ALMA MATER STUDIORUM
UNIVERSITÀ DI BOLOGNA

Automotive Design Models

Faculty of «Electronic Engineering for Intelligent Vehicles» and «Advanced Automotive Engineering»

Nicola Mimmo

Department of Electrical, Electronics and Information Engineering «G. Marconi» - University of Bologna

Contents

- Active Suspensions
- Adaptive Cruise Control
- Wheel Speed Controls
- Electronic Stability Program / Torque Vectoring
- Self-Park Assist

Contents

Active Suspensions Simulation model

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\frac{m_s+m_u}{m_s m_u}k_s(x_1-\ell_s) - \frac{m_s+m_u}{m_s m_u}\beta_s x_2 + \frac{m_s+m_u}{m_s m_u}f_a + \frac{1}{m_s}f_d - \frac{1}{m_u}f_t(x_3)$$

$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = -g + \frac{k_s}{m_u}(x_1-\ell_s) + \frac{\beta_s}{m_u}x_2 - \frac{f_a}{m_u} + \frac{1}{m_u}f_t(x_3) - \ddot{z}_r$$

$$y_p = x_1 + \nu_p$$

$$y_a = -g - \frac{k_s}{m_s}(x_1-\ell_s) - \frac{\beta_s}{m_s}x_2 + \frac{f_a+f_d}{m_s} + \nu_a$$

$$\mathbf{x} = \mathrm{col}(x_1, x_2, x_3, x_4), \quad u = f_a,$$
$$\mathbf{d} = \mathrm{col}(\ddot{z}_r,\ f_d),$$
$$\boldsymbol{\nu} = \mathrm{col}(\nu_p, \nu_a),$$
$$\mathbf{w} = \mathrm{col}(\mathbf{d}, \boldsymbol{\nu}, r),$$

$$e = x_1 + \nu_p - r.$$

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, u, \mathbf{w}) \qquad \mathbf{x}(t_0) = \mathbf{x}_0$$
$$\mathbf{y} = \mathbf{h}(\mathbf{x}, u, \mathbf{w})$$
$$e = h_e(\mathbf{x}, u, \mathbf{w}),$$

Active Suspensions Equilibrium triplet

assume $u_0 = 0$, $\mathbf{d}_0$, $\boldsymbol{\nu}_0 = \mathbf{0}, \mathbf{x}_0 := \mathrm{col}(x_{1_0}, x_{2_0}, x_{3_0}, x_{4_0}), \mathbf{y}_0 := \mathrm{col}(y_{p0}, y_{a0})$, and $r = r_0$

Impose $\dot{\mathbf{x}} = \mathbf{0}$

$$
\begin{aligned}
0 &= x_{2_0} \\
0 &= -\frac{m_s + m_u}{m_s m_u} k_s (x_{1_0} - \ell_s) + \frac{k_t}{m_u}(x_{3_0} - \ell_t) \\
0 &= x_{4_0} \\
0 &= -g + \frac{k_s}{m_u}(x_{1_0} - \ell_s) - \frac{k_t}{m_u}(x_{3_0} - \ell_t) \\
y_{p0} &= x_{1_0} \\
y_{a0} &= -g - \frac{k_s}{m_s}(x_{1_0} - \ell_s) \\
r_0 &= x_{1_0},
\end{aligned}
$$

$$
\begin{bmatrix} x_{1_0} \\ x_{2_0} \\ x_{3_0} \\ x_{4_0} \end{bmatrix} = \begin{bmatrix} \ell_s - g\dfrac{m_s}{k_s} \\ 0 \\ \ell_t - g\dfrac{m_s + m_u}{k_t} \\ 0 \end{bmatrix}
$$

$$
\begin{bmatrix} y_{p0} \\ y_{a0} \end{bmatrix} = \begin{bmatrix} \ell_s - g\dfrac{m_s}{k_s} \\ 0 \end{bmatrix}
$$

$$
r_0 = \ell_s - g\frac{m_s}{k_s}.
$$

Active Suspensions Linearisation

Define the errors to the equilibrium point as

$$\tilde{u} := u - u_0,$$
$$\tilde{\mathbf{d}} := \mathbf{d} - \mathbf{d}_0,\ \tilde{\boldsymbol{\nu}} = \boldsymbol{\nu} - \boldsymbol{\nu}_0,$$
$$\tilde{r} := r - r_0,\ \tilde{\mathbf{w}} = \mathrm{col}(\tilde{d}, \tilde{\boldsymbol{\nu}}, \tilde{\mathbf{r}}),$$

To obtain

$$\dot{\tilde{\mathbf{x}}} = \mathbf{A}\tilde{\mathbf{x}} + \mathbf{B}_1\tilde{u} + \mathbf{B}_2\tilde{\mathbf{w}} \quad \tilde{\mathbf{x}}(t_0) = \tilde{\mathbf{x}}_0$$
$$\tilde{\mathbf{y}} = \mathbf{C}\tilde{\mathbf{x}} + \mathbf{D}_1\tilde{u} + \mathbf{D}_2\tilde{\mathbf{w}}$$
$$\tilde{e} = \mathbf{C}_e\tilde{\mathbf{x}} + \mathbf{D}_{e2}\tilde{\mathbf{w}}.$$

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -k_s\dfrac{m_s + m_u}{m_s m_u} & -\beta_s\dfrac{m_s + m_u}{m_s m_u} & \dfrac{k_t}{m_u} & 0 \\ 0 & 0 & 0 & 1 \\ \dfrac{k_s}{m_u} & \dfrac{\beta_s}{m_u} & -\dfrac{k_t}{m_u} & 0 \end{bmatrix}$$

$$\mathbf{B}_1 = \begin{bmatrix} 0 \\ \dfrac{m_s + m_u}{m_s m_u} \\ 0 \\ -\dfrac{1}{m_u} \end{bmatrix},\ \mathbf{B}_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & m_s^{-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\dfrac{k_s}{m_s} & -\dfrac{\beta_s}{m_s} & 0 & 0 \end{bmatrix},\ \mathbf{C}_e = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{D}_1 = \begin{bmatrix} 0 \\ m_s^{-1} \end{bmatrix},\ \mathbf{D}_2 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & m_s^{-1} & 0 & 1 & 0 \end{bmatrix}$$

$$\mathbf{D}_{e2} = \begin{bmatrix} 0 & 0 & 0 & 1 & -1 \end{bmatrix}.$$

Contents

Adaptive Cruise Control Simulation model

let $\mathbf{x} := \mathrm{col}(d_B, v_B, d_C, v_C)$, $\mathbf{u} := \mathrm{col}(f_B, f_C)$, $\mathbf{d} := \mathrm{col}(v_A, \sin\theta, w)$,
$\boldsymbol{\nu} := \mathrm{col}(\nu_{dB}, \nu_{vB}, \nu_{dC}, \nu_{vC})$, $\mathbf{w} := \mathrm{col}(\mathbf{d}, \boldsymbol{\nu})$, $\mathbf{y} := \mathrm{col}(y_{dB}, y_{vB}, y_{dC}, y_{vC})$, and $\mathbf{e} := \mathrm{col}(e_{dB}, e_{dC})$.

