ALMA MATER STUDIORUM
UNIVERSITÀ DI BOLOGNA

Automotive Design Models
Eig Analysis

Faculty of «Electronic Engineering for Intelligent Vehicles» and «Advanced Automotive Engineering»

Nicola Mimmo

Department of Electrical, Electronics and Information Engineering «G. Marconi» - University of Bologna

Contents

Contents

Active Suspensions

Let us study **A**

How many eigenvalues?

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -k_s \dfrac{m_s + m_u}{m_s m_u} & -\beta_s \dfrac{m_s + m_u}{m_s m_u} & \dfrac{k_t}{m_u} & 0 \\ 0 & 0 & 0 & 1 \\ \dfrac{k_s}{m_u} & \dfrac{\beta_s}{m_u} & -\dfrac{k_t}{m_u} & 0 \end{bmatrix}$$

Active Suspensions

Let us study **A**

How many eigenvalues?

Numerically …

```
K>> eig(A)

ans =

 -11.4354 +60.8968i
 -11.4354 -60.8968i
  -1.6757 + 7.5142i
  -1.6757 - 7.5142i
```

```
A =

   1.0e+03 *

         0    0.0010         0         0
   -0.4196   -0.0262    3.5556         0
         0         0         0    0.0010
    0.3556    0.0222   -3.5556         0
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Let $\lambda_{1,2} = -\alpha_1 \pm \mathrm{i}\beta_1$ and $\lambda_{3,4} = -\alpha_2 \pm \mathrm{i}\beta_2$

$$\begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix} = e^{-\alpha_1 t} \begin{bmatrix} \cos(\beta_1 t) & \sin(\beta_1 t) \\ -\sin(\beta_1 t) & \cos(\beta_1 t) \end{bmatrix} \begin{bmatrix} z_1(0) \\ z_2(0) \end{bmatrix}$$

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$$\mathbf{x}(t) = \mathbf{V}\mathbf{z}(t)$$

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$\mathbf{x}(t) = \mathbf{V}\mathbf{z}(t)$

0.0021	0.0114	-0.0279	-0.1251
-0.7162	0	0.9865	0
-0.0027	-0.0109	0.0035	-0.0127
0.6965	-0.0411	0.0893	0.0477

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$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1.2816 & 6.8250 & 16.7559 & 75.1375 \\ 42.0622 & 0 & 57.9378 & 0 \\ 9.1364 & 36.6087 & 11.7927 & 42.4622 \\ 79.6353 & 4.7001 & 10.2131 & 5.4516 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_4 \\ z_4 \end{bmatrix}$$

Normalised (by row)

$$\bar{V}(i,:) = \frac{|V(i,:)|}{\sum_j |V(i,j)|}$$

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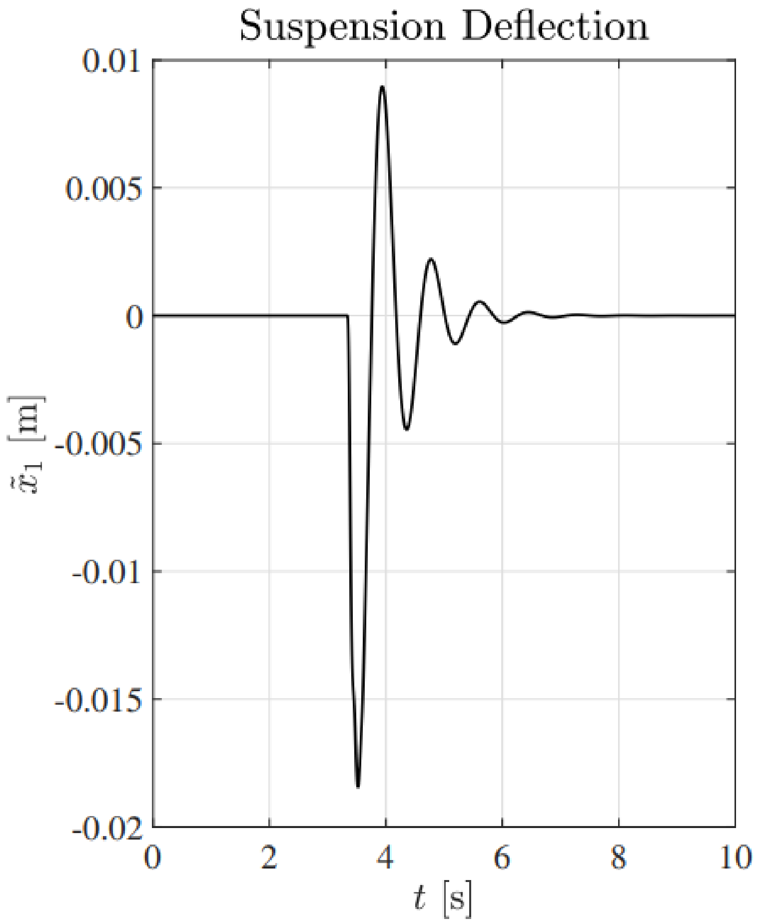

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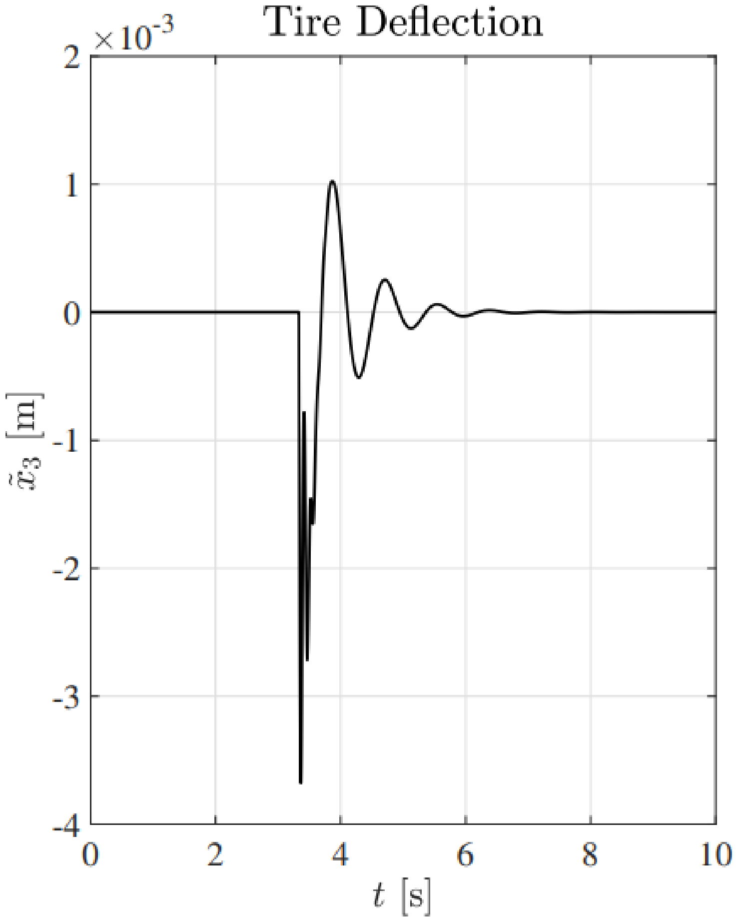

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Contents

- Active Suspensions
- Adaptive Cruise Control
- Wheel Speed Controls
- Electronic Stability Program / Torque Vectoring
- Self-Park Assist

Adaptive Cruise Control

Find the eigenvalues of $$\mathbf{A} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & -\rho S_B C_{D_B} v0/m_B & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & -\rho S_C C_{D_C} v0/m_C \end{bmatrix}$$

Adaptive Cruise Control

Find the eigenvalues of $\mathbf{A} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & -\rho S_B C_{D_B} v0/m_B & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & -\rho S_C C_{D_C} v0/m_C \end{bmatrix}$

$$\begin{matrix} 0 & -1 \\ 0 & -\rho S_B C_{D_B} v_0/m_B \end{matrix}$$

$$\begin{aligned} \lambda_1 &= 0 & a_1 &= 2 \\ \lambda_2 &= -\rho S_B C_{D_B} v_0/m_B & a_2 &= 1 \\ \lambda_3 &= -\rho S_C C_{D_C} v_0/m_C & a_3 &= 1 \end{aligned}$$

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the geometric multiplicity is $g_1 = 2$,

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Find the eigenvalues of $\mathbf{A} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & -\rho S_B C_{D_B} v0/m_B & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & -\rho S_C C_{D_C} v0/m_C \end{bmatrix}$

$$\begin{bmatrix} 0 & -1 \\ 0 & -\rho S_B C_{D_B} v_0/m_B \end{bmatrix} \qquad \begin{bmatrix} 0 & -1 \\ 0 & -\rho S_C C_{D_C} v_0/m_C \end{bmatrix}$$

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$$\mathbf{V}^{-1}\mathbf{A}\mathbf{V} = \mathbf{J} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -\rho S_B C_{D_B} v_0/m_B & 0 \\ 0 & 0 & 0 & -\rho S_C C_{D_C} v_0/m_C \end{bmatrix}$$

Adaptive Cruise Control

$$\dot{\mathbf{z}} = \mathbf{J}\mathbf{z},\ \mathbf{z}(0) = \mathbf{z}_0 \quad \mathbf{J} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -\rho S_B C_{D_B} v_0/m_B & 0 \\ 0 & 0 & 0 & -\rho S_C C_{D_C} v_0/m_C \end{bmatrix}$$

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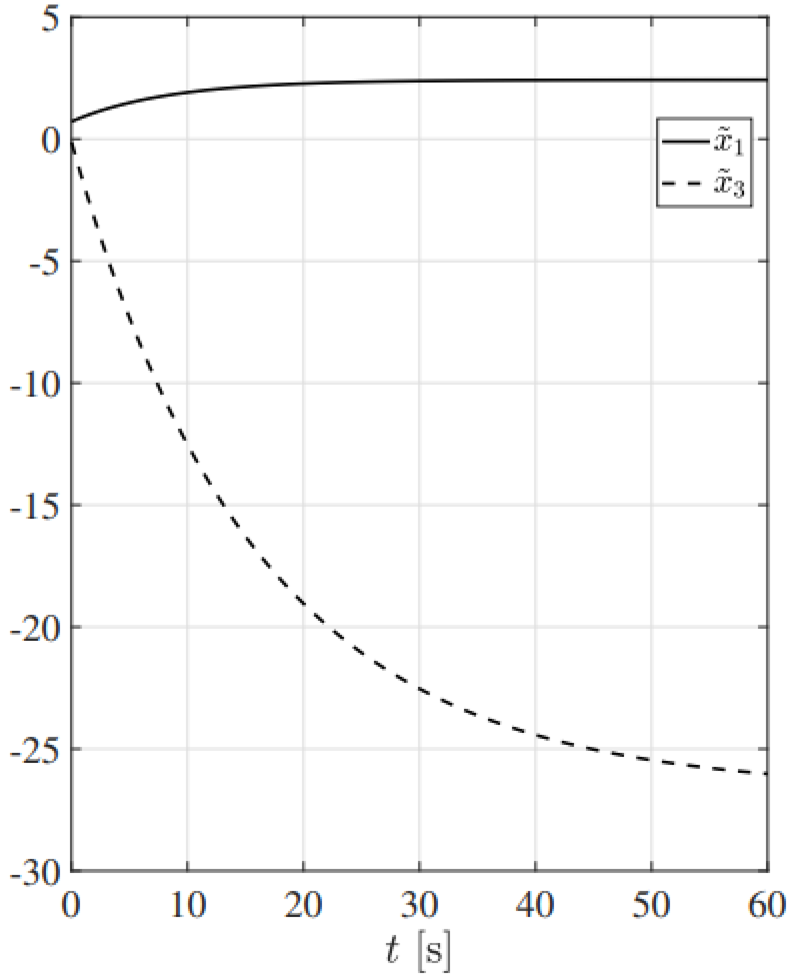

Adaptive Cruise Control

