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PROPERTIES OF DFT

Aim

Write a MATLAB program to verify the following properties of the DFT:

1. Periodicity property
2. Linearity property
3. Time Reversal property
4. Time Shifting property
5. Frequency Shifting property
6. Circular Convolution property
7. Multiplication (Modulation) property
8. Parseval's Theorem

Theory

1. **Periodicity Property:**
The DFT is periodic with period N, where N is the length of the sequence. This means that the DFT coefficients repeat every N samples:
$$X[k+N] = X[k]$$
This property is crucial for understanding the behaviour of signals in the frequency domain.

2. **Linearity Property:**
The DFT is a linear operator. If x[n] and y[n] are two signals, and a and b are constants, then:
$$DFT\{ax[n] + by[n]\} = aX[k] + bY[k]$$
This property allows for the combination of signals before applying the DFT.

3. **Time Reversal Property:**
If a sequence x[n] has a DFT X[k], then the time-reversed sequence x[−n] has a DFT given by:
$$DFT\{x[-n]\} = X[N-k]$$
This property highlights the symmetry in the frequency domain

4. **Time Shifting property:**
Shifting a signal in the time domain results in a phase shift in the frequency domain:
$$DFT\{x[n-n_0]\} = X[k]e^{-jN2\pi kn_0}$$
This property is useful for analysing the effects of time delays on signals.

5. **Frequency Shifting property:**
   Multiplying a signal by a complex exponential causes a shift in the frequency domain:
   $$DFT\{e^{j\frac{2\pi}{N}mn}x[n]\} = X[k-m]$$
   This is important for modulation techniques in communications.

6. **Circular Convolution property:**
   The DFT of the circular convolution of two sequences is the pointwise product of their DFTs:
   $$DFT\{x[n] \circledast y[n]\} = X[k]Y[k]$$
   This property simplifies the computation of convolutions using the DFT.

7. **Multiplication {Modulation) property:**
   The DFT of the product of two time-domain signals corresponds to the convolution of their DFTs in the frequency domain:
   $$DFT\{x[n]y[n]\} = \frac{1}{N}X[k] * Y[k]$$
   This property is useful in signal processing applications where modulation is involved.

8. **Parseval's Theorem:**
   This theorem states that the total energy of a signal in the time domain is equal to the total energy in the frequency domain:
   $$\sum_{n=0}^{N-1} |x[n]|^2 = \frac{1}{N}\sum_{k=0}^{N-1} |X[k]|^2$$
   It establishes the equivalence of energy measures between the two domains, which is fundamental in analysing signals.

Output

-----1. Periodicity Property-----

Enter the sequence : [1 2 3 4]

DFT of input in 1st period (1 to N): 10.0000+0.0000i -2.0000-2.0000i -2.0000+0.0000i -2.0000+2.0000i

DFT of input in 2nd period (N+1 to 2N): 10.0000-0.0000i -2.0000-2.0000i -2.0000+0.0000i -2.0000+2.0000i

Periodicity Property of DFT is verified

Program

1. Periodicity property

```
clc;
close all;
clear all;
x=input("enter the sequence:");
N=length(x);
x=[x x];
for k=1:2*N
   y(k)=0;
   for n=1:N
      y(k)=y(k)+exp(-i*2*pi*(k-1)*(n-1)/N)*x(n);
   end
end
disp((y(1:N))');
disp("DFT of input in 1st period 1 to N:");
disp((y(1:N))');
disp("DFT of input in 2nd period N+1 to 2N:");
disp((y(N+1:2*N))');
if abs(y(1:N)-y(N+1:2*N))<10^(-10)
   disp("periodicity verified");
else
   disp("periodicity not verified");
end
```

Output

---- 2. Linearity Property-----

Enter the 1st sequence : [1 2 3 4]

Enter the 2nd sequence : [1 2 1 2]

Enter 1st scalar value a : 2

Enter 2nd scalar value b : 5

Y1 = 50.0000+0.0000i -4.0000-4.0000i -14.0000+0.0000i -4.0000+4.0000i

Y2 = 50.0000+0.0000i -4.0000-4.0000i -14.0000+0.0000i -4.0000+4.0000i

Linearity property of DFT is verified

Program

2. Linearity property

```
clc;
clear all;
close all;
disp("linearity propert");
clear all;
x1=input("enter the 1st sequence");
x2=input("enter the 2nd sequence");
a=input("enter the 1st scalar");
b=input("enter the 2nd scalar");
y1=dft(x1);
y2=dft(x2);
Y1=(dft(a*x1+b*x2));
Y2=(a*y1+b*y2);
disp(Y1);
disp(Y2);
if abs(Y1-Y2)<10^(-10)
   disp("linearity property is verified");
else
   disp("linearity property is not verified");
end
```

