# 场论知识梳理

## 数量场

梯度 grad $\xrightarrow{\text{向量}}$ 梯度场

方向导数 $\frac{\partial u}{\partial l}=\frac{\partial u}{\partial x}\cos\alpha+\frac{\partial u}{\partial y}\cos\beta+\frac{\partial u}{\partial z}\cos\gamma$

$\frac{\partial u}{\partial l}=\vec{G}\cdot\vec{l^\circ}$

$\vec{G}=\left(\frac{\partial u}{\partial x},\frac{\partial u}{\partial y},\frac{\partial u}{\partial z}\right)$ $\vec{l^\circ}=(\cos\alpha,\cos\beta,\cos\gamma)$

$\vec{G}$ 的方向为方向导数最大的方向

$|\vec{G}|$ 为该方向导数的大小

du 增量

偏导

$\frac{\partial u}{\partial l}$

* 向量

方向导数 $\vec{G}\cdot\vec{l^\circ}$（单位距离↑/↓）

$\vec{G}$ 梯度 ~ 数量场↑/↓最快 ⟨ 方向 / 极值

$\vec{R}$ 旋度 ~ 向量场 ⟳ 最强 ⟨ 方向 / 极值

环量面密度 $\vec{R}\cdot\vec{n^\circ}$（单位面积旋转）

$\mu_n=\lim\limits_{S\to P}\frac{\Gamma}{S}$

* 标量

曲线积分

$\Gamma$ 环量 $\oint_L\vec{A}\cdot d\vec{s}$

散度 ~ 向量场发出 $\vec{v}$ 的强弱

$\lim\limits_{\Delta V\to 0}\frac{\Phi}{\Delta V}$ （闭合）通量体积密度

（闭合曲面）通量 $\Phi=\oint\vec{A}\cdot d\vec{s}$

曲面积分

## 向量场

通量 +/0/- ~ 有无源 标量

$\Phi=\iint_S\vec{A}\cdot d\vec{s}=\iint_S X\,dydz+Y\,dzdx+Z\,dxdy$

散度 divergence ~ 源强弱

$\lim\limits_{\Delta V\to 0}\frac{\Delta\Phi}{\Delta V}=\lim\frac{\oint_S\vec{A}\cdot d\vec{s}}{\Delta V}$ 标量

$\Downarrow$

$\oint_S\vec{A}\cdot d\vec{s}=\iiint_V(\operatorname{div}\vec{A})\cdot dV$

$\operatorname{div}\vec{A}=\frac{\partial X}{\partial x}+\frac{\partial Y}{\partial y}+\frac{\partial Z}{\partial z}$

环量

$\Gamma=\oint_L\vec{A}\cdot d\vec{s}$ 标量

环量面密度

$\mu_n=\lim\limits_{S\to P}\frac{\oint_L\vec{A}\cdot d\vec{s}}{S}$ 标量

方向 向一点收缩

$\oint_L\vec{A}\cdot d\vec{s}=\oint_L X\,dx+Y\,dy+Z\,dz$

$ds\cos\alpha$ $ds\cos\beta$

$\iint_S\left(\frac{\partial Z}{\partial y}-\frac{\partial Y}{\partial z}\right)dydz+\left(\frac{\partial X}{\partial z}-\frac{\partial Z}{\partial x}\right)dzdx$

$+\left(\frac{\partial Y}{\partial x}-\frac{\partial X}{\partial y}\right)dxdy$ $ds\cos\gamma$

$=\iint\cdots+\left(\frac{\partial Y}{\partial x}-\frac{\partial X}{\partial y}\right)\cos\gamma$

$\therefore\mu_n=\left(\frac{\partial Z}{\partial y}-\frac{\partial Y}{\partial z}\right)\cos\alpha+\left(\frac{\partial X}{\partial z}-\frac{\partial Z}{\partial x}\right)\cos\beta$

* 相关符号 ~ 算子

Hamilton

$$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$

$$\nabla(f) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right) = \overrightarrow{\mathrm{grad} f}$$

向量

$f(x,y,z)$

$$\vec{F} = (X, Y, Z)$$

标量

$$\nabla \cdot \vec{F} = \frac{\partial X}{\partial x} + \frac{\partial Y}{\partial y} + \frac{\partial Z}{\partial z} = \mathrm{div}\vec{F}$$

$$\nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ X & Y & Z \end{vmatrix} = \mathrm{rot}\vec{F}$$

向量

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

旋度 向量

$$\vec{R} = \left(\frac{\partial Z}{\partial y} - \frac{\partial Y}{\partial z}, \frac{\partial X}{\partial z} - \frac{\partial Z}{\partial x}, \frac{\partial Y}{\partial x} - \frac{\partial X}{\partial y}\right)$$

$$\vec{n_0} = (\cos\alpha, \cos\beta, \cos\gamma)$$

$$\mu_n = \vec{R} \cdot \vec{n_0}$$

$\vec{R}$ 方向为 环量面密度最大的方向

该最大值为 $|\vec{R}|$

$$\mathrm{rot}\vec{A} = \vec{R} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ X & Y & Z \end{vmatrix}$$

$$\oint_L \vec{A} \cdot d\vec{s} = \iint_S \mathrm{rot}\vec{A} \cdot d\vec{S}$$

$$\mathrm{rot}\vec{A} = 0$$

$$\Leftrightarrow \left\{ \begin{aligned} \frac{\partial Z}{\partial y} &= \frac{\partial Y}{\partial z} \\ \frac{\partial X}{\partial y} &= \frac{\partial Z}{\partial x} \\ \frac{\partial Y}{\partial x} &= \frac{\partial X}{\partial y} \end{aligned} \right\} \begin{aligned} &\Leftrightarrow \oint_L Xdx+Ydy+Zdz= \\ &\Leftrightarrow \text{全微分存在} \\ &\Leftrightarrow \text{梯度存在} \end{aligned}$$

有势(grad)场 $\Leftrightarrow$ 无旋场

调和场 $\Leftrightarrow$ 无源无旋场

$$\mathrm{div}(\mathrm{grad}\, u) = 0$$

$$\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{\partial u}{\partial z}\right) = 0$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$$