$$\dot{\mathbf{z}} = \mathbf{J}\mathbf{z},\ \mathbf{z}(0) = \mathbf{z}_0 \quad \mathbf{J} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -\rho S_B C_{D_B} v_0/m_B & 0 \\ 0 & 0 & 0 & -\rho S_C C_{D_C} v_0/m_C \end{bmatrix}$$

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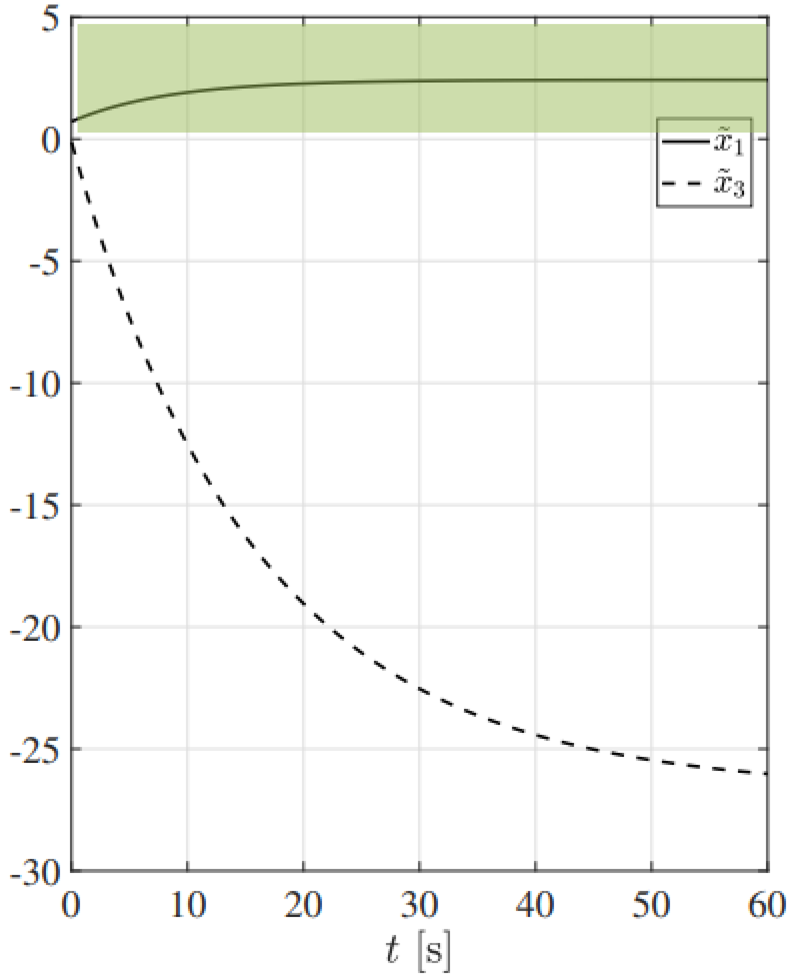

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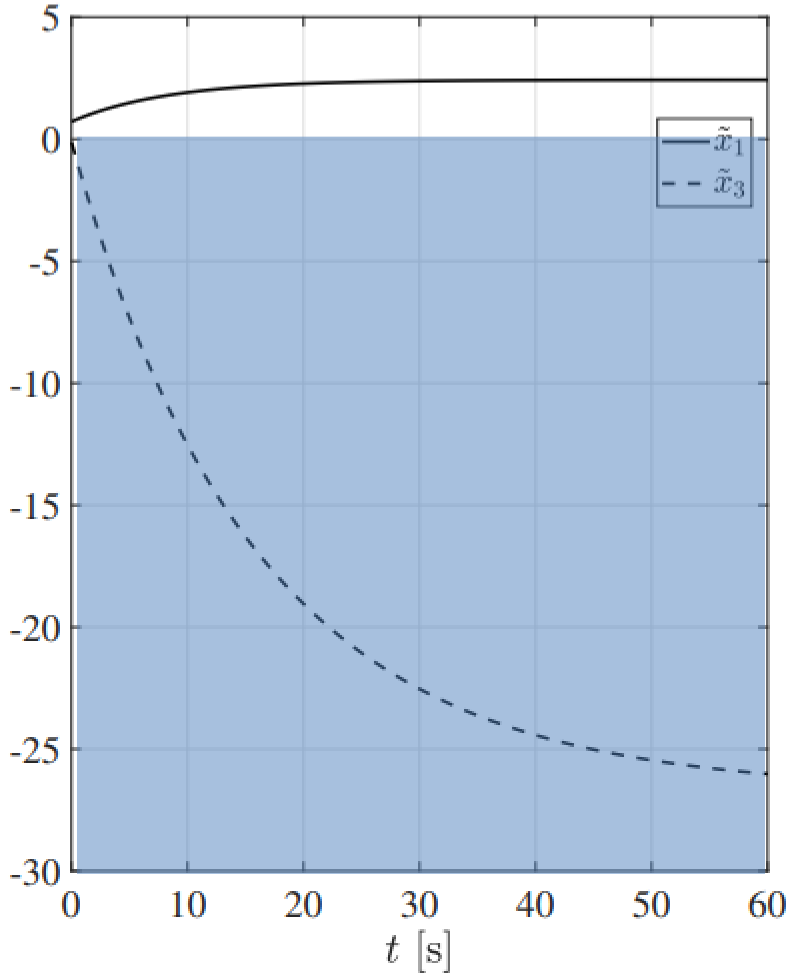
$$\mathbf{z}(t) = \exp(\boldsymbol{J}t)\mathbf{z}(0) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \exp\left(-\frac{\rho S_B C_{D_B} v_0}{m_B} t\right) & 0 \\ 0 & 0 & 0 & \exp\left(-\frac{\rho S_C C_{D_C} v_0}{m_C} t\right) \end{bmatrix} \begin{bmatrix} z_{10} \\ z_{20} \\ z_{30} \\ z_{40} \end{bmatrix}$$

$$\mathbf{x}(t) = \mathbf{T}^{-1}\mathbf{z}(t) \qquad \mathbf{x} := \mathrm{col}(d_B, v_B, d_C, v_C)$$

$$\begin{bmatrix} \tilde{x}_1(t) \\ \tilde{x}_2(t) \\ \tilde{x}_3(t) \\ \tilde{x}_4(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & \frac{\rho S_B C_{D_B} v_0}{m_B} & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & \frac{\rho S_C C_{D_C} v_0}{m_C} \end{bmatrix} \begin{bmatrix} z_{10} \\ z_{20} \\ \exp(-\frac{\rho S_B C_{D_B} v_0}{m_B} t) z_{30} \\ \exp(-\frac{\rho S_C C_{D_C} v_0}{m_C} t) z_{40} \end{bmatrix}$$



Contents

Wheel Speed Controls

Let us have a look at

$$\mathbf{A} = \begin{bmatrix} \frac{1}{m}\left(\frac{\partial f_r + f_f}{\partial v} - \rho S v C_D\right) & \frac{1}{m}\frac{\partial f_r + f_f}{\partial \omega_r} & \frac{1}{m}\frac{\partial f_r + f_f}{\partial \omega_f} \\ -\frac{r_r}{J_r}\frac{\partial f_r}{\partial v} & -\frac{r_r}{J_r}\frac{\partial f_r}{\partial \omega_r} & -\frac{r_r}{J_r}\frac{\partial f_r}{\partial \omega_f} \\ -\frac{r_f}{J_f}\frac{\partial f_f}{\partial v} & -\frac{r_f}{J_f}\frac{\partial f_f}{\partial \omega_r} & -\frac{r_f}{J_f}\frac{\partial f_f}{\partial \omega_f} \end{bmatrix}_{\mathbf{x}=\mathbf{x}_0}$$

Wheel Speed Controls

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And divide the force derivatives in static and load-transfer related contributions such that

$$\mathbf{A} = \mathbf{A}_{\mathrm{s}} + \mathbf{A}_{\mathrm{lt}}$$

$\bar{\mu}_r = \mu_r - c_r;$

$\bar{\mu}_f = \mu_f - c_r$

with

$$\mathbf{A}_{\mathrm{s}} = \begin{bmatrix} \frac{N_r}{m}\frac{\partial \bar{\mu}_r}{\partial v} + \frac{N_f}{m}\frac{\partial \bar{\mu}_f}{\partial v} - \frac{\rho S v C_D}{m} & \frac{N_r}{m}\frac{\partial \bar{\mu}_r}{\partial \omega_r} & \frac{N_f}{m}\frac{\partial \bar{\mu}_f}{\partial \omega_f} \\ -\frac{r_r N_r}{J_r}\frac{\partial \bar{\mu}_r}{\partial v} & -\frac{r_r N_r}{J_r}\frac{\partial \bar{\mu}_r}{\partial \omega_r} & 0 \\ -\frac{r_f N_f}{J_f}\frac{\partial \bar{\mu}_f}{\partial v} & 0 & -\frac{r_f N_f}{J_f}\frac{\partial \bar{\mu}_f}{\partial \omega_f} \end{bmatrix}_{\mathbf{x}=\mathbf{x}_0}$$

Wheel Speed Controls

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$$\mathbf{A} = \begin{bmatrix} \frac{1}{m}\left(\frac{\partial f_r + f_f}{\partial v} - \rho S v C_D\right) & \frac{1}{m}\frac{\partial f_r + f_f}{\partial \omega_r} & \frac{1}{m}\frac{\partial f_r + f_f}{\partial \omega_f} \\ -\frac{r_r}{J_r}\frac{\partial f_r}{\partial v} & -\frac{r_r}{J_r}\frac{\partial f_r}{\partial \omega_r} & -\frac{r_r}{J_r}\frac{\partial f_r}{\partial \omega_f} \\ -\frac{r_f}{J_f}\frac{\partial f_f}{\partial v} & -\frac{r_f}{J_f}\frac{\partial f_f}{\partial \omega_r} & -\frac{r_f}{J_f}\frac{\partial f_f}{\partial \omega_f} \end{bmatrix}_{\mathbf{x}=\mathbf{x}_0}$$

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Numerical evaluations show that

$$\mathbf{A} \approx \mathbf{A}_{\mathrm{s}}$$

Wheel Speed Controls

Therefore, we focus only on

$$\mathbf{A}_s = \begin{bmatrix} \frac{N_r}{m}\frac{\partial \bar{\mu}_r}{\partial v} + \frac{N_f}{m}\frac{\partial \bar{\mu}_f}{\partial v} - \frac{\rho S v C_D}{m} & \frac{N_r}{m}\frac{\partial \bar{\mu}_r}{\partial \omega_r} & \frac{N_f}{m}\frac{\partial \bar{\mu}_f}{\partial \omega_f} \\ -\frac{r_r N_r}{J_r}\frac{\partial \bar{\mu}_r}{\partial v} & -\frac{r_r N_r}{J_r}\frac{\partial \bar{\mu}_r}{\partial \omega_r} & 0 \\ -\frac{r_f N_f}{J_f}\frac{\partial \bar{\mu}_f}{\partial v} & 0 & -\frac{r_f N_f}{J_f}\frac{\partial \bar{\mu}_f}{\partial \omega_f} \end{bmatrix}_{\mathbf{x}=\mathbf{x}_0}$$

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And we note that $m \gg J_r/r_r, J_f/r_f$

Wheel Speed Controls

