Mathematical Modelling, Simulation, and Optimal Vaccination Rate For The COVID-19 Virus in Jakarta

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Nomenclature

Symbol	Definition
N	Total Population
S	Susceptible Population
I	Infected Population
R	Recovered Population
E	Exposed Population
D	Death Population
V	Vaccinated Population
α	Virus Death Proportionality Constant
β	Transmission Proportionality Constant
γ	Recovery Proportionality Constant
μ	Birth Proportionality Constant
δ	Exposed to Infected Proportionality Constant
ν	Vaccination Proportionality Constant
η	Natural Death Proportionality Constant

Abbreviations

SIR	Susceptible-Infected-Recovered
ODE	Ordinary Differential Equations
SEIR	Susceptible-Exposed-Infected-Recovered
SEIRDV	Susceptible-Exposed-Infected-Recovered-Death-Vaccinated
H.O.T	Higher Order Terms
ODE45	**MATLAB** Differential Solver (medium order method)
Eigs()	**MATLAB** Eigenvalue Solver

1 Introduction

1.1 Background Study

COVID-19 is the new lethal virus of the 21st century. It was initially identified in Wuhan, China, in late 2019. As the characteristics of the virus are distinct from others, scientists are having trouble developing vaccines and cures. Even to this day, we have yet to discover a vaccine with 100% potency. The virus can spread in multiple ways, from an infected person's mouth, nose, or even small liquid particles when someone sneezes or speaks [1]. Although most people who carry the COVID-19 virus only display minor symptoms, the moment it reaches a critical point, COVID-19 can cause severe illness and, in most cases, death. One of the current treatments is either quarantine or let our body's immune system dispel the virus. As of 19 November 2020, the World Health Organization (WHO) reported a total of 55,659,785 confirmed cases of COVID-19, including 1,338,769 deaths [9]. With a 3 - 4% mortality rate, it has been the deadly highlights of yet another tragedy of 2020 [2].

Mathematical modeling has become an effective and vital tool for us to understand and predict infectious disease dynamics. It has also become a practical toolkit in helping government agencies and healthcare systems to predict future possibilities. This paper aims to understand the dynamics of the COVID-19 virus spread, particularly in Jakarta and propose an optimal vaccination rate. We will begin by considering local datasets from national public websites and evaluate a system of ODEs (ordinary differential equations) to describe the outbreak. The basic model is categorized into three compartmental populations: the Susceptible, S, the Infected, I, and the Recovered, R. We will then simulate the system of ODEs with our approximated parameters. This paper will also consider other model variations and optimal vaccination rates to reduce mortality rates and flatten the infected curve.

This paper is also a unique way to understand how mathematical models can represent a real-life scenario. I have mainly been interested in identifying possible optimal controls for the COVID-19, a possible extension to this paper. Modeling and simulating data is also of interest to me as ever since I was in primary, data analysis and modeling have repeatedly been introduced. Yet, in most cases the data are not related to any current real-life problems. This paper opens the opportunity for me to broaden my understanding of data, especially in its applications with technology.

1.2 Goals

The goals of this paper are as follows:

1. Introduce basic mathematical model to describe the dynamics of the COVID-19 (SIR)

2. Estimate parameters based on existing local data and simulate the dynamics
3. Suggest a different variation of the dynamic model for COVID-19 (SEIRDV)
4. Develop Jacobian matrix through taylor series expansion (SEIR)
5. Solve for eigenvalues to describe COVID-19 stability in Jakarta (SEIR)
6. Simulate data with presence of vaccination (SEIRDV)
7. Suggest an optimal vaccination rate and voronoi diagrams placement of vaccination stalls. (SEIRDV)

2 Basic Dynamics & Mathematical Model

2.1 Standard Model (SIR Model)

At a standard level, the dynamics is described by the SIR (Susceptible-Infected-Recovery) epidemic model, a system of three ODEs, where the population of a certain group is categorized into three: Susceptible, Infected, and Recovered, denoted by *S, I and R* respectively [10]. With *R* being those who are leaving *I*, regardless whether or not they are cured or dead. With each group increasing/decreasing at a constant rate by proportionality constants of β for $S \rightarrow I$ and γ for $I \rightarrow R$. The total population, which can be assumed to be constant in that period, is given by $N = S + I + R$. A chain model can be made to visualise the dynamics with each as a function of time:

***Figure 1**: Standard Scheme for Susceptible-Infected-Recovered Model Eq(1)-(3)*

To model the dynamics we must first identify the rate equations of each category, that describes its change with respect to time. Considering the fact that the initial population is in *S(t)* which will decrease as they come into contact with the COVID-19 infectives at a proportional rate constant of infection β. This means that the change of population in *S(t)* is proportional to the negative product of *S(t)* and *I(t)*, at rate β [11]. Thus:

$$\frac{dS(t)}{dt} = -\beta\frac{S(t)I(t)}{N} \qquad \textbf{(1)}$$

We know the increase in the *I(t)* population over time, though we must also consider that the *I(t)* population is moved to *R(t)* at a proportional rate constant γ. Thus we must consider:

1) Population leaving from *S(t)* is proportional to population joining *I(t)*, thus the population of *I(t)* at a given time is increasing by, $\beta\frac{S(t)I(t)}{N}$.

2) Population leaving from *I(t)* and joining *R(t)*, reducing *I(t)* at $-\gamma I(t)$.

Thus, *I(t)* is:

$$\frac{dI(t)}{dt} = \beta\frac{S(t)I(t)}{N} - \gamma I(t) \qquad \textbf{(2)}$$

Lastly, the equation that describes the population of *R(t)* is based on the number of individuals recovered from COVID-19 at a rate of γ. Thus, it's the product of γ and *I(t)*:

$$\frac{dR(t)}{dt} = \gamma I(t) \qquad \textbf{(3)}$$

Fig(1), visualises the relationship between these three rate equations. This standard model would model the COVID-19 trend in Jakarta for suitable proportionality parameters β and γ. With initial conditions *S(0), I(0),* and *R(0)*. With the aim to approximate and predict the COVID-19 presence in Jakarta. We now have three ordinary differential equations, Eq(1)-(3), which forms an SIR compartmental equation [11]:

$$\begin{cases} \frac{dS(t)}{dt} = -\beta\frac{S(t)I(t)}{N} \\ \frac{dI(t)}{dt} = \beta\frac{S(t)I(t)}{N} - \gamma I(t) \\ \frac{dR(t)}{dt} = \gamma I(t) \end{cases} \tag{4}$$

2.2 Estimating Parameters & Testing of simulation VS real data

We have shown that, *N* = *S* + *I* + *R*, all dependent on time, *t*. *N* which gives the total population, in this case, the population of Jakarta 10.9 million [3]. With initial scenarios where there are insignificant initial infectives, we could approximate *I(0)* + *R(0)* ≈ *0*, thus initially we can assume that *S*(0) ≈ *N*, so through Eq(2), we can cancel *S* and *N,* and factor *I*:

$$\frac{\mathrm{d}I(t)}{\mathrm{d}t} = I(t)(\beta - \gamma) \tag{5}$$

Through integration of the differential equation Eq(5) we could achieve the following solution by expressing it with euler and *I(0)*. Through integration laws:

$$\begin{aligned} \int\left(\frac{1}{I(t)}\frac{\mathrm{d}I(t)}{\mathrm{d}t}\right)\mathrm{d}t &= \int(\beta-\gamma)\mathrm{d}t \\ \ln\left(I(t)\right) &= (\beta-\gamma)t + C \\ I(t) &= e^{(\beta-\gamma)t + C} \\ I(t) &= e^{(\beta-\gamma)t} \times e^{C} \end{aligned} \tag{6}$$

Where *C* is denoted as the integration constant. Evaluating at *t* = 0, *I(0)*:

$$\begin{aligned} I(0) &= e^{(\beta-\gamma)0} \times e^{C} \\ I(0) &= e^{C} \end{aligned} \tag{7}$$

Thus substituting Eq(7) to Eq(6), our infected populace, *I(t)*, as a function of time, *t*:

$$I(t) = I(0) \times e^{(\beta-\gamma)} \tag{8}$$

Initially the infected populace, *I*, will grow exponentially, if we were to denote *m* = β - γ, *m* is a constant term that shows the difference between infection and recovery rate, thus replacing Eq(6) and linearizing the equation for plot data applying log rule we have:

$$\begin{aligned} I(t) &= I(0) \times e^{mt} \\ \ln\left(I(t)\right) &= \ln\left(e^{mt}\right) + \ln\left(I(0)\right) \\ \ln\left(I(t)\right) &= mt + \ln\left(I(0)\right) \end{aligned} \tag{9}$$

As *m* is β - γ, we can calculate γ by manipulating Eq(3), noting that derivative is simply rate of change or gradient, by replacing the time derivative as gradient between days:

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y_2-y_1}{x_2-x_1} = \frac{y(x+1)-y(x)}{(x+1)-x} \Rightarrow \frac{\mathrm{d}R}{\mathrm{d}t} = \frac{R(t+1)-R(t)}{(t+1)-1} \tag{10}$$

Thus through Eq(3), we can substitute by Eq(10), with our *t* difference being 1:

$$\begin{aligned}\frac{R(t+1)-R(t)}{(t+1)-t} &= \gamma I(t)\\ R(t+1)-R(t) &= \gamma I(t)\\ \frac{R(t+1)-R(t)}{I(t)} &= \gamma\end{aligned} \tag{11}$$

Considering recent Jakarta data [4] we can estimate the constants, by linearizing the exponential data through Eq(9) to calculate *m* and to calculate each γ value using Eq(11), full calculation using excel table is given in Appx(1). Averaging the values we have:

Name	Symbol	Proportionality Constant Values
β - γ	*m*	0.05648026897
Transmission Rate	β	0.07913350836
Recovery Rate	γ	0.02265323939

Figure 2: Estimate Constants from COVID-19 Data Based on https://corona.jakarta.go.id/en

Thus, we can finally use **MATLAB** to integrate Eq(4) using **ODE45**, which is a **MATLAB** toolkit that utilises Euler's method for solving differential equations Fig(3) [5]. By utilising the real data estimates of β and γ in Fig(2). They are rounded to the lowest possible decimal to keep the estimation accurate. With initial numbers susceptible, infected and recovered being *S(0) = 2997, I(0) = 3, R(0) = 0* respectively. The choice of *S(0)* is an estimate from Appx(1) as the highest infected does not exceed 3000. Evaluating at population out of 3000:

```
clear all, close all, clc

beta = 0.079133508366/3000;
gamma = 0.02265323939;

tspan = [1:1:500];
y0 = [3000 3 0];
[t,y] = ode45(@(t,y)odefcn(y,A,B),tspan,y0);

function dydt = odefcn(y,beta,gamma)
  dydt = zeros(3,1);
  dydt(1) = -beta*y(1)*y(2);
  dydt(2) = beta*y(1)*y(2)-gamma*y(2);
  dydt(3) = gamma*y(2);
end
```

***Figure 3:** Simple MATLAB Code for SIR Model Simulation*

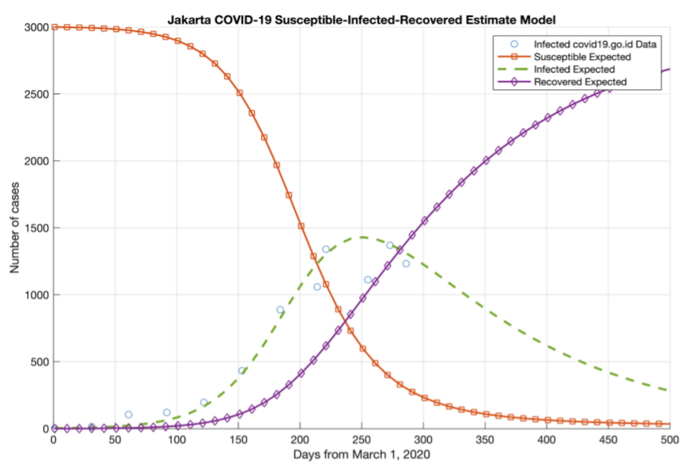


***Figure 4:** All groups, S(t), I(t), and R(t) with β = 0.07913350836, γ = 0.02265323939, S(0) = 3000, I(0) = 3, R(0) = 0 against the real data of confirmed infected cases based on https://corona.jakarta.go.id/en that occurred between March 1, 2020 (t =0) and July 14, 2021 (t =500)*

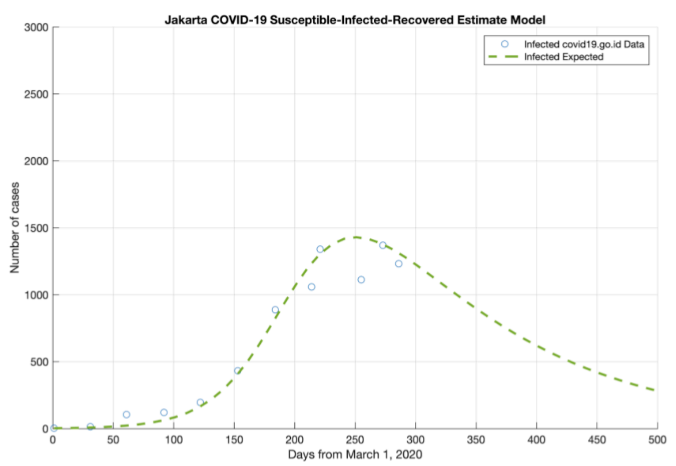


***Figure 5:** Only the infected compartment I(t) simulation parameters β = 0.07913350836, γ = 0.02265323939, S(0) = 3000, I(0) = 3, R(0) = 0 against the real data of confirmed infected cases based on https://corona.jakarta.go.id/en that occurred between March 1, 2020 (t =0) and July 14, 2021 (t =500)*

In Fig(4) we can see each compartment's progression through time and how they are linked together. The susceptible population will start from *S(0)* and gradually decreases to an asymptote, (y = 0), opposite to the recovered population, where it gradually increases to an asymptote, ($y = S(0) - I(500)$). Both have a point of inflection near their midpoint, as the rate of increase/decrease slowly reduces. At the same time, the infected group will rise. Though, it will never reach *S(0)*, forming a parabolic concave down curve, as the recovery rate is counteracting it. Once it reaches the maximum point, the recovery rate in the infected compartment will exceed the rate of infection as the infected population is decreasing. We can also see that the pandemic in Jakarta would most likely end by mid-2021, considering no interventions to the current norm, which would change the proportionality constants. Or if there are new developments in vaccines or cures. Given that we already have a model, (expected data), we must confirm that it follows the same trend as our data (observed data), so it could be used for predictions. Done by comparing the infected values in Fig(6). Only *I,* as its rate equation combines the other two rate equations and it is the only one that follows a normal distribution. Due to its small sample size, **GDC*:2-Sample t-test*** is applicable:

Day From March 1, 2020	Observed Data Fig(5) (u_0)	Expected Data [4] (u_e)
1	3	3
31	14	9
61	105	23
92	121	64
122	196	166
153	432	408
184	888	825
214	1059	1237
245	750	1426
273	1370	1377
286	1232	1312

Figure 6*: Data Comparison With Approximately 1 Month Intervals*

With our **hypothesis test**, for mean of observed, u_0, and expected, u_e:

$$\begin{aligned} H_0 &: u_o = u_e \\ H_1 &: u_o \neq u_e \end{aligned} \tag{12}$$

Testing the hypothesis in Eq(12) through **GDC: *2-Sample t-test*** at **5% significance level** we have:

$$\mathrm{PVal} = 0.80121 > 0.05 \tag{13}$$

As the PVal Eq(13) is not significant, there is insufficient evidence to reject the null hypothesis. Thus, it would mean that the distribution and mean of the observed and expected/simulated infected population are roughly equal. Thus, I can conclude that the estimated parameters in Fig(2) are applicable for further COVID-19 estimation in Jakarta.

