Gas of Different Molar Mass on The Thrust Performance of Ion Thrusters

[redacted]

1 Introduction

1.1 Research Question

How do gases of varying molar mass (Oxygen (O_2), Carbon Dioxide (CO_2), Helium (He), Hydrogen (H_2), Nitrogen (N_2)) affect the thrust generated by the ion thrusters?

1.2 Hypothesis

If a gas of lower molar mass is ionised and accelerated through an electric field in an ion thruster, then it will result in a higher exhaust velocity, but due to its low gas density, a lower thrust. Predictably, with Hydrogen (H_2) and Helium (He) producing the least thrust due to the high ionisation energy required to pull one electron away from the atom. Or since its molar mass is incredibly light its exhaust velocity will be incredibly high but does not produce much thrust due to its mass. Thus, if the ion thruster is isolated with Hydrogen (H_2) and Helium (He) in a vacuum such as space it wouldn't generate sufficient thrust to move the aircraft. Oppositely, if the molar mass of the atom is higher, based on the formula of kinetic energy, and as the electrons is assumed to have the same energy, higher mass means lower velocity, but due to its higher density at the same pressure it would result in a higher thrust. Thus, a gas of higher mass would result in a lower velocity but it will have a higher momentum and as thrust can be defined as the transfer of momentum per unit time. Increasing the molar mass of the gas that is being ionised will increase the ion thruster's thrust performance.

1.3 Introduction

I first became interested in this area of aerospace propulsion when I encountered it in a maker faire Fig(1). So I decided to make one with a few modifications of my own using a 3D Printer rather than wood. What I found fascinating about this is that compared to the traditional chemical rockets, these types of rocket propulsion does not exude any smoke or waste, and added to the fact that the cathode and the anode components don't even touch, and yet they are able to generate thrust. It is certainly unique to other types of propulsion. Then I began to think if it generates thrust by ionising air, what if I change the air into a gas of lighter atomic mass so in space mission it will reduce the spacecraft's overall mass. In theory we should see a considerable difference. Thus, in this investigation paper I will focus on how the varying atomic mass of the ionised gas may affect the thrust generated by the ion thruster.

***Figure 1**: Makerfaire Ionic Space Thruster* [1]

1.4 Background Study

Ion thruster or ion drive is a common form of electric propulsion that is used in the spacecraft industry today. The general idea on how it works is it creates thrust by accelerating ions using electricity; specifically through solar power if it's in space. Currently, there are a wide variety of operations that are conducted using ion thrusters, from communication satellites to propelling a small spacecraft through the solar system for a long period of time. Unlike chemical rockets, these ion thrusters have a high specific impulse, as their ratio of thrust to the rate of propellant used is incredibly efficient. Thus, they require less propellant for a given mission. Ion propulsion is even considered ideal for missions where it requires to be run for a long period of time something chemical propellants lack. While all of the above are advantageous, an ion thruster has a trade off in its thrust, as it generates thrust in millinewtons (mN), which is the same force equivalent to holding a paper on our palm. But, while its thrust is incredibly miniscule it has an advantage of accelerating for a longer period, therefore the amount of thrust will just keep compounding over time, this is generalized by Newton's law $F = ma$. [4]

In the traditional ion thruster as seen in Fig(2) it utilizes the coulomb force to accelerate positively charged ions and create thrust. In the diagram we can see a hollow yellow tube, this represents the cathode, of which emits electrons around the cathode we can see the propellant atoms, commonly spacecraft industries use Xenon (Xe) due to its non-reactive and low ionization properties. Both the electrons and propellant atoms will be mixed in the ionization chamber, where the propellant atoms will be bombarded by electrons. Thus, some of the electrons in the propellant atom will be knocked out resulting in a positive ion. These positive ions will later on be attracted to the negative grid where it will accelerate massively and fly out at the back of the spacecraft, creating an ion beam. Once it's out of the grid the ions will be neutralized again through means of another cathode on the outside. So it will not react and be attracted to other electrical components. [7]

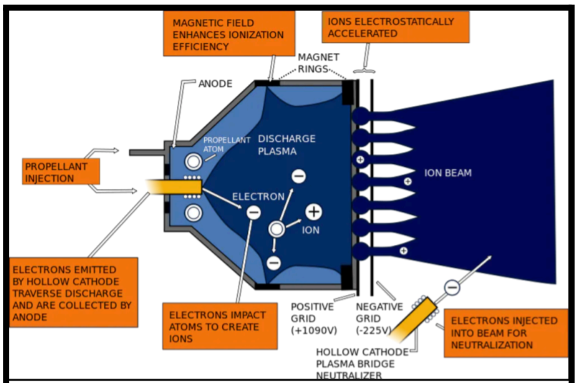


***Figure 2**: Ion Thruster Setup Diagram* [2]

This similar concept can be seen through DIY ion thrusters in both Fig(1-2). Referring to Fig(3), when we attach the sharp edged object (Nail) to the positive terminal of the power supply and the smooth edged object (Pipe) to the negative terminal, and the power supply is turned on, the electrons in the sharp object will be pulled away leaving more positive protons than negative electrons, thus it is positively charged. Similarly, the smooth object as it is connected to the negative terminal more electrons will be added into the object, thus its negatively charged. As they are both connected to the same power supply there will be equal amounts of extra positive and negative, and due to the Coulomb's law, like charges repel and unlike charges attract. Thus, they will have an equal distance in their respective objects, but due to the curvature in the sharp surface, the repulsive force is not directed along the surface, rather a small component in direction along the surface. Forces from the adjacent flatter surfaces will repel charges back to the point. Due to this the sharper edge object will have a higher charge density as the charges are packed closer together. There is an electric field in between the two oppositely charged objects, as seen in Fig(2) it is represented by the blue lines, notice that the lines are closer packed together near the sharp object this refers to the high electric field in that region due to the high charge density there. Due to a phenomenon called the corona discharge effect, the air surrounding the strong electric field will be ionised as the electrons are attracted to the sharp object and at times resulting in blue sparks, thus leaving the air positively charged, and based on Coulomb's law, the positive ions will be attracted to the negatively charged object [10]. In this process it will collide with the neutral atoms pushing it towards the negatively charged object creating a force we know as wind. Elaboration on how mass of the gas particles affect the thrust is displayed through mathematics in the next section. [2]