```
A =

   -7.4857    1.1758    0.9706
  799.9075 -231.2361   -6.9318
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```

$v_0 = 100$ km/h

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$$\mathbf{A}_\mathrm{s} = \begin{bmatrix} \frac{N_r}{m}\frac{\partial \bar{\mu}_r}{\partial v} + \frac{N_f}{m}\frac{\partial \bar{\mu}_f}{\partial v} - \frac{\rho S v C_D}{m} & \frac{N_r}{m}\frac{\partial \bar{\mu}_r}{\partial \omega_r} & \frac{N_f}{m}\frac{\partial \bar{\mu}_f}{\partial \omega_f} \\ -\frac{r_r N_r}{J_r}\frac{\partial \bar{\mu}_r}{\partial v} & -\frac{r_r N_r}{J_r}\frac{\partial \bar{\mu}_r}{\partial \omega_r} & 0 \\ -\frac{r_f N_f}{J_f}\frac{\partial \bar{\mu}_f}{\partial v} & 0 & -\frac{r_f N_f}{J_f}\frac{\partial \bar{\mu}_f}{\partial \omega_f} \end{bmatrix}_{\mathbf{x}=\mathbf{x}_0}$$

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Whose eigenvalues are

Wheel Speed Controls

```
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Aerodynamic Drag

Traction coefficient variation due to wheel speed variations

Whose eigenvalues are

Wheel Speed Controls

$$\bar{\mu}_r = \mu_r - c_r;$$
$$\bar{\mu}_f = \mu_f - c_r$$

More in detail, we investigate

$$\mathbf{A}_s \approx \begin{bmatrix} -\dfrac{\rho S v C_D}{m} & 0 & 0 \\ -\dfrac{r_r N_r}{J_r}\dfrac{\partial \bar{\mu}_r}{\partial v} & -\dfrac{r_r N_r}{J_r}\dfrac{\partial \bar{\mu}_r}{\partial \omega_r} & 0 \\ -\dfrac{r_f N_f}{J_f}\dfrac{\partial \bar{\mu}_f}{\partial v} & 0 & -\dfrac{r_f N_f}{J_f}\dfrac{\partial \bar{\mu}_f}{\partial \omega_f} \end{bmatrix}_{\mathbf{x}=\mathbf{x}_0}$$

Wheel Speed Controls

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$$\left.\frac{\partial \mu(\lambda(v,\omega r),\boldsymbol{\Theta})}{\partial v}\right|_{v=v_0,\omega=\omega_0} = \left.\frac{\partial \mu(\lambda,\boldsymbol{\Theta})}{\partial \lambda}\right|_{\lambda=\lambda_0} \left.\frac{\partial \lambda(v,\omega r)}{\partial v}\right|_{v=v_0,\omega=\omega_0}$$
$$= \left.\frac{\partial \mu(\lambda,\boldsymbol{\Theta})}{\partial \lambda}\right|_{\lambda=\lambda_0} \frac{\lambda_0 - 1}{v_0}$$
$$\left.\frac{\partial \mu(\lambda(v,\omega r),\boldsymbol{\Theta})}{\partial \omega}\right|_{v=v_0,\omega=\omega_0} = \left.\frac{\partial \mu(\lambda,\boldsymbol{\Theta})}{\partial \lambda}\right|_{\lambda=\lambda_0} \left.\frac{\partial \lambda(v,\omega r)}{\partial \omega}\right|_{v=v_0,\omega=\omega_0}$$
$$= \left.\frac{\partial \mu(\lambda,\boldsymbol{\Theta})}{\partial \lambda}\right|_{\lambda=\lambda_0} \frac{r\,(1-\lambda_0)^2}{v_0},$$

Wheel Speed Controls

$$\bar{\mu}_r = \mu_r - c_r;$$
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?

$v_0 \to 0$

Wheel Speed Controls

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$$\begin{aligned} \left.\frac{\partial \mu(\lambda(v, \omega r), \boldsymbol{\Theta})}{\partial v}\right|_{v=v_0, \omega=\omega_0} &= \left.\frac{\partial \mu(\lambda, \boldsymbol{\Theta})}{\partial \lambda}\right|_{\lambda=\lambda_0} \left.\frac{\partial \lambda(v, \omega r)}{\partial v}\right|_{v=v_0, \omega=\omega_0} \\ &= \left.\frac{\partial \mu(\lambda, \boldsymbol{\Theta})}{\partial \lambda}\right|_{\lambda=\lambda_0} \frac{\lambda_0 - 1}{v_0} \\ \left.\frac{\partial \mu(\lambda(v, \omega r), \boldsymbol{\Theta})}{\partial \omega}\right|_{v=v_0, \omega=\omega_0} &= \left.\frac{\partial \mu(\lambda, \boldsymbol{\Theta})}{\partial \lambda}\right|_{\lambda=\lambda_0} \left.\frac{\partial \lambda(v, \omega r)}{\partial \omega}\right|_{v=v_0, \omega=\omega_0} \\ &= \left.\frac{\partial \mu(\lambda, \boldsymbol{\Theta})}{\partial \lambda}\right|_{\lambda=\lambda_0} \frac{r(1-\lambda_0)^2}{v_0}, \end{aligned}$$

When, $v_0 \to 0$ the first row of A_S is negligible

Wheel Speed Controls

When, $v_0 \to 0$ the first row of A_S is negligible

This implies that the vehicle speed dynamics are much slower than the wheel speed dynamics

Wheel Speed Controls

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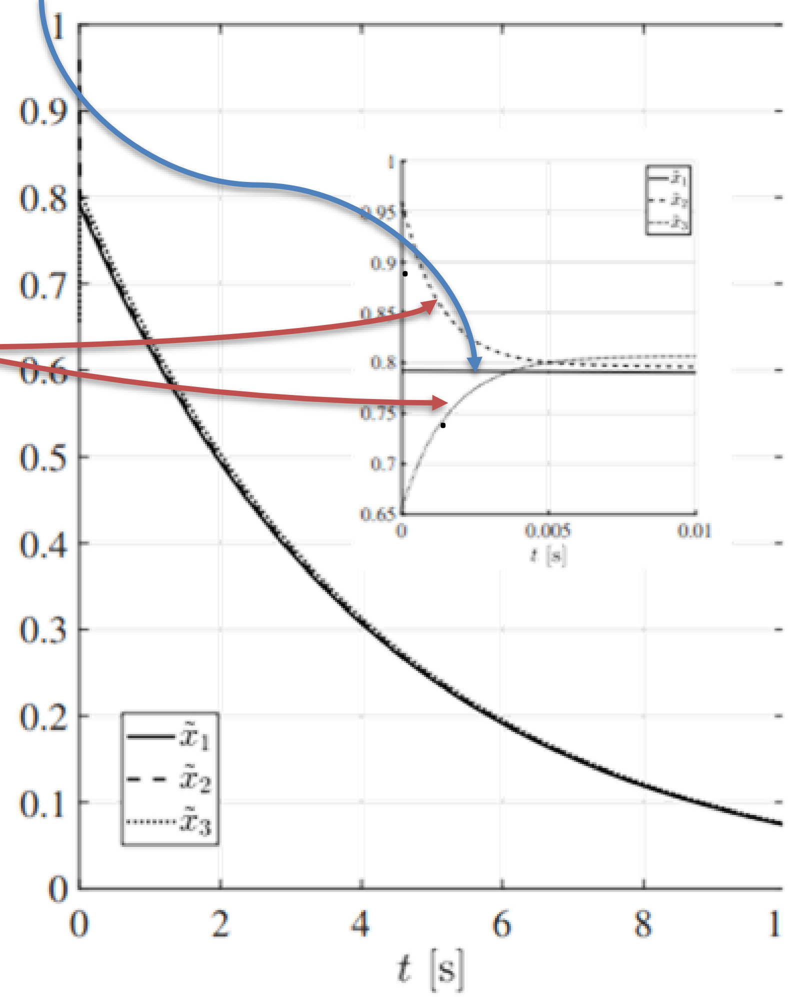

Therefore, we can consider $\dot{v} \approx 0$ while examining the dynamics of $\omega_\#$

Wheel Speed Controls

As a consequence, we can write

$$
\begin{bmatrix} \dot{\tilde{v}} \\ \dot{\tilde{\omega}}_r \\ \dot{\tilde{\omega}}_f \end{bmatrix} \approx \begin{bmatrix} -\dfrac{\rho S v C_D}{m} & 0 & 0 \\ -\dfrac{r_r N_r}{J_r}\dfrac{\partial \bar{\mu}_r}{\partial v} & -\dfrac{r_r N_r}{J_r}\dfrac{\partial \bar{\mu}_r}{\partial \omega_r} & 0 \\ -\dfrac{r_f N_f}{J_f}\dfrac{\partial \bar{\mu}_f}{\partial v} & 0 & -\dfrac{r_f N_f}{J_f}\dfrac{\partial \bar{\mu}_f}{\partial \omega_f} \end{bmatrix}_{\mathbf{x}=\mathbf{x}_0} \begin{bmatrix} \tilde{v} \\ \tilde{\omega}_r \\ \tilde{\omega}_f \end{bmatrix} + \cdots
$$

$\mathbf{A}_{21}$ $\mathbf{A}_{22}$

as

$$