Then

$$
\begin{aligned}
\dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}, \mathbf{u}, \mathbf{w}) \qquad \mathbf{x}(t_0) = \mathbf{x}_0 \\
\mathbf{y} &= \mathbf{h}(\mathbf{x}, \mathbf{u}, \mathbf{w}) \\
\mathbf{e} &= \mathbf{h}_e(\mathbf{x}, \mathbf{u}, \mathbf{w})
\end{aligned}
$$

where

$$
\mathbf{f}(\mathbf{x}, \mathbf{u}, \mathbf{w}) = \begin{bmatrix} v_A - v_B \\ g\sin\theta + \left(f_B - 1/2\rho S_B C_{D_B}(v_B - w)^2\right)/m_B \\ v_B - v_C \\ g\sin\theta + \left(f_C - 1/2\rho S_C C_{D_C}(v_C - w)^2\right)/m_C \end{bmatrix}
$$

$$
\mathbf{h}(\mathbf{x}, \mathbf{u}, \mathbf{w}) = \begin{bmatrix} d_B + \nu_{dB} \\ v_B + \nu_{vB} \\ d_C + \nu_{dC} \\ v_C + \nu_{vC} \end{bmatrix}
$$

$$
\mathbf{h}_e(\mathbf{x}, \mathbf{u}, \mathbf{w}) = \begin{bmatrix} y_{dB} - d_{\min} - y_{vB}^2/k \\ y_{dC} - d_{\min} - y_{vC}^2/k \end{bmatrix}.
$$

Adaptive Cruise Control Equilibrium triplet

Let $v_A = v_B = v_C = v_0 > 0$ and define $D_{B0} = D_B(v_0)$, and $D_{C0} = D_C(v_0)$

Impose $\dot{\mathbf{x}} = \mathbf{0}$, $\theta = 0$, $d_B = d_B^\star$, and $d_C = d_C^\star$, and let $\mathbf{d}_0 = \mathrm{col}(v_0, 0, 0)$

$$\boldsymbol{\nu}_0 = \mathbf{0}, \text{ and define } \mathbf{w}_0 = \mathrm{col}(\mathbf{d}_0, \boldsymbol{\nu}_0),$$

$$D_B(v_B - w) = \frac{1}{2}\rho S_B C_{D_B}(v_B - w)^2$$

$$D_C(v_C - w) = \frac{1}{2}\rho S_C C_{D_C}(v_C - w)^2$$

As a consequence

$$\mathbf{u}_0 := \mathrm{col}(D_{B0}, D_{C0})$$

$$\mathbf{x}_0 = \mathrm{col}(d_{\min} + v_0^2/k, v_0, d_{\min} + v_0^2/k, v_0)$$

$$\mathbf{y}_0 = \mathbf{h}(\mathbf{x}_0, \mathbf{u}_0, \mathbf{w}_0), \text{ and } \mathbf{e}_0 = \mathbf{0}.$$

Adaptive Cruise Control Linearisation

define the errors

$$\tilde{\mathbf{u}} = \mathbf{u} - \mathbf{u}_0, \ \tilde{\mathbf{w}} = \mathbf{w} - \mathbf{w}_0,$$

Then

$$\dot{\tilde{\mathbf{x}}} = \mathbf{A}\tilde{\mathbf{x}} + \mathbf{B}_1\tilde{\mathbf{u}} + \mathbf{B}_2\tilde{\mathbf{w}} \quad \tilde{\mathbf{x}}(t_0) = \tilde{\mathbf{x}}_0$$
$$\tilde{\mathbf{y}} = \mathbf{C}\tilde{\mathbf{x}} + \mathbf{D}_2\tilde{\mathbf{w}}$$
$$\tilde{\mathbf{e}} = \mathbf{C}_e\tilde{\mathbf{x}} + \mathbf{D}_{e_2}\tilde{\mathbf{w}}$$

$$\mathbf{A} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & -\rho S_B C_{D_B} v0/m_B & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & -\rho S_C C_{D_C} v0/m_C \end{bmatrix}$$

$$\mathbf{B}_1 = \begin{bmatrix} 0 & 0 \\ m_B^{-1} & 0 \\ 0 & 0 \\ 0 & m_C^{-1} \end{bmatrix}, \ \mathbf{B}_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & g & \rho S_B C_{D_B} v0/m_B & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & g & \rho S_c C_{D_c} v0/m_c & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \ \mathbf{D}_2 = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{C}_e = \begin{bmatrix} 1 & -2v_0/k & 0 & 0 \\ 0 & 0 & 1 & -2v_0/k \end{bmatrix}$$

$$\mathbf{D}_{e2} = \begin{bmatrix} 0 & 0 & 0 & 1 & -2v_0/k & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -2v_0/k \end{bmatrix}.$$

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Wheel Speed Controls Simulation model

let $\mathbf{x} := \text{col}(v, \omega_r, \omega_f)$, $\mathbf{u} := \text{col}(\tau_r, \tau_f)$, $\mathbf{w} := \text{col}(\theta, w, \nu_v, \nu_r, \nu_f, \omega_r^r, \omega_f^r)$

$\mathbf{y} := \text{col}(v, \omega_r, \omega_f)$, and $\mathbf{e} := \text{col}(e_r, e_f)$

$$
\begin{aligned}
\dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}, \mathbf{u}, \mathbf{w}) \qquad \mathbf{x}(t_0) = \mathbf{x}_0 \\
\mathbf{y} &= \mathbf{h}(\mathbf{x}, \mathbf{u}, \mathbf{w}) \\
\mathbf{e} &= \mathbf{h}_e(\mathbf{x}, \mathbf{u}, \mathbf{w})
\end{aligned}
$$

$$
\mathbf{f}(\mathbf{x}, \mathbf{u}, \mathbf{w}) = \begin{bmatrix} \dfrac{N_f(\mu_f - c_r) + N_r(\mu_r - c_r)}{m} - g \sin\theta - \dfrac{D}{m} \\ J_r^{-1}(\tau_r - N_r r_r(\mu_r - c_r)) \\ J_f^{-1}(\tau_f - N_f r_f(\mu_f - c_r)) \end{bmatrix}
$$

$$
\mathbf{h}(\mathbf{x}, \mathbf{u}, \mathbf{w}) = \begin{bmatrix} v + \nu_v \\ \omega_r(1 + \nu_r) \\ \omega_f(1 + \nu_f) \end{bmatrix}, \qquad \mathbf{h}_e(\mathbf{x}, \mathbf{u}, \mathbf{w}) = \begin{bmatrix} \omega_r(1 + \nu_r) - r_r \\ \omega_f(1 + \nu_f) - r_f \end{bmatrix}
$$

Wheel Speed Controls Simulation model (continued)

Aerodynamic Drag

$$D(v-w) = \frac{1}{2}\rho S \frac{(v-w)^3}{|v-w|} C_D$$

Traction Coefficient (Burckhardt)

$$\mu(\lambda, \boldsymbol{\Theta}) = \mathtt{sign}(\lambda)\theta_1 \left(1 - e^{-|\lambda|\theta_2}\right) - \lambda\theta_3$$

$$\boldsymbol{\Theta} := \mathrm{col}(\theta_1, \theta_2, \theta_3) \in \mathbb{R}^3$$

Vertical Load

$$\begin{bmatrix} N_r \\ N_f \end{bmatrix} = \frac{mg\cos\theta}{1-(\mu_r - \mu_f)\bar{h}} \begin{bmatrix} (\mu_f - c_r)\bar{h} + \bar{a} \\ -(\mu_r - c_r)\bar{h} + \bar{b} \end{bmatrix}$$

Rolling Resistance

$$c_r(v) = c_{r0} + c_{r1}v + c_{r2}v^2$$

Longitudinal Slip Ratio

$$\lambda(v, \omega r) = \frac{\omega r - v}{\epsilon + \max\{|v|, |\omega r|, |\omega_r - v|\}}$$

