Automotive Design Models Reachability Analysis

Faculty of «Electronic Engineering for Intelligent Vehicles» and «Advanced Automotive Engineering»

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Contents

- Active Suspensions
- Adaptive Cruise Control
- Wheel Speed Controls
- Electronic Stability Program / Torque Vectoring
- Self-Park Assist

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Active Suspensions

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -k_s \dfrac{m_s + m_u}{m_s m_u} & -\beta_s \dfrac{m_s + m_u}{m_s m_u} & \dfrac{k_t}{m_u} & 0 \\ 0 & 0 & 0 & 1 \\ \dfrac{k_s}{m_u} & \dfrac{\beta_s}{m_u} & -\dfrac{k_t}{m_u} & 0 \end{bmatrix}$$

$$\mathbf{B}_1 = \begin{bmatrix} 0 \\ \dfrac{m_s + m_u}{m_s m_u} \\ 0 \\ -\dfrac{1}{m_u} \end{bmatrix}$$

Reachability of $(\mathbf{A}, \mathbf{B}_1)$

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Reachability of $(\mathbf{A}, \mathbf{B}_1)$

$$\mathbf{R} = \begin{bmatrix} \mathbf{B}_1 & \cdots & \mathbf{A}^3\mathbf{B}_1 \end{bmatrix} \qquad \bar{m} = (m_s + m_u)/m_s$$

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Fully reachable

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$$\bar{R}(:,i) = \frac{R(:,i)}{|R(:,i)|}$$

Normalised (by column)

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         0    0.0291   -0.0067   -0.0125
    0.7629   -0.7623   -0.7018    0.7351
         0   -0.0246    0.0057    0.0126
   -0.6465    0.6461    0.7123   -0.6777
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Basis for $\dot{\mathbf{x}}$

Active Suspensions

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Basis for $d^2\mathbf{x}/dt^2$

Active Suspensions

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -k_s\dfrac{m_s+m_u}{m_s m_u} & -\beta_s\dfrac{m_s+m_u}{m_s m_u} & \dfrac{k_t}{m_u} & 0 \\ 0 & 0 & 0 & 1 \\ \dfrac{k_s}{m_u} & \dfrac{\beta_s}{m_u} & -\dfrac{k_t}{m_u} & 0 \end{bmatrix}$$

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Basis for $d^3\mathbf{x}/dt^3$

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Basis for $d^4\mathbf{x}/dt^4$

Contents

Adaptive Cruise Control

Reachability

Is this system reachable?

$$\mathbf{A} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & -\rho S_B C_{D_B} v0/m_B & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & -\rho S_C C_{D_C} v0/m_C \end{bmatrix} \quad \mathbf{B}_1 = \begin{bmatrix} 0 & 0 \\ m_B^{-1} & 0 \\ 0 & 0 \\ 0 & m_C^{-1} \end{bmatrix}$$

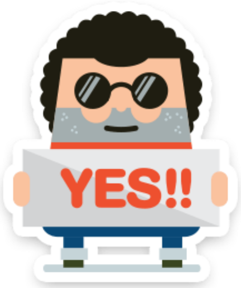
$$\mathbf{R} = [\mathbf{B}_1 \quad \mathbf{AB}_1] = \begin{bmatrix} 0 & 0 & -m_B^{-1} & 0 \\ m_B^{-1} & 0 & -\rho S_B C_{D_B} v_0/m_B^{-2} & 0 \\ 0 & 0 & m_B^{-1} & -m_C^{-1} \\ 0 & m_C^{-1} & 0 & -\rho S_C C_{D_C} v_0/m_C^{-2} \end{bmatrix}$$

Adaptive Cruise Control

Reachability

Is this system reachable?

