows × 4 columns



The graph clearly shows a downward sloping trend for the interest rate during the 31 years we considered.

However, notice the spikes in the early 2000s and during the economic crisis of 2008.

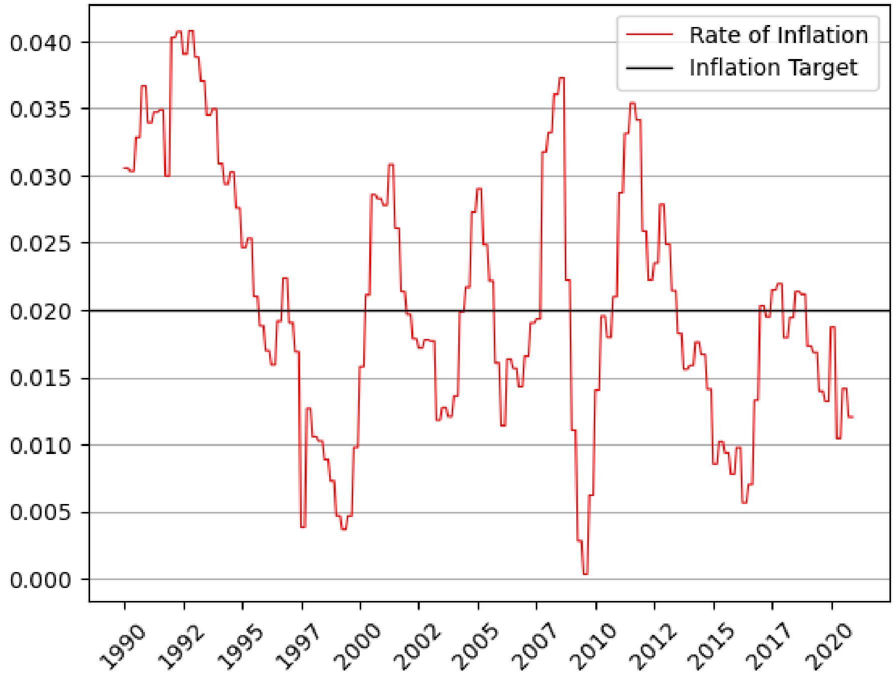
Inflation Gap

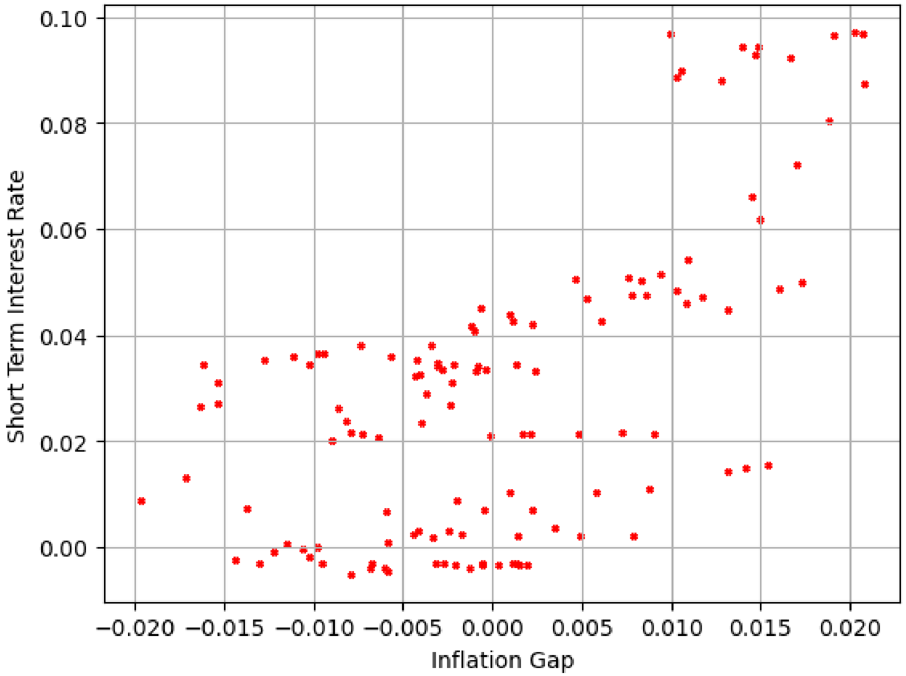
On OECD.Stats, we found quarterly time series for the Rate of Inflation (measure of inflation as change in percentage from previous year) and decided to do interpolation by assuming constant values in each quarter to obtain monthly values.

