ALMA MATER STUDIORUM
UNIVERSITÀ DI BOLOGNA

Automotive Simulation Models & Control Problem Formulation

Faculty of «Electronic Engineering for Intelligent Vehicles» and «Advanced Automotive Engineering»

Nicola Mimmo

Department of Electrical, Electronics and Information Engineering «G. Marconi» - University of Bologna

Contents

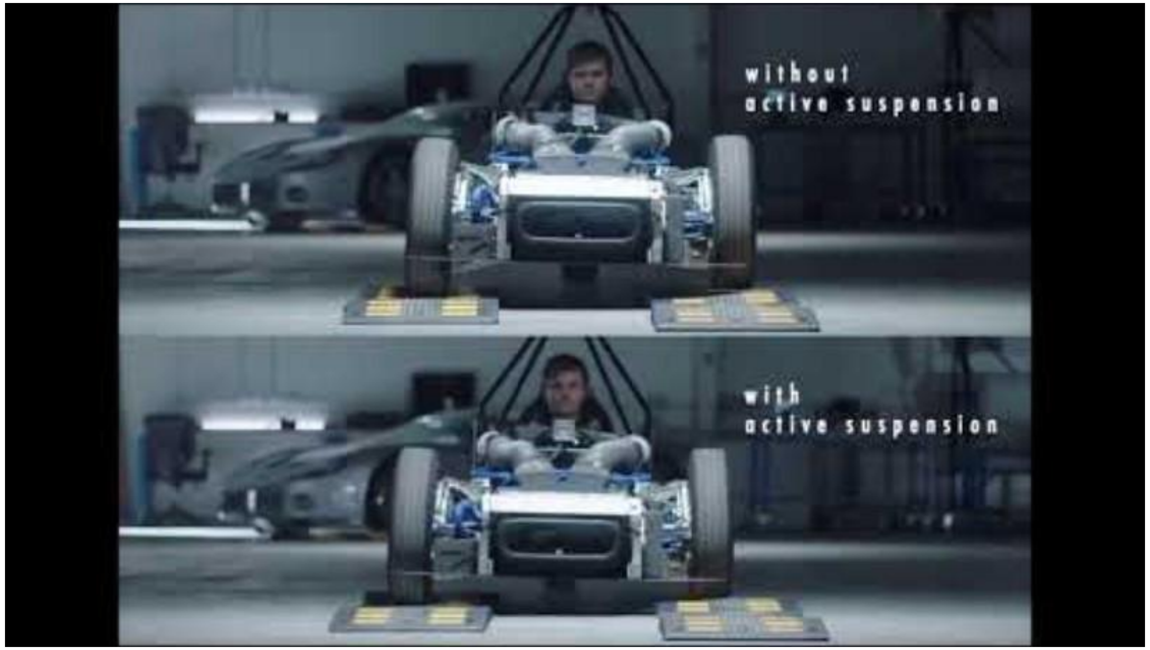
- Active Suspensions
- Adaptive Cruise Control
- Wheel Speed Controls
- Electronic Stability Program / Torque Vectoring
- Self-Park Assist

Contents

Active Suspensions

Motivations & Goals

Improve the ride quality and change the setup.



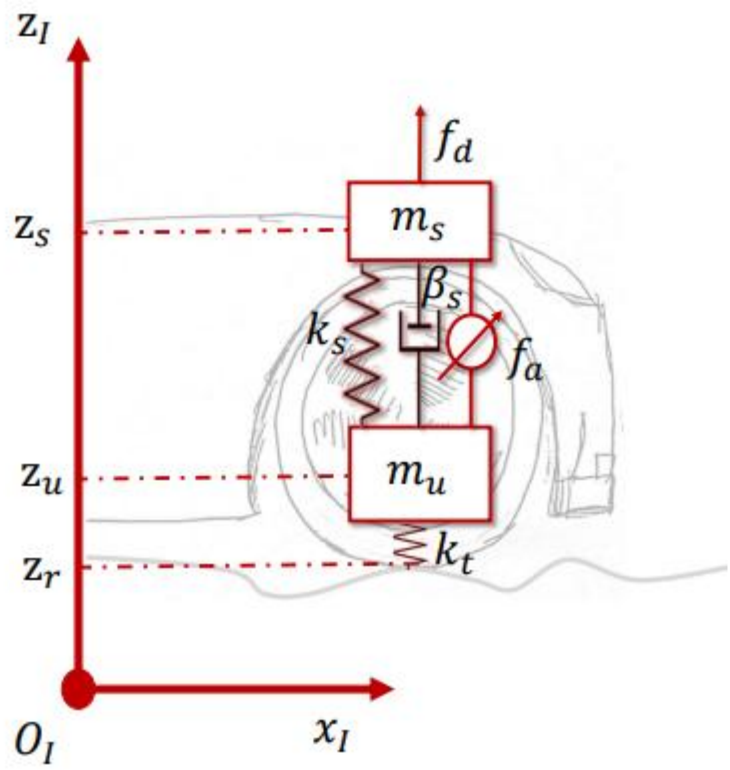
Active Suspensions

Motivations & Goals

Improve the ride quality and change the setup.

Active Suspensions

Single-corner Model



Sprung mass $$m_s \ddot{z}_s = -m_s g + f_s + f_d$$

Unsprung mass $$m_u \ddot{z}_u = -m_u g - f_s + f_t(z_u - z_r)$$

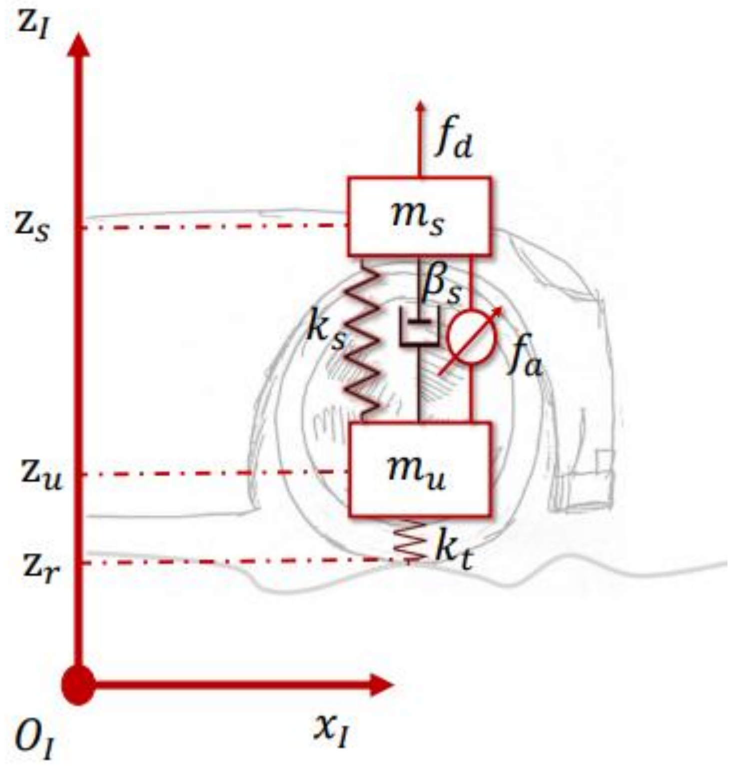
Tire $$f_t(z_u - z_r) = \begin{cases} 0 & z_u - z_r > \ell_t \\ -k_t(z_u - z_r - \ell_t) & z_u - z_r \leq \ell_t \end{cases}$$

Suspension $$f_s := -k_s(z_s - z_u - \ell_s) - \beta_s(\dot{z}_s - \dot{z}_u) + f_a$$

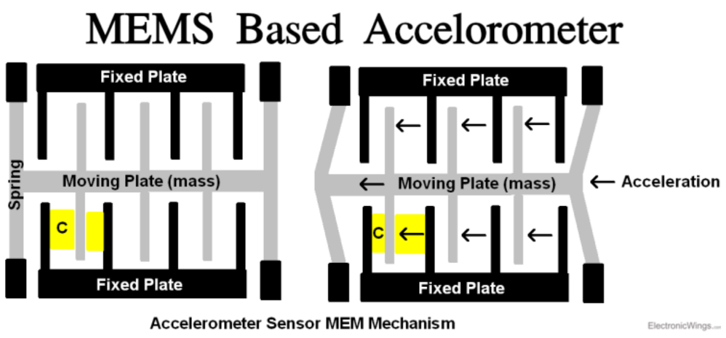
Aerodynamics (downforce) $$f_d = \frac{1}{2} \rho S v^2 C_z$$

Active Suspensions

Single-corner Model



Sensors

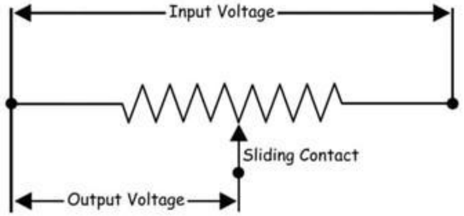


Accelerometer (z)

$$y_a = -g - \frac{k_s}{m_s}(z_s - z_u - \ell_s) - \frac{\beta_s}{m_s}(\dot{z}_s - \dot{z}_u) + \frac{f_a + f_d}{m_s} + \nu_a$$

Potentiometer

$$y_p = z_s - z_u + \nu_p$$



Regulated Output

$$e = y_p - r$$

Active Suspensions Problem Formulation

let $\dot{z}_s := v_s$ and $\dot{z}_u := v_u$

$$
\begin{aligned}
\dot{z}_s &= v_s \\
m_s \dot{v}_s &= -m_s g - k_s(z_s - z_u - \ell_s) - \beta_s(v_s - v_u) + f_a + f_d \\
\dot{z}_u &= v_u \\
m_u \dot{v}_u &= -m_u g + k_s(z_s - z_u - \ell_s) + \beta_s(v_s - v_u) - f_a + f_t(z_u - z_r)
\end{aligned}
$$

Active Suspensions Problem Formulation

Define

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} := \begin{bmatrix} z_s - z_u \\ v_s - v_u \\ z_u - z_r \\ v_u - \dot{z}_r \end{bmatrix}$$

let $\dot{z}_s := v_s$ and $\dot{z}_u := v_u$

$$\begin{aligned} \dot{z}_s &= v_s \\ m_s \dot{v}_s &= -m_s g - k_s(z_s - z_u - \ell_s) - \beta_s(v_s - v_u) + f_a + f_d \\ \dot{z}_u &= v_u \\ m_u \dot{v}_u &= -m_u g + k_s(z_s - z_u - \ell_s) + \beta_s(v_s - v_u) - f_a + f_t(z_u - z_r) \end{aligned}$$

Active Suspensions Problem Formulation

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -\frac{m_s + m_u}{m_s m_u} k_s(x_1 - \ell_s) - \frac{m_s + m_u}{m_s m_u} \beta_s x_2 + \frac{m_s + m_u}{m_s m_u} f_a + \frac{1}{m_s} f_d - \frac{1}{m_u} f_t(x_3)$$
$$\dot{x}_3 = x_4$$
$$\dot{x}_4 = -g + \frac{k_s}{m_u}(x_1 - \ell_s) + \frac{\beta_s}{m_u} x_2 - \frac{f_a}{m_u} + \frac{1}{m_u} f_t(x_3) - \ddot{z}_r$$

Define

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} := \begin{bmatrix} z_s - z_u \\ v_s - v_u \\ z_u - z_r \\ v_u - \dot{z}_r \end{bmatrix}$$

let $\dot{z}_s := v_s$ and $\dot{z}_u := v_u$

$$\dot{z}_s = v_s$$
$$m_s \dot{v}_s = -m_s g - k_s(z_s - z_u - \ell_s) - \beta_s(v_s - v_u) + f_a + f_d$$
$$\dot{z}_u = v_u$$
$$m_u \dot{v}_u = -m_u g + k_s(z_s - z_u - \ell_s) + \beta_s(v_s - v_u) - f_a + f_t(z_u - z_r)$$

Active Suspensions Problem Formulation

$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -\frac{m_s + m_u}{m_s m_u} k_s (x_1 - \ell_s) - \frac{m_s + m_u}{m_s m_u} \beta_s x_2 + \frac{m_s + m_u}{m_s m_u} f_a + \frac{1}{m_s} f_d - \frac{1}{m_u} f_t(x_3) \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= -g + \frac{k_s}{m_u}(x_1 - \ell_s) + \frac{\beta_s}{m_u} x_2 - \frac{f_a}{m_u} + \frac{1}{m_u} f_t(x_3) - \ddot{z}_r
\end{aligned}
$$

$$
\begin{aligned}
y_p &= x_1 + \nu_p \\
y_a &= -g - \frac{k_s}{m_s}(x_1 - \ell_s) - \frac{\beta_s}{m_s} x_2 + \frac{f_a + f_d}{m_s} + \nu_a
\end{aligned}
$$

$$
e = x_1 + \nu_p - r.
$$

Active Suspensions Problem Formulation

$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -\frac{m_s+m_u}{m_s m_u} k_s(x_1-\ell_s) - \frac{m_s+m_u}{m_s m_u}\beta_s x_2 + \frac{m_s+m_u}{m_s m_u} f_a + \frac{1}{m_s} f_d - \frac{1}{m_u} f_t(x_3) \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= -g + \frac{k_s}{m_u}(x_1-\ell_s) + \frac{\beta_s}{m_u} x_2 - \frac{f_a}{m_u} + \frac{1}{m_u} f_t(x_3) - \ddot{z}_r
\end{aligned}
$$

$$
\begin{aligned}
y_p &= x_1 + \nu_p \\
y_a &= -g - \frac{k_s}{m_s}(x_1-\ell_s) - \frac{\beta_s}{m_s} x_2 + \frac{f_a+f_d}{m_s} + \nu_a
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{x} &= \mathrm{col}(x_1, x_2, x_3, x_4), \quad u = f_a, \\
\mathbf{d} &= \mathrm{col}(\ddot{z}_r,\ f_d), \\
\boldsymbol{\nu} &= \mathrm{col}(\nu_p, \nu_a), \\
\mathbf{w} &= \mathrm{col}(\mathbf{d}, \boldsymbol{\nu}, r),
\end{aligned}
$$

$$
e = x_1 + \nu_p - r.
$$

$$
\begin{aligned}
\dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}, u, \mathbf{w}) \qquad \mathbf{x}(t_0) = \mathbf{x}_0 \\
\mathbf{y} &= \mathbf{h}(\mathbf{x}, u, \mathbf{w}) \\
e &= h_e(\mathbf{x}, u, \mathbf{w}),
\end{aligned}
$$

Active Suspensions Problem Formulation

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\frac{m_s+m_u}{m_s m_u}k_s(x_1-\ell_s) - \frac{m_s+m_u}{m_s m_u}\beta_s x_2 + \frac{m_s+m_u}{m_s m_u}f_a + \frac{1}{m_s}f_d - \frac{1}{m_u}f_t(x_3)$$

$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = -g + \frac{k_s}{m_u}(x_1-\ell_s) + \frac{\beta_s}{m_u}x_2 - \frac{f_a}{m_u} + \frac{1}{m_u}f_t(x_3) - \ddot{z}_r$$

$$y_p = x_1 + \nu_p$$

$$y_a = -g - \frac{k_s}{m_s}(x_1-\ell_s) - \frac{\beta_s}{m_s}x_2 + \frac{f_a+f_d}{m_s} + \nu_a$$

$$\begin{aligned} \mathbf{x} &= \mathrm{col}(x_1, x_2, x_3, x_4), \quad u = f_a, \\ \mathbf{d} &= \mathrm{col}(\ddot{z}_r,\ f_d), \\ \boldsymbol{\nu} &= \mathrm{col}(\nu_p, \nu_a), \\ \mathbf{w} &= \mathrm{col}(\mathbf{d}, \boldsymbol{\nu}, r), \end{aligned}$$

$$e = x_1 + \nu_p - r.$$

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}, u, \mathbf{w}) \qquad \mathbf{x}(t_0) = \mathbf{x}_0 \\ \mathbf{y} &= \mathbf{h}(\mathbf{x}, u, \mathbf{w}) \\ e &= h_e(\mathbf{x}, u, \mathbf{w}), \end{aligned}$$

Control Goals

1) Keep all the signals bounded
2) Asymptotically steer the regulated output to zero assuming constant disturbances.

