TITLE

Automatic Control
Electronic Engineering for Intelligent Vehicles
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Abstract

Here briefly detail the aims of the project.

Chapter 1

Introduction

1.0.1 System Linearization

To facilitate the control design of the longitudinal half-car model equipped with active front and rear suspension systems, the initial step involves the linearization of the nonlinear system dynamics. This process is performed by identifying appropriate steady-state operating points (x^*, y^*, w^*), which characterize representative conditions under which the vehicle is expected to operate. Linearizing the system around these equilibrium points enables the derivation of a time-invariant linear approximation of the vehicle dynamics, thereby simplifying the synthesis and analysis of control strategies.

Linearization Around the Operating Point

Consider the nonlinear system model:

$$\begin{aligned} \dot{x} &= f(x, u, w), \quad x(t_0) = x_0 \\ y &= h(x, u, w) \\ e &= h_e(x, u, w) \end{aligned} \tag{1.1}$$

The steady-state operating points $(x^\star, u^\star, w^\star)$ is called *equilibrium triplet* if satisfies the condition:

$$f(x^\star, u^\star, w^\star) = 0 \tag{1.2}$$

and defines the equilibrium output and error as:

$$y^\star := h(x^\star, u^\star, w^\star), \quad e^\star := h_e(x^\star, u^\star, w^\star) \tag{1.3}$$

The variations around the equilibrium point are defined as:

$$\begin{aligned} \tilde{x} &:= x - x^\star \\ \tilde{y} &:= y - y^\star \\ \tilde{e} &:= e - e^\star \\ \tilde{u} &:= u - u^\star \\ \tilde{w} &:= w - w^\star \end{aligned} \tag{1.4}$$

Using the fact that $\dot{x}^\star = 0$, the dynamics of the variations are:

$$\begin{aligned}
\dot{\tilde{x}} &= f(x^\star + \tilde{x}, u^\star + \tilde{u}, w^\star + \tilde{w}), \quad \tilde{x}(t_0) = x_0 - x^\star \\
\tilde{y} &= h(x^\star + \tilde{x}, u^\star + \tilde{u}, w^\star + \tilde{w}) \\
\tilde{e} &= h_e(x^\star + \tilde{x}, u^\star + \tilde{u}, w^\star + \tilde{w})
\end{aligned} \tag{1.5}$$

To obtain a tractable model for controller synthesis, we apply a first-order Taylor expansion around the equilibrium point. The resulting Jacobian matrices are defined as:

$$\begin{aligned}
A &:= \left.\frac{\partial f(x,u,w)}{\partial x}\right|_{\substack{x=x^\star\\u=u^\star\\w=w^\star}} & B_1 &:= \left.\frac{\partial f(x,u,w)}{\partial u}\right|_{\substack{x=x^\star\\u=u^\star\\w=w^\star}} & B_2 &:= \left.\frac{\partial f(x,u,w)}{\partial w}\right|_{\substack{x=x^\star\\u=u^\star\\w=w^\star}} \\
C &:= \left.\frac{\partial h(x,u,w)}{\partial x}\right|_{\substack{x=x^\star\\u=u^\star\\w=w^\star}} & D_1 &:= \left.\frac{\partial h(x,u,w)}{\partial u}\right|_{\substack{x=x^\star\\u=u^\star\\w=w^\star}} & D_2 &:= \left.\frac{\partial h(x,u,w)}{\partial w}\right|_{\substack{x=x^\star\\u=u^\star\\w=w^\star}} \\
C_e &:= \left.\frac{\partial h_e(x,u,w)}{\partial x}\right|_{\substack{x=x^\star\\u=u^\star\\w=w^\star}} & D_{e1} &:= \left.\frac{\partial h_e(x,u,w)}{\partial u}\right|_{\substack{x=x^\star\\u=u^\star\\w=w^\star}} & D_{e2} &:= \left.\frac{\partial h_e(x,u,w)}{\partial w}\right|_{\substack{x=x^\star\\u=u^\star\\w=w^\star}}
\end{aligned} \tag{1.6}$$

Neglecting second-order terms, the linearized system becomes the so-called *design model*:

$$\begin{cases}
\dot{\tilde{x}} = A\tilde{x} + B_1\tilde{u} + B_2\tilde{w}, \quad \tilde{x}(t_0) = x_0 - x^\star \\
\tilde{y} = C\tilde{x} + D_1\tilde{u} + D_2\tilde{w} \\
\tilde{e} = C_e\tilde{x} + D_{e1}\tilde{u} + D_{e2}\tilde{w}
\end{cases} \tag{1.7}$$

This Linear Time-Invariant (LTI) approximation of the nonlinear model is valid in a neighborhood of the equilibrium point, enabling efficient analysis and controller design under small perturbations.

Matrix calculus For the matrix calculation, the procedure described in equations (1.6) was followed. By substituting the equilibrium triplet given in (1.2), obtaining the following matrices:

$$A = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{2k}{m} & -\frac{2\beta}{m} & -\frac{(d_f-d_r)k}{m} & -\frac{(d_f-d_r)\beta}{m} & \frac{d_f k}{m} & -\frac{d_r k}{m} & \frac{d_f\beta}{m} & -\frac{d_r\beta}{m} \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
-\frac{(d_f-d_r)k}{J} & -\frac{(d_f-d_r)\beta}{J} & -\frac{k(d_f^2+d_r^2)}{J} & -\frac{\beta(d_f^2+d_r^2)}{J} & \frac{kd_f^2}{J} & \frac{kd_r^2}{J} & \frac{\beta d_f^2}{J} & \frac{\beta d_r^2}{J} \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} \tag{1.8}$$

$$B_1 = \begin{bmatrix} 0 & 0 \\ \frac{1}{m} & 0 \\ 0 & 0 \\ 0 & \frac{1}{J} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{1.9}$$

$$B_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\ell_0 - 0.177398}{J} & \frac{\ell_0 - 0.177398}{J} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \tag{1.10}$$

$$C = \begin{bmatrix} 0 & 0 & \frac{0.354795\,k}{m} & 0 & 0 & 0 & 0 & 0 \\ -\frac{2k}{m} & -\frac{2\beta}{m} & -\frac{(d_f - d_r)k}{m} & -\frac{(d_f - d_r)\beta}{m} & \frac{d_f k}{m} & -\frac{d_r k}{m} & \frac{d_f \beta}{m} & -\frac{d_r \beta}{m} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & d_f & 0 & -d_f & 0 & 0 & 0 \\ 1 & 0 & -d_r & 0 & 0 & d_r & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{1.11}$$

$$D_1 = \begin{bmatrix} 0 & 0 \\ \frac{1}{m} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{1.12}$$

$$D_2 = \begin{bmatrix} \frac{1}{m} & \frac{1}{m} & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{1.13}$$

$$\mathrm{CE} = \begin{bmatrix} 1 & 0 & 0 & 0 & -\frac{d_f d_r}{d_f + d_r} & \frac{d_f d_r}{d_f + d_r} & 0 & 0 \\ 0 & 0 & \frac{|m|(6.39142 \times 10^{15}\, k + 1.80144 \times 10^{16}\, u_1)}{m\,|6.39142 \times 10^{15}\, k + 1.80144 \times 10^{16}\, u_1|} & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{1.14}$$

$$\mathrm{DE}_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{1.15}$$

$$\mathrm{DE}_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{2.81853\,|m|}{m\,|k|} & \frac{2.81853\,|m|}{m\,|k|} & \frac{2.81853\,|m|}{|k|} & 0 & 0 & 0 & 0 \end{bmatrix} \tag{1.16}$$