Variance, Co-Variance, Mahalanobis distance, Co-Relation, Mean, Standard Deviation

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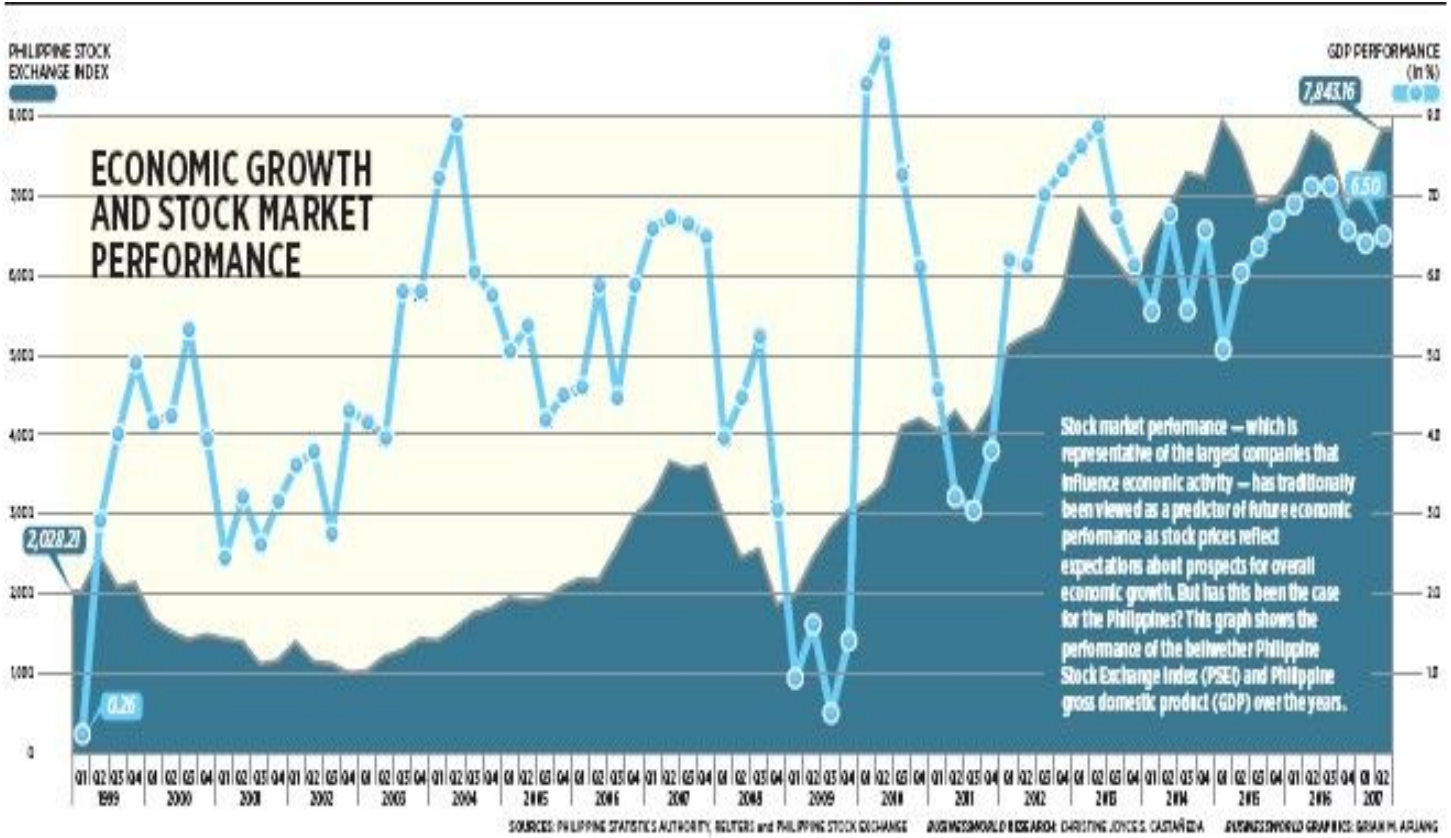
For Machine Learning Class
Target Audience: Sem 6 Students

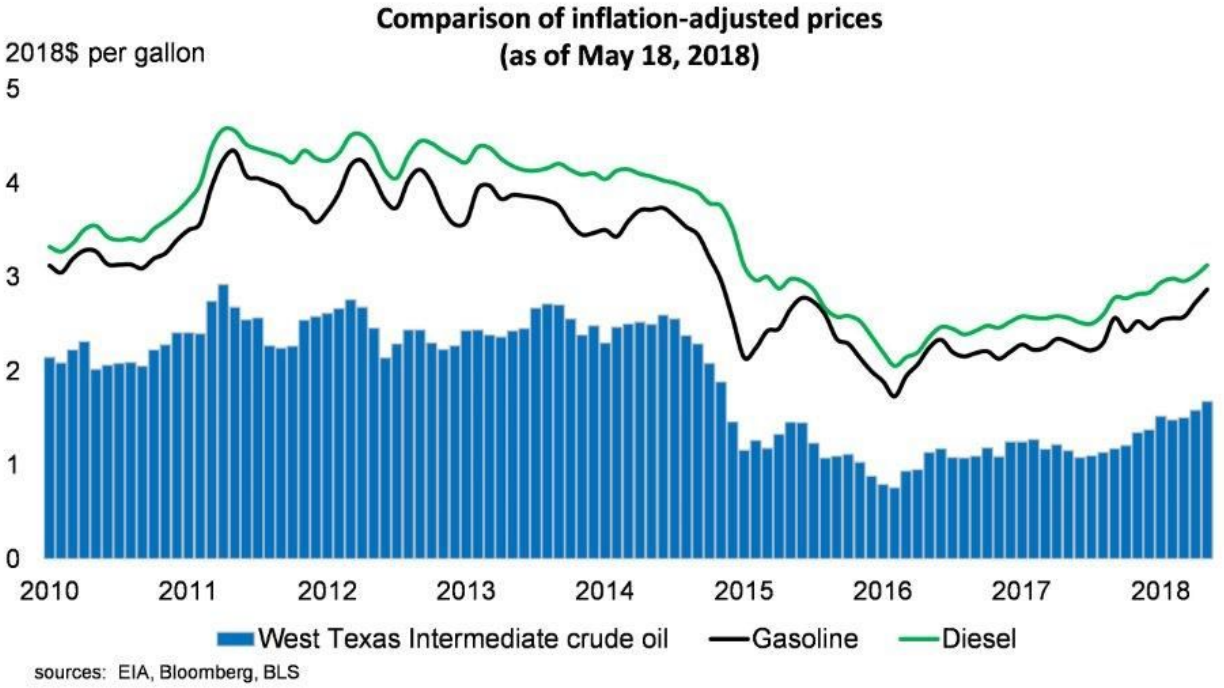
As economic growth increases, stock market returns increase.
E.g. *Philippines Stock Exchange Index*

