

TITLE

Automatic Control
Electronic Engineering for Intelligent Vehicles
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Abstract

Here briefly detail the aims of the project.

Chapter 1

Introduction

1.1 Motivations

Explain why the selected application is important. Describe the application with informal words.

1.2 Contributions

Describe what this project deals with. What has been done to solve the problem presented in the motivations.

1.3 State of art and literature comparison

List the closest works that deal with the same problem and compare the achievement obtained and the strategies exploited in this paper. For the search of the literature use <https://ieeexplore.ieee.org/Xplore/home.jsp> and <https://www.sciencedirect.com/>.

1.4 Organisation of the manuscript

Describe what the reader finds in each of the Sections of this manuscript.

1.5 List of the symbols

Here list all the symbols used in the manuscript and add a description to each of them (Use the International System of Units https://en.wikipedia.org/wiki/International_System_of_Units).

Chapter 2

MAIN BODY

Change the title with the name of the selected application

2.1 Model and Problem Formulation

In this section, we formulate the control problem for the active suspension system of a half-car model. This system aims to regulate both the vertical position and the perceived pitch angle of the vehicle body, enhancing ride comfort and handling. The model is described by a nonlinear dynamic system influenced by road disturbances, actuator forces, and sensor measurements.

The general form of the system is expressed as:

$$\dot{x} = f(x, u, w) \quad (2.1)$$

$$y = h(x, u, w) \quad (2.2)$$

$$e = h_e(x, u, w) \quad (2.3)$$

Where:

- $x \in \mathbb{R}^n$ is the **state vector**,
- $u \in \mathbb{R}^p$ is the **control input vector**,
- $y \in \mathbb{R}^q$ is the **measured output vector**,
- $e \in \mathbb{R}^{l_m}$ is the **control error (goal)**,
- $d \in \mathbb{R}^{l_d}$ is the **disturbance vector**,
- $r \in \mathbb{R}^{l_r}$ is the **reference signal**,
- $\nu \in \mathbb{R}^q$ is the **sensor noise**,
- $w = \text{col}(d, \nu, r)$ is the **exogenous input**.

$$\begin{aligned} \dot{x} &= f(x, u) & x(t_0) &= x_0 \\ y &= h(x, u) \end{aligned} \quad (2.4)$$

Assumptions

To make the problem tractable and to ensure solvability of the control task, we impose the following assumptions:

1. The exogenous input w is not directly measurable.
2. Disturbances d are bounded.
3. Reference signal r and its first derivatives are known.
4. Bounded disturbances imply bounded internal states and outputs.
5. The system has at least as many control inputs as control goals, i.e., $p \geq l_m$.
6. The control error e can be reconstructed from the output y : $\exists E$ such that $e = E(y)$.

These assumptions lay the theoretical foundation required to design a control law capable of driving the error e to zero despite the presence of unknown disturbances and sensor noise.

2.2 Model Analysis

2.2.1 Dynamic Model

The dynamic behavior of the half-car is described using a state vector that captures both translational and rotational aspects of motion, as well as external road-induced disturbances. The state vector consists of six components and is expressed as follows:

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} z - z_g \\ \dot{z} - \dot{z}_g \\ \theta \\ \dot{\theta} \\ \theta_g \\ \omega_g \end{bmatrix} \quad (2.5)$$

Here, z represents the vertical displacement of the vehicle's center of mass (CoM), while z_g denotes the vertical road disturbance. The variable \dot{z} is the vertical velocity of the vehicle body, and θ is the pitch angle, which describes the rotation of the vehicle body about its lateral axis. The pitch rate is given by $\dot{\theta}$. The disturbance inputs include the road pitch angle θ_g and its time derivative $\omega_g = \dot{\theta}_g$, which captures the rate of change of the road gradient. Therefore, the state vector effectively captures the relative vertical and angular positions and velocities of the vehicle body with respect to the road surface.

The suspension system in the half-car model is influenced by two actuators—one at the front and one at the rear. These actuators generate forces that contribute to both the vertical and rotational dynamics of the vehicle body. The control input vector is defined as:

$$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} f_{af} + f_{ar} \\ f_{af}d_f - f_{ar}d_r \end{bmatrix} \quad (2.6)$$

In this expression, f_{af} and f_{ar} are the forces generated by the front and rear suspension actuators, respectively. The parameters d_f and d_r denote the distances from the vehicle's center of mass to the front and rear axles. The first control input, u_1 , represents the total vertical force acting on the vehicle body due to both actuators. The second input, u_2 , represents the net moment about the vehicle's center of mass generated by these forces, which directly influences the pitch motion.