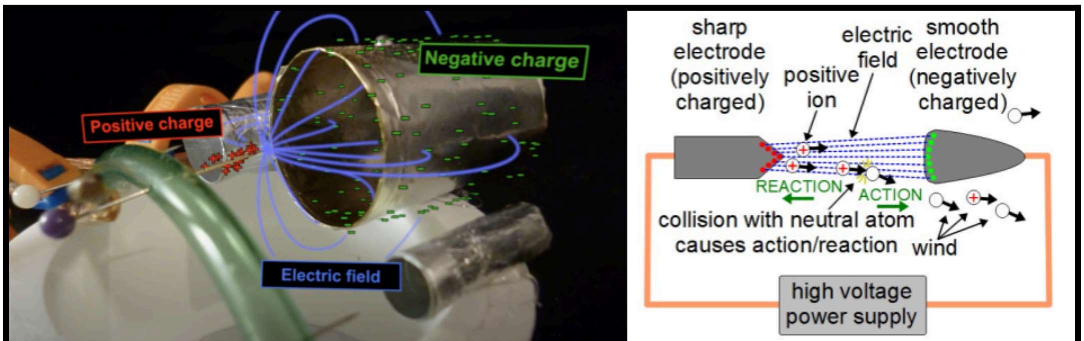


***Figure 3**: DIY Ion Thruster Concept setup* [2] [7]

1.5 Theoretical Expected Data

Given the DIY nature of this experiment execution. We can first identify the theoretical/expected data and observe how our experimental data deviates from it. Through this we can determine whether or not our DIY apparatus is appropriate for real-world comparisons. Thrust is commonly defined as the transfer of momentum per unit time [11]:

$$F = \frac{\mathrm{d}p}{\mathrm{d}t} \tag{1}$$

If we do a theoretical analysis we would assume that the ions in the ion thruster are accelerated mutually by coulomb repulsion based on the Sect(1.4). With this we can assume that the magnitude of the momentum generated by the device is equal to the ion's. Of which we can get the total reachable kinetic energy, *KE*, of our transformer that can generate 10kV in terms of electron volts, ˆ, to be:

$$\text{Possible } KE = 10000 \times 1 = 10\,\mathrm{keV} \tag{2}$$

Deriving the equation of kinetic energy, *KE*, we can then calculate the velocity of each ion that is accelerated by the ion thruster. With its mass, *m*, being the molar mass of the gas times the mass of the proton in energy units from Einstein's energy and mass relation, $E = mc^2$, to derive it, a mass of 1 dalton, $1u$, is equivalent to an energy of 931.5 MeV/c^2 [12] [13].

$$\begin{aligned} KE &= \frac{1}{2}mv^2 \\ v^2 &= \frac{2KE}{m} \\ v &= \sqrt{\frac{2KE}{m}} \end{aligned} \tag{3}$$

<u>**Example calculation** of theoretical velocity of Oxygen (O_2) gas with molar mass of 32.00 *u* [8]</u>**:**

$$\begin{aligned} v &= \sqrt{\frac{2 \times 10000\,\mathrm{eV}}{32.00 \times (931 \times 10^6)\,\mathrm{eV}/c^2}} \\ &= 8.193419 \times 10^{-4} c \end{aligned} \tag{4}$$

To convert to velocity from speed of light, *c*, we have the velocity of Oxygen (O_2) to be [13]:

$$\begin{aligned} v &= 8.193419 \times 10^{-4} \times 3.00 \times 10^8 \\ &= 245802.57\,\mathrm{m\,s^{-1}} \end{aligned} \tag{5}$$

By definition of the derivative of momentum, thrust is equal to the mass and velocity of one gas ion multiplied by the number of possible ions per second from which could derive from the maximum current generated by the flyback transformer. Note that any transformer will do but this is the only type of transformer I have in hand. Thus, we could achieve the equation:

$$\begin{aligned} F &= \frac{\mathrm{d}p}{\mathrm{d}t} \\ &= m_{\mathrm{O_2}} \times v_{\mathrm{O_2}} \times \frac{\mathrm{d}n}{\mathrm{d}t} \\ &= m_{\mathrm{O_2}} \times v_{\mathrm{O_2}} \times I \times C \end{aligned} \tag{6}$$

Where 1 coulomb, 1 C, is the number of charges per second with 1 Ampere current, which is approximately equal to the charge on 6.24×10^{18} protons [14]. Converting everything into SI units through the MKS system, we have a 1 AMU value of 1.66×10^{-27} kg and its current, I, at 100% efficiency given from the data sheet of our flyback transformer to be 30 mA [13].