By what quantity are these two parameters related?
What is the "level of correlation"??



By what quantity are these three parameters related?
What is the "level of correlation"???

Diesel, motor gasoline & crude oil prices move together



- Fuel prices closely track those of crude oil because crude oil is the largest input cost

Formulae

$$Mean = \bar{x} = \frac{\sum x}{n}$$

$$Variance = s^2 = \frac{\sum (x - \bar{x})^2}{n - 1}$$

$$Std\ dev = s = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}}$$

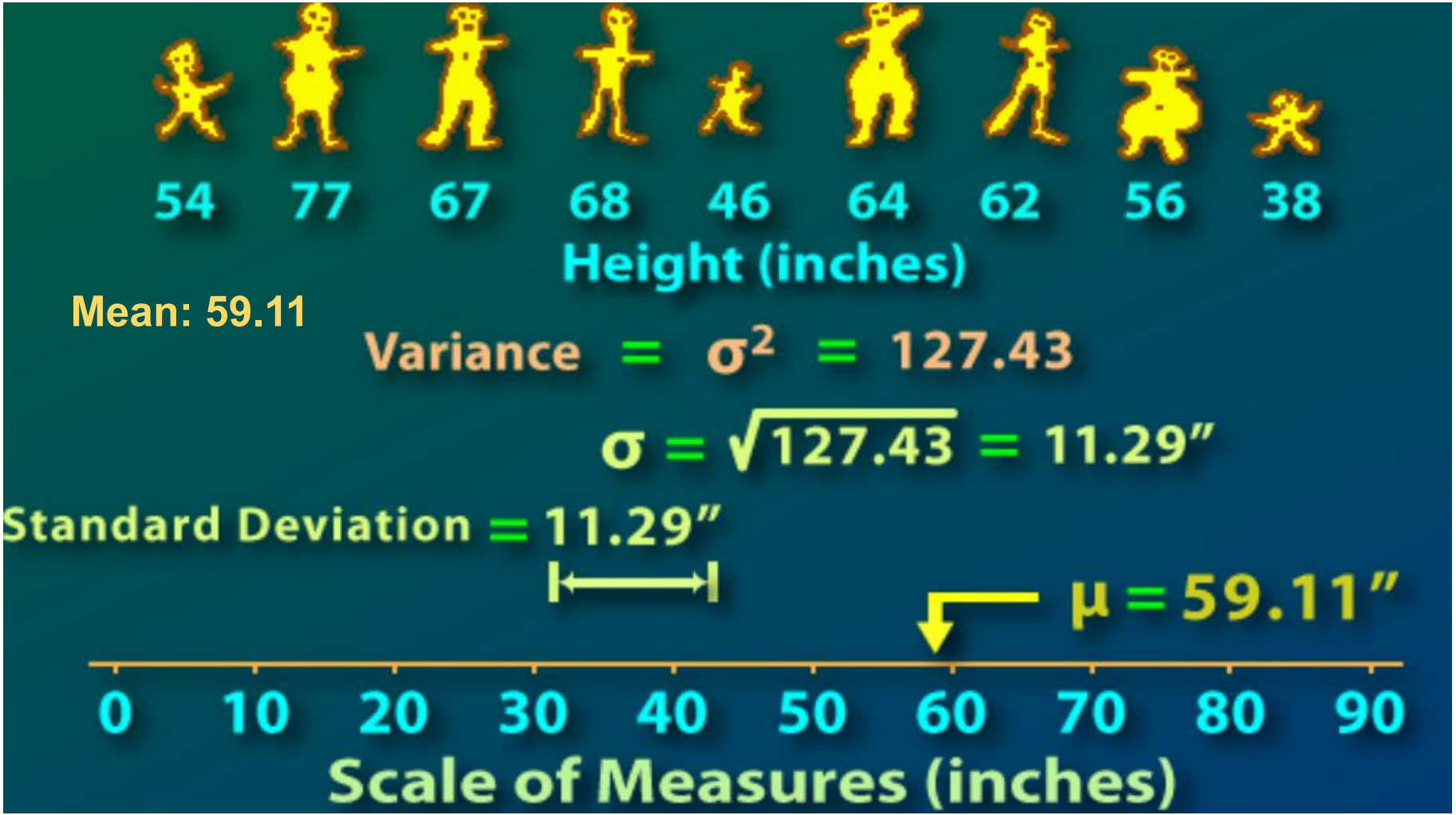
Note: The numerator is **not** a squared deviation, but a **product** of the summation deviation of each data point X from its mean and each data point Y from its mean.