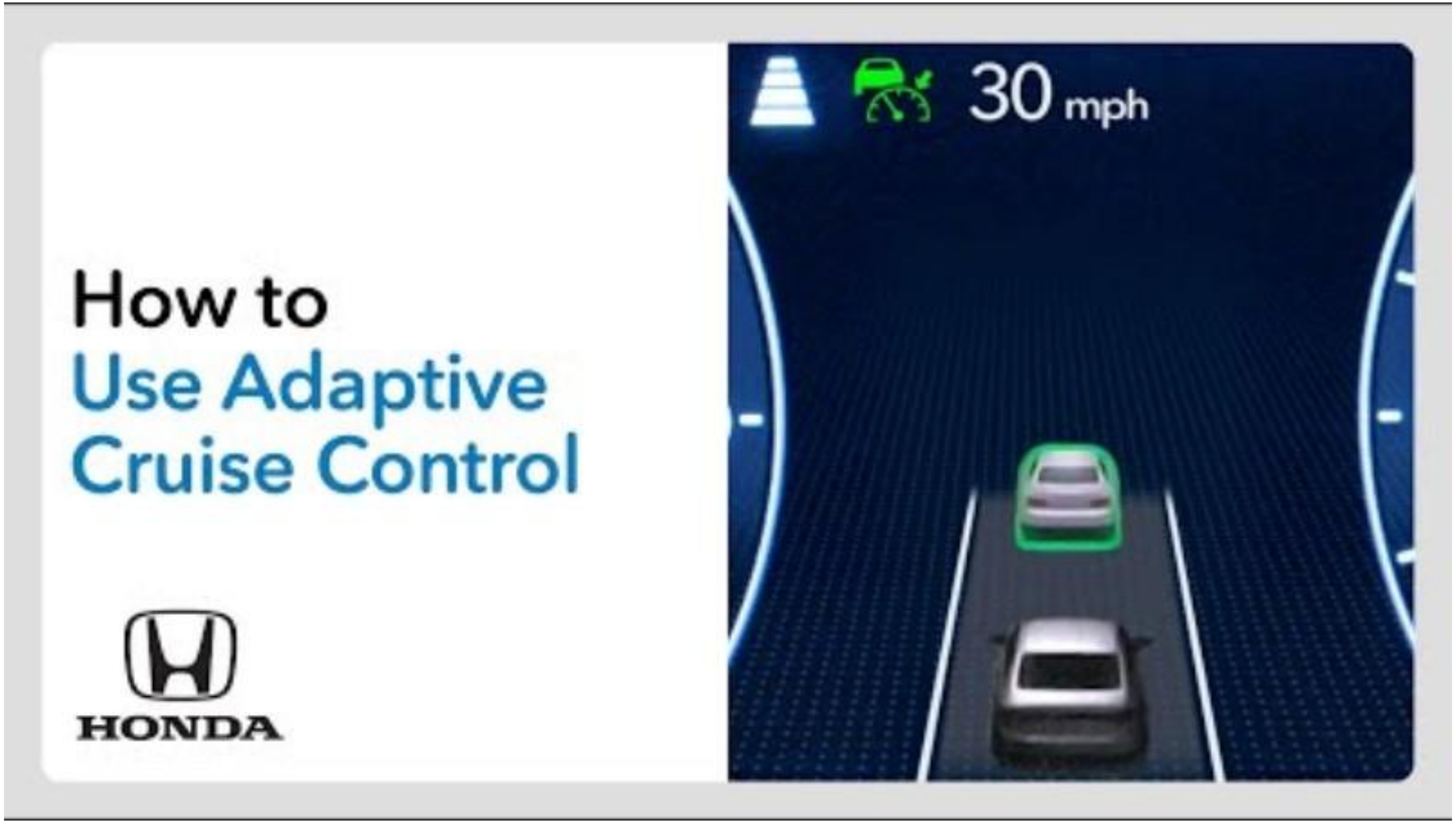
Contents

ALMA MATER STUDIORUM
UNIVERSITÀ DI BOLOGNA

Adaptive Cruise Control

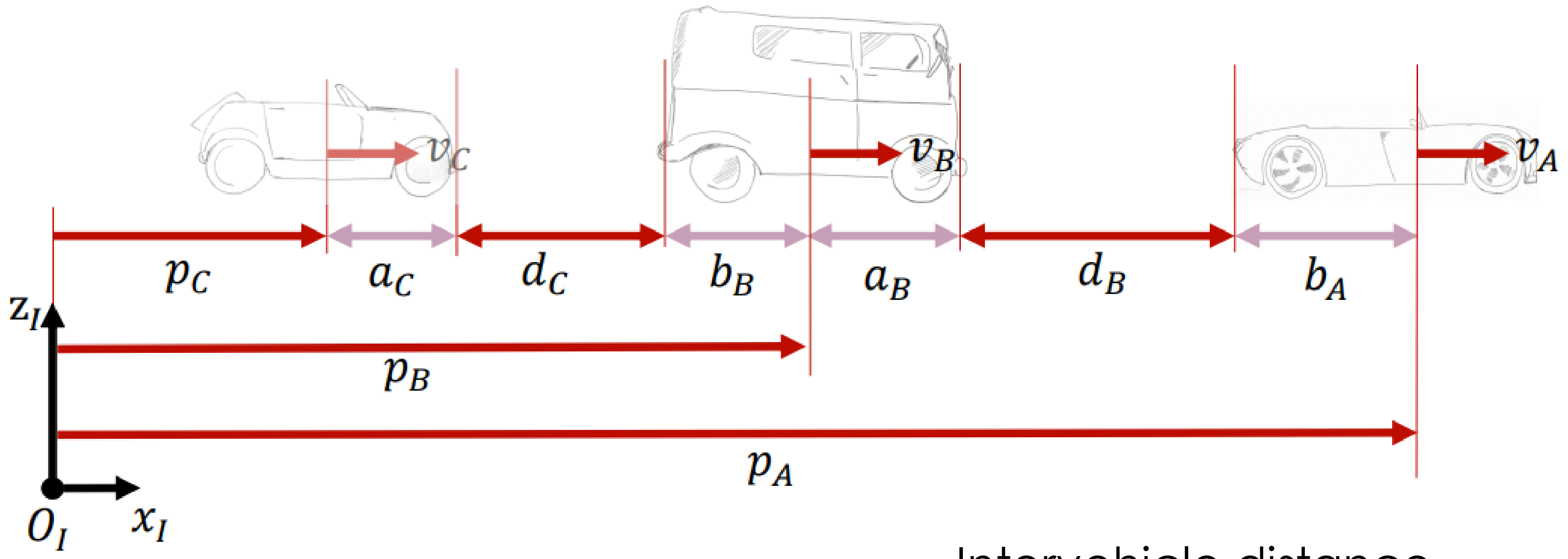
Motivations & Goals

Improve safety and comfort, reduce driver's workload.



Adaptive Cruise Control

1D Model



Platoon dynamics

$$\dot{p}_A = v_A$$
$$\dot{p}_B = v_B$$
$$\dot{v}_B = g\sin\theta + \frac{1}{m_B}(f_B - D_B(v_B - w))$$
$$\dot{p}_C = v_C$$
$$\dot{v}_C = g\sin\theta + \frac{1}{m_C}(f_C - D_C(v_C - w))$$

Intervehicle distance

$$d_B := p_A - p_B - b_A - a_B$$
$$d_C := p_B - p_C - b_B - a_C$$

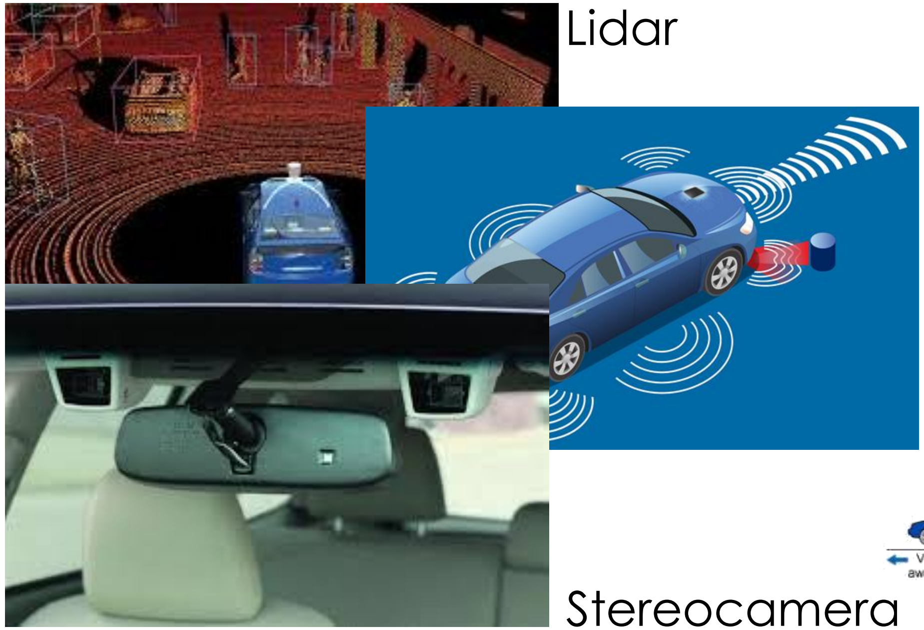
Aerodynamic Drag

$$D_B(v_B - w) = \frac{1}{2}\rho S_B C_{D_B}(v_B - w)^2$$
$$D_C(v_C - w) = \frac{1}{2}\rho S_C C_{D_C}(v_C - w)^2$$

Adaptive Cruise Control

1D Model

Sensors

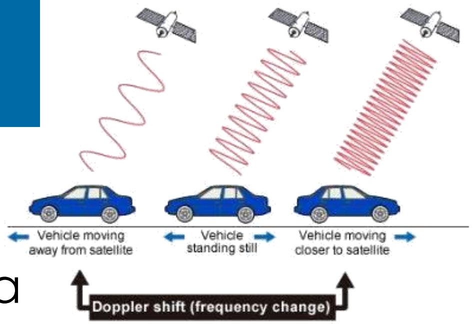


Lidar

Sonar

Stereocamera

GNSS



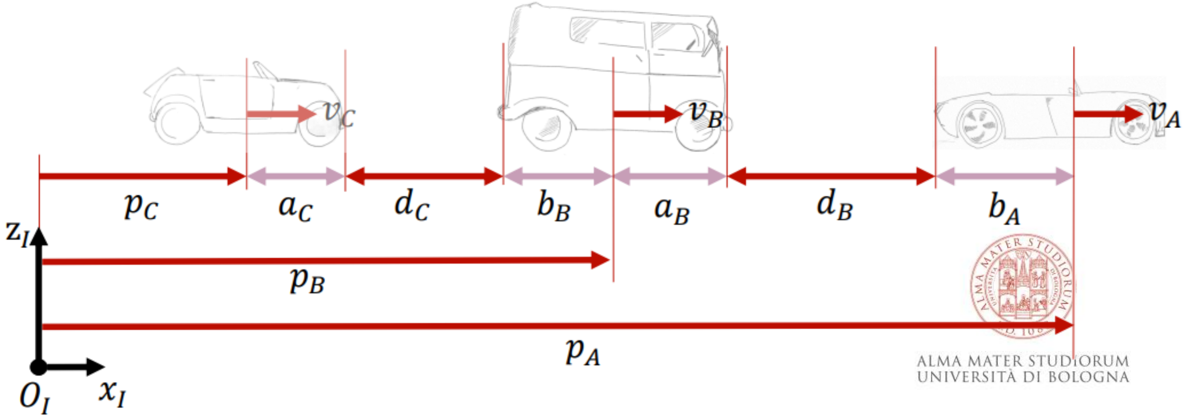
$$
\begin{aligned}
y_{dB} &= d_B + \nu_{dB} && \text{range sensor vehicle B} \\
y_{vB} &= v_B + \nu_{vB} && \text{GNSS vehicle B} \\
y_{dC} &= d_C + \nu_{dC} && \text{range sensor vehicle C} \\
y_{vC} &= v_C + \nu_{vC} && \text{GNSS vehicle C,}
\end{aligned}
$$

Regulated output

$$
\begin{aligned}
e_{dB} &= y_{dB} - d_B^\star \\
e_{dC} &= y_{dC} - d_C^\star
\end{aligned}
$$

Safety distance

$$
\begin{aligned}
d_B^\star &= d_{\min} + \frac{1}{k} v_B^2 \qquad k > 0 \\
d_C^\star &= d_{\min} + \frac{1}{k} v_C^2,
\end{aligned}
$$



Adaptive Cruise Control Problem Formulation

$$\text{let } \mathbf{x} := \mathrm{col}(d_B, v_B, d_C, v_C), \; \mathbf{u} := \mathrm{col}(f_B, f_C), \; \mathbf{d} := \mathrm{col}(v_A, \sin\theta, w),$$
$$\boldsymbol{\nu} := \mathrm{col}(\nu_{dB}, \nu_{vB}, \nu_{dC}, \nu_{vC}), \mathbf{w} := \mathrm{col}(\mathbf{d}, \boldsymbol{\nu}), \mathbf{y} := \mathrm{col}(y_{dB}, y_{vB}, y_{dC}, y_{vC}), \text{ and } \mathbf{e} := \mathrm{col}(e_{dB}, e_{dC}).$$

Then

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}, \mathbf{w}) \qquad \mathbf{x}(t_0) = \mathbf{x}_0$$
$$\mathbf{y} = \mathbf{h}(\mathbf{x}, \mathbf{u}, \mathbf{w})$$
$$\mathbf{e} = \mathbf{h}_e(\mathbf{x}, \mathbf{u}, \mathbf{w})$$

where

$$\mathbf{f}(\mathbf{x}, \mathbf{u}, \mathbf{w}) = \begin{bmatrix} v_A - v_B \\ g\sin\theta + \left(f_B - 1/2\rho S_B C_{D_B}(v_B - w)^2\right)/m_B \\ v_B - v_C \\ g\sin\theta + \left(f_C - 1/2\rho S_C C_{D_C}(v_C - w)^2\right)/m_C \end{bmatrix}$$

$$\mathbf{h}(\mathbf{x}, \mathbf{u}, \mathbf{w}) = \begin{bmatrix} d_B + \nu_{dB} \\ v_B + \nu_{vB} \\ d_C + \nu_{dC} \\ v_C + \nu_{vC} \end{bmatrix}$$

$$\mathbf{h}_e(\mathbf{x}, \mathbf{u}, \mathbf{w}) = \begin{bmatrix} y_{dB} - d_{\min} - y_{vB}^2/k \\ y_{dC} - d_{\min} - y_{vC}^2/k \end{bmatrix}.$$

Control Goals

1) Keep all the signals bounded
2) Asymptotically steer the regulated output to zero assuming constant disturbances.

Contents

- Active Suspensions
- Adaptive Cruise Control
- Wheel Speed Controls
- Electronic Stability Program / Torque Vectoring
- Self-Park Assist

Wheel Speed Controls

Motivations & Goals

How do ground vehicles move?

Wheel Speed Controls

Motivations & Goals

How do ground vehicles move?