Thus, <u>**Example calculation** of theoretical thrust of Oxygen (O_2) gas with molar mass data from [8]</u>**:**

$$\begin{aligned} F &= 32.00 \times 1.66 \times 10^{-27} \times (245802.57)\,\mathrm{ms^{-1}} \times (30 \times 10^{-3})\,\mathrm{A} \times 6.24 \times 10^{18} \\ &= 0.002444\,\mathrm{N} \end{aligned} \tag{7}$$

Repeating the methods above with various gases of different molar mass:

Gas	Molar mass (*u*) [8]	Theoretical Thrust (μN)
Hydrogen (H_2)	2.00	611.07
Helium (He)	4.00	864.18
Nitrogen (N_2)	28.00	2286.41
Oxygen (O_2)	32.00	2444.28
Carbon Dioxide (CO_2)	44.01	2866.49

***Table 1**: Thrust Generated By Theoretical Calculation through Sect(1.5)*

Thus, through a theoretical approach it demonstrates how we could hypothesise our results as it shows in Tab(1) that as the mass of the gas being ionised increases higher thrust is generated. This supports the hypothesis in Sect(1.2).

1.6 Acquisition of The Gases Through Stoichiometry

As acquiring high purity gases is incredibly costly. This experiment took a stoichiometric approach. The chemical equations are balanced and produced through common household liquid oxides and various kitchen ingredients that act as reactants [8]. The gas collection strategy is shown in the methodology section in Sect(3.3):

$$\mathrm{O_2\,(Oxygen)} = \underbrace{2\mathrm{H_2O_2}\,(l)}_{\text{Hydrogen Peroxide}} + \text{Yeast} \longrightarrow \underbrace{2\mathrm{H_2O}\,(l)}_{\text{Water}} + \underbrace{\mathrm{O_2}\,(g)}_{\text{Oxygen}} \tag{8}$$

$$\mathrm{CO_2\,(Carbon\ Dioxide)} = \underbrace{\mathrm{NaHCO_3}\,(s)}_{\text{Baking Soda}} + \underbrace{\mathrm{HC_2H_3O_2}\,(l)}_{\text{Vinegar}} \longrightarrow \underbrace{\mathrm{NaC_2H_3O_2}\,(aq)}_{\text{Sodium Acetate Trihydrate}} + \underbrace{\mathrm{H_2O}\,(l)}_{\text{Water}} + \underbrace{\mathrm{CO_2}\,(g)}_{\text{Carbon Dioxide}} \tag{9}$$

$$\mathrm{H_2\,(Hydrogen)} = \underbrace{2\mathrm{Al}\,(s)}_{\text{Aluminium}} + \underbrace{2\mathrm{NaOH}\,(l)}_{\text{Sodium Hydroxide}} + \underbrace{2\mathrm{H_2O}\,(l)}_{\text{Water}} \longrightarrow \underbrace{2\mathrm{NaAlO_2}\,(aq)}_{\text{Sodium Aluminate}} + \underbrace{3\mathrm{H_2}\,(l)}_{\text{Hydrogen}} \tag{10}$$

$$\mathrm{N_2\,(Nitrogen)} = \text{Air approximately 78\% Nitrogen} \tag{11}$$

$$\mathrm{He\,(Helium)} = \text{Helium Balloons} \tag{12}$$

2 Variables

2.1 Independent Variable:

Molar mass of the gases (Oxygen (O_2), Carbon Dioxide (CO_2), Helium (He), Hydrogen (H_2), Nitrogen (N_2)): The different molar mass of the gases that will be released close to the high electric field is the independent variable. This gas extrusion will be released at a constant rate through a valve. While there are other factors that may affect the thrust generated by an ion thruster, mass of the gas is the most obvious one, as based on the formula of kinetic energy the value of velocity may increase or decrease based on the value of mass, as shown in Sect(1.5).

2.2 Independent Variable:

The thrust generated: The value of the thrust generated will be the dependent variable in this investigation as it will vary depending on the molar mass of the gas being ionized, this can be seen through the kinetic value formula where the velocity of the ions will change and since force is a transfer of momentum the velocity measured by the anemometer will be used to see the difference in force from different gases Sect(1.5). As it is not possible for this variable to be recorded directly. It will be recorded indirectly using an anemometer with intervals of 5 seconds the velocity that is generated, and the thrust values will be calculated. Shown in Sect(4.2).

2.3 Independent Variable:

- Amount of gas released: An increase in the amount of gas within the high electric field region would mean an increase in the number of particles for the ions to collide with as it gets attracted to the oppositely charged metal. If there is an increase in the number of collisions it would possibly result in an increase in thrust. *Controlled using a valve close to the ionisation and high electric field region to give a constant release.*
- Distance between the sharp and the rounded object: As the gas is ionised due to the high electric field in the copper nail and later on attracted to the oppositely charged copper pipe. If the distance is increased this attraction will be weaker. *Controlled by setting the nail and the pipe at equal distance for all trials.*
- Time allotted: As the amount of gas theoretically will run out in 30s with constant outflow from the valve, recording will stop at 25s and record the velocity value every 5s. This is because an increase in the time will result in the lack of that particular gas present in the

system and the device will just ionise the air. *Controlled by using a stopwatch and record the value every 5 seconds*

- Environment (Pressure, temperature, and etc.): Gas may behave differently in different conditions such as the different temperature may result in gases accelerating faster. *Controlled by conducting the experiment in one sitting and in the same location*
- Voltage: As different voltages applied across the device will result in a weaker electric field for it to ionise the gas. *By using a transformer with constant voltage output, verified through high voltage multimeter*
- Current: Similar to voltage, varying the current would mean a different number of ions per second that is charged from the gas within the electric field. *Controlled by using a transformer with constant current output, verified through high voltage multimeter*