Covariance

$$COV(x,y) = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{n - 1}$$

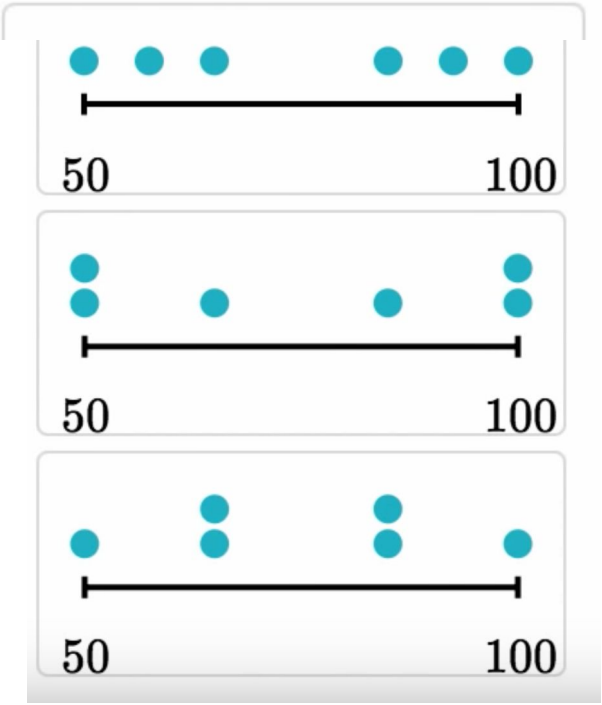
Correlation Coefficient

$$r_{(x,y)} = \frac{COV(x,y)}{Std\ dev}$$


54 77 67 68 46 64 62 56 38
Height (inches)
Mean: 59.11
Variance = σ² = 127.43
σ = √127.43 = 11.29″
Standard Deviation = 11.29″
μ = 59.11″
0 10 20 30 40 50 60 70 80 90
Scale of Measures (inches)

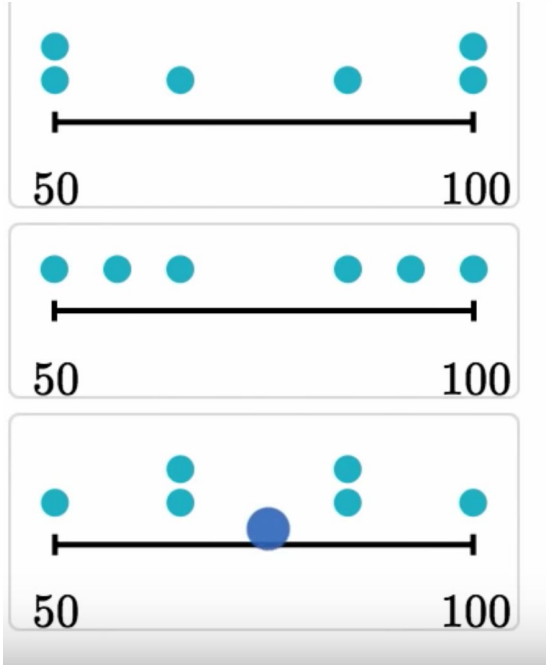
Each dot plot below represents a different set of data.

Order the dot plots from largest standard deviation (top) to smallest standard deviation (bottom).



Standard Deviation in Visual Form
E.g.#1

Answer



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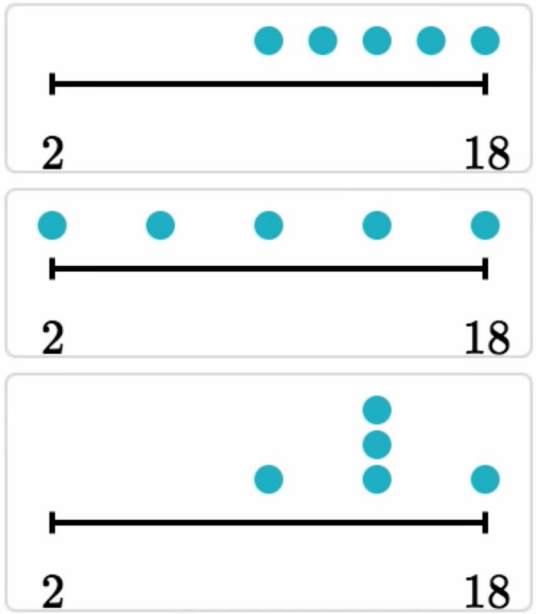
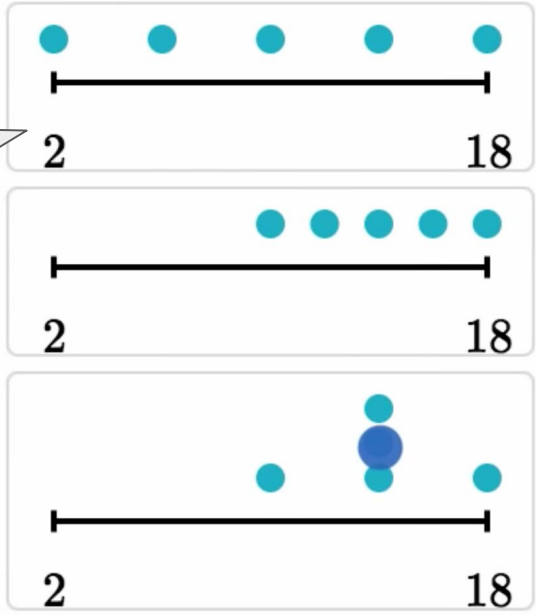


Standard Deviation in Visual Form
E.g.#2

Answer



σ_X

Question#1 : Standard Deviation Calculation

A veterinarian weighed a sample of 6 puppies. Here are each of their weights (in kilograms):