2.3 Reproduction number

An important dimensionless value that plays a key role in understanding the SIR model is the basic reproduction number, R_0. Essentially, it determines how fast a disease will spread from one to another [12]. As in an SIR model we only have one infected compartment, thus the R_0 is simply given by $\frac{\beta}{\gamma}$. Assume that at t_0 the number for *I(0) = 1,* so small that in a large population it is neglectable. Thus we could estimate *S(0)* ≈ *N* thus considering Eq(2):

$$\frac{dI}{dt} = \frac{\beta SI}{N} - \gamma I \tag{14}$$

Cancelling our initial value as *I(0)* = 1, and *S* and *N* and estimates in Eq(14):

$$\frac{dI}{dt} = \beta - \gamma \tag{15}$$

Multiplying by γ, and expanding:

$$\frac{dI}{dt} = (\beta - \gamma)\frac{\gamma}{\gamma} \tag{16}$$

This proves that $\frac{\beta}{\gamma}$ is R_0 as the the rate of infectious will only increase if $R_0 > 1$

$$\frac{dI}{dt} = \gamma\left(\frac{\beta}{\gamma} - 1\right) = \gamma(R_0 - 1) \tag{17}$$

From our parameters we yield R_0 of:

$$R_0 = \frac{\beta}{\gamma} = \frac{0.07913350836}{0.02265323939} = 3.4932 \approx 3.5 \tag{18}$$

With estimated parameters Fig(2) based on local data in Jakarta [4] rounded off to one decimal, we can approximate it to be 3 ~ 4 people infected by COVID-19 from every one (infected). So, if $R_0 < 0$ the disease will eventually die out as less susceptibles are infected over time; whereas, if $R_0 > 1$ the infected population will increase and infect multiple susceptibles from a single infected. The R_0 in Eq(18) is valid as in Jakarta we are seeing an increase in the number of individuals infected. Though how valid is it? R_0 is the mean number of individuals infected, thus, using data collected from [4]; assuming it follows a normal distribution and due to its relatively low sample size we can use **GDC:*t-test***:

Approximated From 30 days Since March 1, 2020 Through Observation of Jakarta Data	
Approximated Infected From 1 Infectee	Frequency
1	12
2	11
3	5
4	2
5	0

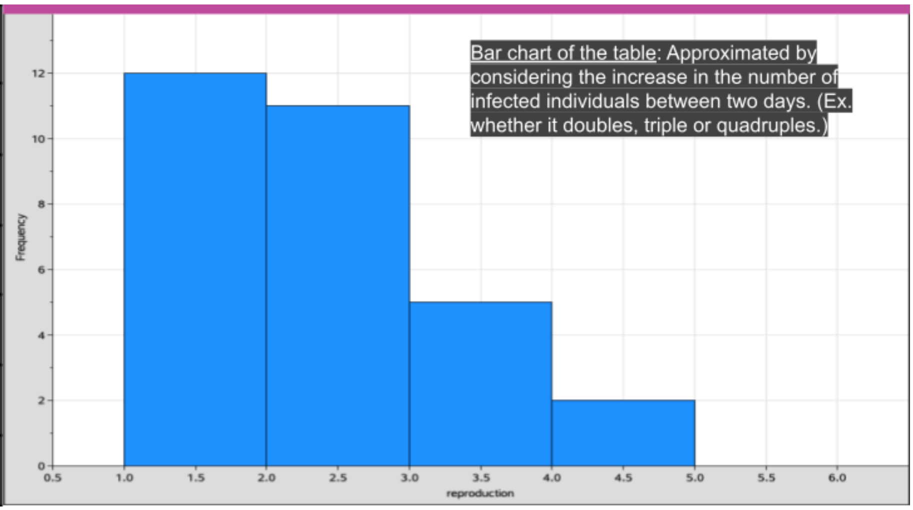


***Figure** 7: Approximated Reproduction Number From the First 30 Days of March [4]*

With our **hypothesis test**, testing for mean, where mean is given as R_0. Since I parsed the data into only 30 days, 30 samples Fig(7) ***t-test*** is applicable:

$$\begin{aligned} H_0 &: R_0 = 3.5 \\ H_1 &: R_0 \neq 3.5 \end{aligned} \tag{19}$$

Through **GDC:** ***t-test*** at **5% significance level**:

$$\mathrm{PVal} = 2.11 \times 10^{-10} < 0.05 \tag{20}$$

As the PVal Eq(20) is significant, there is sufficient evidence to reject the null hypothesis. So while it may be true that R_0 in Jakarta is greater than 1, Eq(18) is an inaccurate estimate. This is most likely due to the lack of other variables and parameters that will be introduced in the next section.

3 Complex Dynamics & Mathematical Model

3.1 Considering More Complex and Accurate Models (SEIR Model)

Most SIR models are usually too simple to model pandemic diseases, thus why the R_0, Eq(18), is inaccurate. There are typically many more parameters that must be taken into account. Such as, individuals moving from infected to recovered are assumed to be cured, and no death will occur due to the virus. We also did not take mortality or death and birth rates into account in the SIR model. We also assumed that susceptible individuals would directly move into the infected population, even though we know that in Jakarta, there is a period where susceptible individuals are exposed to COVID-19 before they are showing actual symptoms. Lastly, as we will be receiving vaccinations soon, so we must also consider that. Thus, Fig(8) will best describe our improved model. Where vaccine individuals are immediately immune [13].

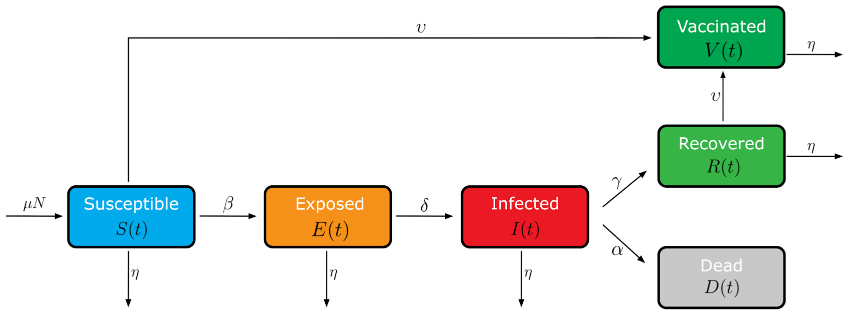


Figure 8: Improved Scheme for Susceptible-Infected-Recovered model to SEIRDV

In this section we have added more compartments to the SIR model from Sect(2). The system of equations, our ODEs that will describe this new model are:

$$\begin{cases} \frac{\mathrm{d}S}{\mathrm{d}t} &= \mu N - S\left(\beta \frac{I}{N} + \eta + \upsilon\right) \\ \frac{\mathrm{d}E}{\mathrm{d}t} &= \beta \frac{SI}{N} - E(\eta + \delta) \\ \frac{\mathrm{d}I}{\mathrm{d}t} &= \delta E - I(\eta + \gamma + \alpha) \\ \frac{\mathrm{d}R}{\mathrm{d}t} &= \gamma I - \upsilon R - \eta R \\ \frac{\mathrm{d}D}{\mathrm{d}t} &= \alpha I \\ \frac{\mathrm{d}V}{\mathrm{d}t} &= \upsilon R + \upsilon S \end{cases} \tag{21}$$

For our adapted model, our total population would be *N* = *S* + *E* + *I* + *R* + *D* + *V*. We have also added the parameters *μ* and *η* describing the natural birth and death rates, commonly called the vital dynamics. In this case, both parameters will have the same rate; thus, our simulation will not increase *N* or result in a population value of < 0. An exposed compartment is added, thus in this model, the chances of actually going into the infected compartment are reduced. The other new addition within this model is the death compartment, describing how individuals may die or recover due to the virus, which are branches from the infected population. The vaccination population is also included, directly branching from the susceptibles and recovered. Values will be taken from the previous simulation, additional parameters for the ones that are not from Jakarta's database will be estimated from other public databases Fig(9). To simplify the simulation, we will use percentage for the population rather than actual numbers, thus 1 denoting total population. For our new parameter α we can estimate it similar to Eq(10):

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y(x+1) - y(x)}{(x+1) - x} \Rightarrow \frac{\mathrm{d}D}{\mathrm{d}t} = \frac{D(t+1) - D(t)}{(t+1) - 1} \tag{22}$$

Since *t* difference is 1, we have:

$$\begin{aligned} \frac{D(t+1) - D(t)}{(t+1) - 1} &= \alpha I(t) \\ D(t+1) - D(t) &= \alpha I(t) \\ \frac{D(t+1) - D(t)}{I(t)} &= \alpha \end{aligned} \tag{23}$$

Thus, using spreadsheet and averaging the values our table of parameters is given in Fig(9). Full calculations are in Appx(1):

Variable / Parameter	Symbol	Value / Initial Value	Data Source
Birth Rate	μ	$\approx 6.25 \times 10^{-3}$	Indexmundi.com [14]
Natural Death Rate	η	$\approx 6.25 \times 10^{-3}$	Indexmundi.com [14]
Vaccination Rate	υ	$0 \sim 0.05$	approx.
Transmission Rate	β	0.07913350836	corona.jakarta.co.id [4]
Recovery Rate	γ	0.02265323939	Corona.jakarta.co.id [4]
E to I Rate	δ	1/7	WHO [15]
Disease Death Rate	α	0.01013392178	Corona.jakarta.co.id [4]

Figure 9: Estimate Constants from COVID-19 For The SEIRDV Model

Simulating the model with parameters in Fig(9) in the absence of vaccination through **MATLAB** with **ODE45** [5] with code adapted from Fig(3):

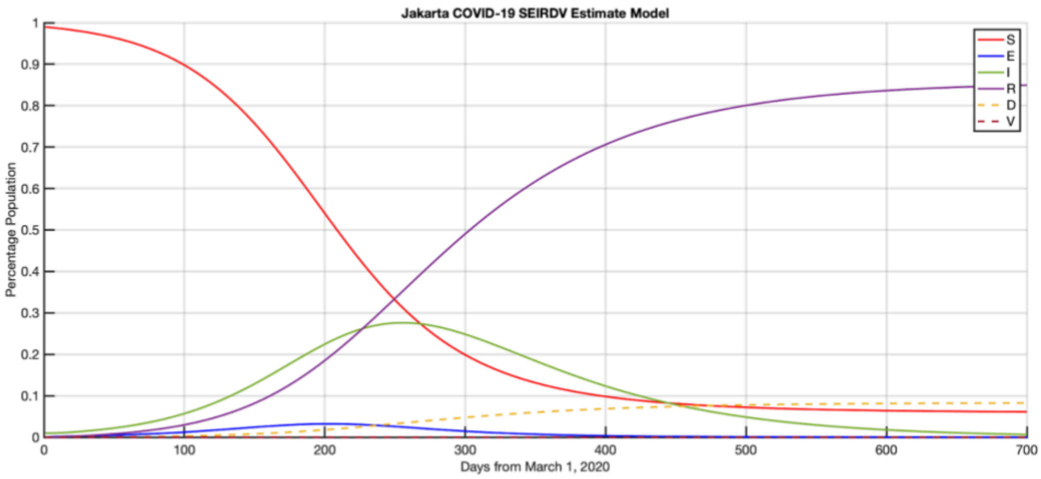


***Figure 10:** All groups, S(t), E(t), I(t), R(t), D(t), and V(t) with S(0) = 0.99, E(0) = 0, I(0) = 0.01, R(0) = 0, D(0) = 0, and V(0) = 0. Integrated for 700 days beginning from March 1, 2020*

Similar to Fig(4) the susceptible population will start from *S(0)* and gradually decreases to an asymptote (*y* = 0), opposite to the recovered population, where it gradually increases to an asymptote (*y = S(0) - E(700) - I(700) - D(700) - V(700)*). Both have a point of inflection near the midpoint, as the rate of increase/decrease slowly reduces. At the same time, infected groups will rise, forming a parabolic concave down curve as the recovery rate is counteracting it, though compared to Fig(4) we can see a reduction due to the exposed compartment that also forms a similar curve. The death compartment increases in the same manner as susceptible and recovered, as it slowly approaches its asymptote as the population of infected slowly decreases. Compared to Fig(4) this new model for Jakarta will reach a lower peak in the infected compartment, though a longer outbreak period. This is likely due to the new exposed compartment which reduces the increase rate of the infected population. Estimated death population can also be seen clearly compared to Fig(4). Thus, from this new model, without any intervention we would expect that COVID-19 in Jakarta will fully end around October 2021 ~ November 2021.

3.2 Equilibrium Analysis for COVID-19 (SEIR Model)

Based on the equation in Eq(19), we can analyse the stability to determine the disease free equilibrium point. This helps Jakarta researchers by giving and understanding whether the disease will equilibrate or not. To achieve this equilibrium point, we know that at equilibrium all rates must be 0 or when $\frac{\mathrm{d}S}{\mathrm{d}t}=0,\ \frac{\mathrm{d}E}{\mathrm{d}t}=0,\ \frac{\mathrm{d}I}{\mathrm{d}t}=0,\ \frac{\mathrm{d}R}{\mathrm{d}t}=0,\ \frac{\mathrm{d}D}{\mathrm{d}t}=0$. Except *V* as it will keep on changing until 100% of the surviving population is vaccinated. Since *D* and *V* are output ODEs; byproduct of other ODEs it's not a fixed point, it can be excluded as it will be dependent on the other compartments [16]. Taking into account that the remaining compartments *(S, E, I, R)* are a fraction of the total population *N* whose value is 1, thus *N* can be removed, Manipulating the ODEs in Eq(21) yields:

$$\frac{\mathrm{d}S}{\mathrm{d}t}=0=\mu-S(\beta I+\eta+v) \quad (24)$$

$$\frac{\mathrm{d}E}{\mathrm{d}t}=0=\beta SI-E(\eta+\delta) \quad (25)$$

$$\frac{\mathrm{d}I}{\mathrm{d}t}=0=\delta E-I(\eta+\gamma+\alpha) \quad (26)$$

$$\frac{\mathrm{d}R}{\mathrm{d}t}=0=\gamma I-vR-\eta R \quad (27)$$

3.3 Equilibrium for Free-Disease (SEIR Model)

For a free-disease equilibrium to occur we know that it means there is no more spread of the COVID-19 virus in Jakarta, or the compartment of exposed and infected must be equal to 0 or *E = I = 0*. Thus isolating *S*, from Eq(24) and applying the condition of *E = I = 0*. We have:

$$\mu=S(\beta(0)+\eta+v) \quad (28)$$

$$S=\frac{\mu}{(\eta+v)} \quad (29)$$

From Eq(27), with condition *E = I = 0* and substituting S to Eq(29):

$$0=\gamma(0)-\frac{v\mu}{(\eta+v)}-\eta R \quad (30)$$

$$R=-\frac{v\mu}{\eta(\eta+v)} \quad (31)$$

As mentioned previously variable D is the product of I and the death constant and V is the product of R and S and the vaccination constant, so both are not fixed points. Thus, the equilibrium point for a disease-free scenario for COVID-19 is given by $Equi_0$ where:

$$Equi_0 = (S,\ E,\ I,\ R)\ =\ \left(\tfrac{\mu}{\eta+v}, 0, 0,\ -\tfrac{v\mu}{\eta(\eta+v)}\right) \tag{32}$$

3.4 Jacobian Matrix & Eigenvalue Stability (SEIR Model)

We have established that from our SEIR model we have a disease-free fixed point given by $Equi_0$, in Eq(32). Intuitively we know that from our SIR model the eigenvalues are likely to be stable, negative, as all compartments equilibrates Fig(4), thus we would assume that the SEIR model is also a stable system [29]. Though, this may not be true as in our SEIR model we have new compartments and we are taking vital dynamics into account. Thus, to verify our assumption we must first linearize the differential equations around a fixed point, specifically by $Equi_0$ given in Eq(32) to determine whether or not this disease in Jakarta will blow up, spread unendingly or will equilibrate and die out, where infected and exposed compartments eventually reach 0 [17]. Linearizing a nonlinear model is often visualised by a vector field, and when we zoom into a particular point, at that specific point it would be linear [19]. Thus, suppose we have a set of ODEs, overdot denoting its first derivative:

$$\dot{x} = f(x) \tag{33}$$

With our fixed point x_0, thus by definition when our x is at fixed point:

$$\dot{x} = f(x_0) = 0 \tag{34}$$

In our case the x_0 the fixed point is $Equi_0$, Eq(32), Now suppose that we take a multivariate Taylor expansion series of the right hand side we will have:

$$\dot{x} = f(x_0) + \left.\frac{\partial f}{\partial x}\right|_{x_0} \cdot (x - x_0) + \left.\frac{\partial^2 f}{\partial x^2}\right|_{x_0} \cdot (x - x_0)^2 + ... + H.O.T \tag{35}$$

In this case we will be neglecting its high order terms **(H.O.T)** as we are zooming into the equilibrium point thus the values will be very small, and as by definition $f(x_0) = 0$ we would have the simplified expression of:

$$\dot{x} = \left.\frac{\partial f}{\partial x}\right|_{x_0} \cdot (x - x_0) \tag{36}$$

With the partial derivative of f with respect to partial x evaluated at equilibrium point can be represented by an $n \times n$, otherwise known as the Jacobian Matrix, $\boldsymbol{J}$. Thus, as we have multiple ODEs, we can represent it in terms of $f(x)$. The following will represent the variables of our compartments:

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} S \\ E \\ I \\ R \end{bmatrix} \tag{37}$$

