1a. Partition $A = \left(\begin{array}{c|c} A_{00} & a_{01} \\ \hline a^T_{10} & \alpha_{11} \end{array}\right)$

Assuming $A_{00} := L_{00}$ has already been computed,
overwrite $a^T_{10} := l^T_{10} = a^T_{10}(L_{00}^T)^{-1}$
and overwrite $\alpha_{11} := \sqrt{\alpha_{11} - l^T_{10} l_{10}}$.

This is justified by $A = LL^T$

$$\Rightarrow \left(\begin{array}{c|c} A_{00} & a_{01} \\ \hline a_{10}^T & \alpha_{11} \end{array}\right) = \left(\begin{array}{c|c} L_{00} & 0 \\ \hline l^T_{10} & \lambda_{11} \end{array}\right)\left(\begin{array}{c|c} L_{00} & 0 \\ \hline l^T_{10} & \lambda_{11} \end{array}\right)^T$$

$$= \left(\begin{array}{c|c} L_{00}L_{00}^T & l_{10} L_{00} \\ \hline l^T_{10} L_{00}^T & l^T_{10} l_{10} + \lambda^2_{11} \end{array}\right)$$

b. For $n=1$, the first 2 steps of the algorithm deal with null matrices and are ignored. Then $\alpha_{11} := \sqrt{\alpha_{11}} = \lambda_{11}$ is well defined since A is SPD. Also if λ_{11} is positive, $\sqrt{\alpha_{11}} = \lambda_{11}$ is unique.

Now assuming the theorem holds for a specific matrix size $n \times n$, let A be an $n+1 \times n+1$ SPD matrix.
Then $A_{00} = L_{00}$ is well defined by the IH and if the diagonals of L must be positive, $A_{00} = L_{00}$ is unique by the IH.
$a^T_{10} := a^T_{10}(L_{00}^T)^{-1}$ is well defined and unique since L_{00}^T is lower triangular, so non-singular.
$\alpha_{11} := \sqrt{\alpha_{11} - l^T_{10} l_{10}}$ is well defined as long as $\alpha_{11} \geq l^T_{10} l_{10}$ and unique since l_{10} is unique (when diagonals of L must be positive).
Therefore, the factorization of an $n+1 \times n+1$ A is well defined and unique if the diagonals must be positive.