$1, 2, 7, 7, 10, 15$

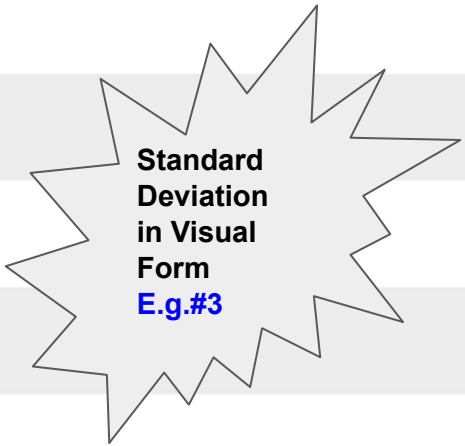
The mean of these weights is $\bar{x} = 7$ kg.

What is the standard deviation?

Standard Deviation in Visual Form
E.g.#3

Question#1 : Standard Deviation Calculation

x_i	Deviation: $(x_i - \bar{x})$	Squared deviation: $(x_i - \bar{x})^2$
1	$1 - 7 = -6$	$(-6)^2 = 36$
2	$2 - 7 = -5$	$(-5)^2 = 25$
7	$7 - 7 = 0$	$0^2 = 0$
7	$7 - 7 = 0$	$0^2 = 0$
10	$10 - 7 = 3$	$3^2 = 9$
15	$15 - 7 = 8$	$8^2 = 64$
Sum:	0	134



Question#1 : Standard Deviation Calculation

$$\frac{134}{n-1} = \frac{134}{6-1} = \frac{134}{5} = 26.8$$

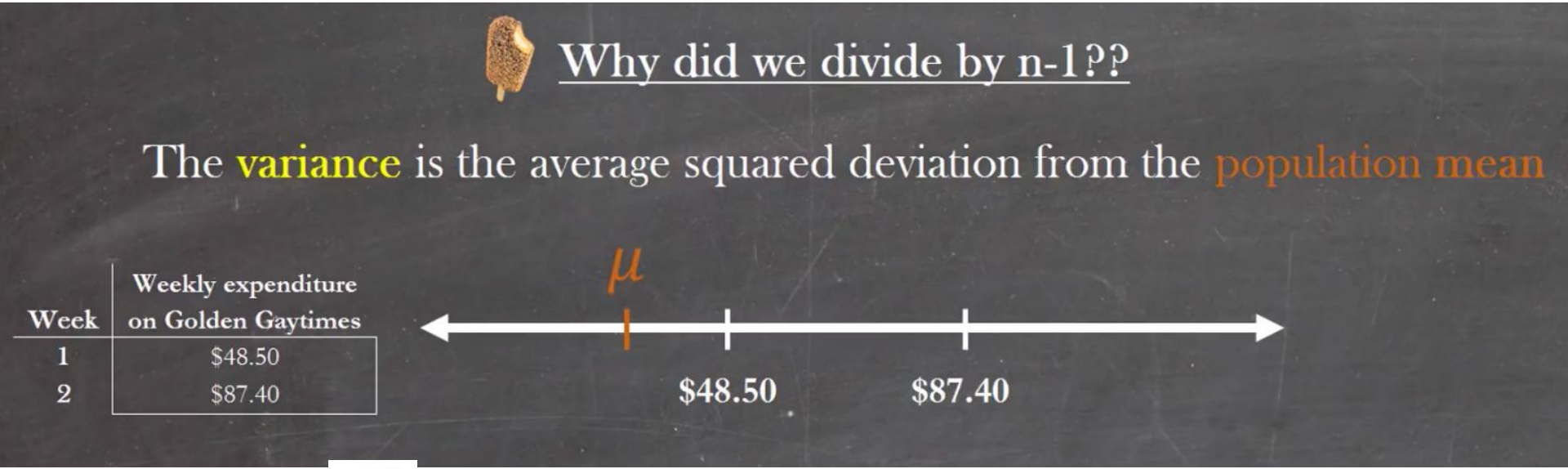
$$s_x = \sqrt{26.8} \approx 5.177 \text{ kg}$$

$$Variance = s^2 = \frac{\sum(x-\bar{x})^2}{n-1}$$

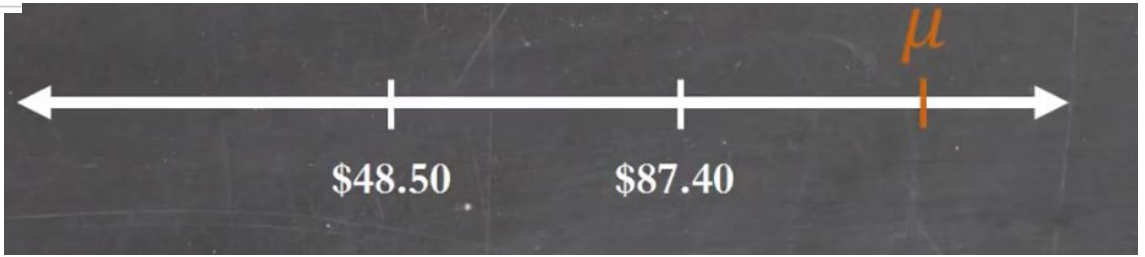
$$Std\ dev = s = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}}$$