Substituting Eq(37) by their respective ODEs we have:

$$\dot{x} = \begin{bmatrix} \mu - x_1(\beta x_3 + \eta + v) \\ \beta x_1 x_3 - x_2(\eta + \delta) \\ \delta x_2 - x_3(\eta + \gamma + \alpha) \\ \gamma x_3 - \eta x_4 - v x_1 \end{bmatrix} = \begin{bmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \\ f_4(x) \end{bmatrix} \tag{38}$$

Thus, the partial derivative in Eq(36) is commonly called the Jacobian Matrix, ***J***, which is a square matrix with state components *x*, (x_1, x_2, x_3, x_4) and the components rate *f*, (f_1, f_2, f_3, f_4), and evaluated at x_0 our fixed point Eq(32), then the Jacobian is [18]:

$$\left.\frac{\partial f}{\partial x}\right|_{x0} = \begin{bmatrix} \left.\frac{\partial f_1}{\partial x_1}\right|_{x0} & \left.\frac{\partial f_1}{\partial x_2}\right|_{x0} & \left.\frac{\partial f_1}{\partial x_3}\right|_{x0} & \left.\frac{\partial f_1}{\partial x_4}\right|_{x0} \\ \left.\frac{\partial f_2}{\partial x_1}\right|_{x0} & \left.\frac{\partial f_2}{\partial x_2}\right|_{x0} & \left.\frac{\partial f_2}{\partial x_3}\right|_{x0} & \left.\frac{\partial f_2}{\partial x_4}\right|_{x0} \\ \left.\frac{\partial f_3}{\partial x_1}\right|_{x0} & \left.\frac{\partial f_3}{\partial x_2}\right|_{x0} & \left.\frac{\partial f_3}{\partial x_3}\right|_{x0} & \left.\frac{\partial f_3}{\partial x_4}\right|_{x0} \\ \left.\frac{\partial f_4}{\partial x_1}\right|_{x0} & \left.\frac{\partial f_4}{\partial x_2}\right|_{x0} & \left.\frac{\partial f_4}{\partial x_3}\right|_{x0} & \left.\frac{\partial f_4}{\partial x_4}\right|_{x0} \end{bmatrix} \tag{39}$$

Differentiating and evaluating Eq(39) with ODEs in Eq(38) at the equilibrium point will yield the Jacobian Matrix. Full differentiation can be found in Appx(2):

$$J = \begin{bmatrix} -(\beta x_3 + \eta + \upsilon) & 0 & -\beta x_1 & 0 \\ \beta x_3 & -(\eta + \delta) & \beta x_1 & 0 \\ 0 & \delta & -(\eta + \gamma + \alpha) & 0 \\ -\upsilon & 0 & \gamma & -\eta \end{bmatrix}$$

$$J = \begin{bmatrix} -(\eta + \upsilon) & 0 & -\frac{\mu\beta}{\eta+\upsilon} & 0 \\ 0 & -(\eta + \delta) & \frac{\mu\beta}{\eta+\upsilon} & 0 \\ 0 & \delta & -(\eta + \gamma + \alpha) & 0 \\ -\upsilon & 0 & \gamma & -\eta \end{bmatrix} \tag{40}$$

Now we can use the characteristic equation for solving eigenvalues to determine whether the system of equations for COVID-19 in Jakarta is stable or unstable. Thus using the characteristic equation to solve the Jacobian Matrix, ***J***, eigenvalues by its determinant:

$$\begin{aligned} Jx &= \lambda x \\ (J - \lambda I)x &= 0 \\ \det(J - \lambda I) &= 0 \end{aligned} \tag{41}$$

With eigenvalues, λ, of matrix ***J*** as the roots of the characteristic equation. Thus, applying to Eq(41) and substituting ***J*** by Eq(40):

$$\left| \begin{bmatrix} -(\eta + \upsilon) & 0 & -\frac{\beta\mu}{(\eta+\upsilon)} & 0 \\ 0 & -(\eta + \delta) & \frac{\beta\mu}{(\eta+\upsilon)} & 0 \\ 0 & \delta & -(\eta + \gamma + \alpha) & 0 \\ -\upsilon & 0 & \gamma & -\eta \end{bmatrix} - \lambda I \right| = 0$$

$$\left| \begin{bmatrix} -((\eta + \upsilon) + \lambda) & 0 & -\frac{\beta\mu}{(\eta+\upsilon)} & 0 \\ 0 & -((\eta + \delta) + \lambda) & \frac{\beta\mu}{(\eta+\upsilon)} & 0 \\ 0 & \delta & -((\eta + \gamma + \alpha) + \lambda) & 0 \\ -\upsilon & 0 & \gamma & -(\eta + \lambda) \end{bmatrix} \right| = 0 \tag{42}$$

To get its determinant we can solve by reducing the matrix, ***J***, to row echelon form because of its unique property that if two rows are interchanged, then the determinant of the matrix is equal to the determinant of the original matrix multiplied by -1 and if we interchange rows to form an upper triangular matrix, the determinant would be the product of its diagonal entries [21]. Thus, by cancelling and swapping, $R_1 \leftrightarrow R_4$, and cancel leading coefficient in R_4 by executing $R_4 \longleftarrow R_4 - \frac{\eta+\upsilon+\lambda}{\upsilon} \cdot R_1$

$$\begin{bmatrix} -\upsilon & 0 & \gamma & -(\eta+\lambda) \\ 0 & -((\eta+\delta)+\lambda) & \frac{\beta\mu}{(\eta+\upsilon)} & 0 \\ 0 & \delta & -((\eta+\gamma+\alpha)+\lambda) & 0 \\ 0 & 0 & \frac{-\beta\upsilon\mu-\gamma(\eta+\upsilon+\lambda)(\upsilon+\eta)}{\upsilon(\upsilon+\eta)} & \frac{(\lambda+\eta)(\upsilon+\lambda+\eta)}{\upsilon} \end{bmatrix} \tag{43}$$

Swapping, $R_2 \leftrightarrow R_3$, and cancel the leading coefficient in R_3 by executing $R_3 \longleftarrow R_3 + \frac{\eta+\delta+\lambda}{\delta} \cdot R_2$

$$\begin{bmatrix} -\upsilon & 0 & \gamma & -(\eta+\lambda) \\ 0 & \delta & -(\eta+\gamma+\alpha+\lambda) & 0 \\ 0 & 0 & \frac{\beta\delta\mu-(\eta+\delta+\lambda)(\eta+\gamma+\alpha+\lambda)(\upsilon+\eta)}{\delta(\upsilon+\eta)} & 0 \\ 0 & 0 & \frac{-\beta\upsilon\mu-\gamma(\eta+\upsilon+\lambda)(\upsilon+\eta)}{\upsilon(\upsilon+\eta)} & \frac{(\lambda+\eta)(\upsilon+\lambda+\eta)}{\upsilon} \end{bmatrix} \tag{44}$$

Lastly, swapping $R_3 \leftrightarrow R_4$, and cancel the leading coefficient of R_4 by executing $R_4 \longleftarrow R_4 - \frac{\upsilon(\beta\delta\mu-(\eta+\delta+\lambda)(\eta+\gamma+\alpha+\lambda)(\upsilon+\eta))}{\delta(-\beta\upsilon\mu-\gamma(\eta+\upsilon+\lambda)(\upsilon+\eta))} \cdot R_3$

$$\begin{bmatrix} -\upsilon & 0 & \gamma & -(\eta+\lambda) \\ 0 & \delta & -(\eta+\gamma+\alpha+\lambda) & 0 \\ 0 & 0 & \frac{-\beta\upsilon\mu-\gamma(\eta+\upsilon+\lambda)(\upsilon+\eta)}{\upsilon(\upsilon+\eta)} & \frac{(\lambda+\eta)(\upsilon+\lambda+\eta)}{\upsilon} \\ 0 & 0 & 0 & -\frac{(\lambda+\eta)(\upsilon+\lambda+\eta)(\beta\delta\mu-(\upsilon+\eta)(\lambda+\delta+\eta)(\gamma+\lambda+\alpha+\eta))}{\delta(-\beta\upsilon\mu-\gamma(\upsilon+\eta)(\upsilon+\lambda+\eta))} \end{bmatrix} \tag{45}$$

From the row echelon property, the determinant of the matrix is its diagonal product:

$$\frac{(\lambda+\eta)(\upsilon+\lambda+\eta)(\beta\delta\mu-(\upsilon+\eta)(\lambda+\delta+\eta)(\gamma+\lambda+\alpha+\eta))}{\upsilon+\eta} \tag{46}$$

From row echelon rule, we have to multiply by negative one, and as we interchanged three row, we raised to three:

$$(-1)^3 \cdot \frac{(\lambda+\eta)(\upsilon+\lambda+\eta)(\beta\delta\mu-(\upsilon+\eta)(\lambda+\delta+\eta)(\gamma+\lambda+\alpha+\eta))}{\upsilon+\eta} \tag{47}$$

As the negative does not chancel in an odd power, and simplifying by cancelling $(\upsilon + \eta)$, our characteristic equation in Eq(42) is reduced to:

$$-(\lambda+\eta)(\upsilon+\lambda+\eta)\left(-(\lambda+\delta+\eta)(\gamma+\lambda+\alpha+\eta)+\frac{\beta\delta\mu}{\upsilon+\eta}\right)=0 \tag{48}$$

Further simplifying to more a basic equation:

$$-(\lambda+A)(\lambda+B)(-(\lambda+C)(\lambda+D)+E)=0 \tag{49}$$

Where,

$$A=\eta,\ B=(\upsilon+\eta),\ C=(\delta+\eta),\ D=(\gamma+\alpha+\eta),\ E=\frac{\beta\delta\mu}{\upsilon+\eta}$$

Similar to a 2×2 matrix, the number of roots of the characteristic will be equal to the number of powers, meaning that we would be certain that Eq(49) has four solutions/eigenvalues. Expanding we have:

$$\begin{aligned} &\lambda^4+D\lambda^3+C\lambda^3+B\lambda^3+A\lambda^3+CD\lambda^2-E\lambda^2+BD\lambda^2+BC\lambda^2 \\ &+AD\lambda^2+AC\lambda^2+AB\lambda^2+BCD\lambda-BE\lambda+ACD\lambda-AE\lambda \\ &+ABD\lambda+ABCD-ABE=0 \end{aligned} \tag{50}$$

Factoring, similar powers:

$$\begin{aligned} &\lambda^4+(A+B+C+D)\lambda^3+(AB+(A+B)(C+D)+CD-E)\lambda^2 \\ &+(AB(D+C)+(A+B)(CD-E))\lambda+AB(CD-E)=0 \end{aligned} \tag{51}$$

If it is simplified in terms of k_i, for ($i = 1, 2, 3, 4$):

$$k_1\lambda^4 + k_2\lambda^3 + k_3\lambda^2 + k_4\lambda + k_5 = 0 \qquad \textbf{(52)}$$

Where,

$$\begin{aligned} k_1 &= 1 \\ k_2 &= A + B + C + D \\ k_3 &= AB + (A+B)(C+D) + CD - E \\ k_4 &= AB(D+C) + (A+B)(CD-E) \\ k_5 &= AB(CD-E) \end{aligned}$$

Thus, based on Descartes's rule for sign change [22], to verify negative values we would use (-λ) thus, we would expect four negative values when k_1, k_2, k_3, k_4, $k_5 > 0$. To verify:

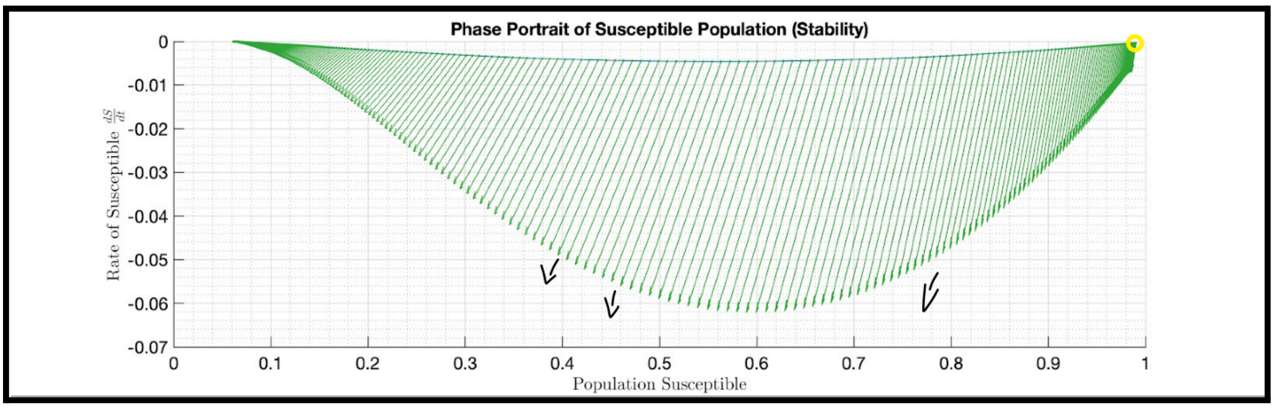
$$\underbrace{(-\lambda^4)\underbrace{k_1}_{+}}_{+} + \underbrace{(-\lambda^3)\underbrace{k_2}_{+}}_{-} + \underbrace{(-\lambda^2)\underbrace{k_3}_{+}}_{+} + \underbrace{(-\lambda)\underbrace{k_4}_{+}}_{-} + \underbrace{k_5}_{+} = 0 \qquad \textbf{(53)}$$

Eq(53) yields four sign changes. Concluding that if CD>E, so there won't be a negative value, and $k_1,k_2,k_3,k_4,k_5>0$, then $\lambda_1,\lambda_2,\lambda_3,\lambda_4<0$. In our model, since we do not have any negative proportionality constants in Fig(9) and based on our substitutions in Eq(49) k_1, k_2, k_3, k_4, k_5 would surely be greater than 0. Thus, because of characteristic values in the system of equations in the COVID-19 model, we can assume that the equilibrium point in Eq(32) is stable and asymptotic meaning that over time, without any intervention it will level out or essentially sink into an equilibrium point. This is the reason as to why our model simulations would always reach an asymptote or 0 after a period of time, *t*. This is why I realize that however bad COVID-19 in Jakarta is, it will eventually level out after a period of time. Proving stability we can use **MATLAB** by using ***eigs()*** [20]:

```
jacobian = [-(eta+upsilon) 0 (-mu*beta)/(eta+upsilon) 0 ;
   0 -(eta+delta) (mu*beta)/(eta+upsilon) 0 ;
   0 delta -(eta+gamma+alpha) 0 ;
   upsilon 0 gamma -eta ;
   ];

>> eigs(jacobian)

ans =

  -0.3654
  -0.1571
  -0.1063
  -0.0063
```

Figure 11: *Verifying the eigenvalues through MATLAB*

This could also be verified by observing the phase portrait behaviors generated in **MATLAB** through the **ODE45** output by plotting the population of each compartment against its rate of change, the following phase portraits are generated:



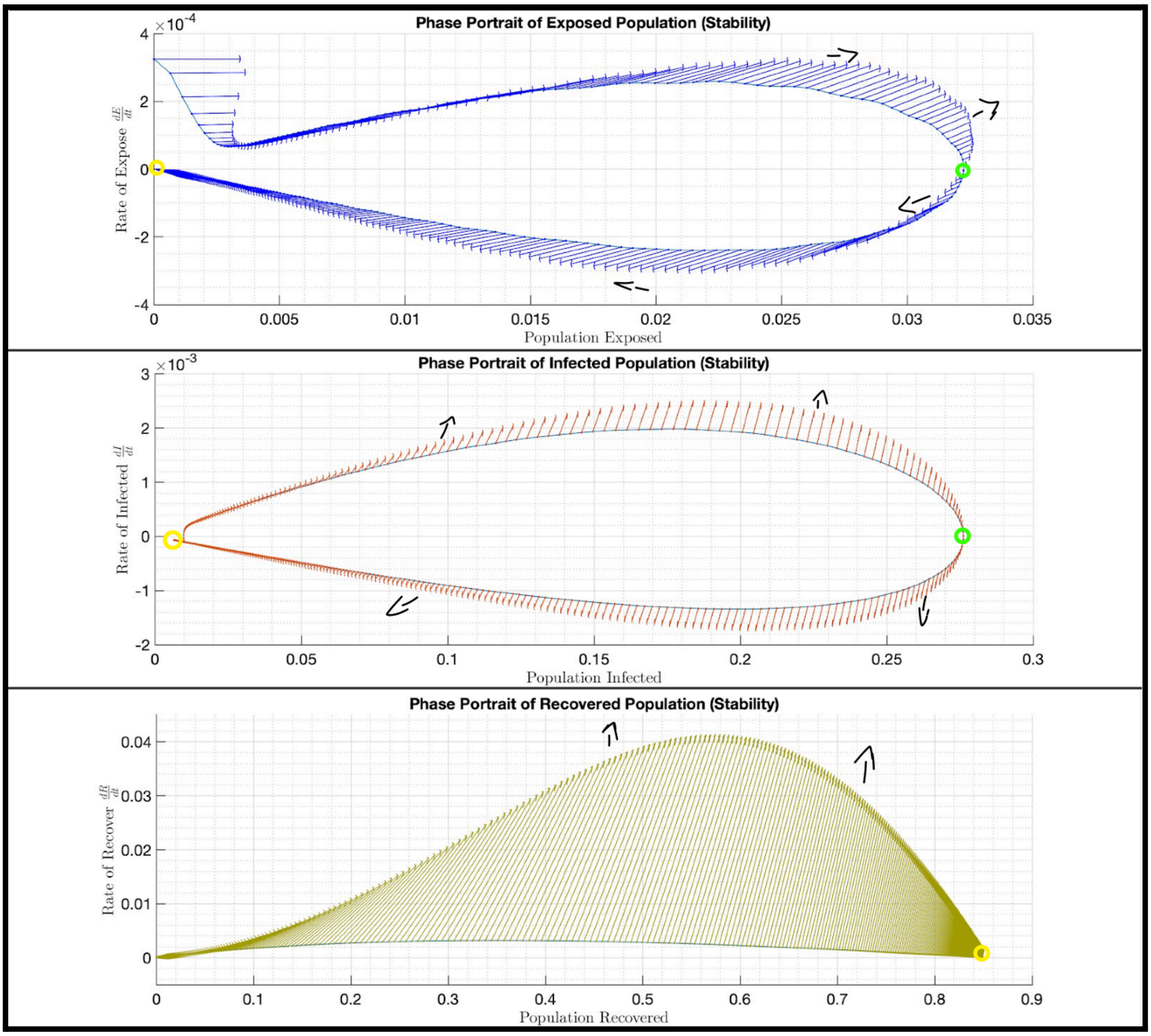


Figure 12: *Phase portraits for S, E, I, and R compartment*

From Fig(12) we can verify that the free-disease in Eq(32) is indeed a stable equilibrium point, as they all sink. While it does not return to its origin point, this can be seen by the point marked in ∘, that is to be expected as in our free-disease equilibrium point condition in Eq(32) only E and I is meant to reach 0 for both their rate of change and its population. Opposite to S and R as they move away from the origin point but noted that their rate will also be 0 at equilibrium. A maximum point is also marked by ∘ in the E and I phase portrait. Referring to COVID-19 in Jakarta, this would mean that with current proportionality parameters of Fig(9), when approximately a quarter of Jakarta's population is infected, the rate of individuals moving into and moving out will be equal. Similar in the exposed compartment. This does assure us, showing that without taking any action, with the virus's current rate of progression in Jakarta, in the end it will equilibrate. Thus, we should focus on reducing the population of the infected through vaccination and increasing the rate of recovery. A point of inflection is also seen in the maximum point of S and R phase portraits. Implying that when 60% of the population is recovered herd immunity [30] will come into effect as Jakarta will already be resistant to COVID-19 thus less individuals are recovered.

3.5 Basic Reproduction Number R_0 (SEIR Model)

Unlike the basic SIR model, in an SEIR model the basic reproduction number, R_0, can be determined through the next generation matrix [24]. Though similar to the basic SIR model only the disease compartments will be taken into account; *E* and *I*. Exploring next generation matrix method is beyond the scope of this paper, so we would proceed to the general equation, where $i = 1$ and 2 denotes the 'disease compartments', *E* and *I* respectively by:

$$\frac{dx_i}{dt} = \mathcal{F}_i(x) - \mathcal{V}_i(x),\ where\ \mathcal{V}_i(x) = \left[\mathcal{V}_i^- - \mathcal{V}_i^+\right] \tag{54}$$

Where $\mathcal{F}_i(x)$,represents the rate of appearance of new infections. $\mathcal{V}_i^+(x)$ the rate of individuals moving into, by all means. $\mathcal{V}_i^-(x)$the rate of individuals moving out. Following Eq(54), since moving into the disease compartment only appears in $\frac{\mathrm{d}E}{\mathrm{d}t}$ as $\frac{\beta SI}{N}$, and not in , $\frac{\mathrm{d}I}{\mathrm{d}t}$ thus $\mathcal{F}_2$is 0:

$$\begin{aligned}
\mathcal{F}_1(x) &= \frac{\beta SI}{N}\\
\mathcal{F}_2(x) &= 0\\
\mathcal{V}_1(x) &= \left[\mathcal{V}_1^- - \mathcal{V}_1^+\right] = -(-E(\eta+\delta))\\
\mathcal{V}_2(x) &= \left[\mathcal{V}_2^- - \mathcal{V}_2^+\right] = (-\delta E) - (-I(\eta+\gamma+\alpha))
\end{aligned} \tag{55}$$

Where $\mathcal{F}_i(x)$and $\mathcal{V}_i(x)$ have values of below, *T* denoting that the matrix is transposed:

$$\mathcal{F}_i(x) = (\mathcal{F}_1(x),\ \mathcal{F}_2(x),\ \ldots,\ \mathcal{F}_m(x))^T \tag{56}$$

$$\mathcal{V}_i(x) = (\mathcal{V}_1(x),\ \mathcal{V}_2(x),\ \ldots,\ \mathcal{V}_m(x))^T \tag{57}$$

Following equations in Eq(56) and Eq(57), replacing by the equations in Eq(55) and transposing we have the values 2 × 1 matrix,

$$\mathcal{F}_i = \begin{bmatrix}\frac{\beta SI}{N}\\ 0\end{bmatrix} \&\ \mathcal{V}_i = \begin{bmatrix}E(\eta+\delta)\\ -\delta E + I(\eta+\gamma+\alpha)\end{bmatrix} \tag{58}$$

Let $x = (E, I) = (x_1, x_2)$

$$\mathcal{F}_i = \begin{bmatrix}\frac{\beta S x_2}{N}\\ 0\end{bmatrix} \&\ \mathcal{V}_i = \begin{bmatrix}x_1(\eta+\delta)\\ -\delta x_1 + x_2(\eta+\gamma+\alpha)\end{bmatrix} \tag{59}$$

Now based on next-generation matrix method we have the equation by Hefferman et al:

$$K = FV^{-1} \tag{60}$$

Where, ***F*** is the rate secondary infections increase the disease compartment and *V* is the rate disease progression (death or immunity) given that both are *n* × *n* matrices, with the R_0 value being the largest eigenvalue of matrix ***K***. With matrix ***F*** and ***V*** described as the partial derivatives as follows:

$$F = \left[\frac{\partial \mathcal{F}_i(x_0)}{\partial x_j}\right],\ V = \left[\frac{\partial \mathcal{V}_i(x_0)}{\partial x_j}\right] \tag{61}$$

Thus, we would form the ***F*** and ***V*** matrices by Eq(59) and evaluated at 0, with initially *S*≈*N*:

$$F = \begin{bmatrix}\frac{\partial \mathcal{F}_1}{\partial x_1} & \frac{\partial \mathcal{F}_1}{\partial x_2}\\ \frac{\partial \mathcal{F}_2}{\partial x_1} & \frac{\partial \mathcal{F}_2}{\partial x_2}\end{bmatrix}\Bigg|_{x_1=0,\,x_2=0,\,S=N},\ V = \begin{bmatrix}\frac{\partial \mathcal{V}_1}{\partial x_1} & \frac{\partial \mathcal{V}_1}{\partial x_2}\\ \frac{\partial \mathcal{V}_2}{\partial x_1} & \frac{\partial \mathcal{V}_2}{\partial x_2}\end{bmatrix}\Bigg|_{x_1=0,\,x_2=0,\,S=N} \tag{62}$$

Evaluating the ***F*** and ***V*** matrix through differentiation laws [23]:

$$F = \begin{bmatrix}0 & \beta\\ 0 & 0\end{bmatrix},\ V = \begin{bmatrix}\eta+\delta & 0\\ -\delta & \eta+\gamma+\alpha\end{bmatrix} \tag{63}$$

Then we found:

$$\begin{aligned}
V^{-1} &= \begin{bmatrix}\eta+\delta & 0\\ -\delta & \eta+\gamma+\alpha\end{bmatrix}^{-1} = \frac{1}{(\eta+\delta)(\eta+\gamma+\alpha)}\begin{bmatrix}\eta+\gamma+\alpha & 0\\ \delta & \eta+\delta\end{bmatrix}\\
V^{-1} &= \begin{bmatrix}\frac{1}{(\eta+\delta)} & 0\\ \frac{\delta}{(\eta+\delta)(\eta+\gamma+\alpha)} & \frac{1}{(\eta+\gamma+\alpha)}\end{bmatrix}
\end{aligned} \tag{64}$$

Then substituting Eq(63) and Eq(64) to Eq(60),

$$K = FV^{-1} = \begin{bmatrix} 0 & \beta \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{(\eta+\delta)} & 0 \\ \frac{\delta}{(\eta+\delta)(\eta+\gamma+\alpha)} & \frac{1}{(\eta+\gamma+\alpha)} \end{bmatrix}$$

$$K = \begin{bmatrix} \frac{\beta\delta}{(\eta+\delta)(\eta+\gamma+\alpha)} & \frac{\beta}{(\eta+\gamma+\alpha)} \\ 0 & 0 \end{bmatrix} \tag{65}$$

The dominant eigenvalue can be found by the characteristic equation similar to Eq(41):

$$|K - \lambda I| = 0$$
$$\left| \begin{bmatrix} \frac{\beta\delta}{(\eta+\delta)(\eta+\gamma+\alpha)} - \lambda & \frac{\beta}{(\eta+\gamma+\alpha)} \\ 0 & -\lambda \end{bmatrix} \right| = 0$$
$$-\lambda\left(\frac{\beta\delta}{(\eta+\delta)(\eta+\gamma+\alpha)} - \lambda\right) = 0 \tag{66}$$

Evaluating Eq(66) we have two possible eigenvalues:

$$\lambda_1 = 0,\ \lambda_2 = \frac{\beta\delta}{(\eta+\delta)(\eta+\gamma+\alpha)} - \lambda \tag{67}$$

Thus, by matrices generation the reproductive number, R_0, with λ_2 being the dominant one:

$$R_0 = \frac{\beta\delta}{(\eta+\delta)(\eta+\gamma+\delta)} \approx 2.1 \tag{68}$$

Rounded to one decimal so it will be a fair test with Sect(2.3). By considering a more complex model Fig(8), R_0 value decreases as there are chances that an infected individual won't infect others if they die or are vaccinated. This is also verified as based on POPSCI the COVID-19 reproductive number worldwide has an average of 2.5, Fig(13) [25]. I would say that Jakarta is doing quite well in handling the COVID-19 as we have an approximate reproductive number of less than 2.5, the global average. Though to verify my model's claim of having a mean $R_0 \approx 2.1$, we can conduct a similar **hypothesis test** to Eq(19). By testing the mean of the data in Fig(7).

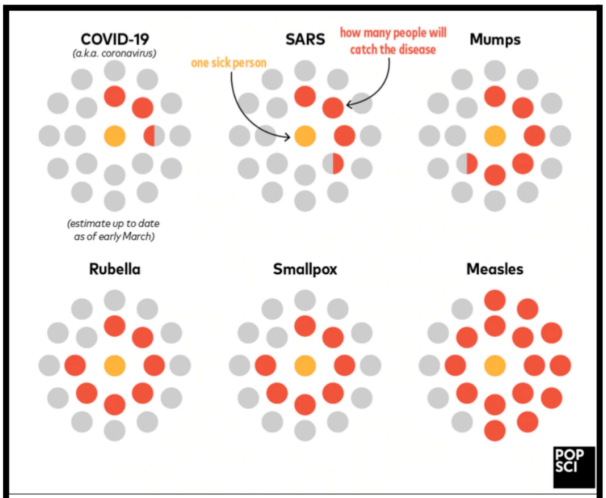


Figure 13: PopSci R0 Value Of Worldwide Diseases

$$H_0 : R_0 = 2.1$$
$$H_1 : R_0 \neq 2.1 \tag{69}$$

Through **GDC: *t-test*** at **5% significance level**:

$$\text{PVal} = 0.269 > 0.05 \tag{70}$$

As the PVal Eq(70) is not significant, there is insufficient evidence to reject the null hypothesis. Thus, Eq(68) is a relatively accurate estimate for the reproduction number, R_0, or the mean of individuals infected from one infectee in Jakarta. This is a more acceptable estimate compared Eq(18). Though the test in Eq(70) still yields a low PVal meaning that the actual R_0 in Jakarta is somewhere around 1.5. Again it would be expected as the number of infectives currently are still increasing.

4 Considering Complex Model With Vaccination

4.1 Numerical Simulation of SEIR Model Jakarta With Vaccination

Model simulations are done using **MATLAB ODE45** with code adapted from Fig(3) with ODEs from Eq(21) including vaccination, population values will be taken in the range of 0 ~ 1 to provide simplicity. We would simulate the SEIRDV model above and evaluate the evolution of each compartment; in the case of no vaccination and vaccination. We would take the initial conditions to be:

Variable / Parameter	Symbol	Value / Initial Value
Total Population	$N(0)$	1
Susceptible Population	$S(0)$	0.99
Exposed Population	$E(0)$	0
Infected Population	$I(0)$	0.01
Recovered Population	$R(0)$	0
Death Population	$D(0)$	0
Vaccinated Population	$V(0)$	0

Figure 14*: Initial values for the compartments*

We would also take vaccination into account, 1% meaning that from the susceptible, population 1% will be vaccinated each day. Even though using a 100% vaccination rate would be most ideal, it is simply not doable as the vaccination centres in Jakarta are limited and cost is also a factor. We would look into more plausible vaccination rate with values of:

Parameter	Simulation 1	Simulation 2	Simulation 3	Simulation 4	Simulation 5	Simulation 6
υ	0%	0.1%	0.5%	1%	5%	10%

Figure 15*: Proportionality vaccination rates for the simulation*

As the parameters were estimated as of *March 1, 2020* it would be ideal that the simulation would also start from then, based on local news reports vaccination in Jakarta began on *15 January, 2021*, for the president, government officials and health workers. Thus, we would assume that optimal vaccination for all citizens will start on *February 1, 2021*. Approximately 337 days after (t = 0). Thus, we would adapt our code to change υ to Fig(15) parameters after 337 days. Simulation graphs are given in Fig(17) - Fig(22). Code is given in Fig(16):

```
function dydt = odefcn(t,y,mu,eta,beta,gamma,delta,alpha)

if t>337
   upsilon = 0.1;
else
   upsilon = 0;
end

dydt = zeros(6,1);
dydt(1) = mu - y(1)*(beta*y(3)+mu+upsilon);
dydt(2) = beta*y(1)*y(3)-y(2)*(mu+delta);
dydt(3) = delta*y(2)-y(3)*(mu+gamma+alpha);
dydt(4) = gamma*y(3)-eta*y(4)-upsilon*y(4);
dydt(5) = alpha*y(3);
dydt(6) = upsilon*y(1)+upsilon*y(4);
end
```

Figure 16*: MATLAB Code Example with 10% vaccination rate on the 337th day after March 1, 2020*

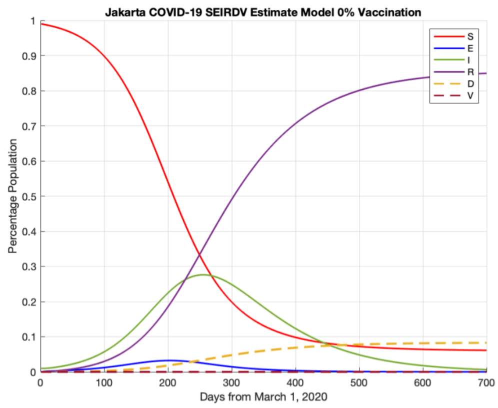


Figure 17: *With vaccination at 0% effective at 337 days with data occurred between March 1, 2020 (t =0) and January 30, 2022 (t =700)*

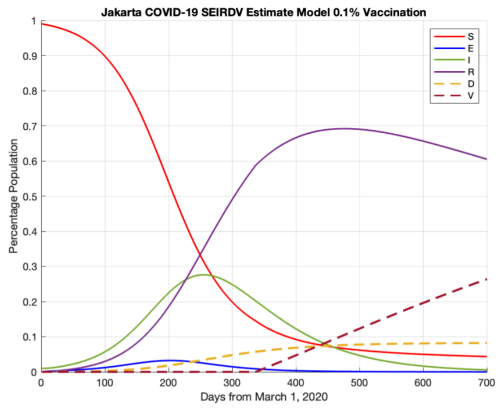


Figure 18: *With vaccination at 0.1% effective at 337 days with data occurred between March 1, 2020 (t =0) and January 30, 2022 (t =700)*