To compute the Inflation Gap we assumed the Inflation Target set by the EU, that is 2%, to be constant throughout the whole time series.

	year	quarter	month	pi
0	1990	1	1	0.030558
1	1990	1	2	0.030558
2	1990	1	3	0.030558
3	1990	2	4	0.030335
4	1990	2	5	0.030335
...	...	...	...	...
367	2020	3	8	0.014136
368	2020	3	9	0.014136
369	2020	4	10	0.012039
370	2020	4	11	0.012039
371	2020	4	12	0.012039

372 rows × 4 columns





The line plot shows how the Rate of Inflation in Austria has been oscillating around the target 2% almost regularly from 1990 to 2020, reaching almost deflation right after the economic crisis.

The scatter plot suggests a strong linear relationship between the Inflation Rate and the Interest Rate.

Output Gap

On OECD.Stats, we found:

- Monthly time series for the GPD (Y);
- Quarterly time series for the Potential Output (Y*), which we interpolate to obtain monthly data by assuming it to be constant during each quarter. (Jan = Feb = Mar = 1/3 Q1)

We then computed the Output Gap as percentage of Potential Output.

	year	quarter	month	y	ystar	outputGap
0	1990	1	1	1.117131e+10	1.125555e+10	-0.007484
1	1990	1	2	1.117131e+10	1.125555e+10	-0.007484
2	1990	1	3	1.117131e+10	1.125555e+10	-0.007484
3	1990	2	4	1.136493e+10	1.125555e+10	0.009718
4	1990	2	5	1.136493e+10	1.125555e+10	0.009718
...	...	...	...	...	...	...
367	2020	3	8	3.176626e+10	3.295882e+10	-0.036183
368	2020	3	9	3.176626e+10	3.295882e+10	-0.036183
369	2020	4	10	2.997283e+10	3.295882e+10	-0.090598
370	2020	4	11	2.997283e+10	3.295882e+10	-0.090598
371	2020	4	12	2.997283e+10	3.295882e+10	-0.090598

372 rows × 6 columns

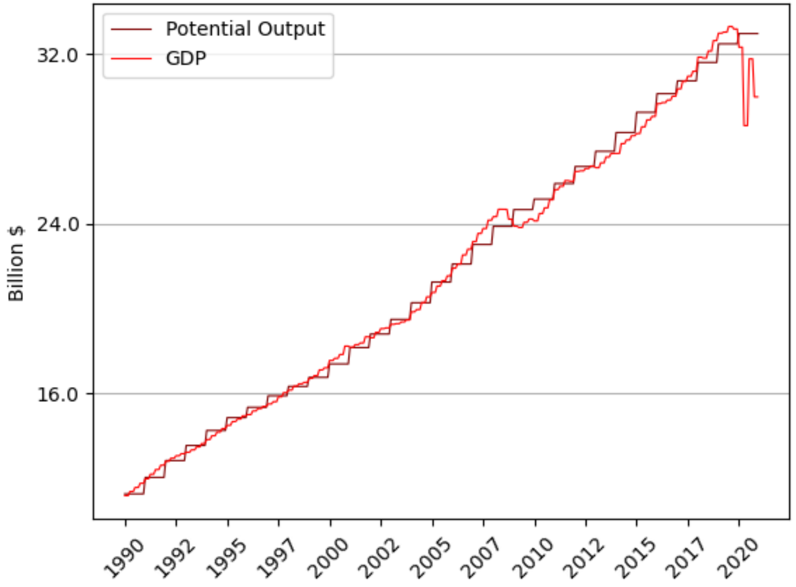
To compute the Output Gap as percentage of Potential Output it is possible to use the following different formulas, however they come up with the same result and so we chose to use the first one as it is more intuitive.