Standard Deviation in Visual Form
E.g.#3

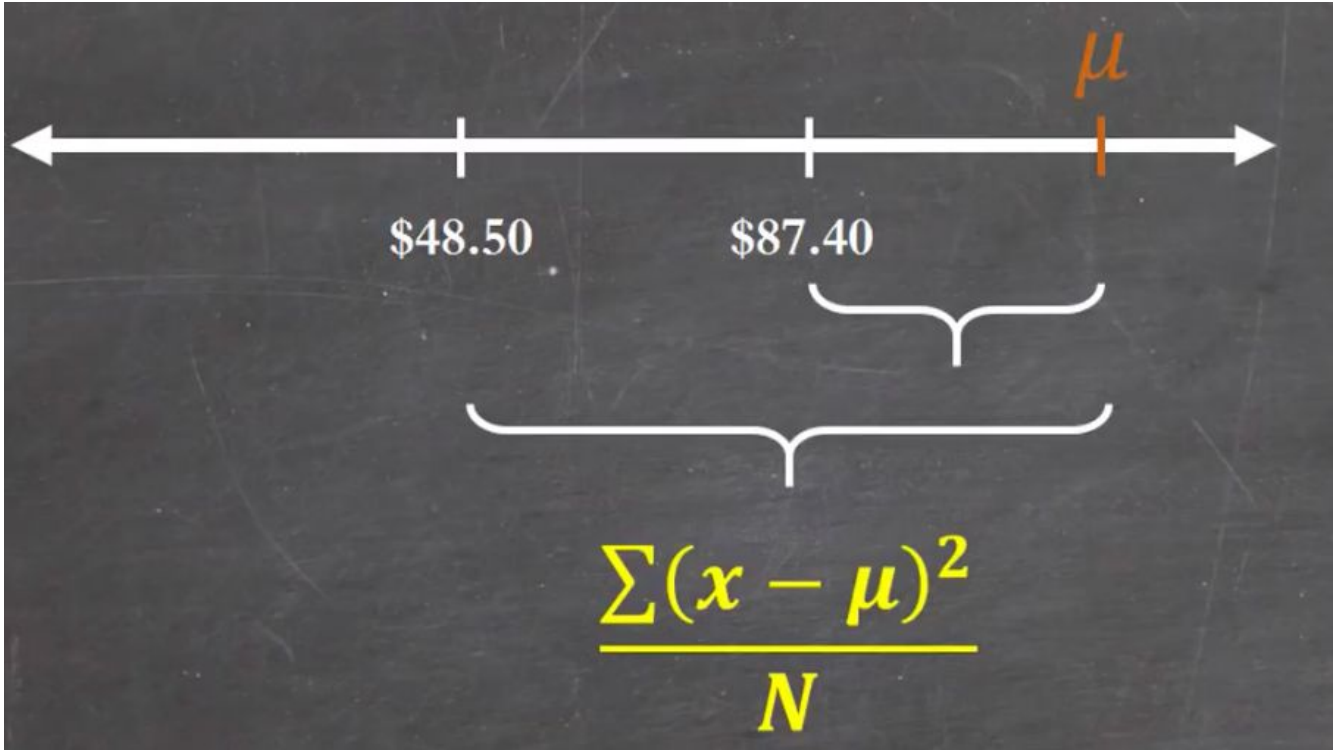
Standard Deviation Formula Explanation
(Why did we divide by n-1 ?)



Population Mean μ could be anywhere on this number line.

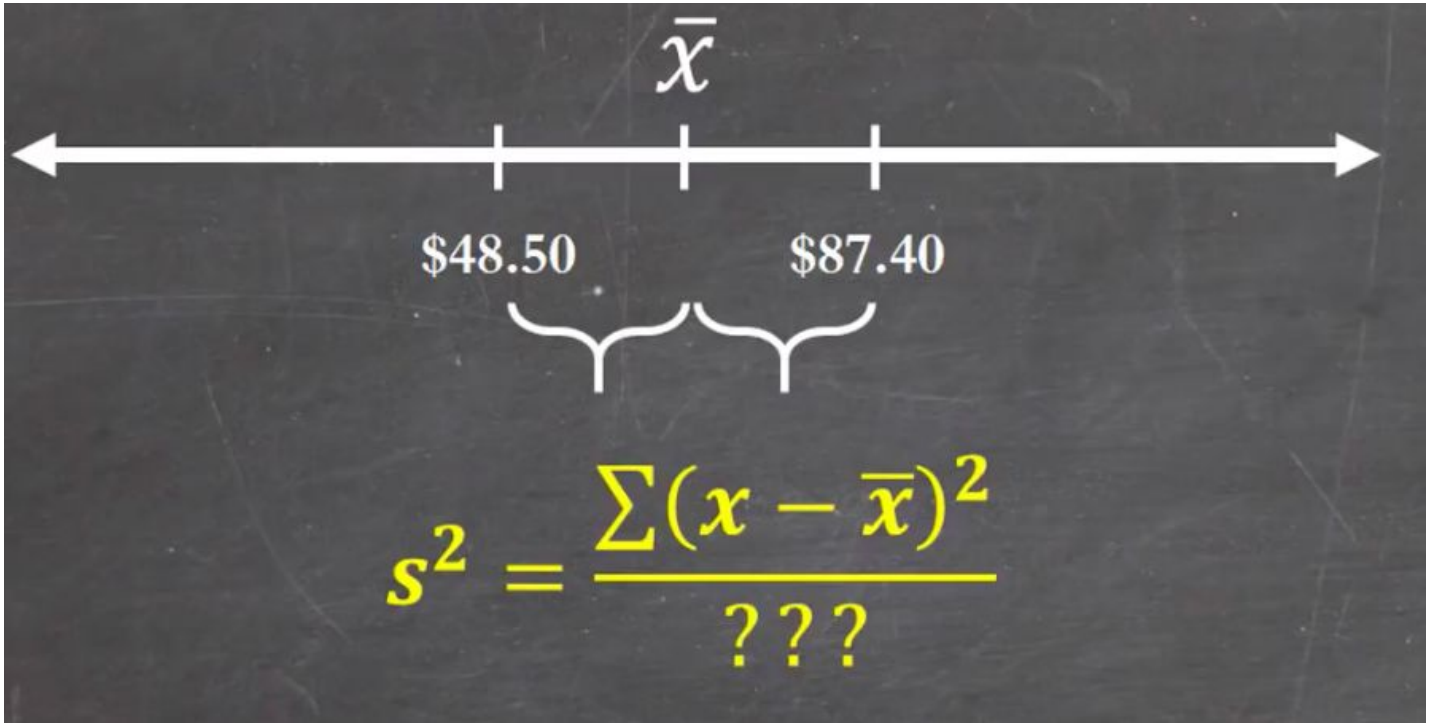


Standard Deviation Formula Explanation
(Why did we divide by n-1 ?)