- Ground **vehicle performance** is related to the **tire-road friction**
- The tire-road friction depends on the **wheel hub speed** and the **wheel rotational speed** (among others such as temperature, road conditions, inflation pressure, tire wearing, vertical load, …)
- Therefore, controlling the wheel rotational speed let us **control the vehicle traction/braking forces**

Wheel Speed Controls

Goals

We can regulate the wheel speed to

Wheel Speed Controls

Goals

We can regulate the wheel speed to

Aim at the desired traction performance (Traction Control)

Wheel Speed Controls

Goals

We can regulate the wheel speed to

Aim at the desired traction performance (Traction Control)

Aim at the best traction performance (Launch Control)

Wheel Speed Controls

Goals

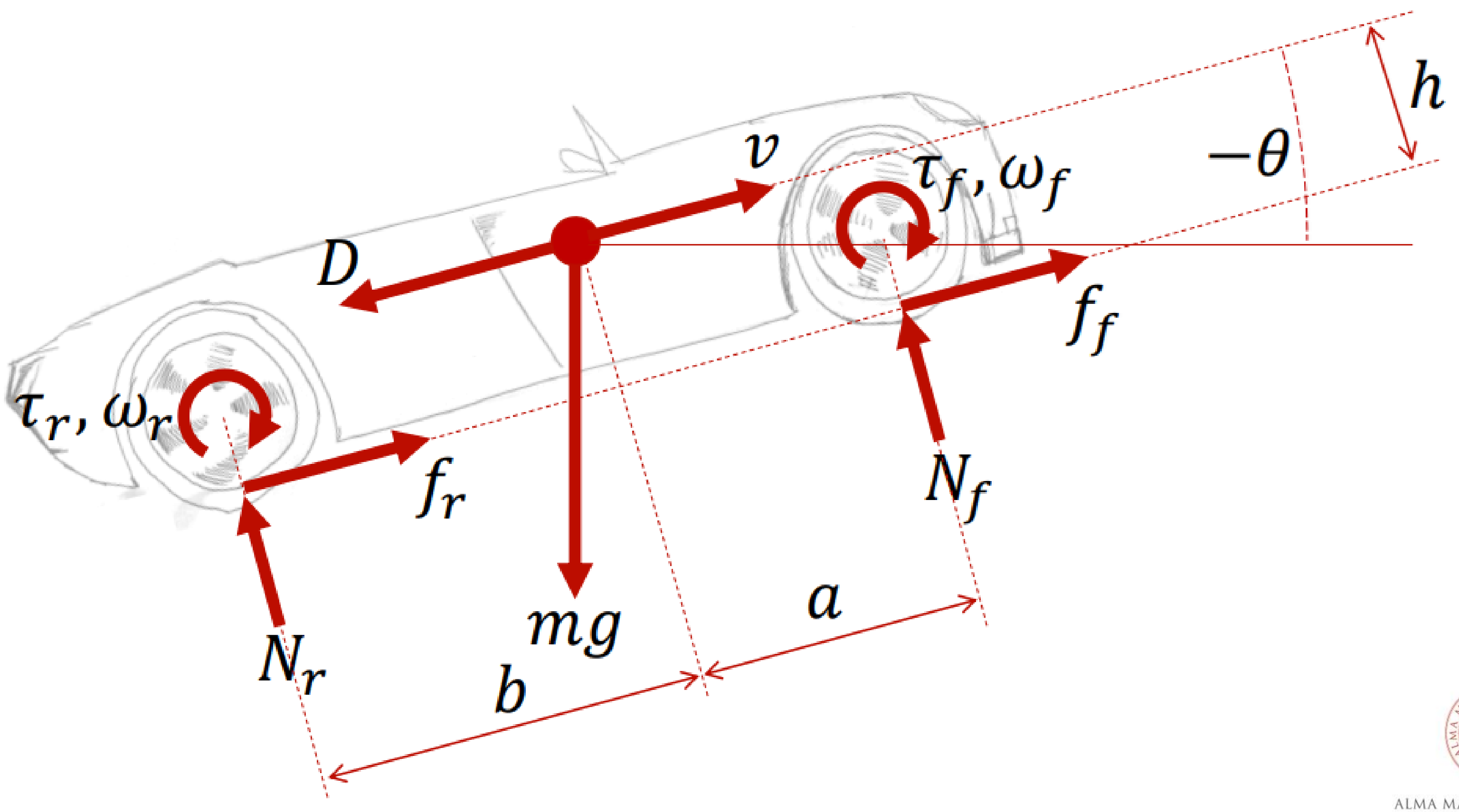
We can regulate the wheel speed to

Aim at the desired traction performance (Traction Control)

Aim at the best traction performance (Launch Control)

Aim at the best braking performance (Anti-lock Braking System)

Wheel Speed Controls 2D Model

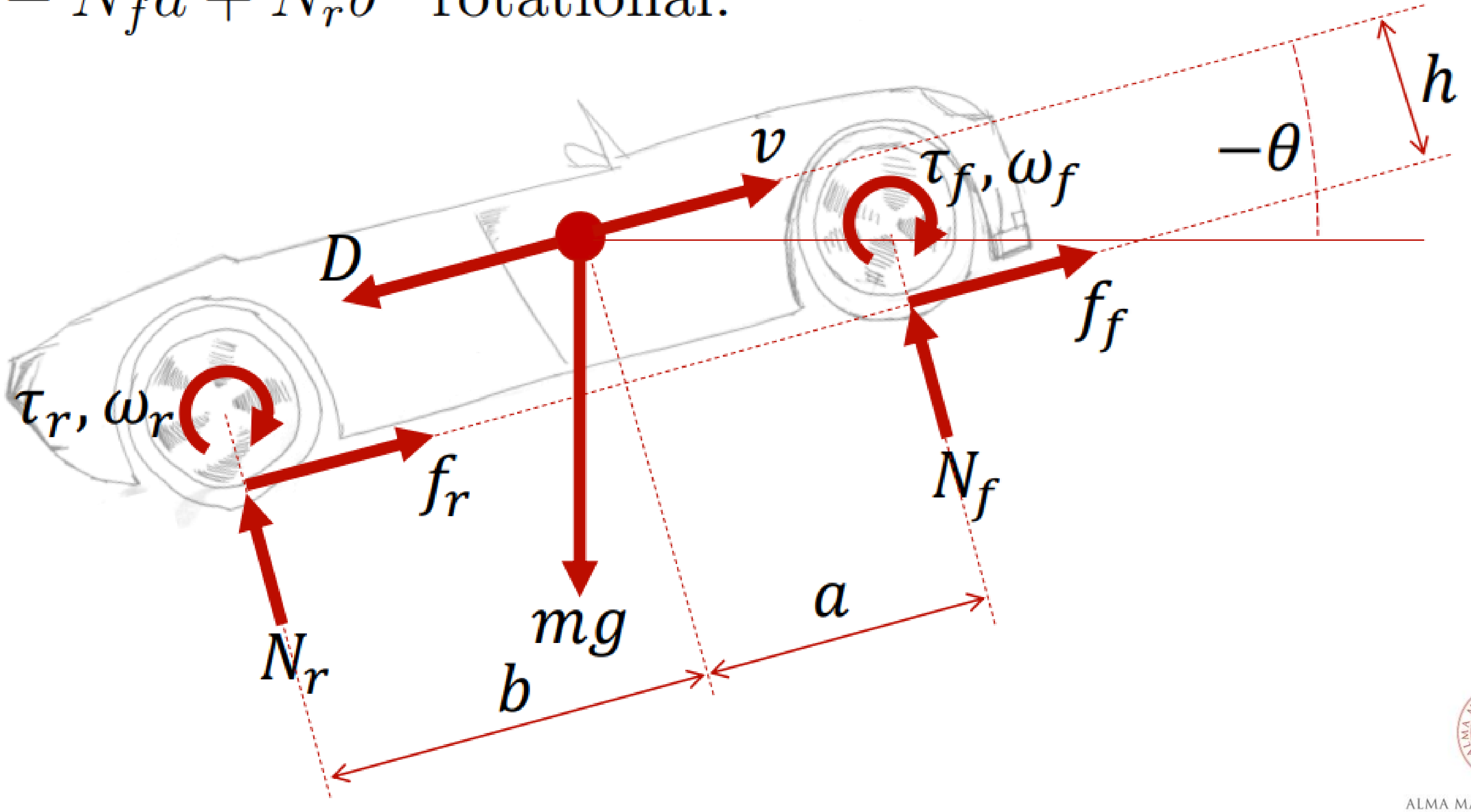


Wheel Speed Controls 2D Model

$$
\begin{aligned}
m\dot{v} &= -\,mg\sin\theta + f_f + f_r - D && \text{longitudinal}\\
0 &= -\,mg\cos\theta + N_f + N_r && \text{vertical}\\
0 &= -\,(f_f + f_r)h - N_f a + N_r b && \text{rotational.}
\end{aligned}
$$

$$J_r\dot{\omega}_r = \tau_r - f_r r_r$$

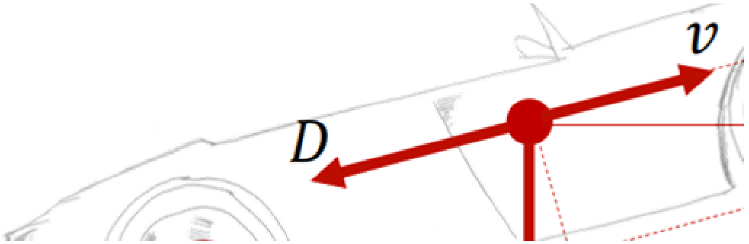
$$J_f\dot{\omega}_f = \tau_f - f_f r_f$$



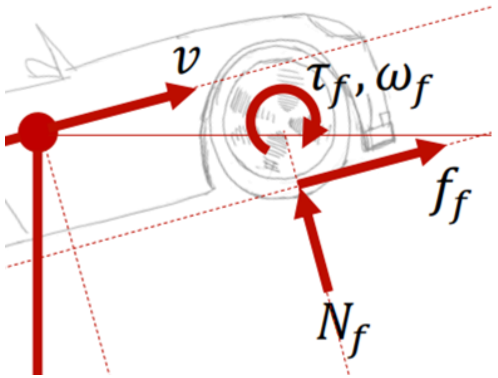
Wheel Speed Controls 2D Model

Aerodynamic Drag

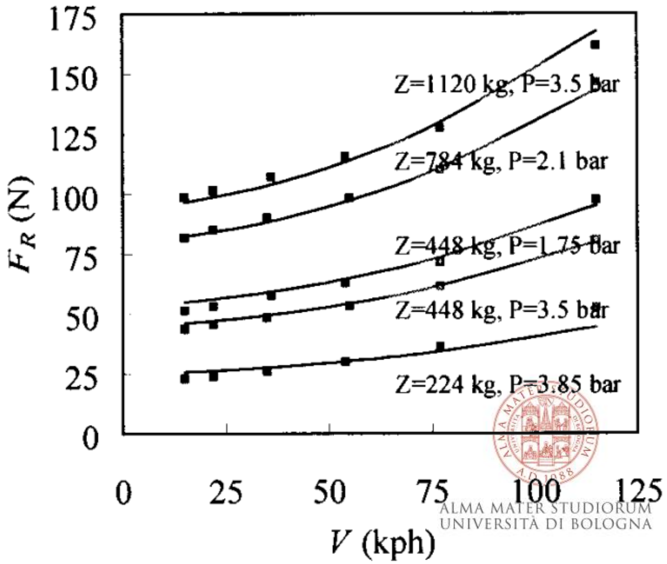
$$D(v-w) = \frac{1}{2}\rho S \frac{(v-w)^3}{|v-w|} C_D$$



Tire Forces

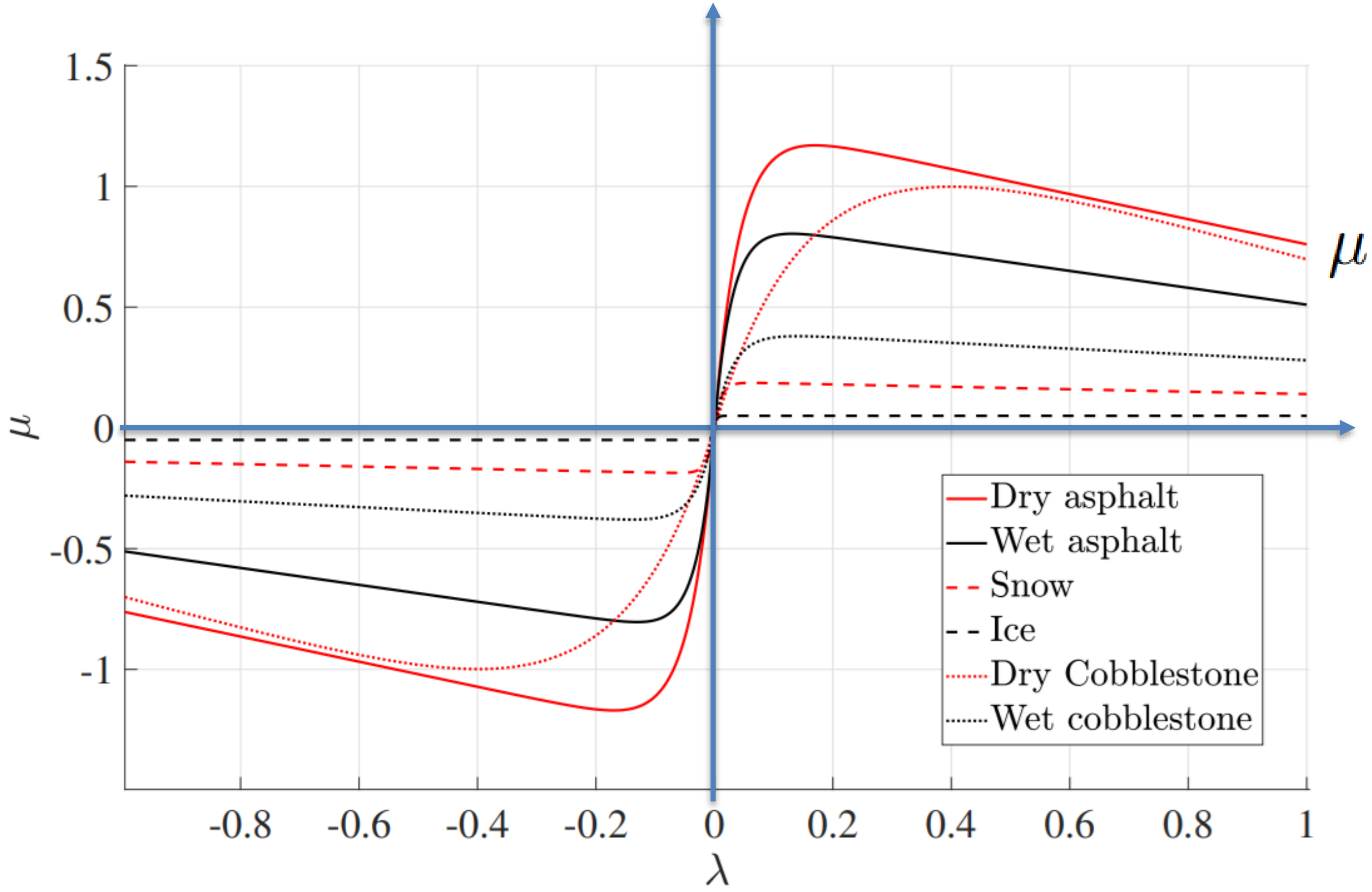


$$f_r = N_r(\mu(\lambda(v, \omega_r r_r), \Theta) - c_r(v))$$
$$f_f = N_f(\mu(\lambda(v, \omega_f r_f), \Theta) - c_r(v))$$



Rolling Resistance $\quad c_r(v) = c_{r0} + c_{r1}v + c_{r2}v^2$

Wheel Speed Controls 2D Model



Traction Coefficient (Burckhardt)

