二元一次方程组．与 解析几何直线的方程．与二阶矩阵

$$\begin{cases} ax+by=e \\ cx+dy=f \end{cases}$$

方程的形式．

解析几何

$l_1: ax+by=e.$

$l_2: cx+dy=f.$ } 与上式等价．

二阶矩阵．

$A=\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ $\vec{x}=\begin{pmatrix} x \\ y \end{pmatrix}$ $\vec{Y}=\begin{pmatrix} e \\ f \end{pmatrix}$

$Y=Ax$ 与上式等价．

唯一解．

l_1 与 l_2 相交．

$\vec{n_1}$ 与 $\vec{n_2}$ 不平行．

$\vec{n_1}=(a,b)=\begin{pmatrix} a \\ b \end{pmatrix}.$

$\vec{n_2}=(c,d)=\begin{pmatrix} c \\ d \end{pmatrix}.$

$ad\neq bc$ $P(x_0,y_0)=l_1\cap l_2.$

A 可逆 $\Rightarrow A^{-1}$ 存在．

$\det(A)=ad-bc\neq 0.$

$\vec{x_0}=A^{-1}\vec{Y}$

无数解．

l_1 与 l_2 重合．

$\vec{n_1}\parallel\vec{n_2}$ 且．$\frac{a}{c}=\frac{b}{d}=\frac{e}{f}.$

A 不可逆（降维）．

$\det(A)=0$．且．$\vec{Y}$ 在 A 值域内．

值域 l: $y=\left(\frac{cx+dy}{ax+by}\right)x=\frac{c}{a}x=\frac{d}{b}x$

$\therefore \frac{f}{e}=\frac{c}{a}=\frac{d}{b}.$

无解．

$l_1\parallel l_2$

$\vec{n_1}\parallel\vec{n_2}$ 且 $\frac{a}{c}=\frac{b}{d}\neq\frac{e}{f}.$

A 不可逆 $\det(A)=0.$

且 $\vec{Y}$ 在 A 值域外．

$\frac{f}{e}\neq k_0=\frac{cx+dy}{ax+by}=\frac{c}{a}=\frac{d}{b}.$

Matrix and Transformation

* Part 1. Kownledge.

Transformation $\vec{Y} = A\vec{X}$ i.e. $\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$

Matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

* internal characteristics

*external characteristics

(the relationship between image and preimage)

Δcharacteristic root: $\lambda_1\ \lambda_2$.

Δcharacteristic vector.

Δcharacteristic polynomial

keep direction while ~~transforming~~

invariant during similar transformation.

TAT^{-1}.

$$\det\begin{pmatrix} \lambda-a & -b \\ -c & \lambda-d \end{pmatrix} = 0$$

the equation to solve $\lambda_1\ \lambda_2$.

* Part 2. Application.

(1) To solve a system of equations.

$$X = A^{-1}Y.$$

$$\Uparrow$$

$$\begin{cases} ax+by=e \\ cx+dy=f \end{cases} \Leftrightarrow \underbrace{\begin{pmatrix} e \\ f \end{pmatrix}}_{Y} = \underbrace{\begin{pmatrix} a & b \\ c & d \end{pmatrix}}_{A}\underbrace{\begin{pmatrix} x \\ y \end{pmatrix}}_{X} \Leftrightarrow Y = AX.$$

$$A^{-1} = \frac{1}{\det(A)}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}.$$

* $\det(A) = 0 \Rightarrow \begin{cases} \text{countless solutions.} \\ \text{no solution.} \end{cases}$

(2). To calculate $A^n\vec{\alpha}$.

$$A \to \begin{cases} \lambda_1 \longrightarrow \vec{\xi_1} \\ \lambda_2 \longrightarrow \vec{\xi_2} \end{cases};\ \vec{\alpha} = s\vec{\xi_1} + t\vec{\xi_2};\ A^n\vec{\alpha} = \lambda_1^n s\vec{\xi_1} + \lambda_2^n t\vec{\xi_2}$$

*. $\lambda_0 = 0 \Leftrightarrow \det(A) = 0$.