Template

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September 27, 2024

Abstract

pass now.

Keywords:

1 Theory

1.1 Meaning of Symbols

To emphasis the time dependence of some variables, I add subscript to them, which is slightly different from the original article.

- C: number of similar **cultures** in our experiment.
- t: time in units of division cycles of the yeasts.
  - thus the growth rate should be exactly 1, both for sensitive yeasts and resistant yeasts.
  - t_0: a certain time, prior to which mutations were not likely to occur in any experimental **cultures**. Specifically speaking the t_0 satisfies:

$$1 = \mathrm{Ex}(Cm_{t_0})) = \mathrm{Ex}(aC(N_{t_0} - N_0))$$

- N_t, *observable*: number of yeasts in **a growing culture** at time t.
  - $N_t - N_0$: increase in the number of yeasts.
  - $N_0 \ll N_t$, thus, $N_0 \approx 0$ in the presence of N_t.
- ρ_t: the **average** number of resistant yeasts present at time t, in one **culture**.
- r_t, *observable*: the **likely average** number of resistant yeasts in a **culture** at time t, gotten from **limited number of samples**.
- a: mutation rate, namely, the chance of mutation per yeast per time unit.
- m_t: the **average** number of mutations at time t, in **a culture**.
- p_0: the fraction of cultures showing **no** mutation in a large series of similar cultures.

1.2 Derivation of the Formulas

1.2.1 Acquired Hereditary Immunity Hypothesis

Assumption: a fixed small chance for each yeast to survive an attack.

$$\begin{aligned}\text{Number of resistant yeasts} &\sim \text{Binomial Distribution} \quad (1)\\ &\sim \text{Poisson Distribution} \quad (2)\end{aligned}$$

$$\mathrm{Var}(r_t) = \mathrm{Ex}(r_t) = r_t \quad (3)$$

$$(4)$$

1.2.2 Mutation Hypothesis

Assumption: a fixed small chance per time unit for each yeast to undergo a mutation to resistance.

$$\frac{dN_t}{dt} = N_t \tag{5}$$
$$N_t = N_0 e^t \tag{6}$$
$$dm_t = adtN_t \tag{7}$$
$$m_t = a(N_t - N_0) \tag{8}$$

$$\text{Number of yeasts mutate during } dt \sim \text{Poisson Distribution} \tag{9}$$

$$p_0 = e^{-m_t} \tag{10}$$
$$\frac{d\rho_t}{dt} = aN_t + \rho \tag{11}$$
$$\text{assume: } \rho_0 = 0 \tag{12}$$
$$\rho_t = taN_t \tag{13}$$
$$r_t = (t - t_0)aN_t \tag{14}$$
$$1 = aC(N_{t_0} - N_0) \approx aCN_{t_0} \tag{15}$$
$$N_{t_0} = N_t e^{-(t-t_0)} \tag{16}$$
$$t - t_0 = \ln(N_t Ca) \tag{17}$$
$$r_t = aN_t \ln(N_t Ca) \tag{18}$$

The following discussion is about **partial distribution**, defined as the distribution during a certain time interval from $t - \tau$ to $t - \tau + d\tau$ with t fixed while τ as parameter:

$$\text{Ex}(dm_t) = aN_\tau d\tau = aN_t e^{-\tau} d\tau \tag{19}$$
$$d\rho_t = e^\tau dm_t \tag{20}$$
$$\text{Ex}(d\rho_t) = e^\tau \text{Ex}(dm_t) = aN_t d\tau \tag{21}$$
$$\text{Var}(d\rho_t) = e^{2\tau} \text{Var}(dm_t) = aN_t e^\tau d\tau \tag{22}$$
$$\tag{23}$$

To find the total distribution from partial distribution:

$$\text{Var}(\rho_t) = \int_0^t \text{Var}(d\rho_t) d\tau \tag{24}$$
$$= aN_t(e^t - 1) \tag{25}$$
$$\text{Var}(r_t) = \int_0^{t-t_0} \text{Var}(dr_t) d\tau \tag{26}$$
$$= aN_t(e^{t-t_0} - 1) \tag{27}$$
$$\approx Ca^2 N_t^2 \tag{28}$$
$$\frac{\text{Var}(r_t)}{r_t} = \frac{CaN_t}{\ln(N_t Ca)} \gg 1 \tag{29}$$

2 Calculating Procedure in Experiment

- observe the number of yeasts in a growing culture N_t at time t.
- determine the number of resistant yeasts in each **sample**.
  - this should be carried out by observing the number of colonies several days after inoculating.
- calculate the number of resistant yeasts r_t in each **culture**:

$$r_t = \frac{\text{volume of culture}}{\text{volume of samples}} \cdot \text{Ex}(r_{t \text{ sample}})$$

Column One	Column Two
Content One	Content One

Table 1: example 3

  - volume of culture = 1 mL, volume of samples = 20 μL in our experiment.