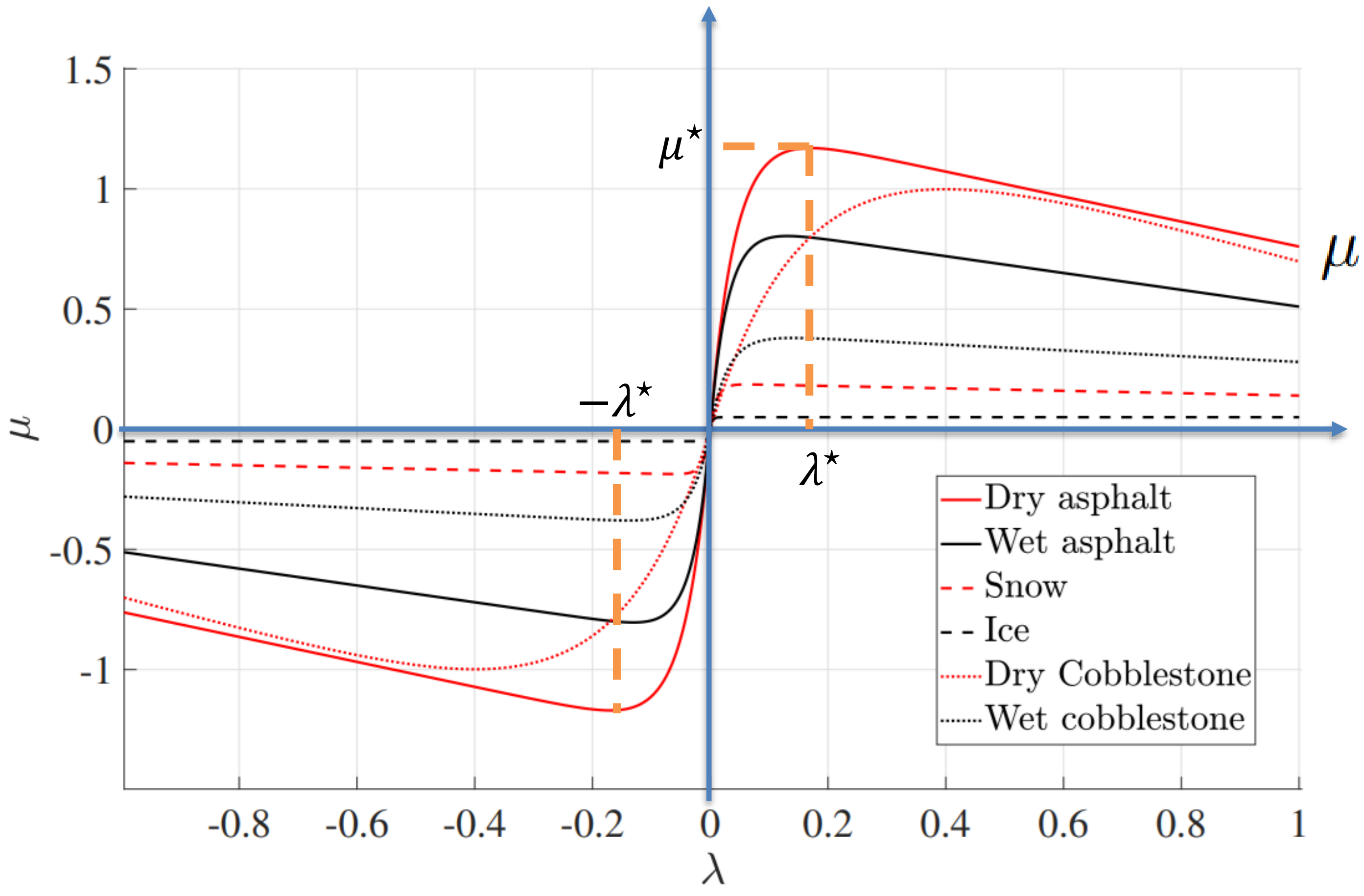
$$\mu(\lambda, \boldsymbol{\Theta}) = \mathtt{sign}(\lambda)\theta_1 \left(1 - e^{-|\lambda|\theta_2}\right) - \lambda\theta_3$$

$$\boldsymbol{\Theta} := \mathrm{col}(\theta_1, \theta_2, \theta_3) \in \mathbb{R}^3$$

Longitudinal Slip Ratio

$$\lambda(v, \omega r) = \frac{\omega r - v}{\epsilon + \max\{|v|, |\omega r|, |\omega_r - v|\}}$$

Wheel Speed Controls 2D Model



Traction Coefficient (Burckhardt)

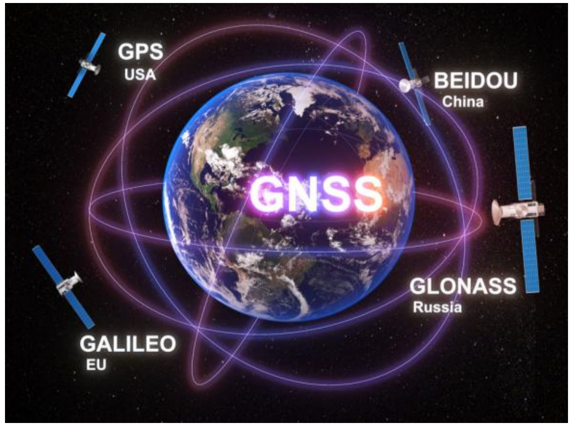
$$\mu(\lambda, \mathbf{\Theta}) = \mathtt{sign}(\lambda)\theta_1 \left(1 - e^{-|\lambda|\theta_2}\right) - \lambda\theta_3$$

$$\mathbf{\Theta} := \mathrm{col}(\theta_1, \theta_2, \theta_3) \in \mathbb{R}^3$$

Longitudinal Slip Ratio

$$\lambda(v, \omega r) = \frac{\omega r - v}{\epsilon + \max\{|v|, |\omega r|, |\omega_r - v|\}}$$

Wheel Speed Controls 2D Model

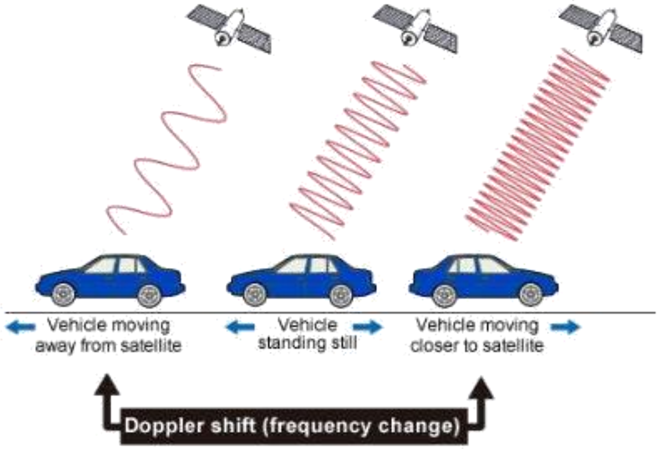


Sensors

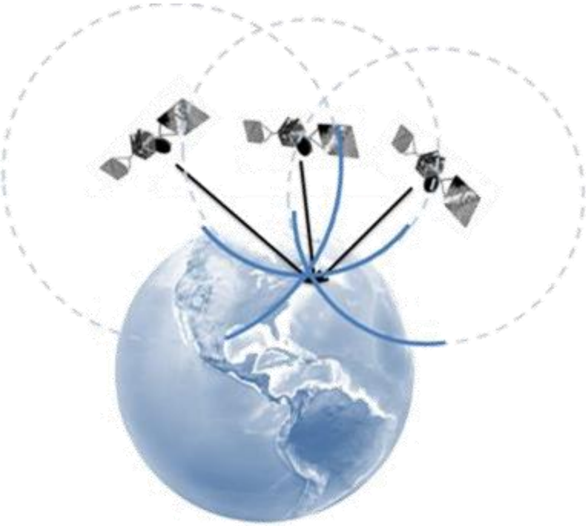
$$y_v = v + \nu_v \qquad \text{GNSS receiver}$$

$$y_r = \omega_r(1 + \nu_r) \qquad \text{rear tonewheel}$$

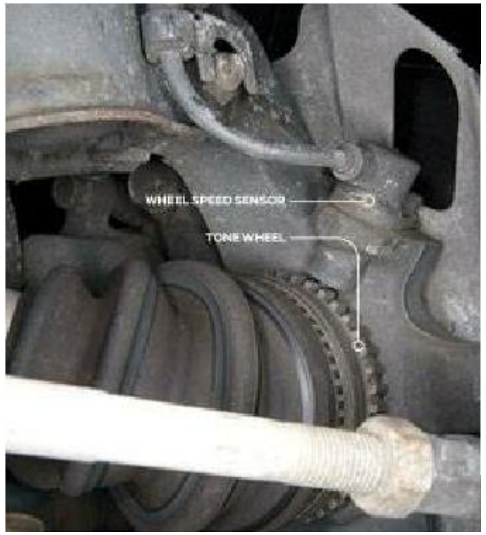
$$y_f = \omega_f(1 + \nu_f) \qquad \text{front tonewheel}$$

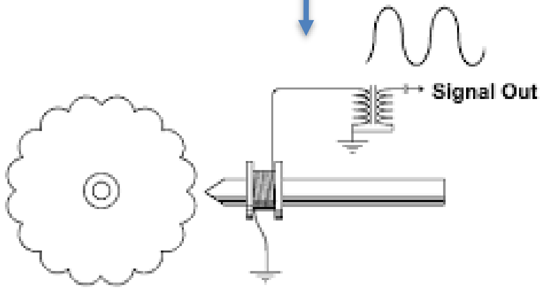


Velocity via Doppler Effect



Position via Trilateration





Wheel Speed Controls Problem Formulation

$$
\begin{aligned}
m\dot{v} &= -\,mg\sin\theta + f_f + f_r - D && \text{longitudinal}\\
0 &= -\,mg\cos\theta + N_f + N_r && \text{vertical}\\
0 &= -\,(f_f + f_r)h - N_f a + N_r b && \text{rotational.}
\end{aligned}
$$

Wheel Speed Controls Problem Formulation

$$
\begin{aligned}
m\dot{v} &= -\,mg\sin\theta + f_f + f_r - D && \text{longitudinal}\\
0 &= -\,mg\cos\theta + N_f + N_r && \text{vertical}\\
0 &= -\,(f_f + f_r)h - N_f a + N_r b && \text{rotational.}
\end{aligned}
$$

$$
\begin{bmatrix} N_r \\ N_f \end{bmatrix} = \frac{mg\cos\theta}{1-(\mu_r-\mu_f)\bar{h}} \begin{bmatrix} (\mu_f - c_r)\bar{h} + \bar{a} \\ -(\mu_r - c_r)\bar{h} + \bar{b} \end{bmatrix}
$$

Wheel Speed Controls Problem Formulation

$$
\begin{aligned}
m\dot{v} &= -\,mg\sin\theta + f_f + f_r - D && \text{longitudinal} \\
0 &= -\,mg\cos\theta + N_f + N_r && \text{vertical} \\
0 &= -\,(f_f + f_r)h - N_f a + N_r b && \text{rotational.}
\end{aligned}
$$

$$
\begin{bmatrix} N_r \\ N_f \end{bmatrix} = \frac{mg\cos\theta}{1-(\mu_r-\mu_f)\bar{h}} \begin{bmatrix} (\mu_f - c_r)\bar{h} + \bar{a} \\ -(\mu_r - c_r)\bar{h} + \bar{b} \end{bmatrix}
$$

$$
\dot{v} = -g\sin\theta + \frac{N_f}{m}(\mu_f - c_r) + \frac{N_r}{m}(\mu_r - c_r) - \frac{D}{m}
$$

Wheel Speed Controls Problem Formulation

$$\begin{aligned} m\dot{v} &= -mg\sin\theta + f_f + f_r - D && \text{longitudinal} \\ 0 &= -mg\cos\theta + N_f + N_r && \text{vertical} \\ 0 &= -(f_f + f_r)h - N_f a + N_r b && \text{rotational.} \end{aligned}$$

$$\begin{bmatrix} N_r \\ N_f \end{bmatrix} = \frac{mg\cos\theta}{1-(\mu_r-\mu_f)\bar{h}} \begin{bmatrix} (\mu_f - c_r)\bar{h} + \bar{a} \\ -(\mu_r - c_r)\bar{h} + \bar{b} \end{bmatrix}$$

$$\dot{v} = -g\sin\theta + \frac{N_f}{m}(\mu_f - c_r) + \frac{N_r}{m}(\mu_r - c_r) - \frac{D}{m}$$

$$\dot{\omega}_r = (\tau_r - f_r r_r)/J_r$$

$$\dot{\omega}_f = (\tau_f - f_f r_f)/J_f$$

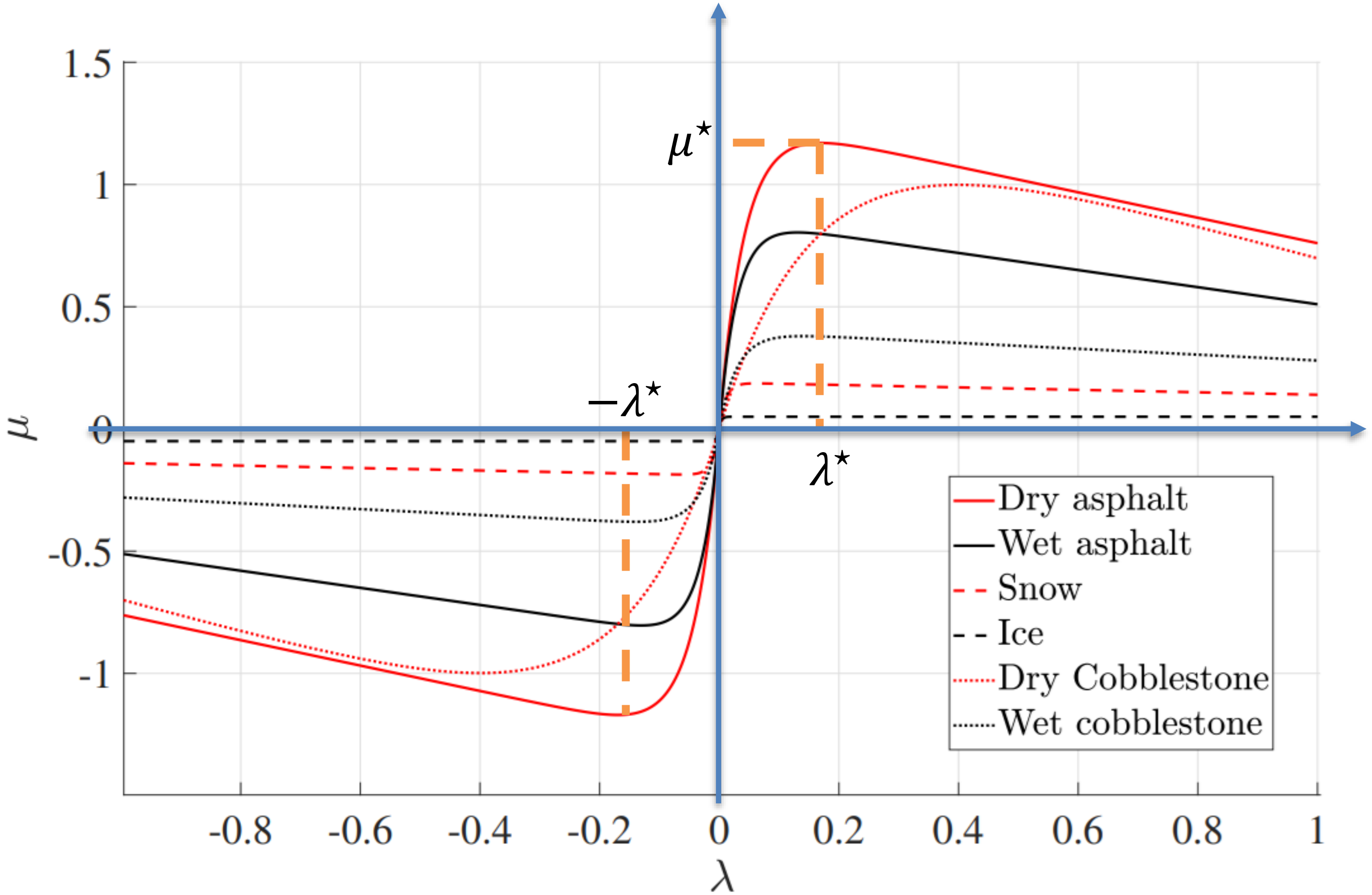
$$f_r = N_r(\mu(\lambda(v, \omega_r r_r), \Theta) - c_r(v))$$

$$f_f = N_f(\mu(\lambda(v, \omega_f r_f), \Theta) - c_r(v))$$

Wheel Speed Controls Problem Formulation

$$\lambda(v, \omega r) = \frac{\omega r - v}{\epsilon + \max\{|v|, |\omega r|, |\omega_r - v|\}}$$

