# TITLE

Automatic Control
Electronic Engineering for Intelligent Vehicles
University of Bologna

A.A. 202X-202X

Student A and Student B and ...

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**Abstract**

Here briefly detail the aims of the project.

# Chapter 1

# Introduction

### 1.0.1 System Linearization

To facilitate the control design of the longitudinal half-car model equipped with active front and rear suspension systems, the initial step involves the linearization of the nonlinear system dynamics. This process is performed by identifying appropriate steady-state operating points $(x^*, y^*, w^*)$, which characterize representative conditions under which the vehicle is expected to operate. Linearizing the system around these equilibrium points enables the derivation of a time-invariant linear approximation of the vehicle dynamics, thereby simplifying the synthesis and analysis of control strategies.

#### Linearization Around the Operating Point

Consider the nonlinear system model:

$$\begin{aligned} \dot{x} &= f(x,u,w), \quad x(t_0) = x_0 \\ y &= h(x,u,w) \\ e &= h_e(x,u,w) \end{aligned} \tag{1.1}$$

The steady-state operating points $(x^\star, u^\star, w^\star)$ is called *equilibrium triplet* if satisfies the condition:

$$f(x^\star, u^\star, w^\star) = 0 \tag{1.2}$$

and defines the equilibrium output and error as:

$$y^\star := h(x^\star, u^\star, w^\star), \quad e^\star := h_e(x^\star, u^\star, w^\star) \tag{1.3}$$

The variations around the equilibrium point are defined as:

$$\begin{aligned} \tilde{x} &:= x - x^\star \\ \tilde{y} &:= y - y^\star \\ \tilde{e} &:= e - e^\star \\ \tilde{u} &:= u - u^\star \\ \tilde{w} &:= w - w^\star \end{aligned} \tag{1.4}$$

Using the fact that $\dot{x}^\star = 0$, the dynamics of the variations are:

$$\begin{aligned} \dot{\tilde{x}} &= f(x^\star + \tilde{x}, u^\star + \tilde{u}, w^\star + \tilde{w}), \quad \tilde{x}(t_0) = x_0 - x^\star \\ \tilde{y} &= h(x^\star + \tilde{x}, u^\star + \tilde{u}, w^\star + \tilde{w}) \\ \tilde{e} &= h_e(x^\star + \tilde{x}, u^\star + \tilde{u}, w^\star + \tilde{w}) \end{aligned} \tag{1.5}$$

To obtain a tractable model for controller synthesis, we apply a first-order Taylor expansion around the equilibrium point. The resulting Jacobian matrices are defined as:

$$\begin{aligned} A &:= \left.\frac{\partial f(x,u,w)}{\partial x}\right|_{\substack{x=x^\star\\u=u^\star\\w=w^\star}} & B_1 &:= \left.\frac{\partial f(x,u,w)}{\partial u}\right|_{\substack{x=x^\star\\u=u^\star\\w=w^\star}} & B_2 &:= \left.\frac{\partial f(x,u,w)}{\partial w}\right|_{\substack{x=x^\star\\u=u^\star\\w=w^\star}} \\ C &:= \left.\frac{\partial h(x,u,w)}{\partial x}\right|_{\substack{x=x^\star\\u=u^\star\\w=w^\star}} & D_1 &:= \left.\frac{\partial h(x,u,w)}{\partial u}\right|_{\substack{x=x^\star\\u=u^\star\\w=w^\star}} & D_2 &:= \left.\frac{\partial h(x,u,w)}{\partial w}\right|_{\substack{x=x^\star\\u=u^\star\\w=w^\star}} \\ C_e &:= \left.\frac{\partial h_e(x,u,w)}{\partial x}\right|_{\substack{x=x^\star\\u=u^\star\\w=w^\star}} & D_{e1} &:= \left.\frac{\partial h_e(x,u,w)}{\partial u}\right|_{\substack{x=x^\star\\u=u^\star\\w=w^\star}} & D_{e2} &:= \left.\frac{\partial h_e(x,u,w)}{\partial w}\right|_{\substack{x=x^\star\\u=u^\star\\w=w^\star}} \end{aligned} \tag{1.6}$$