\begin{bmatrix} \dot{\tilde{\omega}}_r \\ \dot{\tilde{\omega}}_f \end{bmatrix} \approx \begin{bmatrix} -\dfrac{r_r N_r}{J_r}\dfrac{\partial \bar{\mu}_r}{\partial \omega_r} & 0 \\ 0 & -\dfrac{r_f N_f}{J_f}\dfrac{\partial \bar{\mu}_f}{\partial \omega_f} \end{bmatrix}_{\mathbf{x}=\mathbf{x}'} \begin{bmatrix} \tilde{\omega}_r \\ \tilde{\omega}_f \end{bmatrix} + \begin{bmatrix} -\dfrac{r_r N_r}{J_r}\dfrac{\partial \bar{\mu}_r}{\partial v} \\ -\dfrac{r_f N_f}{J_f}\dfrac{\partial \bar{\mu}_f}{\partial v} \end{bmatrix} \tilde{v} + \cdots
$$

Wheel Speed Controls

As a consequence, we can write

$$\begin{bmatrix} \dot{\tilde{v}} \\ \dot{\tilde{\omega}}_r \\ \dot{\tilde{\omega}}_f \end{bmatrix} \approx \left[\begin{array}{c|cc} -\dfrac{\rho S v C_D}{m} & 0 & 0 \\ -\dfrac{r_r N_r}{J_r}\dfrac{\partial \bar{\mu}_r}{\partial v} & -\dfrac{r_r N_r}{J_r}\dfrac{\partial \bar{\mu}_r}{\partial \omega_r} & 0 \\ -\dfrac{r_f N_f}{J_f}\dfrac{\partial \bar{\mu}_f}{\partial v} & 0 & -\dfrac{r_f N_f}{J_f}\dfrac{\partial \bar{\mu}_f}{\partial \omega_f} \end{array}\right]_{\mathbf{x}=\mathbf{x}_0} \begin{bmatrix} \tilde{v} \\ \tilde{\omega}_r \\ \tilde{\omega}_f \end{bmatrix} + \cdots$$

$\mathbf{A}_{21}$ $\mathbf{A}_{22}$

as

$$\begin{bmatrix} \dot{\tilde{\omega}}_r \\ \dot{\tilde{\omega}}_f \end{bmatrix} \approx \begin{bmatrix} -\dfrac{r_r N_r}{J_r}\dfrac{\partial \bar{\mu}_r}{\partial \omega_r} & 0 \\ 0 & -\dfrac{r_f N_f}{J_f}\dfrac{\partial \bar{\mu}_f}{\partial \omega_f} \end{bmatrix}_{\mathbf{x}=\mathbf{x}'} \begin{bmatrix} \tilde{\omega}_r \\ \tilde{\omega}_f \end{bmatrix} + \begin{bmatrix} -\dfrac{r_r N_r}{J_r}\dfrac{\partial \bar{\mu}_r}{\partial v} \\ -\dfrac{r_f N_f}{J_f}\dfrac{\partial \bar{\mu}_f}{\partial v} \end{bmatrix} \tilde{v} + \cdots$$

$$\bar{\mu}_r = \mu_r - c_r;$$
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$$\left.\frac{\partial \mu(\lambda(v,\omega r),\boldsymbol{\Theta})}{\partial \omega}\right|_{v=v_0,\omega=\omega_0} = \left.\frac{\partial \mu(\lambda,\boldsymbol{\Theta})}{\partial \lambda}\right|_{\lambda=\lambda_0} \left.\frac{\partial \lambda(v,\omega r)}{\partial \omega}\right|_{v=v_0,\omega=\omega_0}$$
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Wheel Speed Controls

As a consequence, we can write

$$\begin{bmatrix}\dot{\tilde v}\\ \dot{\tilde\omega}_r\\ \dot{\tilde\omega}_f\end{bmatrix} \approx \begin{bmatrix} -\frac{\rho S v C_D}{m} & 0 & 0 \\ -\frac{r_rN_r}{J_r}\frac{\partial\bar\mu_r}{\partial v} & -\frac{r_rN_r}{J_r}\frac{\partial\bar\mu_r}{\partial\omega_r} & 0 \\ -\frac{r_fN_f}{J_f}\frac{\partial\bar\mu_f}{\partial v} & 0 & -\frac{r_fN_f}{J_f}\frac{\partial\bar\mu_f}{\partial \omega_f}\end{bmatrix}_{\mathbf{x}=\mathbf{x}_0} \begin{bmatrix}\tilde v\\ \tilde\omega_r\\ \tilde\omega_f\end{bmatrix} + \cdots$$

$\mathbf{A}_{21}$ $\mathbf{A}_{22}$

$$\begin{bmatrix}\dot{\tilde\omega}_r\\ \dot{\tilde\omega}_f\end{bmatrix} \approx \begin{bmatrix} -\frac{r_rN_r}{J_r}\frac{\partial\bar\mu_r}{\partial\omega_r} & 0 \\ 0 & -\frac{r_fN_f}{J_f}\frac{\partial\bar\mu_f}{\partial \omega_f}\end{bmatrix}_{\mathbf{x}=\mathbf{x}'} \begin{bmatrix}\tilde\omega_r\\ \tilde\omega_f\end{bmatrix} + \begin{bmatrix} -\frac{r_rN_r}{J_r}\frac{\partial\bar\mu_r}{\partial v} \\ -\frac{r_fN_f}{J_f}\frac{\partial\bar\mu_f}{\partial v}\end{bmatrix} \tilde v + \cdots$$

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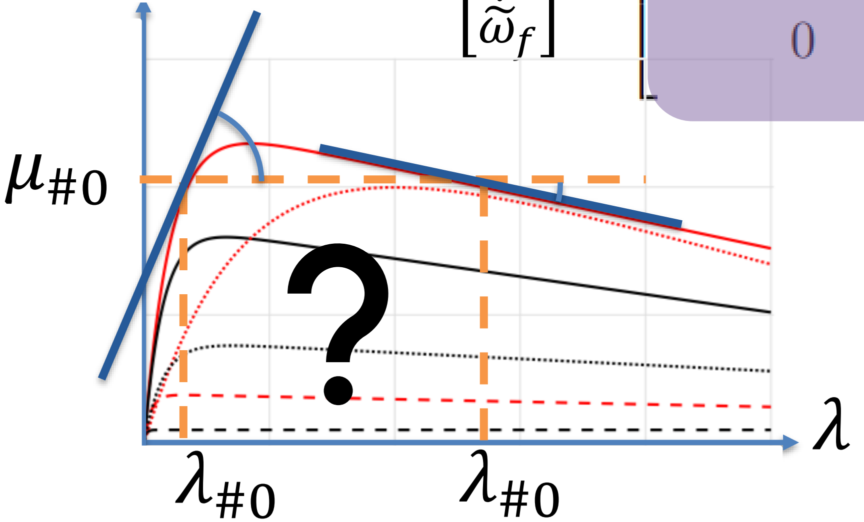


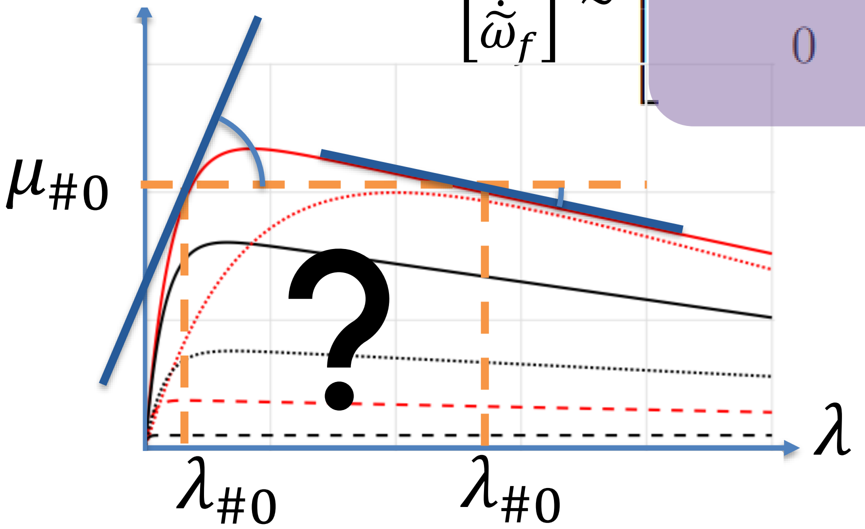
$$\left.\frac{\partial\mu(\lambda(v,\omega r),\boldsymbol{\Theta})}{\partial\omega}\right|_{v=v_0,\omega=\omega_0} = \left.\frac{\partial\mu(\lambda,\boldsymbol{\Theta})}{\partial\lambda}\right|_{\lambda=\lambda_0} \left.\frac{\partial\lambda(v,\omega r)}{\partial\omega}\right|_{v=v_0,\omega=\omega_0}$$
$$= \boxed{\left.\frac{\partial\mu(\lambda,\boldsymbol{\Theta})}{\partial\lambda}\right|_{\lambda=\lambda_0}} \frac{r\,(1-\lambda_0)^2}{v_0},$$

Wheel Speed Controls

As a consequence, we can write

$$
\begin{bmatrix} \dot{\tilde{v}} \\ \dot{\tilde{\omega}}_r \\ \dot{\tilde{\omega}}_f \end{bmatrix} \approx \left. \begin{bmatrix} -\dfrac{\rho S v C_D}{m} & 0 & 0 \\ \mathbf{A}_{21}\; -\dfrac{r_r N_r}{J_r}\dfrac{\partial \bar{\mu}_r}{\partial v} & \mathbf{A}_{22}\; -\dfrac{r_r N_r}{J_r}\dfrac{\partial \bar{\mu}_r}{\partial \omega_r} & 0 \\ -\dfrac{r_f N_f}{J_f}\dfrac{\partial \bar{\mu}_f}{\partial v} & 0 & -\dfrac{r_f N_f}{J_f}\dfrac{\partial \bar{\mu}_f}{\partial \omega_f} \end{bmatrix} \right|_{\mathbf{x}=\mathbf{x}_0} \begin{bmatrix} \tilde{v} \\ \tilde{\omega}_r \\ \tilde{\omega}_f \end{bmatrix} + \cdots
$$

$$
\begin{bmatrix} \dot{\tilde{\omega}}_r \\ \dot{\tilde{\omega}}_f \end{bmatrix} \approx \left. \begin{bmatrix} -\dfrac{r_r N_r}{J_r}\dfrac{\partial \bar{\mu}_r}{\partial \omega_r} & 0 \\ 0 & -\dfrac{r_f N_f}{J_f}\dfrac{\partial \bar{\mu}_f}{\partial \omega_f} \end{bmatrix} \right|_{\mathbf{x}=\mathbf{x}'} \begin{bmatrix} \tilde{\omega}_r \\ \tilde{\omega}_f \end{bmatrix} + \begin{bmatrix} -\dfrac{r_r N_r}{J_r}\dfrac{\partial \bar{\mu}_r}{\partial v} \\ -\dfrac{r_f N_f}{J_f}\dfrac{\partial \bar{\mu}_f}{\partial v} \end{bmatrix} \tilde{v} + \cdots
