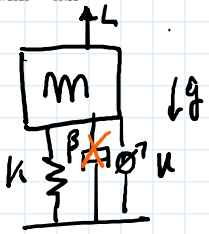
19 March 2025 09:11



$$A(x,u,w)=\begin{bmatrix} 0 & 1 \\ -\frac{K}{m} & -\frac{3\beta}{m}V^2 \end{bmatrix}$$

$$A(x^*,u^*,w^*) = A := \begin{bmatrix} 0 & 1 \\ -\frac{K}{m} & 0 \end{bmatrix}$$

$V^* = 0$

TO COMPUTE THE EIGENVALUES OF A

$$\det(A-\lambda I)=0$$

$$\det\left(\begin{bmatrix} 0 & 1 \\ -\frac{K}{m} & 0 \end{bmatrix} - \lambda\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) = \det\left(\begin{bmatrix} -\lambda & 1 \\ -\frac{K}{m} & -\lambda \end{bmatrix}\right)$$

$$= (-\lambda)(-\lambda) + \left(\frac{K}{m}\right)(1) = \lambda^2 + \frac{K}{m} = 0$$

$$\lambda_{1,2} = \frac{\pm\sqrt{-4K/m}}{2} = \pm\sqrt{-K/m}$$

$$\lambda_{1,2} = \pm i\sqrt{K/m}$$

$\lambda_1 = \alpha + i\beta$ ($\alpha = 0$)

$\lambda_2 = \alpha - i\beta$

$$J = \begin{bmatrix} \alpha & \beta \\ -\beta & \alpha \end{bmatrix} = \begin{bmatrix} 0 & \sqrt{K/m} \\ \sqrt{K/m} & 0 \end{bmatrix}$$

$V = [a \quad b]$ $\quad v := a + ib$

$$(A-\lambda_i I)v = 0$$

$\lambda_1 \Rightarrow$

$$\left(\begin{bmatrix} 0 & 1 \\ -\frac{K}{m} & 0 \end{bmatrix} - \sqrt{\frac{K}{m}}\,i\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)v =$$

$$\begin{bmatrix} -\sqrt{K/m}\,i & 1 \\ \cdots & \cdots \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ \cdots \end{bmatrix}$$

$$\begin{bmatrix} -\sqrt{K/m}\,i & 1 \\ -\frac{K}{m} & -\sqrt{\frac{K}{m}}\,i \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} x = 1 \\ y = -i\sqrt{\frac{K}{m}} \end{cases} \qquad v_1 = \begin{bmatrix} 1 \\ -i\sqrt{\frac{K}{m}} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{a} + i \underbrace{\begin{bmatrix} 0 \\ -\sqrt{\frac{K}{m}} \end{bmatrix}}_{b}$$

$$(A - \lambda_2 I)\, v_2 \qquad v_2 = a - ib$$

$$V := [\,a \;\; b\,] = \begin{bmatrix} 1 & 0 \\ 0 & -\sqrt{\frac{K}{m}} \end{bmatrix}$$

$$V^{-1} A V = J = \begin{bmatrix} 0 & \sqrt{\frac{K}{m}} \\ -\sqrt{\frac{K}{m}} & 0 \end{bmatrix}$$

$$x(t) = V\, z(t)$$

$$z(t) = \begin{bmatrix} \cos(\beta t) & -\sin(\beta t) \\ \sin(\beta t) & \cos(\beta t) \end{bmatrix} \begin{bmatrix} z_{01} \\ z_{02} \end{bmatrix}$$

$$x(t) = \begin{bmatrix} 1 & 0 \\ 0 & -\sqrt{\frac{K}{m}} \end{bmatrix} \begin{bmatrix} \cos(\beta t) & -\sin(\beta t) \\ \sin(\beta t) & \cos(\beta t) \end{bmatrix} \begin{bmatrix} z_{01} \\ z_{02} \end{bmatrix}$$

$$\begin{bmatrix} p(t) \\ v(t) \end{bmatrix} = \begin{bmatrix} \cos(\beta t) & -\sin(\beta t) \\ \sqrt{\frac{K}{m}}\sin(\beta t) & \sqrt{\frac{K}{m}}\cos(\beta t) \end{bmatrix} \begin{bmatrix} z_{01} \\ z_{02} \end{bmatrix}$$

$$x = Vz \qquad x(t) = V\, z(t) = V \exp(Jt)\, z_0$$

$$\dot{z} = Jz \qquad z(t) = \exp(Jt)\, z_0$$