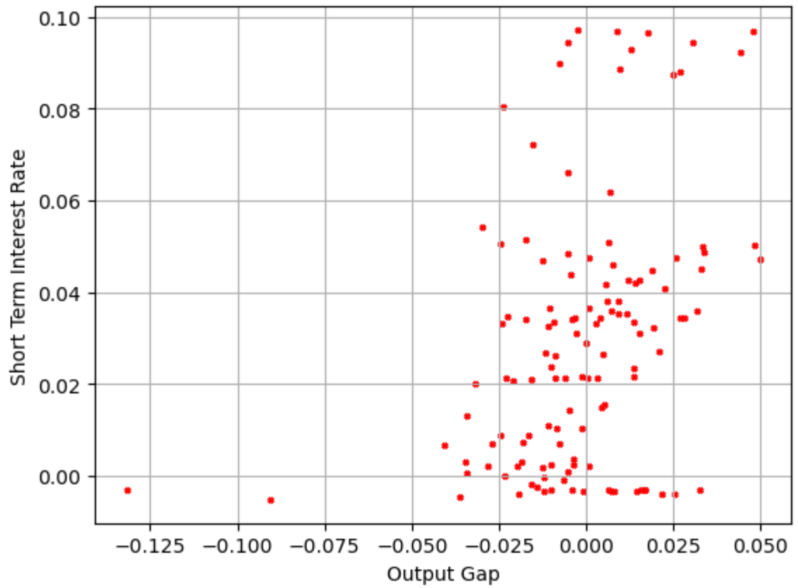
The first plot shows how overall the potential and actual output stayed close to each other during the years we observed.

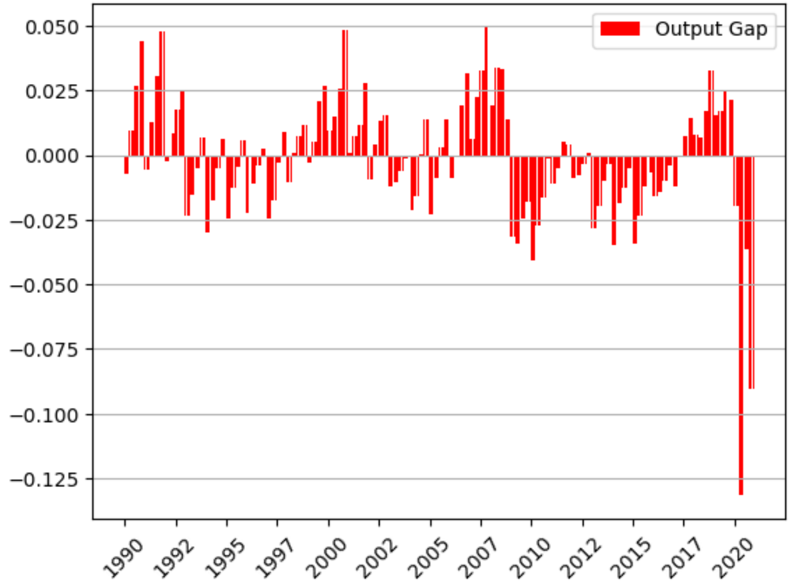
Surpisingly, the scatterplot suggests quite a non perfect linear relationship between the output gap and the interest rate which potentially contradicts the Taylor's Rule.

$$Y\% = \frac{Y - Y^*}{Y^*} = ln(Y) - ln(Y^*)$$

$$using: ln(x) = x - 1 \; as \; x \to 1$$







Unemployment Rate (alternative model)

On OECD.Stats, we found quarterly time series for the Unemployment Rate from 1993 to 2020 and decided to do interpolation by assuming constant values in each quarter to obtain monthly values.

Whereas for the 1990-1992 period we found only yearly time seris and decided to do interpolation by assuming constant values in each year to determine monthly values.

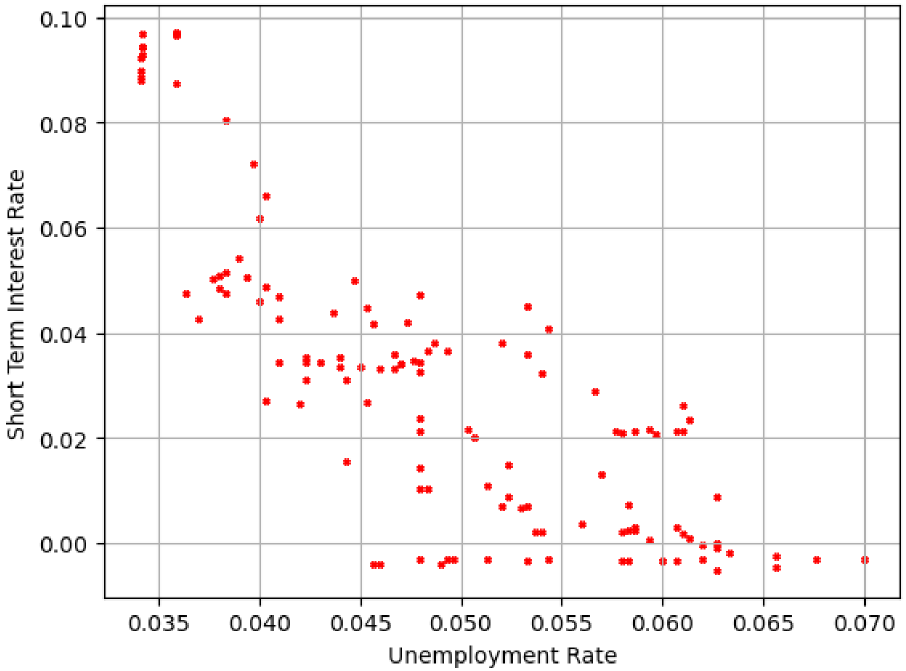
	year	quarter	month	u
0	1990	1	1	0.034100
1	1990	1	2	0.034100
2	1990	1	3	0.034100
3	1990	2	4	0.034100
4	1990	2	5	0.034100
...	...	...	...	...
367	2020	3	8	0.065667
368	2020	3	9	0.065667
369	2020	4	10	0.062667
370	2020	4	11	0.062667
371	2020	4	12	0.062667

