## Bayes' Theorem

**Bayes' Theorem** is a fundamental theorem in probability that describes how to update the probability of a hypothesis when new evidence or information becomes available. Essentially, it allows you to **reverse a conditional probability**.

### The Formula

For two events, A and B, Bayes' Theorem is expressed as:

P(A∣B)=P(B)P(B∣A)⋅P(A)​

### Terminology

To understand the formula, it's helpful to define the terms:

| Term | Name | Description |
| --- | --- | --- |
| $\mathbf{P(A | B)}$ | **Posterior Probability** |
| $\mathbf{P(B | A)}$ | **Likelihood** |
| P(A) | **Prior Probability** | The initial probability of hypothesis A being true **before** observing the evidence B. |
| P(B) | **Marginal Probability / Evidence** | The total probability of observing the evidence B under all possible scenarios (including A and its complement Ac). |

### Expanded Formula (for the Denominator)

In many practical applications, the term P(B) (the total probability of the evidence B) must be calculated using the **Law of Total Probability**. If A and its complement Ac are the only two possibilities:

P(B)=P(B and A)+P(B and Ac)

P(B)=P(B∣A)P(A)+P(B∣Ac)P(Ac)

Substituting this into the main formula gives the common expanded version of Bayes' Theorem:

P(A∣B)=P(B∣A)P(A)+P(B∣Ac)P(Ac)P(B∣A)P(A)​

**Example Context:** This formula is most famously used in medical testing, where you want to find the probability that a person **has a disease (A) given a positive test result (B)**.

* P(A): **Prior** probability of having the disease (prevalence).
* P(B∣A): **Likelihood** of a positive test given the disease (test sensitivity).
* P(B∣Ac): **False Positive** rate (likelihood of a positive test given *no* disease).
* P(A∣B): **Posterior** probability of having the disease given the positive test.