

Thoughts on the Mani Project

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Example settings

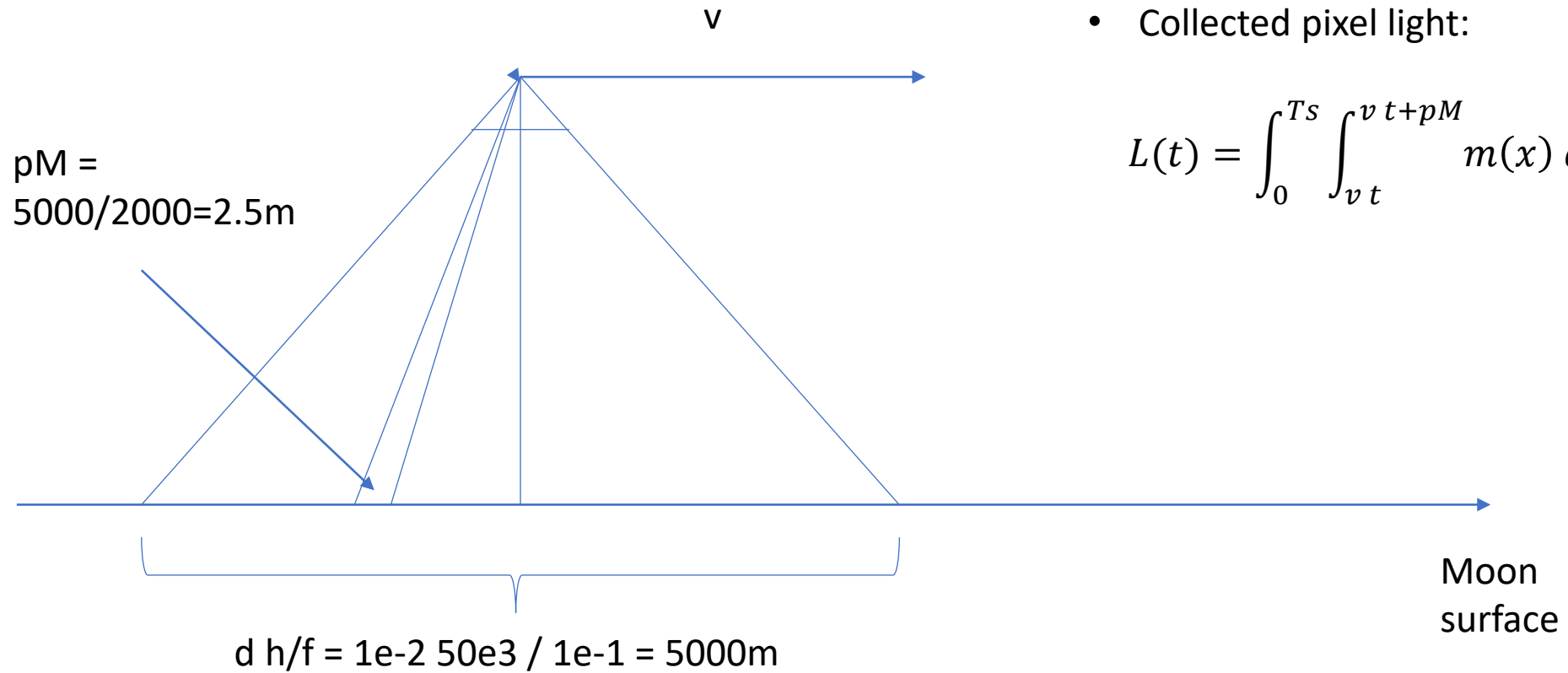
- Camera

- 2k X 2k pixels
- Focal length 100mm
- Shutter speed $T_s=1\text{mS}$
- CCD dimensions: $d \times d = 1\text{cm} \times 1\text{cm}$

- Orbit

- Altitude $h = 50\text{km}$
- Orbital period: $T = 2\pi \sqrt{\frac{(50e3+1738e3)^3}{\mu=5e12}} \sim 111 \text{ min}$
- Orbital speed: $v = 2\pi \frac{1738e3}{T} = 1.6e3 \frac{\text{m}}{\text{s}}$
- Moon surface speed: $2\pi \cdot 1738e3 / 27 / 24 / 60 / 60 = 4.7 \text{ m/s (negl)}$