3 Methodology

3.1 Apparatus

1. **[5]** Hydrogen (H_2) filled party balloons
2. **[5]** Helium (He) filled party balloons
3. **[5]** Nitrogen (N_2) filled party balloons
4. **[5]** Oxygen (O_2) filled party balloons
5. **[5]** Carbon Dioxide (CO_2) filled party balloons
6. **[1]** Anemometer (± 0.1 m/s)
7. **[3]** 250ml Beaker (± 12.5 ml)
8. **[1]** Digital Scale (± 1 g)
9. **[1]** Ruler (± 0.05)
10. **[25]** Party balloons
11. **[1]** Transformer 10,000 volts @ 30ma (Ex. flyback transformer)
12. **[1]** Goggles
13. **[1]** High Voltage Electric resistant gloves
14. **[7]** Copper nail
15. **[1 m]** Copper pipe
16. **[1]** Solder wire and solder
17. **[1]** Digital scale (± 1 g)
18. **[1]** 12v DC power supply
19. **[1]** 2N3055 Transistor
20. **[3]** 220Ω & 22Ω Resistor
21. **[1]** DIY Ion Thruster [1]

3.2 Safety Considerations

- Refrain from running and eating around the experimental space
- Use high voltage gloves when handling the transformer
- Always disconnect the circuit from the power source before handling
- Do the experiment in an open environment
- Use respirator masks as high purity gas can be dangerous
- Use safety goggles
- Keep away from any conductive material
- Keep away from any water source
- Keep a dry workspace at all times
- Keep a fire extinguisher at standby incase the transformer or circuit short circuits

3.3 Experimental Procedure

Disclaimer: This procedure is under the assumption that the DIY ion thruster model has been made. Experimental setup Fig[4]. For reference this was done based on Makezine's design [1].

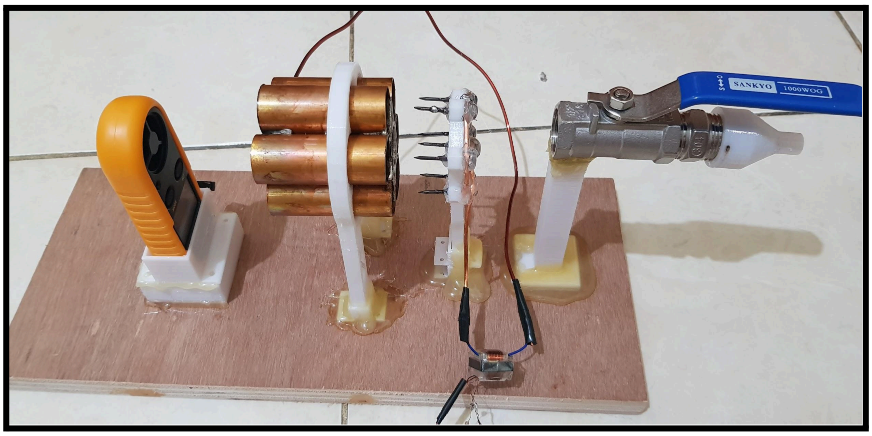


Figure 4: DIY Ion Thruster Experimental Setup

1. Prepare 5 balloons of the same color and label them for the corresponding gas that will be put inside.
2. For the gas reaction setup prepare plastic bottles and label them so there won't be any mishaps Fig(5). Production of the gas can be done through the chemical reactions in Sect(1.6).
3. For the circuitry a 10kV @ 30mA flyback transformer is utilised as it is the only one that I have on hand. Full circuit can be done through a PCB. Full circuit diagram refer to Fig(6)
4. Initially, attach an air filled balloon to the valve. Open the valve slowly until a small air is released, keep air output low so it won't affect the anemometer reading. Mark that valve point using a marker.
5. Attach a balloon with one of the experimental gases into the valve throat. Fig(7)
6. Start the timer when the valve is opened to the marked point Step 5.
7. Record the air velocity output in the timer every 5 seconds until it reaches a total run time of 25 seconds for a total of 5 readings. Immediately close the valve so no gas is wasted.
8. If the balloon air is insufficient for a new trial, replace it with another balloon filled with the same gas and repeat the trial.
9. Repeat step 5 and 8 four more times
10. Repeat step 5 to 9 for every different type of experimental gas