Output

------3. Time Reversal Property----

Enter the sequence : [1 2 3 4]

DFT of x(n) : 10.0000+0.0000i -2.0000-2.0000i -2.0000+0.0000i -2.0000+2.0000i

DFT of x(N-n) : 10.0000+0.0000i -2.0000+2.0000i -2.0000+0.0000i -2.0000-2.0000i

DFT of x(N-n) by property : 10.0000+0.0000i -2.0000+2.0000i -2.0000+0.0000i -2.0000-2.0000i

Time reversal property of DFT is verified

Program

3. Time Reversal property

```
clc;
clear all;
close all;
disp('------3. Time Reversal Property----')
x1=input('Enter the sequence : ');
N=length(x1);
y1=dft(x1);
n=1:N-1;
x2(1)=x1(1);
x2(n+1)=x1(N-(n-1));
y2=dft(x2);
y(1)=y1(1);
y(n+1)=y1(N-(n-1));
disp('DFT of x(n) : ');
disp(y1');
disp('DFT of x(N-n) : ');
disp(y2');
disp('DFT of x(N-n) by property : ');
disp(y');
if abs(y-y2)<10^(-10)
   disp('Time reversal property of DFT is verified');
else
   disp('Time reversal property of DFT is not verified');
end
```

Output

------4. Time Shifting Property----

Enter the sequence : [1 2 3 4]

Enter the shift m : 1 DFT of x(n) : 10.0000+0.0000i -2.0000-2.0000i -2.0000+0.0000i -2.0000+2.0000i

DFT of x(n-m) by direct method : 10.0000+0.0000i 2.0000-2.0000i 2.0000+0.0000i 2.0000+2.0000i

DFT of x(n-m) by property : 10.0000+0.0000i 2.0000-2.0000i 2.0000-0.0000i 2.0000+2.0000i

Time shifting property of DFT is verified

Program

4. Time Shifting property

```
clc;
clear all;
close all;
disp('------4. Time shifting Property----')
x1=input('Enter the sequence : ');
m=input("enter the shift m:");
N=length(x1);
y1=dft(x1);
x2=circshift(x1',m);
y2=dft(x2);
for k=1:N
   y(k)=y1(k)*exp(-i*2*pi*(k-1)*m/N);
end
disp("dft of x(n):");
disp(y1');
disp("dft of x(n-m) by direct method:");
disp(y2');
disp("dft of x(n-m) by property:");
disp(y');
if abs(y-y2)<10^(-10)
   disp('Time shifting property of DFT is verified');
else
   disp('Time shifting property of DFT is not verified');
end
```

Output

-----5. Frequency Shifting property-----

Enter the sequence : [1 2 3 4]

Enter the shift m : 2 DFT of x(n) : 10.0000+0.0000i -2.0000-2.0000i -2.0000+0.0000i -2.0000+2.0000i

DFT of e^(j2pi*kn/N)*x(n-m) by direct method : -2.0000-0.0000i -2.0000+2.0000i 10.0000+0.0000i - 2.0000-2.0000i

DFT of e^(j2pi*kn/N)*x(n-m) by property : -2.0000+0.0000i -2.0000+2.0000i 10.0000+0.0000i - 2.0000-2.0000i

Frequency shifting property of DFT is verified

Program

5. Frequency Shifting property

```
clc;
clear all;
close all;
disp('------5. Frequency shifting Property----');
x1=input('Enter the sequence : ');
m=input("enter the shift m:");
N=length(x1);
y1=dft(x1);
for n=1:N
   x2(n)=exp(i*2*pi*m*(n-1)/N)*x1(n);
end
y2=dft(x2);
y=circshift(y1',m);
disp("dft of x(n):");
disp(y1');
disp("dft of e^(i2pim*kn/N)*x(n-m) by direct method:");
disp(y2');
disp("dft of e^(i2pim*kn/N)*x(n-m) by property:");
disp(y);
if abs(y'-y2)<10^(-10)
   disp('frequency shifting property of DFT is verified');
else
   disp('frequency shifting property of DFT is not verified');
end
```