Pixel input



- Moon surface light intensity: $m(x)$
- Collected pixel light:

$$L(t) = \int_0^{Ts} \int_{v_t}^{v_t + pM} m(x) dx dt$$

Pixel collected light

$$\begin{aligned} L(t) &= \int_t^{t+Ts} \int_{vt'}^{vt'+pM} m(x) dx dt' = \\ &= \int \delta_{Ts}(t' - t) \int \delta_{pM}(x - vt') m(x) dx dt' = \\ &= \int m(x) \int \delta_{pM}(x - vt') \delta_{Ts}(t' - t) dt' dx \end{aligned}$$

$$\delta_{pM}(x) = I_{0 \leq x \leq pM}$$

$$\delta_{Ts}(t) = I_{0 \leq t \leq Ts}$$

Pixel collected light

$$\int m(x) \int \delta_{pM}(x - vt') \delta_{Ts}(t' - t) dt' dx$$

$$\delta_{pM}(x) = I_{0 \leq x \leq pM}$$

$$\delta_{Ts}(t) = I_{0 \leq t \leq Ts}$$

$$\int \delta_{pM}(x - vt') \delta_{Ts}(t' - t) dt' \sim \delta_{pM + Ts v}(x - vt)$$

$$Ts v = 1e - 3 * 1.6 e3 = 1.6 m$$

$$pM + Ts v = 2.5 + 1.6 = 4.1 m$$

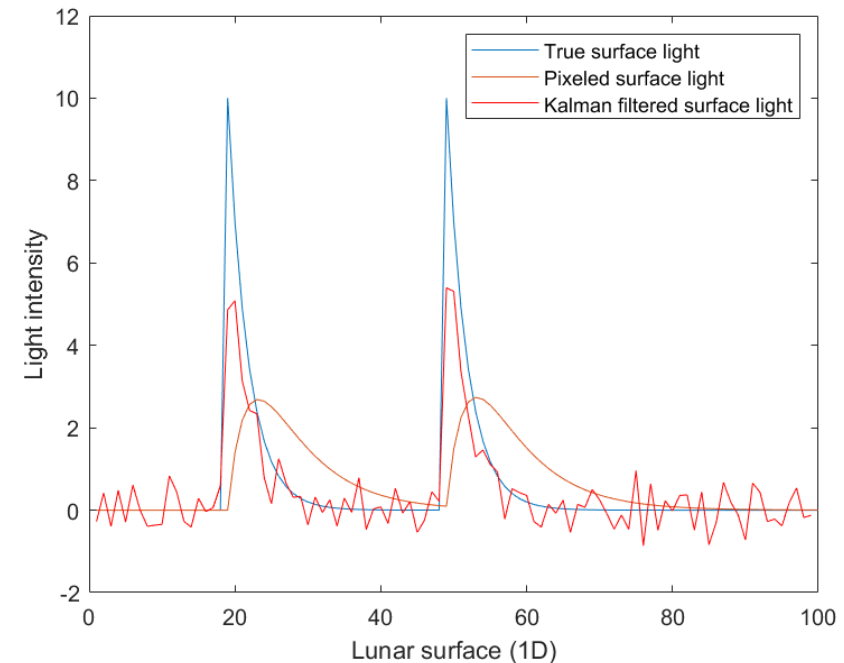
Fast sampling

- Let $T_s = 1e-4$
- $pM + Ts v \sim pM = 2.5m$
- We sample $m(x)$ each $xS = 16cm$
- Each sample is $s(x) = k * \sum_{x'=x}^{x+2.5} m(x) = \frac{1}{16} * \sum_{x'=x+0*0.16}^{x+15*0.16} m(x)$
- $a^{16} = 0.1 \Rightarrow 16 \log(a) = \log(0.1) \Rightarrow \log(a) = \frac{\log(0.1)}{16} \Rightarrow$
- $a = \exp\left(\frac{\log(0.1)}{16}\right) = 0.866$
- Equivalent IIR filter (1. ord AR):
$$s(x - 0.16) = a s(x) + (1 - a) m(x)$$

Kalman filtering

- $s(n + 1) = a s(n) + (1 - a) m(n)$ (Pixeled and blurred surface light)
- $m(n + 1) = b m(n) + \varepsilon(n)$, $0 \leq b < a$ (True surface light)

- $x(n) = \begin{bmatrix} m(n) \\ s(n) \end{bmatrix}$
- $x(n + 1) = \begin{bmatrix} b & 0 \\ 1 - a & a \end{bmatrix} x(n)$
- $y(n) = C x(n) = [0 \ 1] x(n)$



ADCS

- Pointing accuracy: $0.1 \text{ grad} = 0.0017 \text{ rad}$
- Pointing accuracy on lunar surface $0.0017 * 50e3 = 87 \text{ m} !!$
- IMU gyroscope precision $1e-3 \text{ grad/h} = 5e-9 \text{ rad/sek}$
- Gyroscope on lunar surface: $2.4e-4 \text{ m (negl)}$
- What about lunar satellite positioning ??