Figure 5: Production of the gases

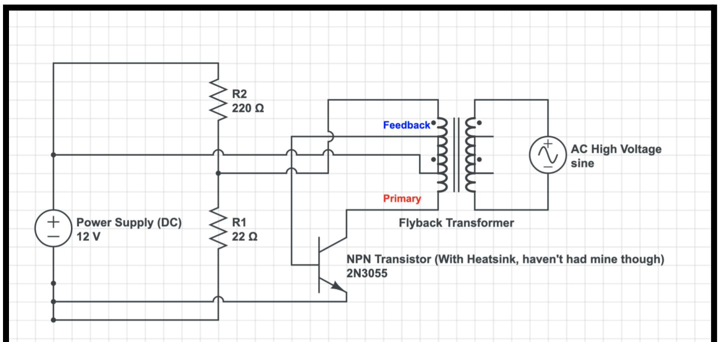


Figure 6: Circuit Diagram of Ion Thruster

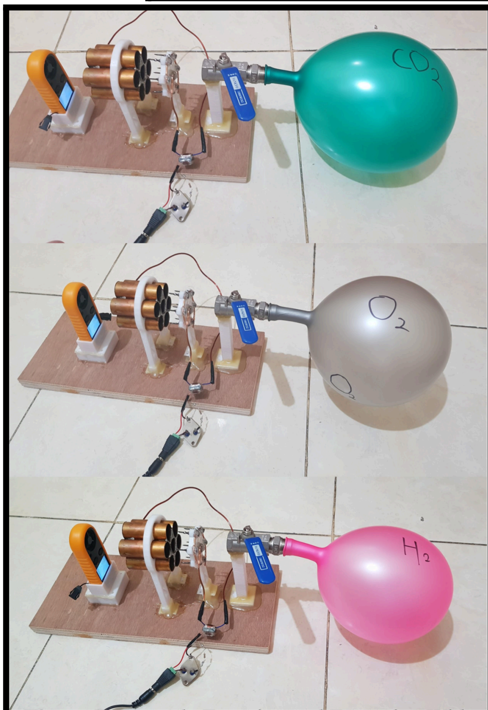


Figure 7: Example Experimental Execution With Anemometer Recording

4 Data

4.1 Raw Experimental Datas

Gas	Exhaust Velocity (m s^{-1})																								
	Trial 1					Trial 2					Trial 3					Trial 4					Trial 5				
	5s	10s	15s	20s	25s	5s	10s	15s	20s	25s	5s	10s	15s	20s	25s	5s	10s	15s	20s	25s	5s	10s	15s	20s	25s
Hydrogen (H_2)	2.70	2.70	2.80	2.80	2.80	2.80	2.80	2.80	2.80	2.90	2.80	2.80	2.80	2.80	2.80	2.70	2.80	2.80	2.80	2.80	2.70	2.70	2.80	2.80	2.80
Helium (He)	2.50	2.60	2.60	2.60	2.60	2.60	2.60	2.60	2.60	2.60	2.60	2.60	2.60	2.60	2.60	2.50	2.60	2.60	2.60	2.60	2.50	2.50	2.60	2.60	2.60
Nitrogen (N_2)	1.70	1.70	1.70	1.70	1.70	1.80	1.70	1.70	1.70	1.70	1.70	1.70	1.80	1.70	1.70	1.70	1.80	1.70	1.70	1.70	1.70	1.70	1.70	1.70	1.70
Oxygen (O_2)	1.60	1.60	1.60	1.70	1.70	1.70	1.60	1.60	1.70	1.70	1.60	1.70	1.70	1.70	1.70	1.60	1.60	1.60	1.60	1.70	1.60	1.60	1.70	1.70	1.70
Carbon Dioxide (CO_2)	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.40	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.40	1.50	1.50	1.50	1.50	1.50

***Table 2**: Raw Experimental Data With Time Intervals*

Gas	Stable Exhaust Velocity (m s^{-1}) for each trial					Average
	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	
Hydrogen (H_2)	2.76	2.82	2.80	2.78	2.76	2.78
Helium (He)	2.58	2.60	2.60	2.58	2.56	2.58
Nitrogen (N_2)	1.70	1.72	1.72	1.72	1.70	1.71
Oxygen (O_2)	1.64	1.66	1.68	1.62	1.66	1.65
Carbon Dioxide (CO_2)	1.50	1.48	1.50	1.48	1.50	1.49

***Table 3**: Raw Experimental Averaged Per Trial From The Time Intervals*

4.2 Processed Experimental Data

To calculate the force generated by the wind velocity data from the anemometer we can use $F = ma$. Assuming that the experiment is done under vacuum and the air accelerated through the ion thruster is purely only one type of gas, its density can be determined [9]. The area of the anemometer is also measured. Thus the mass of air is simply a density times area. Acceleration can be found by squaring our velocity value. Thus we have a general equation in the MKS system [13]:

$$F = \text{area} \times \text{density} \times (\text{velocity})^2 \tag{13}$$

The anemometer have a radius of 0.0125m (± 0.00050m), thus area is:

$$\begin{aligned} \text{Area} &= 0.0125^2 \times \pi \\ &= 4.908 \times 10^{-4}\,\mathbf{m}^2 \end{aligned} \tag{14}$$

<u>Example calculation displaying the experimental thrust of Hydrogen through</u> Eq(13):

$$\begin{aligned} F &= 4.908 \times 10^{-4} \times 0.0899 \times 2.78^2 \\ &= 0.000340995\,\text{N} \\ &\approx 341.00\,\mu\text{N} \end{aligned} \tag{15}$$

Calculating the other values we have:

Gas	Molar mass (g mol^{-1}) [8]	Average Exhaust Velocity (m s^{-1}) Tab(3)	Gas Density at STP (kg m^{-3}) [9]	Experimental Thrust (μN)
Hydrogen (H_2)	2.00	2.78	0.0899	341.00
Helium (He)	4.00	2.58	0.18	588.05
Nitrogen (N_2)	28.00	1.71	1.25	1793.94
Oxygen (O_2)	32.00	1.65	1.43	1910.77
Carbon Dioxide (CO_2)	44.01	1.49	1.96	2135.67

***Table 4**: Thrust Generated By Experimental Ion Thruster*

4.3 Calculation of Error and Uncertainty

The average percentage uncertainty of the velocity recorded across all time intervals was found by finding the percentage uncertainty and subsequently dividing them by 5. Example calculation displaying uncertainty of the trial 1 velocity for Hydrogen (H_2) gas in Tab(2).