Output

------6. Circular Convolution Property-----

Enter the 1st sequence : [1 2 3 4]

Enter the 2nd sequence : [1 2 1 2]

DFT of x1 : 10.0000+0.0000i -2.0000-2.0000i -2.0000+0.0000i -2.0000+2.0000i

DFT of x2 : 6.0000+0.0000i -0.0000+0.0000i -2.0000+0.0000i 0.0000+0.0000i

DFT of convolution of x1 and x2 : 60.0000+0.0000i -0.0000+0.0000i 4.0000+0.0000i 0.0000+0.0000i

DFT of convolution of x1 and x2 by property : 60.0000+0.0000i 0.0000+0.0000i 4.0000-0.0000i -0.0000+0.0000i

Circular Convolution property of DFT is verified

Program

6. Circular Convolution property

```
clc;
clear all;
close all;
disp('------6. Circular Convolution Property-----');
x1=input('Enter the 1st sequence : ');
x2=input('Enter the 2nd sequence : ');
N=length(x1);
M=length(x2);
x1=[x1 zeros(1,M-N)];
x2=[x2 zeros(1,N-M)];
x=cconv(x1,x2,max(N,M));
y1=dft(x1);
y2=dft(x2);
y=y1.*y2;
Y=dft(x);
disp('DFT of x1 : ');
disp(y1');
disp('DFT of x2 : ');
disp(y2');
disp('DFT of convolution of x1 and x2 : ');
disp(Y');
disp('DFT of convolution of x1 and x2 by property : ');
disp(y');
if abs(y-Y)<10^(-10)
   disp('Circular Convolution property of DFT is verified');
else
   disp('Circular Convolution property of DFT is not verified');
end
```

Output

------7. Multiplication (Modulation) property-----

Enter the 1st sequence : [1 2 3 4]

Enter the 2nd sequence : [1 2 1 2]

DFT of x1 : 10.0000+0.0000i -2.0000-2.0000i -2.0000+0.0000i -2.0000+2.0000i

DFT of x2 : 6.0000+0.0000i -0.0000+0.0000i -2.0000+0.0000i 0.0000+0.0000i

DFT of multiplication of x1 and x2 : 16.0000+0.0000i -2.0000-4.0000i -8.0000+0.0000i -
2.0000+4.0000i

DFT of multiplication of x1 and x2 by property : 16.0000-0.0000i -2.0000-4.0000i -8.0000+0.0000i -
2.0000+4.0000i

Multiplication property of DFT is verified

Program

7. Multiplication (Modulation) property

```
clc;
clear all;
close all;
disp('----7. Multiplication (Modulation) property-----');
x1=input('Enter the 1st sequence : ');
x2=input('Enter the 2nd sequence : ');
N=length(x1);
M=length(x2);
x1=[x1 zeros(1,M-N)];
x2=[x2 zeros(1,N-M)];
x=x1.*x2;
y1=dft(x1);
y2=dft(x2);
y=cconv(y1,y2,max(N,M))/N;
Y=dft(x);
disp('DFT of x1 : ');
disp(y1');
disp('DFT of x2 : ');
disp(y2');
disp('DFT of multiplication of x1 and x2 : ');
disp(Y');
disp('DFT of multiplication of x1 and x2 by property : ');
disp(y');
if abs(y-Y)<10^(-10)
   disp('Multiplication property of DFT is verified');
else
   disp('Multiplication property of DFT is not verified');
end
```

Output

------8. Parsevals Theorem----

Enter the sequence : [1 2 3 4]

DFT of x : 10.0000+0.0000i -2.0000-2.0000i -2.0000+0.0000i -2.0000+2.0000i

Sum of |x|^2 : 30

1/N * Sum of |DFT(x)|^2 : 30

Parsevals Theorem of DFT is verifie

Program

8. Parseval's Theorem

```
clc;
clear all;
close all;
disp("---8. Parseval's theorem -----");
x=input('Enter the sequence : ');
N=length(x);
y=dft(x);
X=sum((abs(x)).^2);
Y=sum((abs(y)).^2)/N;
disp('DFT of x:');
disp(y');
disp("sum of |x|^2:");
disp(X);
disp("1/N * sum of |DFT(x)|^2:");
disp(Y);
if X==Y
   disp("Parseval's theorem verified");
else
    disp("Parseval's theorem not verified");
end
```

Result

Verified the properties of DFT using MATLAB.