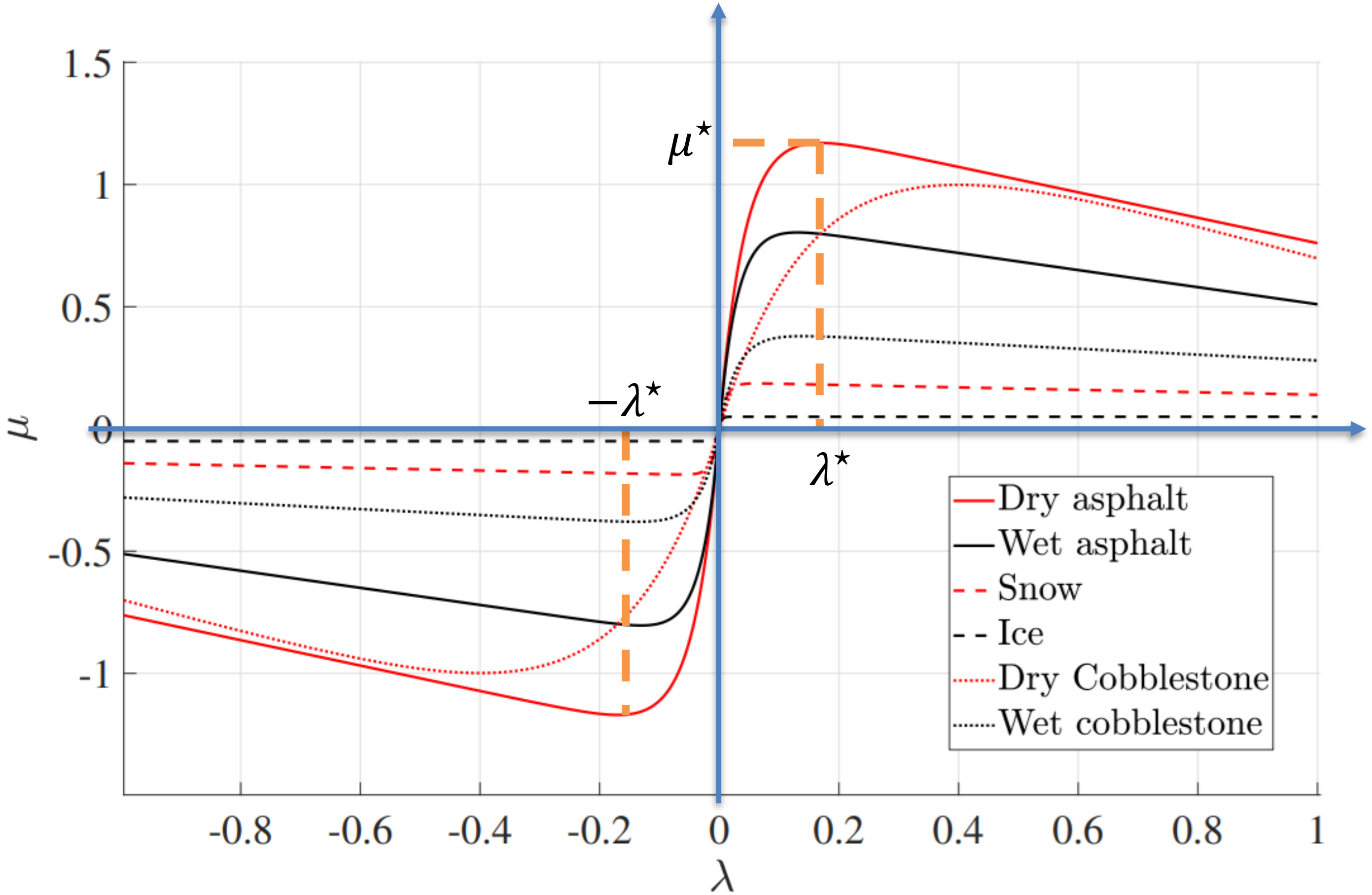
$$\begin{cases} \omega_{\#}^{r} = \dfrac{v}{(1-\lambda)r} & \text{Driving} \\ \omega_{\#}^{r} = (1+\lambda)\dfrac{v}{r} & \text{Braking} \end{cases}$$



Wheel Speed Controls Problem Formulation

$$\lambda(v,\omega r) = \frac{\omega r - v}{\epsilon + \max\{|v|, |\omega r|, |\omega_r - v|\}}$$

$$\begin{cases} \omega_{\#}^{r} = \dfrac{v}{(1-\lambda)r} & \text{Driving} \\ \omega_{\#}^{r} = (1+\lambda)\dfrac{v}{r} & \text{Braking} \end{cases}$$



Regulated Output

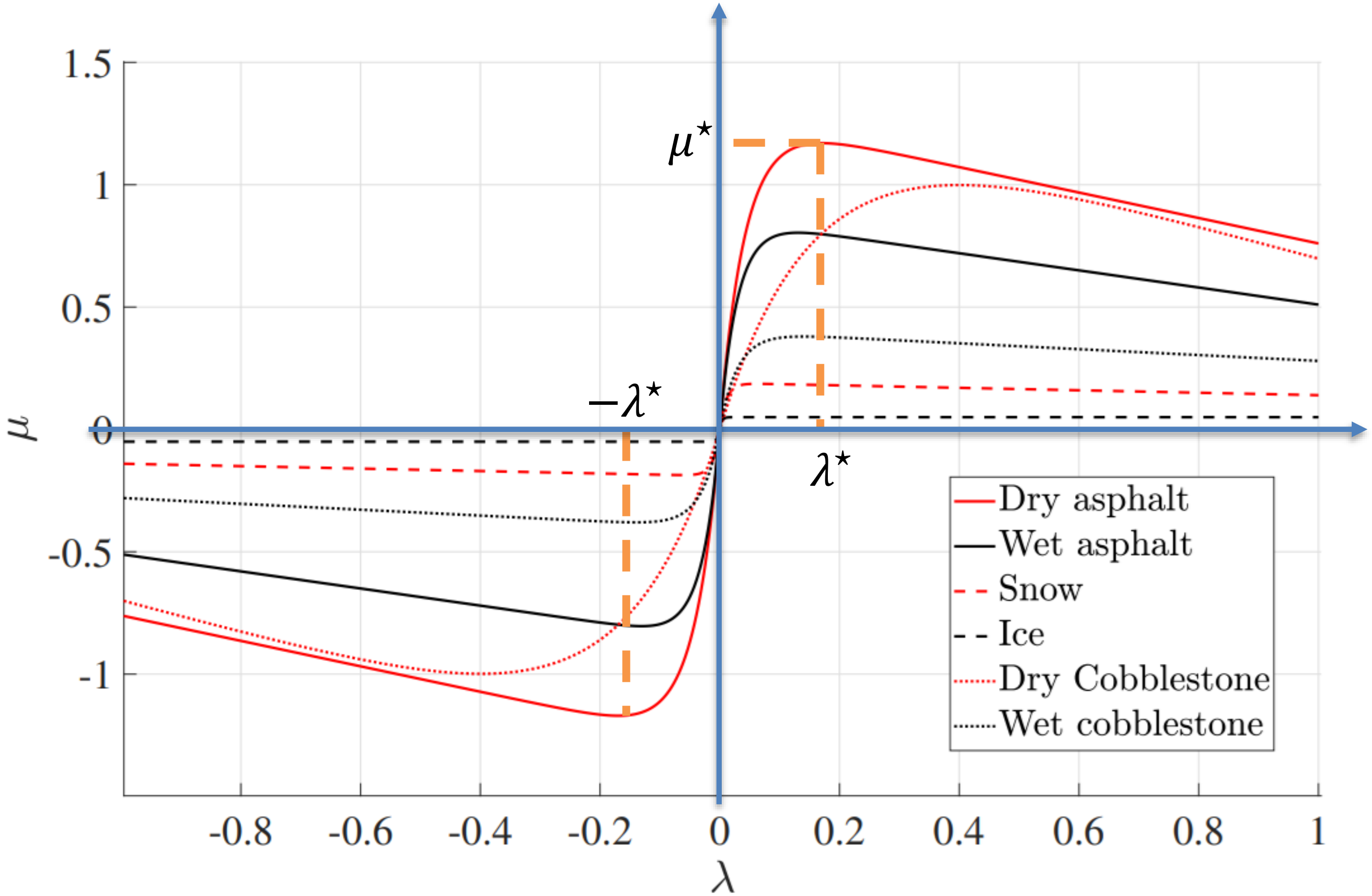
$$e_r = \omega_r - \omega_r^r(t)$$

$$e_f = \omega_f - \omega_f^r(t)$$

Wheel Speed Controls Problem Formulation

$$\lambda(v, \omega r) = \frac{\omega r - v}{\epsilon + \max\{|v|, |\omega r|, |\omega_r - v|\}}$$

$$\begin{cases} \omega_{\#}^{r} = \dfrac{v}{(1-\lambda)r} & \text{Driving} \\ \omega_{\#}^{r} = (1+\lambda)\dfrac{v}{r} & \text{Braking} \end{cases}$$

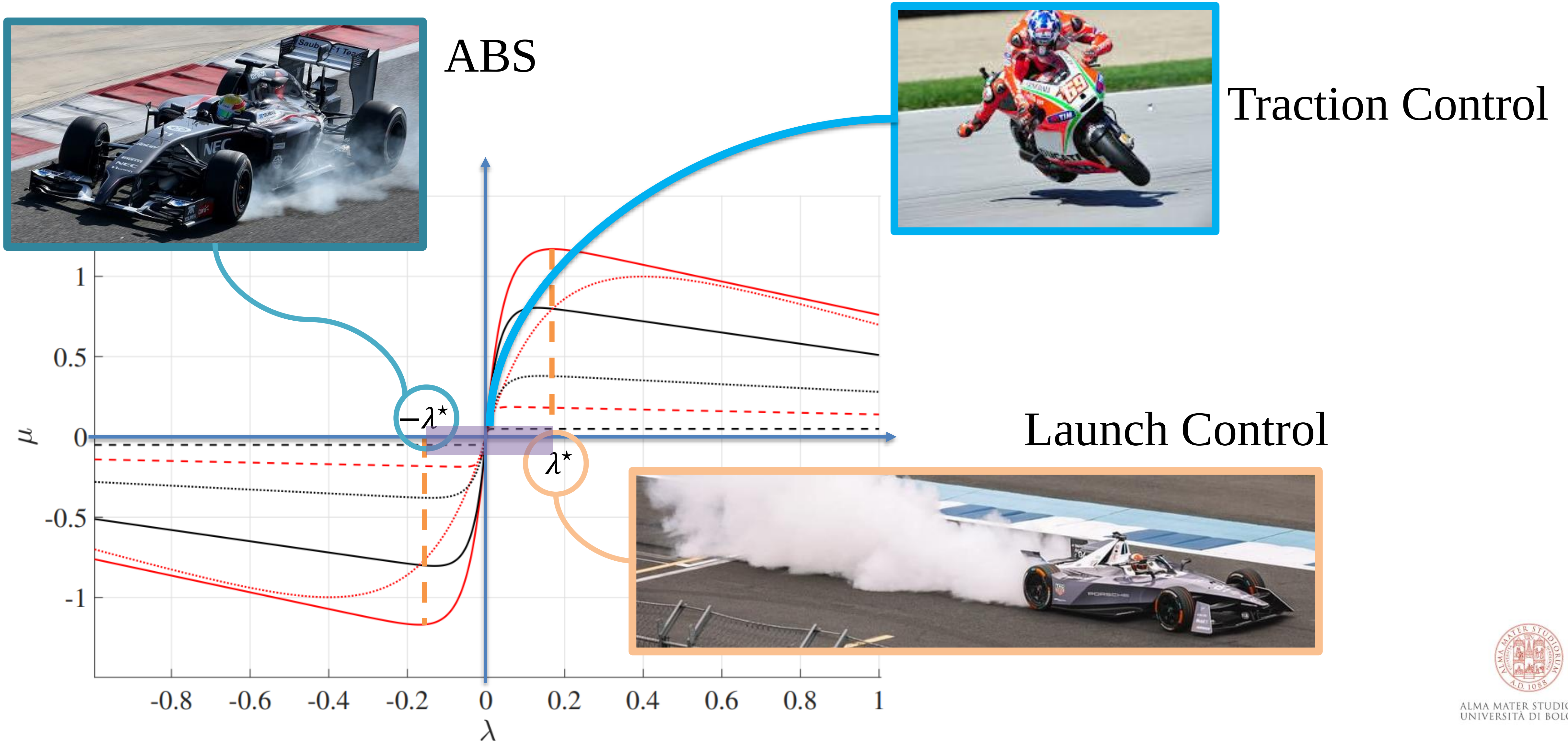


Regulated Output

$$e_r = \omega_r - \omega_r^r(t)$$
$$e_f = \omega_f - \omega_f^r(t)$$

$$\begin{cases} \lambda \in (-\lambda^\star, \lambda^\star) & \text{Traction Control} \\ \lambda = -\lambda^\star & \text{ABS} \\ \lambda = \lambda^\star & \text{Launch Control} \end{cases}$$

Wheel Speed Controls Problem Formulation



Wheel Speed Controls Problem Formulation

let $\mathbf{x} := \mathrm{col}(v, \omega_r, \omega_f)$, $\mathbf{u} := \mathrm{col}(\tau_r, \tau_f)$, $\mathbf{w} := \mathrm{col}(\theta, w, \nu_v, \nu_r, \nu_f, \omega_r^r, \omega_f^r)$
$\mathbf{y} := \mathrm{col}(v, \omega_r, \omega_f)$, and $\mathbf{e} := \mathrm{col}(e_r, e_f)$

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}, \mathbf{u}, \mathbf{w}) \qquad \mathbf{x}(t_0) = \mathbf{x}_0 \\ \mathbf{y} &= \mathbf{h}(\mathbf{x}, \mathbf{u}, \mathbf{w}) \\ \mathbf{e} &= \mathbf{h}_e(\mathbf{x}, \mathbf{u}, \mathbf{w}) \end{aligned}$$

$$\mathbf{f}(\mathbf{x}, \mathbf{u}, \mathbf{w}) = \begin{bmatrix} \dfrac{N_f(\mu_f - c_r) + N_r(\mu_r - c_r)}{m} - g \sin\theta - \dfrac{D}{m} \\ J_r^{-1}(\tau_r - N_r r_r(\mu_r - c_r)) \\ J_f^{-1}(\tau_f - N_f r_f(\mu_f - c_r)) \end{bmatrix}$$

$$\mathbf{h}(\mathbf{x}, \mathbf{u}, \mathbf{w}) = \begin{bmatrix} v + \nu_v \\ \omega_r(1 + \nu_r) \\ \omega_f(1 + \nu_f) \end{bmatrix}, \qquad \mathbf{h}_e(\mathbf{x}, \mathbf{u}, \mathbf{w}) = \begin{bmatrix} \omega_r(1 + \nu_r) - \omega_r^r \\ \omega_f(1 + \nu_f) - \omega_f^r \end{bmatrix}$$

Wheel Speed Controls Problem Formulation

let $\mathbf{x} := \mathrm{col}(v, \omega_r, \omega_f)$, $\mathbf{u} := \mathrm{col}(\tau_r, \tau_f)$, $\mathbf{w} := \mathrm{col}(\theta, w, \nu_v, \nu_r, \nu_f, \omega_r^r, \omega_f^r)$
$\mathbf{y} := \mathrm{col}(v, \omega_r, \omega_f)$, and $\mathbf{e} := \mathrm{col}(e_r, e_f)$

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}, \mathbf{u}, \mathbf{w}) \qquad \mathbf{x}(t_0) = \mathbf{x}_0 \\ \mathbf{y} &= \mathbf{h}(\mathbf{x}, \mathbf{u}, \mathbf{w}) \\ \mathbf{e} &= \mathbf{h}_e(\mathbf{x}, \mathbf{u}, \mathbf{w}) \end{aligned}$$

$$\mathbf{f}(\mathbf{x}, \mathbf{u}, \mathbf{w}) = \begin{bmatrix} \dfrac{N_f(\mu_f - c_r) + N_r(\mu_r - c_r)}{m} - g \sin\theta - \dfrac{D}{m} \\ J_r^{-1}(\tau_r - N_r r_r(\mu_r - c_r)) \\ J_f^{-1}(\tau_f - N_f r_f(\mu_f - c_r)) \end{bmatrix}$$

$$\mathbf{h}(\mathbf{x}, \mathbf{u}, \mathbf{w}) = \begin{bmatrix} v + \nu_v \\ \omega_r(1 + \nu_r) \\ \omega_f(1 + \nu_f) \end{bmatrix}, \qquad \mathbf{h}_e(\mathbf{x}, \mathbf{u}, \mathbf{w}) = \begin{bmatrix} \omega_r(1 + \nu_r) - \omega_r^r \\ \omega_f(1 + \nu_f) - \omega_f^r \end{bmatrix}$$

Control Goals

1) Keep all the signals bounded
2) Asymptotically steer the regulated output to zero assuming constant disturbances.

