TITLE

Automatic Control
Electronic Engineering for Intelligent Vehicles
University of Bologna

A.A. 202X-202X

Student A and Student B and ...

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Abstract

Here briefly detail the aims of the project.

Chapter 1

Introduction

1.0.1 System Linearization

To facilitate the control design of the longitudinal half-car model equipped with active front and rear suspension systems, the initial step involves the linearization of the nonlinear system dynamics. This process is performed by identifying appropriate steady-state operating points (x^*, y^*, w^*), which characterize representative conditions under which the vehicle is expected to operate. Linearizing the system around these equilibrium points enables the derivation of a time-invariant linear approximation of the vehicle dynamics, thereby simplifying the synthesis and analysis of control strategies.

Linearization Around the Operating Point

Consider the nonlinear system model:

$$\begin{aligned} \dot{x} &= f(x, u, w), \quad x(t_0) = x_0 \\ y &= h(x, u, w) \\ e &= h_e(x, u, w) \end{aligned} \tag{1.1}$$

The steady-state operating points $(x^\star, u^\star, w^\star)$ is called *equilibrium triplet* if satisfies the condition:

$$f(x^\star, u^\star, w^\star) = 0 \tag{1.2}$$

and defines the equilibrium output and error as:

$$y^\star := h(x^\star, u^\star, w^\star), \quad e^\star := h_e(x^\star, u^\star, w^\star) \tag{1.3}$$

The variations around the equilibrium point are defined as:

$$\begin{aligned} \tilde{x} &:= x - x^\star \\ \tilde{y} &:= y - y^\star \\ \tilde{e} &:= e - e^\star \\ \tilde{u} &:= u - u^\star \\ \tilde{w} &:= w - w^\star \end{aligned} \tag{1.4}$$

Using the fact that $\dot{x}^\star = 0$, the dynamics of the variations are:

$$\begin{aligned}
\dot{\tilde{x}} &= f(x^\star + \tilde{x}, u^\star + \tilde{u}, w^\star + \tilde{w}), \quad \tilde{x}(t_0) = x_0 - x^\star \\
\tilde{y} &= h(x^\star + \tilde{x}, u^\star + \tilde{u}, w^\star + \tilde{w}) \\
\tilde{e} &= h_e(x^\star + \tilde{x}, u^\star + \tilde{u}, w^\star + \tilde{w})
\end{aligned} \tag{1.5}$$

To obtain a tractable model for controller synthesis, we apply a first-order Taylor expansion around the equilibrium point. The resulting Jacobian matrices are defined as:

$$\begin{aligned}
A &:= \left.\frac{\partial f(x,u,w)}{\partial x}\right|_{\substack{x=x^\star\\u=u^\star\\w=w^\star}} & B_1 &:= \left.\frac{\partial f(x,u,w)}{\partial u}\right|_{\substack{x=x^\star\\u=u^\star\\w=w^\star}} & B_2 &:= \left.\frac{\partial f(x,u,w)}{\partial w}\right|_{\substack{x=x^\star\\u=u^\star\\w=w^\star}} \\
C &:= \left.\frac{\partial h(x,u,w)}{\partial x}\right|_{\substack{x=x^\star\\u=u^\star\\w=w^\star}} & D_1 &:= \left.\frac{\partial h(x,u,w)}{\partial u}\right|_{\substack{x=x^\star\\u=u^\star\\w=w^\star}} & D_2 &:= \left.\frac{\partial h(x,u,w)}{\partial w}\right|_{\substack{x=x^\star\\u=u^\star\\w=w^\star}} \\
C_e &:= \left.\frac{\partial h_e(x,u,w)}{\partial x}\right|_{\substack{x=x^\star\\u=u^\star\\w=w^\star}} & D_{e1} &:= \left.\frac{\partial h_e(x,u,w)}{\partial u}\right|_{\substack{x=x^\star\\u=u^\star\\w=w^\star}} & D_{e2} &:= \left.\frac{\partial h_e(x,u,w)}{\partial w}\right|_{\substack{x=x^\star\\u=u^\star\\w=w^\star}}
\end{aligned} \tag{1.6}$$

Neglecting second-order terms, the linearized system becomes the so-called *design model*:

$$\begin{cases}
\dot{\tilde{x}} = A\tilde{x} + B_1\tilde{u} + B_2\tilde{w}, \quad \tilde{x}(t_0) = x_0 - x^\star \\
\tilde{y} = C\tilde{x} + D_1\tilde{u} + D_2\tilde{w} \\
\tilde{e} = C_e\tilde{x} + D_{e1}\tilde{u} + D_{e2}\tilde{w}
\end{cases} \tag{1.7}$$

This Linear Time-Invariant (LTI) approximation of the nonlinear model is valid in a neighborhood of the equilibrium point, enabling efficient analysis and controller design under small perturbations.

TODO: Inserire le matrici calcolate con matlab e verificare che rispettino quello che abbiamo scritto (verifica che il codice del .m e quello che fa rispetta queste cose dette nella aprte di teoria)