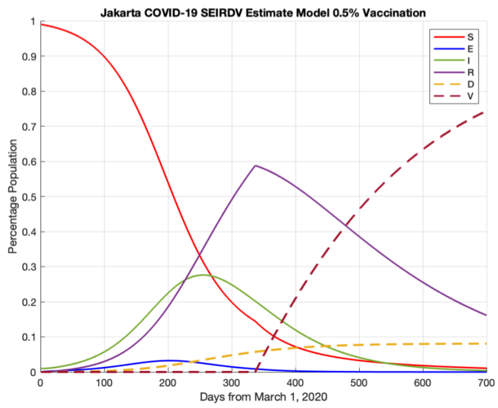


Figure 19: *With vaccination at 0.5% effective at 337 days with data occurred between March 1, 2020 (t =0) and January 30, 2022 (t =700)*

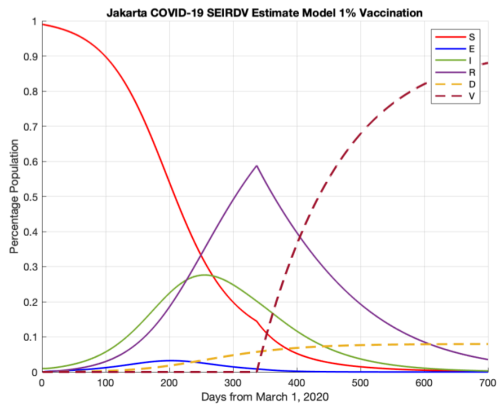


Figure 20: *With vaccination at 1% effective at 337 days with data occurred between March 1, 2020 (t =0) and January 30, 2022 (t =700)*

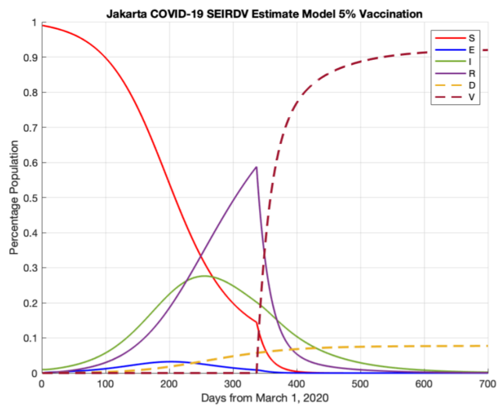


Figure 21: *With vaccination at 5% effective at 337 days with data occurred between March 1, 2020 (t =0) and January 30, 2022 (t =700)*

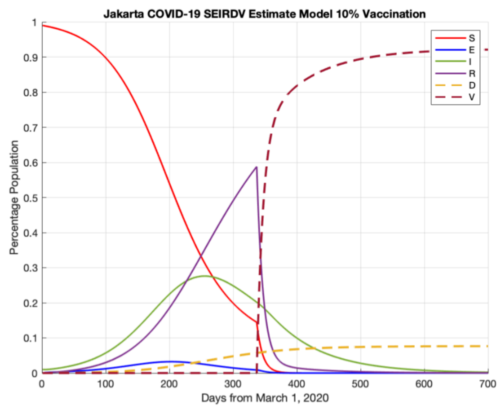


Figure 22: *With vaccination at 10% effective at 337 days with data occurred between March 1, 2020 (t =0) and January 30, 2022 (t =700)*

Variable / Parameter	v	% Recovered / Vaccinated (R)	% Death (D)	% Individuals Vaccinated	Individuals Vaccinated (millions) [V]
Simulation 1 Fig(17)	0%	0.84935	0.082646	0	0
Simulation 2 Fig(18)	0.1%	0.60486	0.082123	0.26391	2.95769
Simulation 3 Fig(19)	0.5%	0.16132	0.080666	0.74353	8.11563
Simulation 4 Fig(20)	1%	0.035405	0.079686	0.88016	9.60694
Simulation 5 Fig(21)	5%	0.00070733	0.077116	0.91987	10.04038
Simulation 6 Fig(22)	10%	0.00028734	0.076584	0.92104	10.05315

Figure 23: *Simulation Data Recorded of Fig(17) - Fig(22) rounded to five decimals to keep it accurate*

From what we could see in Fig(17) - Fig(22), the susceptible and recovered population would spike at exactly 337 days as more of the population move to the vaccinated compartment. However, the lower vaccination rates are not so profound. As minuscule rates will not affect the recovered compartment in the short term. Though with 10% and 5% rates, we can see a significant jump after 337 days as everyone starts to move to the vaccination population, *V*. Though they all share one characteristic where it will all reach an equilibrium point after a certain period. Verifying our predictions in Sect(3.3) that the model is stable. Either it reaches 0 in the case of *E* and *I* or reaches an asymptote, verifying that our eigenvalues are stable as they all reach a steady point. Other than that everything else seems to follow the same trend as in Fig(10) From Fig(23) we can see the data collected on the recovered, death, and vaccinated populations, visually we can tell that it has a negative correlation. As when vaccination rate rises, the amount of recovered and death will decrease, since more individuals move to the vaccinated compartment.

Assuming Jakarta imported the Pfizer vaccine, a linear cost function with vaccination gradient cost of IDR 300 thousand [27] for one individual can be made. August last year, Indonesia's president, Jokowi Widodo, announced that the COVID-19 vaccination would be funded entirely by the state [7]. It was reported that Jakarta allocated IDR 3 trillion for COVID-19 [6]. Utilizing the data Fig(23), by multiplying the percentage vaccinated by the population of Jakarta of 10915000 [3], we can get the number of individuals vaccinated. With *C* being cost expended and *V* the number of individuals vaccinated, we can create a linear function expressing this:

$$C(V) = 300000V \tag{71}$$

Plotting Eq(71) we could generate graph plot:

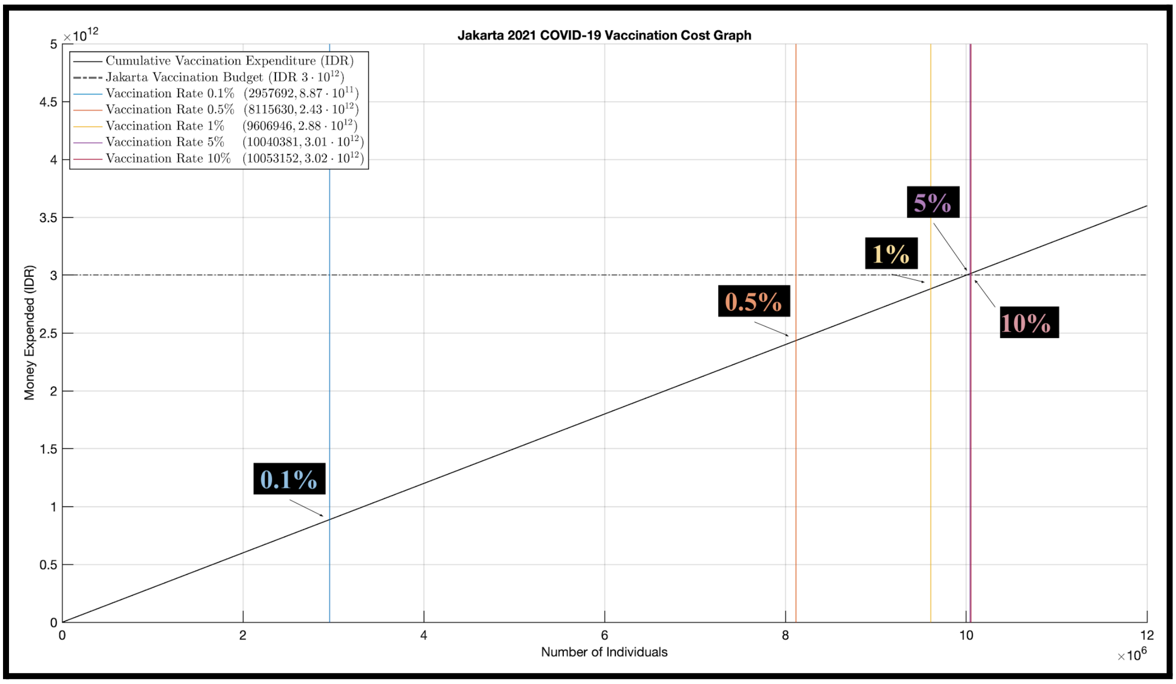


Figure 24: *Linear cost function analysing the possible vaccination rate based on budget*

From Fig(124), the critical point of which is the government budget is denoted by the line C = IDR 3×10^{12}. We can conclude that a 5% and 10% vaccination rate is improbable as it will yield an expenditure beyond the budget. Although 0.1% only consume approximately a quarter of the total budget due to its sparse vaccination population and high death population, it is unlikely to be accepted. 0.5% and 1% are most promising as they are below the critical region and have a high vaccination population and low death population. However, 1% would be most optimal as there is a considerable difference compared to 0.5% in the vaccination population and death population, and it's always crucial to reduce death population as best as possible. With approximately a few of the budget left over for other expenses, such as vaccination stalls, free masks, sanitation, and etc.

4.2 Voronoi Diagram for Vaccination Stall Placement

We have established that a 1% vaccination rate would be most ideal for Jakarta, when we consider its budget and our simulation results. But, what would be considered a 1% vaccination rate? One way to look at this is developing a voronoi diagram of vaccination stalls or hospitals at optimal distances so it would be most optimal. Thus from our Jakarta's population of 10.915 million [3] on the first day we want to have:

$$1\%\ \text{Vaccination Rate}\ (t = 1) = 10915000 \times 1\% = 109150 \tag{72}$$

Recently, it mentioned to the public that Jakarta could reach approximately 600 - 700 vaccinations a day at a single hospital. Taking lower bound as it is most likely we could infer that all hospitals vaccinate at a rate of 600 per day [8]. Assuming that clinics and vaccination stalls have similar capacities if the health workers are evenly distributed throughout each vaccination point. We would need approximately:

$$\text{Vaccination Locations Needed} = \frac{109150\ (\text{First } 1\%)}{600/\text{day}} = 181.9 \approx 182 \tag{73}$$

Rounded off to a whole number as it is then number of hospitals. Through the use of google maps and national recordings, DKI Jakarta has approximately 130 hospitals and health institutes [28]. Thus by subtracting Eq(73), Jakarta requires 52 more vaccination locations to achieve our proposed 1% vaccination rate. Though these 52 additional locations can't just be placed randomly, they need to be spaced far away from other locations so it could cover more areas. Locations of the hospitals [28], Appx(3):

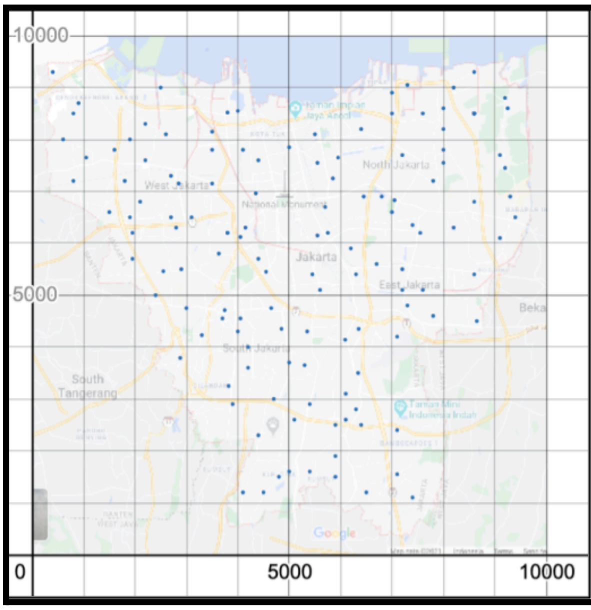


Figure 25: Marked locations of the hospitals that will be considered in this exploration. Locations are taken from google maps and national archives.
← ← ← ← ← ←

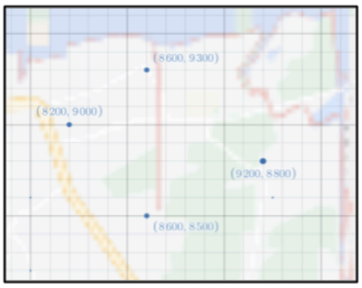


Figure 26: Coordinates tha will be taken int consideration fo the example.
← ← ← ←

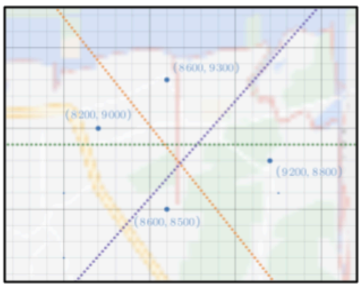


Figure 27: Example of one voronoi cell. Perpendicular lines of
•••(8600,9300) & (8200,9000)
•••(8600,9300) & (9200,8800)
•••(8600,9300) & (8600,8500)
← ← ← ←

Example generation of voronoi cells are as follows. If we consider coordinates Fig(26) we can make (8600,9300) our subject. Steps are:

$$\underbrace{\text{Gradient of two coordinates}}_{\text{(Ex. [8600,9300]\&[8200,9000])}} \Rightarrow \underbrace{\text{Negative reciprocal of gradient}}_{\text{Gradient of perpendicular}} \Rightarrow \underbrace{\text{Midpoint}}_{\text{Pass through perpendicular}} \Rightarrow \underbrace{\text{Solve for line}}_{\text{Slope-intercept form}}$$

For example if we consider (8600,9300) and (8200,9000):

$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_2} = \frac{9300 - 9000}{8600 - 8200} = \frac{3}{4} \quad \textbf{(74)}$$

$$\text{Negative Reciprocal} = -\frac{4}{3} \quad \textbf{(75)}$$

$$\begin{aligned} \text{Midpoint} &= \left(\frac{8600 + 8200}{2}, \frac{9300 + 9000}{2}\right) = (8400, 9150) \\ \text{Perpendicular Line} &= (y - 9150) = -\frac{4}{3}(x - 8400) \end{aligned} \quad \textbf{(76)}$$

$$y(x) = -\frac{4}{3}x + 20350 \quad \textbf{(77)}$$

Thus, the line is given by Eq(77), the **orange line**. A similar process is done for the other three coordinates. Where the green **line** is $y = 8900$ and the purple **line** is $y(x) = \frac{6}{5}x - 1630$. Plotting all the lines it will result in Fig(27). The voronoi cells of the hospital in coordinates (8600,9300) are cut off at the intersection points of each line. Calculating for each line point will result in a voronoi diagram Fig(28):

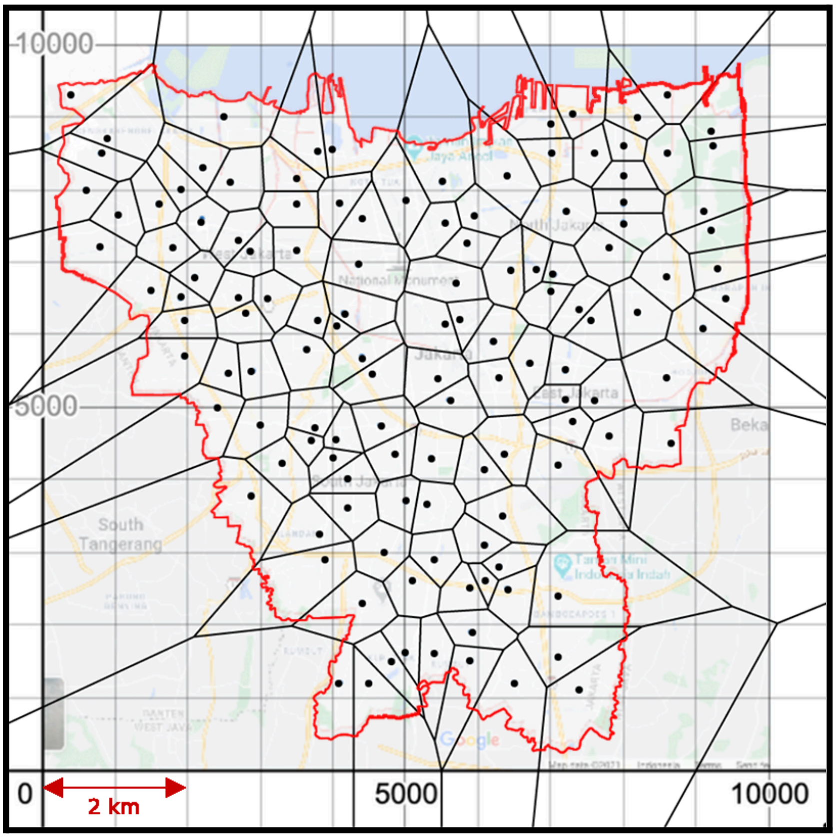


Figure 28: *First voronoi diagram for 130 different locations*

So that's 130 locations and we need an additional 52 locations to achieve our proposed 1% vaccination rate. A way to do this is by solving the toxic-waste-dump. Steps are as follows:

1. Identify the three coordinates with the largest area ([400,9300], [2500,9000] & [900,8700])
2. Find the equation of lines connecting the three coordinates and make sure there are no other coordinates in between the bounded region.