Neglecting second-order terms, the linearized system becomes the so-called *design model*:

$$\begin{cases} \dot{\tilde{x}} = A\tilde{x} + B_1\tilde{u} + B_2\tilde{w}, \quad \tilde{x}(t_0) = x_0 - x^\star \\ \tilde{y} = C\tilde{x} + D_1\tilde{u} + D_2\tilde{w} \\ \tilde{e} = C_e\tilde{x} + D_{e1}\tilde{u} + D_{e2}\tilde{w} \end{cases} \tag{1.7}$$

This Linear Time-Invariant (LTI) approximation of the nonlinear model is valid in a neighborhood of the equilibrium point, enabling efficient analysis and controller design under small perturbations.

**TODO**: Inserire le matrici calcolate con matlab e verificare che rispettino quello che abbiamo scritto (verifica che il codice del .m e quello che fa rispetta queste cose dette nella aprte di teoria)

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{2k}{m} & -\frac{2\beta}{m} & -\frac{(\mathrm{df}-\mathrm{dr})k}{m} & -\frac{(\mathrm{df}-\mathrm{dr})\beta}{m} & \frac{\mathrm{df}k}{m} & -\frac{\mathrm{dr}k}{m} & \frac{\mathrm{df}\beta}{m} & -\frac{\mathrm{dr}\beta}{m} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ -\frac{(\mathrm{df}-\mathrm{dr})k}{J} & -\frac{(\mathrm{df}-\mathrm{dr})\beta}{J} & -\frac{k(\mathrm{df}^2+\mathrm{dr}^2)}{J} & -\frac{\beta(\mathrm{df}^2+\mathrm{dr}^2)}{J} & \frac{k\mathrm{df}^2}{J} & \frac{k\mathrm{dr}^2}{J} & \frac{\beta\mathrm{df}^2}{J} & \frac{\beta\mathrm{dr}^2}{J} \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 & 0 \\ \frac{1}{m} & 0 \\ 0 & 0 \\ 0 & \frac{1}{J} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ -1.0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\mathrm{ell}_0 - 0.177398}{J} & \frac{\mathrm{ell}_0 - 0.177398}{J} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1.0 & 0 & 0 & 0 \\ 0 & 0 & 1.0 & 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & \frac{0.354795\,k}{m} & 0 & 0 & 0 & 0 & 0 \\ -\frac{2.0\,k}{m} & -\frac{2.0\,\beta}{m} & -\frac{(\mathrm{df}-\mathrm{dr})k}{m} & -\frac{(\mathrm{df}-\mathrm{dr})\beta}{m} & \frac{\mathrm{df}k}{m} & -\frac{\mathrm{dr}k}{m} & \frac{\mathrm{df}\beta}{m} & -\frac{\mathrm{dr}\beta}{m} \\ 0 & 0 & 0 & 1.0 & 0 & 0 & 0 & 0 \\ 1.0 & 0 & \mathrm{df} & 0 & -\mathrm{df} & 0 & 0 & 0 \\ 1.0 & 0 & -\mathrm{dr} & 0 & 0 & \mathrm{dr} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$D_1 = \begin{bmatrix} 0 & 0 \\ \frac{1}{m} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$D_2 = \begin{bmatrix} \frac{1}{m} & \frac{1}{m} & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathrm{CE} = \begin{bmatrix} 1.0 & 0 & 0 & 0 & -\frac{\mathrm{df}\,\mathrm{dr}}{\mathrm{df}+\mathrm{dr}} & \frac{\mathrm{df}\,\mathrm{dr}}{\mathrm{df}+\mathrm{dr}} & 0 & 0 \\ 0 & 0 & \frac{|m|\left(6.39142\times 10^{15}\,k + 1.80144\times 10^{16}\,u_1\right)}{m\,|6.39142\times 10^{15}\,k + 1.80144\times 10^{16}\,u_1|} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathrm{DE}_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\mathrm{DE}_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{2.81853\,|m|}{m\,|k|} & \frac{2.81853\,|m|}{m\,|k|} & \frac{2.81853\,|m|}{|k|} & 0 & 0 & 0 & 0 \end{bmatrix}$$