- Model validity and traction distribution

Use

$$\dot{v} = -g\sin\theta + \frac{N_f}{m}(\mu_f - c_r) + \frac{N_r}{m}(\mu_r - c_r) - \frac{D}{m}$$

$$\begin{bmatrix} N_r \\ N_f \end{bmatrix} = \frac{mg\cos\theta}{1-(\mu_r-\mu_f)\bar{h}} \begin{bmatrix} (\mu_f - c_r)\bar{h} + \bar{a} \\ -(\mu_r - c_r)\bar{h} + \bar{b} \end{bmatrix}$$

- Model validity and traction distribution

Use

$$\dot{v} = -g\sin\theta + \frac{N_f}{m}(\mu_f - c_r) + \frac{N_r}{m}(\mu_r - c_r) - \frac{D}{m}$$

$$\begin{bmatrix} N_r \\ N_f \end{bmatrix} = \frac{mg\cos\theta}{\boxed{1-(\mu_r-\mu_f)\bar{h}}} \begin{bmatrix} (\mu_f - c_r)\bar{h} + \bar{a} \\ -(\mu_r - c_r)\bar{h} + \bar{b} \end{bmatrix}$$

where $\boxed{1-(\mu_r-\mu_f)\bar{h} > 0}$

Wheel Speed Controls Equilibrium triplet

- Model validity and traction distribution

Use

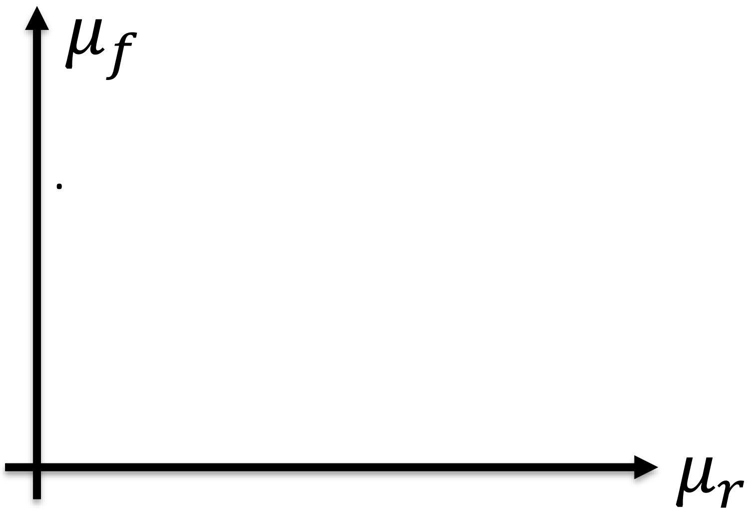
$$\dot{v} = -g\sin\theta + \frac{N_f}{m}(\mu_f - c_r) + \frac{N_r}{m}(\mu_r - c_r) - \frac{D}{m}$$

$$\begin{bmatrix} N_r \\ N_f \end{bmatrix} = \frac{mg\cos\theta}{1-(\mu_r-\mu_f)\bar{h}} \begin{bmatrix} (\mu_f - c_r)\bar{h} + \bar{a} \\ -(\mu_r - c_r)\bar{h} + \bar{b} \end{bmatrix}$$

where $1-(\mu_r-\mu_f)\bar{h} > 0$

to obtain

$$\mu_f = \frac{mgc_r + \dot{v}_0 + D + mg\sin\theta}{mg\bar{b} - (\dot{v}_0 + D + mg\sin\theta)\bar{h}} - \mu_r\frac{(\dot{v}_0 + D + mg\sin\theta)\bar{h} + mg\bar{a}}{mg\bar{b} - (\dot{v}_0 + D + mg\sin\theta)\bar{h}}$$



Wheel Speed Controls Equilibrium triplet

- Model validity and traction distribution

Use

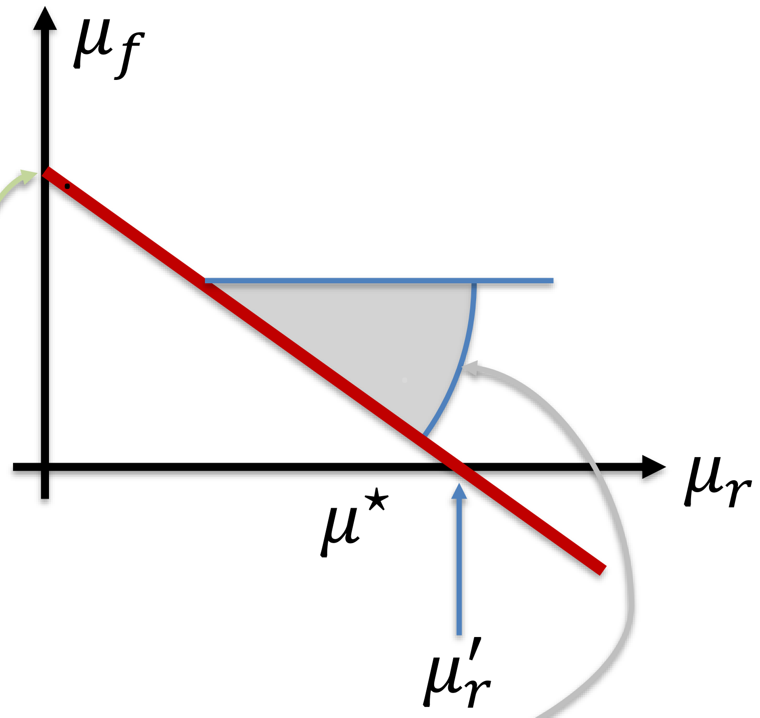
$$\dot{v} = -g\sin\theta + \frac{N_f}{m}(\mu_f - c_r) + \frac{N_r}{m}(\mu_r - c_r) - \frac{D}{m}$$

$$\begin{bmatrix} N_r \\ N_f \end{bmatrix} = \frac{mg\cos\theta}{1-(\mu_r-\mu_f)\bar{h}} \begin{bmatrix} (\mu_f - c_r)\bar{h} + \bar{a} \\ -(\mu_r - c_r)\bar{h} + \bar{b} \end{bmatrix}$$

where $1-(\mu_r-\mu_f)\bar{h} > 0$

to obtain

$$\mu_f = \frac{mgc_r + \dot{v}_0 + D + mg\sin\theta}{mg\bar{b} - (\dot{v}_0 + D + mg\sin\theta)\bar{h}} - \mu_r \frac{(\dot{v}_0 + D + mg\sin\theta)\bar{h} + mg\bar{a}}{mg\bar{b} - (\dot{v}_0 + D + mg\sin\theta)\bar{h}}$$



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Wheel Speed Controls Equilibrium triplet