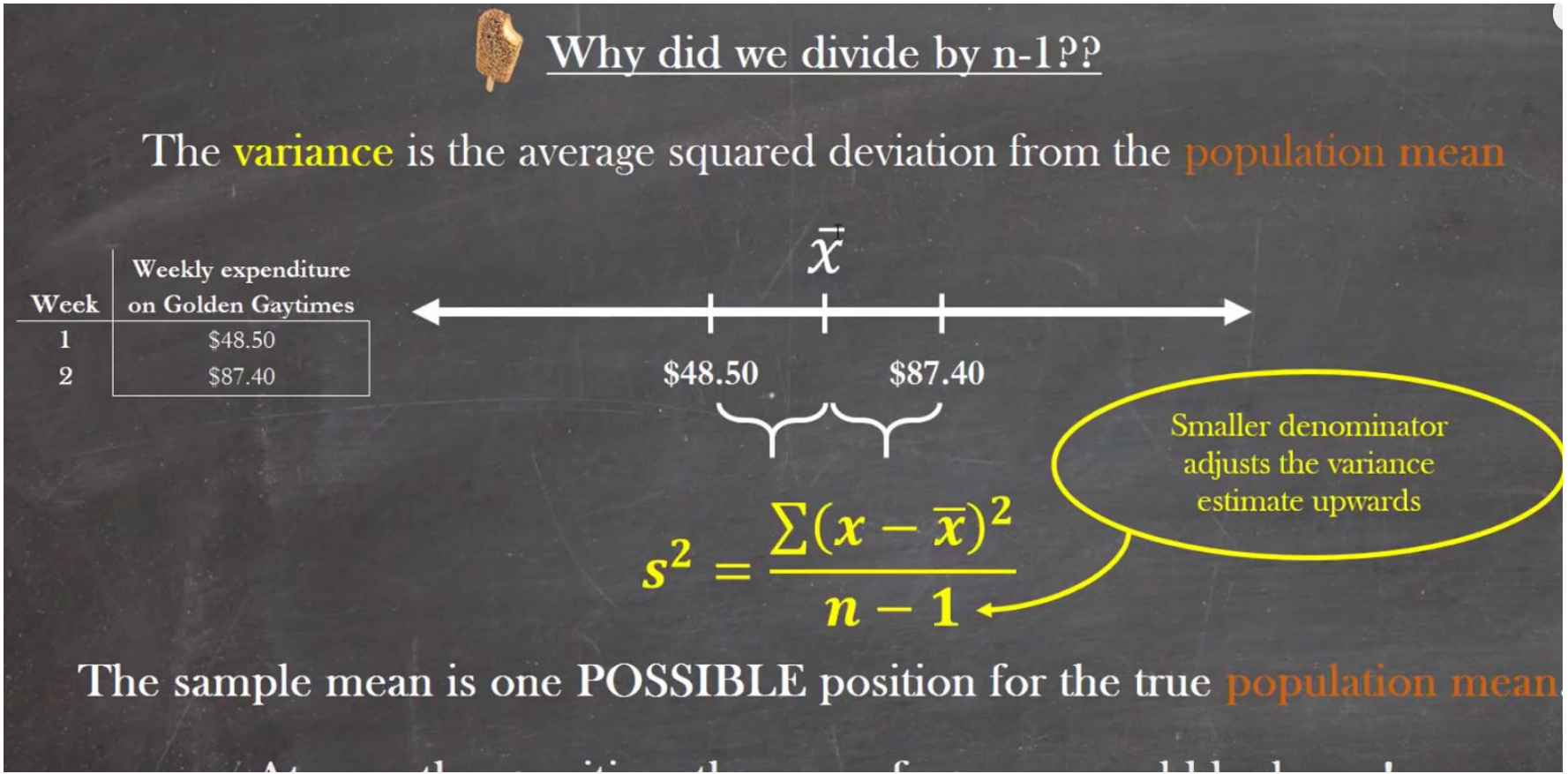
This is the **Population Variance**

Standard Deviation Formula Explanation
(Why did we divide by n-1 ?)



This is the **Sample Variance**

Standard Deviation Formula Explanation
(Why did we divide by n-1 ?)