$$\frac{\Sigma\left(\frac{0.1}{v_{5s}} \times 100\%\right)+\ldots+\left(\frac{0.1}{v_{25s}} \times 100\%\right)}{5} = \frac{\left(\frac{0.1}{2.7} \times 100\%\right) + \left(\frac{0.1}{2.7} \times 100\%\right) + \left(\frac{0.1}{2.8} \times 100\%\right) + \left(\frac{0.1}{2.8} \times 100\%\right) + \left(\frac{0.1}{2.8} \times 100\%\right)}{5} = \pm 3.6243386\% \tag{15}$$

Example calculation displaying uncertainty of the Trial 1 velocity for Hydrogen (H_2) gas in Tab(3).

$$\frac{\Sigma(\Delta v_{t_1} + \Delta v_2 + \Delta v_3 + \Delta v_4 + \Delta v_5)}{5} = \frac{3.6243386\% + 3.5467980\% + 3.57142865 + 3.5978836\% + 3.6243386\%}{5} = 3.5929575\% \tag{16}$$

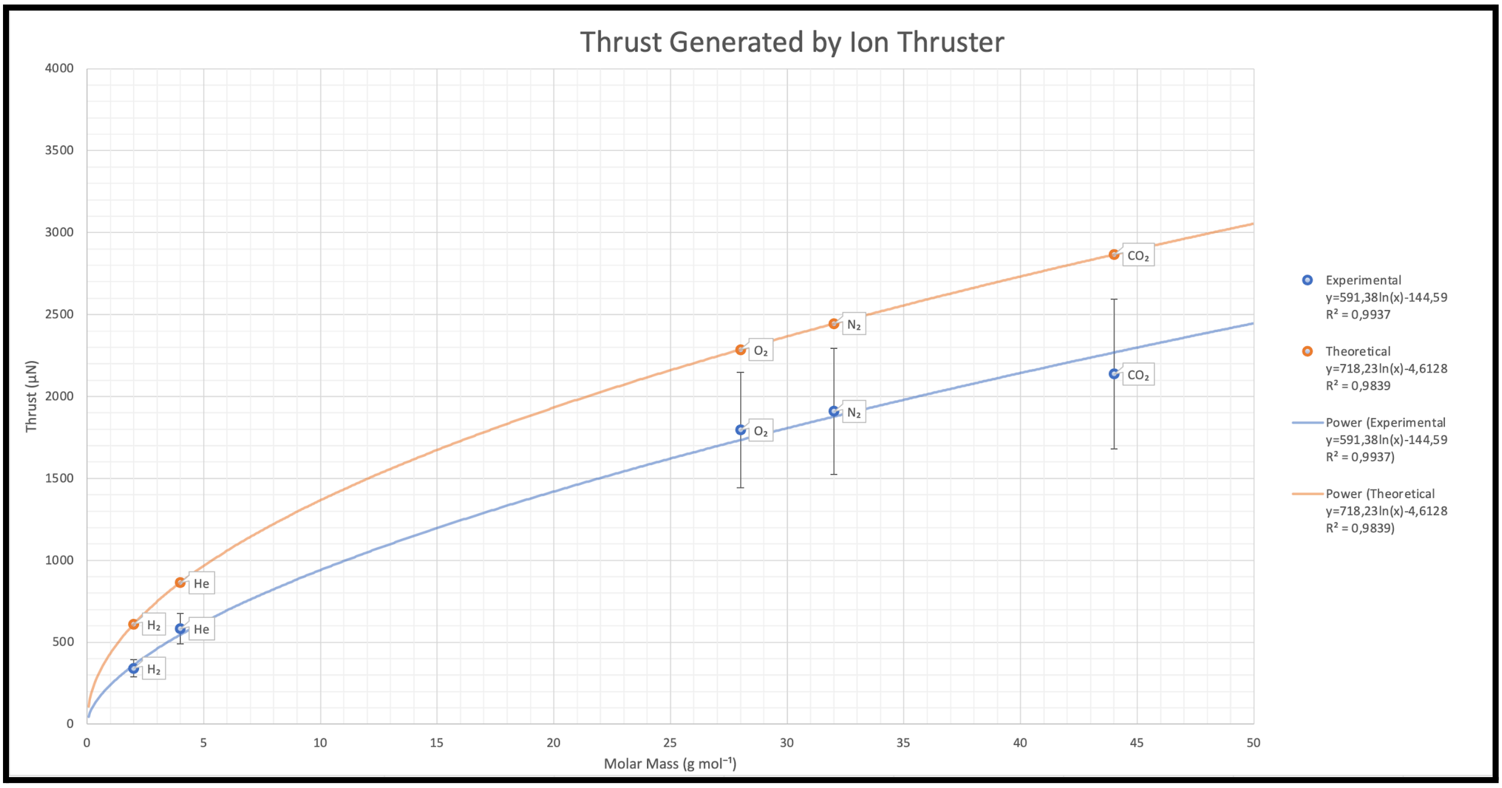
Example calculation displaying uncertainty of the final thrust value for Hydrogen (H_2) gas in Tab(4)

$$\begin{aligned} F &= \text{area} \times \text{density} \times (\text{velocity})^2 \\ &= \left(0.0125^2\left(\pm\frac{0.0005}{0.0125} \times 100\% \times 2\right) \times \pi\right) \times 0.0899 \times 2.78^2(\pm 3.5929575\%) \\ &= 0.0003410509\text{N } (\pm 15.185915\%) \\ &= 341\mu\text{N } (\pm 15.185915\%) \end{aligned} \tag{17}$$

Gas	Molar mass (g mol^{-1}) [8]	Experimental Thrust (μN) Tab(4)	Percentage Uncertainty Sect(4.5)
Hydrogen (H_2)	2.00	341.00	±15.19%
Helium (He)	4.00	588.05	±15.74%
Nitrogen (N_2)	28.00	1793.94	±19.69%
Oxygen (O_2)	32.00	1910.77	±20.12%
Carbon Dioxide (CO_2)	44.01	2135.67	±21.41%

***Table 5**: Raw Experimental Averaged Per Trial From The Time Intervals*

4.4 Graphing The Results



***Graph 1**: Graphing Thrust Against The Gas Molar Mass With Data Tab[1 & 4]*

The graph displays how varying molar mass affects the thrust generated from the ion thruster. It displays both the experimental and theoretical values from Tab[1 & 4], it shows how our theoretical values are higher. The graph also shows a positive correlation between the variables and gives a strong power trend. This strong correlation is also further supported by the high R^2 value.