Contents

- Active Suspensions
- Adaptive Cruise Control
- Wheel Speed Controls
- Electronic Stability Program / Torque Vectoring
- Self-Park Assist

ALMA MATER STUDIORUM
UNIVERSITÀ DI BOLOGNA

ESP/TV

Motivations & Goals

Improve safety (through stabilisation: ESP)

ESP/TV

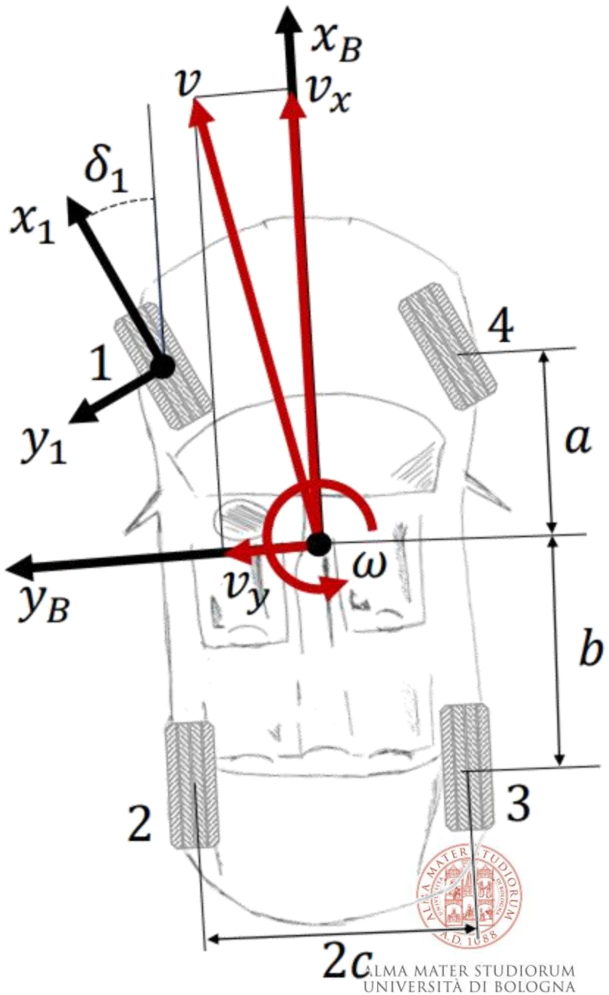
Motivations & Goals

Improve safety (through reference tracking: Torque Vectoring)

ESP/TV 2D Model

let $v_x, v_y \in \mathbb{R}$ be the vehicle inertial speed expressed in the body axes, let $\omega \in \mathbb{R}$ be the yaw rate,

$$\begin{bmatrix} \dot{v}_x \\ \dot{v}_y \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} \omega v_y \\ -\omega v_x \\ 0 \end{bmatrix} + \begin{bmatrix} (f_x - D(v_x - w))/m \\ f_y/m \\ \tau/J \end{bmatrix}$$

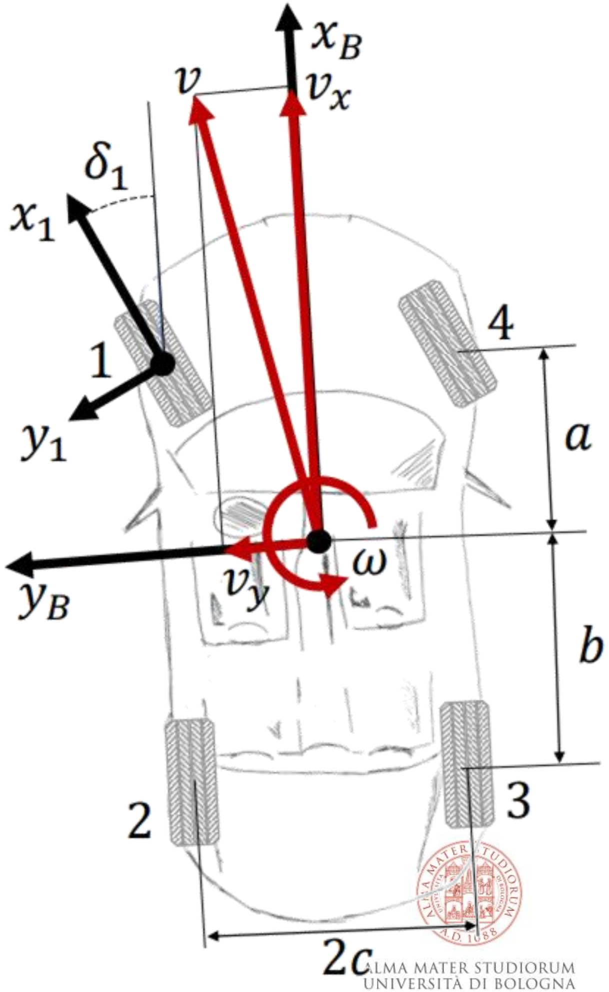


ESP/TV 2D Model

let $v_x, v_y \in \mathbb{R}$ be the vehicle inertial speed expressed in the body axes, let $\omega \in \mathbb{R}$ be the yaw rate,

$$\begin{bmatrix} \dot{v}_x \\ \dot{v}_y \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} \omega v_y \\ -\omega v_x \\ 0 \end{bmatrix} + \begin{bmatrix} (f_x - D(v_x - w))/m \\ f_y/m \\ \tau/J \end{bmatrix}$$

Aerodynamic Drag $\quad D(v - w) = \frac{1}{2}\rho S \frac{(v-w)^3}{|v-w|} C_D$



ESP/TV 2D Model

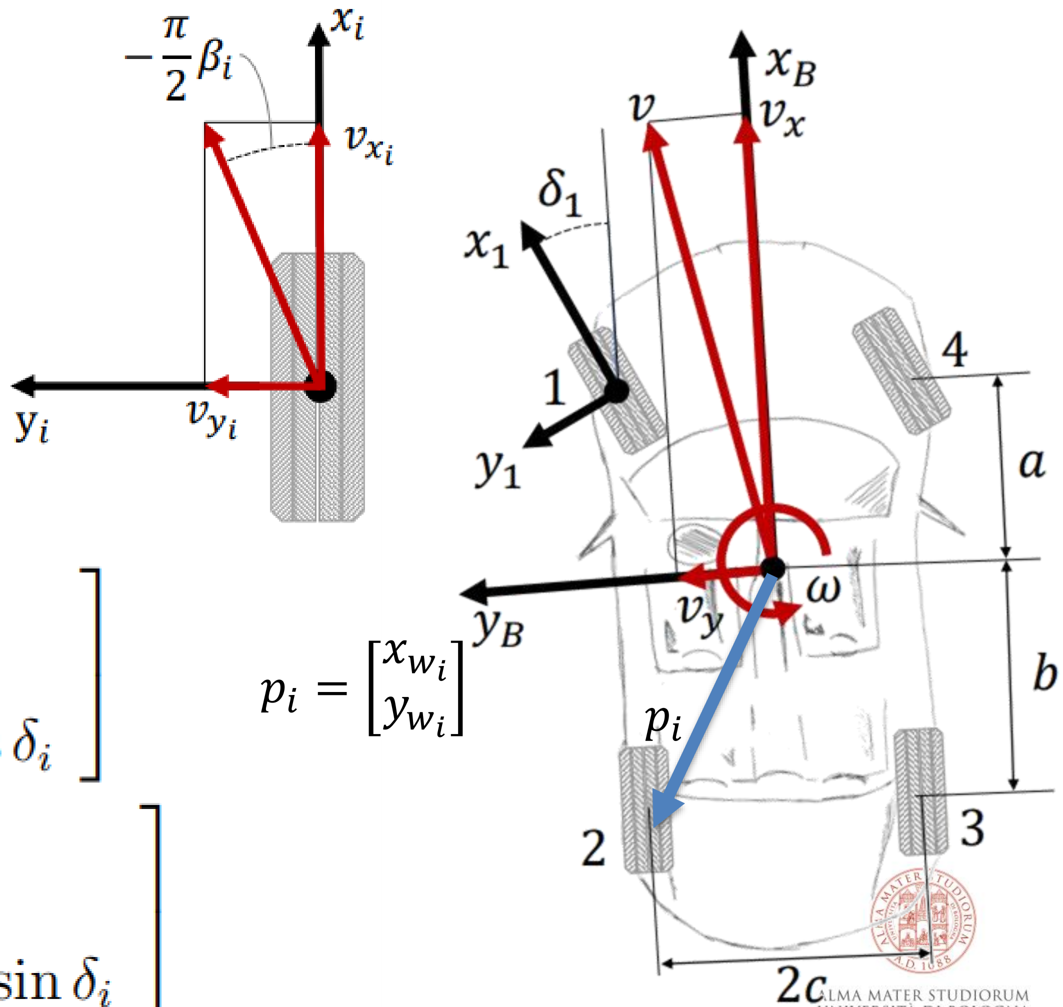
let $v_x, v_y \in \mathbb{R}$ be the vehicle inertial speed expressed in the body axes, let $\omega \in \mathbb{R}$ be the yaw rate,

$$\begin{bmatrix} \dot{v}_x \\ \dot{v}_y \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} \omega v_y \\ -\omega v_x \\ 0 \end{bmatrix} + \begin{bmatrix} (f_x - D(v_x - w))/m \\ f_y/m \\ \tau/J \end{bmatrix}$$

Aerodynamic Drag $\qquad D(v-w) = \frac{1}{2}\rho S \frac{(v-w)^3}{|v-w|} C_D$

Tire forces and torques

$$\begin{bmatrix} f_x \\ f_y \\ \tau \end{bmatrix} = \sum_{i=1}^{4} N_i \mu(\lambda_i, \mathbf{\Theta}_i) \begin{bmatrix} \cos\delta_i \\ \sin\delta_i \\ x_{w_i}\sin\delta_i - y_{w_i}\cos\delta_i \end{bmatrix}$$
$$+ \sum_{i=1}^{4} N_i \mu(\beta_i, \mathbf{\Theta}_i) \begin{bmatrix} -\sin\delta_i \\ \cos\delta_i \\ x_{w_i}\cos\delta_i + y_{w_i}\sin\delta_i \end{bmatrix}$$



ESP/TV 2D Model

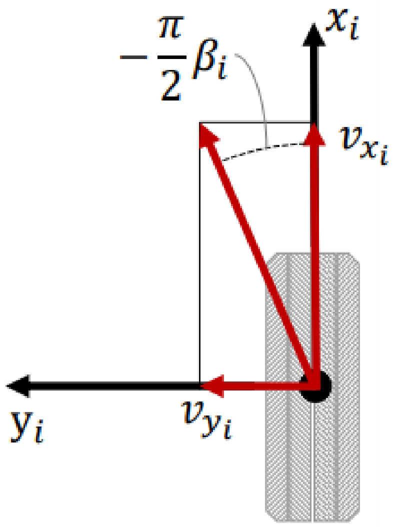
let $v_x, v_y \in \mathbb{R}$ be the vehicle inertial speed expressed in the body axes, let $\omega \in \mathbb{R}$ be the yaw rate,

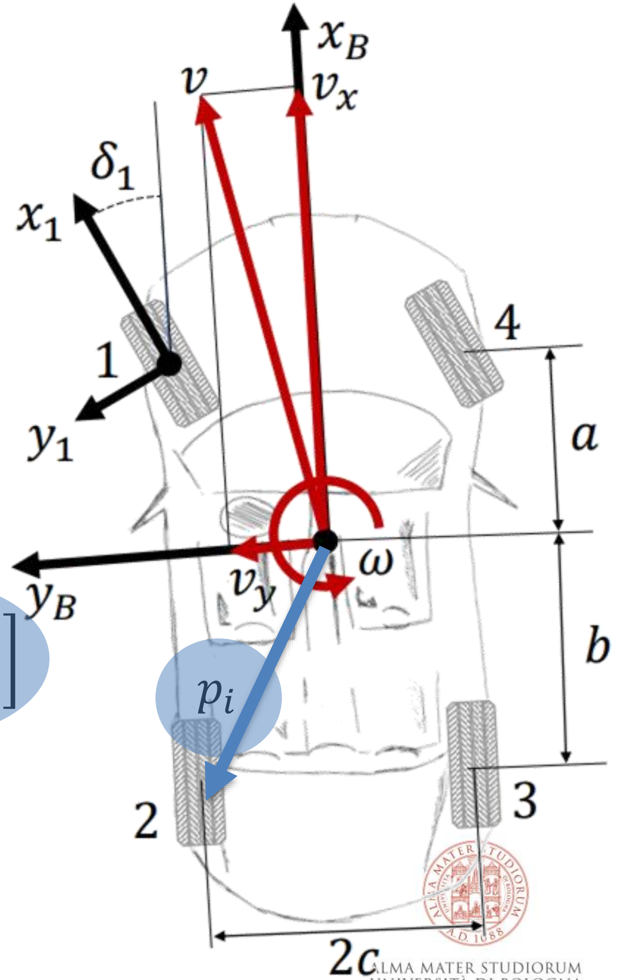
$$\begin{bmatrix} \dot{v}_x \\ \dot{v}_y \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} \omega v_y \\ -\omega v_x \\ 0 \end{bmatrix} + \begin{bmatrix} (f_x - D(v_x - w))/m \\ f_y/m \\ \tau/J \end{bmatrix}$$

Aerodynamic Drag $\quad D(v-w) = \frac{1}{2}\rho S \frac{(v-w)^3}{|v-w|} C_D$

Tire forces and torques

$$\begin{bmatrix} f_x \\ f_y \\ \tau \end{bmatrix} = \sum_{i=1}^{4} N_i \mu(\lambda_i, \mathbf{\Theta}_i) \begin{bmatrix} \cos\delta_i \\ \sin\delta_i \\ x_{w_i} \sin\delta_i - y_{w_i}\cos\delta_i \end{bmatrix} + \sum_{i=1}^{4} N_i \mu(\beta_i, \mathbf{\Theta}_i) \begin{bmatrix} -\sin\delta_i \\ \cos\delta_i \\ x_{w_i}\cos\delta_i + y_{w_i}\sin\delta_i \end{bmatrix}$$