- Model validity and traction distribution

Use

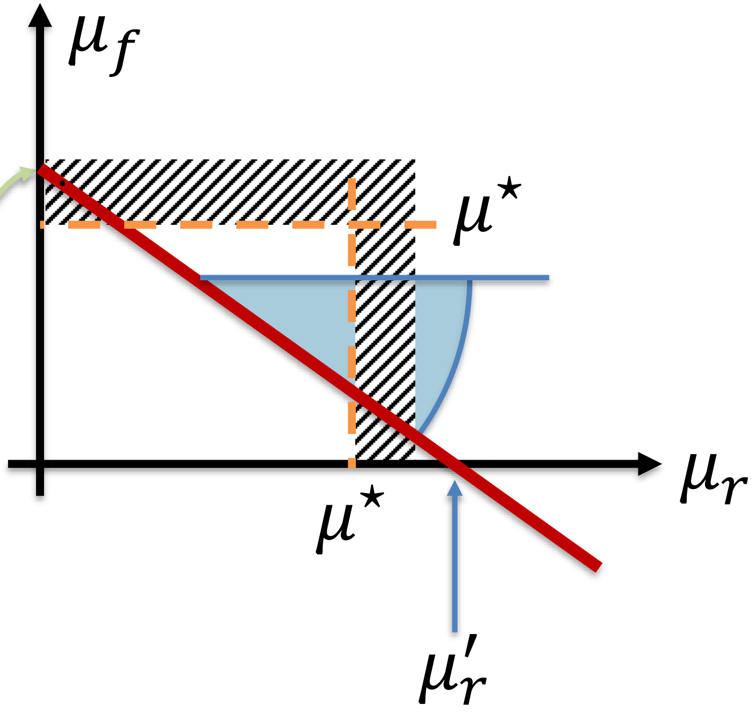
$$\dot{v} = -g\sin\theta + \frac{N_f}{m}(\mu_f - c_r) + \frac{N_r}{m}(\mu_r - c_r) - \frac{D}{m}$$

$$\begin{bmatrix} N_r \\ N_f \end{bmatrix} = \frac{mg\cos\theta}{1-(\mu_r-\mu_f)\bar{h}} \begin{bmatrix} (\mu_f - c_r)\bar{h} + \bar{a} \\ -(\mu_r - c_r)\bar{h} + \bar{b} \end{bmatrix}$$

where $1-(\mu_r-\mu_f)\bar{h} > 0$

to obtain

$$\mu_f = \frac{mgc_r + \dot{v}_0 + D + mg\sin\theta}{mg\bar{b} - (\dot{v}_0 + D + mg\sin\theta)\bar{h}} - \mu_r \frac{(\dot{v}_0 + D + mg\sin\theta)\bar{h} + mg\bar{a}}{mg\bar{b} - (\dot{v}_0 + D + mg\sin\theta)\bar{h}}$$



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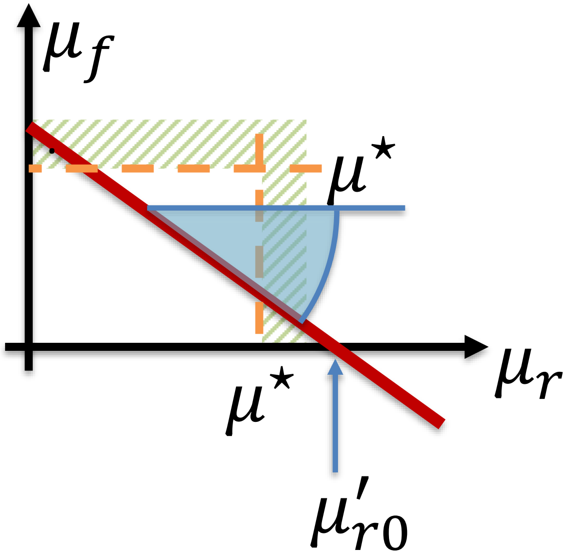
Wheel Speed Controls Equilibrium triplet

Set $v_0 > 0$, $\dot{v} = 0, v = v_0, w = \theta = 0$ (const. speed, no wind, zero slope)

And evaluate

$$\mu_f = \frac{mgc_r + \cancel{\dot{v}_0} + D + \cancel{mg\sin\theta}}{mg\bar{b} - (\cancel{\dot{v}_0} + D + \cancel{mg\sin\theta})\bar{h}} - \mu_r \frac{(\cancel{\dot{v}_0} + D + \cancel{mg\sin\theta})\bar{h} + mg\bar{a}}{mg\bar{b} - (\cancel{\dot{v}_0} + D + \cancel{mg\sin\theta})\bar{h}}$$



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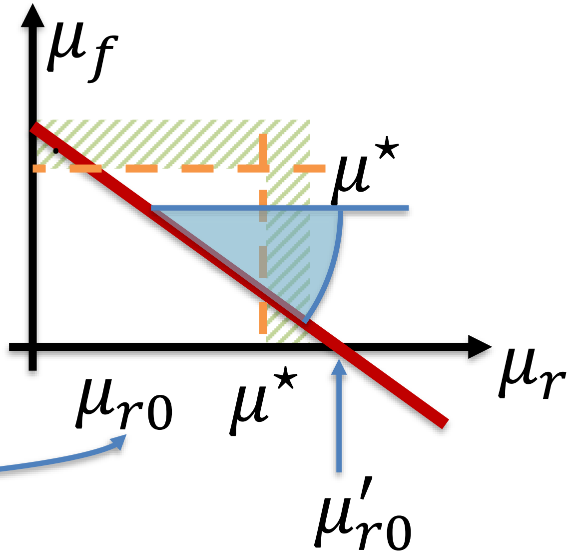
Set $v_0 > 0$, $\dot{v} = 0, v = v_0, w = \theta = 0$ (const. speed, no wind, zero slope)

And evaluate

$$\mu_f = \frac{mgc_r + \cancel{\dot{v}_0} + D + \cancel{mg\sin\theta}}{mg\bar{b} - (\cancel{\dot{v}_0} + D + \cancel{mg\sin\theta})\bar{h}} - \mu_r \frac{(\cancel{\dot{v}_0} + D + \cancel{mg\sin\theta})\bar{h} + mg\bar{a}}{mg\bar{b} - (\cancel{\dot{v}_0} + D + \cancel{mg\sin\theta})\bar{h}}$$

at $\mu_{r0} \in [0, \min\{\mu^\star, \mu'_{r0}\}]$



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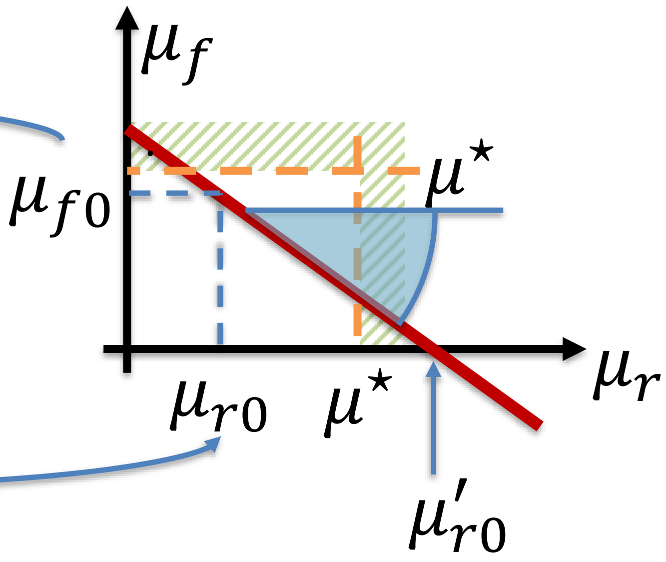
Wheel Speed Controls Equilibrium triplet

Set $v_0 > 0$ $\dot{v} = 0, v = v_0, w = \theta = 0$ (const. speed, no wind, zero slope)