372 rows × 4 columns

The Unemployment Rate shows an overall upward trend, even though deeply volatile.

The scatter plot suggest a negative linear correlation with the Interest Rate, although a non linear relation, such as negative exponential, may even fit better.

The negative correlation can be explained by the fact that lower interest rates lead to more capital injected in the economy, which means higher prices, which implies higher inflation and a decrease in real wages, therefore higher unemployment.
Conversely, higher interest rates lead to a shrinkage of the economy, which drives prices down, real wages up and therefore more employment.



Residential Property Price Index (alternative model)

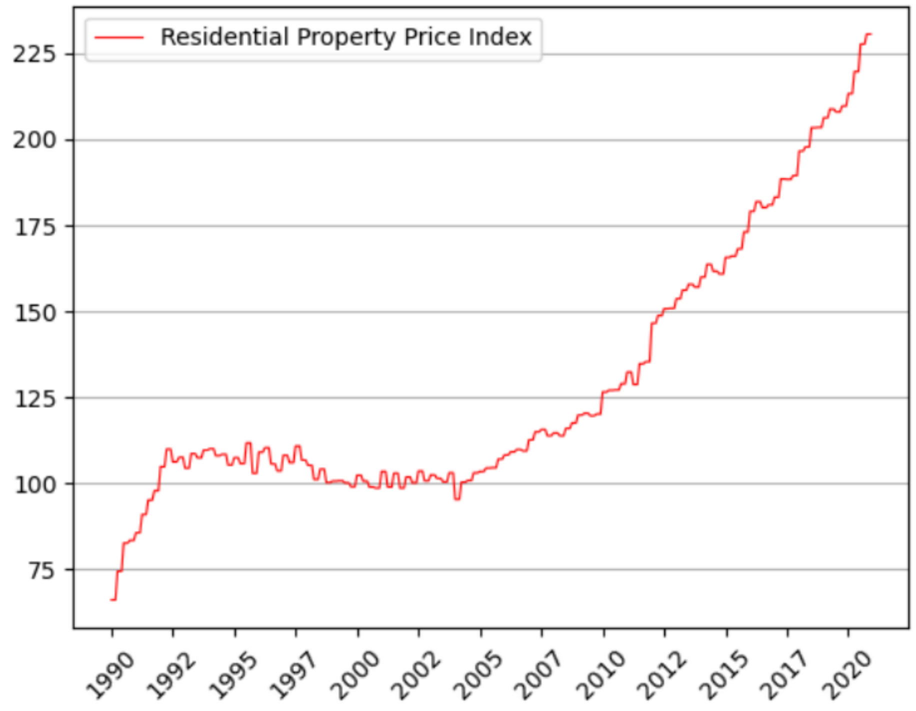
From the Austrian National Bank we retrieved the quarterly time series for the overall Austrian RPPI from 2000 to 2020.

The years from 1990 to 1999 were covered approximating the Austrian RPPI using the Vienna's overall RPPI, which was available in quarterly time series.

We then interpolated assuming the index to be constant during each quarter to obtain monthly data.