$$\mathbf{A} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & -\rho S_B C_{D_B} v0/m_B & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & -\rho S_C C_{D_C} v0/m_C \end{bmatrix} \quad \mathbf{B}_1 = \begin{bmatrix} 0 & 0 \\ m_B^{-1} & 0 \\ 0 & 0 \\ 0 & m_C^{-1} \end{bmatrix}$$



$$\mathbf{R} = [\mathbf{B}_1 \quad \mathbf{AB}_1] = \begin{bmatrix} 0 & 0 & -m_B^{-1} & 0 \\ m_B^{-1} & 0 & -\rho S_B C_{D_B} v_0/m_B^{-2} & 0 \\ 0 & 0 & m_B^{-1} & -m_C^{-1} \\ 0 & m_C^{-1} & 0 & -\rho S_C C_{D_C} v_0/m_C^{-2} \end{bmatrix}$$

Contents

- Active Suspensions
- Adaptive Cruise Control
- Wheel Speed Controls
- Electronic Stability Program / Torque Vectoring
- Self-Park Assist

Wheel Speed Controls

$$\mathbf{A}_{22} = \begin{bmatrix} -\dfrac{r_r N_r}{J_r}\dfrac{\partial \bar{\mu}_r}{\partial \omega_r} & 0 \\ 0 & -\dfrac{r_f N_f}{J_f}\dfrac{\partial \bar{\mu}_f}{\partial \omega_f} \end{bmatrix}_{\mathbf{x}=\mathbf{x}'}$$

Reachability Analysis

Let $\mathbf{B}_1 = \begin{bmatrix} 0 & 0 \\ J_r^{-1} & 0 \\ 0 & J_f^{-1} \end{bmatrix}$ nd write

$\mathbf{B}_{12}$

$$\begin{aligned} \dot{\tilde{\boldsymbol{\omega}}} &= \mathbf{A}_{22}\tilde{\boldsymbol{\omega}} + \mathbf{B}_{12}\mathbf{u} + \mathbf{A}_{21}\tilde{v} \\ \mathbf{e} &= \tilde{\boldsymbol{\omega}} - \mathbf{r} \end{aligned},$$

And compute the reachability matrix associated with the couple $(\mathbf{A}_{22}, \mathbf{B}_{12})$

Wheel Speed Controls

$$\mathbf{A}_{22} = \begin{bmatrix} -\dfrac{r_r N_r}{J_r}\dfrac{\partial \bar{\mu}_r}{\partial \omega_r} & 0 \\ 0 & -\dfrac{r_f N_f}{J_f}\dfrac{\partial \bar{\mu}_f}{\partial \omega_f} \end{bmatrix}_{\mathbf{x}=\mathbf{x}'}$$

Reachability Analysis

Let $\mathbf{B}_1 = \begin{bmatrix} 0 & 0 \\ \boxed{\begin{matrix} J_r^{-1} & 0 \\ 0 & J_f^{-1} \end{matrix}} \end{bmatrix}$ nd write

$$\underbrace{\begin{matrix} J_r^{-1} & 0 \\ 0 & J_f^{-1} \end{matrix}}_{\mathbf{B}_{12}}$$

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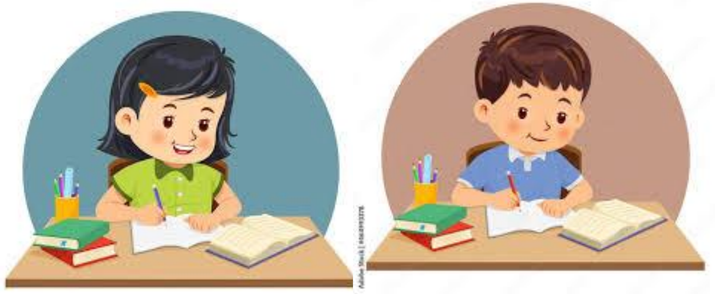

And compute the reachability matrix associated with the couple $(\mathbf{A}_{22}, \mathbf{B}_{12})$

Is the following system stabilisable/reachable at $\lambda = \lambda^{\star}$? (ABS & Launch C.)