And evaluate

$$\mu_f = \frac{mgc_r + \cancel{\dot{v}_0} + D + \cancel{mg\sin\theta}}{mg\bar{b} - (\cancel{\dot{v}_0} + D + \cancel{mg\sin\theta})\bar{h}} - \mu_r \frac{(\cancel{\dot{v}_0} + D + \cancel{mg\sin\theta})\bar{h} + mg\bar{a}}{mg\bar{b} - (\cancel{\dot{v}_0} + D + \cancel{mg\sin\theta})\bar{h}}$$

at $\mu_{r0} \in [0, \min\{\mu^\star, \mu'_{r0}\}]$



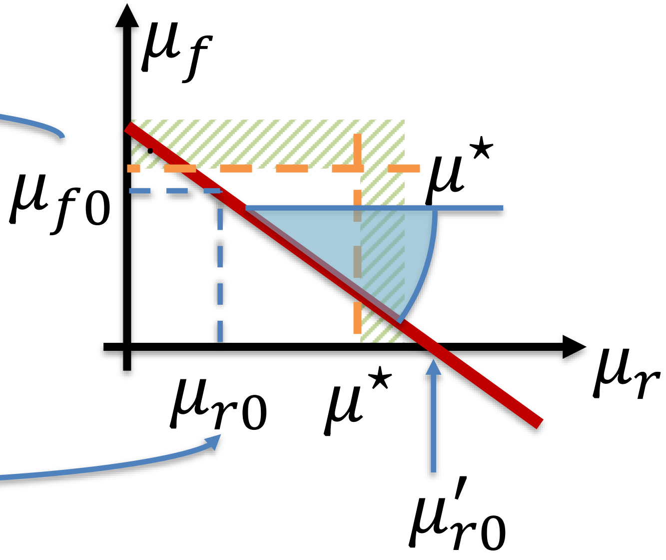
Wheel Speed Controls Equilibrium triplet

Set $v_0 > 0$, $\dot{v} = 0, v = v_0, w = \theta = 0$ (const. speed, no wind, zero slope)

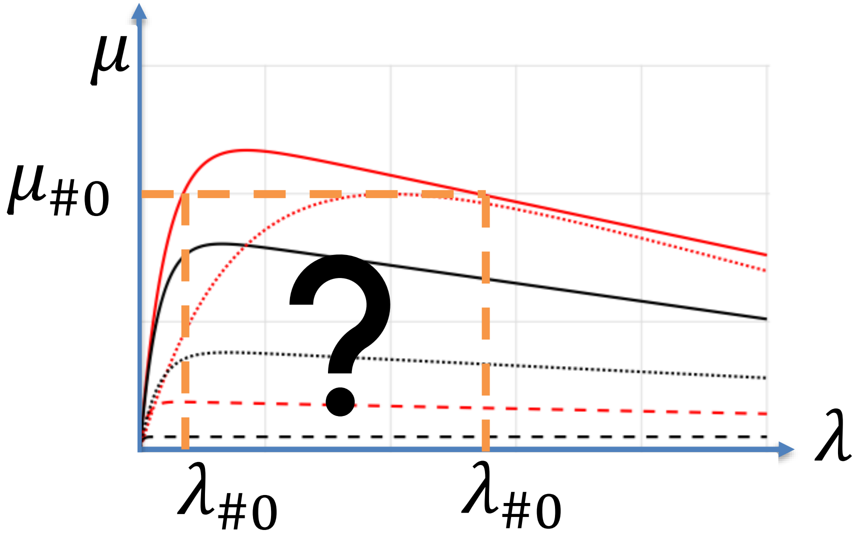
And evaluate

$$\mu_f = \frac{mgc_r + \cancel{\dot{v}_0} + D + \cancel{mg\sin\theta}}{mg\bar{b} - (\cancel{\dot{v}_0} + D + \cancel{mg\sin\theta})\bar{h}} - \mu_r \frac{(\cancel{\dot{v}_0} + D + \cancel{mg\sin\theta})\bar{h} + mg\bar{a}}{mg\bar{b} - (\cancel{\dot{v}_0} + D + \cancel{mg\sin\theta})\bar{h}}$$

at $\mu_{r0} \in [0,\ \min\{\mu^\star,\ \mu'_{r0}\}]$



Define the equilibrium lambda



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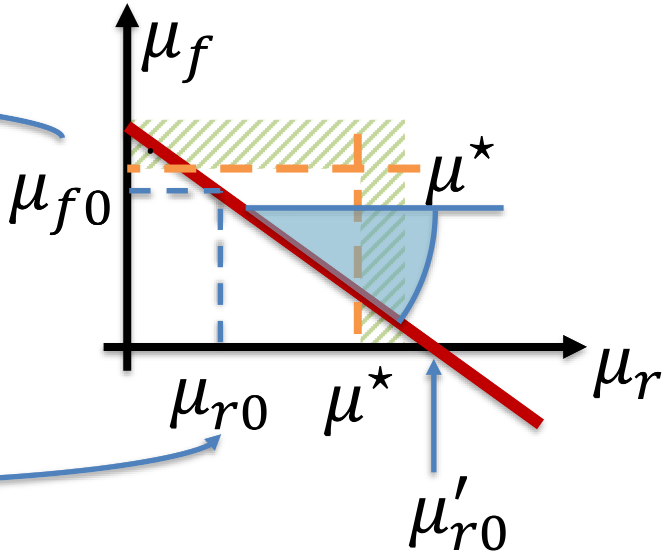
Wheel Speed Controls Equilibrium triplet

Set $v_0 > 0$, $\dot{v} = 0, v = v_0, w = \theta = 0$ (const. speed, no wind, zero slope)

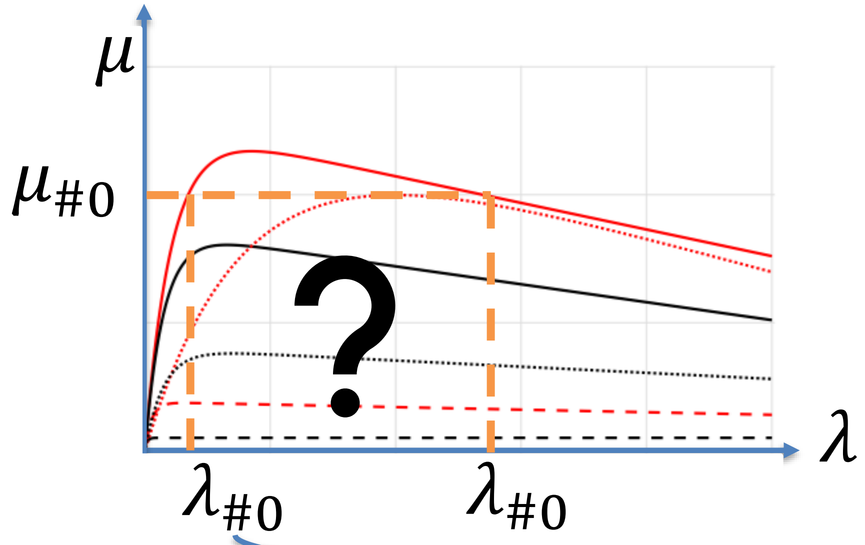
And evaluate

$$\mu_f = \frac{mgc_r + \cancel{\dot{v}_0} + D + \cancel{mg\sin\theta}}{mg\bar{b} - (\cancel{\dot{v}_0} + D + \cancel{mg\sin\theta})\bar{h}} - \mu_r \frac{(\cancel{\dot{v}_0} + D + \cancel{mg\sin\theta})\bar{h} + mg\bar{a}}{mg\bar{b} - (\cancel{\dot{v}_0} + D + \cancel{mg\sin\theta})\bar{h}}$$

at $\mu_{r0} \in [0, \min\{\mu^\star, \mu'_{r0}\}]$



Define the equilibrium lambda



Define the equilibrium wheel speed

$$\omega_{\#0} = \frac{v_0}{(1 - \lambda_{\#0})r} \quad \text{Driving}$$

Wheel Speed Controls Equilibrium triplet

Define $f_{\#0} \coloneqq N_{\#0}\mu_{\#0}$ with $$\begin{bmatrix} N_{r0} \\ N_{f0} \end{bmatrix} = \frac{mg}{1-(\mu_{r0}-\mu_{f0})\bar{h}} \begin{bmatrix} (\mu_{f0}-c_r)\bar{h}+\bar{a} \\ -(\mu_{r0}-c_r)\bar{h}+\bar{b} \end{bmatrix}$$