- evaluate the mutation rate a by the following formula:

$$r_t = aN_t \ln(N_t Ca)$$

- figure out the likely variance $\mathrm{Var}(r_t)$:
  - by mutation rate a according to the theory:

$$\mathrm{Var}(r_t) \approx Ca^2 N_t^2$$

  - by statistical methods:

$$\mathrm{Var}(r_{t\ \tilde{\text{sample}}}) = \mathrm{Var}(r_{t\ \text{sample}}) - \mathrm{Var}(\text{sampling})$$

$$\mathrm{Var}(\text{sampling}) \approx \mathrm{Ex}(\text{sampling}) = r_t$$

$$\mathrm{Var}(r_t) = \left(\frac{\text{volume of culture}}{\text{volume of samples}}\right)^2 \cdot \mathrm{Var}(r_{t\ \tilde{\text{sample}}})$$

    Thus we calculate the modified statistical variance by:

$$\mathrm{Var}(r_t) = \left(\frac{\text{volume of culture}}{\text{volume of samples}}\right)^2 \cdot (\mathrm{Var}(r_{t\ \text{sample}}) - \mathrm{Ex}(r_{t\ \text{sample}}))$$

  - compare the ratio of variance and average to one (Poisson distribution in Acquired hereditary immunity hypothesis):

$$\frac{\mathrm{Var}(r_t)}{\mathrm{Ex}(r_t)}$$

    both by the statistical method (just based on the final result) and by the theoretical method (based on the mutation rate a).

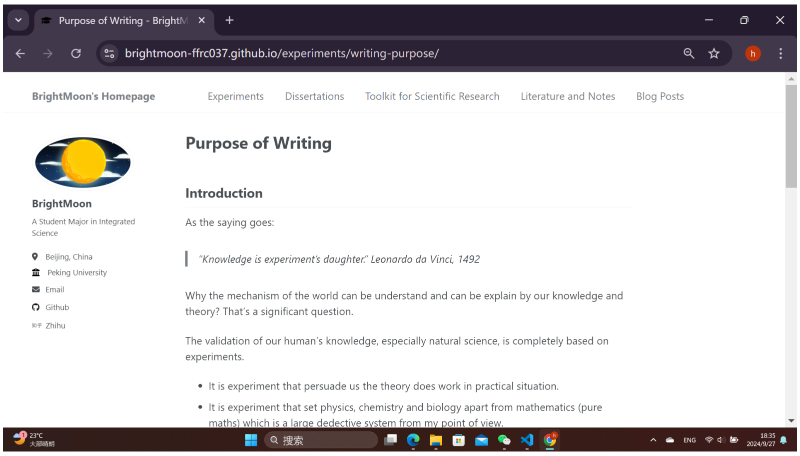
3 Part One

This is a template for convenient writing process. The text book helps [1] me a lot. We should use bibtex to compile the document,
μm
$\mu = 12345$

- item 1
- item 2

References

[1] Bruce Alberts, Rebecca Heald, Alexander Johnson, David Morgan, Martin Raff, Keith Roberts, and Peter Walter. *Molecular Biology of the Cell.* W. W. Norton and Company, 7th edition, 2022.



(a) fig 1

(b) fig 2

Figure 1: example 1

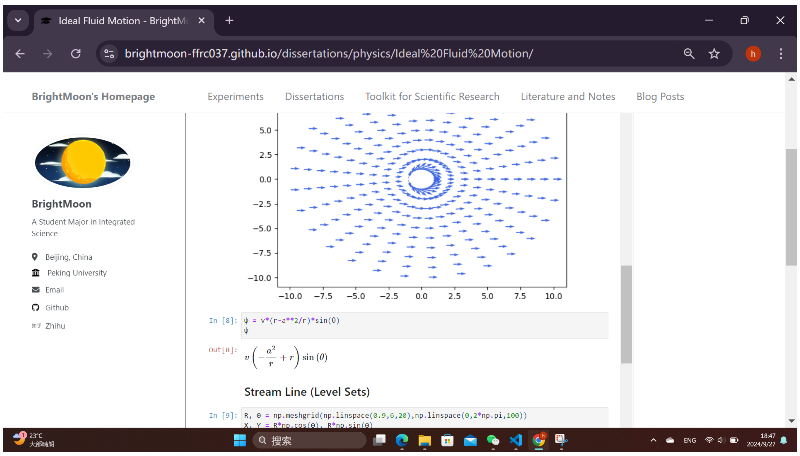


Figure 2: example 2