$$



$$
\left.\frac{\partial \mu(\lambda(v,\omega r),\boldsymbol{\Theta})}{\partial \omega}\right|_{v=v_0,\omega=\omega_0} = \left.\frac{\partial \mu(\lambda,\boldsymbol{\Theta})}{\partial \lambda}\right|_{\lambda=\lambda_0} \left.\frac{\partial \lambda(v,\omega r)}{\partial \omega}\right|_{v=v_0,\omega=\omega_0}
$$

$$
= \boxed{\left.\frac{\partial \mu(\lambda,\boldsymbol{\Theta})}{\partial \lambda}\right|_{\lambda=\lambda_0}} \frac{r\,(1-\lambda_0)^2}{v_0},
$$

?

$v_0 \rightarrow 0$

Contents

ESP/TV

Starting from

$$\begin{bmatrix} \dot{v}_x \\ \dot{v}_y \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} \omega v_y \\ -\omega v_x \\ 0 \end{bmatrix} + \begin{bmatrix} (f_x - D(v_x - w))/m \\ f_y/m \\ \tau/J \end{bmatrix}$$

$\xrightarrow{\text{define}}$

$$\frac{1}{m}\frac{\partial f_x}{\partial v_x} = \frac{1}{v_0}\frac{\partial \bar{f}_x}{\partial v_x},$$

$$\frac{1}{m}\frac{\partial f_y}{\partial v_y} = \frac{1}{v_0}\frac{\partial \bar{f}_y}{\partial v_y}, \quad \frac{1}{m}\frac{\partial f_y}{\partial \omega} = \frac{1}{v_0}\frac{\partial \bar{f}_y}{\partial \omega},$$

$$\frac{1}{m}\frac{\partial \tau}{\partial v_y} = \frac{1}{v_0}\frac{\partial \bar{\tau}}{\partial v_y}, \quad \frac{1}{m}\frac{\partial \tau}{\partial \omega} = \frac{1}{v_0}\frac{\partial \bar{\tau}}{\partial \omega}.$$

to short the notation

$$\mathbf{A} = \begin{bmatrix} \frac{1}{v_0}\frac{\partial \bar{f}_x}{\partial v_x} - \frac{\rho S C_D}{m} v_0 & 0 & 0 \\ 0 & \frac{1}{v_0}\frac{\partial \bar{f}_y}{\partial v_y} & \frac{1}{v_0}\frac{\partial \bar{f}_y}{\partial \omega} - v_0 \\ 0 & \frac{m}{J v_0}\frac{\partial \bar{\tau}}{\partial v_y} & \frac{m}{J v_0}\frac{\partial \bar{\tau}}{\partial \omega} \end{bmatrix}_{\mathbf{x}=\mathbf{x}_0, \mathbf{u}=\mathbf{u}_0}$$

Compute the eigenvalues of **A**

$$\mathbf{A} = \begin{bmatrix} \dfrac{1}{v_0}\dfrac{\partial \bar{f}_x}{\partial v_x} - \dfrac{\rho S C_D}{m} v_0 & 0 & 0 \\ 0 & \dfrac{1}{v_0}\dfrac{\partial \bar{f}_y}{\partial v_y} & \dfrac{1}{v_0}\dfrac{\partial \bar{f}_y}{\partial \omega} - v_0 \\ 0 & \dfrac{m}{J v_0}\dfrac{\partial \bar{\tau}}{\partial v_y} & \dfrac{m}{J v_0}\dfrac{\partial \bar{\tau}}{\partial \omega} \end{bmatrix}_{\mathbf{x}=\mathbf{x}_0, \mathbf{u}=\mathbf{u}_0}$$

ESP/TV

$$\lambda_1 = \frac{1}{v_0}\frac{\partial \bar{f}_x}{\partial v_x} - \frac{\rho S C_D}{m} v_0$$

$$\lambda_{2,3} = \frac{1}{2v_0}\left(\frac{\partial \bar{f}_y}{\partial v_y} + \frac{m}{J}\frac{\partial \bar{\tau}}{\partial \omega}\right) \pm \frac{1}{2v_0}\sqrt{\left(\frac{\partial \bar{f}_y}{\partial v_y} - \frac{m}{J}\frac{\partial \bar{\tau}}{\partial \omega}\right)^2 + \frac{4m}{J}\frac{\partial \bar{\tau}}{\partial v_y}\left(\frac{\partial \bar{f}_y}{\partial \omega} - v_0^2\right)}.$$

Whose eigenvalues are

$$\mathbf{A} = \begin{bmatrix} \frac{1}{v_0}\frac{\partial \bar{f}_x}{\partial v_x} - \frac{\rho S C_D}{m} v_0 & 0 & 0 \\ 0 & \frac{1}{v_0}\frac{\partial \bar{f}_y}{\partial v_y} & \frac{1}{v_0}\frac{\partial \bar{f}_y}{\partial \omega} - v_0 \\ 0 & \frac{m}{J v_0}\frac{\partial \bar{\tau}}{\partial v_y} & \frac{m}{J v_0}\frac{\partial \bar{\tau}}{\partial \omega} \end{bmatrix}_{\mathbf{x}=\mathbf{x}_0,\mathbf{u}=\mathbf{u}_0}$$

ESP/TV

$$\frac{\partial \bar{f}_x}{\partial v_x} = -\frac{g}{[1-\bar{h}(\mu_{r_0}-\mu_{f_0})]^2}\left[(\bar{a}+\bar{h}\mu_{f_0})\left.\frac{\partial \mu(\lambda,\mathbf{\Theta})}{\partial \lambda}\right|_{\lambda=\lambda_{r_0}}(1-\lambda_{r_0}) + (\bar{b}-\bar{h}\mu_{r_0})\left.\frac{\partial \mu(\lambda,\mathbf{\Theta})}{\partial \lambda}\right|_{\lambda=\lambda_{f_0}}(1-\lambda_{f_0})\right] < 0,$$

Rolling Resistance
+
Traction variation

$$\lambda_1 = \frac{1}{v_0}\frac{\partial \bar{f}_x}{\partial v_x} - \frac{\rho S C_D}{m} v_0$$

Drag Resistance

Whose eigenvalues are

$$\mathbf{A} = \begin{bmatrix} \frac{1}{v_0}\frac{\partial \bar{f}_x}{\partial v_x} - \frac{\rho S C_D}{m} v_0 & 0 & 0 \\ 0 & \frac{1}{v_0}\frac{\partial \bar{f}_y}{\partial v_y} & \frac{1}{v_0}\frac{\partial \bar{f}_y}{\partial \omega} - v_0 \\ 0 & \frac{m}{J v_0}\frac{\partial \bar{\tau}}{\partial v_y} & \frac{m}{J v_0}\frac{\partial \bar{\tau}}{\partial \omega} \end{bmatrix}_{\mathbf{x}=\mathbf{x}_0,\mathbf{u}=\mathbf{u}_0}$$

ESP/TV

Rolling Resistance
+
Traction variation

$$\frac{\partial \bar{f}_x}{\partial v_x} = -\frac{g}{[1-\bar{h}(\mu_{r_0}-\mu_{f_0})]^2}\Bigg[(\bar{a}+\bar{h}\mu_{f_0})\,\frac{\partial \mu(\lambda,\mathbf{\Theta})}{\partial \lambda}\bigg|_{\lambda=\lambda_{r_0}}(1-\lambda_{r_0})$$