3. Find the gradients and negative reciprocals of the gradients for its perpendicular line
4. Find the midpoint and solve the line using slope-intercept form
5. Plot the three lines and the intersection of the three lines is the new ideal location for the new vaccination spot.

For example if we consider ([400,9300], [2500,9000] & [900,8700]), for the first line considering coordinates [400,9300] & [2500,9000] Fig(29) - Fig(32):

$$\text{Mid} = \left(\frac{400+2500}{2}, \frac{9300+9000}{2}\right) = (1450, 9150) \tag{78}$$

$$\text{Gradient} = \frac{9300-9000}{400-2500} = -\frac{1}{7} \tag{79}$$

$$\text{Negative Reciprocal} = 7 \tag{80}$$

$$\text{Perpendicular Line} = (y - 9150) = 7(x - 1450)$$

$$y(x) = 7x - 1000 \tag{81}$$

The equations of the other two lines are:

$$\text{For } ([900{,}8700] \,\&\, [400, 9300]) \Rightarrow y(x) = \frac{5}{6}x + \frac{25375}{3} \tag{77}$$

$$\text{For } ([2500{,}9000] \,\&\, [900, 8700]) \Rightarrow y(x) = -\frac{16}{3}x + \frac{53750}{3} \tag{77}$$

Plotting and finding the intersect:

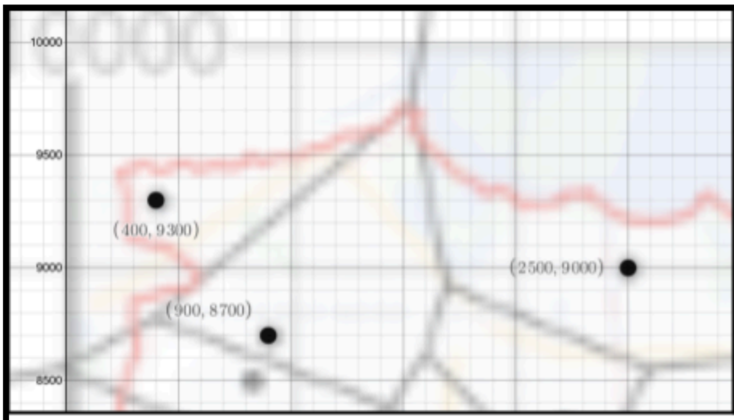


Figure 29: Indentifying the three coordinates with the largest area in between

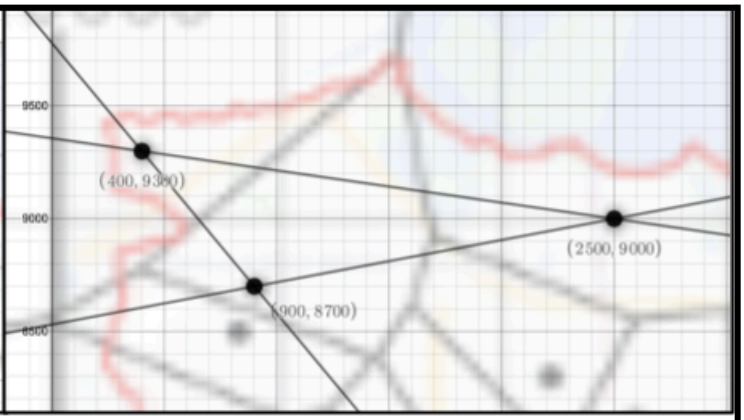


Figure 30: Solving for the equations of line connecting the three coordinates

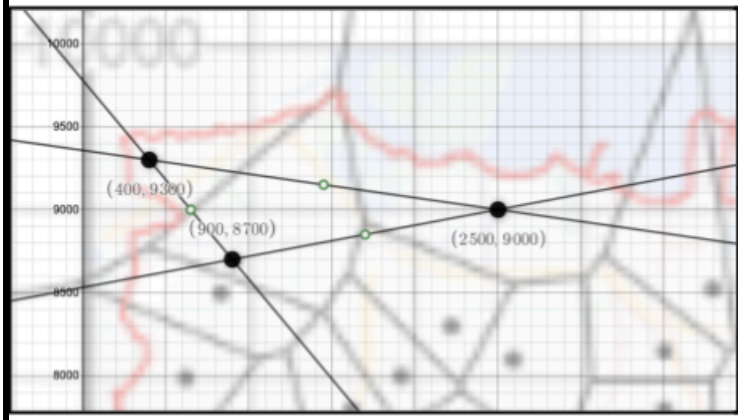


Figure 31: Solving for the midpoint between two coordinates

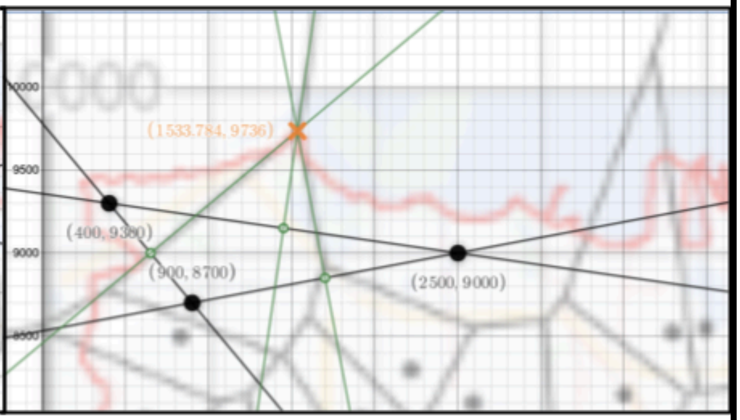


Figure 32: Solving for the negative reciprocal of the gradient from the line equations and using the midpoint solve for the perpeendicular line. Indentify intercept. ❌ is the new coordinate location

Solving for the intersect of the three lines the ideal placement of the new vaccination stall so that it would be optimally distanced from other locations would be in coordinates (1533.784 , 9736). Repeating this same process 52 other times we would achieve the final voronoi diagram for the proposed 1% vaccination rate to be:

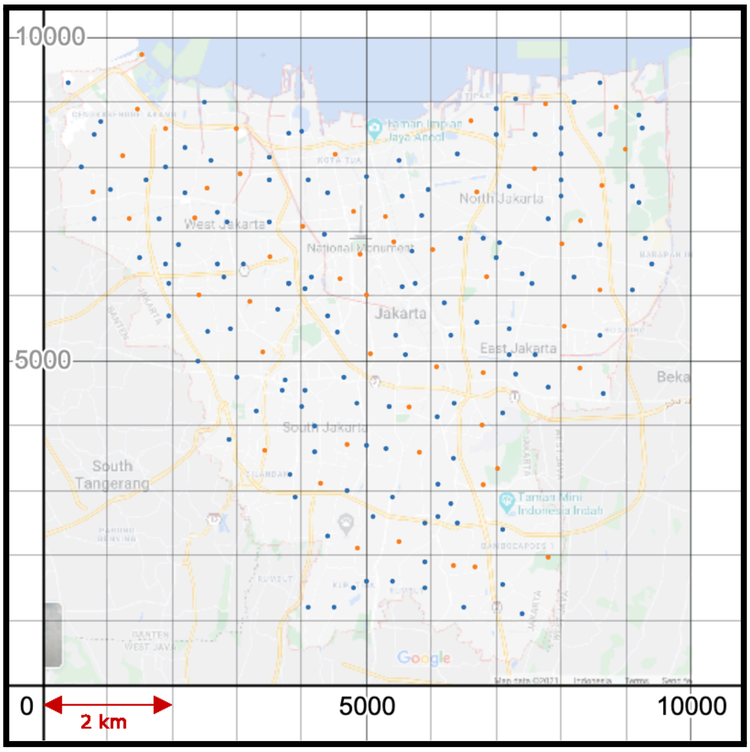


Figure 29: *New coordinates (Orange is new voronoi sites)*

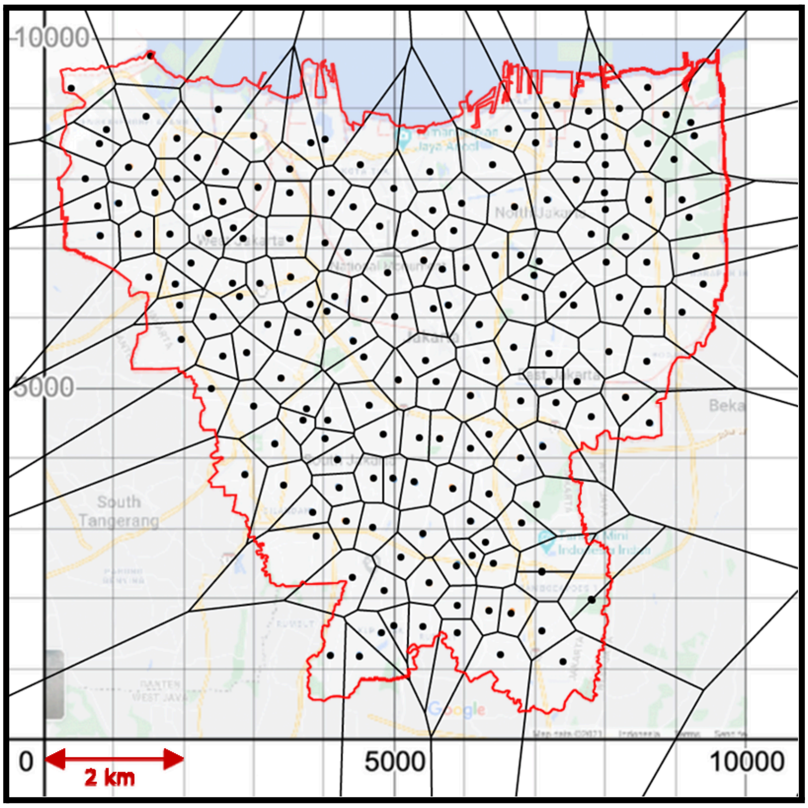


Figure 30: *182 Voronoi cells with for the proposed 1% vaccination rate*

Compared to Fig(28), Fig(30) is much more dense, with each cell covering a smaller area, meaning with the approximately even distribution of hospital area reach. The efficiency of the distribution of vaccines should be optimal for our proposed 1% vaccination rate.

5 Conclusion

Based on our exploration results, it was concluded that a mathematical model could be used to model the COVID-19 situation in Jakarta. The most basic, the SIR, while it did not give us an accurate representation of COVID-19 in Jakarta, will provide us with a rough sketch of what to expect Fig(4). Similarly, our estimated parameters would also not be completely accurate as the parameters were evaluated during the peak first months of the virus, meaning that the rates should have already decreased by now. Though considering the parameters' validity and accuracy, I would safely say it's relatively accurate since it passed the Two-Sample t-test hypothesis with an insignificant PVal Eq(13). Thus the mean of our expected and observed is highly similar. From the general SIR model, we can see that we should've expected at least 90% of our population to recover or succumb to the virus, with an expected reproductive number of 3.5, which was a not so accurate estimate as it failed its *t-test* when tested against the observed data Eq(20). But again, there are still flaws with the SIR model as we can't really tell whether the individuals in the recovered compartment died or really recovered and are immune. I developed the modified SEIR model with a *D* and *V* compartment for death and vaccinations. Similarly, we estimated the parameters in this section Fig(9). The equilibrium analysis through the expanding Taylor series Eq(35) and forming the Jacobian matrix Eq(40) also told us that we should expect a stable model, as the data will eventually reach an asymptote due to the negative eigenvalues Eq(53). This was also verified by the phase portraits Fig(12). Thus, without intervention, we know that ultimately, the disease will die out. The new model's reproduction number was also calculated through the following generation matrix method and approximated to have a reproductive number of 2.09 Eq(70). A lower value than the global average of 2.5. This was also a verified mean R_0 as it passed its t-test when tested against the observed data. A numerical simulation through MATLAB was also done with different vaccination scenarios, and I proposed that it be most ideal with a 1% vaccination rate to achieve maximum effectiveness without crippling Jakarta's economy Fig(23). The Voronoi diagrams also illustrated where the new vaccination sites should be placed to complete the proposed 1% vaccination rate Fig(30).

Though there are certain limitations to this paper, which is mostly in the assumptions. Since our proportionality parameters are assumed to be constant. Even though the parameters could change if a cure is found or, in this case, vaccination. Our SEIRDV model is also not yet 100% accurate as it does not take many parameters into accounts, such as quarantine and home-based work/learning. Extensions to this paper could potentially go to control theory. One of them would be governing quarantine. Given a specific threshold, the quarantine will be mandatory, and when it reaches below the point, the quarantine will be more lenient. This would indeed also flatten the curve while giving more flexibility to the citizens. All in all, this paper does provide government officials with predictions based on the current COVID-19 behavior, and they could apply preventive measures before it worsens. My proposed vaccination rate would also be applicable as it is best for efficiency but will not cripple Jakarta's economy. Given another opportunity I would surely try to incorporate more compartments to increase the accuracy of my model

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7 Appendix

Appendix 1

Data From https://corona.jakarta.go.id/en "Parameter Estimation"

Date	Infected	Recovered	Dead	ln(Infected)	m	γ	α
3/1/2020	3	0	0	0.00000	0.00000	0.00000	0.00000
3/2/2020	1	0	0	0.00000	0.00000	0.00000	1.00000
3/3/2020	3	0	1	1.09861	0.36620	0.00000	-0.33333
3/4/2020	1	0	0	0.00000	0.00000	0.00000	2.00000
3/5/2020	4	0	2	1.38629	0.27726	0.00000	-0.50000
3/6/2020	1	0	0	0.00000	0.00000	0.00000	0.00000
3/7/2020	1	0	0	0.00000	0.00000	0.00000	0.00000
3/8/2020	1	0	0	0.00000	0.00000	0.00000	0.00000
3/9/2020	27	0	0	3.29584	0.36620	0.00000	0.00000
3/10/2020	1	0	0	0.00000	0.00000	0.00000	0.00000
3/11/2020	2	0	0	0.69315	0.06301	0.00000	1.00000
3/12/2020	26	0	2	3.25810	0.27151	0.00000	0.00000
3/13/2020	10	0	2	2.30259	0.17712	0.00000	0.00000
3/14/2020	7	0	2	1.94591	0.13899	0.00000	0.00000
3/15/2020	16	0	2	2.77259	0.18484	0.00000	-0.06250
3/16/2020	2	0	1	0.69315	0.04332	0.00000	-0.50000
3/17/2020	25	0	0	3.21888	0.18935	0.48000	0.12000
3/18/2020	38	12	3	3.63759	0.20209	-0.28947	0.02632
3/19/2020	50	1	4	3.91202	0.20590	-0.02000	-0.06000
3/20/2020	14	0	1	2.63906	0.13195	0.28571	0.28571
3/21/2020	44	4	5	3.78419	0.18020	0.00000	-0.02273
3/22/2020	36	4	4	3.58352	0.16289	-0.08333	-0.05556
3/23/2020	52	1	2	3.95124	0.17179	0.00000	0.01923
3/24/2020	70	1	3	4.24850	0.17702	0.04286	0.08571
3/25/2020	46	4	9	3.82864	0.15315	-0.04348	-0.06522
3/26/2020	43	2	6	3.76120	0.14466	0.00000	0.04651
3/27/2020	51	2	8	3.93183	0.14562	0.19608	-0.05882
3/28/2020	37	12	5	3.61092	0.12896	-0.18919	0.00000
3/29/2020	98	5	5	4.58497	0.15810	-0.04082	0.06122
3/30/2020	26	1	11	3.25810	0.10860	-0.03846	-0.19231
3/31/2020	14	0	6	2.63906	0.08513	0.14286	0.00000

