

# TITLE

Automatic Control  
Electronic Engineering for Intelligent Vehicles  
University of Bologna

A.A. 202X-202X

Student A and Student B and ...

May 14, 2025

### **Abstract**

Here briefly detail the aims of the project.

# Chapter 1

## Introduction

### 1.0.1 System Linearization

To facilitate the control design of the longitudinal half-car model equipped with active front and rear suspension systems, the initial step involves the linearization of the nonlinear system dynamics. This process is performed by identifying appropriate steady-state operating points  $(x^*, y^*, w^*)$ , which characterize representative conditions under which the vehicle is expected to operate. Linearizing the system around these equilibrium points enables the derivation of a time-invariant linear approximation of the vehicle dynamics, thereby simplifying the synthesis and analysis of control strategies.

#### Linearization Around the Operating Point

Consider the nonlinear system model:

$$\begin{aligned}\dot{x} &= f(x, u, w), & x(t_0) &= x_0 \\ y &= h(x, u, w) \\ e &= h_e(x, u, w)\end{aligned}\tag{1.1}$$

The steady-state operating points  $(x^*, u^*, w^*)$  is called *equilibrium triplet* if satisfies the condition:

$$f(x^*, u^*, w^*) = 0\tag{1.2}$$

and defines the equilibrium output and error as:

$$y^* := h(x^*, u^*, w^*), \quad e^* := h_e(x^*, u^*, w^*)\tag{1.3}$$

The variations around the equilibrium point are defined as:

$$\begin{aligned}\tilde{x} &:= x - x^* \\ \tilde{y} &:= y - y^* \\ \tilde{e} &:= e - e^* \\ \tilde{u} &:= u - u^* \\ \tilde{w} &:= w - w^*\end{aligned}\tag{1.4}$$

Using the fact that  $\dot{x}^* = 0$ , the dynamics of the variations are:

$$\begin{aligned}\dot{\tilde{x}} &= f(x^* + \tilde{x}, u^* + \tilde{u}, w^* + \tilde{w}), & \tilde{x}(t_0) &= x_0 - x^* \\ \tilde{y} &= h(x^* + \tilde{x}, u^* + \tilde{u}, w^* + \tilde{w}) \\ \tilde{e} &= h_e(x^* + \tilde{x}, u^* + \tilde{u}, w^* + \tilde{w})\end{aligned}\tag{1.5}$$

To obtain a tractable model for controller synthesis, we apply a first-order Taylor expansion around the equilibrium point. The resulting Jacobian matrices are defined as:

$$\begin{aligned}A &:= \left. \frac{\partial f(x, u, w)}{\partial x} \right|_{\substack{x=x^* \\ u=u^* \\ w=w^*}} & B_1 &:= \left. \frac{\partial f(x, u, w)}{\partial u} \right|_{\substack{x=x^* \\ u=u^* \\ w=w^*}} & B_2 &:= \left. \frac{\partial f(x, u, w)}{\partial w} \right|_{\substack{x=x^* \\ u=u^* \\ w=w^*}} \\ C &:= \left. \frac{\partial h(x, u, w)}{\partial x} \right|_{\substack{x=x^* \\ u=u^* \\ w=w^*}} & D_1 &:= \left. \frac{\partial h(x, u, w)}{\partial u} \right|_{\substack{x=x^* \\ u=u^* \\ w=w^*}} & D_2 &:= \left. \frac{\partial h(x, u, w)}{\partial w} \right|_{\substack{x=x^* \\ u=u^* \\ w=w^*}} \\ C_e &:= \left. \frac{\partial h_e(x, u, w)}{\partial x} \right|_{\substack{x=x^* \\ u=u^* \\ w=w^*}} & D_{e1} &:= \left. \frac{\partial h_e(x, u, w)}{\partial u} \right|_{\substack{x=x^* \\ u=u^* \\ w=w^*}} & D_{e2} &:= \left. \frac{\partial h_e(x, u, w)}{\partial w} \right|_{\substack{x=x^* \\ u=u^* \\ w=w^*}}\end{aligned}\tag{1.6}$$

Neglecting second-order terms, the linearized system becomes the so-called *design model*:

$$\begin{cases} \dot{\tilde{x}} = A\tilde{x} + B_1\tilde{u} + B_2\tilde{w}, & \tilde{x}(t_0) = x_0 - x^* \\ \tilde{y} = C\tilde{x} + D_1\tilde{u} + D_2\tilde{w} \\ \tilde{e} = C_e\tilde{x} + D_{e1}\tilde{u} + D_{e2}\tilde{w} \end{cases}\tag{1.7}$$

This Linear Time-Invariant (LTI) approximation of the nonlinear model is valid in a neighborhood of the equilibrium point, enabling efficient analysis and controller design under small perturbations.

**TODO:** Inserire le matrici calcolate con matlab e verificare che rispettino quello che abbiamo scritto (verifica che il codice del .m e quello che fa rispetta queste cose dette nella aprte di teoria)