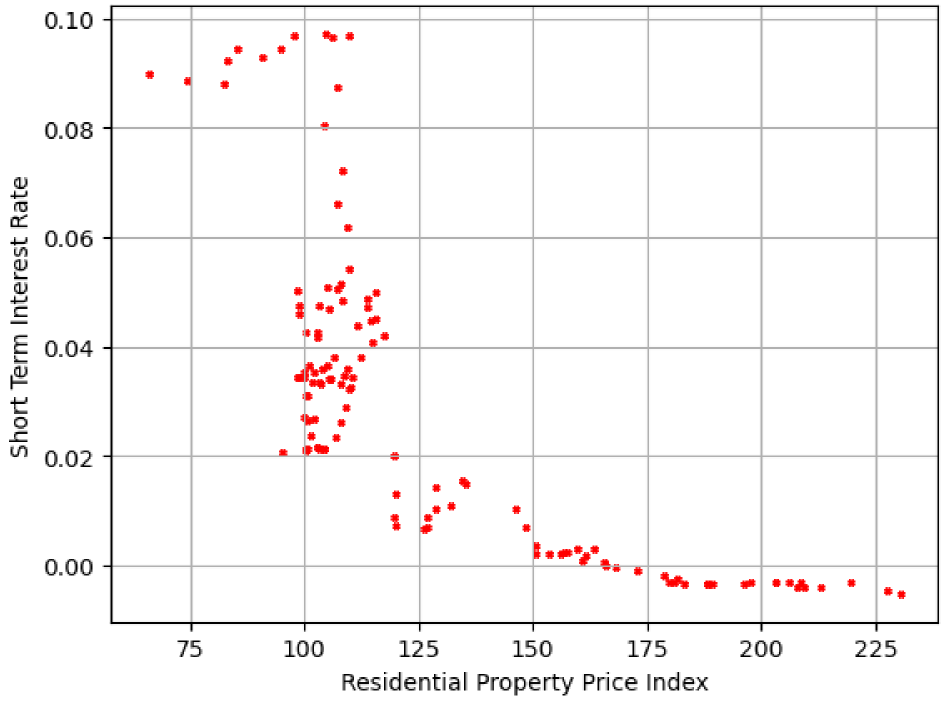
	year	quarter	month	rppi
0	1990	1	1	66.0
1	1990	1	2	66.0
2	1990	1	3	66.0
3	1990	2	4	74.3
4	1990	2	5	74.3
...	...	...	...	...
367	2020	3	8	227.5
368	2020	3	9	227.5
369	2020	4	10	230.4
370	2020	4	11	230.4
371	2020	4	12	230.4

372 rows × 4 columns



The RPPI shows an overall uptrend. In particular, it spiked in the early '90s, stabilizing just above 100 for a decade. In the early 2000s it started increasing quite steeply and has been doing so since then.

The scatter hints at a negative correlation with the interest rate, which is intuitively explained by the fact that lower interest rates make people more interested in different investement opportunities, such as residential properties, driving the prices up, while higher interest rates move people away from the housing to the bond market.



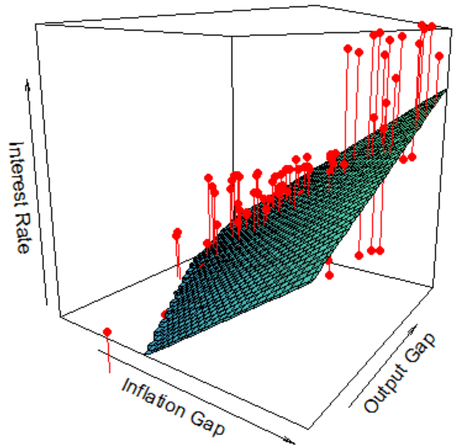
Linear Regression by OLS

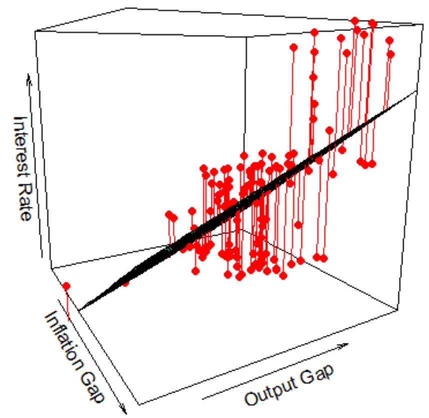
The estimators we found are statistically significant as both the T-Tests and the F-Test show low p-values.

However the p-value related to the estimator of the Output Gap coefficient is much bigger the the one associated to the Inflation Gap, hinting at the stronger correlation between the latter, which the scatter plot showed as well.