$$+\,(\bar{b}-\bar{h}\mu_{r_0})\,\frac{\partial \mu(\lambda,\mathbf{\Theta})}{\partial \lambda}\bigg|_{\lambda=\lambda_{f_0}}(1-\lambda_{f_0})\Bigg] < 0,$$

$$\lambda_1 = \frac{1}{v_0}\frac{\partial \bar{f}_x}{\partial v_x} - \frac{\rho S C_D}{m} v_0$$

Drag Resistance

$\lim_{v_0 \to 0} \lambda_1 = -\infty$ due to the raised sensitivity of the slip ratio function $\lambda(\cdot,\cdot)$. In addition, $\lim_{v_0 \to \infty} \lambda_1 = -\infty$ because of the increased sensitivity of the drag resistance.

ESP/TV

To study $$\lambda_{2,3} = \frac{1}{2v_0}\left(\frac{\partial \bar{f}_y}{\partial v_y} + \frac{m}{J}\frac{\partial \bar{\tau}}{\partial \omega}\right) \pm \frac{1}{2v_0}\sqrt{\left(\frac{\partial \bar{f}_y}{\partial v_y} - \frac{m}{J}\frac{\partial \bar{\tau}}{\partial \omega}\right)^2 + \frac{4m}{J}\frac{\partial \bar{\tau}}{\partial v_y}\left(\frac{\partial \bar{f}_y}{\partial \omega} - v_0^2\right)}.$$

ESP/TV

To study $$\lambda_{2,3} = \frac{1}{2v_0}\left(\frac{\partial \bar{f}_y}{\partial v_y} + \frac{m}{J}\frac{\partial \bar{\tau}}{\partial \omega}\right) \pm \frac{1}{2v_0}\sqrt{\left(\frac{\partial \bar{f}_y}{\partial v_y} - \frac{m}{J}\frac{\partial \bar{\tau}}{\partial \omega}\right)^2 + \frac{4m}{J}\frac{\partial \bar{\tau}}{\partial v_y}\left(\frac{\partial \bar{f}_y}{\partial \omega} - v_0^2\right)}.$$

Note that $$\frac{\partial \bar{f}_y}{\partial v_y} = -g\frac{2}{\pi}\left.\frac{\partial \mu(\beta, \boldsymbol{\Theta}_0)}{\partial \beta}\right|_{\beta=0} < 0$$

$$\frac{\partial \bar{f}_y}{\partial \omega} = g\ell\bar{h}\frac{\mu_{r_0}\bar{a} + \mu_{f_0}\bar{b}}{1 - (\mu_{r_0} - \mu_{f_0})\bar{h}}\frac{2}{\pi}\left.\frac{\partial \mu(\beta, \boldsymbol{\Theta}_0)}{\partial \beta}\right|_{\beta=0} > 0$$

because, by assumption, at least one between μ_{r_0} and μ_{f_0} is greater than zero

ESP/TV

To study $$\lambda_{2,3} = \frac{1}{2v_0}\left(\frac{\partial \bar{f}_y}{\partial v_y} + \frac{m}{J}\frac{\partial \bar{\tau}}{\partial \omega}\right) \pm \frac{1}{2v_0}\sqrt{\left(\frac{\partial \bar{f}_y}{\partial v_y} - \frac{m}{J}\frac{\partial \bar{\tau}}{\partial \omega}\right)^2 + \frac{4m}{J}\frac{\partial \bar{\tau}}{\partial v_y}\left(\frac{\partial \bar{f}_y}{\partial \omega} - v_0^2\right)}.$$

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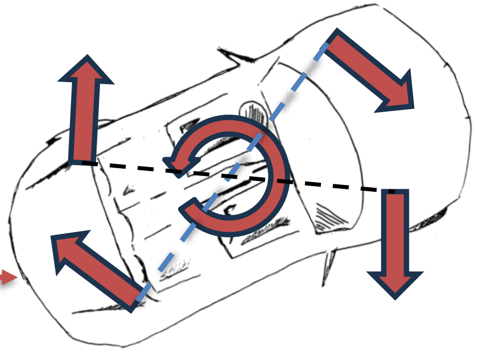
because, by assumption, at least one between μ_{r_0} and μ_{f_0} is greater than zero

Moreover $$\frac{\partial \bar{\tau}}{\partial \omega} = \frac{\partial \bar{\tau}_S}{\partial \omega} + \frac{\partial \bar{\tau}_L}{\partial \omega}$$ $$\frac{\partial \bar{\tau}_S}{\partial \omega} < 0 \qquad \frac{\partial \bar{\tau}_L}{\partial \omega} \in \mathbb{R}$$

Static

Load Transfer



ESP/TV

Consider $$\lambda_{2,3} = \frac{1}{2v_0}\left(\frac{\partial \bar{f}_y}{\partial v_y} + \frac{m}{J}\frac{\partial \bar{\tau}}{\partial \omega}\right) \pm \frac{1}{2v_0}\sqrt{\left(\frac{\partial \bar{f}_y}{\partial v_y} - \frac{m}{J}\frac{\partial \bar{\tau}}{\partial \omega}\right)^2 + \frac{4m}{J}\frac{\partial \bar{\tau}}{\partial v_y}\left(\frac{\partial \bar{f}_y}{\partial \omega} - v_0^2\right)}.$$

ESP/TV

Consider $$\lambda_{2,3} = \frac{1}{2v_0}\left(\underbrace{\frac{\partial \bar{f}_y}{\partial v_y}}_{<0} + \underbrace{\frac{m}{J}\frac{\partial \bar{\tau}}{\partial \omega}}_{\in \mathbb{R}}\right) \pm \frac{1}{2v_0}\sqrt{\underbrace{\left(\frac{\partial \bar{f}_y}{\partial v_y} - \frac{m}{J}\frac{\partial \bar{\tau}}{\partial \omega}\right)^2}_{>0} + \frac{4m}{J}\frac{\partial \bar{\tau}}{\partial v_y}\underbrace{\left(\frac{\partial \bar{f}_y}{\partial \omega} - v_0^2\right)}_{>0}}.$$

ESP/TV

Consider $$\lambda_{2,3} = \frac{1}{2v_0}\Bigg(\underbrace{\frac{\partial \bar{f}_y}{\partial v_y}}_{<0} + \frac{m}{J}\underbrace{\frac{\partial \bar{\tau}}{\partial \omega}}_{\in \mathbb{R}}\Bigg) \pm \frac{1}{2v_0}\sqrt{\underbrace{\left(\frac{\partial \bar{f}_y}{\partial v_y} - \frac{m}{J}\frac{\partial \bar{\tau}}{\partial \omega}\right)^2}_{>0} + \frac{4m}{J}\frac{\partial \bar{\tau}}{\partial v_y}\Bigg(\underbrace{\frac{\partial \bar{f}_y}{\partial \omega}}_{>0} - v_0^2\Bigg)}.$$

Assume $\partial\bar{\tau}/\partial\omega < 0$, then $\lambda_{2,3}$ have positive/negative real parts and eventually possess an imaginary part according to the sign of $\partial\bar{\tau}/\partial v_y$ and the magnitude of v_0:

ESP/TV

Consider $$\lambda_{2,3} = \frac{1}{2v_0}\left(\underbrace{\frac{\partial \bar{f}_y}{\partial v_y}}_{<0} + \frac{m}{J}\underbrace{\frac{\partial \bar{\tau}}{\partial \omega}}_{\in \mathbb{R}}\right) \pm \frac{1}{2v_0}\sqrt{\underbrace{\left(\frac{\partial \bar{f}_y}{\partial v_y} - \frac{m}{J}\frac{\partial \bar{\tau}}{\partial \omega}\right)^2}_{>0} + \frac{4m}{J}\frac{\partial \bar{\tau}}{\partial v_y}\left(\underbrace{\frac{\partial \bar{f}_y}{\partial \omega}}_{>0} - v_0^2\right)}.$$