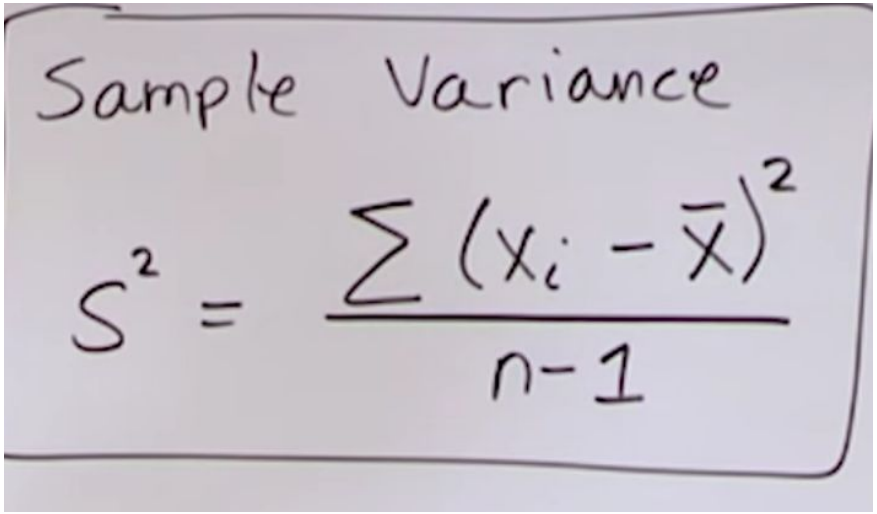


σ^2

$$\mathrm{Var}(X) = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2$$

Population Variance

Difference between Population and Sample Variance



How “connected” or “correlated” are these two parameters?

Sl#	Temperature	Ice-Cream Sales
1	66	8
2	72	11
3	77	15
4	84	20
5	83	21
6	71	11
7	65	8
8	70	10
Mean	73.5	13
Std Deviation	7.19	5.13

How “connected” or “correlated” are these two parameters?

$(x - \bar{x})$

Sample Variance

$$S^2 = \frac{\sum (x_i - \bar{x})^2}{n-1}$$

Sl#	Deviation	
1	-7.5	56.25
2	-1.5	3
3	3.5	12.25
4	10.5	110.25
5	9.5	90.25
6	-2.5	5
7	-8.5	17
8	-3.5	12.25

$(x - \bar{x})$	$(y_i - \bar{y})$	$(x_i - \bar{x})(y_i - \bar{y})$
-7.5	-5	37.5
-1.5	-2	3
3.5	2	7
10.5	7	73.5
9.5	8	76
-2.5	-2	5
-8.5	-5	42.5
-3.5	-3	10.5
		255

Temperature Deviation

Ice Cream Sales Deviation

Economic Growth % (x_i)	S & P 500 Returns % (y_i)
2.1	8
2.5	12
4.0	14
3.6	10

$$COV(x,y) = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{n-1}$$

x = the independent variable
y = the dependent variable
n = number of data points in the sample
$\bar{x}$ = the mean of the independent variable x
$\bar{y}$ = the mean of the dependent variable y

$$COV(x,y) = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{n-1}$$

= 255 / 7

= **36.43**

Covariance is a large positive number. This indicates *positive correlation.*

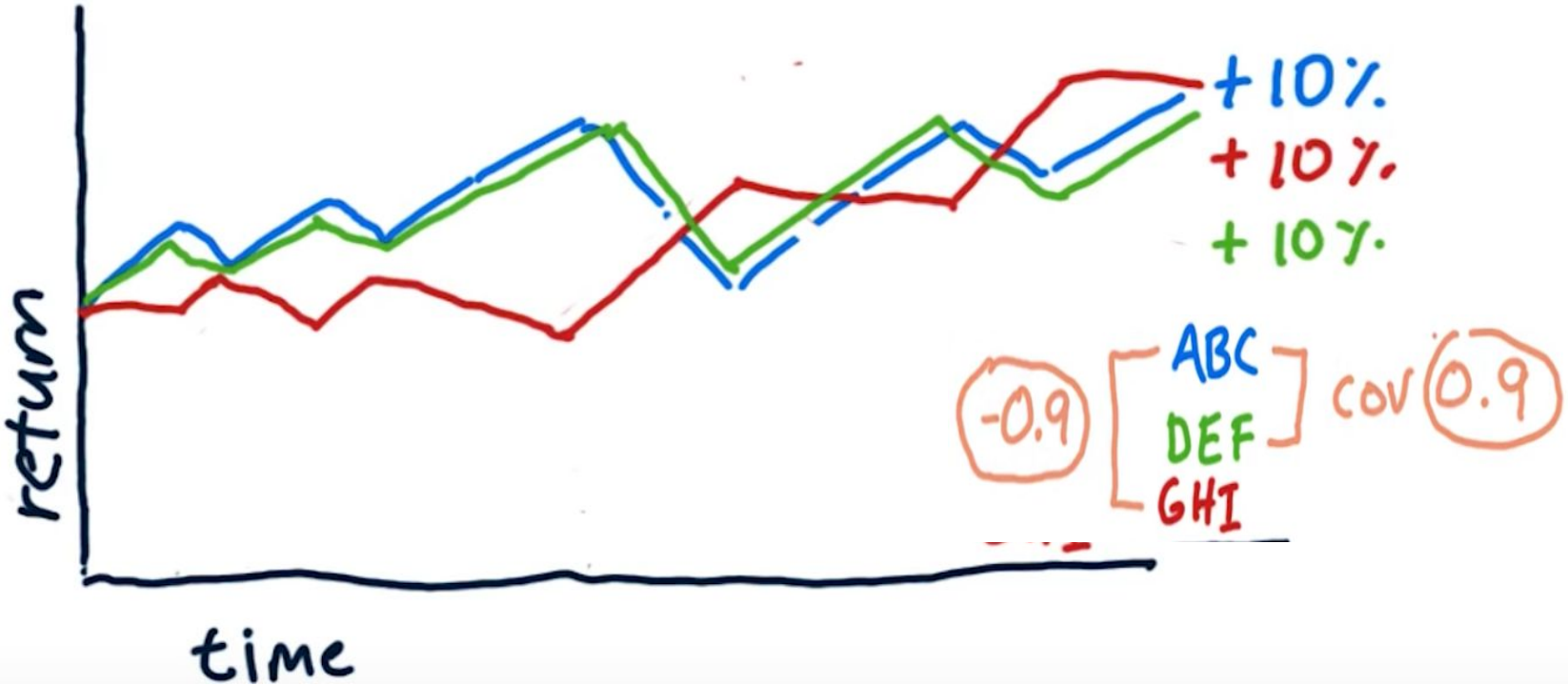
$$r_{(x,y)} = \frac{COV(x,y)}{s_x s_y}$$