Although the coefficients are not the ones theorized by Taylor, specifically 0.5 for both apha and beta, the results do not seem unreasonable and seem to hint at a stronger response by the Austrian Banks to unitary variations in inflation rates compared to the weaker response to deviations in percentage output gap.

```
Call:
lm(formula = i ~ inflationGap + outputGap, data = df)

Residuals:
      Min        1Q    Median        3Q       Max
-0.041281 -0.016726  0.004032  0.014294  0.044697

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.028688   0.001121  25.602  < 2e-16 ***
inflationGap 1.719920   0.123648  13.910  < 2e-16 ***
outputGap    0.228375   0.049294   4.633 5.01e-06 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' '

Residual standard error: 0.02149 on 369 degrees of freedom
Multiple R-squared:  0.4362,	Adjusted R-squared:  0.4331
F-statistic: 142.7 on 2 and 369 DF,  p-value: < 2.2e-16
```





Diagnostic Checks

In order to test the assumptions of the OLS model, we ran:

- Ramsey-Reset test for Linearity;
- Durbin-Watson test for Correlation;
- Breusch-Pagan test for Homoskedasticity;
- Jarque-Bera test for Normality.

```
> ## Linearity: Ramsey (Reset)
> resettest(lm)

	RESET test

data:  lm
RESET = 33.403, df1 = 2, df2 = 367, p-value = 4.703e-14
```

Given the low p-value, this test actually rejects the model's linearity, we therefore decided to run an alternative regression, using the square of the Inflation Gap.

```
> ## Autocorrelation: Durbin-Watson
> dwtest(lm)

	Durbin-Watson test

data:  lm
DW = 0.098975, p-value < 2.2e-16
alternative hypothesis: true autocorrelation is greater than 0
```

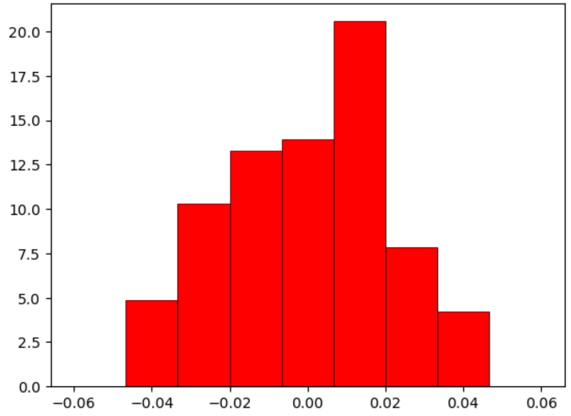
Similarly, the low p-value of the DW test suggests autocorrelation among the errors, and this may come from the strong assumption we made when interpolating the quarterly data to obtain monthly values, hence having exactly equal values for triplets of months.

```
> ## Homoscedasticity: Breusch-Pagan, White, Goldfeld-Quandt
> bptest(lm)

	studentized Breusch-Pagan test

data:  lm
BP = 39.632, df = 2, p-value = 2.478e-09
```

The low p-value of the BP test suggests that there may be heteroskedasticity among the errors.

```
> jarque.bera.test(resid(lm))

	Jarque Bera Test

data:  resid(lm)
X-squared = 11.281, df = 2, p-value = 0.003551
```

This test shows that we can reject the Normality assumption only up to 99.5% confidence.
In fact, at 0.001 significance we cannot state that the errors are not normal, and in fact the histogram of the residuals resembles a Gaussian distribution.



Alternative Model 1: Squared Inflation Gap

The reset test, as previously discussed, returned a negative result. Therefore we tried adding the square of the Inflation Gap, and tested this new model.

```
Call:
lm(formula = i ~ inflationGap + outputGap + sqrinflationGap,
    data = df)

Residuals:
      Min        1Q    Median        3Q       Max
-0.049717 -0.014897  0.003174  0.013448  0.046045

Coefficients:
                Estimate Std. Error t value Pr(>|t|)
(Intercept)      0.02032    0.00132  15.390  < 2e-16 ***
inflationGap     1.39105    0.11553  12.040  < 2e-16 ***
outputGap        0.23231    0.04404   5.275 2.27e-07 ***
sqrinflationGap 94.37059    9.71453   9.714  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.0192 on 368 degrees of freedom
Multiple R-squared:  0.5513,    Adjusted R-squared:  0.5476
F-statistic: 150.7 on 3 and 368 DF,  p-value: < 2.2e-16
```

```
> ## Linearity: Ramsey (Reset)
> resettest(lm)

        RESET test

data:  lm
RESET = 1.1655, df1 = 2, df2 = 366, p-value = 0.3129
```

This new model with a square explanatory variable is now linear according to the Ramsey test. We can conclude that there may be preferrable non-linear models than the Taylor's Rule to forecast the Interest Rate in Austria.