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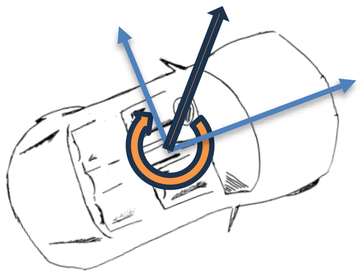
1) $\partial\bar{\tau}/\partial v_y < 0.$



ESP/TV

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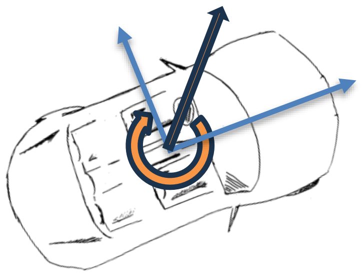
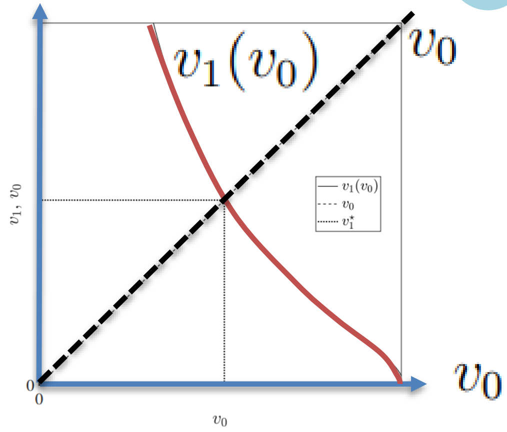
1) $\partial\bar{\tau}/\partial v_y < 0$. Define



$$v_1(v_0) = \sqrt{\frac{\partial \bar{f}_y}{\partial \omega} - \left(\frac{\partial \bar{\tau}}{\partial v_y}\right)^{-1}\frac{\partial \bar{f}_y}{\partial v_y}\frac{\partial \bar{\tau}}{\partial \omega}},$$



ESP/TV

Consider $\lambda_{2,3} = \frac{1}{2v_0}\Big(\underbrace{\frac{\partial \bar f_y}{\partial v_y}}_{<0} + \frac{m}{J}\underbrace{\frac{\partial \bar\tau}{\partial \omega}}_{\in\,\mathbb{R}}\Big) \pm \frac{1}{2v_0}\sqrt{\underbrace{\Big(\frac{\partial \bar f_y}{\partial v_y} - \frac{m}{J}\frac{\partial \bar\tau}{\partial \omega}\Big)^2}_{>0} + \frac{4m}{J}\frac{\partial \bar\tau}{\partial v_y}\Big(\underbrace{\frac{\partial \bar f_y}{\partial \omega}}_{>0} - v_0^2\Big)}.$

Assume $\partial\bar\tau/\partial\omega < 0$, then $\lambda_{2,3}$ have positive/negative real parts and eventually possess an imaginary part according to the sign of $\partial\bar\tau/\partial v_y$ and the magnitude of v_0:

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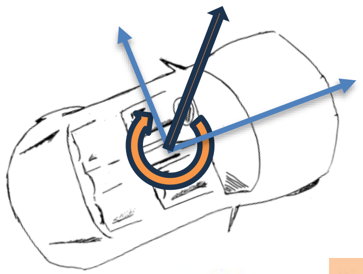
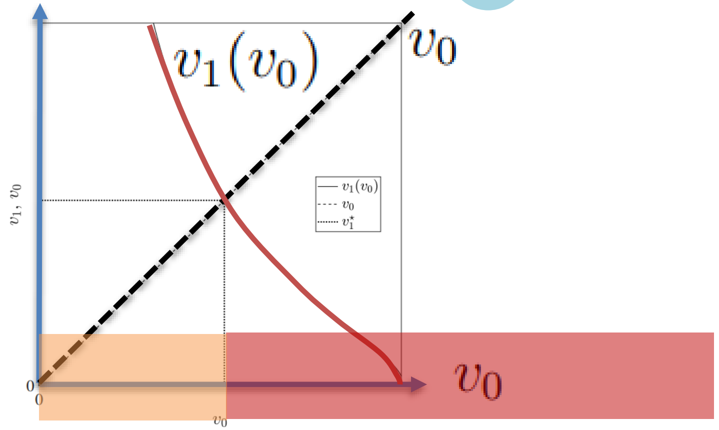


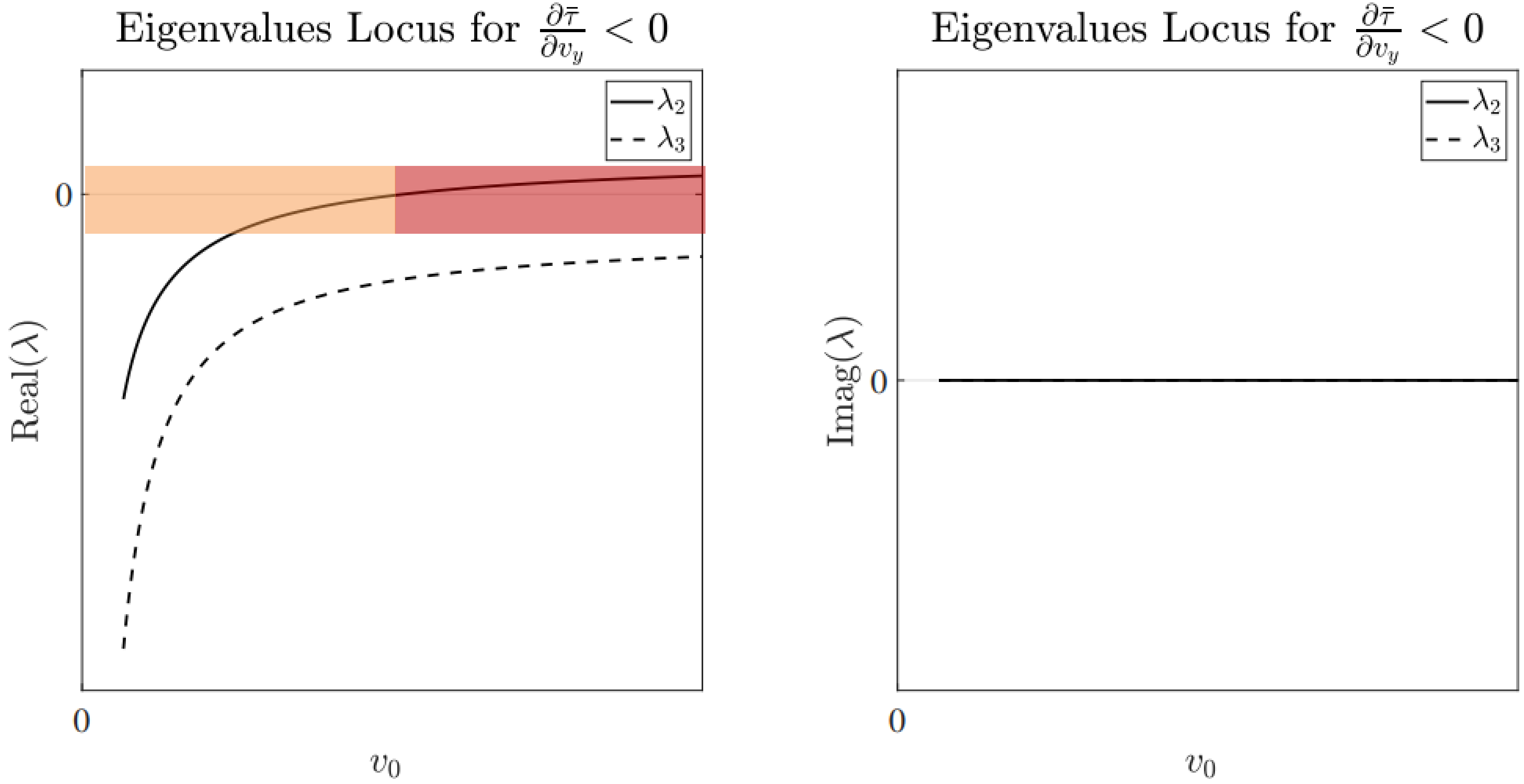
$$v_1(v_0) = \sqrt{\frac{\partial \bar f_y}{\partial \omega} - \Big(\frac{\partial \bar\tau}{\partial v_y}\Big)^{-1}\frac{\partial \bar f_y}{\partial v_y}\frac{\partial \bar\tau}{\partial \omega}},$$



then $\lambda_{2,3}$ are distinct negative reals for $v_0 > 0$ such that $v_1(v_0) > v_0$. At $v_0 = v_1^\star > 0 : v_1^\star = v_1(v_1^\star)$, it is $\lambda_2 < 0$ and $\lambda_3 = 0$. Lastly, $\lambda_2 < 0,\ \lambda_3 > 0$ for $v_0 > 0 : v_0 > v_1(v_0)$;

ESP/TV

1) $\partial\bar{\tau}/\partial v_y < 0.$



ESP/TV

$$\begin{bmatrix} \dot{p}_x \\ \dot{p}_y \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} v_0 \cos\psi - \tilde{v}_y \sin\psi \\ v_0 \sin\psi + \tilde{v}_y \cos\psi \\ \tilde{\omega} \end{bmatrix}$$

$$\dot{\tilde{\mathbf{x}}} = \mathbf{A}\tilde{\mathbf{x}} \qquad \tilde{\mathbf{x}} = [\tilde{v}_x \quad \tilde{v}_y \quad \widetilde{\omega}]^\top$$

ESP/TV

$$\begin{bmatrix} \dot{p}_x \\ \dot{p}_y \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} v_0 \cos\psi - \tilde{v}_y \sin\psi \\ v_0 \sin\psi + \tilde{v}_y \cos\psi \\ \tilde{\omega} \end{bmatrix}$$

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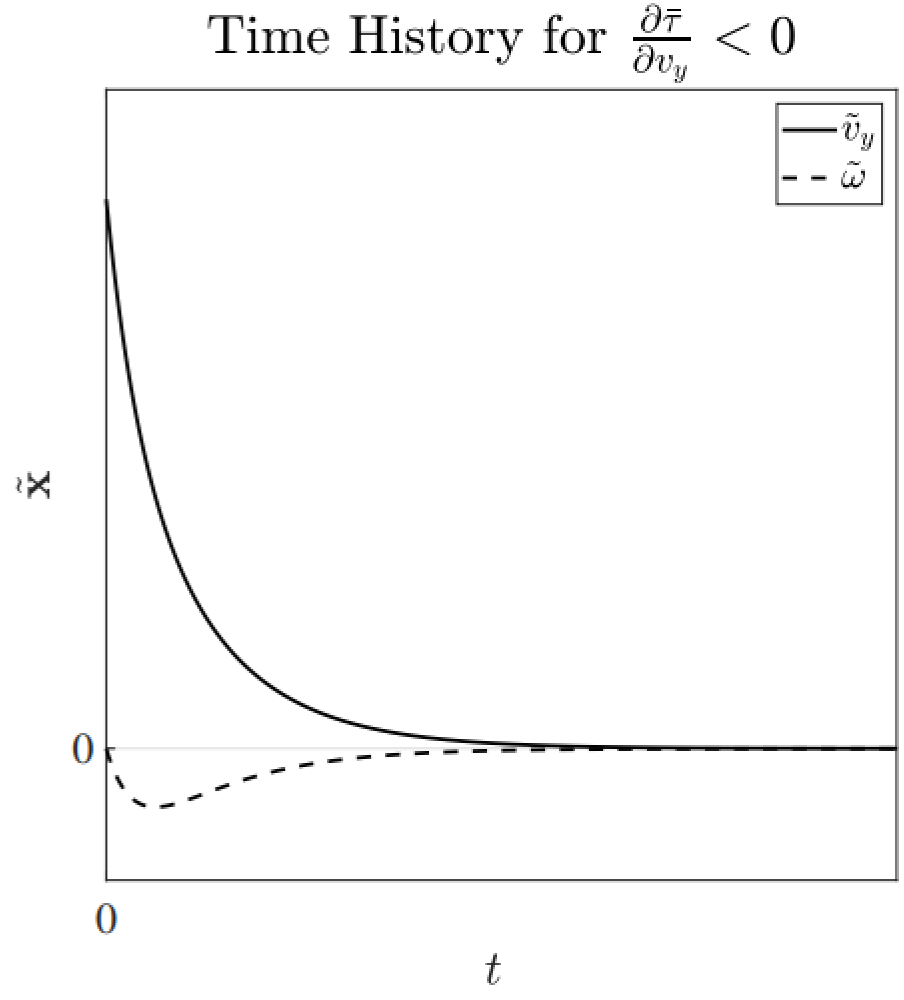
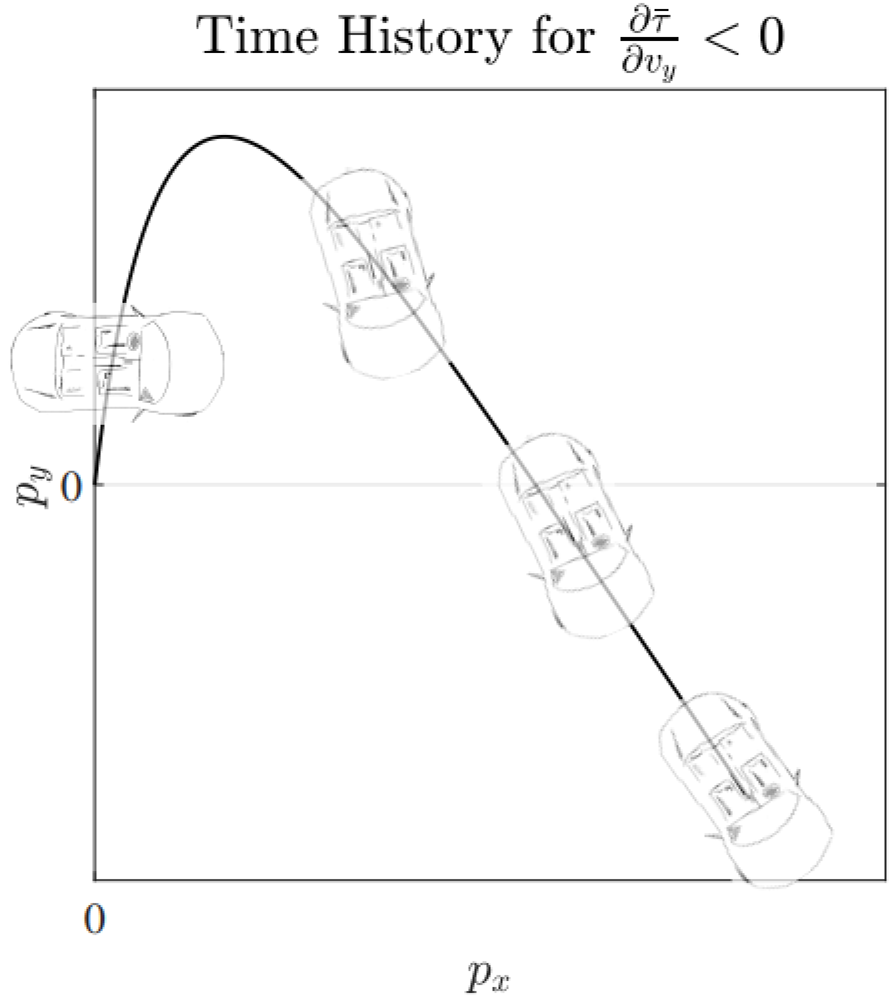
1a) $\partial\bar{\tau}/\partial v_y < 0.$ $v_1(v_0) > v_0$





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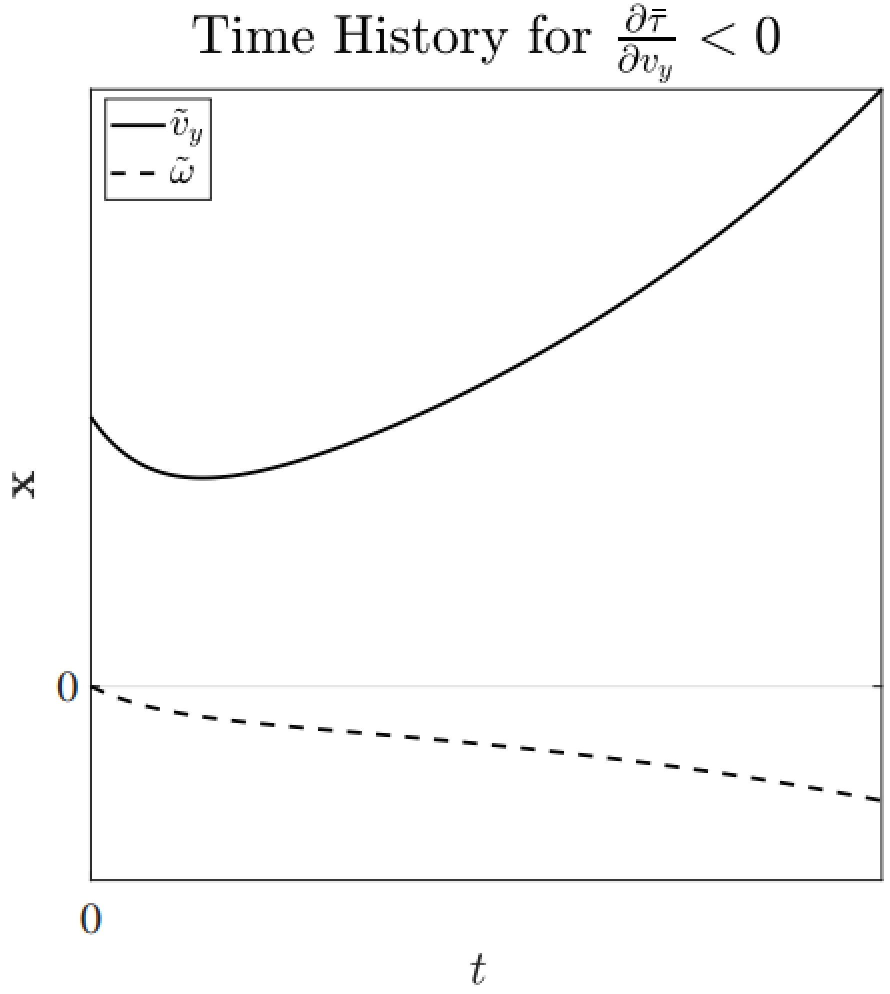
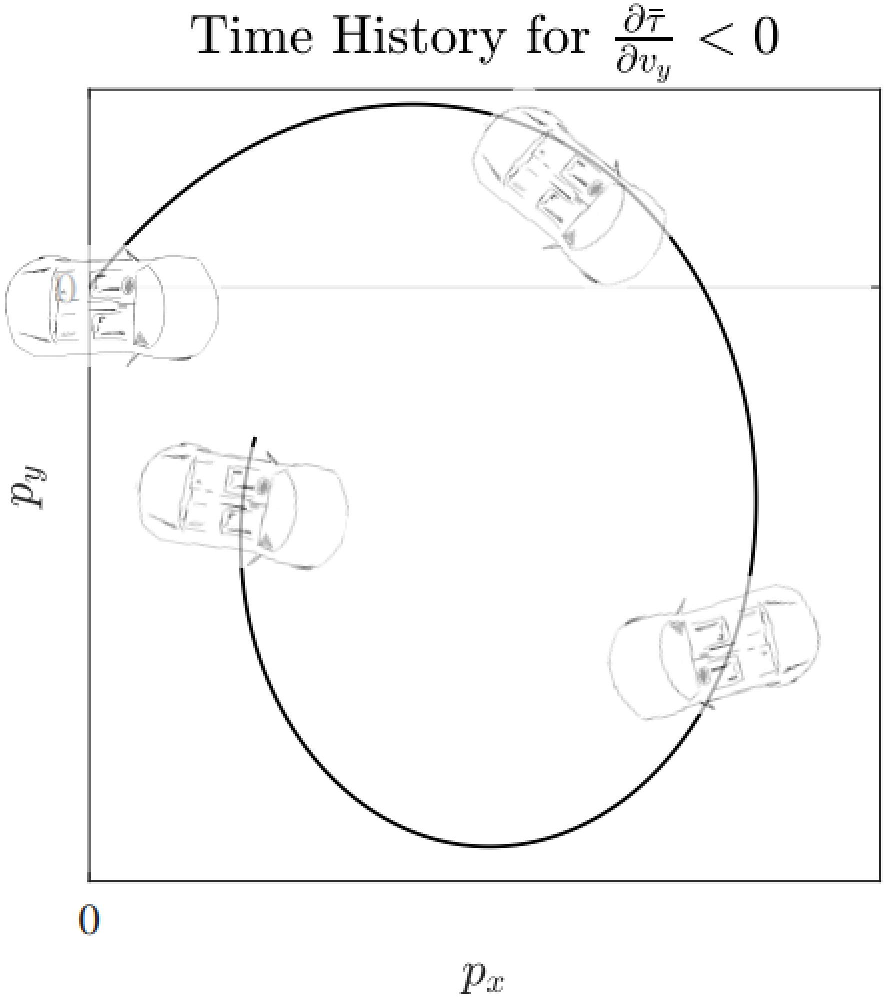
ESP/TV

$$\begin{bmatrix} \dot{p}_x \\ \dot{p}_y \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} v_0 \cos\psi - \tilde{v}_y \sin\psi \\ v_0 \sin\psi + \tilde{v}_y \cos\psi \\ \tilde{\omega} \end{bmatrix}$$

$\dot{\tilde{\mathbf{x}}} = \mathbf{A}\tilde{\mathbf{x}}$ $\quad \tilde{\mathbf{x}} = [\tilde{v}_x \quad \tilde{v}_y \quad \tilde{\omega}]^\top$

1b) $\partial\bar{\tau}/\partial v_y < 0.$ $v_1(v_0) < v_0$



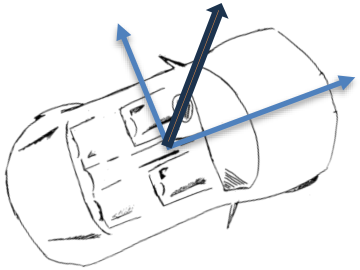


ESP/TV

Consider $$\lambda_{2,3} = \frac{1}{2v_0}\left(\underbrace{\frac{\partial \bar{f}_y}{\partial v_y}}_{<0} + \underbrace{\frac{m}{J}\frac{\partial \bar{\tau}}{\partial \omega}}_{\in \mathbb{R}}\right) \pm \frac{1}{2v_0}\sqrt{\underbrace{\left(\frac{\partial \bar{f}_y}{\partial v_y} - \frac{m}{J}\frac{\partial \bar{\tau}}{\partial \omega}\right)^2}_{>0} + \frac{4m}{J}\frac{\partial \bar{\tau}}{\partial v_y}\left(\underbrace{\frac{\partial \bar{f}_y}{\partial \omega}}_{>0} - v_0^2\right)}.$$

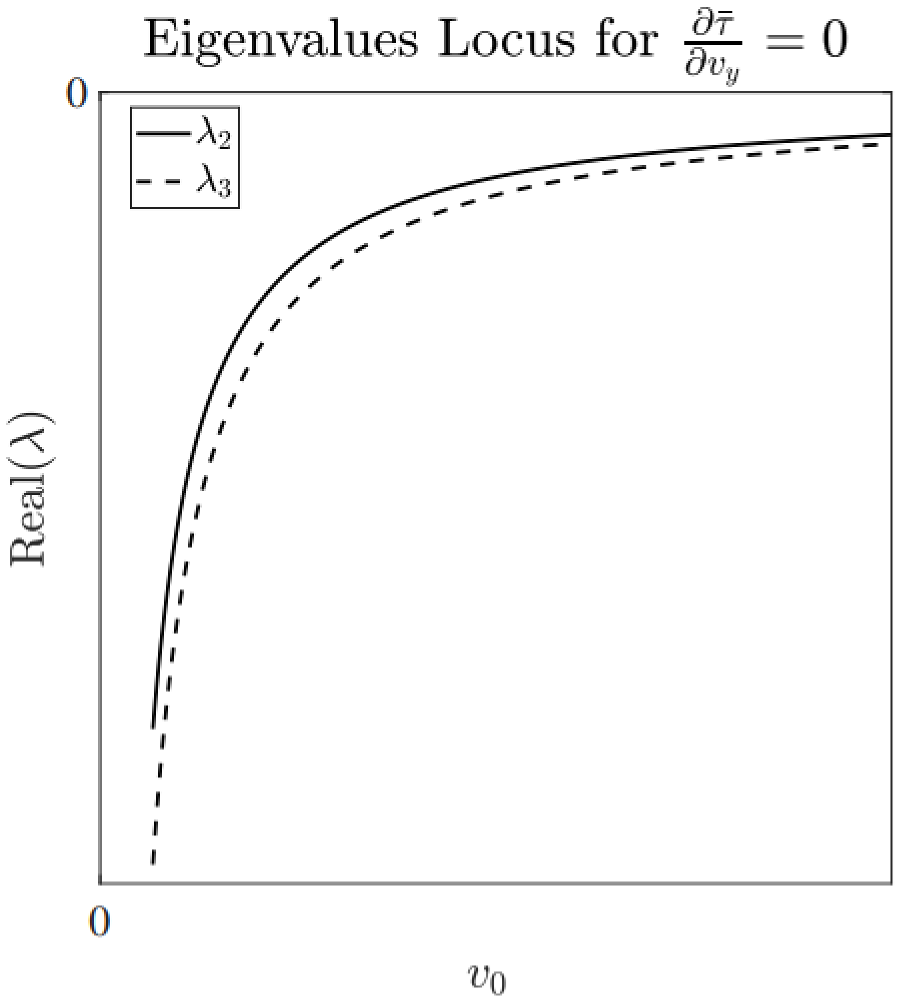
Assume $\partial\bar{\tau}/\partial\omega < 0$, then $\lambda_{2,3}$ have positive/negative real parts and eventually possess an imaginary part according to the sign of $\partial\bar{\tau}/\partial v_y$ and the magnitude of v_0:

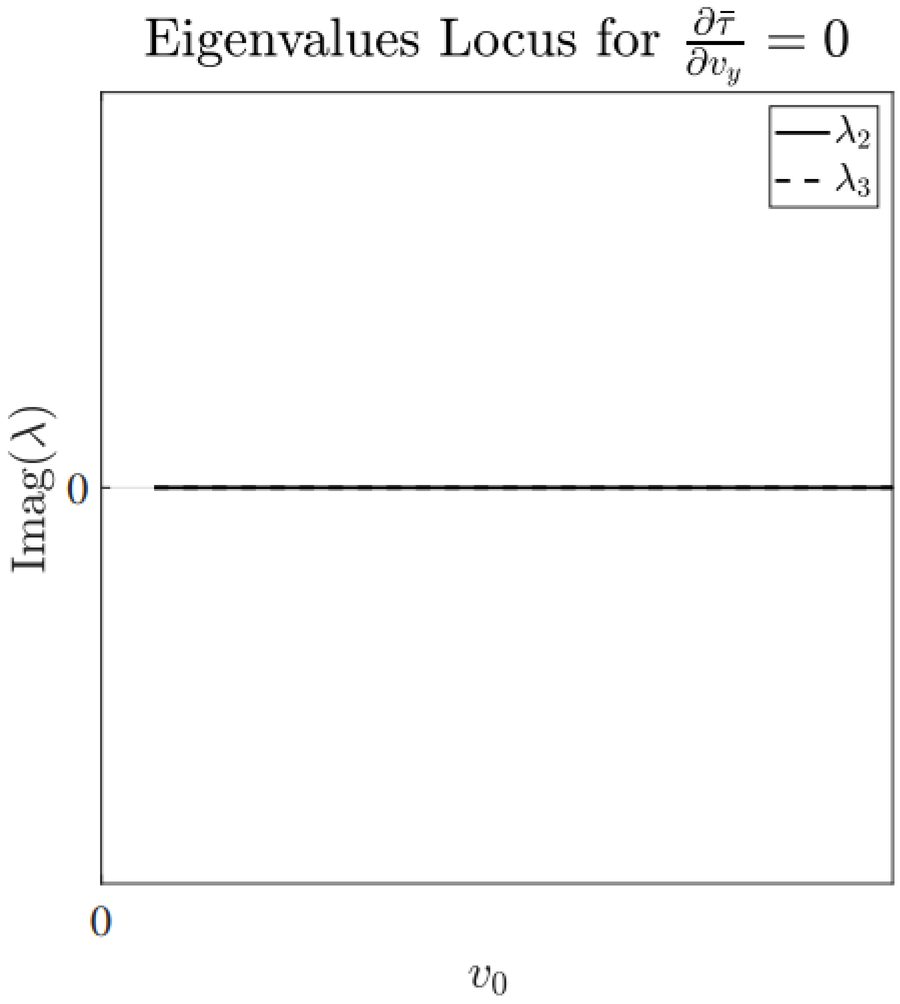
2) $\partial\bar{\tau}/\partial v_y = 0$. In this case, $\lambda_{2,3}$ are two distinct negative real roots whose magnitude decreases with v_0;



ESP/TV

2) $\partial\bar{\tau}/\partial v_y \; = \; 0.$





ESP/TV

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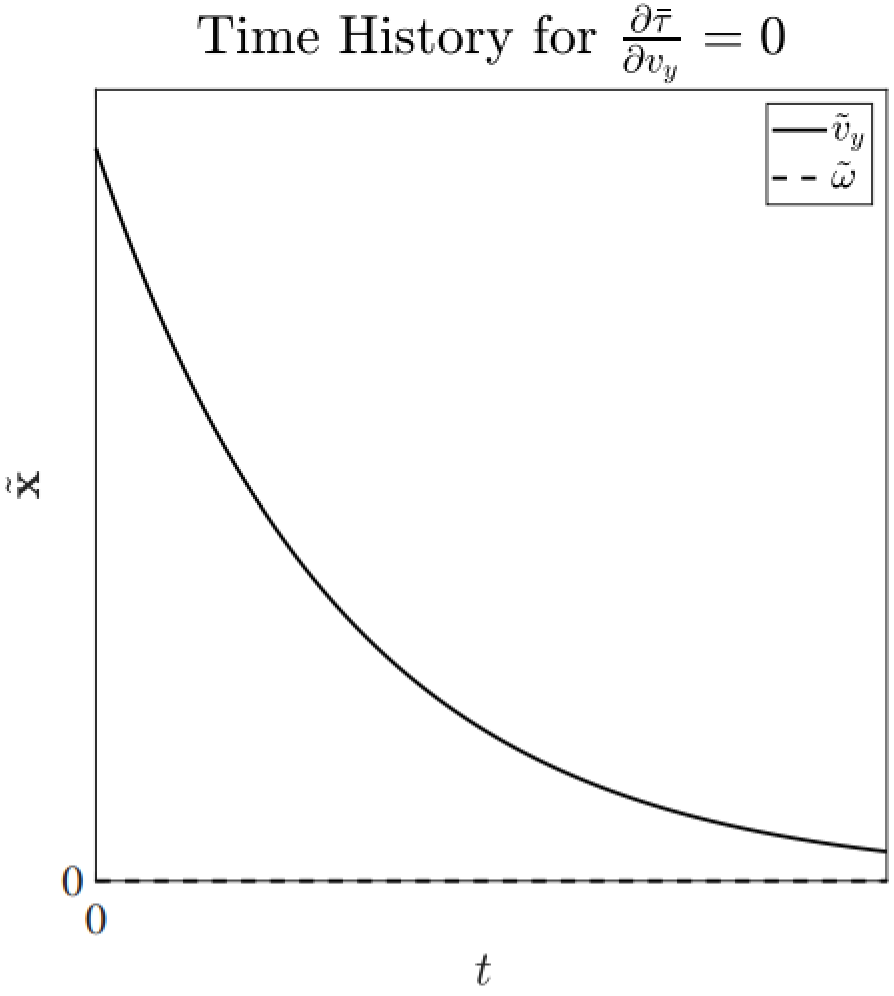
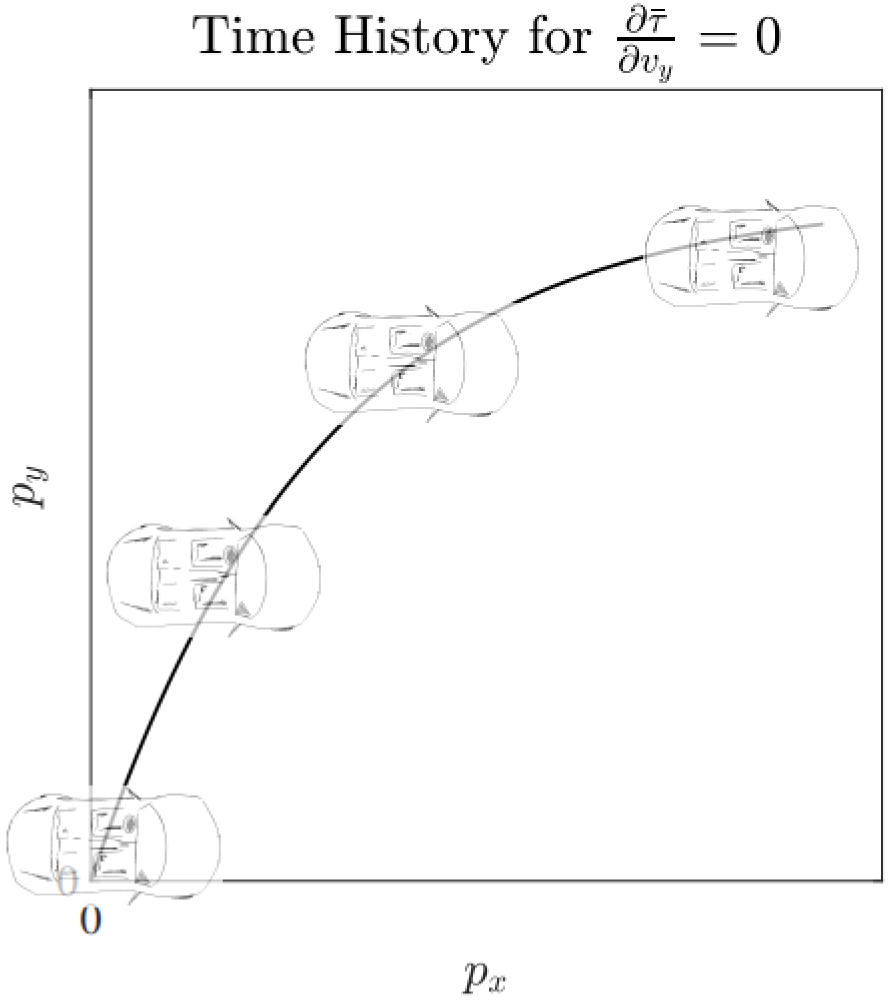
2) $\partial\bar{\tau}/\partial v_y = 0.$



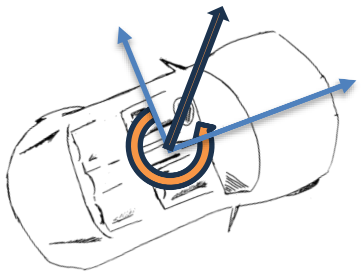


ESP/TV