$p_i = \begin{bmatrix} x_{w_i} \\ y_{w_i} \end{bmatrix}$

ESP/TV 2D Model

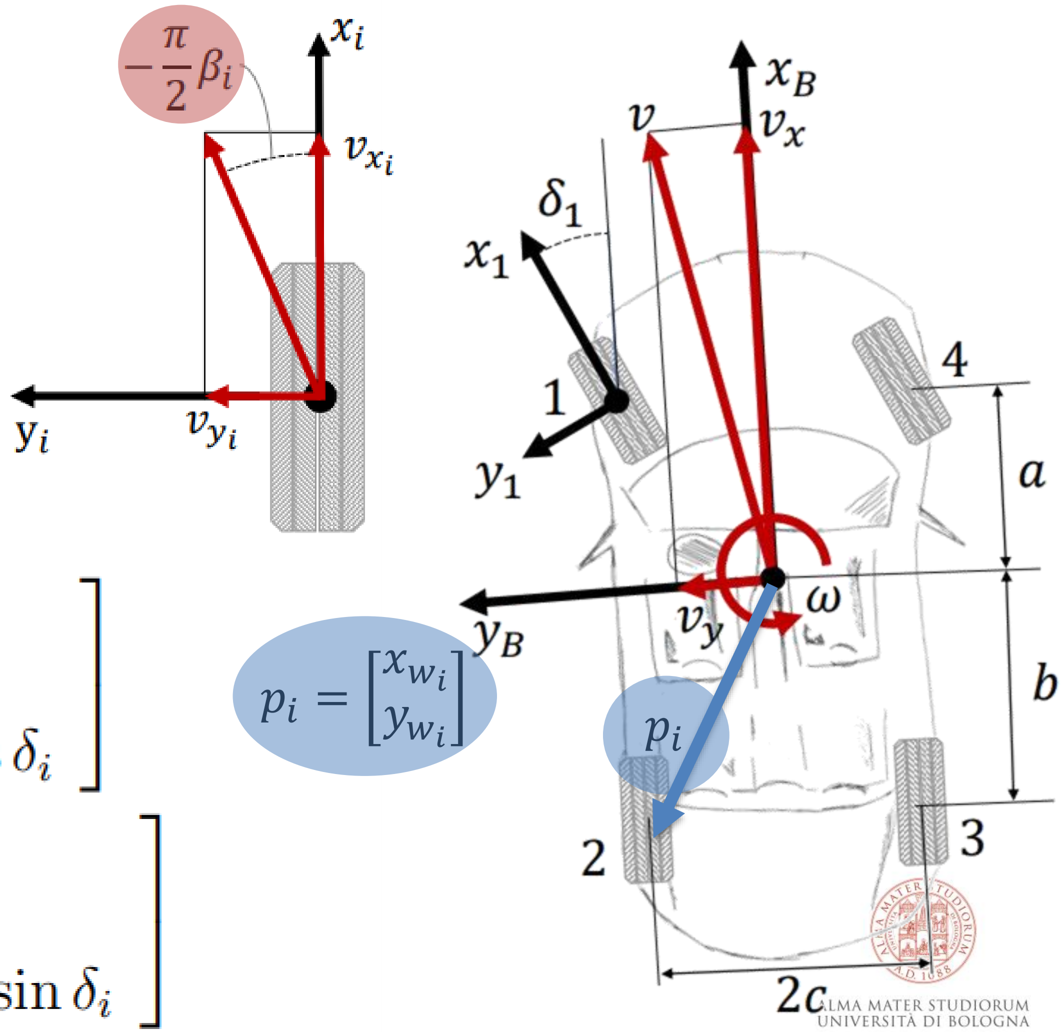
let $v_x, v_y \in \mathbb{R}$ be the vehicle inertial speed expressed in the body axes, let $\omega \in \mathbb{R}$ be the yaw rate,

$$\begin{bmatrix} \dot{v}_x \\ \dot{v}_y \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} \omega v_y \\ -\omega v_x \\ 0 \end{bmatrix} + \begin{bmatrix} (f_x - D(v_x - w))/m \\ f_y/m \\ \tau/J \end{bmatrix}$$

Aerodynamic Drag $\quad D(v-w) = \frac{1}{2}\rho S \frac{(v-w)^3}{|v-w|} C_D$

Tire forces and torques

$$\begin{bmatrix} f_x \\ f_y \\ \tau \end{bmatrix} = \sum_{i=1}^{4} N_i \mu(\lambda_i, \mathbf{\Theta}_i) \begin{bmatrix} \cos\delta_i \\ \sin\delta_i \\ x_{w_i}\sin\delta_i - y_{w_i}\cos\delta_i \end{bmatrix} + \sum_{i=1}^{4} N_i \mu(\beta_i, \mathbf{\Theta}_i) \begin{bmatrix} -\sin\delta_i \\ \cos\delta_i \\ x_{w_i}\cos\delta_i + y_{w_i}\sin\delta_i \end{bmatrix}$$



ESP/TV 2D Model

Tire forces and torques

$$\begin{bmatrix} f_x \\ f_y \\ \tau \end{bmatrix} = \sum_{i=1}^{4} N_i \mu(\lambda_i, \boldsymbol{\Theta}_i) \begin{bmatrix} \cos\delta_i \\ \sin\delta_i \\ x_{w_i}\sin\delta_i - y_{w_i}\cos\delta_i \end{bmatrix} + \sum_{i=1}^{4} N_i \mu(\beta_i, \boldsymbol{\Theta}_i) \begin{bmatrix} -\sin\delta_i \\ \cos\delta_i \\ x_{w_i}\cos\delta_i + y_{w_i}\sin\delta_i \end{bmatrix}$$

ESP/TV 2D Model

$$\sum_{i=1}^{4} \begin{bmatrix} 1 \\ y_{w_i} + h(\mu(\beta_i, \mathbf{\Theta}_i)\cos\delta_i + \mu(\lambda_i, \mathbf{\Theta}_i)\sin\delta_i) \\ h(\mu(\lambda_i, \mathbf{\Theta}_i)\cos\delta_i - \mu(\beta_i, \mathbf{\Theta}_i)\sin\delta_i) + x_{w_i} \end{bmatrix} N_i = \begin{bmatrix} mg \\ 0 \\ 0 \end{bmatrix}$$

Tire forces and torques

$$\begin{bmatrix} f_x \\ f_y \\ \tau \end{bmatrix} = \sum_{i=1}^{4} N_i \mu(\lambda_i, \mathbf{\Theta}_i) \begin{bmatrix} \cos\delta_i \\ \sin\delta_i \\ x_{w_i}\sin\delta_i - y_{w_i}\cos\delta_i \end{bmatrix} + \sum_{i=1}^{4} N_i \mu(\beta_i, \mathbf{\Theta}_i) \begin{bmatrix} -\sin\delta_i \\ \cos\delta_i \\ x_{w_i}\cos\delta_i + y_{w_i}\sin\delta_i \end{bmatrix}$$

ESP/TV 2D Model

$$\mathbf{v}_i := \begin{bmatrix} 1 \\ y_{w_i} + h(\mu(\beta_i)\cos\delta_i + \mu(\lambda_i)\sin\delta_i) \\ h(\mu(\lambda_i)\cos\delta_i - \mu(\beta_i)\sin\delta_i) + x_{w_i} \end{bmatrix} \qquad \mathbf{H} := \begin{bmatrix} \mathbf{v}_1 & \dots & \mathbf{v}_n \end{bmatrix}$$

$$\sum_{i=1}^{4} \begin{bmatrix} 1 \\ y_{w_i} + h(\mu(\beta_i, \mathbf{\Theta}_i)\cos\delta_i + \mu(\lambda_i, \mathbf{\Theta}_i)\sin\delta_i) \\ h(\mu(\lambda_i, \mathbf{\Theta}_i)\cos\delta_i - \mu(\beta_i, \mathbf{\Theta}_i)\sin\delta_i) + x_{w_i} \end{bmatrix} N_i = \begin{bmatrix} mg \\ 0 \\ 0 \end{bmatrix}$$

Tire forces and torques

$$\begin{bmatrix} f_x \\ f_y \\ \tau \end{bmatrix} = \sum_{i=1}^{4} N_i \mu(\lambda_i, \mathbf{\Theta}_i) \begin{bmatrix} \cos\delta_i \\ \sin\delta_i \\ x_{w_i}\sin\delta_i - y_{w_i}\cos\delta_i \end{bmatrix} + \sum_{i=1}^{4} N_i \mu(\beta_i, \mathbf{\Theta}_i) \begin{bmatrix} -\sin\delta_i \\ \cos\delta_i \\ x_{w_i}\cos\delta_i + y_{w_i}\sin\delta_i \end{bmatrix}$$

ESP/TV 2D Model

$$\mathbf{v}_i := \begin{bmatrix} 1 \\ y_{w_i} + h(\mu(\beta_i)\cos\delta_i + \mu(\lambda_i)\sin\delta_i) \\ h(\mu(\lambda_i)\cos\delta_i - \mu(\beta_i)\sin\delta_i) + x_{w_i} \end{bmatrix} \qquad \mathbf{H} := \begin{bmatrix} \mathbf{v}_1 & \dots & \mathbf{v}_n \end{bmatrix} \qquad \mathbf{H}\begin{bmatrix} N_1 \\ \vdots \\ N_4 \end{bmatrix} = \begin{bmatrix} mg \\ 0 \\ 0 \end{bmatrix}$$

$$\sum_{i=1}^{4} \begin{bmatrix} 1 \\ y_{w_i} + h(\mu(\beta_i, \mathbf{\Theta}_i)\cos\delta_i + \mu(\lambda_i, \mathbf{\Theta}_i)\sin\delta_i) \\ h(\mu(\lambda_i, \mathbf{\Theta}_i)\cos\delta_i - \mu(\beta_i, \mathbf{\Theta}_i)\sin\delta_i) + x_{w_i} \end{bmatrix} N_i = \begin{bmatrix} mg \\ 0 \\ 0 \end{bmatrix}$$

Vertical translation
Roll
Pitch

Tire forces and torques

$$\begin{bmatrix} f_x \\ f_y \\ \tau \end{bmatrix} = \sum_{i=1}^{4} N_i \mu(\lambda_i, \mathbf{\Theta}_i) \begin{bmatrix} \cos\delta_i \\ \sin\delta_i \\ x_{w_i}\sin\delta_i - y_{w_i}\cos\delta_i \end{bmatrix} + \sum_{i=1}^{4} N_i \mu(\beta_i, \mathbf{\Theta}_i) \begin{bmatrix} -\sin\delta_i \\ \cos\delta_i \\ x_{w_i}\cos\delta_i + y_{w_i}\sin\delta_i \end{bmatrix}$$

ESP/TV 2D Model

$$\mathbf{v}_i := \begin{bmatrix} 1 \\ y_{w_i} + h(\mu(\beta_i)\cos\delta_i + \mu(\lambda_i)\sin\delta_i) \\ h(\mu(\lambda_i)\cos\delta_i - \mu(\beta_i)\sin\delta_i) + x_{w_i} \end{bmatrix} \qquad \mathbf{H} := \begin{bmatrix} \mathbf{v}_1 & \dots & \mathbf{v}_n \end{bmatrix} \qquad \mathbf{H}\begin{bmatrix} N_1 \\ \vdots \\ N_4 \end{bmatrix} = \begin{bmatrix} mg \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} N_1 \\ \vdots \\ N_4 \end{bmatrix} = \mathbf{H}^\top \left[\mathbf{H}\mathbf{H}^\top\right]^{-1} \begin{bmatrix} mg \\ 0 \\ 0 \end{bmatrix}$$

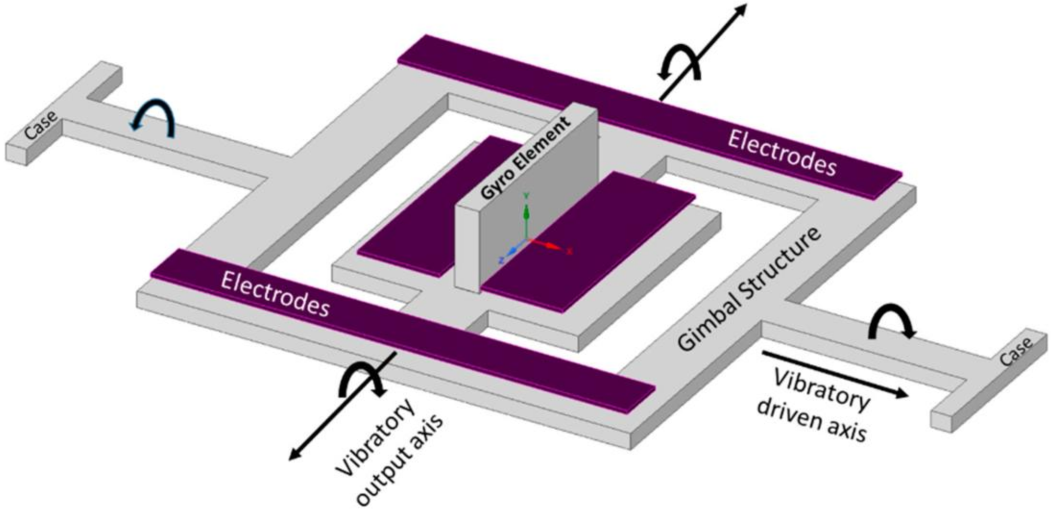
Tire forces and torques

$$\begin{bmatrix} f_x \\ f_y \\ \tau \end{bmatrix} = \sum_{i=1}^{4} N_i \mu(\lambda_i, \mathbf{\Theta}_i) \begin{bmatrix} \cos\delta_i \\ \sin\delta_i \\ x_{w_i}\sin\delta_i - y_{w_i}\cos\delta_i \end{bmatrix} + \sum_{i=1}^{4} N_i \mu(\beta_i, \mathbf{\Theta}_i) \begin{bmatrix} -\sin\delta_i \\ \cos\delta_i \\ x_{w_i}\cos\delta_i + y_{w_i}\sin\delta_i \end{bmatrix}$$

2D Model

Sensor (Gyroscope)

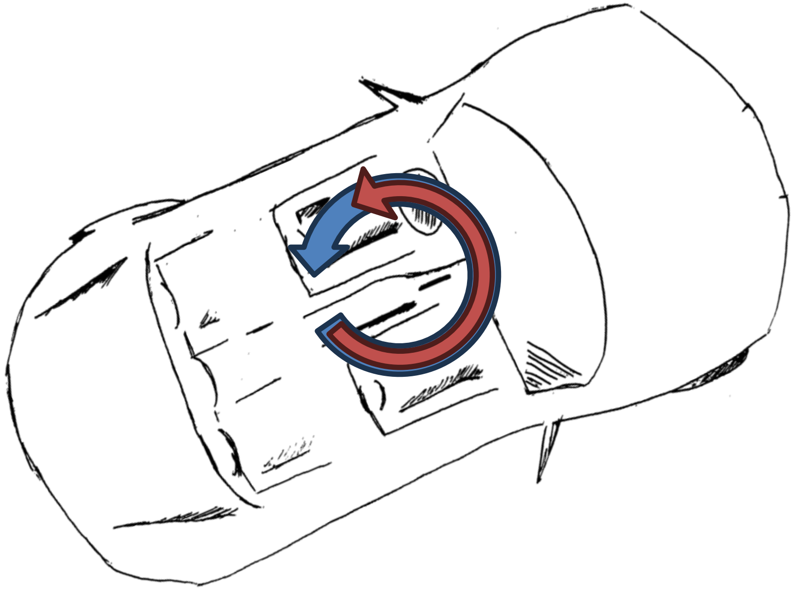
$$y = \omega + \nu.$$



Micro-ElectroMechanical Systems (MEMS)

Regulated output

$$e := \omega - r(t).$$



Problem Formulation

$$\text{Let } \mathbf{x} := \mathrm{col}(v_x, v_y, \omega), \ \mathbf{u} := \mathrm{col}(\delta_1, \ldots, \delta_4, \omega_1, \ldots, \omega_4)$$
$$\mathbf{d} := \mathrm{col}(w, \boldsymbol{\Theta}_1, \boldsymbol{\Theta}_2, \boldsymbol{\Theta}_3, \boldsymbol{\Theta}_4), \ \mathbf{w} := \mathrm{col}(\mathbf{d}, \nu, r)$$

Then

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}, \mathbf{u}, \mathbf{w}) \qquad \mathbf{x}(t_0) = \mathbf{x}_0 \\ y &= h(\mathbf{x}, \mathbf{u}, \mathbf{w}) \\ e &= h_e(\mathbf{x}, \mathbf{u}, \mathbf{w}) \end{aligned}$$

with

$$\mathbf{f}(\mathbf{x}, \mathbf{u}, \mathbf{w}) := \begin{bmatrix} \omega v_y \\ -\omega v_x \\ 0 \end{bmatrix} + \begin{bmatrix} (f_x - D(v_x - w))/m \\ f_y/m \\ \tau/J \end{bmatrix}$$

$$h(\mathbf{x}, \mathbf{u}, \mathbf{w}) := \omega + \nu$$

$$h_e(\mathbf{x}, \mathbf{u}, \mathbf{w}) := y - r(t).$$

Control Goals

1) Keep all the signals bounded
2) Asymptotically steer the regulated output to zero assuming constant disturbances.