4.5 Validity of DIY Ion Thruster By Efficiency Analysis

Assuming our theoretical values are when the ion thruster system is 100% efficient; no energy or voltage loss in transmission. We can calculate how efficient our experimental setup is relative to what we would've expected the values to be. Note that NASA states that they're ion thrusters are approximately 90% efficient [15]. Thus, anywhere nearing this value would mean that our experiment conclusion can be applicable as the yield is similar, although at a much smaller scale. **Example calculation** of efficiency of Oxygen (O_2) gas with data from Tab[1 & 4]**:**

$$\begin{aligned} \text{Percentage Efficiency} &= \frac{\text{Experimental}}{\text{Theoretical}} \times 100\% \\ &= \frac{1910.77}{2444.28} \times 100\% \\ &= 78.17\% \end{aligned} \tag{17}$$

Gas	Molar mass ($g\ mol^{-1}$)	Experimental Thrust (μN)	Theoretical Thrust (μN)	Percentage Efficiency (%)
Hydrogen (H_2)	2.00	341.00	611.07	55.80
Helium (He)	4.00	588.05	864.18	68.05
Nitrogen (N_2)	28.00	1793.94	2286.41	78.46
Oxygen (O_2)	32.00	1910.77	2444.28	78.17
Carbon Dioxide (CO_2)	44.01	2135.67	2866.49	74.50

***Table 6**: Experimental Device Efficiency With Data Tab[1 & 4]*

5 Analysis

5.1 Data Evaluation

Based on raw data observation in Tab(1) we can see that the values are relatively consistent. We can already see a trend, they only slightly vary by the uncertainty value of ($\pm$ 0.1 $m\ s^{-1}$) as they progress in different time intervals. Though when we averaged the five time intervals into a singular trial we can see that the values are consistent and do not diverge too far. From our raw velocity data in Tab(2) already see a trend with Hydrogen (H_2) having the highest velocity followed by Helium (He), Nitrogen (N_2), Oxygen (O_2), and lastly Carbon Dioxide (CO_2). Oppositely, in Tab(3) we see that Carbon Dioxide (CO_2) has the highest thrust followed by Oxygen (O_2), Nitrogen (N_2), Helium (He), and Hydrogen (H_2). A similar trend can also be seen in from the theoretical calculated data in Tab(1). Observing Graph(1) we can see that the experimental has a lower value than the theoretical. Though we can see that both follow a power function and as the molar mass of a gas increases so will its thrust generated by the ion thruster. In Tab(5) we can see the difference in efficiency on which the theoretical values are assumed to be 100% efficient, meaning no loss in energy or voltage drops. Compared to the experimental values of which the DIY ion thruster experiment operates within an efficiency of 50% - 80%.

5.2 Conclusion

Looking into the results of the experiments conducted above, I can safely say that the hypothesis can be accepted. There is sufficient evidence that if a gas of lower molar mass is ionised and accelerated through an electric field in an ion thruster it would result in a higher exhaust velocity due to its light mass, but result in a lower thrust due to its density. Oppositely, a gas of higher molar mass would result in a lower velocity, but due to its high density at the same pressure it will produce a higher thrust. This relation can be defined in terms momentum which is $p = mv$ and consequently thrust can be defined as the transfer of momentum per unit time given by $F = \frac{\mathrm{d}p}{\mathrm{d}t}$. Going over our velocity data, our hypothesis states that gas of lower mass will yield higher velocity and gas of higher mass will yield lower velocity. If we refer to the processed data in Tab(3) this is proven to be true with Hydrogen (H_2) as the lightest diatomic gas with molar mass of 2.02 g mol^{-1} and an average recorded exhaust velocity of 2.78 m s^{-1} followed by Helium (He) as the second lightest gas. As the molar mass difference between Helium (He) and Nitrogen (N_2) gas widens we can also see a significant drop in its exhaust velocity. Followed by Oxygen (O_2) gas and lastly Carbon Dioxide (CO_2) gas the lowest. Intuitively, this would make sense as a gas of higher mass would certainly take a longer time to accelerate over the same distance towards the anemometer. Hence, a gas of higher molar mass would be expected to have a lower velocity. By kinetic theory our transformer could supply 10 keV for all the experimental setup meaning that it is controlled, furthermore as they are tested in the same location and environment, temperature difference would not increase the velocity of a particular gas. Thus, for all the experimental gases it will have the same kinetic energy. It is given that $KE = \frac{1}{2}mv^2$ thus it would be reasonable for lighter gas molecules such as Hydrogen (H_2) to have a greater velocity than Oxygen (O_2) molecules. Example we could write it in terms of an kinetic energy expression:

$$\begin{aligned} KE\,(\mathrm{H_2}) &= KE\,(\mathrm{O_2}) \\ \frac{1}{2}m(\mathrm{H_2})v^2 &= \frac{1}{2}m(\mathrm{O_2})v^2 \\ (2.02)v_{\mathrm{H_2}}^2 &= (32.00)v_{\mathrm{O_2}}^2 \\ \frac{v_{\mathrm{H_2}}}{v_{\mathrm{O_2}}} &= \sqrt{\frac{32.00}{2.02}} \\ \frac{v_{\mathrm{H_2}}}{v_{\mathrm{O_2}}} &= 4 \end{aligned}$$