= 255 / (7.19 * 5.13)

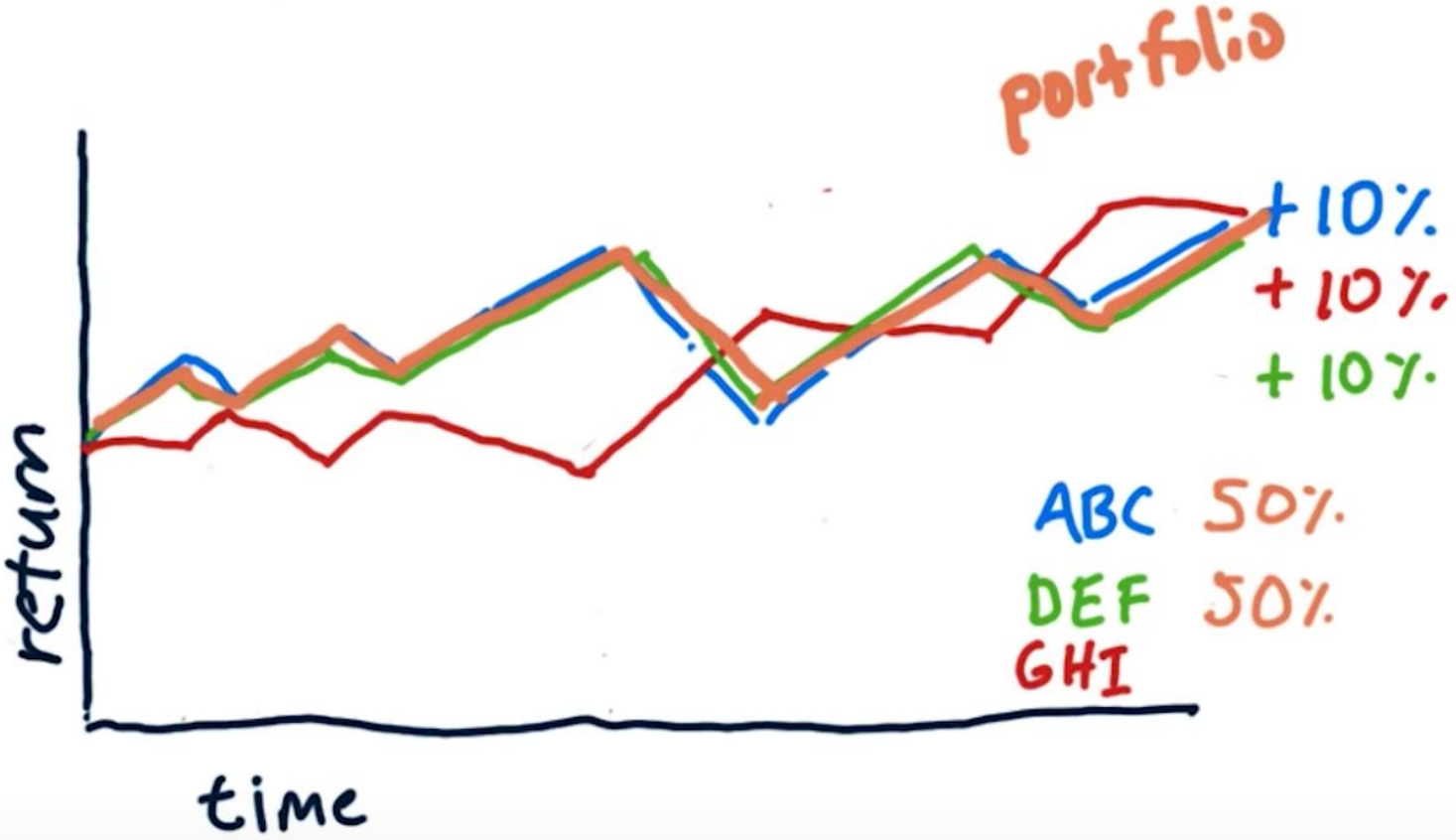
= **0.99**

Correlation Coefficient is number close to 1. This indicates *correlation is very high between X and Y. .*

The importance of covariance



The importance of covariance



The importance of covariance

3. Given the following return information, what is the covariance between the return of Stock A and the return of the market index?

Month	Return of Stock A	Return of Market Index
1	2.3	1.3
2	2.5	5.0
3	1.9	0.8
4	2.4	1.9
5	2.1	1.1

Sources:

1) **Picture of Std Deviation, Mean, Variance etc with class height example:** http://www.differencebetween.net/science/mathematics-statistics/difference-between-sample-variance-population-variance/
2) **All Formulae - blackboard style :** https://www.youtube.com/watch?v=wpY9o_OyxoQ
3) **Std Deviation Visually & Question** https://www.khanacademy.org/math/ap-statistics/summarizing-quantitative-data-ap/measuring-spread-quantitative/v/visual-standard-deviation
4) **Sample and Population Variance difference:** https://www.youtube.com/watch?v=sOb9b_AtwDg
5) **Covariance and Correlation Coefficient - Iceream versus temperature:** https://www.youtube.com/watch?v=0XKIDqB4Wug
6) **Portfolio :** https://www.youtube.com/watch?v=qOl04hw7f9g
7) **Answers to the questions**: http://ci.columbia.edu/ci/premba_test/c0331/s7/s7_5.html