$$\mathbf{A} = \begin{bmatrix} \dfrac{1}{m}\left(\dfrac{\partial f_r + f_f}{\partial v} - \rho S v C_D\right) & \dfrac{1}{m}\dfrac{\partial f_r + f_f}{\partial \omega_r} & \dfrac{1}{m}\dfrac{\partial f_r + f_f}{\partial \omega_f} \\ -\dfrac{r_r}{J_r}\dfrac{\partial f_r}{\partial v} & -\dfrac{r_r}{J_r}\dfrac{\partial f_r}{\partial \omega_r} & -\dfrac{r_r}{J_r}\dfrac{\partial f_r}{\partial \omega_f} \\ -\dfrac{r_f}{J_f}\dfrac{\partial f_f}{\partial v} & -\dfrac{r_f}{J_f}\dfrac{\partial f_f}{\partial \omega_r} & -\dfrac{r_f}{J_f}\dfrac{\partial f_f}{\partial \omega_f} \end{bmatrix}_{\mathbf{x}=\mathbf{x}_0} \qquad \mathbf{B}_1 = \begin{bmatrix} 0 & 0 \\ J_r^{-1} & 0 \\ 0 & J_f^{-1} \end{bmatrix}$$

Contents

ESP/TV

Reachability

Assumptions:

1) Front-wheel steering vehicle (0-toe rear wheels)
2) Toe angles and wheel speed of front wheels under driver control (disturbance)

ESP/TV

Reachability

Assumptions:

1) Front-wheel steering vehicle (0-toe rear wheels)
2) Toe angles and wheel speed of front wheels under driver control (disturbance)

let $\overline{\omega} := (\omega_2 + \omega_3)/2$, $\Delta\omega := (\omega_3 - \omega_2)/2$, and $\mathbf{w}_{\mathrm{ESP}} := \mathrm{col}(\mathbf{d}, \delta_1, \delta_4, \omega_1, \overline{\omega}, \omega_4, \nu, r)$.

Then, use $\mathbf{B}_1$ and $\mathbf{B}_2$ to define $\mathbf{B}_{1_{\mathrm{ESP}}}$, $\mathbf{B}_{2_{\mathrm{ESP}}}$ such that $\mathbf{B}_1\mathbf{u} + \mathbf{B}_2\mathbf{w} = \mathbf{B}_{1_{\mathrm{ESP}}}\Delta\omega + \mathbf{B}_{2_{\mathrm{ESP}}}\mathbf{w}_{\mathrm{ESP}}$.

In particular, $\mathbf{B}_{1_{\mathrm{ESP}}} = \mathrm{col}(0, 0, k)$ where

$$k := \left.\frac{\partial\mu(\lambda, \mathbf{\Theta}_0)}{\partial\lambda}\right|_{\lambda=\lambda_{r0}} \frac{r(1-\lambda_{r_0})^2}{v_0}\frac{c}{J}\left(2N_{2_0} + \frac{\bar{h}\left(\mu_{f_0} - \mu_{r_0}\right)\left(N_{2_0} - N_{1_0}\right)}{1 - \bar{h}(\mu_{r_0} - \mu_{f_0})}\right) > 0$$

ESP/TV

Reachability of $(\mathbf{A}, \mathbf{B}_{1_{\mathrm{ESP}}})$

$$\mathbf{R} = \begin{bmatrix} \mathbf{B}_{1_{\mathrm{ESP}}} & \mathbf{A}\mathbf{B}_{1_{\mathrm{ESP}}} & \mathbf{A}^2\mathbf{B}_{1_{\mathrm{ESP}}} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & k\left(\frac{1}{v_0}\frac{\partial \bar{f}_y}{\partial \omega} - v_0\right) & \frac{k}{v_0}\left(\frac{\partial \bar{f}_y}{\partial v_y} + \frac{m}{J}\frac{\partial \bar{\tau}}{\partial \omega}\right)\left(\frac{1}{v_0}\frac{\partial \bar{f}_y}{\partial \omega} - v_0\right) \\ k & k\frac{m}{Jv_0}\frac{\partial \bar{\tau}}{\partial \omega} & k\frac{m}{Jv_0}\left(\frac{\partial \bar{\tau}}{\partial v_y}\left(\frac{1}{v_0}\frac{\partial \bar{f}_y}{\partial \omega} - v_0\right) + \frac{m}{Jv_0}\left(\frac{\partial \bar{\tau}}{\partial \omega}\right)^2\right) \end{bmatrix}$$

Is this system reachable?

