

2a. For  $n=1$ ,  $A = (\alpha_{11})$ ,  $L = (1)$ ,  $U = (\alpha_{11})$ . This does not take any computations, so  $|\Delta A| \leq \gamma_1 |L| |U|$ .

b. Assume for  $A_{00} \in \mathbb{R}^{n \times n}$  the algorithm computes  $L_{00}$  and  $U_{00}$  where  $A_{00} \Delta A_{00} = L_{00} U_{00}$  and  $|A_{00}| \leq \gamma_n |L_{00}| |U_{00}|$

Now let  $A \in \mathbb{R}^{(n+1) \times (n+1)}$  with  $A = \begin{pmatrix} A_{00} & a_{01} \\ a_{10}^T & \alpha_{11} \end{pmatrix}$

$$\text{let } \begin{pmatrix} A_{00} & a_{01} \\ a_{10}^T & \alpha_{11} \end{pmatrix} \rightarrow \begin{pmatrix} \Delta A_{00} & \delta a_{01} \\ \delta a_{10}^T & \delta \alpha_{11} \end{pmatrix} = \begin{pmatrix} L_{00} & 0 \\ l_{10}^T & 1 \end{pmatrix} \begin{pmatrix} U_{00} & u_{01} \\ 0 & u_{11} \end{pmatrix}$$

be the result of the algorithm.

Note that  $|\Delta A_{00}| \leq \gamma_n |L_{00}| |U_{00}|$  by the IH.

$a_{01}$  is computed from the solution to  $L_{00} u_{01} = a_{01}$

and the error from triangle solve is bounded:  $|\delta a_{01}| \leq \max(\gamma_2, \gamma_{n+1}) |L_{00}| |u_{01}|$

Similarly,  $a_{10}^T$  is computed from the solution to

$l_{10}^T U_{00} = a_{10}^T$  which is equivalent to solving the

lower triangular system:  $U_{00}^T l_{10} = a_{10}$  and the error

is bounded:  $|\delta a_{10}^T| \leq \max(\gamma_2, \gamma_{n+1}) |l_{10}^T| |U_{00}|$

$\alpha_{11}$  is computed from  $\alpha_{11} := u_{11} = \alpha_{11} - a_{10}^T a_{01}$

The error from the dot product  $K := a_{10}^T a_{01}$  is

bounded:  $|\delta K| \leq \gamma_n |a_{10}^T| |a_{01}|$  so the error from

the subtraction is bounded:  $|\delta \alpha_{11}| \leq \gamma_{n+1} |a_{10}^T a_{01} + u_{11}|$

$$\text{So } |\Delta A| \leq \begin{pmatrix} \gamma_n |L_{00}| |U_{00}| & \max(\gamma_2, \gamma_{n+1}) |L_{00}| |u_{01}| \\ \max(\gamma_2, \gamma_{n+1}) |l_{10}^T| |U_{00}| & \gamma_{n+1} |a_{10}^T a_{01} + u_{11}| \end{pmatrix}$$

$$\leq \gamma_{n+1} \begin{pmatrix} |L_{00} U_{00}| & |L_{00} u_{01}| \\ |l_{10}^T U_{00}| & |a_{10}^T a_{01} + u_{11}| \end{pmatrix} \leq \gamma_{n+1} |L| |U|$$