Wheel Speed Controls Equilibrium triplet

Define $f_{\#0} \coloneqq N_{\#0}\mu_{\#0}$ with $\begin{bmatrix} N_{r0} \\ N_{f0} \end{bmatrix} = \frac{mg}{1-(\mu_{r0}-\mu_{f0})\bar{h}} \begin{bmatrix} (\mu_{f0}-c_r)\bar{h}+\bar{a} \\ -(\mu_{r0}-c_r)\bar{h}+\bar{b} \end{bmatrix}$

Solve

$$J_r\dot{\omega}_r = \tau_r - f_r r_r$$
$$J_f\dot{\omega}_f = \tau_f - f_f r_f$$

$v_0 > 0$ $\dot{v} = 0, v = v_0, w = \theta = 0$ $f_{\#} = f_{\#0}$, $\dot{\omega}_{\#} = 0$

Wheel Speed Controls Equilibrium triplet

Define $f_{\#0} \coloneqq N_{\#0}\mu_{\#0}$ with $\begin{bmatrix} N_{r0} \\ N_{f0} \end{bmatrix} = \dfrac{mg}{1-(\mu_{r0}-\mu_{f0})\bar{h}} \begin{bmatrix} (\mu_{f0}-c_r)\bar{h}+\bar{a} \\ -(\mu_{r0}-c_r)\bar{h}+\bar{b} \end{bmatrix}$

Solve $\begin{aligned} J_r\dot{\omega}_r &= \tau_r - f_r r_r \\ J_f\dot{\omega}_f &= \tau_f - f_f r_f \end{aligned}$ $\quad v_0 > 0 \;\; \dot{v}=0, v=v_0, w=\theta=0 \;\; f_{\#}=f_{\#0}, \dot{\omega}_{\#}=0$

To define the equilibrium torques as

$$\tau_{r0} = r_r(\mu_{r0}-c_r)\frac{mg((\mu_{f0}-c_r)\bar{h}+\bar{a})}{1-(\mu_{r0}-\mu_{f0})\bar{h}}$$

$$\tau_{f0} = r_f(\mu_{f0}-c_r)\frac{mg(-(\mu_{r0}-c_r)\bar{h}+\bar{b})}{1-(\mu_{r0}-\mu_{f0})\bar{h}}$$

Wheel Speed Controls Equilibrium triplet

Define $f_{\#0} := N_{\#0}\mu_{\#0}$ with $\begin{bmatrix} N_{r0} \\ N_{f0} \end{bmatrix} = \dfrac{mg}{1-(\mu_{r0}-\mu_{f0})\bar{h}} \begin{bmatrix} (\mu_{f0}-c_r)\bar{h}+\bar{a} \\ -(\mu_{r0}-c_r)\bar{h}+\bar{b} \end{bmatrix}$

Solve
$$J_r\dot{\omega}_r = \tau_r - f_r r_r$$
$$J_f\dot{\omega}_f = \tau_f - f_f r_f$$
$v_0 > 0$ $\dot{v} = 0, v = v_0, w = \theta = 0$ $f_{\#} = f_{\#0}$, $\dot{\omega}_{\#} = 0$

To define the equilibrium torques as

$$\tau_{r0} = r_r(\mu_{r0}-c_r)\frac{mg((\mu_{f0}-c_r)\bar{h}+\bar{a})}{1-(\mu_{r0}-\mu_{f0})\bar{h}}$$
$$\tau_{f0} = r_f(\mu_{f0}-c_r)\frac{mg(-(\mu_{r0}-c_r)\bar{h}+\bar{b})}{1-(\mu_{r0}-\mu_{f0})\bar{h}}$$

Define the equilibrium triplet

$$\mathbf{x}_0 = \mathrm{col}(v_0, \omega_{r0}, \omega_{f0})$$
$$\mathbf{w}_0 = \mathrm{col}(0,0,0,0,0,\omega_{r0},\omega_{f0})$$
$$\mathbf{u}_0 := \mathrm{col}(\tau_{r0}, \tau_{f0})$$

Wheel Speed Controls Linearisation

Define the variations $\tilde{\mathbf{u}} = \mathbf{u} - \mathbf{u}_0$, $\tilde{\mathbf{w}} = \mathbf{w} - \mathbf{w}_0$,

And linearise the plant to get

$$\dot{\tilde{\mathbf{x}}} = \mathbf{A}\tilde{\mathbf{x}} + \mathbf{B}_1\tilde{\mathbf{u}} + \mathbf{B}_2\tilde{\mathbf{w}} \quad \tilde{\mathbf{x}}(t_0) = \tilde{\mathbf{x}}_0$$
$$\tilde{\mathbf{y}} = \mathbf{C}\tilde{\mathbf{x}} + \mathbf{D}_2\tilde{\mathbf{w}}$$
$$\tilde{\mathbf{e}} = \mathbf{C}_e\tilde{\mathbf{x}} + \mathbf{D}_{e_2}\tilde{\mathbf{w}}$$

The expressions of matrices A, B, etc., are too complex (ugly!) to be included here. However, if you wish to see them, please refer to the lecture notes.

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Contents

- Active Suspensions
- Adaptive Cruise Control
- Wheel Speed Controls
- Electronic Stability Program / Torque Vectoring
- Self-Park Assist

ESP/TV Simulation model

Let $\mathbf{x} := \mathrm{col}(v_x, v_y, \omega)$, $\mathbf{u} := \mathrm{col}(\delta_1, \dots, \delta_4, \omega_1, \dots, \omega_4)$

$\mathbf{d} := \mathrm{col}(w, \boldsymbol{\Theta}_1, \boldsymbol{\Theta}_2, \boldsymbol{\Theta}_3, \boldsymbol{\Theta}_4)$, $\mathbf{w} := \mathrm{col}(\mathbf{d}, \nu, r)$

Then

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}, \mathbf{u}, \mathbf{w}) \qquad \mathbf{x}(t_0) = \mathbf{x}_0 \\ y &= h(\mathbf{x}, \mathbf{u}, \mathbf{w}) \\ e &= h_e(\mathbf{x}, \mathbf{u}, \mathbf{w}) \end{aligned}$$

with

$$\mathbf{f}(\mathbf{x}, \mathbf{u}, \mathbf{w}) := \begin{bmatrix} \omega v_y \\ -\omega v_x \\ 0 \end{bmatrix} + \begin{bmatrix} (f_x - D(v_x - w))/m \\ f_y/m \\ \tau/J \end{bmatrix}$$

$$h(\mathbf{x}, \mathbf{u}, \mathbf{w}) := \omega,$$

$$h_e(\mathbf{x}, \mathbf{u}, \mathbf{w}) := y - r(t).$$

Simulation model (continued)

$$\mathbf{H} := \begin{bmatrix} \mathbf{v}_1 & \dots & \mathbf{v}_n \end{bmatrix} \quad \mathbf{v}_i := \begin{bmatrix} 1 \\ y_{w_i} + h(\mu(\beta_i)\cos\delta_i + \mu(\lambda_i)\sin\delta_i) \\ h(\mu(\lambda_i)\cos\delta_i - \mu(\beta_i)\sin\delta_i) + x_{w_i} \end{bmatrix}$$

$$\begin{bmatrix} N_1 \\ \vdots \\ N_4 \end{bmatrix} = \mathbf{H}^\top \left[\mathbf{H}\mathbf{H}^\top\right]^{-1} \begin{bmatrix} mg \\ 0 \\ 0 \end{bmatrix}$$