4/1/2020	75	2	6	4.31749	0.13492	0.01333	-0.01333
4/2/2020	93	3	5	4.53260	0.13735	-0.01075	-0.02151
4/3/2020	81	2	3	4.39445	0.12925	0.00000	-0.02469
4/4/2020	81	2	1	4.39445	0.12556	0.04938	0.28395
4/5/2020	80	6	24	4.38203	0.12172	-0.02500	-0.20000
4/6/2020	148	4	8	4.99721	0.13506	-0.02027	0.01351
4/7/2020	144	1	10	4.96981	0.13078	0.03472	-0.04861
4/8/2020	109	6	3	4.69135	0.12029	0.00917	0.07339
4/9/2020	167	7	11	5.11799	0.12795	-0.04192	-0.05988
4/10/2020	91	0	1	4.51086	0.11002	0.65934	0.12088
4/11/2020	93	60	12	4.53260	0.10792	-0.64516	0.16129
4/12/2020	179	0	27	5.18739	0.12064	0.00000	-0.07263
4/13/2020	160	0	14	5.07517	0.11534	0.13125	0.12500
4/14/2020	107	21	34	4.67283	0.10384	-0.18692	-0.28972
4/15/2020	98	1	3	4.58497	0.09967	0.37755	-0.01020
4/16/2020	223	38	2	5.40717	0.11505	-0.16592	0.00000
4/17/2020	153	1	2	5.03044	0.10480	0.01307	0.03268
4/18/2020	79	3	7	4.36945	0.08917	-0.02532	0.35443
4/19/2020	131	1	35	4.87520	0.09750	0.22137	-0.22901
4/20/2020	79	30	5	4.36945	0.08568	0.24051	0.03797
4/21/2020	167	49	8	5.11799	0.09842	-0.26347	-0.02994
4/22/2020	120	5	3	4.78749	0.09033	-0.03333	0.04167
4/23/2020	107	1	8	4.67283	0.08653	0.31776	0.06542
4/24/2020	99	35	15	4.59512	0.08355	-0.28283	0.04040
4/25/2020	76	7	19	4.33073	0.07733	-0.03947	-0.15789
4/26/2020	65	4	7	4.17439	0.07323	-0.06154	0.16923
4/27/2020	86	0	18	4.45435	0.07680	0.03488	-0.16279
4/28/2020	118	3	4	4.77068	0.08086	0.57627	-0.01695
4/29/2020	83	71	2	4.41884	0.07365	-0.85542	-0.02410
4/30/2020	105	0	0	4.65396	0.07629	0.14286	0.11429
5/1/2020	145	15	12	4.97673	0.08027	0.82759	-0.03448
5/2/2020	72	135	7	4.27667	0.06788	-1.04167	0.04167
5/3/2020	62	60	10	4.12713	0.06449	-0.51613	-0.12903
5/4/2020	55	28	2	4.00733	0.06165	0.60000	0.00000
5/5/2020	169	61	2	5.12990	0.07773	-0.34911	0.02367
5/6/2020	68	2	6	4.21951	0.06298	0.04412	0.05882
5/7/2020	66	5	10	4.18965	0.06161	0.60606	-0.13636

5/8/2020	126	45	1	4.83628	0.07009	-0.32540	0.03968
5/9/2020	57	4	6	4.04305	0.05776	0.56140	0.01754
5/10/2020	182	36	7	5.20401	0.07330	-0.01648	0.01099
5/11/2020	55	33	9	4.00733	0.05566	7.14545	-0.09091
5/12/2020	108	426	4	4.68213	0.06414	-3.80556	0.00000
5/13/2020	134	15	4	4.89784	0.06619	-0.09701	0.00746
5/14/2020	180	2	5	5.19296	0.06924	0.02778	0.01667
5/15/2020	62	7	8	4.12713	0.05430	-0.01613	-0.11290
5/16/2020	116	6	1	4.75359	0.06173	-0.02586	0.01724
5/17/2020	127	3	3	4.84419	0.06210	0.02362	0.01575
5/18/2020	74	6	5	4.30407	0.05448	1.48649	-0.01351
5/19/2020	57	116	4	4.04305	0.05054	-1.89474	0.03509
5/20/2020	97	8	6	4.57471	0.05648	1.06186	-0.01031
5/21/2020	70	111	5	4.24850	0.05181	-1.27143	-0.02857
5/22/2020	96	22	3	4.56435	0.05499	0.07292	0.00000
5/23/2020	127	29	3	4.84419	0.05767	-0.17323	-0.01575
5/24/2020	118	7	1	4.77068	0.05613	0.39831	0.00000
5/25/2020	67	54	1	4.20469	0.04889	-0.35821	0.01493
5/26/2020	61	30	2	4.11087	0.04725	-0.16393	0.00000
5/27/2020	137	20	2	4.91998	0.05591	0.00730	0.01460
5/28/2020	103	21	4	4.63473	0.05208	0.65049	-0.00971
5/29/2020	124	88	3	4.82028	0.05356	0.87097	-0.00806
5/30/2020	98	196	2	4.58497	0.05038	-0.98980	-0.01020
5/31/2020	121	99	1	4.79579	0.05213	0.37190	0.00000
6/1/2020	111	144	1	4.70953	0.05064	0.13514	0.02703
6/2/2020	76	159	4	4.33073	0.04607	-0.39474	0.00000
6/3/2020	80	129	4	4.38203	0.04613	-0.70000	-0.03750
6/4/2020	61	73	1	4.11087	0.04282	1.16393	0.01639
6/5/2020	84	144	2	4.43082	0.04568	-0.65476	0.01190
6/6/2020	102	89	3	4.62497	0.04719	2.36275	-0.00980
6/7/2020	160	330	2	5.07517	0.05126	-1.84375	-0.00625
6/8/2020	91	35	1	4.51086	0.04511	1.41758	0.08791
6/9/2020	239	164	9	5.47646	0.05422	-0.06695	-0.02092
6/10/2020	147	148	4	4.99043	0.04893	-0.00680	0.00000
6/11/2020	129	147	4	4.85981	0.04718	-0.24031	0.01550
6/12/2020	76	116	6	4.33073	0.04164	-0.73684	-0.03947
6/13/2020	120	60	3	4.78749	0.04560	1.59167	0.03333

6/14/2020	115	251	7	4.74493	0.04476	-1.25217	0.01739
6/15/2020	105	107	9	4.65396	0.04349	0.22857	-0.05714
6/16/2020	124	131	3	4.82028	0.04463	-0.12903	0.01613
6/17/2020	117	115	5	4.76217	0.04369	0.28205	0.00855
6/18/2020	176	148	6	5.17048	0.04700	-0.32955	-0.00568
6/19/2020	140	90	5	4.94164	0.04452	0.35000	-0.00714
6/20/2020	178	139	4	5.18178	0.04627	0.52809	0.04494
6/21/2020	127	233	12	4.84419	0.04287	-1.25197	-0.07087
6/22/2020	127	74	3	4.84419	0.04249	0.20472	-0.01575
6/23/2020	166	100	1	5.11199	0.04445	-0.03614	0.04819
6/24/2020	154	94	9	5.03695	0.04342	0.12338	-0.03896
6/25/2020	195	113	3	5.27300	0.04507	-0.03077	-0.01026
6/26/2020	168	107	1	5.12396	0.04342	-0.23214	-0.00595
6/27/2020	213	68	0	5.36129	0.04505	0.87793	0.01408
6/28/2020	132	255	3	4.88280	0.04069	-0.01515	-0.01515
6/29/2020	95	253	1	4.55388	0.03764	1.48421	0.04211
6/30/2020	196	394	5	5.27811	0.04326	-1.15306	-0.01020
7/1/2020	206	168	3	5.32788	0.04332	0.11165	-0.00485
7/2/2020	195	191	2	5.27300	0.04252	0.24103	0.00000
7/3/2020	147	238	2	4.99043	0.03992	0.20408	0.00000
7/4/2020	215	268	2	5.37064	0.04262	0.08372	0.02791
7/5/2020	256	286	8	5.54518	0.04366	0.32813	-0.03125
7/6/2020	231	370	0	5.44242	0.04252	-0.54545	0.02597
7/7/2020	199	244	6	5.29330	0.04103	-0.46231	-0.01508
7/8/2020	344	152	3	5.84064	0.04493	0.19186	0.02035
7/9/2020	290	218	10	5.66988	0.04328	-0.13793	-0.01034
7/10/2020	239	178	7	5.47646	0.04149	0.15481	-0.00418
7/11/2020	359	215	6	5.88332	0.04424	-0.15320	0.01671
7/12/2020	404	160	12	6.00141	0.04479	0.11881	-0.00990
7/13/2020	278	208	8	5.62762	0.04169	-0.31655	-0.01439
7/14/2020	275	120	4	5.61677	0.04130	0.26545	0.00727
7/15/2020	259	193	6	5.55683	0.04056	-0.22008	-0.01544
7/16/2020	293	136	2	5.68017	0.04116	0.00341	0.02389
7/17/2020	241	137	9	5.48480	0.03946	-0.05809	0.00000
7/18/2020	331	123	9	5.80212	0.04144	0.61631	-0.00302
7/19/2020	313	327	8	5.74620	0.04075	-0.53994	-0.02236
7/20/2020	361	158	1	5.88888	0.04147	0.28809	0.02216

7/21/2020	441	262	9	6.08904	0.04258	0.13832	-0.00227
7/22/2020	376	323	8	5.92959	0.04118	-0.55319	-0.01862
7/23/2020	416	115	1	6.03069	0.04159	0.40385	0.00000
7/24/2020	285	283	1	5.65249	0.03872	-0.53684	0.00000
7/25/2020	393	130	1	5.97381	0.04064	0.11196	0.02290
7/26/2020	378	174	10	5.93489	0.04010	-0.17725	-0.01852
7/27/2020	472	107	3	6.15698	0.04132	0.57203	0.02119
7/28/2020	412	377	13	6.02102	0.04014	-0.33252	0.02913
7/29/2020	585	240	25	6.37161	0.04220	-0.08889	-0.04103
7/30/2020	299	188	1	5.70044	0.03750	0.73244	0.04682
7/31/2020	432	407	15	6.06843	0.03966	0.62963	0.00231
8/1/2020	374	679	16	5.92426	0.03847	-1.44118	-0.04278
8/2/2020	379	140	0	5.93754	0.03831	-0.00528	0.03958
8/3/2020	489	138	15	6.19236	0.03969	0.15951	-0.00409
8/4/2020	466	216	13	6.14419	0.03913	0.34979	0.00429
8/5/2020	357	379	15	5.87774	0.03720	-0.37255	-0.00560
8/6/2020	597	246	13	6.39192	0.04020	-0.08543	0.00168
8/7/2020	658	195	14	6.48920	0.04056	0.47720	-0.00304
8/8/2020	721	509	12	6.58064	0.04087	0.06796	-0.00971
8/9/2020	472	558	5	6.15698	0.03801	-0.80508	-0.00847
8/10/2020	479	178	1	6.17170	0.03786	0.63257	0.02505
8/11/2020	471	481	13	6.15486	0.03753	-0.12527	0.00425
8/12/2020	578	422	15	6.35957	0.03854	0.11592	-0.00346
8/13/2020	621	489	13	6.43133	0.03874	0.32367	-0.01449
8/14/2020	575	690	4	6.35437	0.03805	-0.42435	0.00348
8/15/2020	598	446	6	6.39359	0.03806	0.48161	-0.00334
8/16/2020	518	734	4	6.24998	0.03698	-1.01544	0.02317
8/17/2020	538	208	16	6.28786	0.03699	0.70818	0.00186
8/18/2020	505	589	17	6.22456	0.03640	-0.04950	0.00198
8/19/2020	565	564	18	6.33683	0.03684	0.28673	-0.00531
8/20/2020	595	726	15	6.38856	0.03693	-0.49244	0.00000
8/21/2020	641	433	15	6.46303	0.03714	0.33697	0.00000
8/22/2020	601	649	15	6.39859	0.03656	0.06822	0.00166
8/23/2020	637	690	16	6.45677	0.03669	1.89325	-0.01727
8/24/2020	659	1896	5	6.49072	0.03667	-2.08346	0.01821
8/25/2020	636	523	17	6.45520	0.03627	0.37893	-0.00314
8/26/2020	711	764	15	6.56667	0.03669	1.08861	-0.01688

8/27/2020	820	1538	3	6.70930	0.03727	-0.80122	0.00488
8/28/2020	816	881	7	6.70441	0.03704	-0.34559	0.01348
8/29/2020	888	599	18	6.78897	0.03730	-0.26239	-0.00450
8/30/2020	1114	366	14	7.01571	0.03834	0.03411	0.00180
8/31/2020	1029	404	16	6.93634	0.03770	0.31584	0.00097
9/1/2020	941	729	17	6.84694	0.03701	-0.27099	0.00106
9/2/2020	1053	474	18	6.95940	0.03742	0.19848	-0.00190
9/3/2020	1406	683	16	7.24850	0.03876	0.10882	-0.00640
9/4/2020	895	836	7	6.79682	0.03615	-0.11732	0.01117
9/5/2020	842	731	17	6.73578	0.03564	0.01900	-0.00594
9/6/2020	1245	747	12	7.12689	0.03751	-0.04337	0.01365
9/7/2020	1105	693	29	7.00760	0.03669	0.29593	-0.01538
9/8/2020	1015	1020	12	6.92264	0.03606	-0.22266	0.00493
9/9/2020	1026	794	17	6.93342	0.03592	0.18226	0.00097
9/10/2020	1450	981	18	7.27932	0.03752	-0.06345	-0.00069
9/11/2020	1034	889	17	6.94119	0.03560	0.17311	0.00484
9/12/2020	1440	1068	22	7.27240	0.03710	-0.16458	-0.01111
9/13/2020	1103	831	6	7.00579	0.03556	0.43518	0.02176
9/14/2020	1062	1311	30	6.96791	0.03519	-0.31073	-0.00188
9/15/2020	1027	981	28	6.93440	0.03485	-0.03505	0.00195
9/16/2020	1505	945	30	7.31655	0.03658	0.00731	-0.00997
9/17/2020	1014	956	15	6.92166	0.03444	0.07101	0.00690
9/18/2020	1403	1028	22	7.24637	0.03587	-0.00214	-0.00784
9/19/2020	932	1025	11	6.83733	0.03368	0.99142	0.00429
9/20/2020	1079	1949	15	6.98379	0.03423	-1.41613	0.01483
9/21/2020	1310	421	31	7.17778	0.03501	0.32214	0.00076
9/22/2020	1122	843	32	7.02287	0.03409	0.23351	-0.00535
9/23/2020	1187	1105	26	7.07918	0.03420	0.04971	-0.01011
9/24/2020	1133	1164	14	7.03262	0.03381	0.39365	-0.00088
9/25/2020	1289	1610	13	7.16162	0.03427	-0.47479	-0.00853
9/26/2020	1257	998	2	7.13648	0.03398	0.05171	0.00875
9/27/2020	1186	1063	13	7.07834	0.03355	0.22344	-0.00084
9/28/2020	807	1328	12	6.69332	0.03157	-0.25279	0.00248
9/29/2020	1132	1124	14	7.03174	0.03301	0.29240	-0.00088
9/30/2020	1059	1455	13	6.96508	0.03255	-0.31256	-0.00661
10/1/2020	1153	1124	6	7.05012	0.03279	-0.25065	-0.00260
10/2/2020	1098	835	3	7.00125	0.03241	0.15665	0.00000

10/3/2020	1165	1007	3	7.06048	0.03254	0.02232	0.01288
10/4/2020	1430	1033	18	7.26543	0.03333	-0.03986	-0.00490
10/5/2020	822	976	11	6.71174	0.03065	0.05353	0.02798
10/6/2020	1007	1020	34	6.91473	0.03143	-0.02483	-0.02085
10/7/2020	1340	995	13	7.20042	0.03258	0.03507	0.00448
10/8/2020	1009	1042	19	6.91672	0.03116	-0.18930	0.00297
10/9/2020	972	851	22	6.87936	0.03085	0.44547	-0.00514
10/10/2020	1253	1284	17	7.13330	0.03185	-0.17717	0.00559
10/11/2020	1389	1062	24	7.23634	0.03216	0.01584	-0.00216
10/12/2020	1168	1084	21	7.06305	0.03125	0.01370	0.00086
10/13/2020	1054	1100	22	6.96035	0.03066	0.08634	-0.00474
10/14/2020	1038	1191	17	6.94505	0.03046	-0.13584	0.00578
10/15/2020	1071	1050	23	6.97635	0.03046	-0.06349	0.00093
10/16/2020	1045	982	24	6.95177	0.03023	0.11866	0.00000
10/17/2020	974	1106	24	6.88141	0.02979	-0.03285	-0.00513
10/18/2020	971	1074	19	6.87833	0.02965	0.05252	-0.00618
10/19/2020	926	1125	13	6.83087	0.02932	-0.30130	0.00972
10/20/2020	964	846	22	6.87109	0.02936	0.23340	-0.00311
10/21/2020	1000	1071	19	6.90776	0.02939	0.08900	-0.00400
10/22/2020	989	1160	15	6.89669	0.02922	-0.06876	0.00303
10/23/2020	952	1092	18	6.85857	0.02894	0.06723	-0.00315
10/24/2020	1062	1156	15	6.96791	0.02928	0.06874	-0.00377
10/25/2020	771	1229	11	6.64769	0.02781	-0.08690	0.01297
10/26/2020	906	1162	21	6.80904	0.02837	-0.08720	-0.01214
10/27/2020	781	1083	10	6.66058	0.02764	0.01793	0.00768
10/28/2020	844	1097	16	6.73815	0.02784	-0.02251	-0.00237
10/29/2020	713	1078	14	6.56948	0.02703	-0.00140	0.00000
10/30/2020	612	1077	14	6.41673	0.02630	1.70752	0.00327
10/31/2020	750	2122	16	6.62007	0.02702	-2.31600	0.00267
11/1/2020	608	385	18	6.41017	0.02606	1.10526	0.00000
11/2/2020	1024	1057	18	6.93147	0.02806	-0.03027	-0.00879
11/3/2020	617	1026	9	6.42487	0.02591	-0.15397	0.00972
11/4/2020	774	931	15	6.65157	0.02671	0.05426	0.00129
11/5/2020	791	973	16	6.67330	0.02669	0.06448	0.00126
11/6/2020	672	1024	17	6.51026	0.02594	-0.05655	-0.00893
11/7/2020	1118	986	11	7.01930	0.02785	-0.00984	-0.00358