The evolution of the system over time is governed by a set of first-order differential equations derived from Newton's second law for translational and rotational motion. The dynamic model is written as:

$$\dot{x} = \begin{bmatrix} x_2 \\ f_2 - \ddot{z}_g \\ x_4 \\ f_4 \\ x_6 \\ \alpha_g \end{bmatrix} \quad (2.7)$$

In this representation, f_2 is the vertical acceleration of the vehicle body, given by:

$$f_2 = -g + \frac{1}{m}(f_{sf} + f_{sr}) + \frac{u_1}{m} \quad (2.8)$$

$$(2.9)$$

The pitch angular acceleration f_4 is given by:

$$f_4 = \frac{1}{J}(f_{sf}d_f - f_{sr}d_r + u_2 + f_{wf}l_f + f_{wr}l_r) \quad (2.10)$$

The suspension deflections and velocities are:

$$s_1 = x_1 + d_f(\sin x_3 - \sin x_5), \quad s_3 = x_1 - d_r(\sin x_3 - \sin x_5) \quad (2.11)$$

$$s_2 = x_2 + d_f(x_4 \cos x_3 - x_6 \cos x_5), \quad s_4 = x_2 - d_r(x_4 \cos x_3 - x_6 \cos x_5) \quad (2.12)$$

Suspension forces are modeled as spring-damper systems:

$$f_s(p, v) = -kp - \beta v \quad (2.13)$$

2.2.2 Sensor Model

The measurement vector is given by:

$$y = \begin{bmatrix} y_y \\ y_z \\ y_g \\ y_l \\ y_r \end{bmatrix} = \begin{bmatrix} \sin x_3(f_2 + g) + \cos x_3(f_{wr} + f_{wf})/m \\ \cos x_3(f_2 + g) - \sin x_3(f_{wr} + f_{wf})/m \\ x_4 \\ s_1 \\ s_3 \end{bmatrix} + \nu \quad (2.14)$$

2.2.3 Control Objectives

We define the apparent pitch angle θ_a using accelerometer data:

$$\theta_a = \sin^{-1} \left(\frac{y_y}{\sqrt{y_y^2 + y_z^2}} \right) \quad (2.15)$$

The control error vector is defined as:

$$e = \begin{bmatrix} \frac{y_f d_r + y_r d_f}{d_r + d_f} - r_z \\ \theta_a - r_\theta \end{bmatrix} \quad (2.16)$$

This error describes deviations from the desired vertical height and perceived pitch. The control task is to design u to drive $e \rightarrow 0$ in the presence of disturbances and noise.

The second component of the error vector $he(x, u, w)$ captures the deviation from a desired apparent pitch angle θ_a , which reflects the passenger-perceived acceleration. It is defined through the nonlinear relationship:

$$\sin(\theta_a) = \frac{y_y}{\sqrt{y_y^2 + y_z^2}} \Rightarrow \theta_a = \sin^{-1} \left(\frac{y_y}{\sqrt{y_y^2 + y_z^2}} \right),$$

where y_y and y_z are the accelerometer outputs along the body-frame y and z axes, respectively. This formulation provides an estimate of the pitch experienced by passengers during acceleration, braking, or uneven ground contact.

The complete output error function thus becomes:

$$he(x, u, w) = \begin{bmatrix} \frac{(s_1 + \nu_f)d_r + (s_3 + \nu_r)d_f}{d_r + d_f} - r_z \\ \sin^{-1} \left(\frac{h_1 + \nu_y}{\sqrt{(h_1 + \nu_y)^2 + (h_2 + \nu_z)^2}} \right) - r_\theta \end{bmatrix},$$

where s_1, s_3 are suspension deflections, h_1, h_2 are derived from dynamic equations, and ν_i are sensor noise terms. This formulation ensures the controller minimizes both height and perceptual pitch deviations.

2.3 Proposed Solution

Here describe the proposed solution: Control system architecture (draw a block scheme!), mathematical description of the solution, listings of the MATLAB code implemented to obtain the solution

To include a picture use the environment *figure*. Use the environment *ref* to

Figure 2.1: Add the caption to each figure! The caption should completely describe the figure so that the reader should be able to understand it without the need of reading the main text.

add a hyperlink to the figure. As example Figure 2.1.

Application

3.2 Simulation results

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Chapter 4

Conclusions and further investigation

Recap the main results obtained in the project and highlight eventual further investigation directions along which the performance could be improved.

Bibliography

List the papers/books cited.

Appendix

Use appendices to add technical parts which are instrumental for the completeness of the manuscript but are too heavy to be included inside the main text. Basically, appendices are exploited to let the main text cleaner and smoother. As example, the complete MATLAB listings can be reported in appendix.

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