Thus, we should expect a hydrogen (H_2) gas accelerated through the ion thruster to have an exhaust velocity of 4 times greater than oxygen (O_2) gas. However, our recorded values are less this is possibly due to the fact that the experiment was not conducted in vacuum thus air molecules could affect the system. But, understand that this is only velocity, and our paper is focusing on thrust generated by gas of different molar mass. By momentum with $p = mv$, we would expect gas of higher mass to have a higher momentum due to the difference in molar mass is more significant than its velocity. Furthermore, we should also take into account the gas density and how many molecules could each collide between the cathode and the anode where the gas is ionised and accelerated. From the thrust experimental data in Tab(4) we can see an opposite trend to its velocity, with gases of higher molar mass having higher thrust. Which should have been expected as the basis of force can be defined as the transfer of momentum per unit time and since gas of higher mass will yield higher momentum consequently so will its thrust. Thus, it would be safe to say that concludingly from the experimental data I confidently say that gases of higher molar mass will yield a higher thrust compared to gas of lower molar mass. Further verifying the claim of our hypothesis.

Similar results can be seen in our theoretical or expected data Tab(1). Through the derivation of the kinetic energy equation and the definition of thrust as momentum per unit time. With the assumption that there is no energy loss or voltage drops due to resistance in the copper wires. We reached the same conclusion where gases of higher molar mass having higher theoretical thrust than gases of lower molar masses. Though notice that even if they both follow the same trend our theoretical estimation yields a higher thrust value than our experimental which can be seen in Tab(5) if we calculate for my experimental device efficiency we could average them to about 70% efficient with the experimental values of Hydrogen (H_2) being only 55.80% efficient. This would be expected as the experiment was not conducted under vacuum thus air molecules that consist of Nitrogen (N_2) or Oxygen (O_2) gas could potentially deviate the thrust of the Hydrogen (H_2) gas. Though the relatively high efficiency of the experimental device further assures that the data achieved and the conclusion extracted that gas of higher molar mass will yield higher thrust is also true for life-sized ion thrusters in NASA. Our experimental data is expected to have a lower value as previously mentioned; our theoretical value assumes that all electrical power generated by the flyback transformer is used for ionisation. Which is not the case as there are many places where power could go to. A single 10keV electron is not likely to efficiently ionise all the gas molecules within the boundary as it slowly decelerates. While the theory of cascade ionisation certainly happened, it's not even close to being 100% efficient. Lastly, if we go over Graph(1) we can see the trend more clearly with the difference between theoretical and experimental value clearly visible, with the trend following a power function and as the molar mass increases so will the thrust generated.

Conclusively, this would most likely be the reason as to why Xenon (Xe) is the most common gas used as fuel by ion thrusters. Not only is it the heaviest inert gas, it's also non-radioactive. The heavier mass would allow for it to have a denser packing at less pressure. Since mass is one of the limiting factors in ion thruster application today, having a denser gas which would yield a higher thrust is certainly ideal. Furthermore, as stated before heavier mass also allows for more momentum provided to the entire system, and consequently a higher thrust. While it may be true that gas of heavier mass may take longer time to accelerate, proven by the experimental velocity datas, it would also mean that more momentum is exerted on the spacecraft, which is advantageous as the thrust of ion thrusters compounds over time.

5.3 Strengths

My method of investigation on how gas of different molar masses affect the thrust generated by an ion thruster is effective as I created a real model with actual applications of ion thrusters. The experiments also show a low level of error due to the low uncertainty values of my apparatus as shown in Tab(5) and as the line of best fit passes through the vertical error bars, the data is certainly accurate. While it is true that it diverges from 10% to 20%, it is largely contributed by the uncertainty in the area measurement in Eq(14) which could be improved through the use of a vernier caliper. Furthermore, the recorded data was also not manipulated or derived numerous times thus it would minimize the likelihood of the data losing any sort of importance due to the numerous rounding offs. The multiple number of trials and the recorded data per 5 second intervals also made the experimental data more accurate. With the final graph trend in Graph(1) having a high correlation value in both experimental and theoretical with $R^2 = 0.984$ and $R^2 = 0.994$ respectively. So, my resulting data is confidently consistent and accurate to my hypothesis and physic theories.

5.4 Weaknesses

As my experimental ion thruster setup is DIY the efficiency or power losses is significantly reduced. Compared to a professional ion thruster it can be seen clearly. By NASA data their experimental and theoretical values only diverge by 10% meaning that their devices are 90% efficient compared to mine which are only 60%-70% efficient. The gas density that was used in our experimental processed calculations are also values at STP thus there should be an inaccuracy in the actual thrust value as my experiments were not conducted under STP. The derivation using $F = ma$ to actually calculate for thrust from velocity could also be a factor of inaccuracy as the values are already rounded off once. The overall uncertainty values are also quite high ranging from 15% - 20%.

5.5 Extension

Explore a wider variety of gas fuels, possibly testing the inert gases. Radioactivity levels could also be monitored as in space an ion aircraft could be susceptible to it. Experimental data could've been conducted under vacuum to simulate a space environment.

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