Tire forces and torques

$$\begin{bmatrix} f_x \\ f_y \\ \tau \end{bmatrix} = \sum_{i=1}^{4} N_i \mu(\lambda_i, \boldsymbol{\Theta}_i) \begin{bmatrix} \cos\delta_i \\ \sin\delta_i \\ x_{w_i}\sin\delta_i - y_{w_i}\cos\delta_i \end{bmatrix} + \sum_{i=1}^{4} N_i \mu(\beta_i, \boldsymbol{\Theta}_i) \begin{bmatrix} -\sin\delta_i \\ \cos\delta_i \\ x_{w_i}\cos\delta_i + y_{w_i}\sin\delta_i \end{bmatrix}$$

 Equilibrium triplet

Equilibrium: straight path, constant speed

let $v_0 > 0$, $\mathbf{x}_0 = \mathrm{col}(v_0, 0, 0)$ and $\mathbf{d}_0 = \mathrm{col}(0, \boldsymbol{\Theta}_0, \boldsymbol{\Theta}_0, \boldsymbol{\Theta}_0, \boldsymbol{\Theta}_0)$ for some $\boldsymbol{\Theta}_0 \in \mathbb{R}^3$.

Define

$$\mu_{f_0} = \frac{D(v_0)}{mg\bar{b} - D(v_0)\bar{h}} - \mu_{r_0}\frac{D(v_0)\bar{h} + mg\bar{a}}{mg\bar{b} - D(v_0)\bar{h}}$$

$$\mathbf{u}_0 = \mathrm{col}\left(0, 0, 0, 0, \frac{v_0}{r_w(1-\lambda_{f_0})}, \frac{v_0}{r_w(1-\lambda_{r_0})}, \frac{v_0}{r_w(1-\lambda_{r_0})}, \frac{v_0}{r_w(1-\lambda_{f_0})}\right)$$

with $\lambda_{r_0}, \lambda_{f_0} \in (0,\ 1)$ such that

$$\mu_{r_0} = \mu(\lambda_{r_0}, \boldsymbol{\Theta}_0), \qquad \mu_{f_0} = \mu(\lambda_{f_0}, \boldsymbol{\Theta}_0).$$

the equilibrium output is $y_0 = 0$ under the assumption $\nu = 0$

the equilibrium error is $e_0 = 0$

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Linearisation

Let $\tilde{\mathbf{u}} := \mathbf{u} - \mathbf{u}_0$; $\tilde{\mathbf{w}} := \mathbf{w} - \mathbf{w}_0$

Then
$$\dot{\tilde{\mathbf{x}}} = \mathbf{A}\tilde{\mathbf{x}} + \mathbf{B}_1\tilde{\mathbf{u}} + \mathbf{B}_2\tilde{\mathbf{w}} \quad \tilde{\mathbf{x}}(t_0) = \tilde{\mathbf{x}}_0$$
$$\tilde{y} = \mathbf{C}\tilde{\mathbf{x}} + \mathbf{D}_2\tilde{\mathbf{w}}$$
$$\tilde{e} = \mathbf{C}_e\tilde{\mathbf{x}} + \mathbf{D}_e\tilde{\mathbf{w}},$$

Let $\tilde{\mathbf{u}} := \mathbf{u} - \mathbf{u}_0; \ \tilde{\mathbf{w}} := \mathbf{w} - \mathbf{w}_0$

Then

$$\dot{\tilde{\mathbf{x}}} = \mathbf{A}\tilde{\mathbf{x}} + \mathbf{B}_1\tilde{\mathbf{u}} + \mathbf{B}_2\tilde{\mathbf{w}} \quad \tilde{\mathbf{x}}(t_0) = \tilde{\mathbf{x}}_0$$

$$\tilde{y} = \mathbf{C}\tilde{\mathbf{x}} + \mathbf{D}_2\tilde{\mathbf{w}}$$

$$\tilde{e} = \mathbf{C}_e\tilde{\mathbf{x}} + \mathbf{D}_e\tilde{\mathbf{w}},$$

Where

$$\mathbf{A} = \begin{bmatrix} m^{-1}\rho S C_D v_0 & 0 & 0 \\ 0 & 0 & -v0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} m^{-1} & 0 & 0 \\ 0 & m^{-1} & 0 \\ 0 & 0 & J^{-1} \end{bmatrix} \left(\sum_{i=1}^{4} \mu(\lambda_{i_0}, \mathbf{\Theta}_i) \begin{bmatrix} 1 \\ 0 \\ -y_{w_i} \end{bmatrix} \frac{\partial N_i}{\partial \mathbf{x}} + \sum_{i=1}^{4} N_{i_0} \begin{bmatrix} 1 \\ 0 \\ -y_{w_i} \end{bmatrix} \frac{\partial \mu(\lambda_i, \mathbf{\Theta}_i)}{\partial \lambda_i} \frac{\partial \lambda_i}{\partial \mathbf{x}} + \sum_{i=1}^{4} N_{i_0} \begin{bmatrix} 0 \\ 1 \\ x_{w_i} \end{bmatrix} \frac{\partial \mu(\beta_i, \mathbf{\Theta}_i)}{\partial \beta_i} \frac{\partial \beta_i}{\partial \mathbf{x}} \right)_{(\mathbf{x},\mathbf{u},\mathbf{w})=(\mathbf{x}_0,\mathbf{u}_0,\mathbf{w}_0)}$$

$$\begin{bmatrix} N_{1_0} \\ N_{2_0} \\ N_{3_0} \\ N_{4_0} \end{bmatrix} = \frac{mg}{2(1-(\mu_{r_0}-\mu_{f_0})\bar{h})} \begin{bmatrix} \bar{b} - \mu_{r_0}\bar{h} \\ \mu_{f_0}\bar{h} + \bar{a} \\ \mu_{f_0}\bar{h} + \bar{a} \\ \bar{b} - \mu_{r_0}\bar{h} \end{bmatrix}$$

Let $\tilde{\mathbf{u}} := \mathbf{u} - \mathbf{u}_0$, $\tilde{\mathbf{w}} := \mathbf{w} - \mathbf{w}_0$

Then
$$\dot{\tilde{\mathbf{x}}} = \mathbf{A}\tilde{\mathbf{x}} + \mathbf{B}_1\tilde{\mathbf{u}} + \mathbf{B}_2\tilde{\mathbf{w}} \quad \tilde{\mathbf{x}}(t_0) = \tilde{\mathbf{x}}_0$$
$$\tilde{y} = \mathbf{C}\tilde{\mathbf{x}} + \mathbf{D}_2\tilde{\mathbf{w}}$$
$$\tilde{e} = \mathbf{C}_e\tilde{\mathbf{x}} + \mathbf{D}_e\tilde{\mathbf{w}},$$