Consider $\lambda_{2,3} = \frac{1}{2v_0}\left(\underset{<0}{\frac{\partial \bar{f}_y}{\partial v_y}} + \frac{m}{J}\underset{\in\,\mathbb{R}}{\frac{\partial \bar{\tau}}{\partial \omega}}\right) \pm \frac{1}{2v_0}\sqrt{\underbrace{\left(\frac{\partial \bar{f}_y}{\partial v_y} - \frac{m}{J}\frac{\partial \bar{\tau}}{\partial \omega}\right)^2}_{>0} + \frac{4m}{J}\frac{\partial \bar{\tau}}{\partial v_y}\left(\underset{>0}{\frac{\partial \bar{f}_y}{\partial \omega}} - v_0^2\right)}.$

Assume $\partial\bar{\tau}/\partial\omega < 0$, then $\lambda_{2,3}$ have positive/negative real parts and eventually possess an imaginary part according to the sign of $\partial\bar{\tau}/\partial v_y$ and the magnitude of v_0:

3) $\partial\bar{\tau}/\partial v_y > 0.$



ESP/TV

Consider $$\lambda_{2,3} = \frac{1}{2v_0}\Big(\underbrace{\frac{\partial \bar f_y}{\partial v_y}}_{<0} + \underbrace{\frac{m}{J}\frac{\partial \bar\tau}{\partial \omega}}_{\in\mathbb{R}}\Big) \pm \frac{1}{2v_0}\sqrt{\underbrace{\Big(\frac{\partial \bar f_y}{\partial v_y} - \frac{m}{J}\frac{\partial \bar\tau}{\partial \omega}\Big)^2}_{>0} + \frac{4m}{J}\frac{\partial \bar\tau}{\partial v_y}\Big(\underbrace{\frac{\partial \bar f_y}{\partial \omega}}_{>0} - v_0^2\Big)}.$$

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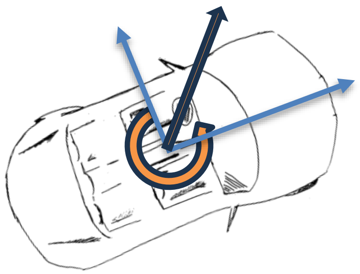
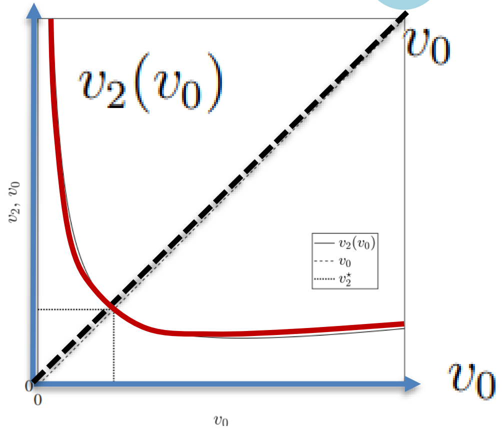
3) $\partial\bar\tau/\partial v_y > 0.$ Define



$$v_2(v_0) = \sqrt{\frac{\partial \bar f_y}{\partial \omega} + \Big(\frac{\partial \bar\tau}{\partial v_y}\Big)^{-1}\frac{J}{4m}\Big(\frac{\partial \bar f_y}{\partial v_y} - \frac{m}{J}\frac{\partial \bar\tau}{\partial \omega}\Big)^2},$$

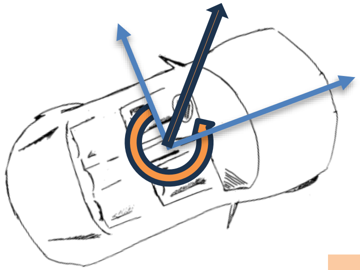


ESP/TV

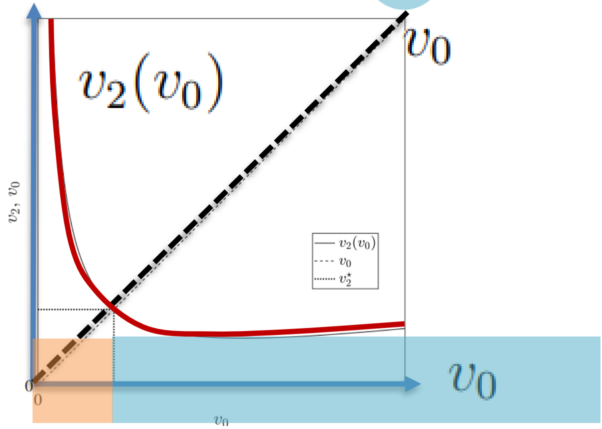
Consider
$$\lambda_{2,3} = \frac{1}{2v_0}\left(\underset{<0}{\frac{\partial \bar{f}_y}{\partial v_y}} + \frac{m}{J}\underset{\in\,\mathbb{R}}{\frac{\partial \bar{\tau}}{\partial \omega}}\right) \pm \frac{1}{2v_0}\sqrt{\underbrace{\left(\frac{\partial \bar{f}_y}{\partial v_y} - \frac{m}{J}\frac{\partial \bar{\tau}}{\partial \omega}\right)^2}_{>0} + \frac{4m}{J}\frac{\partial \bar{\tau}}{\partial v_y}\left(\underset{>0}{\frac{\partial \bar{f}_y}{\partial \omega}} - v_0^2\right)}.$$

Assume $\partial\bar{\tau}/\partial\omega < 0$, then $\lambda_{2,3}$ have positive/negative real parts and eventually possess an imaginary part according to the sign of $\partial\bar{\tau}/\partial v_y$ and the magnitude of v_0:

3) $\partial\bar{\tau}/\partial v_y > 0$. Define



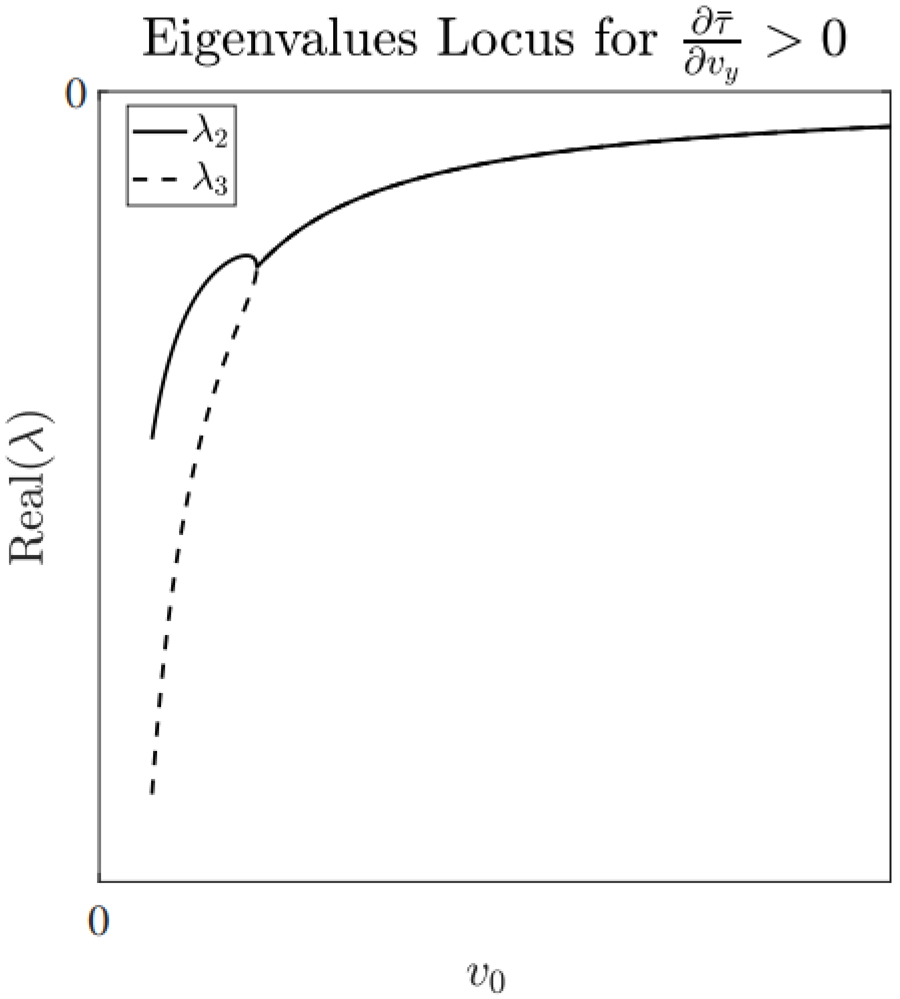
$$v_2(v_0) = \sqrt{\frac{\partial \bar{f}_y}{\partial \omega} + \left(\frac{\partial \bar{\tau}}{\partial v_y}\right)^{-1}\frac{J}{4m}\left(\frac{\partial \bar{f}_y}{\partial v_y} - \frac{m}{J}\frac{\partial \bar{\tau}}{\partial \omega}\right)^2},$$

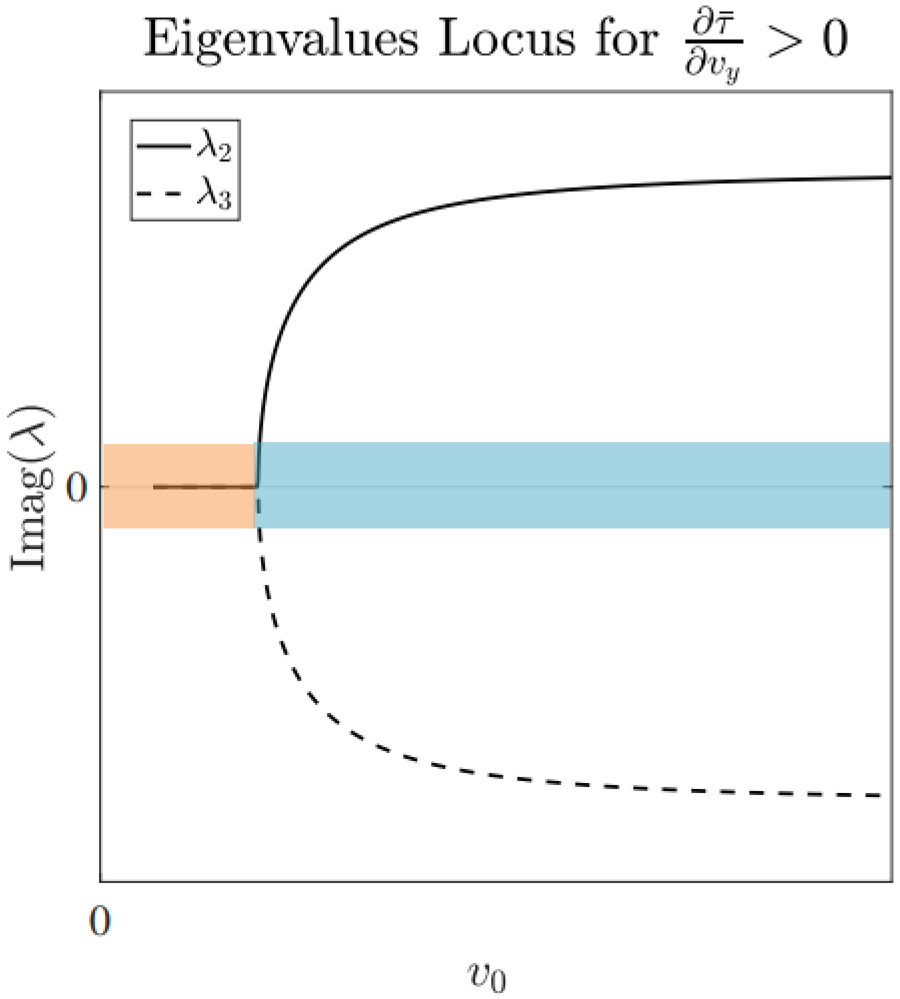


then $\lambda_{2,3}$ are distinct negative reals for $v_0 > 0 \,:\, v_0 < v_2(v_0)$. At $v_0 = v_2^\star > 0 \,:\, v_2^\star = v_2(v_2^\star)$, it is $\lambda_2 = \lambda_3 < 0$. To conclude, $\lambda_{2,3}$ are complex conjugated with negative real parts for $v_0 > 0 \,:\, v_0 > v_2(v_0)$.

ESP/TV

3) $\partial\bar{\tau}/\partial v_y > 0.$



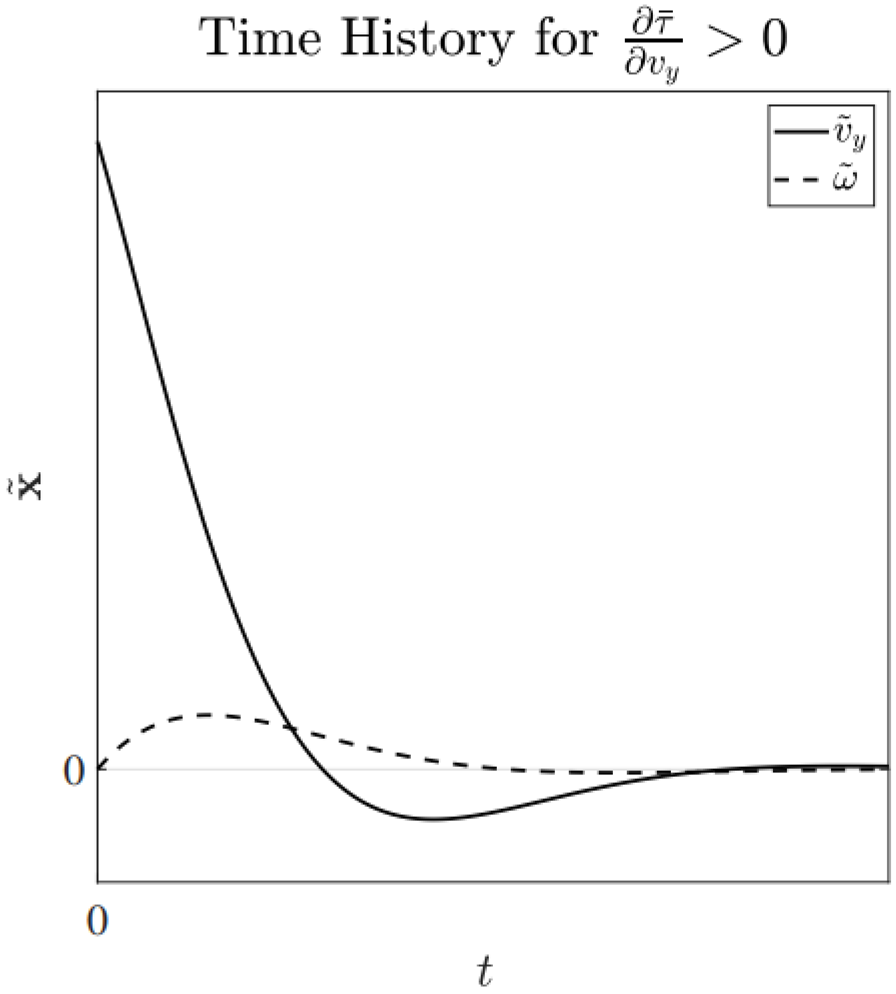


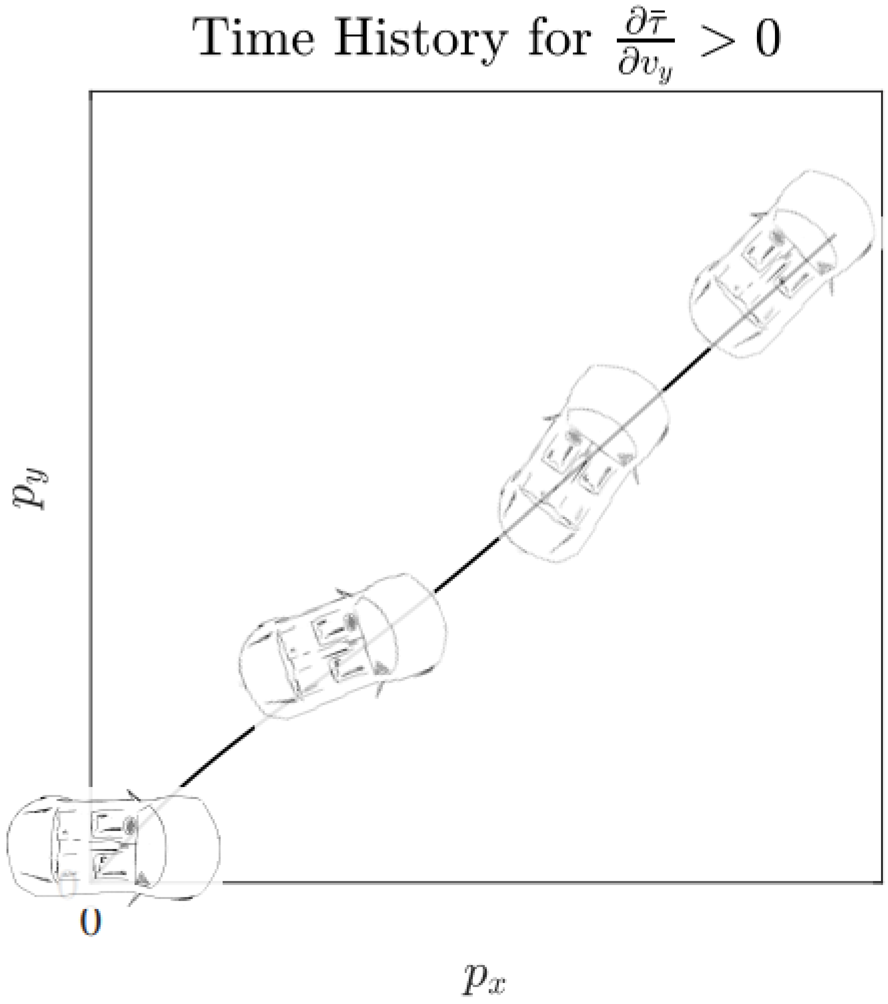
ESP/TV

$$\begin{bmatrix} \dot{p}_x \\ \dot{p}_y \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} v_0 \cos\psi - \tilde{v}_y \sin\psi \\ v_0 \sin\psi + \tilde{v}_y \cos\psi \\ \tilde{\omega} \end{bmatrix}$$

$\dot{\tilde{\mathbf{x}}} = \mathbf{A}\tilde{\mathbf{x}}$ $\tilde{\mathbf{x}} = [\tilde{v}_x \quad \tilde{v}_y \quad \tilde{\omega}]^\top$

3) $\partial\bar{\tau}/\partial v_y > 0.$ $v_0 > v_2(v_0)$





Contents

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- Is this system open-loop BIBS?

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  No.

Self-Park Assist

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Self-Park Assist

Let us have a look at $\quad \mathbf{A} = \begin{bmatrix} 0 & -v_0 \\ 0 & 0 \end{bmatrix}$

- Is this system open-loop BIBS?

  No.

- What eigenvalues/eigenvectors?
  - Eigenvalues $\lambda_1 = 0$
  - Algebraic multiplicity $a_1 = 2$
  - Geometric multiplicity $g_1 = 1$
  - Length of the chain of generalised eigenvectors $q_{11} = 2 \Rightarrow \mathbf{V} := \begin{bmatrix} 1 & 0 \\ 0 & -v_0^{-1} \end{bmatrix}, \mathbf{J} := \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

Exponential

$\tilde{\mathbf{x}} = \mathrm{col}(\tilde{\rho}, \tilde{\psi}_r)$

$$\begin{bmatrix} \tilde{\rho}(t) \\ \tilde{\psi}_r(t) \end{bmatrix} = \begin{bmatrix} 1 & -tv_0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \tilde{\rho}(0) \\ \tilde{\psi}_r(0) \end{bmatrix}$$

$$\tilde{\mathbf{x}}(\mathrm{t}) = \mathbf{V}\mathbf{z}(\mathrm{t}) \qquad \mathbf{z}(t) = \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix} \mathbf{z}(0)$$

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Nicola Mimmo

Department of Electrical, Electronics and Information Engineering «G. Marconi»

nicola.mimmo2@unibo.it