11/8/2020	826	975	7	6.71659	0.02655	0.09443	0.00484
11/9/2020	716	1053	11	6.57368	0.02588	0.34497	0.00419
11/10/2020	1013	1300	14	6.92067	0.02714	-0.32280	-0.00197
11/11/2020	587	973	12	6.37502	0.02490	0.16865	-0.00170
11/12/2020	831	1072	11	6.72263	0.02616	-0.13718	0.00361
11/13/2020	1033	958	14	6.94022	0.02690	0.10068	-0.00097
11/14/2020	1255	1062	13	7.13489	0.02755	-0.07171	-0.00478
11/15/2020	1165	972	7	7.06048	0.02716	0.05837	0.00000
11/16/2020	1006	1040	7	6.91374	0.02649	-0.16402	-0.00298
11/17/2020	1038	875	4	6.94505	0.02651	-0.02216	0.00674
11/18/2020	1147	852	11	7.04491	0.02679	0.02877	0.00262
11/19/2020	1185	885	14	7.07750	0.02681	0.01772	0.00253
11/20/2020	1240	906	17	7.12287	0.02688	0.17581	-0.00242
11/21/2020	1579	1124	14	7.36455	0.02769	-0.03040	0.00127
11/22/2020	1342	1076	16	7.20192	0.02697	-0.00894	0.00075
11/23/2020	1009	1064	17	6.91672	0.02581	-0.00496	0.00198
11/24/2020	1015	1059	19	6.92264	0.02573	-0.02167	-0.00197
11/25/2020	1273	1037	17	7.14913	0.02648	0.11862	-0.00314
11/26/2020	1064	1188	13	6.96979	0.02572	-0.36936	0.00376
11/27/2020	1436	795	17	7.26962	0.02673	0.31407	0.00070
Average m		0.05648026897					
Average γ		0.02265323939					
Average α		0.01013					

Appendix 2

Derivation of the Jacobian matrix in Sect(3.3)

From Eq(38):

$$\dot{x} = \begin{bmatrix} \mu - x_1(\beta x_3 + \eta + v) \\ \beta x_1 x_3 - x_2(\eta + \delta) \\ \delta x_2 - x_3(\eta + \gamma + \alpha) \\ \gamma x_3 - \eta x_4 - v x_1 \end{bmatrix} = \begin{bmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \\ f_4(x) \end{bmatrix}$$

With the Jacobian matrix in the form of ***J***:

$$\frac{\partial f}{\partial x}\Big|_{x_0} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1}\Big|_{x_0} & \frac{\partial f_1}{\partial x_2}\Big|_{x_0} & \frac{\partial f_1}{\partial x_3}\Big|_{x_0} & \frac{\partial f_1}{\partial x_4}\Big|_{x_0} \\ \frac{\partial f_2}{\partial x_1}\Big|_{x_0} & \frac{\partial f_2}{\partial x_2}\Big|_{x_0} & \frac{\partial f_2}{\partial x_3}\Big|_{x_0} & \frac{\partial f_2}{\partial x_4}\Big|_{x_0} \\ \frac{\partial f_3}{\partial x_1}\Big|_{x_0} & \frac{\partial f_3}{\partial x_2}\Big|_{x_0} & \frac{\partial f_3}{\partial x_3}\Big|_{x_0} & \frac{\partial f_3}{\partial x_4}\Big|_{x_0} \\ \frac{\partial f_4}{\partial x_1}\Big|_{x_0} & \frac{\partial f_4}{\partial x_2}\Big|_{x_0} & \frac{\partial f_4}{\partial x_3}\Big|_{x_0} & \frac{\partial f_4}{\partial x_4}\Big|_{x_0} \end{bmatrix}$$

The derivation of entries in matrix ***J*** are as follows (matrix location denoted by $J_{i,j}$):

$J_{1,1}$: Exponent rule

$$\begin{aligned} \frac{\partial f_1}{\partial x_1}|_{x_0} &= \mu - x_1(\beta x_3 + \eta + v) \\ \text{Exponent Rule } &= -(\beta x_3 + \eta + v) \\ \text{Evaluated at } x_0 &= -(\beta(0) + \eta + v) \\ &= -(\eta + v) \end{aligned}$$

$J_{1,2}$: Not in equation

$$\frac{\partial f_1}{\partial x_2}|_{x_0} = \mu - x_1(\beta x_3 + \eta + v) = 0$$

$J_{1,3}$: Exponent rule

$$\begin{aligned} \frac{\partial f_1}{\partial x_3}|_{x_0} &= \mu - x_1(\beta x_3 + \eta + v) \\ \text{Exponent Rule } &= -\beta x_1 \\ \text{Evaluated at } x_0 &= -\beta\left(\frac{\mu}{\eta + v}\right) \\ &= -\frac{\beta\mu}{\eta + v} \end{aligned}$$

$J_{1,4}$: Not in equation

$$\frac{\partial f_1}{\partial x_4}|_{x_0} = \mu - x_1(\beta x_3 + \eta + v) = 0$$

***J*$_{2,1}$**: Exponent rule

$$\begin{aligned}\frac{\partial f_2}{\partial x_1}|_{x_0} &= \beta x_1 x_3 - x_2(\eta + \delta)\\ \text{Exponent Rule} &= \beta x_3\\ \text{Evaluated at } x_0 &= \beta(0)\\ &= 0\end{aligned}$$

***J*$_{2,2}$**: Exponent rule

$$\begin{aligned}\frac{\partial f_2}{\partial x_2}|_{x_0} &= \beta x_1 x_3 - x_2(\eta + \delta)\\ \text{Exponent Rule} &= -(\eta + \delta)\end{aligned}$$

***J*$_{2,3}$**: Exponent rule

$$\begin{aligned}\frac{\partial f_2}{\partial x_3}|_{x_0} &= \beta x_1 x_3 - x_2(\eta + \delta)\\ \text{Exponent Rule} &= \beta x_1\\ \text{Evaluated at } x_0 &= \beta\left(\frac{\mu}{\eta + \upsilon}\right)\\ &= \frac{\beta\mu}{\eta + \upsilon}\end{aligned}$$

***J*$_{2,4}$**: Not in equation

$$\frac{\partial f_2}{\partial x_4}|_{x_0} = \beta x_1 x_3 - x_2(\eta + \delta) = 0$$

***J*$_{3,1}$**: Not in equation

$$\frac{\partial f_3}{\partial x_1}|_{x_0} = \delta x_2 - x_3(\eta + \gamma + \alpha) = 0$$

***J*$_{3,2}$**: Exponent rule

$$\begin{aligned}\frac{\partial f_3}{\partial x_2}|_{x_0} &= \delta x_2 - x_3(\eta + \gamma + \alpha)\\ \text{Exponent Rule} &= \delta\end{aligned}$$

***J*$_{3,3}$**: Exponent rule

$$\begin{aligned}\frac{\partial f_3}{\partial x_3}|_{x_0} &= \delta x_2 - x_3(\eta + \gamma + \alpha) = 0\\ \text{Exponent Rule} &= -(\eta + \gamma + \alpha)\end{aligned}$$

***J*$_{3,4}$**: Not in equation

$$\frac{\partial f_3}{\partial x_4}|_{x_0} = \delta x_2 - x_3(\eta + \gamma + \alpha) = 0$$

***J*$_{4,1}$**: Exponent rule

$$\begin{aligned}\frac{\partial f_4}{\partial x_1}|_{x_0} &= \gamma x_3 - \eta x_4 - \upsilon x_1\\ \text{Exponent Rule} &= -\upsilon\end{aligned}$$

***J*$_{4,2}$**: Not in equation

$$\frac{\partial f_4}{\partial x_2}|_{x_0} = \gamma x_3 - \eta x_4 - \upsilon x_1 = 0$$

$\boldsymbol{J}_{4,3}$: Exponent rule

$$\frac{\partial f_4}{\partial x_3}|_{x_0} = \gamma x_3 - \eta x_4 - \upsilon x_1$$
$$\text{Exponent Rule } = \gamma$$

$\boldsymbol{J}_{4,4}$: Exponent rule

$$\frac{\partial f_4}{\partial x_1}|_{x_0} = \gamma x_3 - \eta x_4 - \upsilon x_1$$
$$\text{Exponent Rule } = -\eta$$

Appendix 3

Location coordinates for voronoi diagram in Sect(4.2), from google maps and [28]

No.	Institute Name	X	Y
1	Klinik BPS Bidan Ika Retna	400	9300
2	Rumah Sakit Hermina	900	8700
3	Rumah Sakit Ciputra Hospital	800	8500
4	Puskesmas Kelurahan Kalideres	600	8000
5	Hermina Daan Mogot Hospital	1050	7650
6	Semanan Urban Village Puskesmas	800	7200
7	Pantai Indah Kapuk Hospital	2500	9000
8	Puskesmas Kapuk I	2200	8300
9	RSUD Cengkareng	1900	8000
10	Puskesmas Cengkareng Barat	1600	7800
11	Klinik Mulia Metod	2600	8100
12	Klinik Medikana	2200	7600
13	Puskesmas Rawa Buaya	1800	7200
14	Puskesmas Duri Kosambi I	1500	6600
15	Klinik Taman Kota	2700	7300
16	Puskesmas Kecamatan Kembangan	2100	6800
17	Pondok Indah Puri Indah Hospital	1900	6500
18	Kembangan Regional General Hospital	1950	6200
19	Puskesmas Kelurahan Meruya Selatan	1950	5700
20	Siloam Hospital Kebon Jeruk	2800	6300
21	Rumah sakit Cendana	2700	6500
22	Rumah Sakit Geraha Kedoya	2850	7150
23	Puskesmas Kelurahan Jelambar Baru	3500	7800
24	Duta Indah Hospital	3500	8150
25	Rumah Sakit Atamajaya Bonaventura	3800	8520
26	Rumah Sakit Pluit	4000	8550
27	Klinik Umum Yakrija	2400	5000
28	Klinik Pratama	2550	5460
29	Klinik Nadira	2900	5500
30	Royal Taruma Hospital	3500	7150
31	Puskesmas Kecamatan Tambora	4100	7800
32	Kartini Hospital	3000	4750
33	Puskesmas Kecamatan Palmerah	3630	5800
34	Patria Hospital IKKT	3800	6200
35	Tarakan Regional General Hospital	4350	6960
36	Puskesmas Kelurahan Krukut	4400	7600
37	Dr. Suyoto Hospital	2880	3790

38	Klinik Tanah Kusir	3300	4230
39	Gandaria Hospital	3700	4550
40	Pertamina Central Hospital	3745	4710
41	Dr. Mintohardjo Naval Hospital	4400	5700
42	Bhakti Mulia Hospital	4050	6120
43	Pelni Hospital	4150	6300
44	Rumah Sakit Husada	5000	7850
45	Hermina Hospital Kemayoran	5550	7550
46	Puskesmas Kecamatan Pademangan	5500	8100
47	Angsamerah Klinik Fatmawati	4000	4300
48	Rumah Sakit Ibu dan Anak ASIH	4050	4550
49	MRCCC Siloam Hospital Semanggi	4550	5450
50	Puskesmas Kelurahan Utan Panjang	5850	7250
51	RS Mitra Keluarga Kemayoran	5950	7650
52	Puskesmas Sunter Agung	6400	8200
53	RSUP Fatmawati	3900	2900
54	Puskesmas Kecamatan Cilandak	3650	3250
55	Puskesmas South Cipete	4200	3600
56	Puskesmas Kelurahan Pela Mampang	4850	4350
57	Puskesmas Kuningan Barat	4650	4750
58	Puskesmas Kelurahan Menteng Atas	5450	5400
59	RSCM Kencana	5550	6150
60	Rumah Sakit St. Carolus	5750	6200
61	RSUD Johar Baru	5700	6700
62	Pertamina Jaya Hospital	6450	6900
63	Puri Medika Hospital	7000	8500
64	Hospital Tanjung Priok	7000	8900
65	Koja District Hospital	7300	9050
66	Regional General Hospital Jagakarsa	4400	2300
67	RSUD Pasar Minggu	4700	3000
68	Jakarta Medical Center Hospital	5000	3700
69	Siaga Raya Hospital	5300	3650
70	Tria Dipa Hospital	5350	4300
71	Klinik Sehati	5600	5100
72	Puskesmas Kelurahan Rawa Bungga	6300	5400
73	Regional Hospital Matraman	6200	5900
74	Columbia Asia Hospital Pulomas	7000	6600
75	Mediros Hospital	7050	6830
76	Rumah Sakit Kartika Pulomas	6800	6900
77	Mitra Keluarga Kelapa Gading	7200	7700
78	Sukapura Jakarta Islamic Hospital	8000	8200

79	Rumah Sakit Umum Daerah Tugu Koja	7600	8500
80	Rumah Sakit Pelabuhan Jakarta	8000	8600
81	RSUD Cilincing	8200	9000
82	Puskesmas Cilincing	8600	9300
83	Puskesmas Kecamatan Jagakarsa	4100	1200
84	Hospital Ali Sibroh Malisi	4500	1200
85	Rumah Sakit Umum Zahirah	4800	1500
86	RSU Aulia	5000	1600
87	Klinik Bhakti Asih	5100	2600
88	Puskesmas Pasar Minggu	5400	2900
89	RSUD Budhi Asih	6090	4140
90	RumaH Sakit UKI	6350	4350
91	RSUP Persahabatan	6700	5600
92	Puskesmas Kelurahan Klender	7200	5500
93	Rumah Sakit Harapan Jayakarta	7550	6200
94	Antam Medika Hospital	7400	6350
95	Gading Pluit Hospital	7800	7200
96	Rumah Sakit Firdaus	8000	7550
97	Puskesmas Sukapura	8000	7800
98	PHC Rawa Malang	8600	8500
99	Poliklinik Marunda	9200	8800
100	Puskesmas Ujung Menteng	9400	6500
101	Puskesmas Cakung Timur	9300	6900
102	Puskesmas Pondok Rangon	7400	1100
103	Setia Mitra Hospital	3820	3250
104	Radjak Hospital Salemba	5850	6250
105	Rumah Sakit Tamansari	4600	8000
106	RSU Al-Fauzan Islamic Hospital	6300	2800
107	Puskesmas Batu Ampar	6100	3100
108	Rumah Sakit Polri Sukanto	6340	3500
109	Dr. Esnawan Antariksa Air Force Hospital	7100	4200
110	Yadika Pondok Bambu Hospital	7300	4800
111	Puskesmas Pondok Bambu	7800	4600
112	Puskesmas Cipinang Muara	7200	5100
113	Rumah Sakit Duren Sawit	7600	5100
114	Klinik Wahana Penggilingan	8200	6300
115	Klinik Pelita Insani	8600	6800
116	Puskesmas Rorotan	9200	7450
117	Klinik Dania Baru	9100	7700
118	Puskesmas Jefry Marunda	9250	8600
119	RSUD Ciracas	6500	1200

120	Puskesmas Kelurahan Cipayung	7100	1550
121	Puskesmas Kelurahan Ceger	7100	2400
122	Puskesmas Pondok Kelapa	8650	4500
123	Pharmacies Jakarta Islamic Hospital	8600	5400
124	Klinik Millenium Sehat	9100	6100
125	Puskesmas Kelurahan Lenteng	5400	1600
126	Puskesmas Pasar Rebo	5900	1500
127	Puskesmas Kelurahan Cijantung	5900	1900
128	Hospital Cijantung Kesdam Jaya	5900	2500
129	Regional General Hospital Pasar Rebo	6100	2600
130	Binawaluya Heart Hospital	6400	2500