Where
$$\mathbf{A} = \begin{bmatrix} m^{-1}\rho S C_D v_0 & 0 & 0 \\ 0 & 0 & -v0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} m^{-1} & 0 & 0 \\ 0 & m^{-1} & 0 \\ 0 & 0 & J^{-1} \end{bmatrix} \left(\sum_{i=1}^{4} \mu(\lambda_{i_0}, \boldsymbol{\Theta}_i) \begin{bmatrix} 1 \\ 0 \\ -y_{w_i} \end{bmatrix} \frac{\partial N_i}{\partial \mathbf{x}} + \sum_{i=1}^{4} N_{i_0} \begin{bmatrix} 1 \\ 0 \\ -y_{w_i} \end{bmatrix} \frac{\partial \mu(\lambda_i, \boldsymbol{\Theta}_i)}{\partial \lambda_i} \frac{\partial \lambda_i}{\partial \mathbf{x}} + \sum_{i=1}^{4} N_{i_0} \begin{bmatrix} 0 \\ 1 \\ x_{w_i} \end{bmatrix} \frac{\partial \mu(\beta_i, \boldsymbol{\Theta}_i)}{\partial \beta_i} \frac{\partial \beta_i}{\partial \mathbf{x}} \right)_{(\mathbf{x},\mathbf{u},\mathbf{w})=(\mathbf{x}_0,\mathbf{u}_0,\mathbf{w}_0)}$$

Load Transfer

$$\begin{bmatrix} N_{1_0} \\ N_{2_0} \\ N_{3_0} \\ N_{4_0} \end{bmatrix} = \frac{mg}{2(1-(\mu_{r_0}-\mu_{f_0})\bar{h})} \begin{bmatrix} \bar{b} - \mu_{r_0}\bar{h} \\ \mu_{f_0}\bar{h} + \bar{a} \\ \mu_{f_0}\bar{h} + \bar{a} \\ \bar{b} - \mu_{r_0}\bar{h} \end{bmatrix}$$

 Linearisation

$$\mathbf{B}_1 = \begin{bmatrix} m^{-1} & 0 & 0 \\ 0 & m^{-1} & 0 \\ 0 & 0 & J^{-1} \end{bmatrix} \left(\sum_{i=1}^{4} \mu(\lambda_{i_0}, \mathbf{\Theta}_i) \begin{bmatrix} 1 \\ 0 \\ -y_{w_i} \end{bmatrix} \frac{\partial N_i}{\partial \mathbf{u}} \right.$$

Load Transfer

$$+ \sum_{i=1}^{4} N_{i_0} \begin{bmatrix} 1 \\ 0 \\ -y_{w_i} \end{bmatrix} \frac{\partial \mu(\lambda_i, \mathbf{\Theta}_i)}{\partial \lambda_i} \frac{\partial \lambda_i}{\partial \mathbf{u}}$$

$$+ \sum_{i=1}^{4} N_{i_0} \begin{bmatrix} 0 \\ 1 \\ x_{w_i} \end{bmatrix} \frac{\partial \mu(\beta_i, \mathbf{\Theta}_i)}{\partial \beta_i} \frac{\partial \beta_i}{\partial \mathbf{u}}$$

$$\left. + \sum_{i=1}^{4} N_i \mu(\lambda_i, \mathbf{\Theta}_i) \frac{\partial}{\partial \mathbf{u}} \begin{bmatrix} \cos \delta_i \\ \sin \delta_i \\ x_{w_i} \sin \delta_i - y_{w_i} \cos \delta_i \end{bmatrix} \right) \Bigg|_{(\mathbf{x},\mathbf{u},\mathbf{w})=(\mathbf{x}_0,\mathbf{u}_0,\mathbf{w}_0)}$$

$$\mathbf{C} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{D} = \begin{bmatrix} \mathbf{0} & 1 & 0 \end{bmatrix},$$

$$\mathbf{C}_e = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{D}_e = \begin{bmatrix} \mathbf{0} & 1 & -1 \end{bmatrix}$$

Contents

Self-Park Assist Simulation model

Let

$$\mathbf{x} \coloneqq \mathrm{col}(\rho, \psi_r)$$
$$u \coloneqq \delta$$
$$d \coloneqq \chi$$
$$\boldsymbol{\nu} \coloneqq \mathrm{col}(\nu_\rho, \nu_\psi)$$
$$\mathbf{w} \coloneqq \mathrm{col}(d, \boldsymbol{\nu})$$
$$\boldsymbol{y} = \mathrm{col}(\rho, \psi_r, d) + \boldsymbol{\nu}$$
$$e \coloneqq \rho$$

Then

$$\mathbf{f}(\mathbf{x}, u, \mathbf{w}) = V \begin{bmatrix} \tan\psi_r \left(\frac{\rho}{\ell}\sin\delta - \cos\delta\right) - \frac{x_A}{\ell}\sin\delta \\ \frac{\sin\delta}{\ell}\left(1 + \rho\frac{\chi(\mathbf{p}_L^I)}{\cos\psi_r}\right) - \chi(\mathbf{p}_L^I)\frac{\cos\delta}{\cos\psi_r} \end{bmatrix}$$

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, u, \mathbf{w}) \qquad \mathbf{x}(t_0) = \mathbf{x}_0$$
$$\mathbf{y} = \mathbf{h}(\mathbf{x}, u, \mathbf{w})$$
$$e = h_e(\mathbf{x}, u, \mathbf{w})$$

$$\mathbf{h}(\mathbf{x}, u, \mathbf{w}) := \mathrm{col}\big(\rho + \nu_\rho, \psi_r + \nu_\psi, d\big)$$

$$h_e(\mathbf{x}, u, \mathbf{w}) := \rho + \nu_\rho$$

Self-Park Assist Equilibrium triplet & Linearisation

Linearisation Conditions $\mathbf{x_0}, \mathbf{w_0} = \mathbf{0} \;\Rightarrow u_0, e_0 = 0, \boldsymbol{y}_0 = \mathbf{0}$

Define $\tilde{u} = u,\; \tilde{\mathbf{w}} = \mathbf{w}$

and compute

$$\dot{\tilde{\mathbf{x}}} = \mathbf{A}\tilde{\mathbf{x}} + \mathbf{B}_1\tilde{u} + \mathbf{B}_2\tilde{\mathbf{w}} \quad \tilde{\mathbf{x}}(t_0) = \tilde{\mathbf{x}}_0$$

$$\tilde{\mathbf{y}} = \mathbf{C}\tilde{\mathbf{x}} + \mathbf{D}_2\tilde{\mathbf{w}}$$

$$\tilde{e} = \mathbf{C}_e\tilde{\mathbf{x}} + \mathbf{D}_{2e}\tilde{\mathbf{w}}.$$

in which

$$\mathbf{A} = \begin{bmatrix} 0 & -v_0 \\ 0 & 0 \end{bmatrix}, \mathbf{B}_1 = \begin{bmatrix} -v_0 x_A/\ell \\ v_0/\ell \end{bmatrix}, \mathbf{B}_2 = \begin{bmatrix} 0 & 0 & 0 \\ -v_0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \mathbf{D}_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix},$$

$$\mathbf{C}_e = \begin{bmatrix} 1 & 0 \end{bmatrix}, \; \mathbf{D}_{2e} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$$

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UNIVERSITÀ DI BOLOGNA

Nicola Mimmo

Department of Electrical, Electronics and Information Engineering «G. Marconi»

nicola.mimmo2@unibo.it