ESP/TV

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Is this system reachable?

Non-completely reachable

$$\frac{1}{v_0}\frac{\partial \bar{f}_y}{\partial \omega} - v_0 = 0$$

$$v_{\text{lc}}(v_0) := \sqrt{\partial \bar{f}_y / \partial \omega},$$

$v_{\text{lc}}(v_0) \neq v_0$, the non-reachable part corresponds to the longitudinal speed dynamics.

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ESP/TV

Reachability of $(\mathbf{A}, \mathbf{B}_{1_{\mathrm{ESP}}})$

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ESP/TV

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$$v_{\mathrm{lc}}(v_0) = v_0$$

Is this system stabilisable?

$\lambda_{2,3}$ real and negative.

Reachability of $(\mathbf{A}, \mathbf{B}_{1_{\mathrm{ESP}}})$

For $v_0 \neq \sqrt{\partial \bar{f}_y / \partial \omega}$, identify the reachable state as $\mathbf{z}_{\mathrm{R}} := \mathrm{col}(v_y, \omega)$. Then, for the design of the state feedback, refer to the dynamics $\dot{\mathbf{z}}_{\mathrm{R}} = \bar{\mathbf{A}}\mathbf{z}_{\mathrm{R}} + \bar{\mathbf{B}}_1 \Delta\omega$ in which

$$\mathbf{A} = \begin{bmatrix} \frac{1}{v_0}\frac{\partial \bar{f}_x}{\partial v_x} - \frac{\rho S C_D}{m} v_0 & 0 & 0 \\ 0 & \frac{1}{v_0}\frac{\partial \bar{f}_y}{\partial v_y} & \frac{1}{v_0}\frac{\partial \bar{f}_y}{\partial \omega} - v_0 \\ 0 & \frac{m}{J v_0}\frac{\partial \bar{\tau}}{\partial v_y} & \frac{m}{J v_0}\frac{\partial \bar{\tau}}{\partial \omega} \end{bmatrix}_{\mathbf{x}=\mathbf{x}_0, \mathbf{u}=\mathbf{u}_0}$$

$$\bar{\mathbf{A}} = \begin{bmatrix} \frac{1}{v_0}\frac{\partial \bar{f}_y}{\partial v_y} & \frac{1}{v_0}\frac{\partial \bar{f}_y}{\partial \omega} - v_0 \\ \frac{m}{J v_0}\frac{\partial \bar{\tau}}{\partial v_y} & \frac{m}{J v_0}\frac{\partial \bar{\tau}}{\partial \omega} \end{bmatrix}_{\mathbf{x}=\mathbf{x}_0, \mathbf{u}=\mathbf{u}_0}$$

$$\mathbf{B}_{1_{\mathrm{ESP}}} = \mathrm{col}(0, 0, k)$$

$$\bar{\mathbf{B}}_1 = \begin{bmatrix} 0 \\ k \end{bmatrix}$$

Contents

Self-Park Assist

Let us have a look at $\mathbf{A} = \begin{bmatrix} 0 & -v_0 \\ 0 & 0 \end{bmatrix}$, $\mathbf{B}_1 = \begin{bmatrix} -v_0 x_A/\ell \\ v_0/\ell \end{bmatrix}$

- Is this system reachable/stabilisable?

Self-Park Assist

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- Is this system reachable/stabilisable?

    Yes, it's reachable

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