2 - Linearisation

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$$\dot{x} = f(x, u, w)$$

$$f := \begin{bmatrix} v \\ -\frac{K}{m}(p - \ell_0) - \frac{\beta}{m} v^3 + \frac{1}{m} u + \frac{1}{m} L - g \end{bmatrix} =: \begin{bmatrix} f_1(x,u,w) \\ f_2(x,u,w) \end{bmatrix}$$

$$y = h(x,u,w) := p + \nu \qquad x := \begin{bmatrix} p \\ v \end{bmatrix}, d := \begin{bmatrix} L \\ g \end{bmatrix}, w := \begin{bmatrix} d \\ \nu \\ r \end{bmatrix}$$

$$e = h_e(x,u,w) := p + \nu - r$$

$$A(x,u,w) := \frac{\partial f}{\partial x} = \begin{bmatrix} \frac{\partial f}{\partial p} & \frac{\partial f}{\partial v} \end{bmatrix} = \begin{bmatrix} \partial f_1/\partial p & \partial f_1/\partial v \\ \partial f_2/\partial p & \partial f_2/\partial v \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ -\frac{K}{m} & -\frac{3\beta}{m} v^2 \end{bmatrix}$$

$$B_1(x,u,w) := \frac{\partial f}{\partial u} = \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} \qquad B_2(x,u,w) := \frac{\partial f}{\partial w} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \frac{1}{m} & -1 & 0 & 0 \end{bmatrix}$$

$$C(x,u,w) := \frac{\partial h}{\partial x} = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$D_1(x,u,w) := \frac{\partial h}{\partial u} = 0 \qquad D_2(x,u,w) := \frac{\partial h}{\partial w} = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}$$

$$C_e(x,u,w) := \frac{\partial h_e}{\partial x} = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$D_{e_1}(x,u,w) := \frac{\partial h_e}{\partial u} = 0 \qquad D_{e_2}(x,u,w) := \frac{\partial h_e}{\partial w} = \begin{bmatrix} 0 & 0 & 1 & -1 \end{bmatrix}$$

EQUILIBRIUM TRIPLET

$$\dot{x}^* = f(x^*, u^*, w^*) = 0$$

$$f := \begin{bmatrix} v^* \\ -\frac{K}{m}(p^* - \ell_0) - \frac{\beta}{m}[v^*]^3 + \frac{1}{m}u^* + \frac{1}{m}L^* - g^* \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$v^* = 0$$

$$r^* = 0$$

$$e^* = h_e(x^*, u^*, w^*) := \underset{=}{p^*} + r^* - \underset{=}{r^*} = 0$$

GIVEN $r^* \Rightarrow p^* = r^* \Rightarrow e^* = 0$ if $r^* = 0$

$$-\frac{K}{m}(p^* - \ell_0) - \frac{\beta}{m}[v^*]^3 + \frac{1}{m}u^* + \frac{1}{m}L^* - g^* = 0$$

$$\cancel{m}\cancel{\frac{1}{m}}u^* = -\left[-\frac{K}{m}(\underbrace{p^*}_{= r^*} - \ell_0) - \frac{\beta}{m}[\underbrace{v^*}_{= 0}]^3 + \frac{1}{m}L^* - g^*\right] m$$

$$u^* = K(r^* - \ell_0) - L^* + mg^*$$

$g^* \approx 9.8\ m/s^2$

$L^* = 0$