Contents

- Active Suspensions
- Adaptive Cruise Control
- Wheel Speed Controls
- Electronic Stability Program / Torque Vectoring
- Self-Park Assist

Self-Park Assist

Motivations & Goals

Facilitate Parking

Self-Park Assist

Motivations & Goals

Facilitate Parking

Jokes apart!
Self-Park Assist is fundamental for impaired people

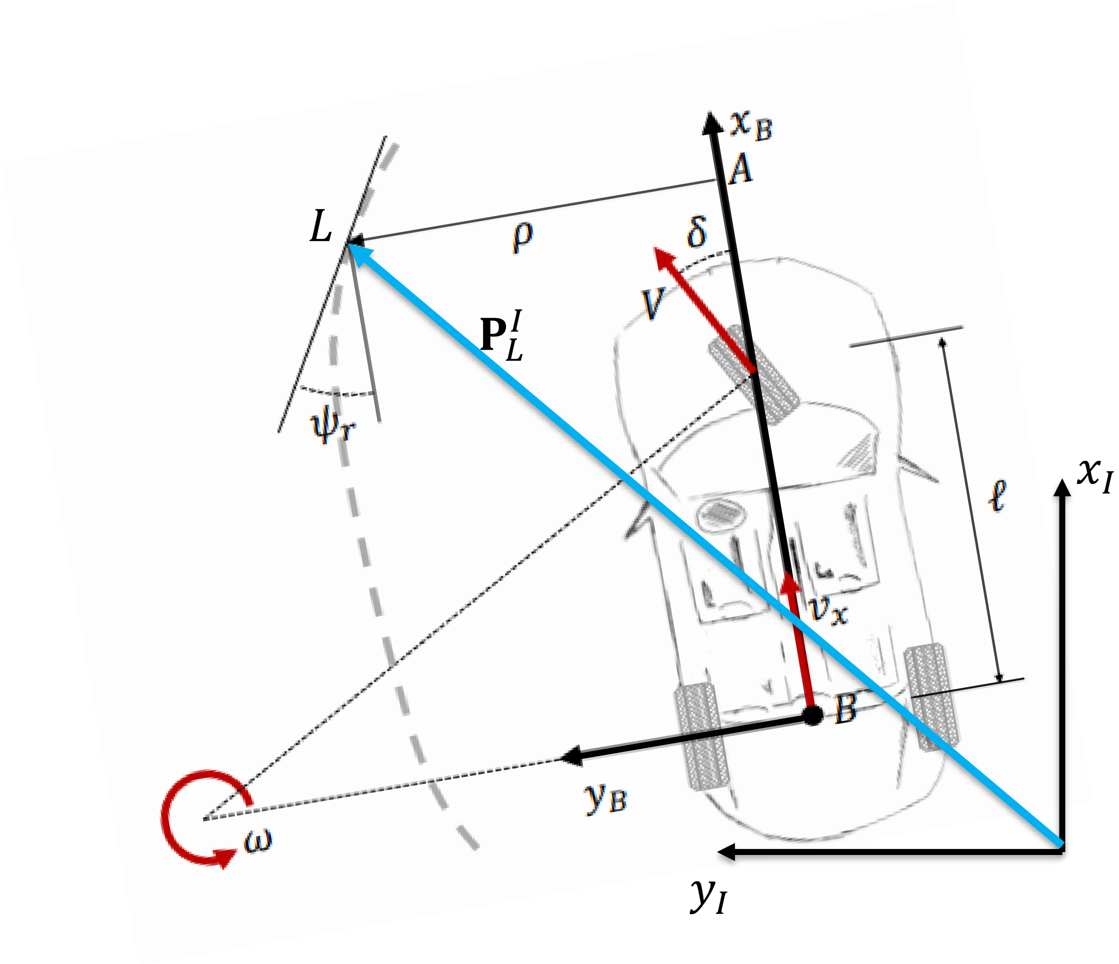
Self-Park Assist 2D Model

Bicycle model with Ackerman Steering

$$v_x = V\cos\delta$$
$$v_y = 0$$
$$\omega = \frac{V}{\ell}\sin\delta$$

Let $\chi : \mathbb{R}^2 \to \mathbb{R}$ be the line curvature. Then

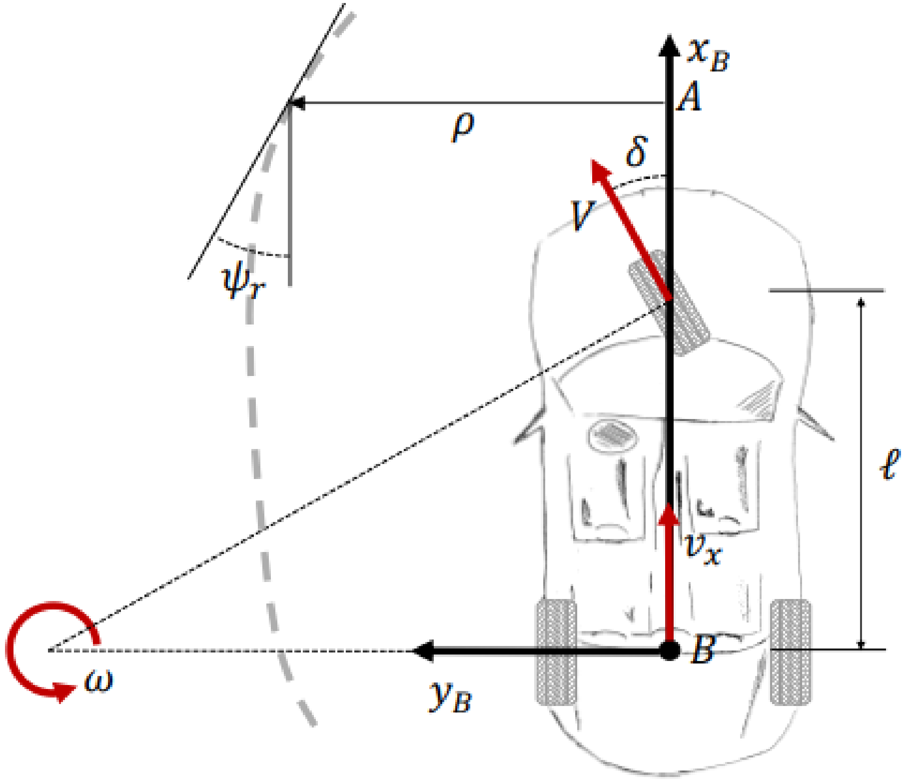
$$\dot{\rho} = V\left(\tan\psi_r\left(\frac{\rho}{\ell}\sin\delta - \cos\delta\right) - \frac{x_A}{\ell}\sin\delta\right)$$
$$\dot{\psi}_r = V\left(\frac{\sin\delta}{\ell}\left(1 + \rho\frac{\chi(\mathbf{p}_L^I)}{\cos\psi_r}\right) - \chi(\mathbf{p}_L^I)\frac{\cos\delta}{\cos\psi_r}\right)$$

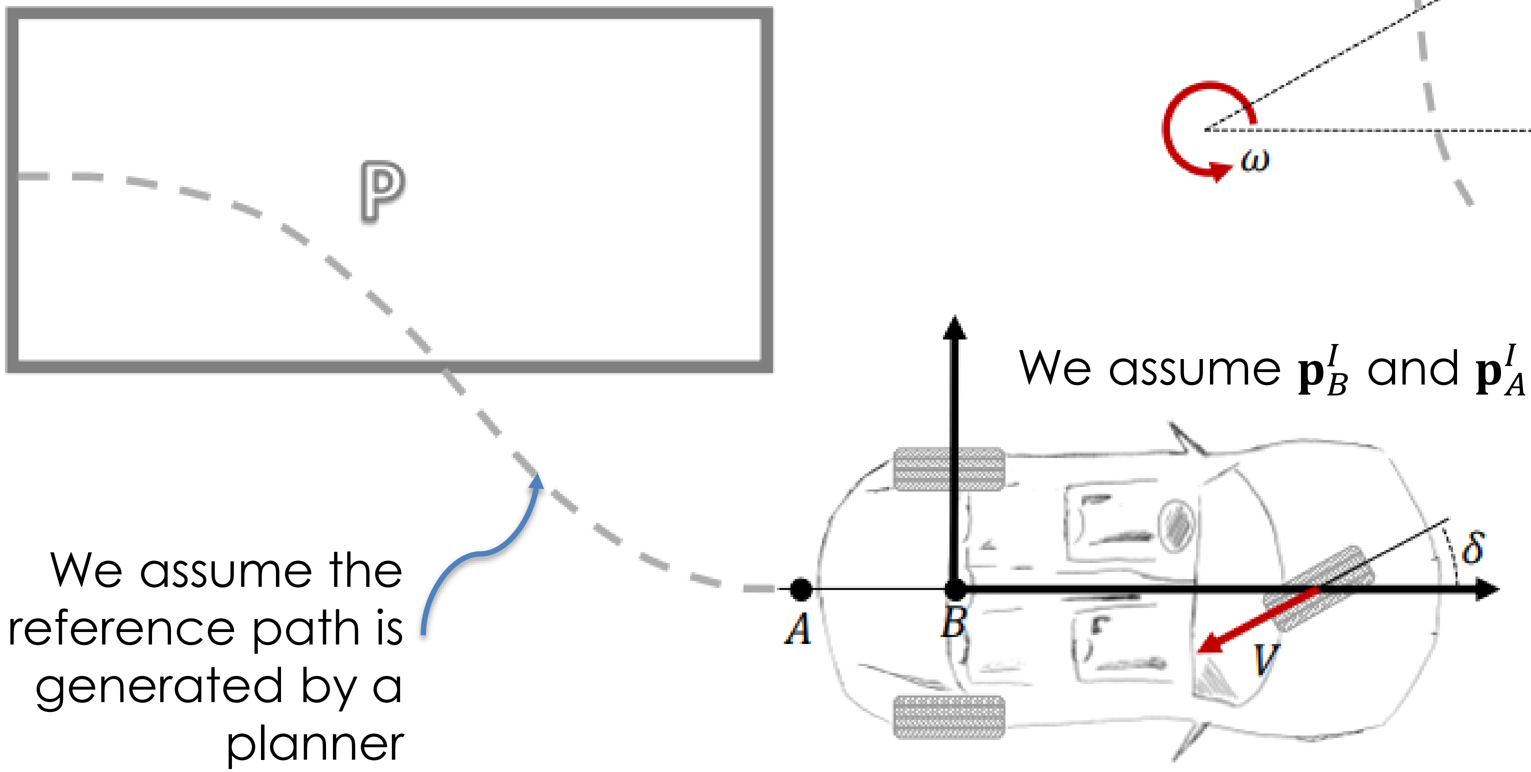


Self-Park Assist 2D Model

We assume ρ and ψ_r are estimated and χ is known

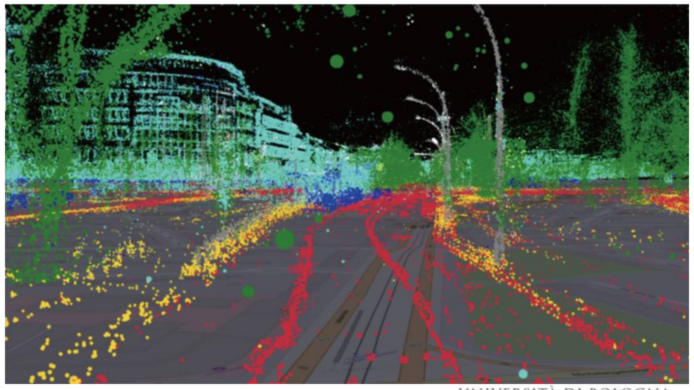
$$\boldsymbol{y} = \mathrm{col}(\rho + \nu_\rho, \psi_r + \nu_\psi, \chi)$$



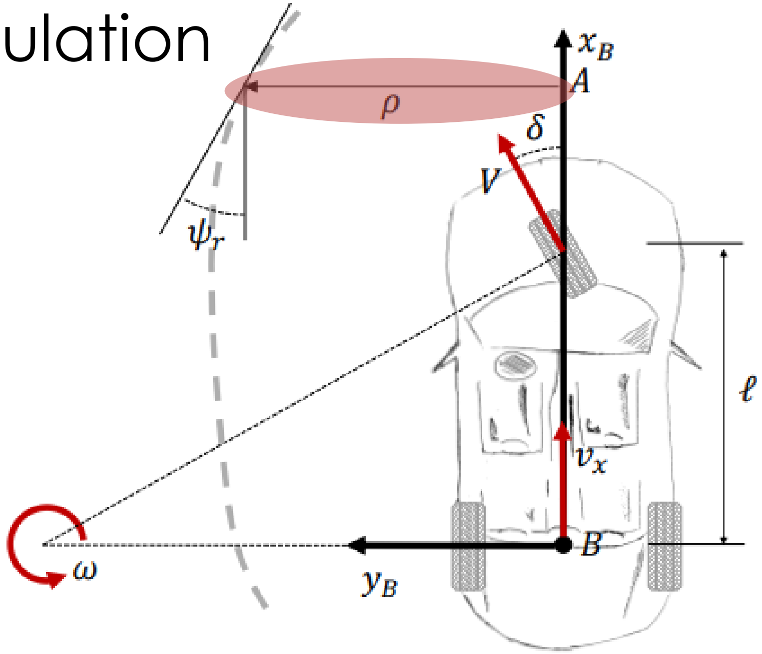


We assume the reference path is generated by a planner

We assume $\mathbf{p}_B^I$ and $\mathbf{p}_A^I$ are obtained via SLAM



Self-Park Assist Problem Formulation



Let

$$\mathbf{x} := \mathrm{col}(\rho, \psi_r)$$
$$u := \delta$$
$$d := \chi$$
$$\boldsymbol{\nu} := \mathrm{col}(\nu_\rho, \nu_\psi)$$
$$\mathbf{w} := \mathrm{col}(d, \boldsymbol{\nu})$$
$$\boldsymbol{y} = \mathrm{col}(\rho, \psi_r, d) + \boldsymbol{\nu}$$
$$e := \rho$$

Control Goals

1) Keep all the signals bounded
2) Asymptotically steer the regulated output to zero assuming constant disturbances.

Then

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, u, \mathbf{w}) \qquad \mathbf{x}(t_0) = \mathbf{x}_0$$
$$\mathbf{y} = \mathbf{h}(\mathbf{x}, u, \mathbf{w})$$
$$e = h_e(\mathbf{x}, u, \mathbf{w})$$

$$\mathbf{f}(\mathbf{x}, u, \mathbf{w}) = V \begin{bmatrix} \tan\psi_r \left(\frac{\rho}{\ell}\sin\delta - \cos\delta\right) - \frac{x_A}{\ell}\sin\delta \\ \frac{\sin\delta}{\ell}\left(1 + \rho\frac{\chi(\mathbf{p}_L^I)}{\cos\psi_r}\right) - \chi(\mathbf{p}_L^I)\frac{\cos\delta}{\cos\psi_r} \end{bmatrix}$$

$$\mathbf{h}(\mathbf{x}, u, \mathbf{w}) := \mathrm{col}\big(\rho + \nu_\rho, \psi_r + \nu_\psi, d\big)$$

$$h_e(\mathbf{x}, u, \mathbf{w}) := \rho + \nu_\rho$$

ALMA MATER STUDIORUM
UNIVERSITÀ DI BOLOGNA

Nicola Mimmo

Department of Electrical, Electronics and Information Engineering «G. Marconi»

nicola.mimmo2@unibo.it