Alternative Model 2: Unemployment Rate & RPPI

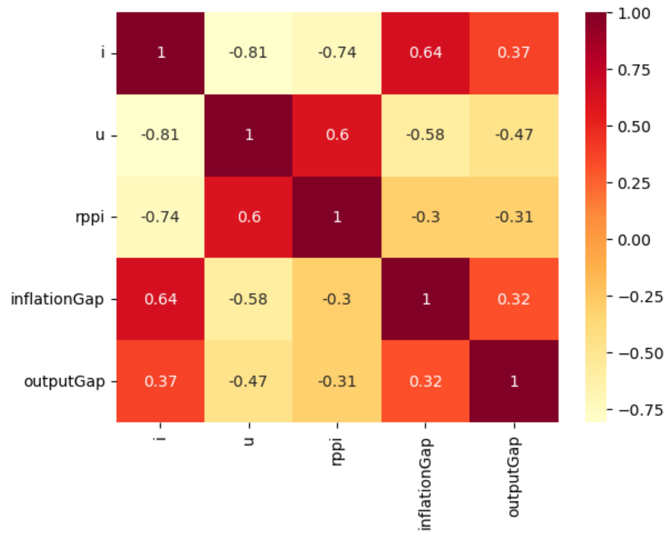
Even though our sqrinflationGap model suggests that the relationship between the variables we tested is not linear and a second degree model may actually be better, we wondered whether the Interest Rate could be linearly correlated to other economic variables.
There is a moltitude of macroeconomics variables that could increase the explanatory power of our model and indeed we found two interesting variables that correlated well with the interest rate

In particular, we decided to test the Unemployment Rate and the Residential Property Price Index, a measure for the houses' price in Austria.

```
Call:
lm(formula = i ~ inflationGap + outputGap + rppi + u, data = df)

Residuals:
      Min        1Q    Median        3Q       Max 
-0.030306 -0.008493 -0.001935  0.007977  0.032636 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept)  1.331e-01  4.654e-03  28.604   <2e-16 ***
inflationGap 8.487e-01  8.401e-02  10.102   <2e-16 ***
outputGap   -4.693e-02  3.079e-02  -1.525    0.128    
rppi        -3.184e-04  2.154e-05 -14.778   <2e-16 ***
u           -1.290e+00  1.085e-01 -11.892   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.01249 on 367 degrees of freedom
Multiple R-squared:  0.8106,	Adjusted R-squared:  0.8085 
F-statistic: 392.7 on 4 and 367 DF,  p-value: < 2.2e-16
```

```
> ## Linearity: Ramsey (Reset)
> resettest(lm)

	RESET test

data:  lm
RESET = 246.67, df1 = 2, df2 = 365, p-value < 2.2e-16
```

From the results of the test we conclude that there is a strong correlation between all the variables we considered but the outputGap, which becomes negligible when other variables such as the Unemployment Rate or the RPPI that correlate more are included.
This result is visually shown by the correlation matrix heatmap.



Conclusions

- Although the data collection process was not complicated and we managed to find all the data we needed, we still had to do some interpolation to extend the available information.
- We chose an EU country, with a relative stable economy and in fact the behaviour of the explanatory variable was regular and in line with what we expected.
- The OLS Regression's significance tests (t & F) were positive and showed low p-value, statistically supporting the results we obtained.
- Unfortunately, none of the diagnostic tests we carried out returned a positive feedback, suggesting that in our case the OLS assumptions may be too strong to hold true, and in general reality is not always coherent with what we expect from theory.
- We decided to test an alternative model in response to the non-linearity suggested by the RESET test, and we indeed confirmed our doubt, finding that the model is linear when introducing the square of the Inflation Gap.
- Finally, we proceeded to test a model we deemed reasonable, using a well known macroeconomic indicator, the Unemployment Rate, but also a "minor" one, the Residential Property Price Index, and discovered that they both correlate well with the Interest Rate and actually pushed the Output Gap away, making it's contribution to the model statistically insignificant.